Quantum Sensors and the Fundamental Limits of Electromagnetic Axion and Hidden-Photon Searches

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3rd Workshop on Workshops on Microwave Cavities and Detectors for Axion Research

Livermore, CA
August 23, 2018

1. Fundamental limits
    - What is the fundamental limit of sensitivity of a search using a single electromagnetic mode, passive impedance matching, and a phase-insensitive amplifier at the quantum limit?
    - What is the optimal scan strategy, given a set of priors?
    - Is a single-pole resonant circuit optimal, or can we do better?
    - *The above strongly motivate quantum sensors*

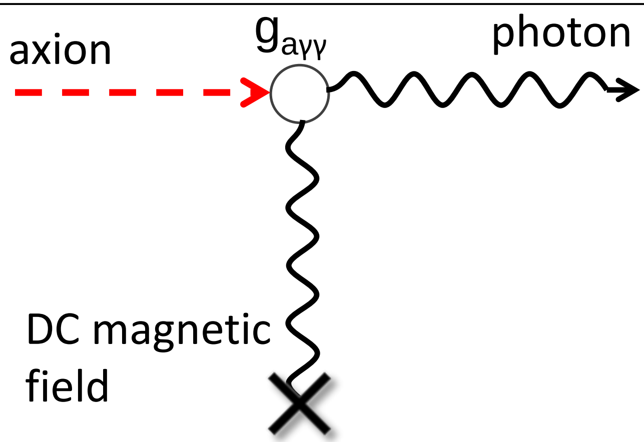
2. Quantum sensors < 300 MHz:
    - backaction evasion and the Zappe Photon Upconverter (ZPU)

Stanford: Arran Phipps, Dale Li, Saptarshi Chaudhuri, Peter Graham, Jeremy Mardon, Hsiao-Mei Cho, Connor FitzGerald, Stephen Kuenstner, Carl Dawson, Betty Young, Cyndia Yu, Kent Irwin

Arxiv:1803.01627, "Quantum Limits for Electromagnetic Axion and Hidden-Photon Dark Matter Searches"
S. Chaudhuri, K. Irwin, P. Graham, J. Mardon

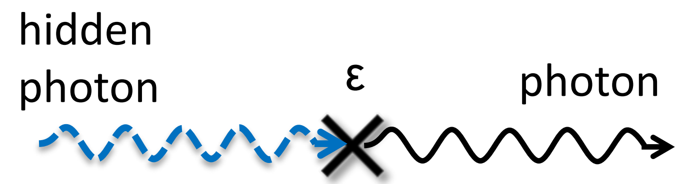
HEISING-SIMONS FOUNDATION   KIPAC   SLAC

Axions (spin 0)



- Solves strong CP problem
- Converts to photon via inverse Primakoff effect- requires background EM field
- Photon frequency gives mass, $h\nu=mc^2$
- ~10^{-6} bandwidth set by DM virial velocity

Hidden Photons (spin 1)



- Appears in generic extensions of Standard Model, may be produced by cosmic inflation
- Converts via kinetic mixing
- Photon frequency gives mass, $h\nu=mc^2$
- ~10^{-6} bandwidth set by DM virial velocity

Ultralight, high number density → Look for classical, oscillating EM field

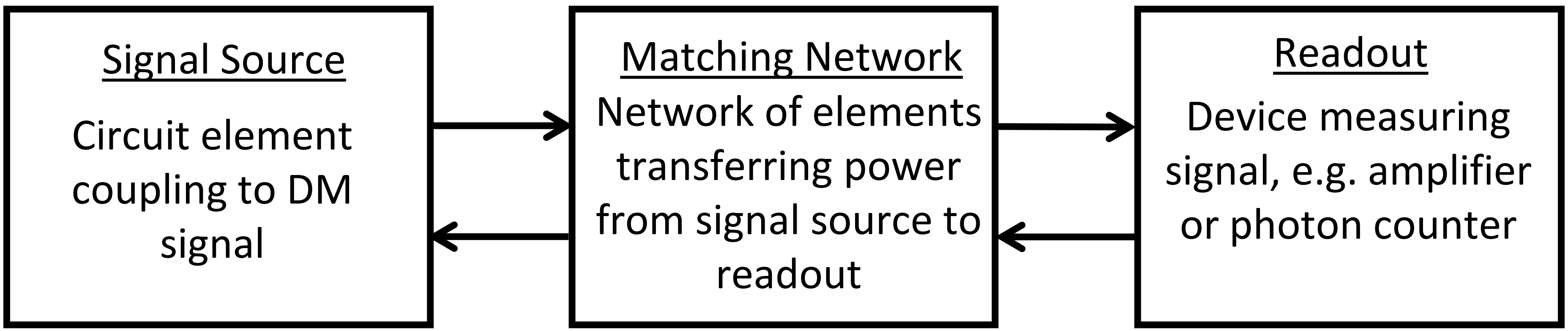
Outline

- Receiver circuit model
- Standard quantum limit
- Optimizing the matching network
  - Integrated sensitivity
  - Bode-Fano Limit
  - Single-pole resonators are 75% of Bode-Fano Limit
- How do we improve our science limit?
  - Use multiple modes
  - Get colder
  - Use active feedback matching circuits
  - *Measure below SQL with quantum sensors*
- Quantum sensors below 300 MHz

Receiver circuit model: schematic

What is the fundamental limit of sensitivity of a search using

- a single electromagnetic mode
- passive impedance matching, and
- a phase-insensitive amplifier at the quantum limit?



To arrive at fundamental limits, optimize each block and interactions across blocks.

Model for axion / hidden photon detection through electromagnetism

Signal Source

1) Inductive coupling to DM signal
2) Residual loss and associated thermal noise

→ / ←

Matching Network

Examples:

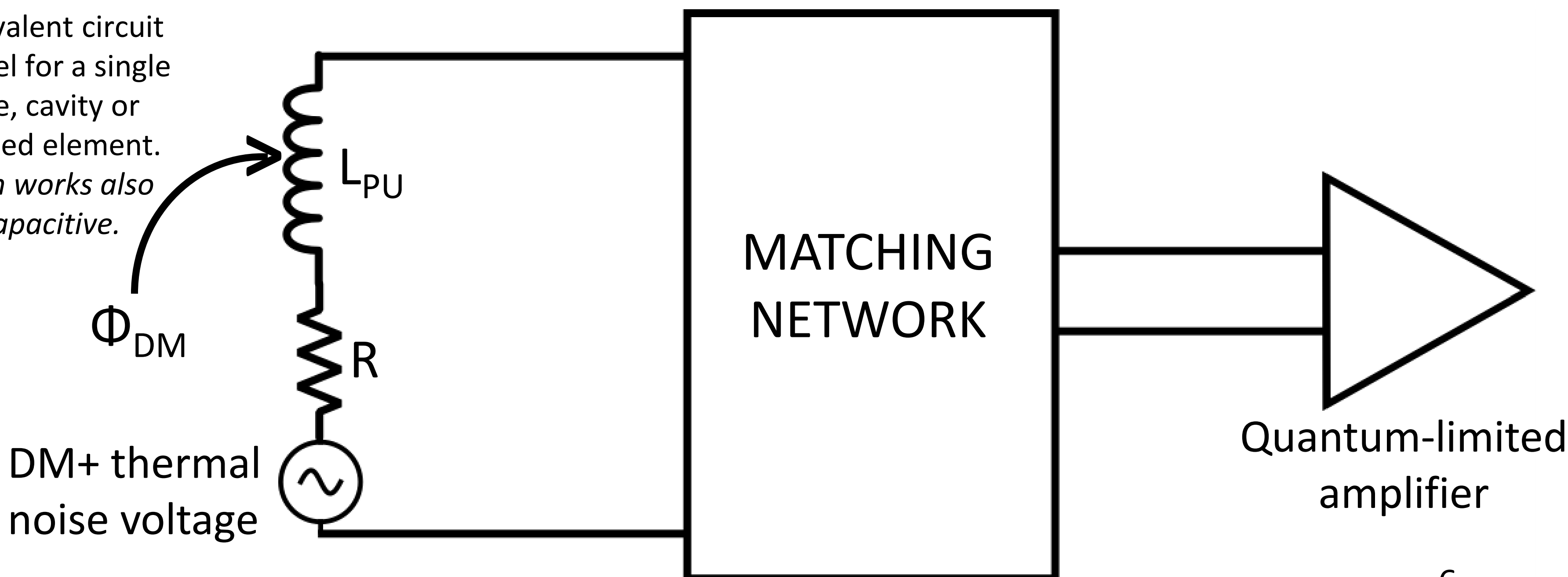
1) Single-pole LC resonator
2) Broadband inductive
3) Multi-pole resonator

→ / ←

Readout

1) Phase-insensitive amplifier
2) Quantum limit on performance
3) Imprecision and backaction noise

Equivalent circuit model for a single mode, cavity or lumped element. *Math works also for capacitive.*



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Standard Quantum Limit (SQL) on amplification

- Phase-insensitive amplifier: both sine and cosine components of signal (“quadratures”) are amplified equally
- Subject to Standard Quantum Limit: Heisenberg uncertainty on noise performance
  - H.A. Haus and J.A. Mullen, Phys. Rev. **128**, 407 (1962)
  - Caves, PRL **26**, 1817 (1982)
  - Modern review: Clerk et al, RMP **82**, 1155 (2010)
- SQL=1 photon of noise added by the measurement
  - 1 photon= increase required in thermal occupation number of circuit for change in thermal noise to equal amplifier noise

Standard Quantum Limit (SQL) on amplification

SQL=1 photon

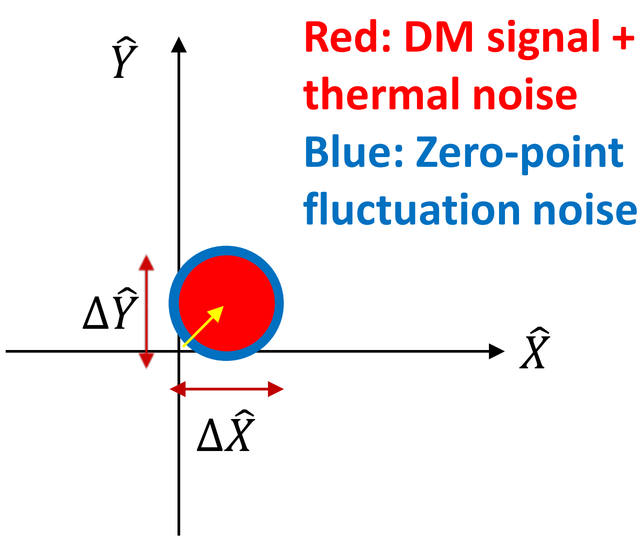
<u>Zero-point fluctuation noise (1/2)</u>

- Quadrature measurements $\hat{X}$ (cosine) and $\hat{Y}$ (sine) applied to vacuum have nonzero variance →noise

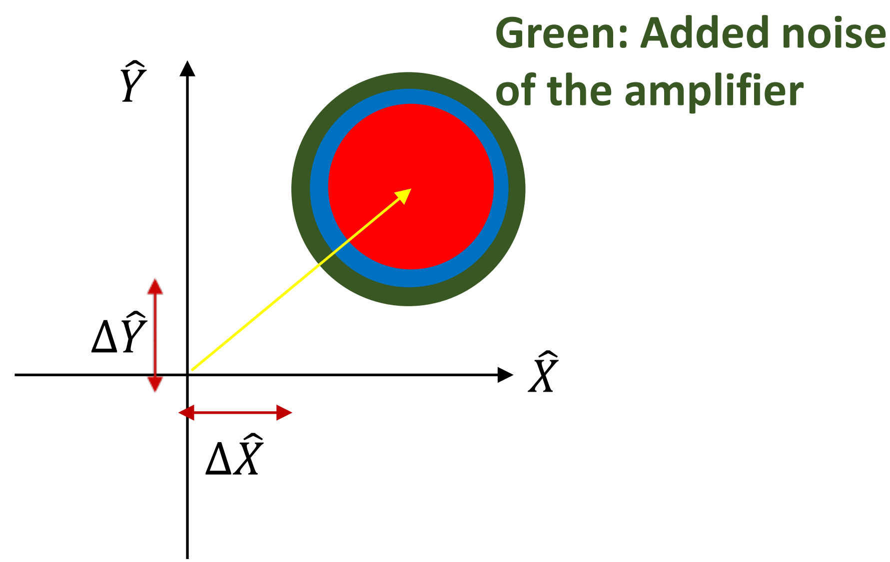
<u>Amplifier noise (1/2)</u>

- Noise added upon amplification from simultaneously measuring two noncommuting operators, $[\hat{X}, \hat{Y}] = i$

Amplifier Input



Amplifier Output



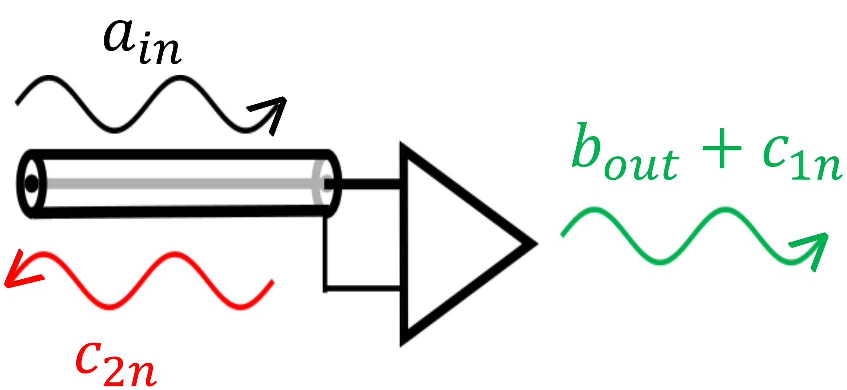
Amplifier noise = imprecision + backaction

Amplifier has two effective noise modes

- Imprecision noise: independent of input circuit
- Backaction noise: dependent on input circuit

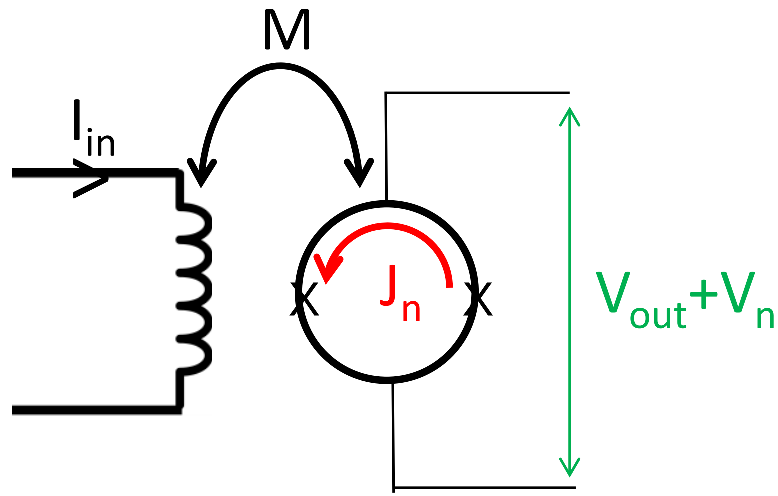
Amplifier noise = imprecision + backaction

Scattering-mode Amplifier



- E.g. JPAs, used in ADMX, HAYSTAC
- Incoming wave a_{in} amplified, giving output wave b_{out}
- Imprecision noise: intrinsic noise wave c_{1n} at output
- Backaction noise: noise wave c_{2n} injected into input circuit
- Reflects off input circuit, appears as more noise at output

Flux-to-Voltage Amplifier



- E.g. SQUIDs, used in DM Radio, ABRACADABRA
- Input current I_{in} feeds flux into loop, giving output voltage V_{out}
- Imprecision noise: intrinsic voltage fluctuations V_n at output
- Backaction noise: circulating noise currents J_n couple voltage to input
- Creates noise currents in input, appears as more noise at output

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How do we optimize matching network?

Signal Source

1) Inductive coupling to DM signal is optimal
2) Residual loss and associated thermal noise

Matching Network

Examples:

1) Single-pole LC resonator
2) Broadband inductive
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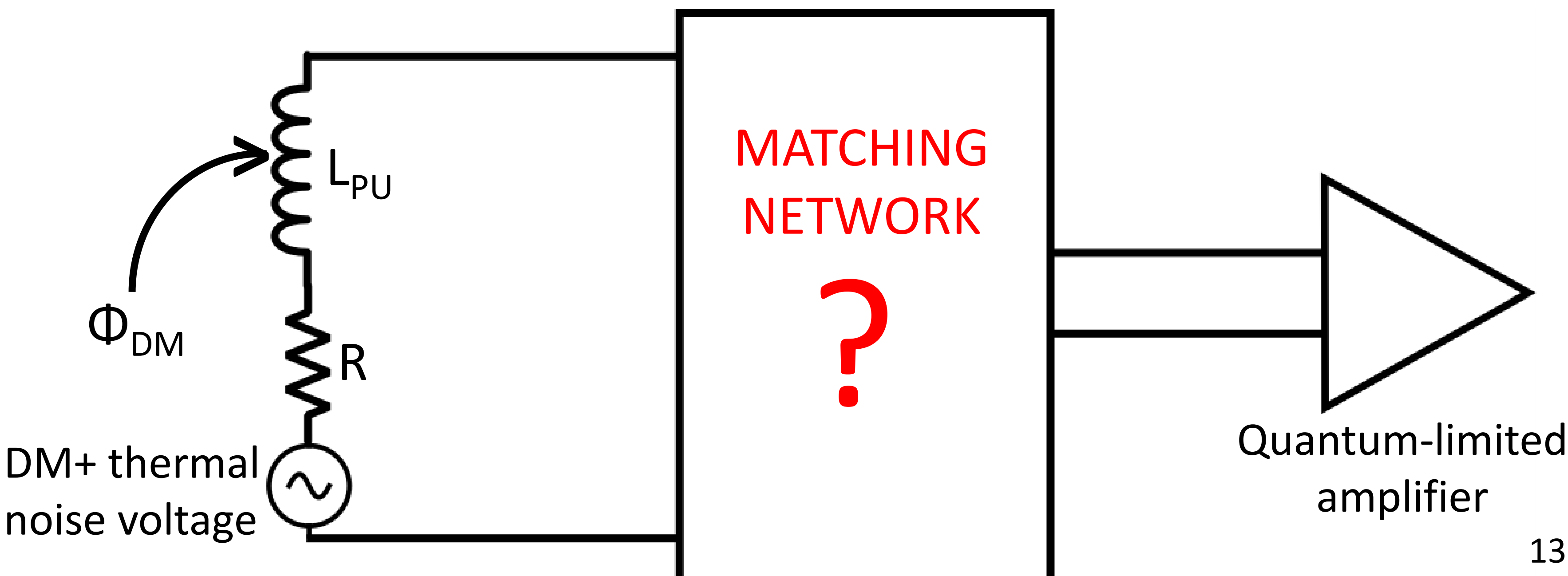
Readout

1) Phase-insensitive amplifier
2) Imprecision and backaction noise
3) Quantum limit on performance



Value function for matching optimization

- Value function needs to reflect:
  - Signal-to-noise ratio (SNR)
  - Priors- Favored mass or coupling range? Candidate signal to validate?
- Value function is expectation value of SNR squared:

$$U[S(\nu)] = E[SNR^2\,[S(\nu)]]$$

- $S(\nu)$=scattering matrix for the network
- Expectation is evaluated with user-defined preference functions for DM properties, e.g. mass
- *Log-uniform search*
  - Uninformative priors on DM
  - DM mass uniformly likely in log space
  - Want sensitivity as large as possible over as wide a bandwidth as possible

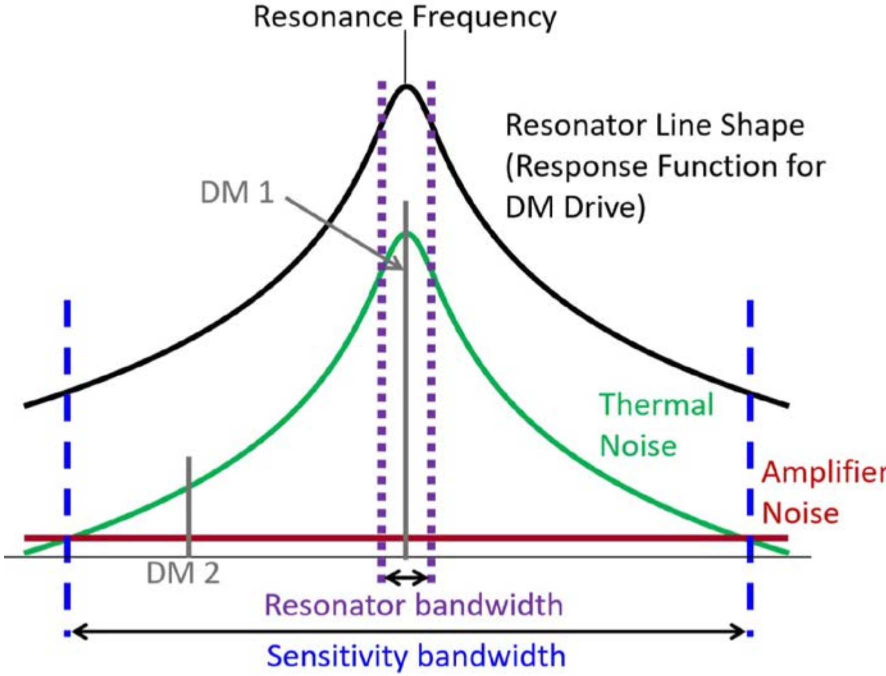
Log-uniform search: optimize integrated sensitivity

Example: single-pole resonator

- Maximize integrated sensitivity across search band, between ν_l and ν_h
- Figure of merit with quantum-limited amplifier:

$$U[S(\nu)] = \int_{\nu_l}^{\nu_h} d\nu \left(\frac{|S_{21}(\nu)|^2}{|S_{21}(\nu)|^2 n(\nu) + 1} \right)^2$$

- $n(\nu)$= signal source thermal occupation number
- "+1" is standard quantum limit



Substantial sensitivity available outside of resonator bandwidth for thermal resonator states. Substantial scan rate enhancement.

Quantum-limited amplifiers highly desirable even for thermal states hf<kT. (Measuring below SQL even better)

How large can sensitivity *U* be? Bode-Fano limit

- Constraint provided by Bode-Fano criterion for matching LR to a quantum-limited amplifier with a real noise impedance:
  - H.W. Bode, ``Network Analysis and Feedback Amplifier Design" (1946)
  - R.M. Fano, *Journal of the Franklin Institute* (1950)
- *Assume matching network is linear, passive, and reciprocal.*

Bode-Fano

$$\int_{\nu_l}^{\nu_h} d\nu \, ln\left(\frac{1}{|S_{22}(\nu)|}\right) \leq \frac{R}{2L_{PU}} \quad \Rightarrow$$

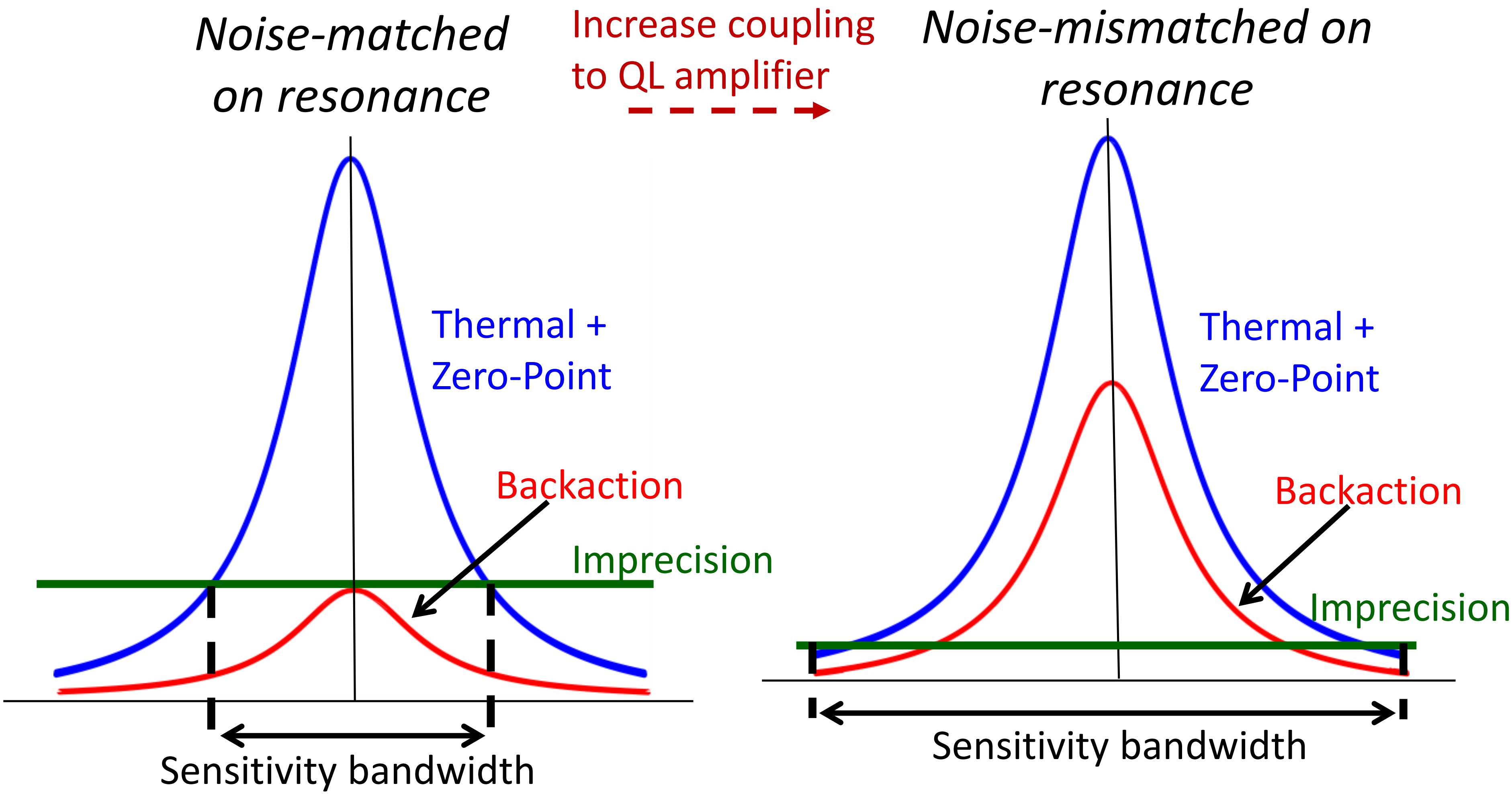
Bode-Fano-limited *U*

$$U[S(\nu)] \leq \begin{cases} \frac{1}{4n(\nu_h)} \frac{R}{L_{PU}}, & n(\nu_h) \gg 1 \\ 0.41 \frac{R}{L_{PU}}, & n(\nu_h) \ll 1 \end{cases}$$

- Analogous constraint for RC signal source

An ***optimal*** single-pole resonator can have a figure of merit *U* that is ~75% of the fundamental limit (pretty good!)

Optimize coupling strength with respect to integrated sensitivity



- Increased coupling: reduced imprecision, increased backaction
- 50% on-resonance noise penalty. Much larger sensitivity bandwidth

Completing our optimal detector!

Signal Source

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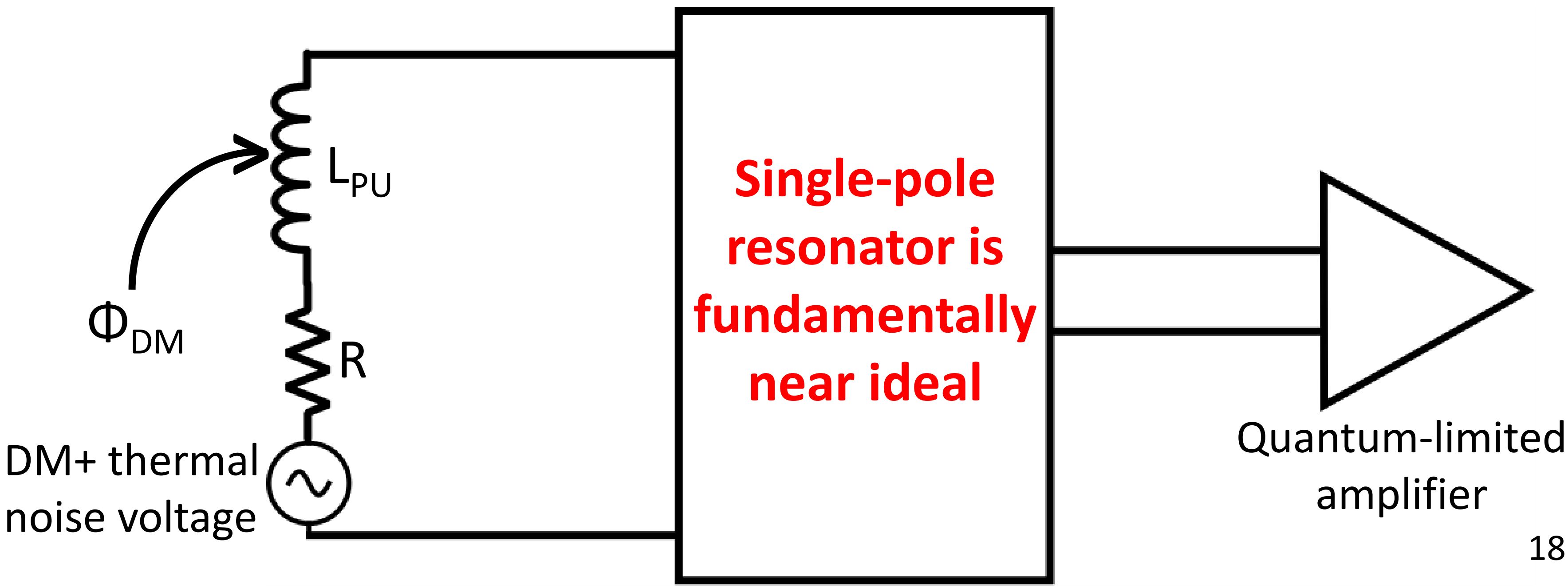
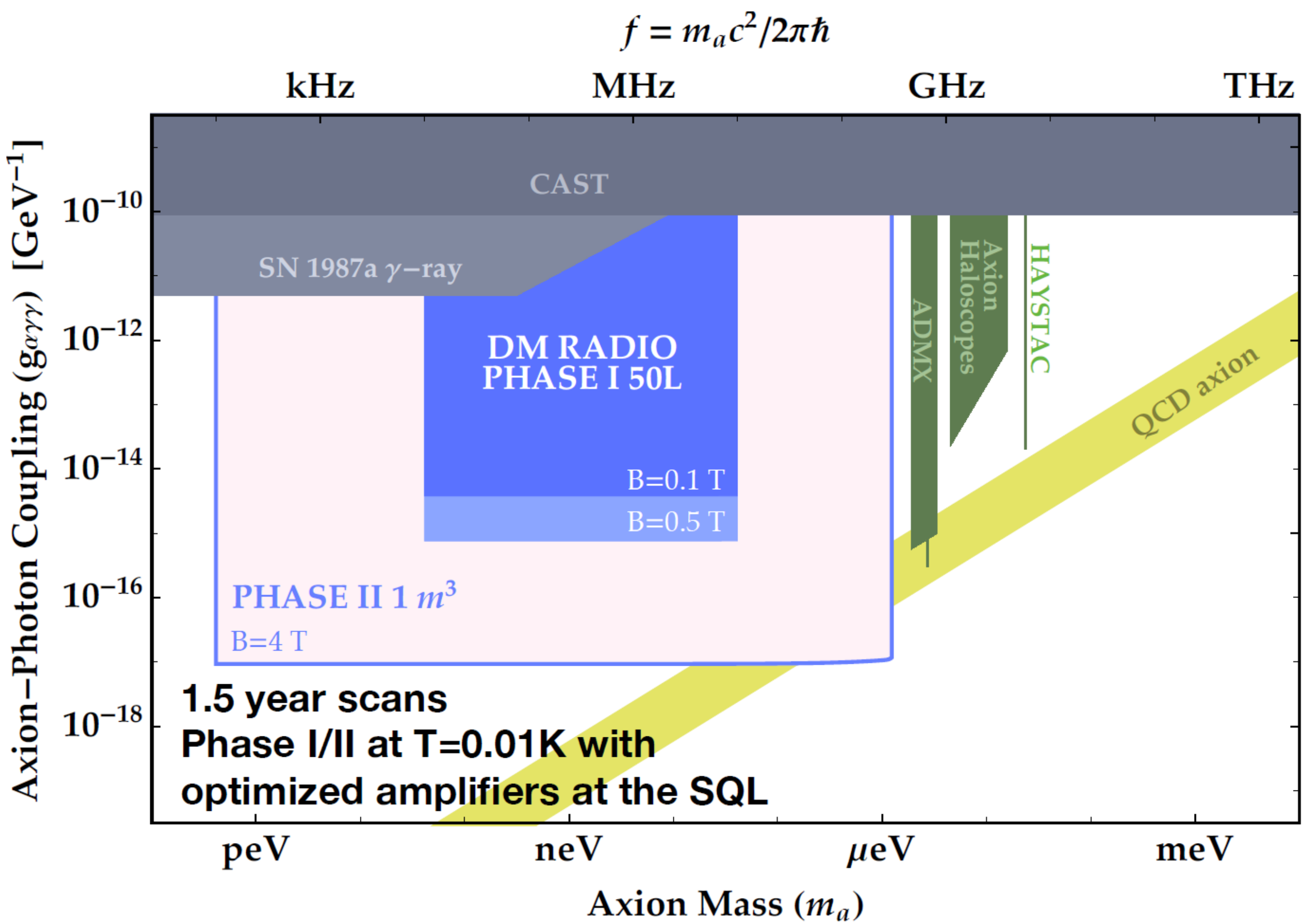


SQL for DM Radio from Arran's Talk

Axion Sensitivity



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Quantum noise in a harmonic oscillator

The Hamiltonian of a harmonic oscillator is

$$\hat{H} = \hbar\omega\left(a^{\dagger}a + 1/2\right)$$

The Hamiltonian can be written in the cosine component ($\hat{X}$) and the sine component ($\hat{Y}$)

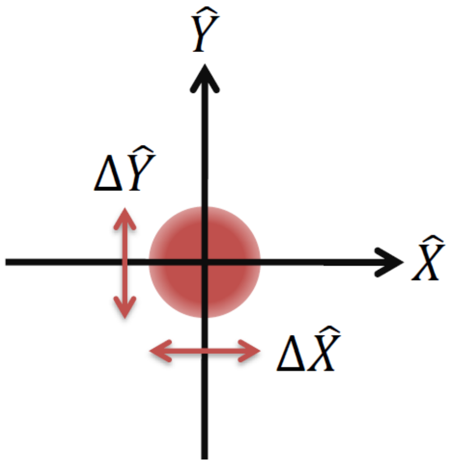
$$\hat{H} = \frac{\hbar\omega}{2}\left(\hat{X}^2 + \hat{Y}^2\right)$$

$$\left[\hat{X}, \hat{Y}\right] = i$$

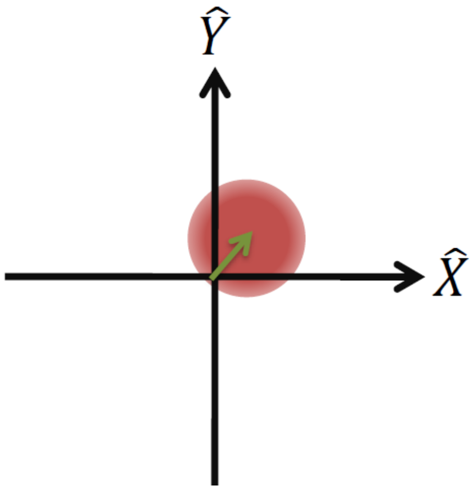
$$\Delta\hat{X}\Delta\hat{Y} \geq \frac{1}{2}$$ vacuum noise

When amplified, add one more ½ quantum

$$N_{add} \geq \frac{1}{2}$$



With nonzero expectation value



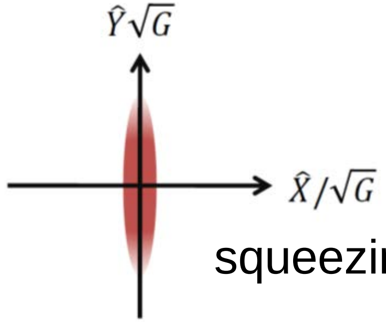
Quantum sensing

- If we don't need to measure both quadratures of a field, we don't have to be limited by the standard quantum limit.

- The standard quantum limit can be evaded using quantum correlations. These techniques are deeply related:

  - Photon counting
    / quantum nondemolition
  - Squeezing
  - *Backaction evasion*
  - Entanglement
  - Cooling
  - Quantum nondemolition

$$\hat{H} = \hbar\omega\left(a^\dagger a + 1/2\right)$$



squeezing

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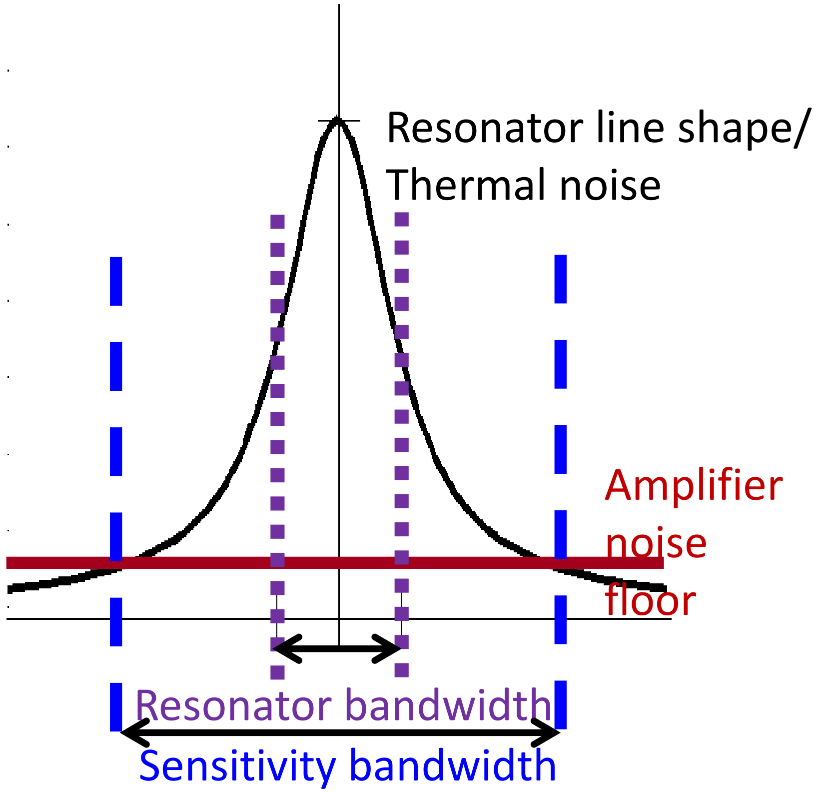
Quantum sensing of thermal states

$\hbar\omega < k_B T$ Thermal state

Why would we use a quantum sensor for a thermal state?

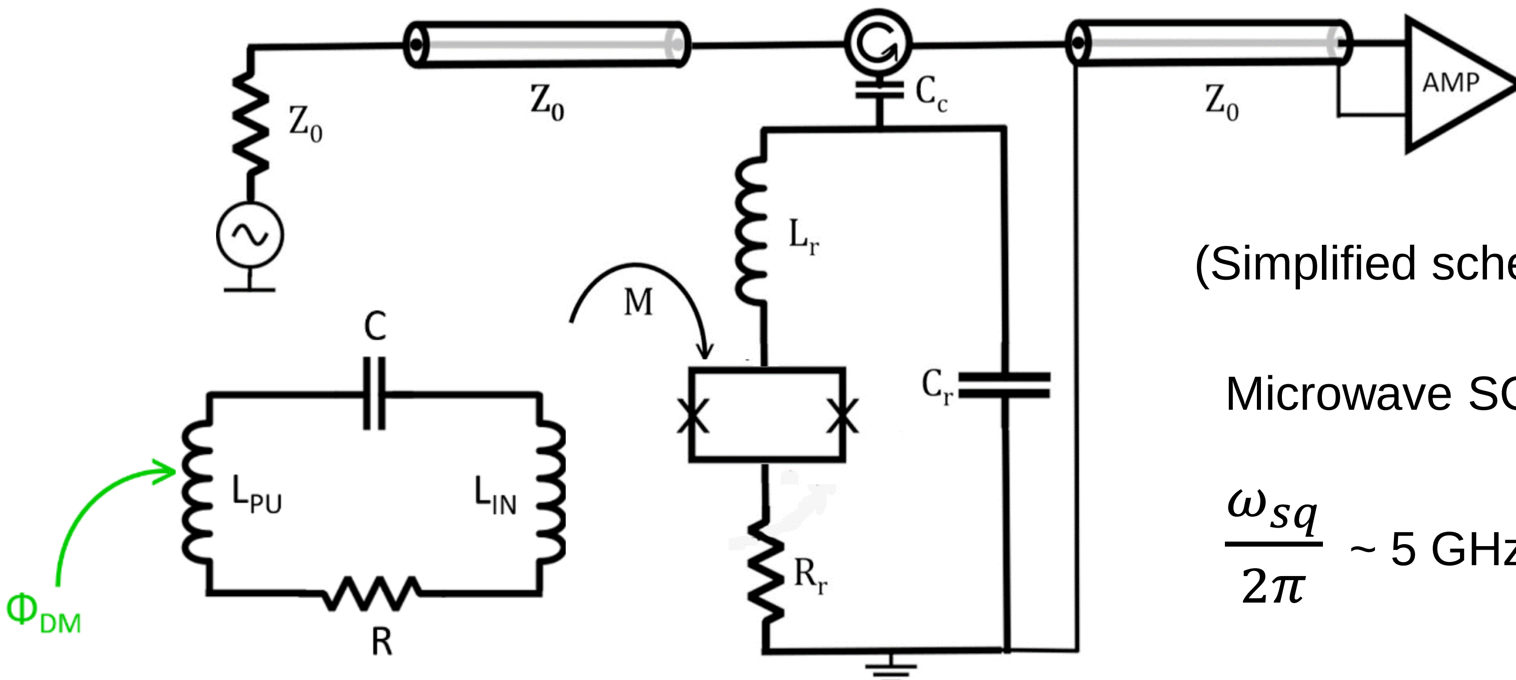
- The signal to noise within the resonator bandwidth is not helped by a better amplifier.
- The sensitivity of the amplifier determines the *sensitivity bandwidth*, and thus the sensitivity of a search for an unknown signal frequency.
- Very large speedup possible for a sensor operating below the standard limit even if $\hbar\omega < k_B T$



Quantum sensors are needed for low-frequency thermal states too

Measuring a resonator with a dissipationless microwave SQUID frequency upconverter

Dissipationless microwave SQUID flux amplifier



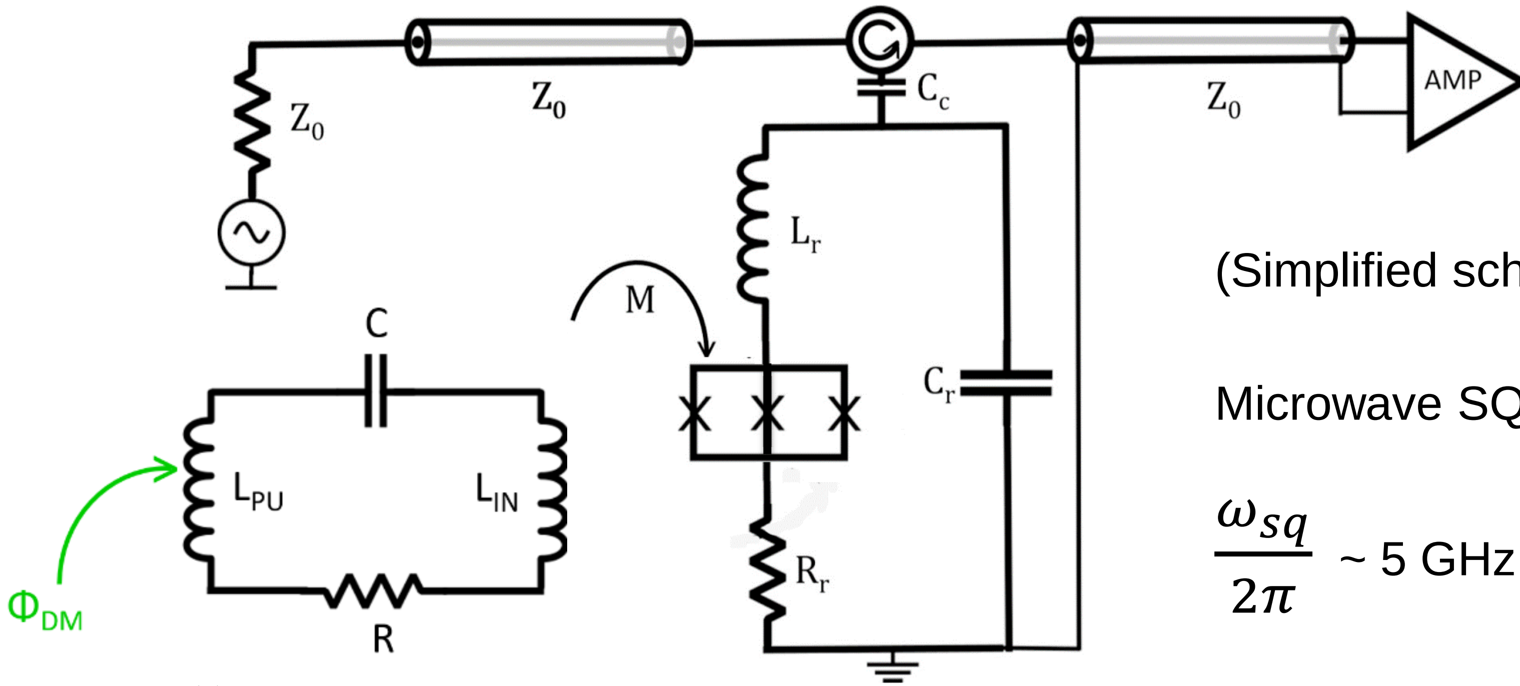
(Simplified schematic)

Microwave SQUID:

$$\frac{\omega_{sq}}{2\pi} \sim 5 \text{ GHz}$$

DM Radio: $\frac{\omega_r}{2\pi}$ = 1 kHz – 100 MHz

Measuring a resonator with a dissipationless Zappe Photon Upconverter (ZPU)



(Simplified schematic)

Microwave SQUID:

$\frac{\omega_{sq}}{2\pi}$ ~ 5 GHz

DM Radio: $\frac{\omega_r}{2\pi}$ = ~300 Hz – ~300 MHz

Uncoupled Hamiltonian: $\hat{H}_0 = \hbar\omega_{sq}\hat{a}^\dagger\hat{a} + \hbar\omega_r\hat{b}^\dagger\hat{b}$

Interaction Hamiltonian: $\hat{H}_{int} = -\hbar G\hat{\Phi}_{in}\hat{a}^\dagger\hat{a} = -\hbar g_0\hat{a}^\dagger\hat{a}\left(\hat{b} + \hat{b}^\dagger\right)$

Hamiltonian maps onto optomechanical system

DM Radio: $\frac{\omega_r}{2\pi}$ = ~300 Hz – ~300 MHz   Microwave resonator: $\frac{\omega_{sq}}{2\pi}$ ~ 5 GHz

Uncoupled Hamiltonian: $\hat{H}_0 = \hbar\omega_{sq}\hat{a}^\dagger\hat{a} + \hbar\omega_r\hat{b}^\dagger\hat{b}$

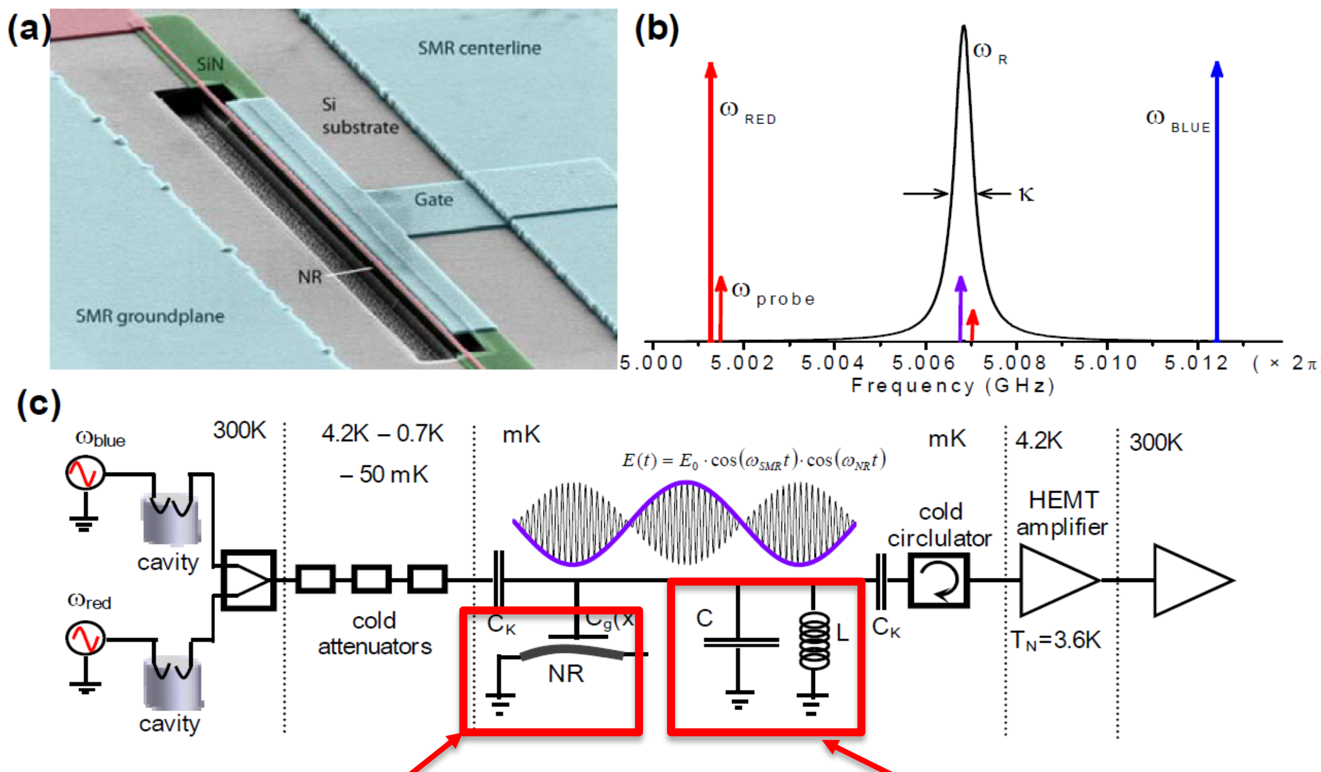
Interaction Hamiltonian: $\hat{H}_{int} = -\hbar G\hat{\Phi}_{in}\hat{a}^\dagger\hat{a} = -\hbar g_0\hat{a}^\dagger\hat{a}\left(\hat{b} + \hat{b}^\dagger\right)$

This maps onto the Hamiltonian of on optomechanical resonator with:

Displacement r	⟷	Flux Φ
Momentum p	⟷	Charge Q
Inverse spring constant 1/k	⟷	Inductance L
Mass m	⟷	Capacitance C

Nonlinear interaction upconverts photons from the DM Radio resonator to the uwave SQUID, downconverts uwave SQUID photons to the DM Radio, leading to backaction

Hamiltonian maps onto optomechanical system



Low-frequency mechanical resonance
(Maps onto DM radio Hamiltonian)

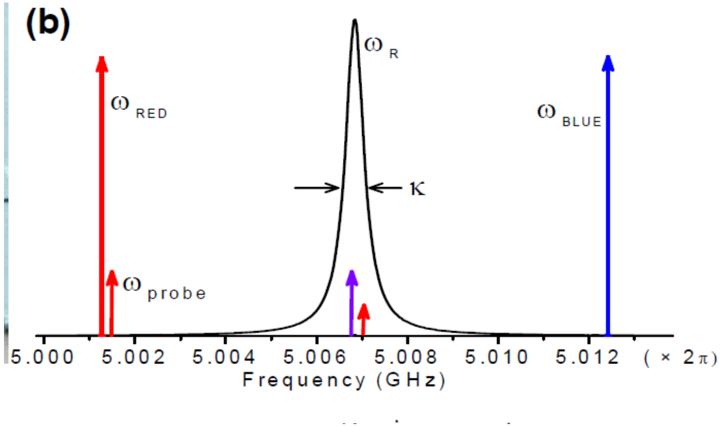
High-frequency LC resonance
(Maps onto uwave SQUID Hamiltonian)

Hertzberg, J. B., Rocheleau, T., Ndukum, T., Savva, M., Clerk, A. A., & Schwab, K. C. (2010). Back-action-evading measurements of nanomechanical motion. *Nature Physics*, *6*(3), 213-217.

Back-action Evasion

- Originally proposed by Braginsky (1980) for gravitational wave detectors.

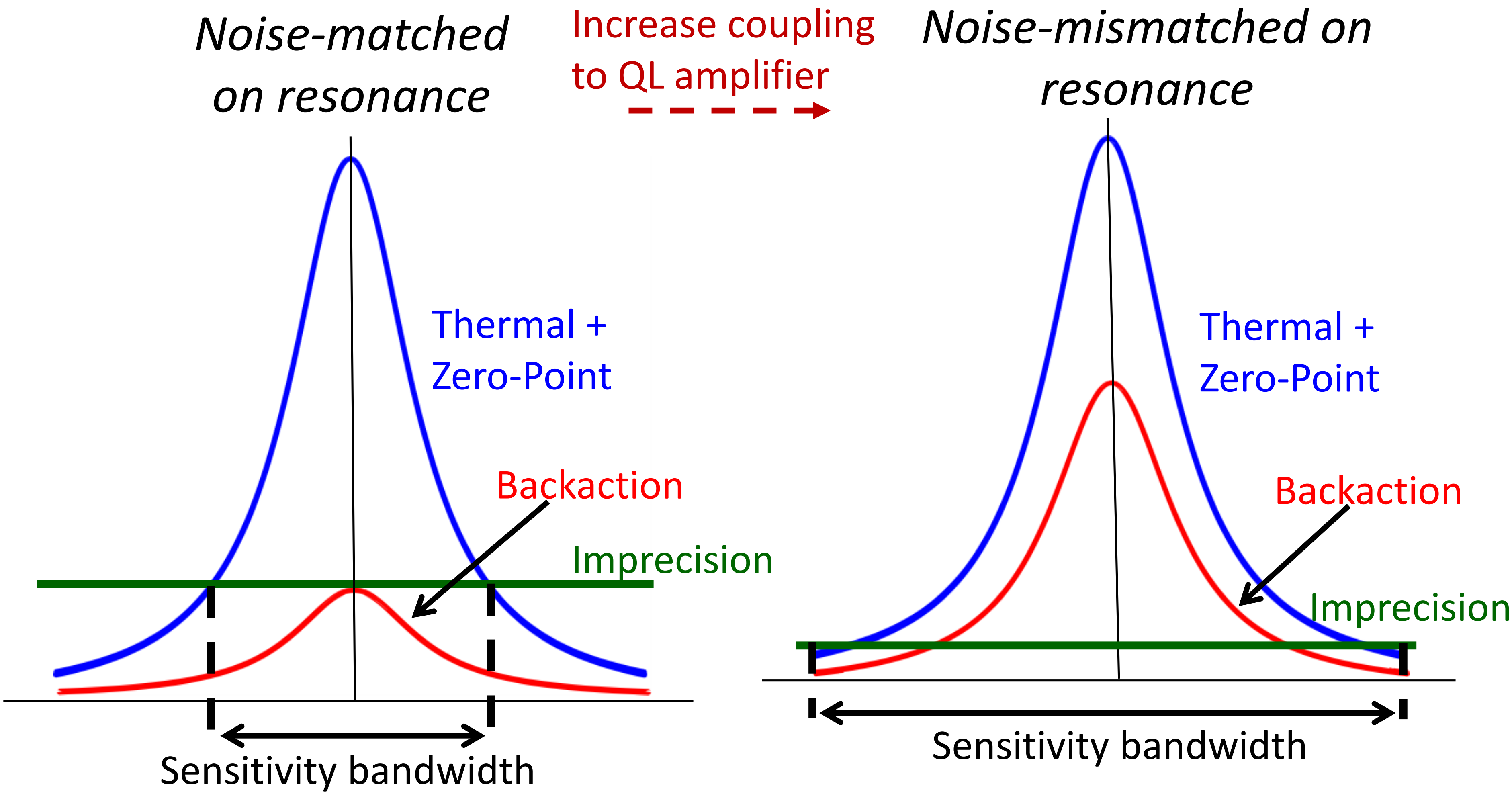
- With proper device symmetry, when both sidebands are pumped, the back-action is applied only to the unmeasured quadrature. Allows much stronger coupling, and reduction of both imprecision and back-action noise.



- Squeezing, cooling, other quantum protocols possible

Back-action Evasion with microwave SQUID requency upconverters is a promising quantum protocol for DM Radio

Optimize coupling strength with respect to integrated sensitivity



- Increased coupling: reduced imprecision, increased backaction
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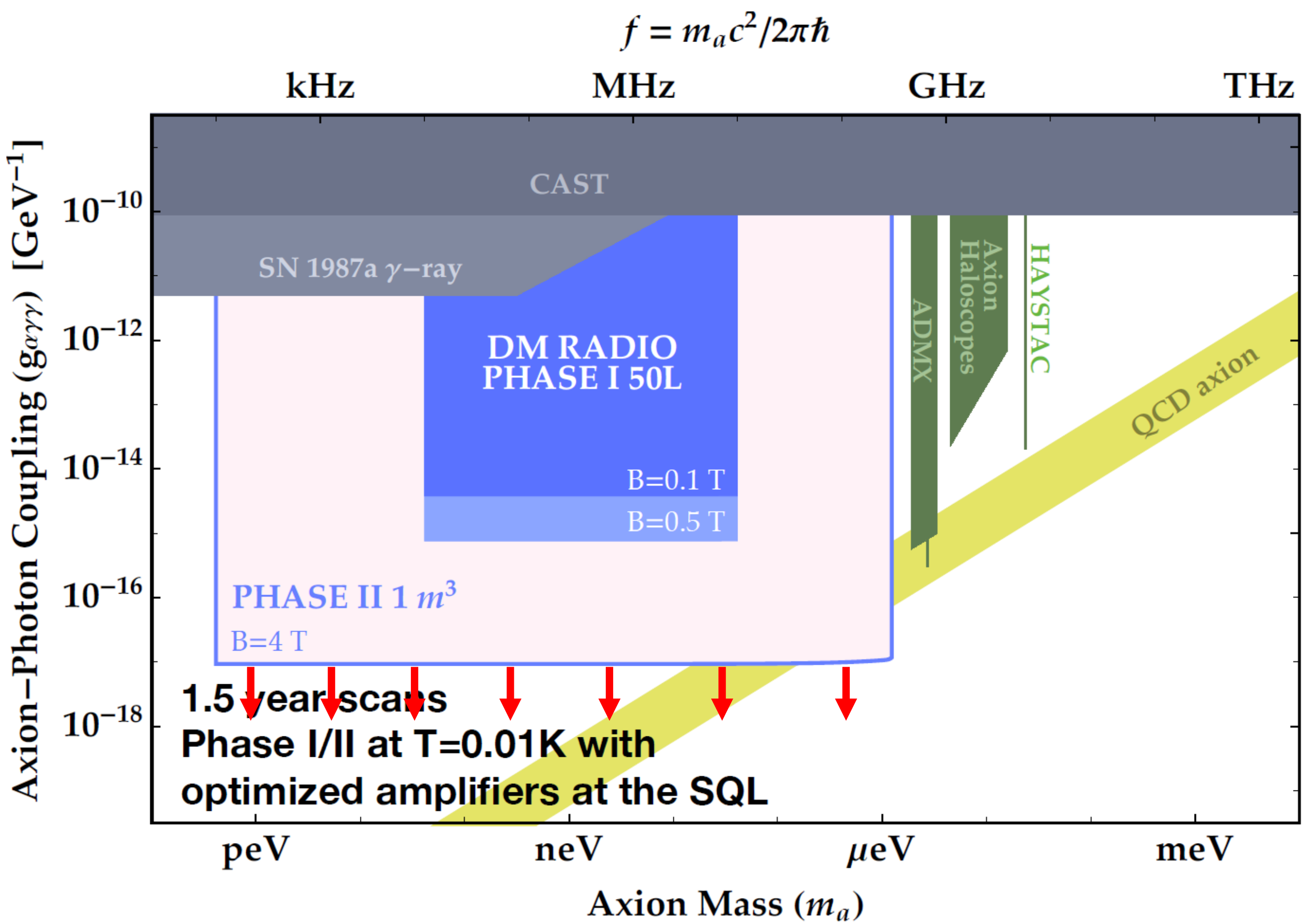
Axion Sensitivity



Conclusions

- One-pole resonators are nearly optimal for single-mode dark-matter searches (75% saturation of Bode-Fano Limit)
- Significant sensitivity outside of the resonator bandwidth
  - Larger scan steps possible: with Q~1e6, at 1 MHz SQL, we would likely have 40 Hz scan steps, rather than 1 Hz.
- Strong encouragement to improve limits with quantum sensors, even for resonators in a thermal state (< 300 MHz)
- Zappe Photon Upconverters promising for backaction-evasion to measure below the SQL in experiments < 300 MHz, including DM Radio (and others).


STOP