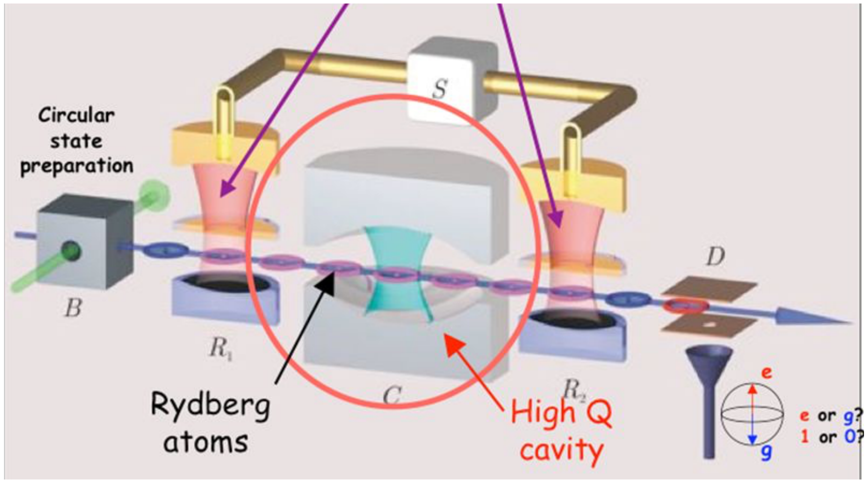
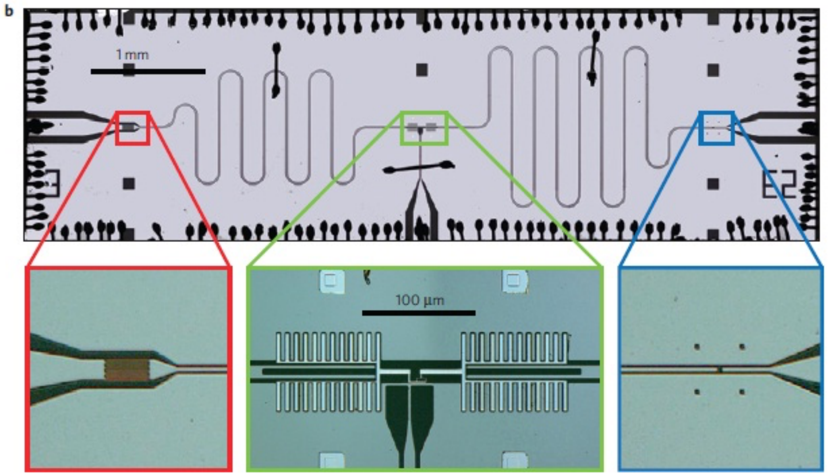
Quantum Metrology Techniques for Axion DM above 10 GHz

Aaron S. Chou (FNAL)

LLNL Microwave Cavity Workshop

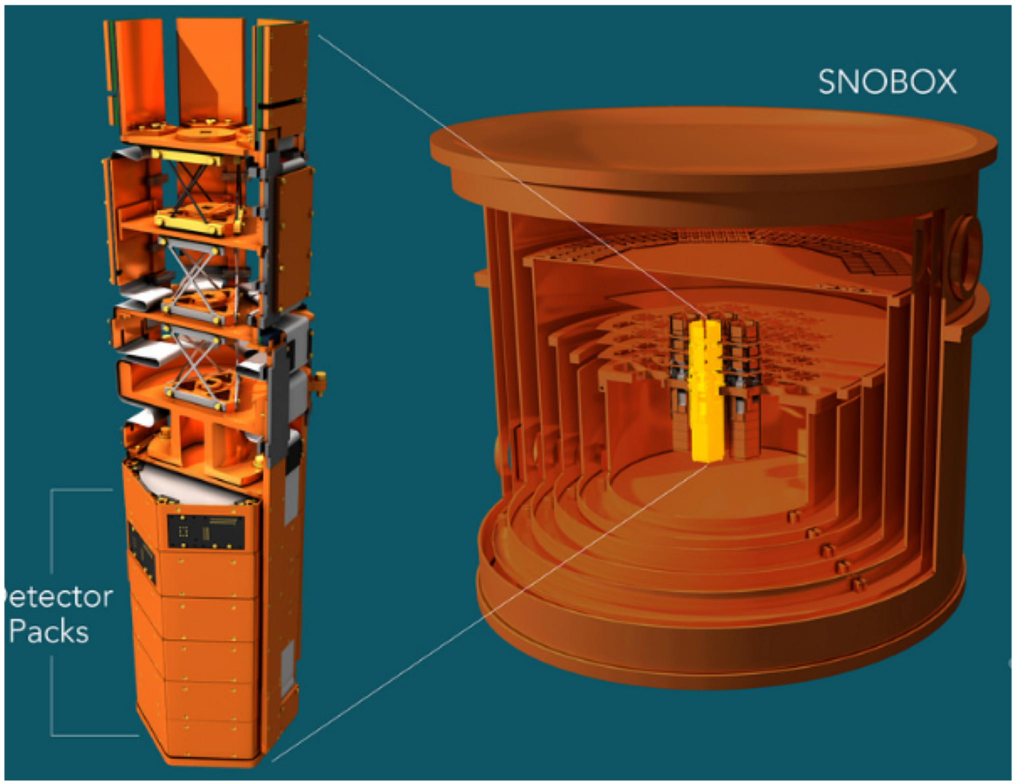
8/23/2018





Review panel question:
Why do axion searches take so long???

CDMS: 1 puck (1997)
→ SuperCDMS: **24 pucks in 4 towers**



Can we similarly conduct simultaneously operate many haloscopes using **identical** technology, each assigned a different frequency range?

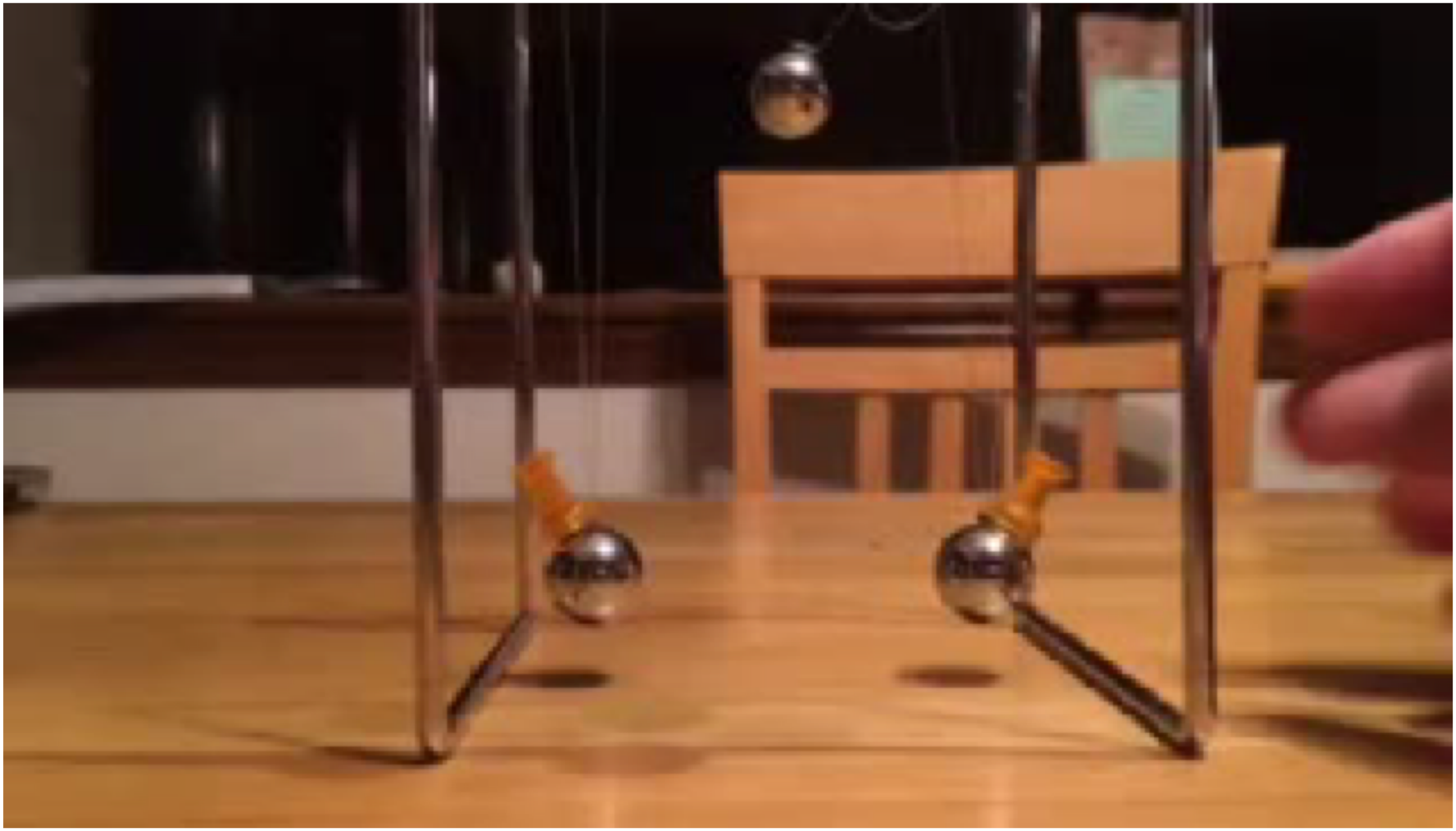
Turn-key dilution refrigerators and magnets are cheap and low risk.

How to enable an experiment based on commodity refrigerators and magnets???

- Backgrounds due to thermal photons and readout noise can be reduced using single photon detection
  - Superconducting qubits for QND (Akash Dixit talk)
    - 2018 DOE Early Career Award to Daniel Bowring
  - Rydberg atoms (Reina Maruyama)

- Only need to boost signal strength to compensate for low-ish B^2V.
  - High-Q cavities (Ankur Agrawal talk)
  - Stimulated emission (Konrad Lehnert, David Schuster, Aaron Chou)

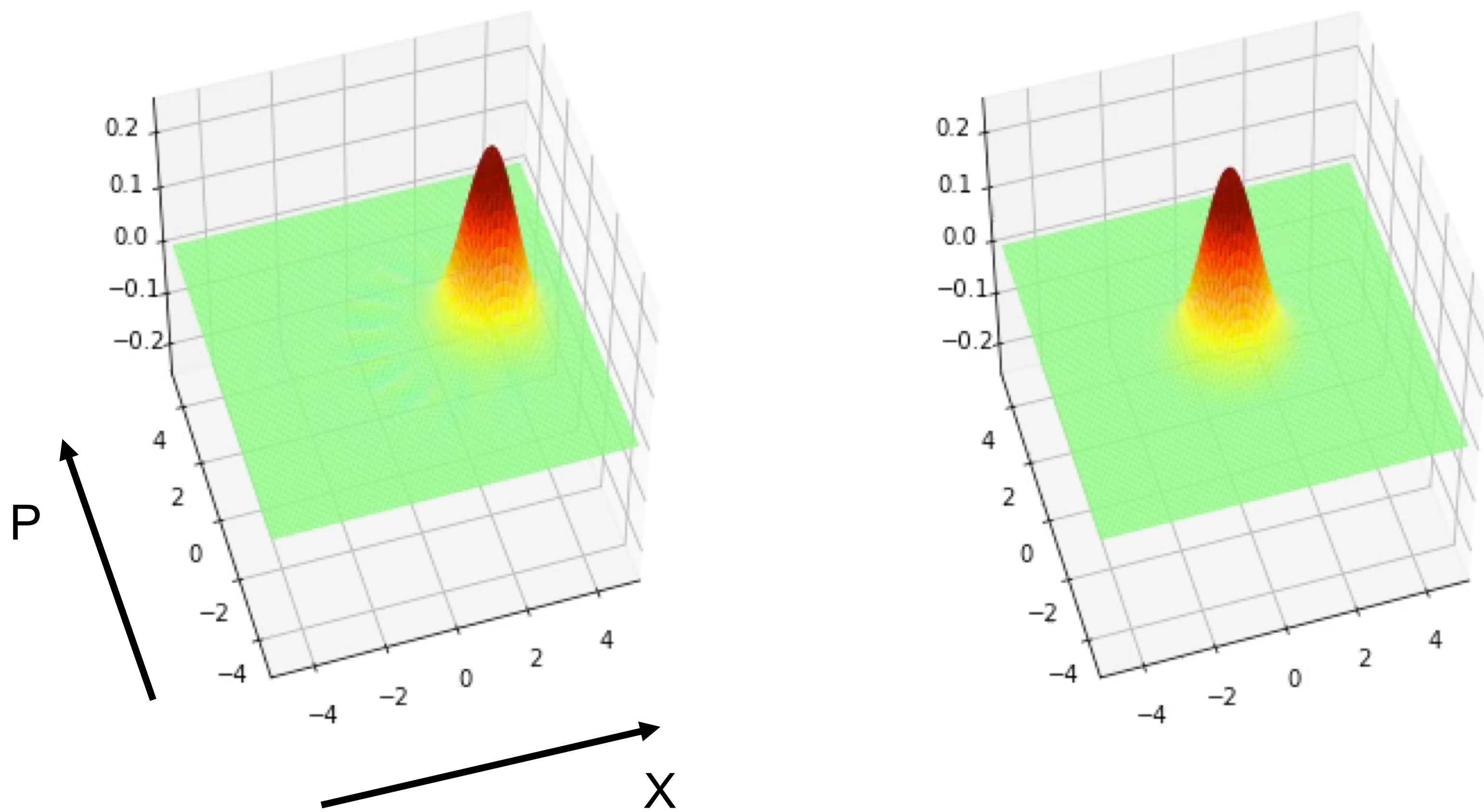
Energy transfer between two coupled oscillators



Weak coupling -- takes many swings to fully transfer the wave amplitude.
In real life, the number of swings is limited by coherence time.
Narrowband cavity response → iterative scan through frequency space.

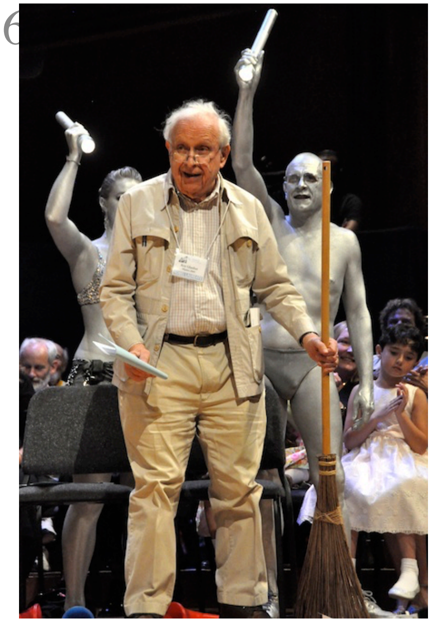
Classical pendulum system: $|\alpha = 3\rangle \otimes |\alpha = 0\rangle$

Time evolution of Wigner distributions in X-P “phasor” space



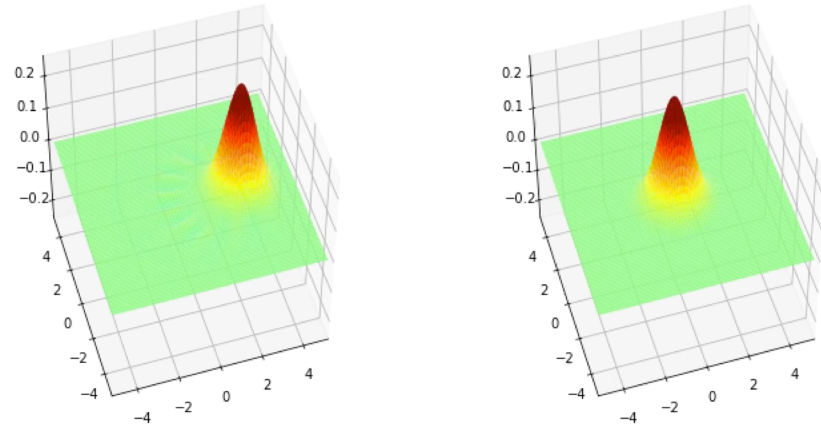
The two pendula swap their coherent states.

Simulated with QuTIP



Classically-driven quantum harmonic oscillator

Roy Glauber
Nobel Prize
2005,
"Keeper of the
Broom"



Phasor space evolution of coupled oscillators

classical sine wave drive: $f(t) = f_0 e^{-i\omega t}$

$$\boxed{U_{\mathrm{I}}(t) = \exp\left[(f_0 a^\dagger - f_0^* a)t\right]}$$

$$|\psi(t)\rangle_{\mathrm{I}} = \exp[(f_0 a^\dagger - f_0^* a)t]|0\rangle = e^{-|f_0|^2 t^2/2} e^{f_0 a^\dagger t}|0\rangle$$

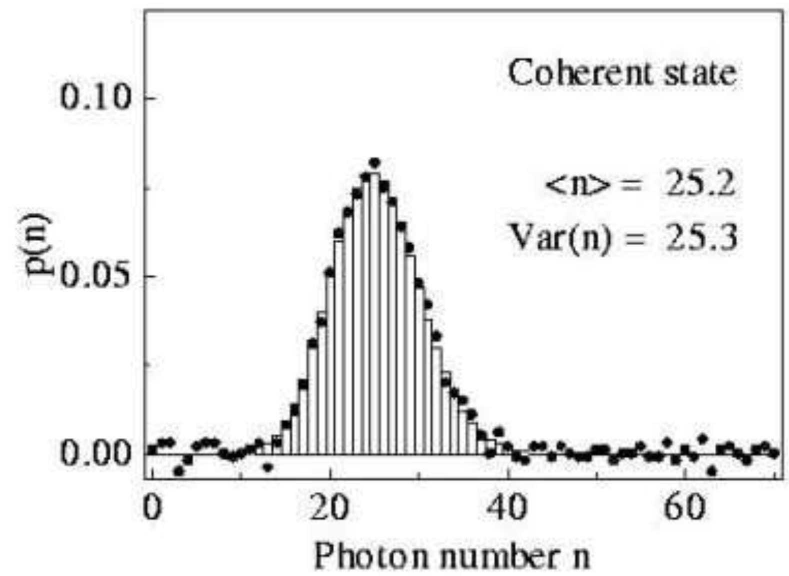
$= D(f_0 t)|0\rangle$ **Glauber displacement operator**

$\equiv |\alpha = f_0 t\rangle$ **Glauber coherent state:** quantum description of a classical sine wave, eigenstate of the annihilation operator: $a|\alpha\rangle = \alpha|\alpha\rangle$

Coherent states in the Fock basis

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}^\dagger} |0\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle} \frac{\langle n\rangle^n}{n!}$$



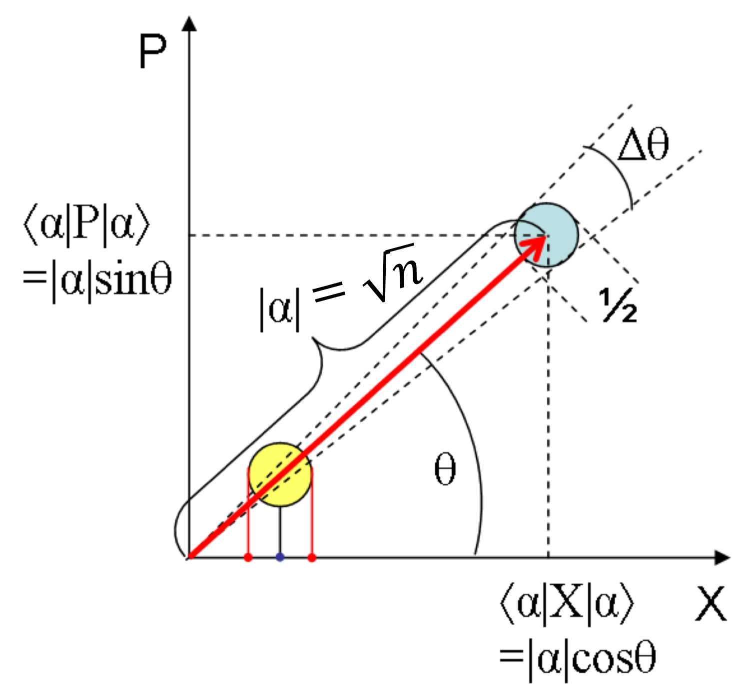
QM knows that the squared wavefunction describes a Poisson process:

$$\langle n\rangle = \langle \hat{a}^\dagger \hat{a}\rangle = |\alpha|^2$$

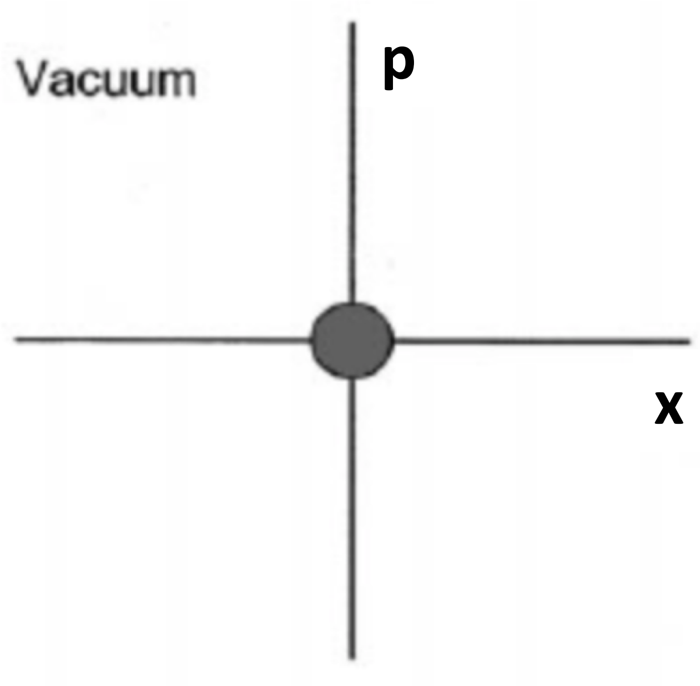
$$(\Delta n)^2 = \mathrm{Var}\left(\hat{a}^\dagger \hat{a}\right) = |\alpha|^2$$

Like the zero-point fluctuations, the Poisson shot noise in classical wave intensity is a consequence of the Heisenberg uncertainty principle.

The shot noise enforces the standard quantum limit in parametric amplifiers.



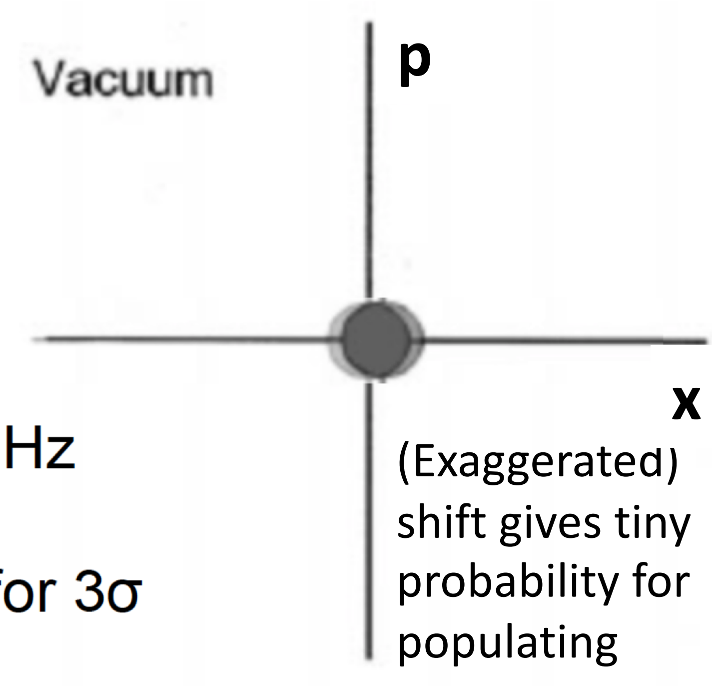
The axion wave displaces the cavity vacuum state by an amount much smaller than the zero-point vacuum noise



$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

Axion-photon mixing induces tiny coherent state displacement

0.01% shift @ 2 GHz
(20 kHz BW)
Avg >900M FFTs for 3σ

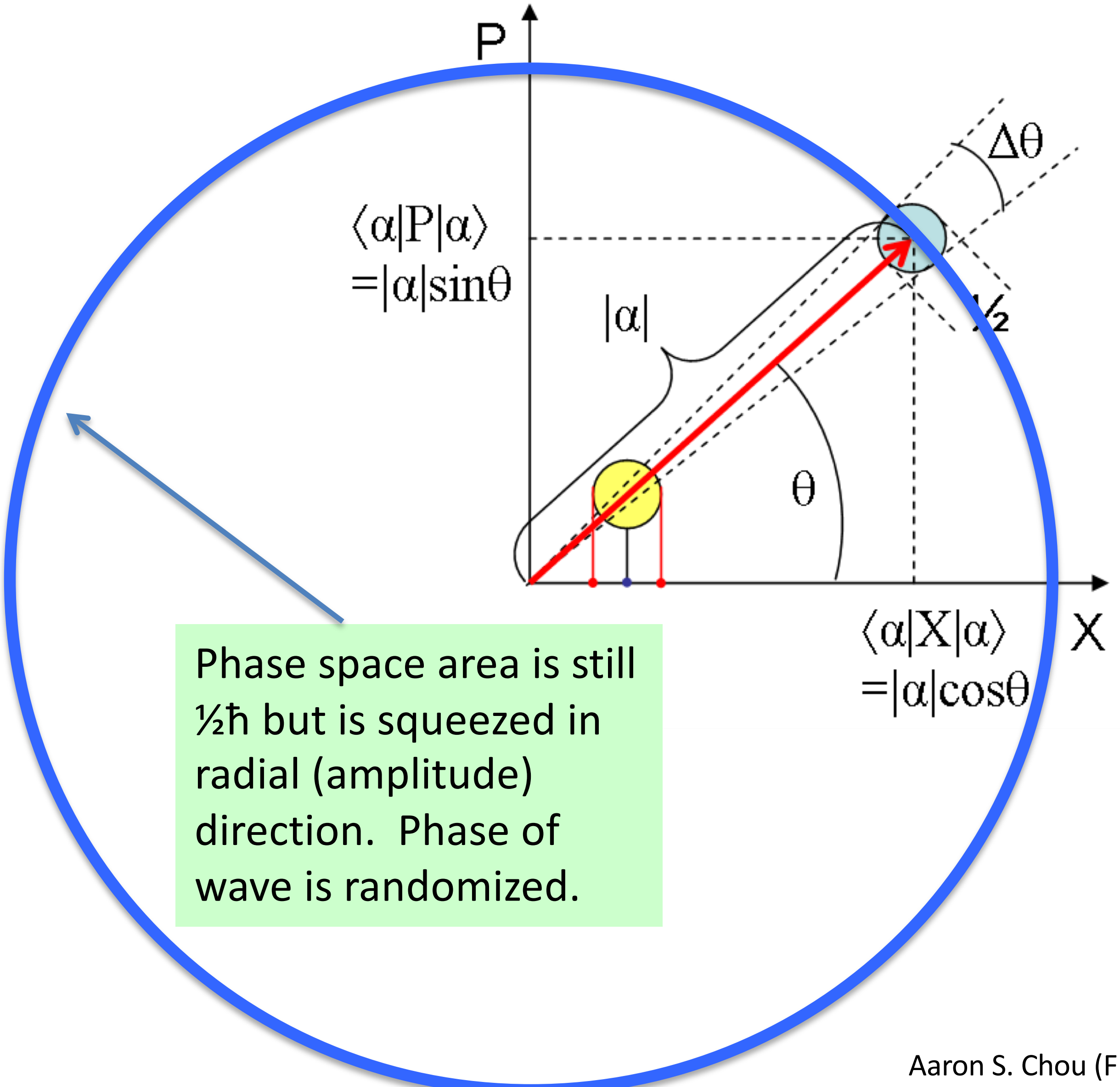


(Exaggerated) shift gives tiny probability for populating N=1 state

Cannot easily see the tiny shift above the zero-point noise of the vacuum…

Quantum non-demolition (QND) single photon detectors can do much better than SQL amplifiers

Number operator commutes with the Hamiltonian→ all backreaction is put into the unobserved phase – which we don't care about...



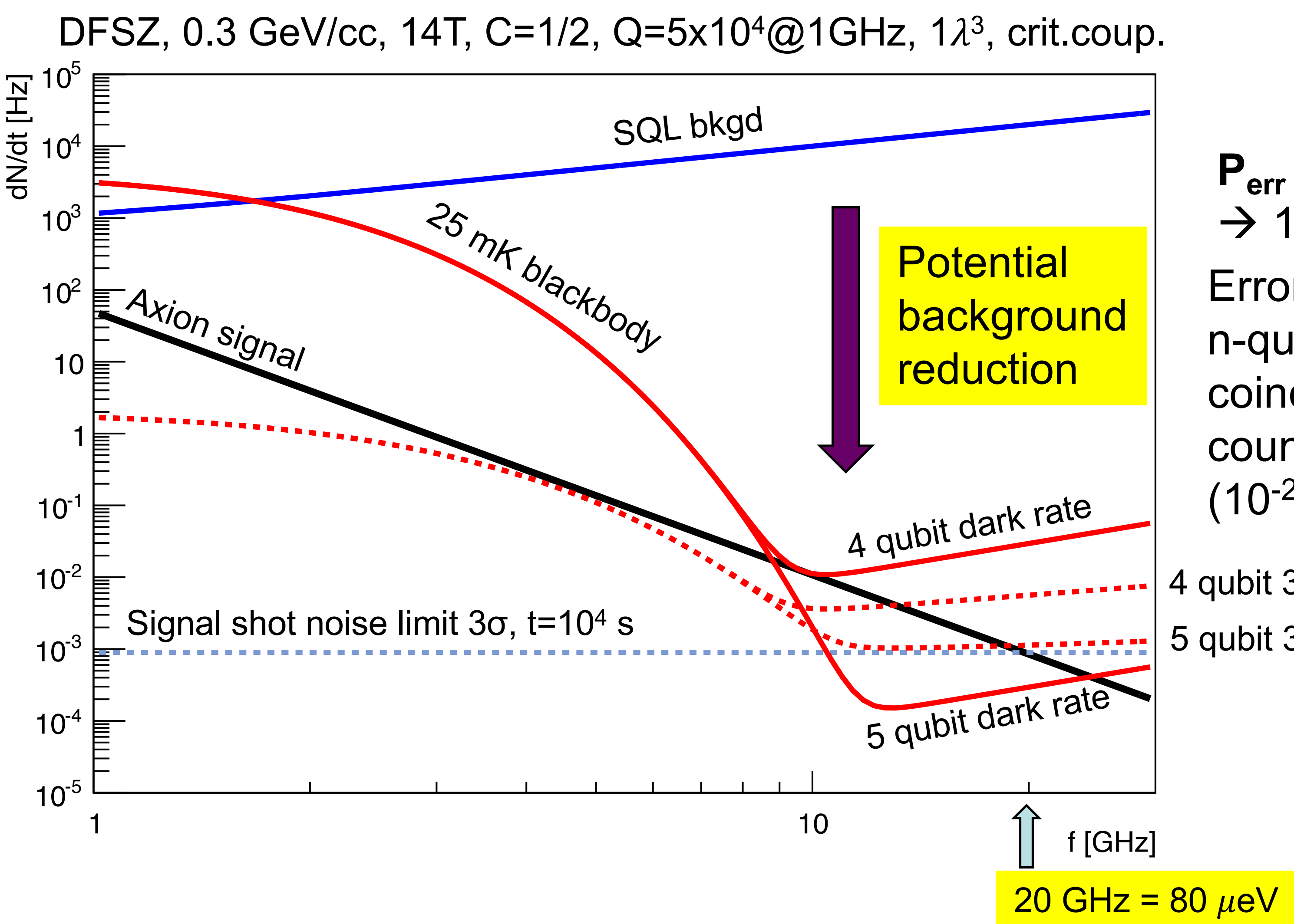
Avoid the zero-point noise by making measurements in the Fock basis instead of the coherent state basis.

Demonstrated with Rydberg atoms, (Serge Haroche Nobel Prize 2012)

Implemented using solid state artificial atom qubits, D.Schuster et.al, 2007

Proposed for axion search: Lamoreaux, Lehnert, et.al, 2013, Zheng, Lehnert, et.al, 2016

Multi-qubit sensors can achieve **vanishing** background rates



P_{err} = 10^{-2}

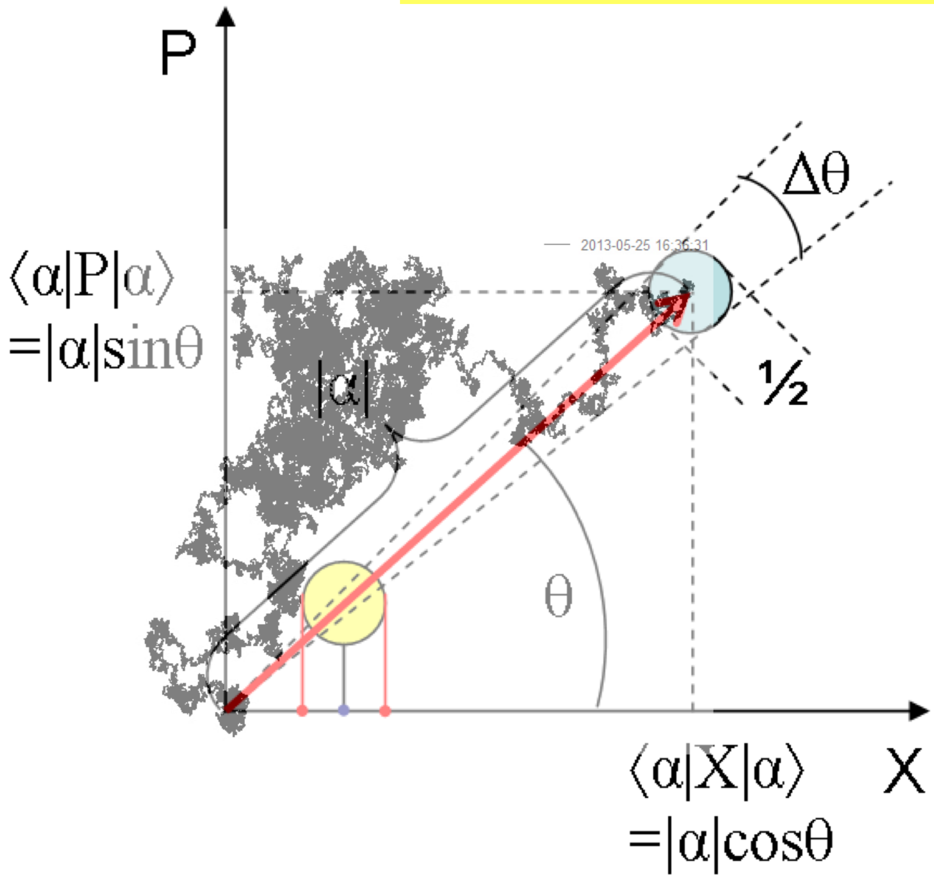
→ 1% of SQL.

Error prob. for n-qubit coincidence counting = $(10^{-2})^n$

Sensitivity is only limited by **signal shot noise.** Signal rate can be increased with: 1) higher B field; 2) power-combined cavities; 3) higher Q cavities

Use high-Q_c cavity ($Q_c > Q_a = 10^6$) as signal integrator to match noisy readout rate to the expected signal rate

Why read out at 10^6 Hz when the signal rate is 10^{-2} Hz?



Displacement operator product gives **random walk** accumulation of signal amplitude (1 step per signal coherence time):

$$\hat{D}(\alpha)\hat{D}(\beta) = e^{(\alpha\beta^* - \alpha^*\beta)/2}\hat{D}(\alpha+\beta)$$

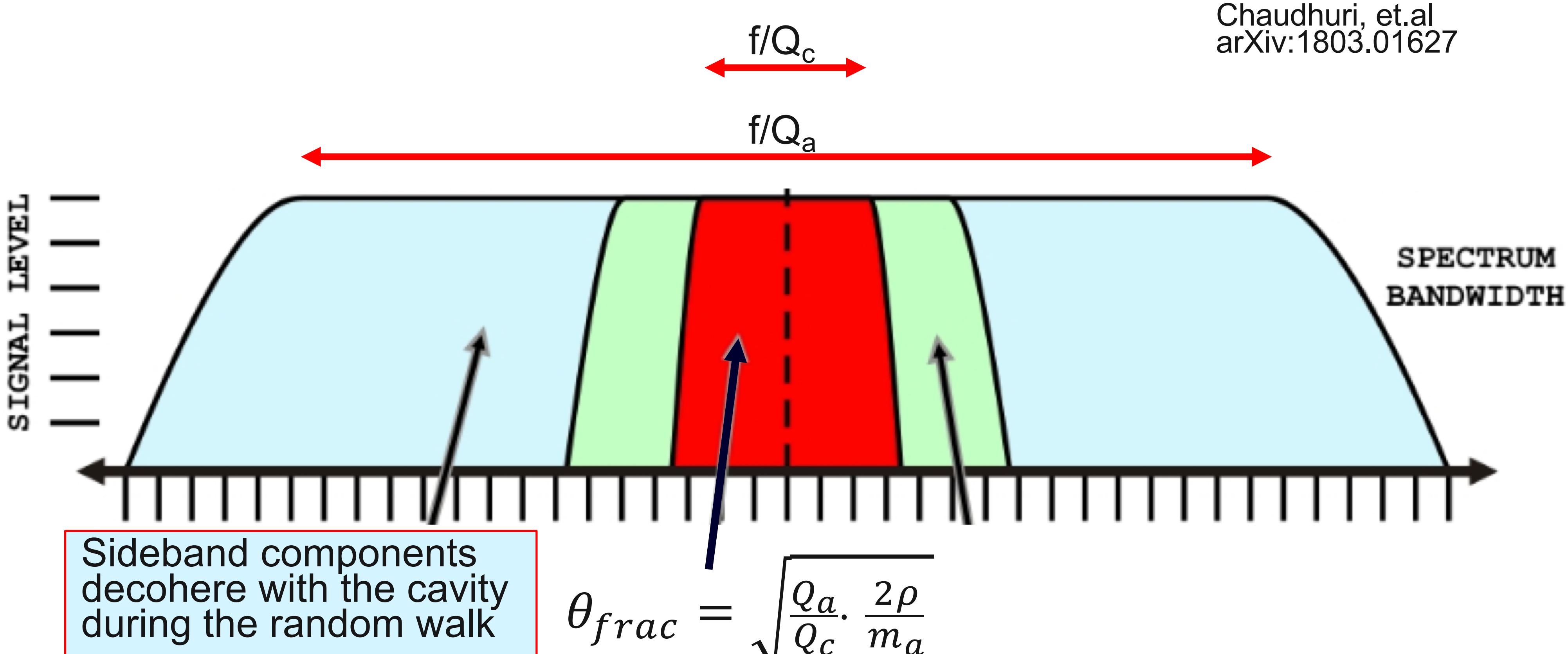
$$\rightarrow \Pi_i\, D(\Delta\alpha_i) = D(\underbrace{\Sigma_i \Delta\alpha_i}_{\sqrt{N_{steps}}\Delta\alpha})e^{i(phase)}$$

$$E = \sqrt{N_{steps}} \cdot \Delta\alpha = \sqrt{\frac{Q_c}{Q_a}} \cdot g\theta B Q_a$$

$$\rightarrow \quad P_{signal} \sim E^2 V \cdot \frac{m}{Q_c} \propto Q_a$$

Can also view as detecting the power in a small but highly coherent fraction of the axion broadcast bandwidth

Chaudhuri, et.al
arXiv:1803.01627



$$\theta_{frac} = \sqrt{\frac{Q_a}{Q_c} \cdot \frac{2\rho}{m_a}}$$

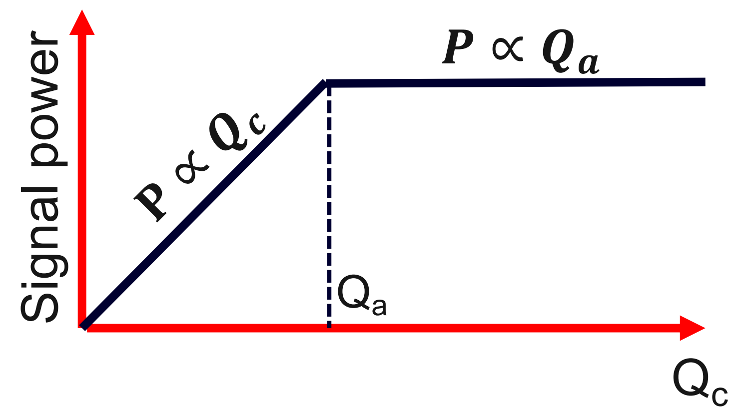
$$\rightarrow \; P_{signal} \sim E^2 V \cdot \frac{m}{Q_c} = (g\theta_{frac} B Q_c)^2 \; V \cdot \frac{m}{Q_c} \propto Q_a$$

Require cavity coherence time T2 > axion coherence time T_a.
Cavity frequency can drift as long as it stays within the axion signal band on the time scale of the accumulation time = cavity lifetime T1.

Accumulate signal at the resulting constant power plateau

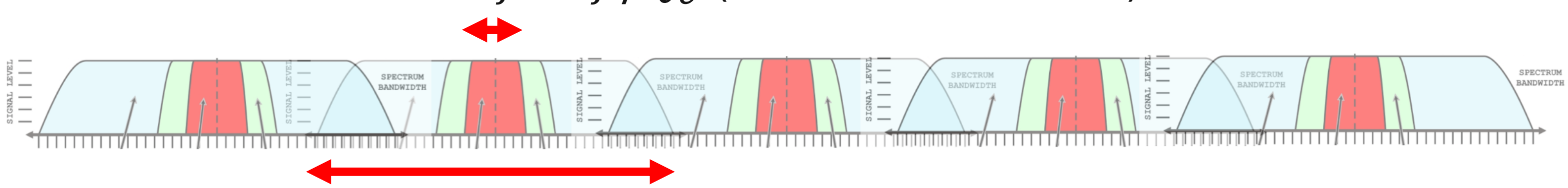
$$E = \sqrt{N_{steps}} \cdot \Delta\alpha = \sqrt{\frac{Q_c}{Q_a}} \cdot g\theta B Q_a$$

$$\rightarrow \quad P \sim E^2 V \cdot \frac{m}{Q_c} \propto Q_a$$



Need only one “sample point” per axion linewidth, i.e. Q_a cavity tunings per octave.

$\Delta f = f/Q_c$ (detection bandwidth)



f/Q_a (axion linewidth = tuning step size)

For a fixed total run time $t_{tot} \sim 1$ year, the allocated time per tuning is:

$$t_{tune} = t_{tot}/Q_a$$

$$SNR = \frac{P_{signal}}{PSD_{noise}\Delta f}\sqrt{2\Delta f \cdot t_{tune}} \propto P_{signal} \cdot \sqrt{\frac{t_{tune}}{\Delta f}} \propto \sqrt{Q_a Q_c}$$

Increases for arbitrarily large Q_c !

Stimulated emission

Axion haloscopes are classically-driven quantum harmonic oscillators

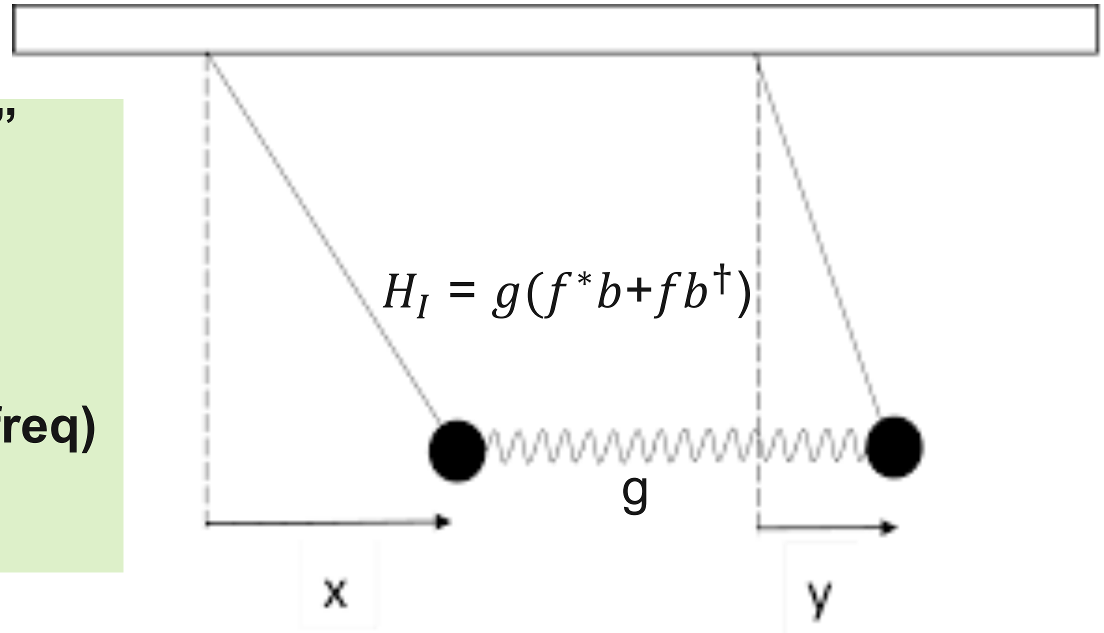
"Real world values"

$|\alpha\rangle$ **= 10¹¹**

$\frac{\omega}{2\pi}=$ **10^{10} Hz**

$\frac{2g}{2\pi}=$ **10^{-5} Hz (beat freq)**

$t_{coherence}$ = 10^{-4} s



f(ω_1)=classical sine wave.

Look for resonant transfer of power at $\omega_1 = \omega_2$ due to 2-mode mixing at beat frequency = 2g

Can we get enhanced Power = (Force × velocity) if we start the detection pendulum swinging with some initial velocity?

Pushing the swing at the wrong phase

Good!

Waiting…

Oops

Power = $\overrightarrow{Force} \cdot \overrightarrow{velocity}$

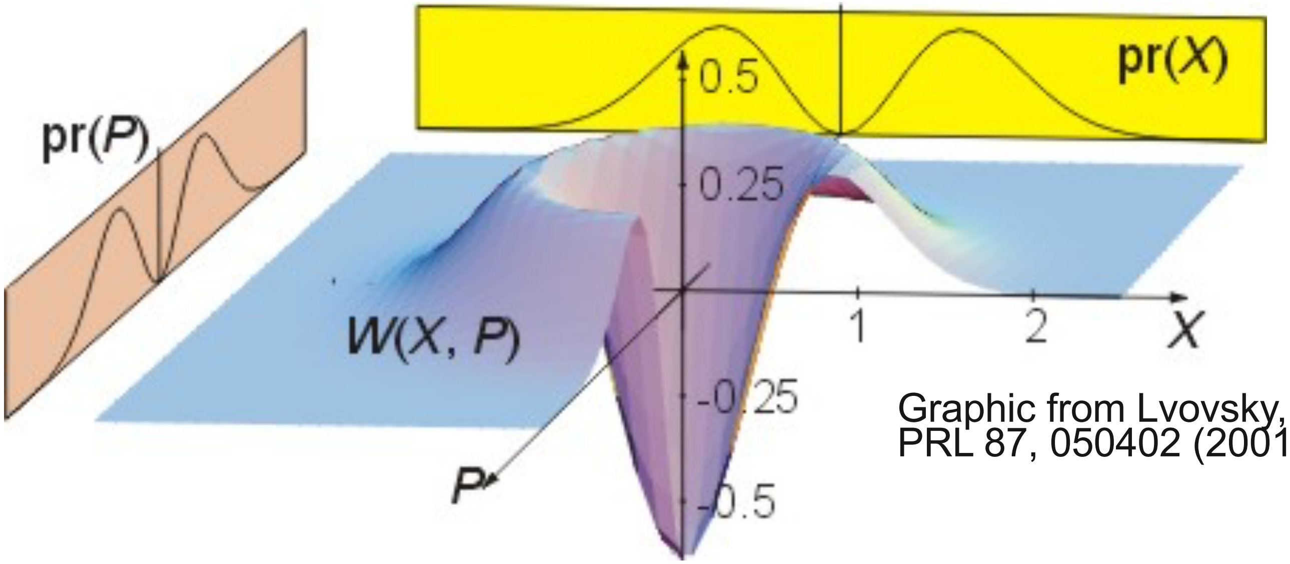
Phase offset determines the direction of energy flow.

But the axion wave is a coherent state of unknown phase which changes every millisecond… How do we prepare the cavity oscillator???

Solution: Prepare the cavity in a Fock state which has definite photon number but maximally indeterminate phase

Schrodinger's cat state of the cavity mode in a symmetric superposition of all possible oscillation phases. Just like the vacuum state, this Fock state will respond equally well to any arbitrary phase of the axion wave…**and it has no Poisson noise!**

N=1 Fock state

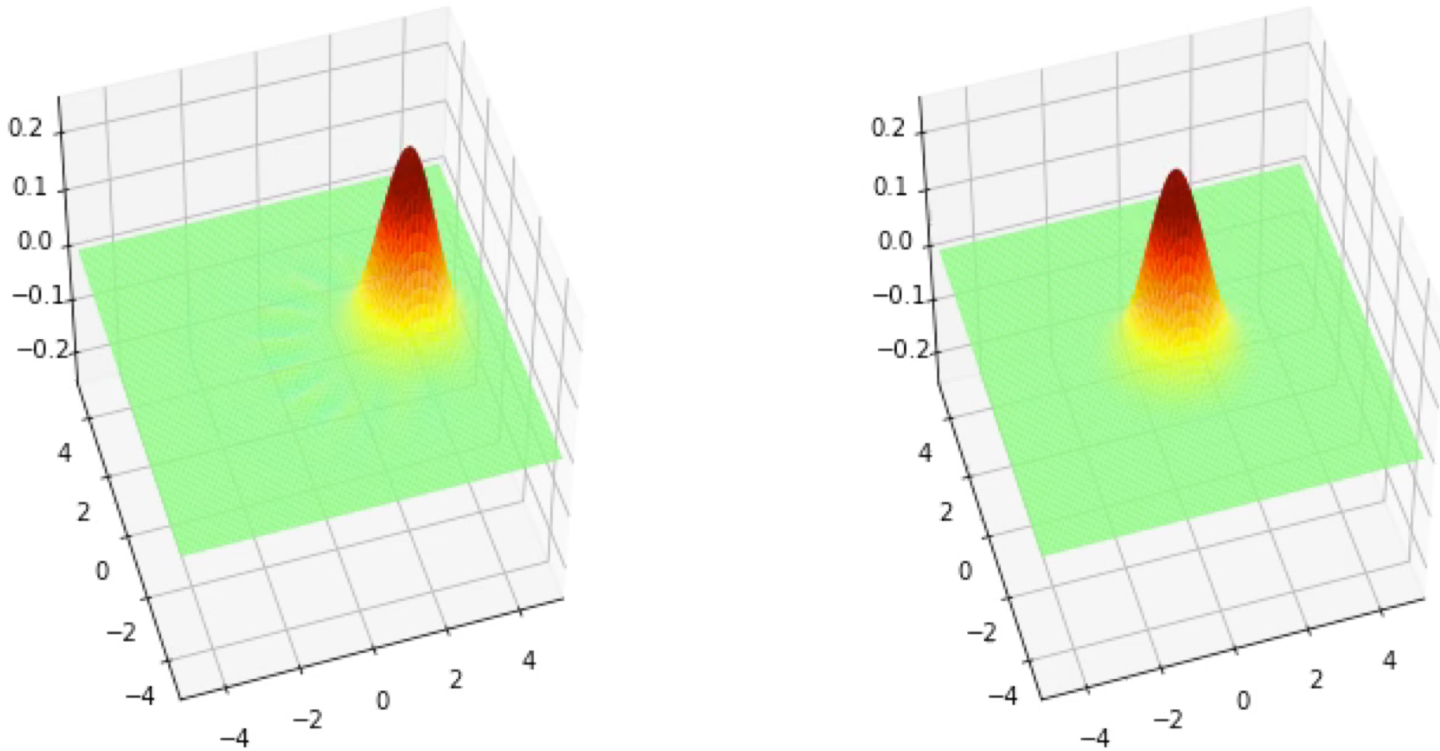


Graphic from Lvovsky, et.al, PRL 87, 050402 (2001)

Think of the phase space distribution as standing waves inside a 2-dimensional potential well. **Wigner distributions are Laguerre-Gauss functions.**

Classical pendulum system: $|\alpha = 3\rangle \otimes |\alpha = 0\rangle$

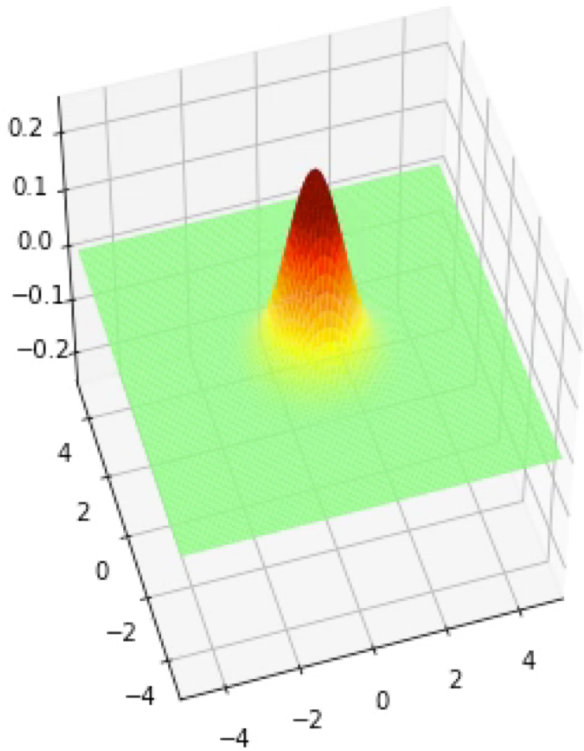
Time evolution of Wigner distributions in X-P "phasor" space

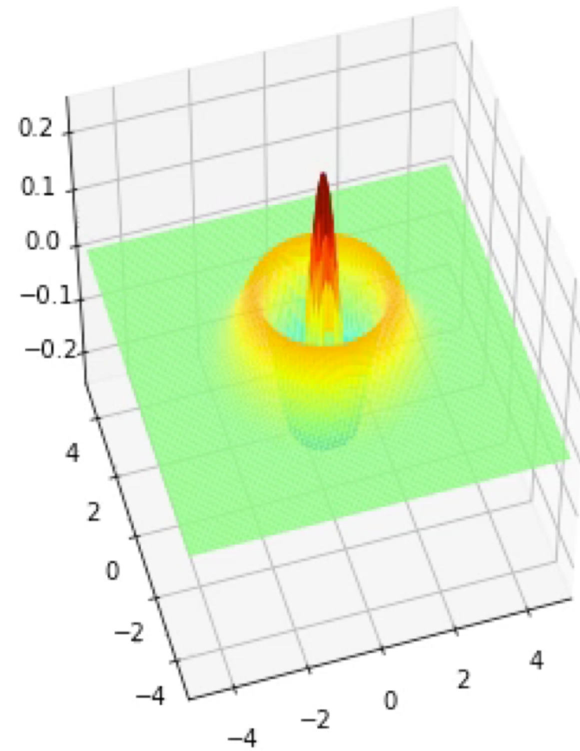


The two pendula swap their coherent states.

Simulated with QuTIP

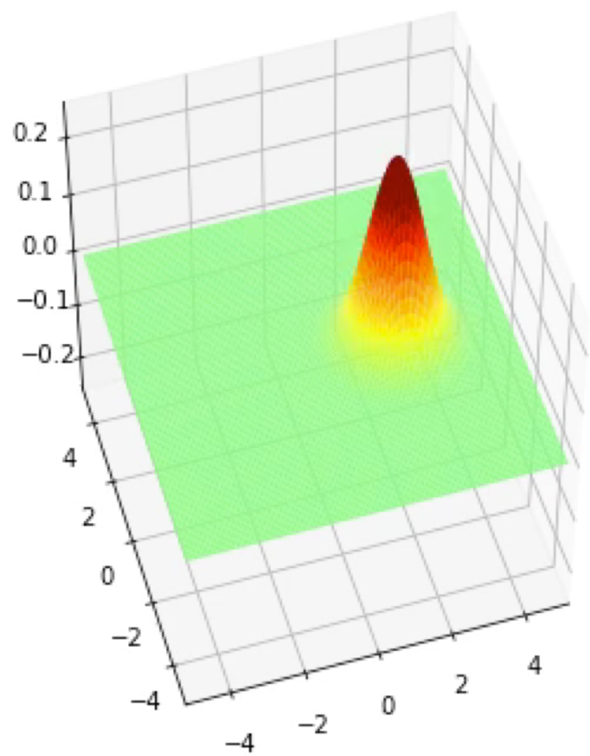
$$|\alpha = 0\rangle \otimes |N = 2\rangle$$

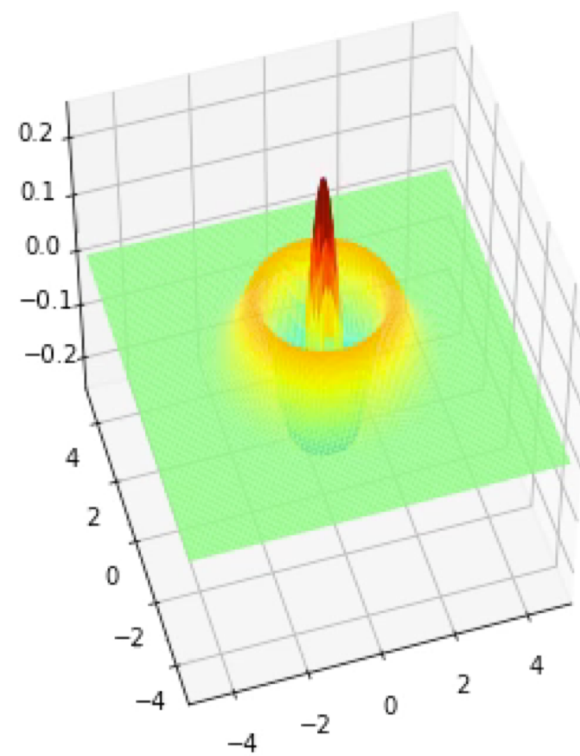




Vacuum and Fock states swap places

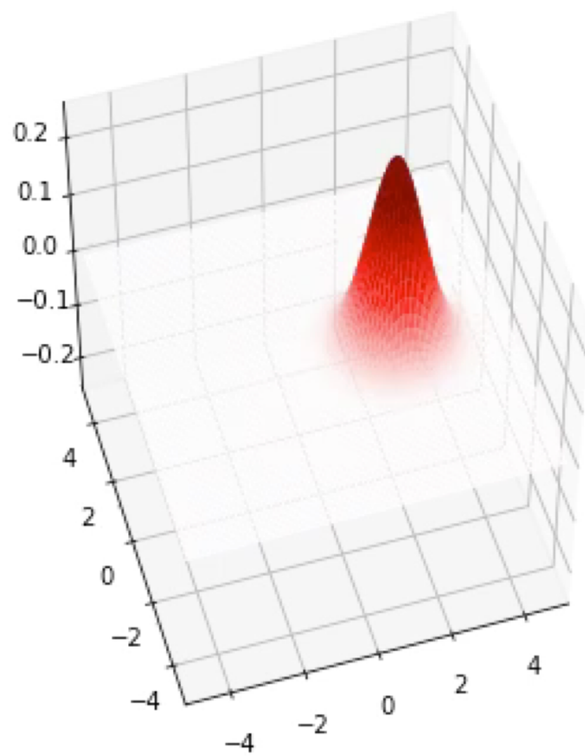
$$|\alpha = \sqrt{2}\rangle \otimes |N = 2\rangle$$

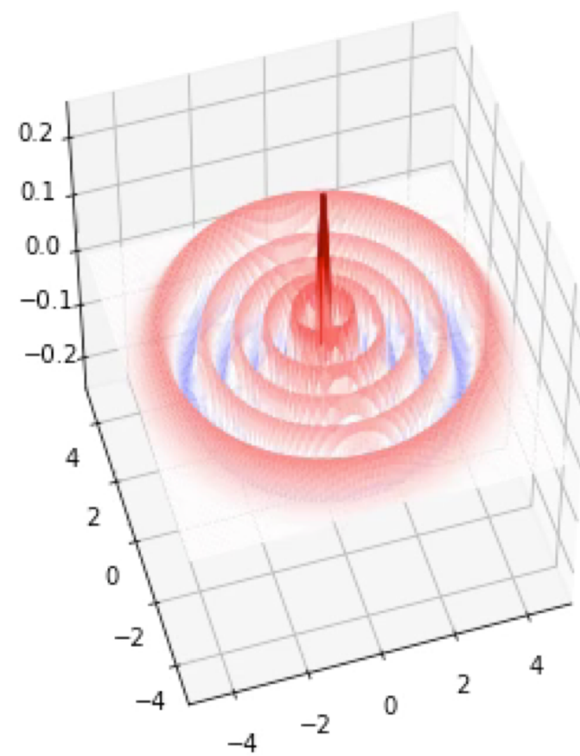




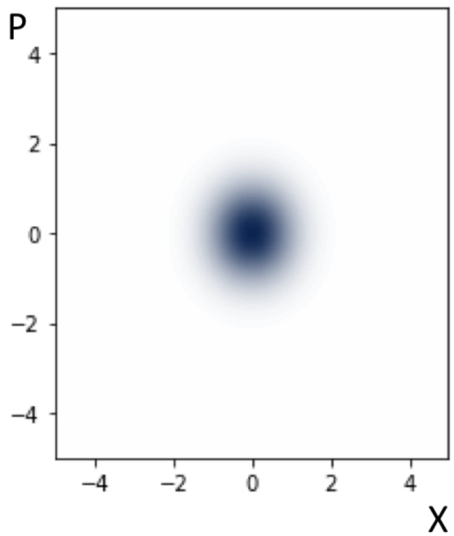
Coherent and Fock states swap places

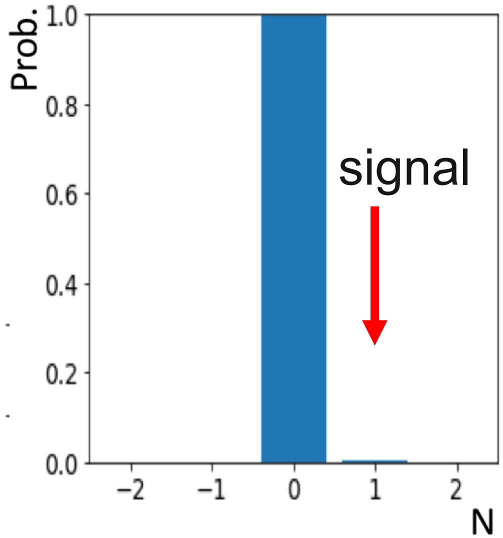
$$|\alpha = \sqrt{2}\rangle \otimes |N = 10\rangle$$

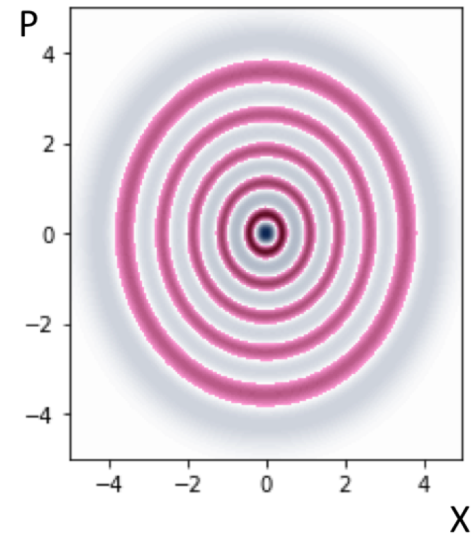


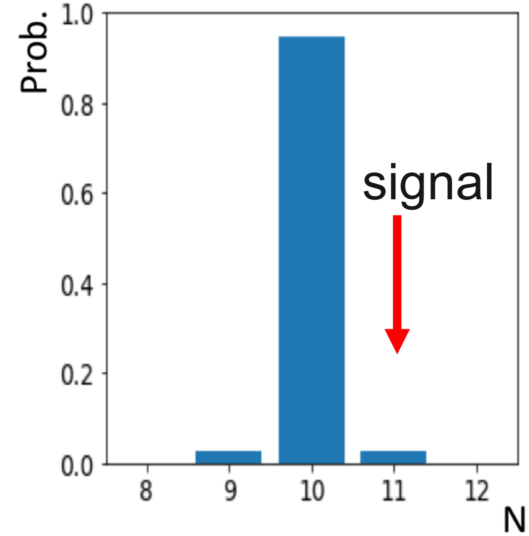


Spontaneous vs Stimulated Emission









Cavity prepared in vacuum state N=0. Interaction with axions displaces this from the origin.

Spontaneous emission gives small population of N=1 state

Cavity prepared in N=10 Fock state. Displacement due to axions moves some components to smaller radius and some components to larger radius, corresponding to stimulated absorption and stimulated emission, respectively

Factor of 10 in transition rates gives much larger signal in the N=11 state

Again, use qubit QND photon counting to measure the final state Fock number.

Cavity Fock-state preparation gives stimulated emission/absorption between axion and cavity modes

Interaction Hamiltonian for direct product of 2 bosonic modes:

$$H_I = g(a^\dagger b + a b^\dagger)$$

Start with axion coherent state displacement α and cavity photon Fock number N:

$$\langle \alpha, N+1 | H_I | \alpha, N \rangle = g\alpha\sqrt{N+1}$$

Matrix element is enhanced by factor $\sqrt{N+1}$ ($\propto$ oscillator velocity)

Signal rate in linear mixing regime is enhanced by factor $N+1$
]
Faster exchange of quanta to/from axion wave via stimulated emission/absorption.

- Laser: stimulated emission rapidly erases a population inversion in a lasing medium to reach a Boltzmann equilibrium between atom and photon states

- Axion cavity: stimulated absorption/emission more rapidly smears out the initial "delta function" Fock state of the cavity.
  - **Also need higher Q_c cavity resonator since $Q_c \rightarrow Q_c/(N+1)$ and thus increases the noise bandwidth**

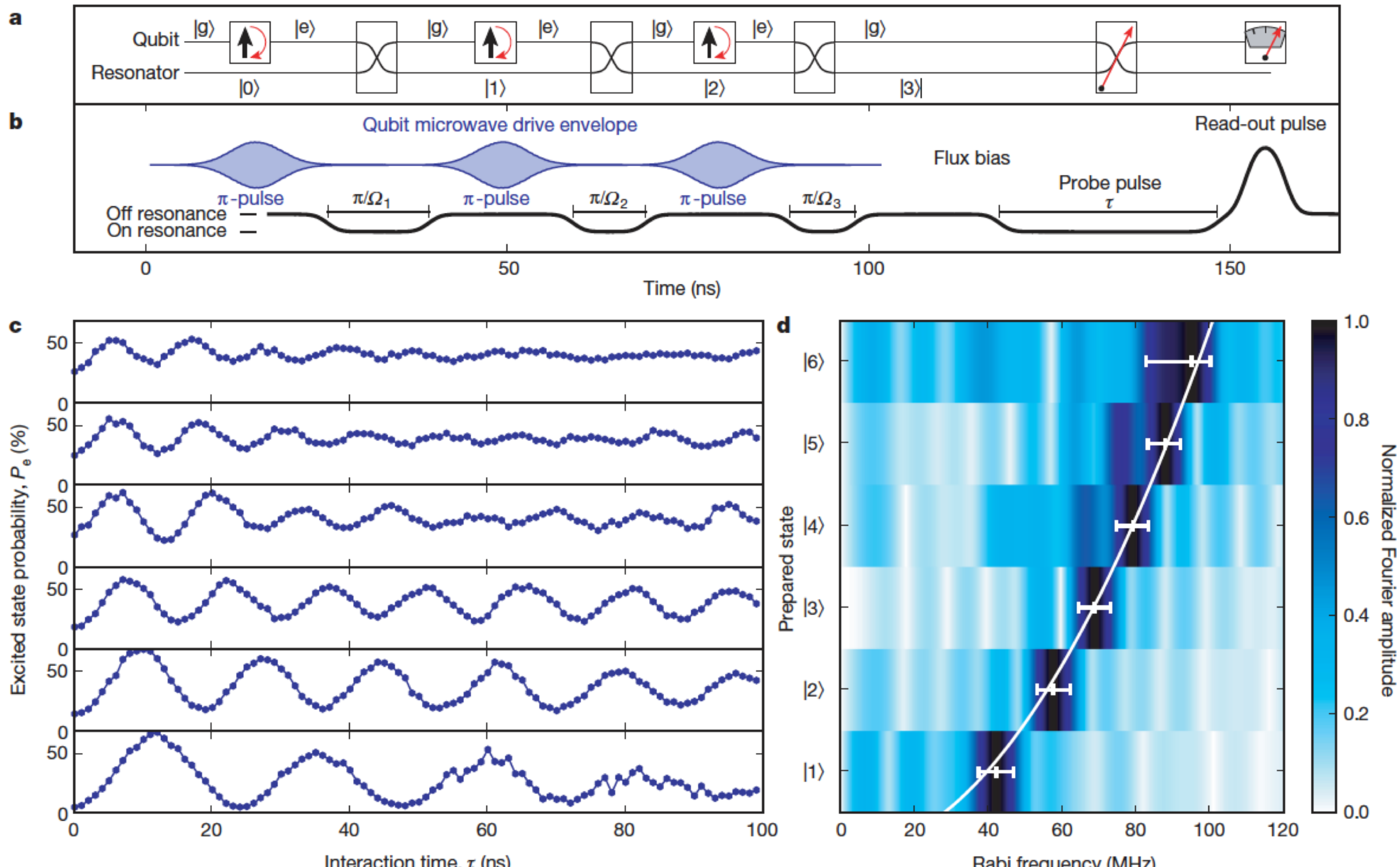
nature | Vol 454 | 17 July 2008 | doi:10.1038/nature07136

LETTERS

Generation of Fock states in a superconducting quantum circuit

Max Hofheinz[1], E. M. Weig[1]†, M. Ansmann[1], Radoslaw C. Bialczak[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], H. Wang[1], John M. Martinis[1] & A. N. Cleland[1]

Loading one quantum at a time using swept-frequency qubits as buckets



Signal/Noise scaling with Fock enhancement

$$\mathrm{SNR}_{\mathrm{SPD,Qc/(Nfock+1)>Qa}} = \frac{R_s t_{\mathrm{tune}}}{\sqrt{R_b t_{\mathrm{tune}}}}$$

$$\propto \frac{Q_a (N_{\mathrm{Fock}}+1) \times 1/Q_a}{\sqrt{p_{\mathrm{err}}((N_{\mathrm{Fock}}+1)/Q_c) \times (1/Q_a)}}$$

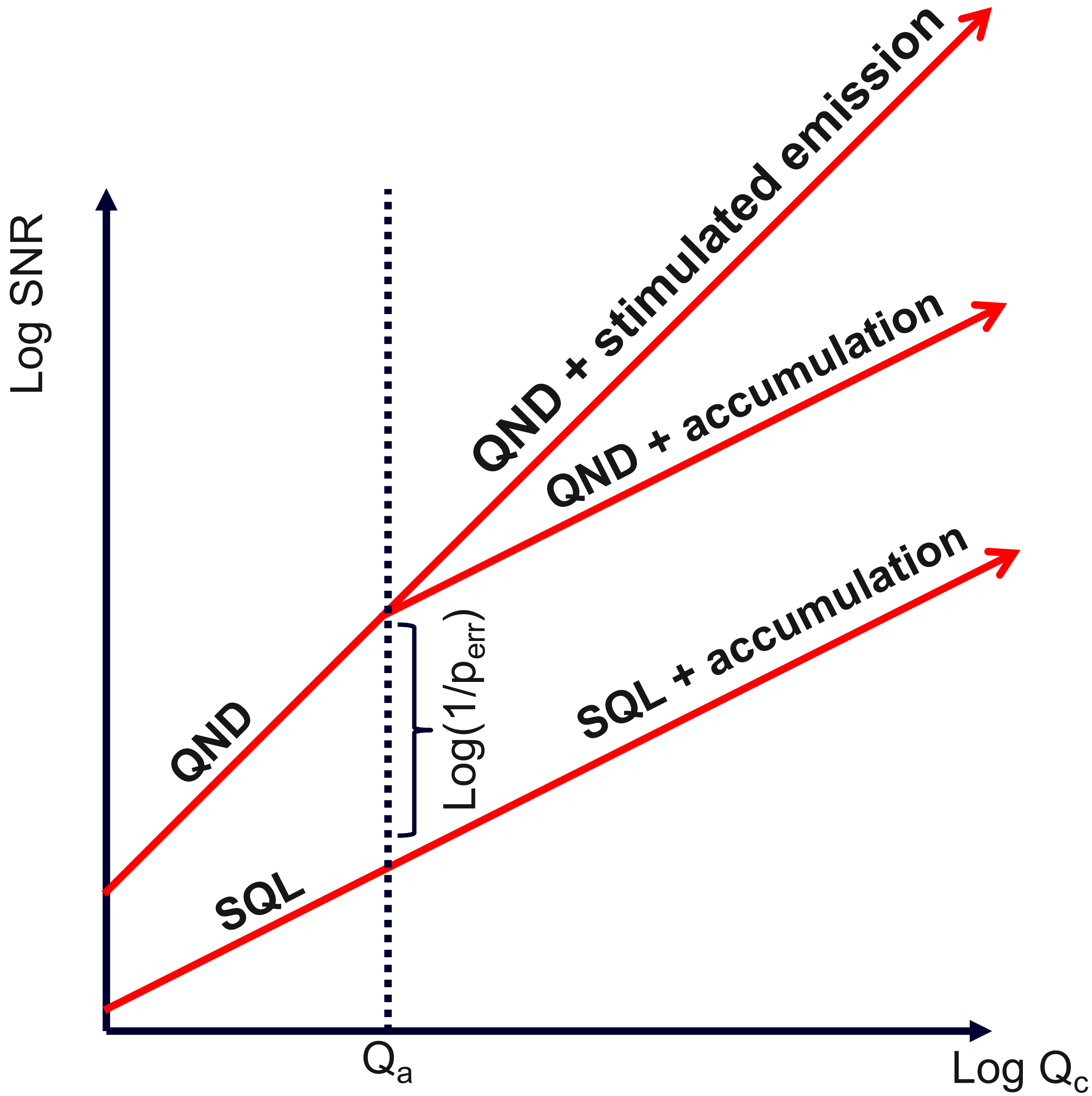
$$\propto \sqrt{\frac{Q_a Q_c (N_{\mathrm{Fock}}+1)}{p_{\mathrm{err}}}}$$

Set Fock state lifetime = axion coherence time: $N_{\mathrm{Fock}}^{\mathrm{optimal}} = \frac{Q_c}{Q_a} - 1$

$$\rightarrow \quad SNR \propto \frac{Q_c}{\sqrt{p_{err}}}$$

Linear scaling with Q_c!!!

Roadmap



Log SNR
QND + stimulated emission
QND + accumulation
SQL + accumulation
QND
SQL
Log(1/p_err)
Q_a
Log Q_c

Take-aways

Axion searches do not need to be dismally slow.
A next-generation experiment should employ many haloscopes. Why operate a single pixel photodiode when one could operate a CCD? Each haloscope should use identical technology but be responsible for different frequency ranges

The individual haloscopes should be cheap, easy to operate and low-risk: Commercial dilution refrigerator + 14T solenoid ~$1M

Multi-QND single photon counting by itself could enable sensitivity to DFSZ axions with this setup, but signal is shot noise-limited.

Higher-Q cavities will:

- Make qubit- and atom-based QND much easier
- Enable accumulation of DM signal over long times to reduce the noise bandwidth
- Enable stimulated enhancement of axion-photon transition rates

New Consortium: Quantum Metrology for Dark Matter Axion Detection

- Aaron Chou, Daniel Bowring (FNAL), David Schuster(Chicago): Qubit QND sensors
- Konrad Lehnert(Colorado/JILA): Stimulated emission, photon transport
- Reina Maruyama (Yale): Rydberg atom single photon detection
- You?