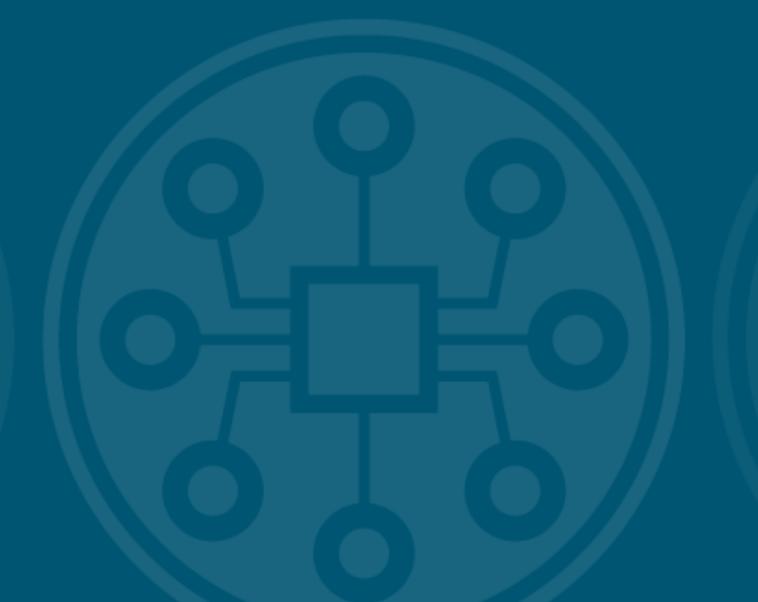
# Axion Detection with Precision Frequency Metrology



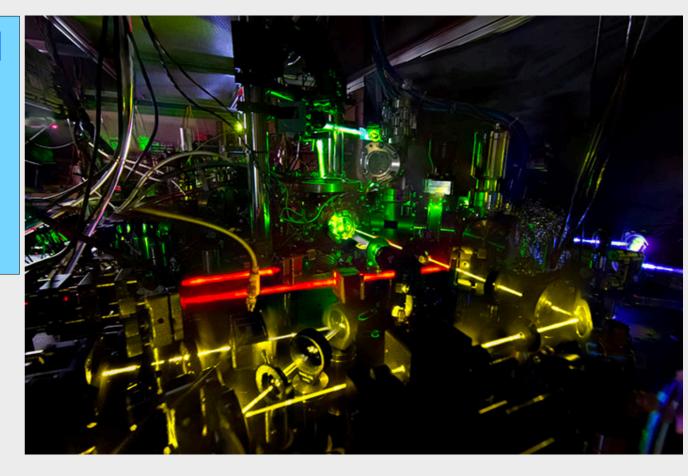


Maxim Goryachev
Ben McAllister
Mike Tobar



## Why Frequency Metrology?

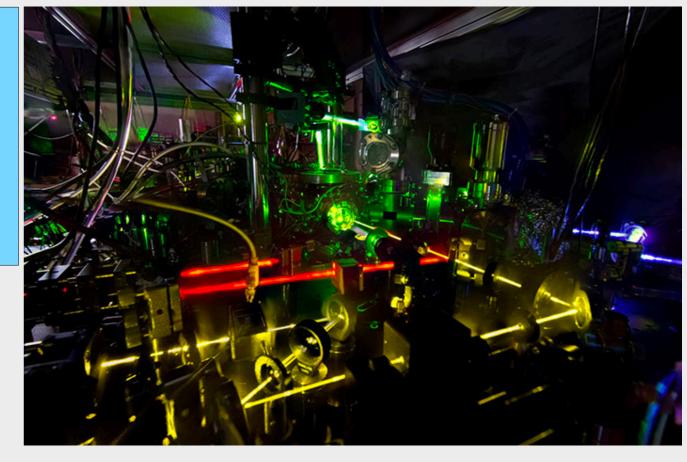
Frequency and time can be controlled with a nearly **IO-I8** accuracy, meaning that such a clock created at the time of the big bang would be today accurate within I second.





## Why Frequency Metrology?

Frequency and time can be controlled with a nearly 10-18 accuracy, meaning that such a clock created at the time of the big bang would be today accurate within 1 second.



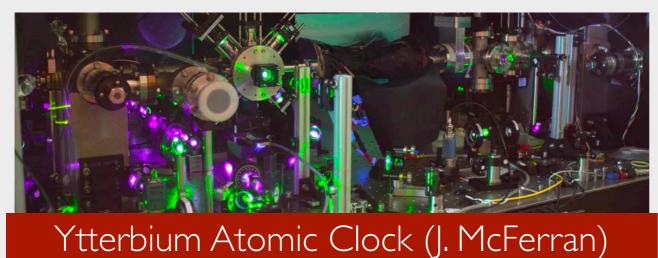


In the same time, accuracy of Josephson Junction voltage standards achieve only I 0-8 uncertainty level

It seems that frequency is a better "observable" than voltage/amplitude

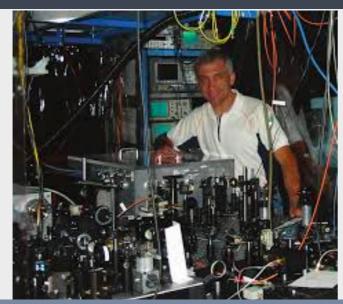


## Frequency Metrology @ UWA





Ultra-Stable Acoustic Oscillators





Cryogenic Sapphire Oscillators with frequency stability of **IO-16** 



Ultra-Stable Frequency Transfer (S. Schediwy)



Low Phase Noise Measurements (E. Ivanov)

## Frequency Metrology for Fundamental Physics (UWA)



High Sensitivity Gravitational Wave Antenna with Parametric Transducer Readout

Phys. Rev. Lett. **74**, 1908 – Published 13 March 1995

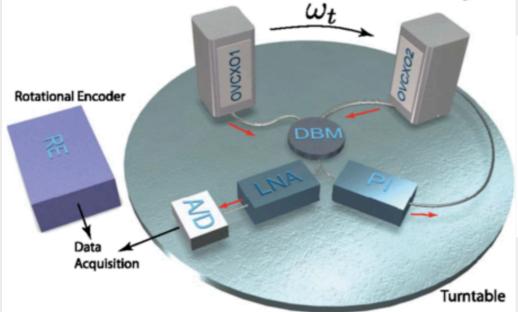
A high- Q niobium resonant mass gravitational radiation antenna with a superconducting parametric transducer and noncontacting readout is shown to achieve a noise temperature of about 2 mK using a zero order predictor filter.

## Direct terrestrial test of Lorentz symmetry in electrodynamics to 10<sup>-18</sup> DOI: 10.1038/ncomms9174

Here we use ultrastable oscillator frequency sources

to perform a modern Michelson-Morley experiment and make the most precise direct terrestrial test to date of Lorentz symmetry for the photon, constraining Lorentz violating orientation-dependent relative frequency changes  $\Delta v/v$  to  $9.2 \pm 10.7 \times 10^{-19}$  (95%) confidence interval). This order of magnitude improvement over previous Michelson-Morley experiments allows us to set comprehensive simultaneous bounds on nine boost and rotation anisotropies of the speed of light, finding no significant violations of Lorentz symmetry.





Acoustic Tests of Lorentz Symmetry Using Quartz Oscillators Phys. Rev. X 6, 011018 - Published 24 February 2016

realization of such a "phonon-sector" test of Lorentz symmetry using room-temperature stresscompensated-cut crystals yields 120 h of data at a frequency resolution of  $2.4 \times 10^{-15}$  and a limit of  $ilde{c}_{O}^{\,n}=(-1.8\pm2.2) imes10^{-14}~{
m GeV}$  on the most weakly constrained neutron-sector c coefficient of the standard model extension. Future experiments with cryogenic oscillators promise significant improvements in accuracy, opening up the potential for improved limits on Lorentz violation in the neutron, proton, electron, and photon sector.



### Frequency Metrology in Paraphoton Detection

#### New alternative to Light Shining through a Wall

PHYSICAL REVIEW D 87, 115008 (2013)

#### **Hidden sector photon coupling of resonant cavities**

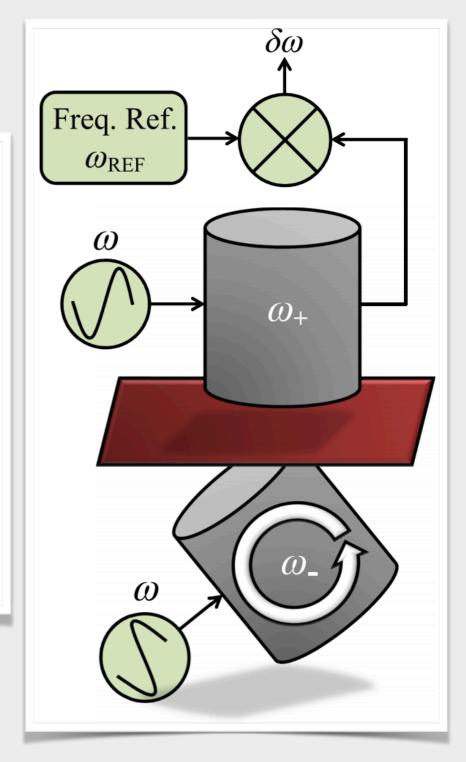
Stephen R. Parker,<sup>1,\*</sup> Gray Rybka,<sup>2</sup> and Michael E. Tobar<sup>1</sup>

<sup>1</sup>School of Physics, The University of Western Australia, Crawley 6009, Australia

<sup>2</sup>University of Washington, Seattle, Washington 98195, USA

(Received 25 April 2013; published 7 June 2013)

Many beyond the standard model theories introduce light paraphotons, a hypothetical spin-1 field that kinetically mixes with photons. Microwave cavity experiments have traditionally searched for paraphotons via transmission of power from an actively driven cavity to a passive receiver cavity, with the two cavities separated by a barrier that is impenetrable to photons. We extend this measurement technique to account for two-way coupling between the cavities and show that the presence of a paraphoton field can alter the resonant frequencies of the coupled cavity pair. We propose an experiment that exploits this effect and uses measurements of a cavity's resonant frequency to constrain the paraphoton-photon mixing parameter  $\chi$ . We show that such an experiment can improve the sensitivity to  $\chi$  over existing experiments for paraphoton masses less than the resonant frequency of the cavity, and that it can eliminate some of the most common systematics for resonant cavity experiments.



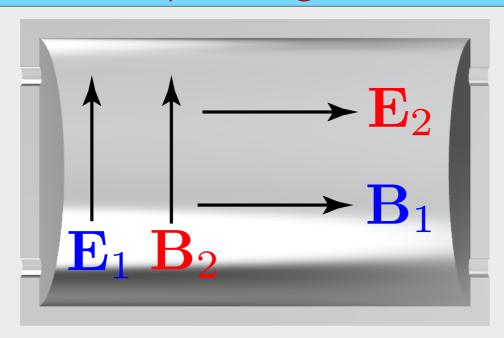
$$\omega_{\pm} \approx \omega_{0} \left( \frac{1}{1 - \frac{x^{2}}{2}} \left( 1 + \frac{1}{2Q_{1}Q_{2}} + \frac{x^{2}}{4} + \frac{m_{\gamma \prime}^{2} \chi^{2}}{\omega_{0}^{2}} - \frac{m_{\gamma \prime}^{4} \chi^{2} G_{S}}{\omega_{0}^{4}} \right) \pm \left( \frac{1}{Q_{1}Q_{2}} + x^{2} + \frac{2m_{\gamma \prime}^{2} x^{2} \chi^{2}}{\omega_{0}^{2}} - \frac{2m_{\gamma \prime}^{4} x^{2} \chi^{2} G_{S}}{\omega_{0}^{4}} + \frac{m_{\gamma \prime}^{8} \chi^{4} G}{\omega_{0}^{8}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}},$$

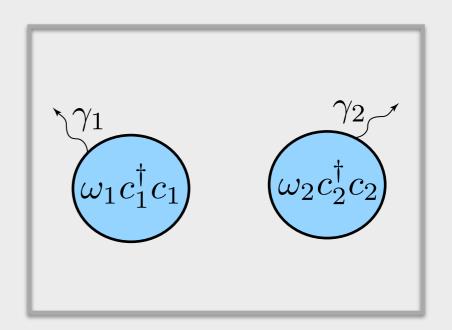


## System for Axion Detection

photonic cavity with two mutually orthogonal modes

optical or microwave





Axion Electrodynamics

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} a)^2 - \frac{1}{2} m_a^2 a^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Hamiltonian Density

$$\mathcal{H} = \mathcal{H}_{\mathrm{EM}} + \mathcal{H}_a + \mathcal{H}_{\mathrm{int}}$$

$$\mathcal{H}_{\rm EM} = \frac{\varepsilon_0}{2} \left[ \mathbf{E}^2 + c^2 \mathbf{B}^2 \right] \qquad \mathcal{H}_{\rm a} = \frac{\phi^2}{2m_a} + V(\theta)$$

$$\mathcal{H}_{\mathrm{a}} = \frac{\phi^2}{2m_a} + V(\theta)$$

axion

$$\mathcal{H}_{\text{int}} = \varepsilon_0 c g_{a\gamma\gamma} \theta \ \mathbf{E} \cdot \mathbf{B}$$

interaction



normal ED

arXiv:1806.07141

based on axion Electrodynamics we derive axion induced coupling between two cavity modes

$$\mathbf{E}_{n}(\mathbf{r}) = -\frac{1}{\varepsilon_{0}} \Pi_{n} \mathbf{u}_{n}(\mathbf{r}) = i E_{V,n} (c_{n} - c_{n}^{\dagger}) \mathbf{e}_{n}(\mathbf{r}),$$

$$E_{V,n} = \sqrt{\frac{\hbar \omega_{n}}{2\varepsilon_{0} V_{n}}}$$

$$\mathbf{B}_{n}(\mathbf{r}) = A_{i} \nabla \times \mathbf{u}_{n}(\mathbf{r}) = \frac{1}{\varepsilon} E_{V,n} (c_{n} + c_{n}^{\dagger}) \mathbf{b}_{n}(\mathbf{r}),$$

$$E_{V,n} = \sqrt{\frac{\hbar \omega_{n}}{2\varepsilon_{0} V_{n}}}$$

$$H_{\text{int}} = i\hbar g_{\text{eff}} \theta \left[ \xi_{-} (c_1 c_2^{\dagger} - c_1^{\dagger} c_2) + \xi_{+} (c_1^{\dagger} c_2^{\dagger} - c_1 c_2) \right]$$

Dimensionless Orthogonality Form Factors

$$\xi_{1} = \frac{1}{\sqrt{V_{1}V_{2}}} \int_{V} d^{3}r(\mathbf{e}_{1} \cdot \mathbf{b}_{2}),$$

$$\xi_{2} = \frac{1}{\sqrt{V_{1}V_{2}}} \int_{V} d^{3}r(\mathbf{e}_{2} \cdot \mathbf{b}_{1}).$$

$$\xi_{\pm} = \xi_{1} \pm \xi_{2}$$

**Effective Coupling** 

$$g_{\text{eff}} = \frac{g_{a\gamma\gamma}}{2} \sqrt{\omega_1 \omega_2}$$



based on axion Electrodynamics we derive axion induced coupling between two cavity modes

$$H_{\text{int}} = i\hbar g_{\text{eff}} \theta \left[ \xi_{-} (c_1 c_2^{\dagger} - c_1^{\dagger} c_2) + \xi_{+} (c_1^{\dagger} c_2^{\dagger} - c_1 c_2) \right]$$

#### Dimensionless Orthogonality Form Factors

$$\xi_1 = \frac{1}{\sqrt{V_1 V_2}} \int_V d^3 r(\mathbf{e}_1 \cdot \mathbf{b}_2),$$

$$\xi_2 = \frac{1}{\sqrt{V_1 V_2}} \int_V d^3 r(\mathbf{e}_2 \cdot \mathbf{b}_1).$$

$$\xi_{\pm} = \xi_1 \pm \xi_2$$

#### Rotating Wave Approximation

allows optical search at microwaves and mm-wave

allows microwave search at mm-wave

#### **Axion UpConversion**

$$\omega_a = \omega_2 - \omega_1$$

$$H_{\mathrm{U}} = i\hbar g_{\mathrm{eff}} \xi_{-} (a^* c_1 c_2^{\dagger} - a c_1^{\dagger} c_2)$$

beam splitter

#### **Axion DownConversion**

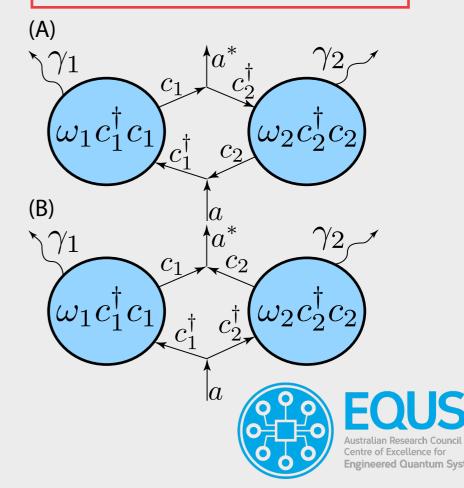
$$\omega_a = \omega_2 + \omega_1$$

$$H_{\rm D} = i\hbar g_{\rm eff} \xi_+ (ac_1^{\dagger} c_2^{\dagger} - a^* c_1 c_2)$$

parametric amplification

#### **Effective Coupling**

$$g_{\text{eff}} = \frac{g_{a\gamma\gamma}}{2} \sqrt{\omega_1 \omega_2}$$



arXiv:1806.07141

$$H_{\rm U} = i\hbar g_{\rm eff} \xi_{-} (a^* c_1 c_2^{\dagger} - a c_1^{\dagger} c_2)$$

 $H_{\rm D} = i\hbar g_{\rm eff} \xi_+ (ac_1^{\dagger} c_2^{\dagger} - a^* c_1 c_2)$ 

beam splitter

parametric amplification

Experimental Approaches



$$H_{\rm U} = i\hbar g_{\rm eff} \xi_{-} (a^* c_1 c_2^{\dagger} - a c_1^{\dagger} c_2)$$

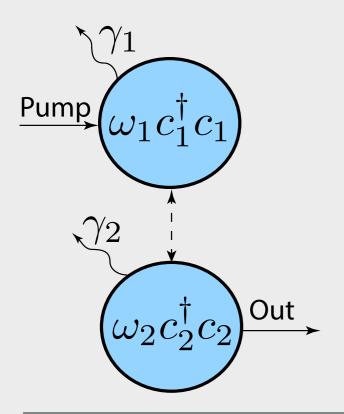
 $H_{\rm D} = i\hbar g_{\rm eff} \xi_+ (ac_1^{\dagger}c_2^{\dagger} - a^*c_1c_2)$ 

beam splitter

parametric amplification

#### Experimental Approaches

#### Power Detection



P. Sikivie: arXiv:1009.0762



$$H_{\rm U} = i\hbar g_{\rm eff} \xi_{-} (a^* c_1 c_2^{\dagger} - a c_1^{\dagger} c_2)$$

 $H_{\rm D} = i\hbar g_{\rm eff} \xi_+ (ac_1^{\dagger}c_2^{\dagger} - a^*c_1c_2)$ 

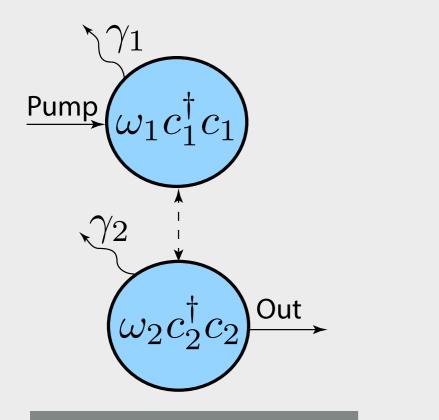
beam splitter

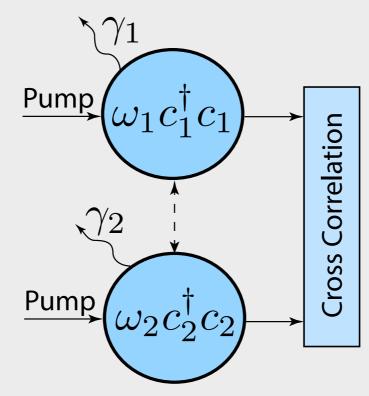
parametric amplification



#### Power Detection

#### Cross Correlation





P. Sikivie: arXiv: 1009.0762



$$H_{\rm U} = i\hbar g_{\rm eff} \xi_{-} (a^* c_1 c_2^{\dagger} - a c_1^{\dagger} c_2)$$

 $H_{\rm D} = i\hbar g_{\rm eff} \xi_+ (ac_1^{\dagger} c_2^{\dagger} - a^* c_1 c_2)$ 

beam splitter

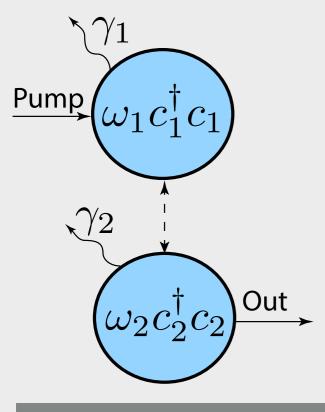
parametric amplification

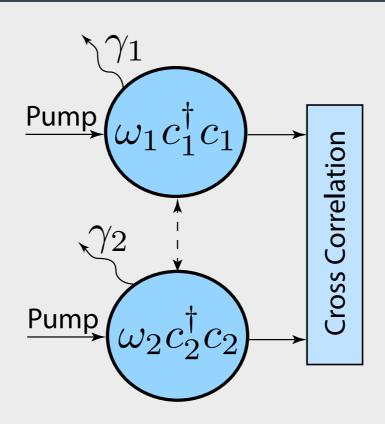
#### Experimental Approaches

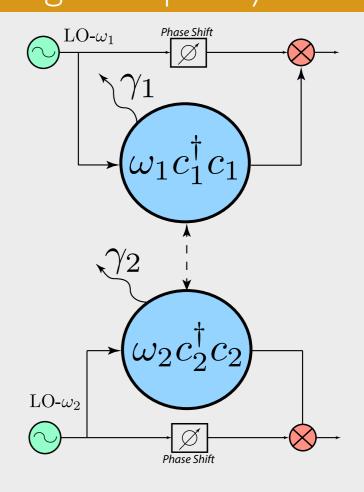
#### Power Detection

#### Cross Correlation

#### Eigenfrequency Shift







P. Sikivie: arXiv: 1009.0762



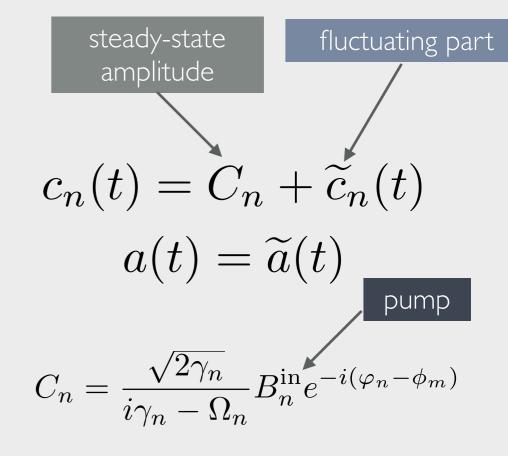
#### Perturbation Analysis

Decompose the solution into large steady state and small fluctuating parts

Solve for Steady State components neglecting fluctuating components

Linearise the EoM for small fluctuating parts around steady state solutions

Solve linear EoMs in the frequency domain

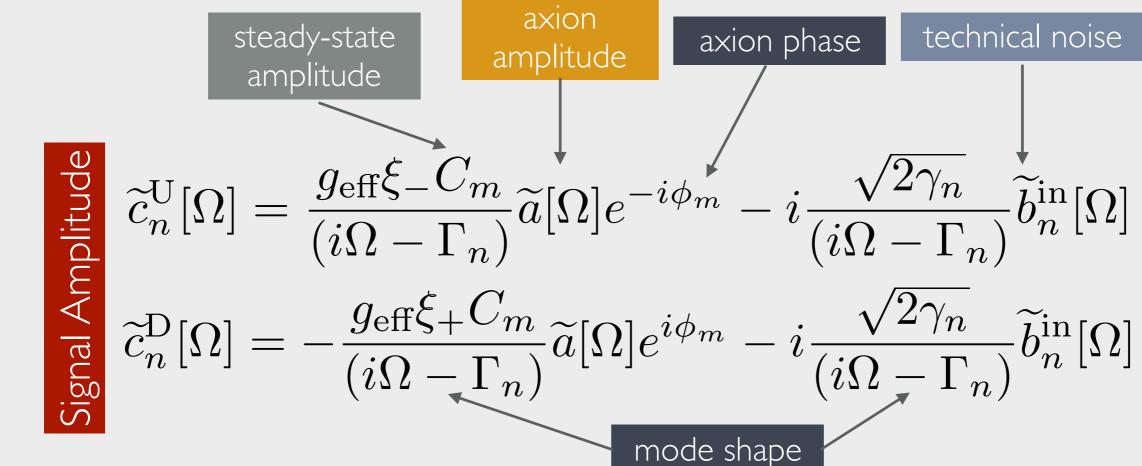


$$\frac{d}{dt}\widetilde{c}_n = -\Gamma_n\widetilde{c}_n + g_{\text{eff}}\xi_+\widetilde{a}C_m e^{-i\Delta_D t + i\phi_m}$$
$$-i\sqrt{2\gamma_n}\widetilde{b}_n^{\text{in}}$$

Note: take into account axion & signal phases



#### Power Detection and Cross Correlation



the approach gives a phase sensitive result

generalisation of the traditional DC-field Haloscope approach

potentially less sensitive ( $C_m$  vs.  $B_0$ )

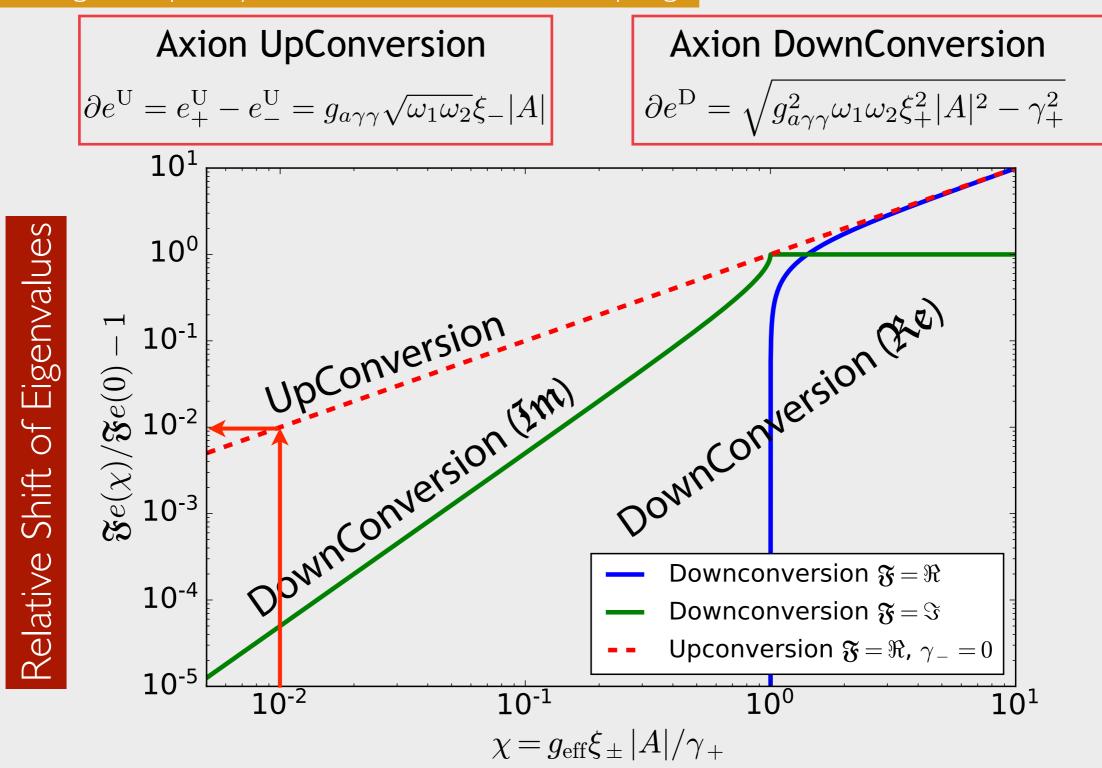
#### Crosscorrelation:

$$S_{12}^{\text{D/U}}[\Omega] = \frac{g_{\text{eff}}^2 \xi_{\pm}^2 C_1 C_2}{|(i\Omega - \Gamma_1)(i\Omega - \Gamma_2)|} e^{i(\pm \phi_2 \mp \phi_1)} S_a[\Omega]$$

arXiv:1806.07141

## Axion Induced Frequency Shifts

calculate eigenfrequency as a function of axion coupling



Normalised Axion Mediated Coupling



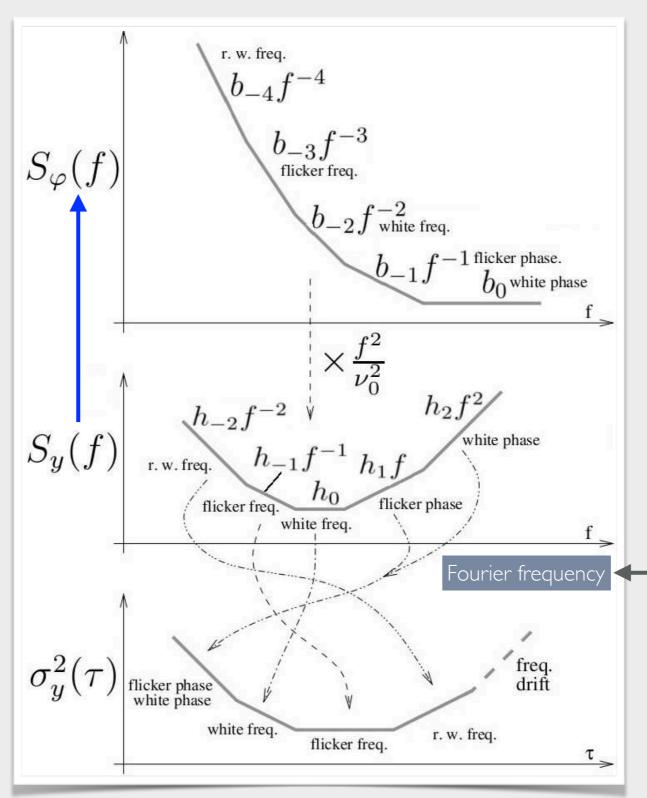
## From Frequency To Phase Fluctuations

quite often direct frequency measurements are not practical

phase fluctuations

fractional frequency fluctuations

Allan Variance



Also we want RF
measurements rather
than DC frequency shifts

Search in the Fourier
Spectrum

offset from carrier



arXiv:1806.07141

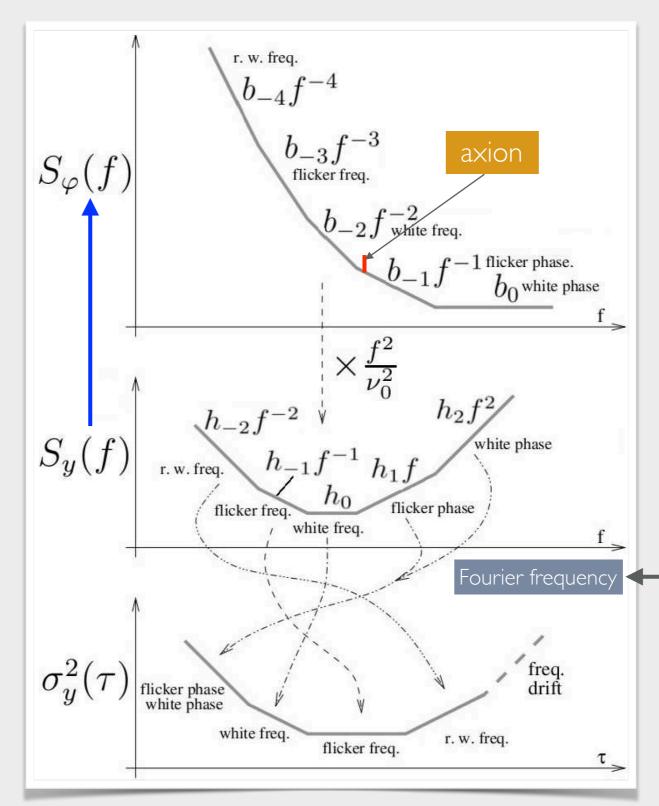
## From Frequency To Phase Fluctuations

quite often direct frequency measurements are not practical

phase noise

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Also we want RF
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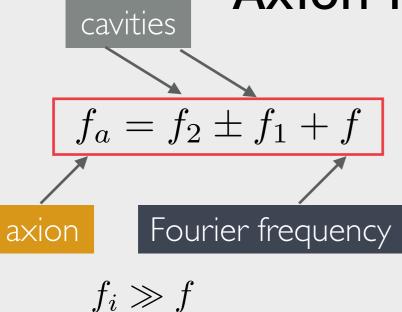
Search in the Fourier
Spectrum

offset from carrier

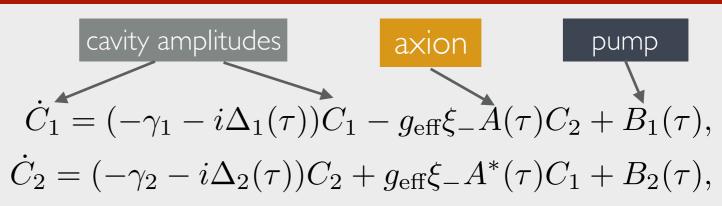


arXiv:1806.07141

### **Axion Induced Phase Fluctuations**



#### EoMs in the Rotating Frame (complex amplitudes)



#### EoMs in terms of phases and magnitudes:

$$\dot{x}_1 = -\gamma_1 x_1 - x_2 Q_c(\tau) + y_1 \cos(\theta_1 - \varphi_1),$$

$$\dot{\varphi}_1 = \Delta_1(\tau) - \frac{x_2}{x_1} Q_s(\tau) + \frac{y_1}{x_1} \sin(\theta_1 - \varphi_1),$$

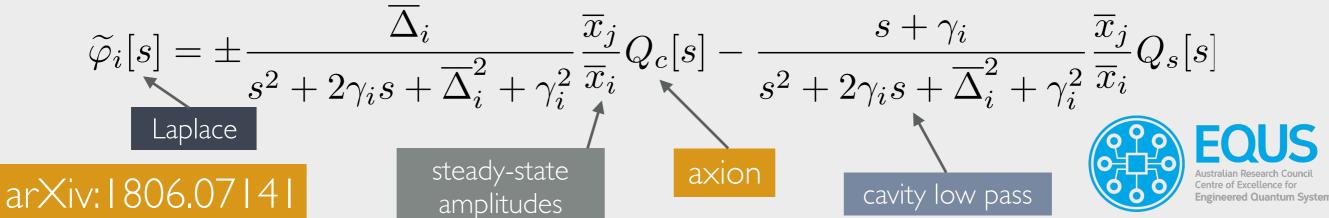
$$\dot{x}_2 = -\gamma_2 x_2 + x_1 Q_c(\tau) + y_2 \cos(\theta_2 - \varphi_2),$$

$$\dot{\varphi}_2 = \Delta_2(\tau) - \frac{x_1}{x_2} Q_s(\tau) + \frac{y_2}{x_2} \sin(\theta_2 - \varphi_2),$$

#### Axion quadratures:

$$\begin{aligned} Q_c(\tau) &= g_{\text{eff}} \xi_- |A| \cos(\theta_a(\tau) + \varphi_2 - \varphi_1), \\ Q_s(\tau) &= g_{\text{eff}} \xi_- |A| \sin(\theta_a(\tau) + \varphi_2 - \varphi_1), \\ \end{aligned}$$
 phase sensitivity

#### can be solved for steady-state and small fluctuation:



#### Axion Induced Phase Fluctuations

Cavity phase noise measurements

 $f_a = f_2 \pm f_1 + f$  $f_i \gg f$ 

basic phase noise

technical noise

Spectrum of Cavity Phase Fluctuations:

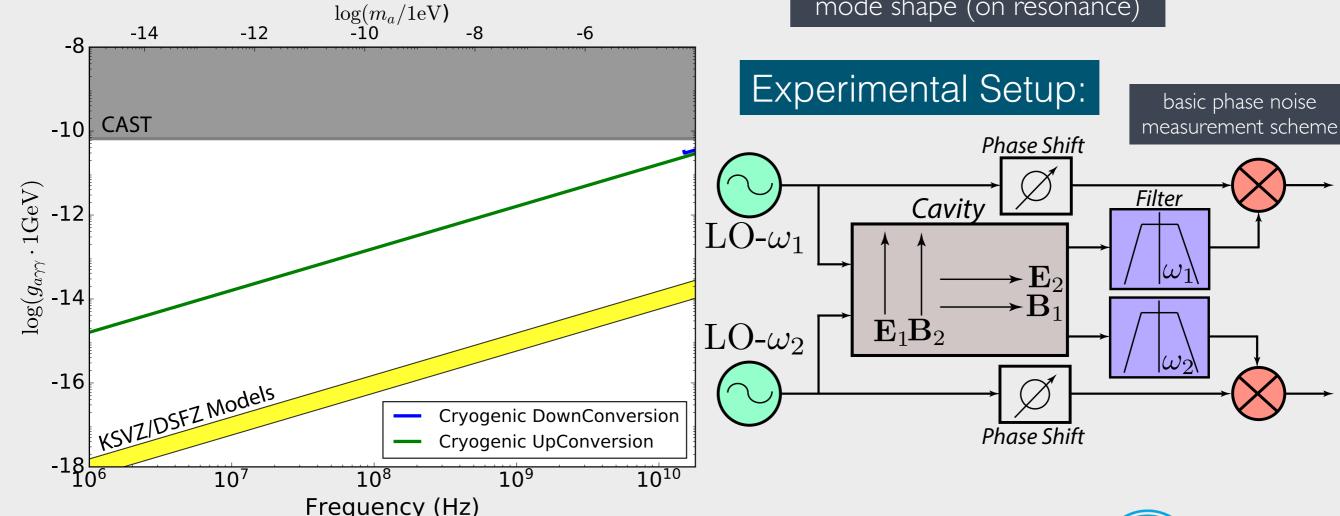
$$S_{\varphi,i}^{\text{U/D}}(f) = \frac{g_{\text{eff}}^2 \xi_{\pm}^2}{f^2 + \gamma_i^2} \left| \frac{\overline{x}_j}{\overline{x}_i} \right|^2 S_a(f) + \frac{\gamma_i^2}{f^2 + \gamma_i^2} S_{\theta}(f)$$

Steady-state

amplitudes

mode shape (on resonance)

axion



search for axion in the Fourier spectrum of cavity phase noise



#### **Axion Induced Phase Fluctuations**

Dual Loop Oscillator phase noise measurements

 $\begin{array}{c} f_a = f_2 \pm f_1 + f \\ f_i \gg f \end{array}$ 

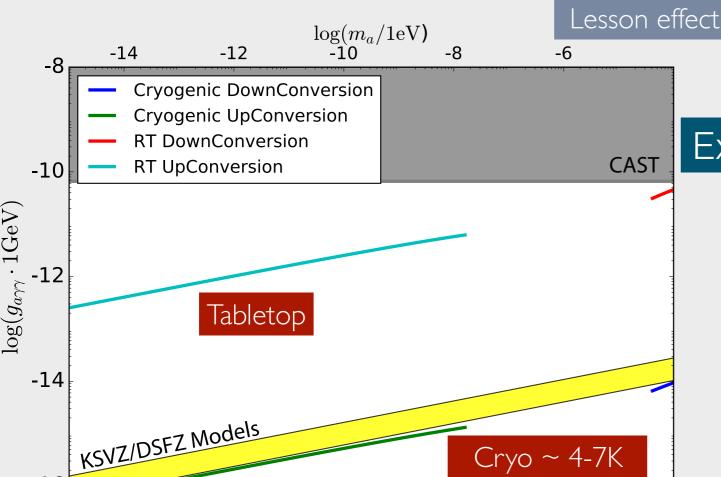
technical noise

amplitudes

Spectrum of Oscillator Phase Fluctuations:

$$S_{\varphi,i}^{\mathrm{U/D}}(f) = \left[1 + \frac{\gamma_i^2}{f^2}\right] \left(\frac{g_{\mathrm{eff}}^2 \xi_\pm^2}{f^2 + \gamma_i^2} \Big| \frac{\overline{x}_j}{\overline{x}_i} \Big|^2 S_a(f) + S_\theta(f)\right),$$
 Lesson effect mode shape Steady-state

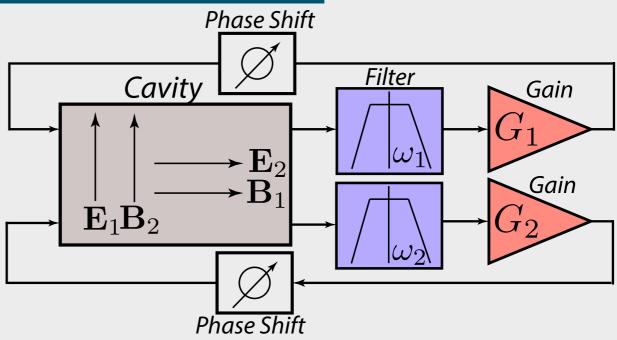
axion



10<sup>9</sup>

Frequency (Hz)

#### Experimental Setup:



by turning a cavity in a feedback oscillator, we can get rid of technical fluctuations due to external pumps



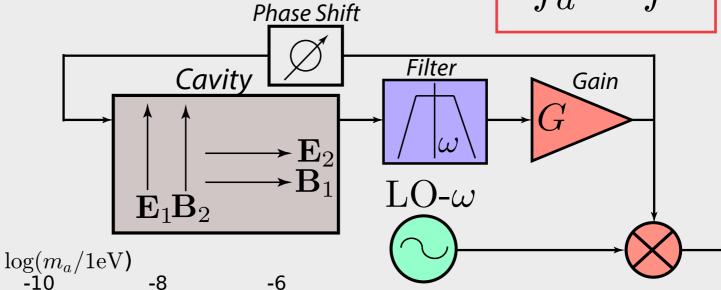
Australian Research Council Centre of Excellence for Engineered Quantum System

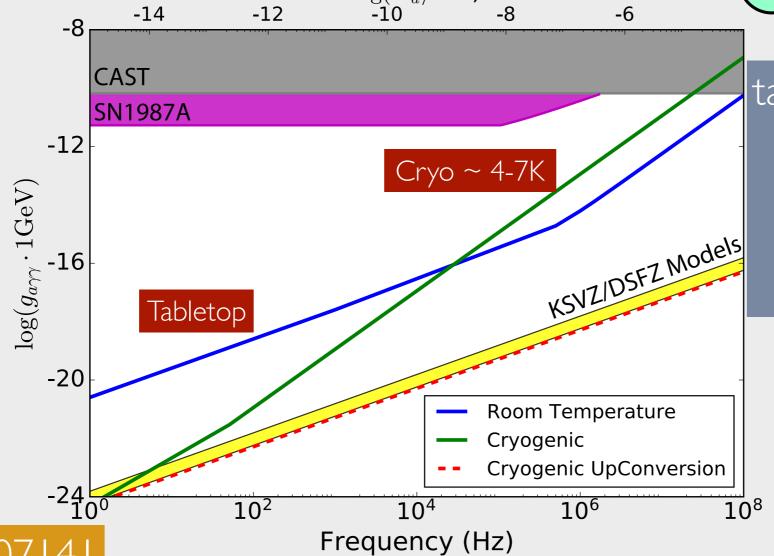
-16<sup>6</sup>

#### BroadBand Search

 $f_1 = f_2$  $f_a = f$ 

to do a broadband search for low axion masses we consider the mode degenerate case



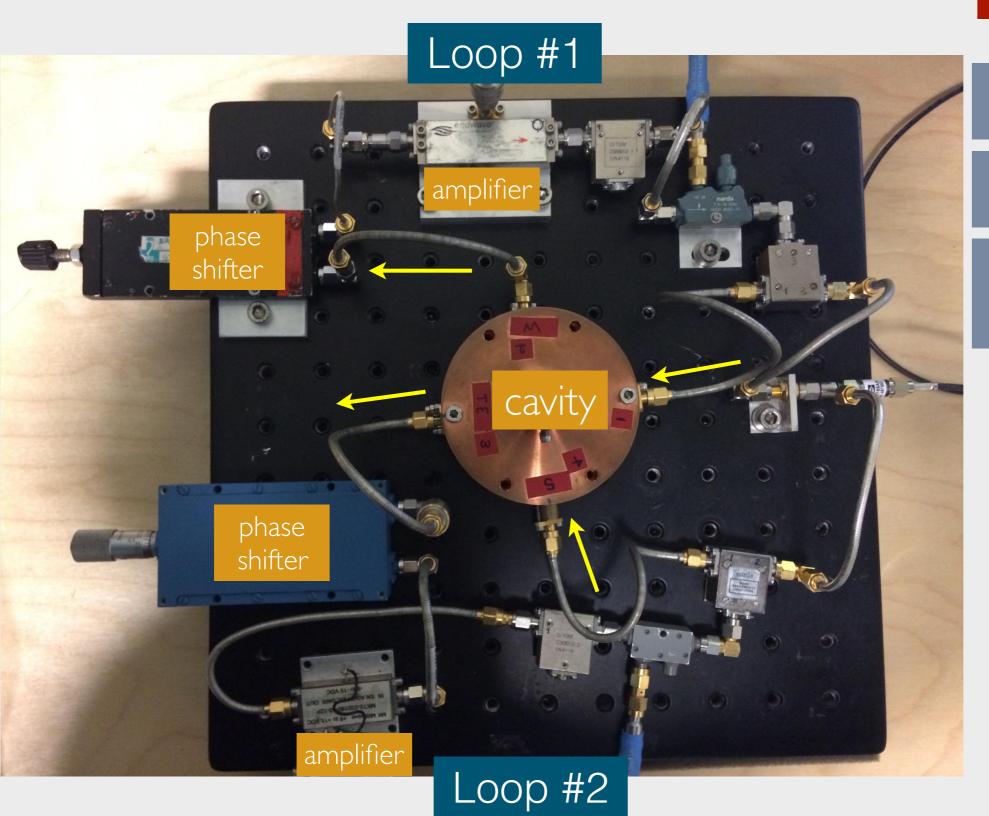


tabletop and cryogenic experiments are complementary due to flicker/white noise competition



arXiv:1806.07141

## Experiment



#### Dual Loop Oscillator

R=22mm, H = 18.5-83.6 mm cylindrical copper cavity

TM<sub>022</sub> mode (9GHz) TM<sub>011</sub> mode (6.5-9GHz)

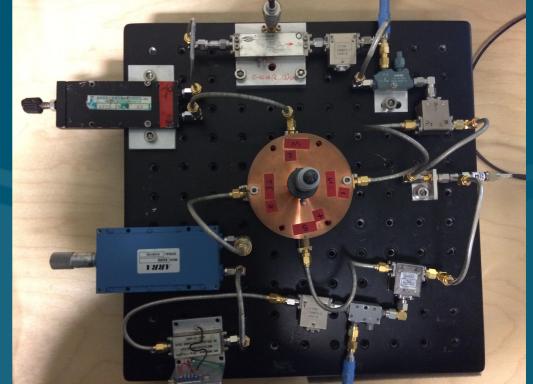
$$\xi_{-} = -0.39.. - 0.5$$

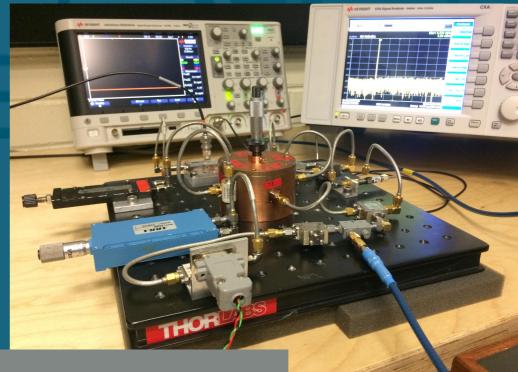
$$\xi_{+} = -0.46.. - 0.57$$



## Advantages of Frequency Metrology

- Magnet-free
- SQUID-free
- Superconductor-free
- -Semiconductor amplifiers
  - Volume independent
    - P<100 uW signals
- Liquid-Helium only (>4K)
- Only cavity/amplifier at 4K
- -High/Low frequency ranges
- Optical implementation possible
  - Broadband search is possible
    - Axion phase sensitive
    - KSVZ/DSFZ achievable
  - Tabletop search worth doing
  - -Room for improvement!





Catriona Thomson





## One more thing...

$$f \sim g_{a\gamma\gamma}\theta$$

Can we do better?



## One more thing...

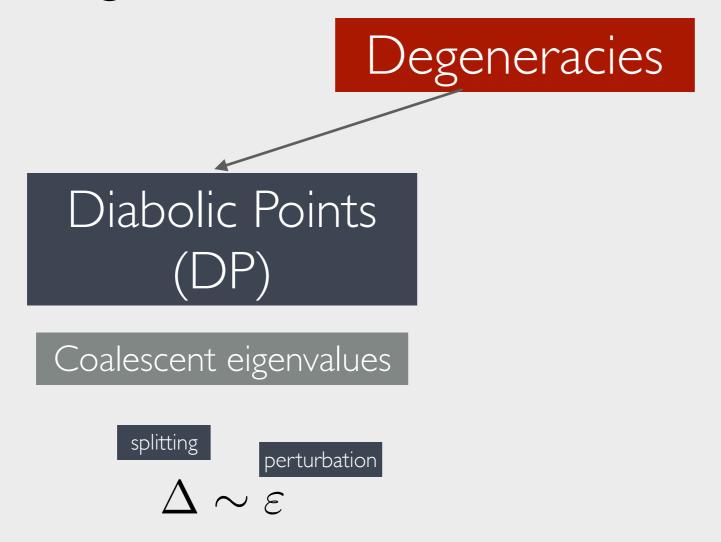
$$f \sim \sqrt{g_{a\gamma\gamma}\theta}$$



## One more thing...

$$f \sim \sqrt[3]{g_{a\gamma\gamma}\theta}$$







Degeneracies

Diabolic Points (DP)

Coalescent eigenvalues

splitting

perturbation to one of the modes

 $\Delta \sim \varepsilon$ 

Exceptional Points (EP)

Coalescent eigenvalues

Coalescent eigenstates

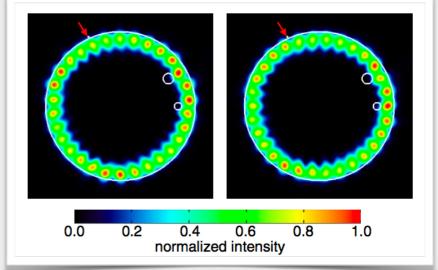
$$\Delta \sim \sqrt{\varepsilon}$$

Enhancing the Sensitivity of Frequency and Energy Splitting Detection by Using Exceptional Points: Application to Microcavity Sensors for Single-Particle Detection

Jan Wiersig

Phys. Rev. Lett. **112**, 203901 – Published 20 May 2014

Several types of sensors used in physics are based on the detection of splittings of resonant frequencies or energy levels. We propose here to operate such sensors at so-called exceptional points, which are degeneracies in open wave and quantum systems where at least two resonant frequencies or energy levels and the corresponding eigenstates coalesce. We argue that this has great potential for enhanced sensitivity provided that one is able to measure both the frequency splitting as well as the linewidth splitting. We apply this concept to a microcavity sensor for single-particle detection. An analytical theory and numerical simulations prove a more than threefold enhanced sensitivity. We discuss the possibility to resolve individual linewidths using active optical microcavities.





## Degeneracies

#### Diabolic Points

$$f \sim g_{a\gamma\gamma}\theta$$

$$\begin{array}{c} \omega_0 a_L^{\dagger} a_L \\ -\gamma_L \end{array} \begin{array}{c} \omega_0 a_R^{\dagger} a_R \\ -\gamma_R \end{array}$$

#### **Probing Dark Universe with Exceptional Points**

Maxim Goryachev, 1 Ben McAllister, 1 Jason Twamley, 2 and Michael E. Tobar 1, a)

ARC Centre of Excellence for Engineered Quantum Systems, School of Physics, University of Western Australia, 35 Stirling Highway, Crawley WA 6009, Australia

<sup>2)</sup>ARC Centre of Excellence for Engineered Quantum Systems, Macquarie University, Sydney, Australia

(Dated: 16 August 2018)

It is demonstrated that detection of putative particles such as paraphotons and axions constituting the dark sector of the universe can be reduced to detection of extremely weak links or couplings between cavities and modes. This method allows utilisation of extremely sensitive frequency metrology methods that are not limited by traditional requirements on ultra low temperatures, strong magnetic fields and sophisticated superconducting technology. We show that exceptional points in the eigenmode structure of coupled modes may be used to boost the sensitivity of dark matter mediated weak links. We find observables that are proportional to fractional powers of fundamental coupling constants. Particularly, in case of axion detection, it is demonstrated that resonance frequency scaling with  $\sim \sqrt{g_{a\gamma\gamma}}$  and  $\sim \sqrt[3]{g_{a\gamma\gamma}}$  dependencies can be realised in a ternary photonic cavity system, which is beneficial as these coupling constants are extremely small.

#### **Exceptional Points**

$$f \sim \sqrt{g_{a\gamma\gamma}\theta}$$

$$g$$

$$g$$

$$+\gamma_C$$

$$Gain or Loss$$

$$-\gamma_L$$

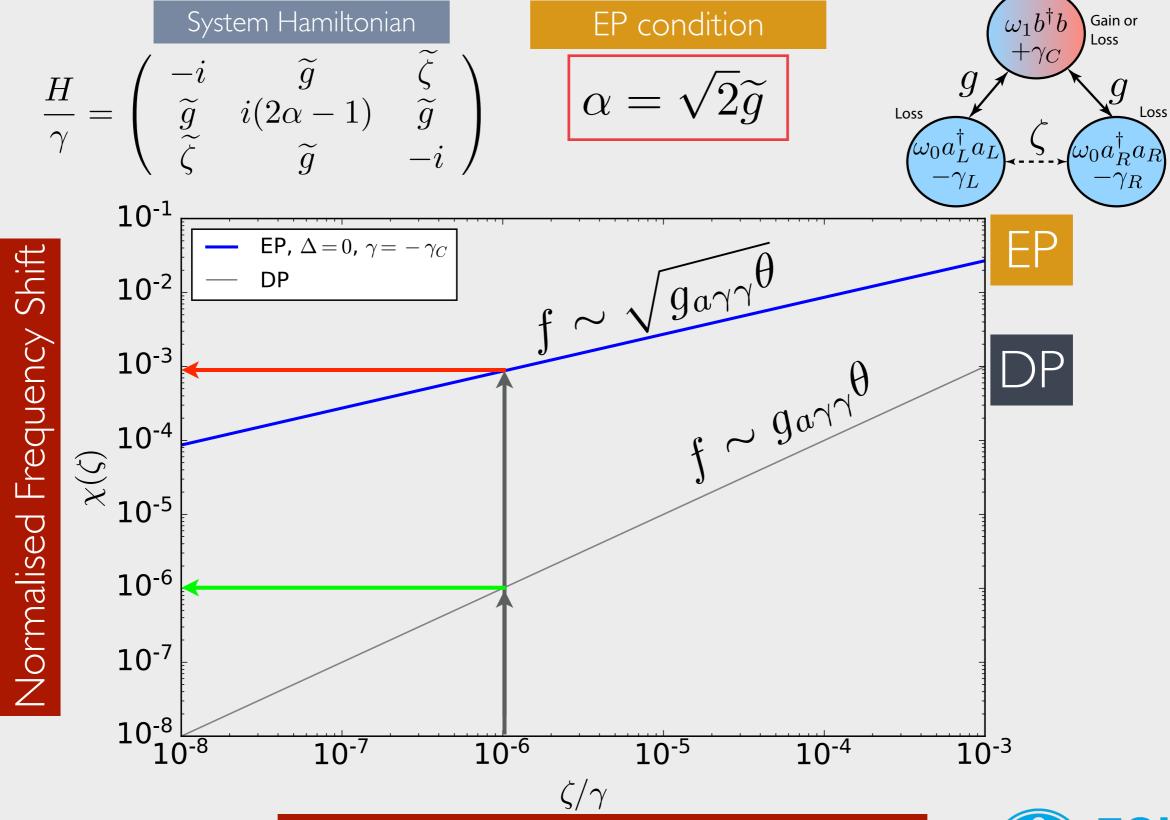
$$\omega_0 a_L^{\dagger} a_L$$

$$-\gamma_R$$

$$\omega_0 a_R^{\dagger} a_R$$

$$-\gamma_R$$





Normalised Axion Mediated Coupling

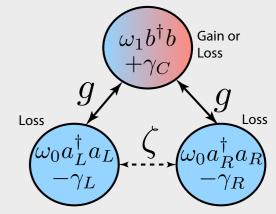


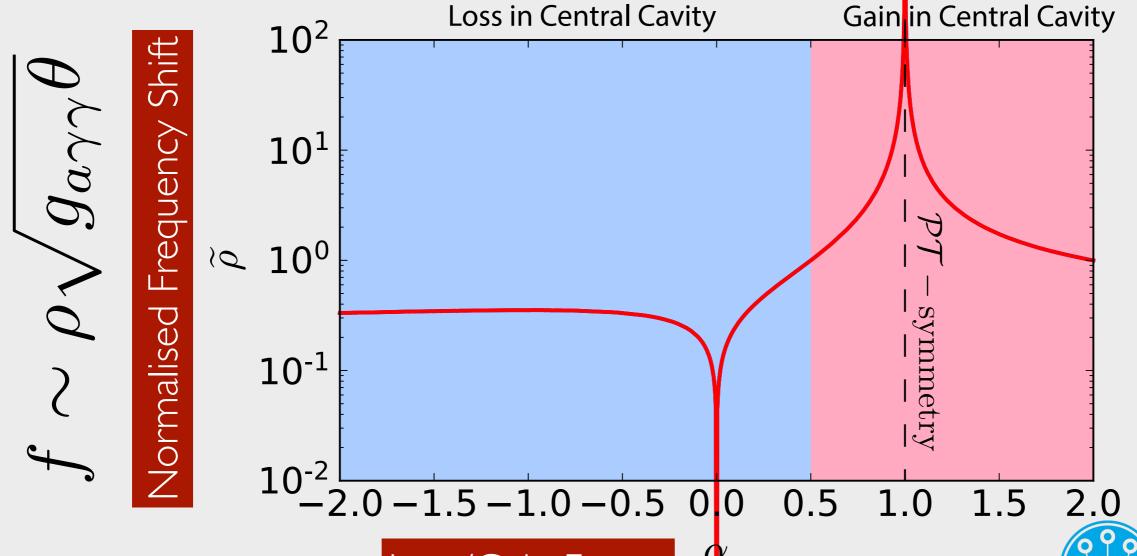
#### System Hamiltonian

$$\frac{H}{\gamma} = \begin{pmatrix} -i & \widetilde{g} & \widetilde{\zeta} \\ \widetilde{g} & i(2\alpha - 1) & \widetilde{g} \\ \widetilde{\zeta} & \widetilde{g} & -i \end{pmatrix}$$

#### EP condition

$$\alpha = \sqrt{2}\widetilde{g}$$





arXiv:{almost there}

Loss/Gain Factor



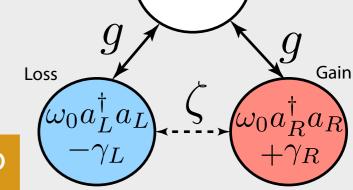
Australian Research Council Centre of Excellence for Engineered Quantum Systems Higher Order Exceptional Points

#### System Hamiltonian

$$\frac{H}{\gamma} = \left( \begin{array}{ccc} -i & \widetilde{g} & \widetilde{\zeta} \\ \widetilde{g} & 0 & \widetilde{g} \\ \widetilde{\zeta} & \widetilde{g} & +i \end{array} \right)$$

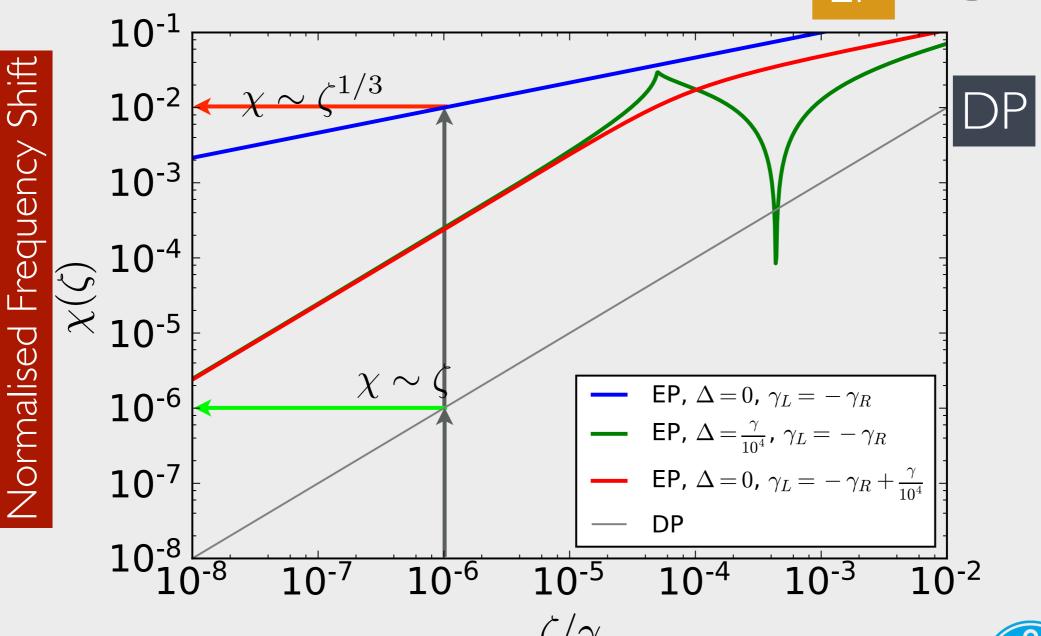
#### EP condition

$$\sqrt{2}g = \gamma$$



 $\omega_1 b^\dagger b$ 

Neutral





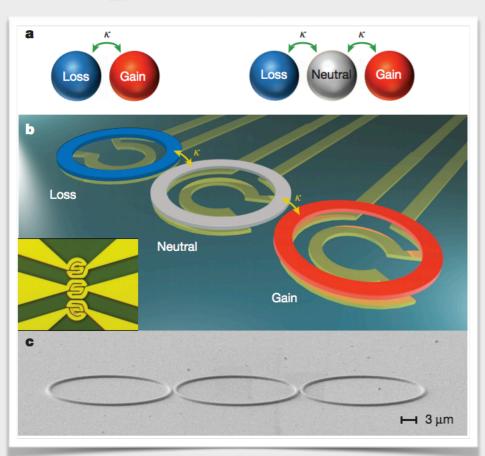
## Is this experimentally feasible?

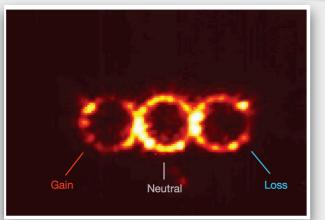
## **LETTER**

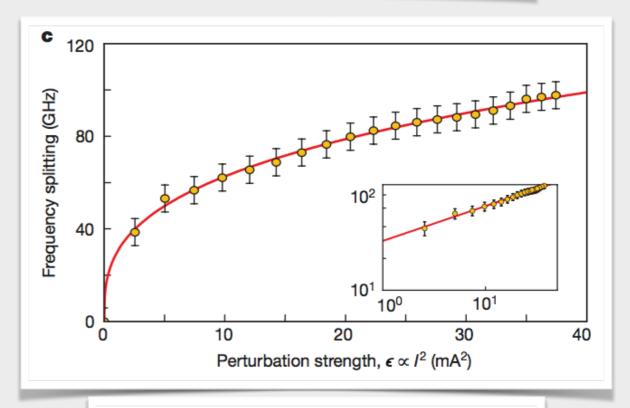
doi:10.1038/nature23280

## Enhanced sensitivity at higher-order exceptional points

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 $\Delta\omega_{\mathrm{EP3}}/\epsilon\propto(\kappa/\epsilon)^{2/3}$ 

perturbation to one of the modes



## Advantages of Frequency Metrology

- Magnet-free
- SQUID-free
- -Semiconductor amplifiers
  - Volume independent
    - Low Power signals
  - Superconductor-free
- Liquid-Helium only (>4K)
- Only cavity/amplifier at 4K
- -High/Low frequency ranges
- Optical implementation possible
  - Broadband search is possible
    - Axion phase sensitive
    - KSVZ/DSFZ achievable
  - Tabletop search worth doing
- Could be fractional in axion coupling
  - -Room for improvement!



arXiv:1806.07141

arXiv:{almost there}

