

Using a quantum annealer for particle tracking at the LHC



Lucy Linder
December 9, 2018

<http://bit.ly/hepqpr-cpad>



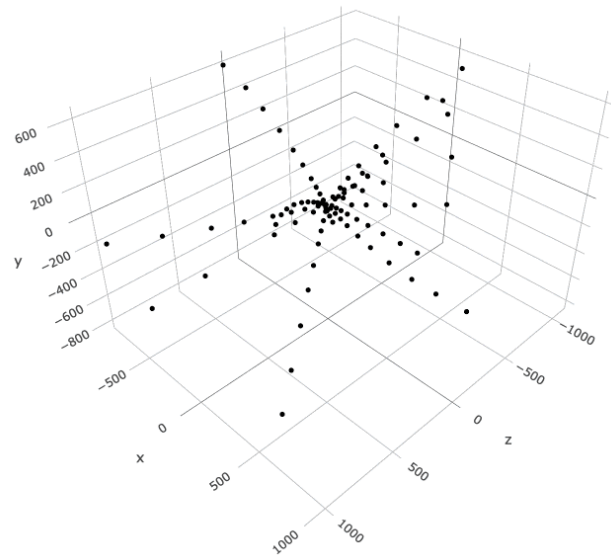
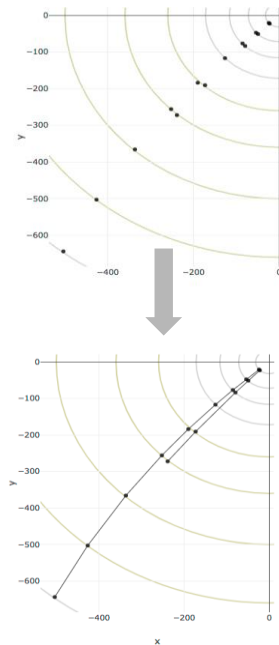
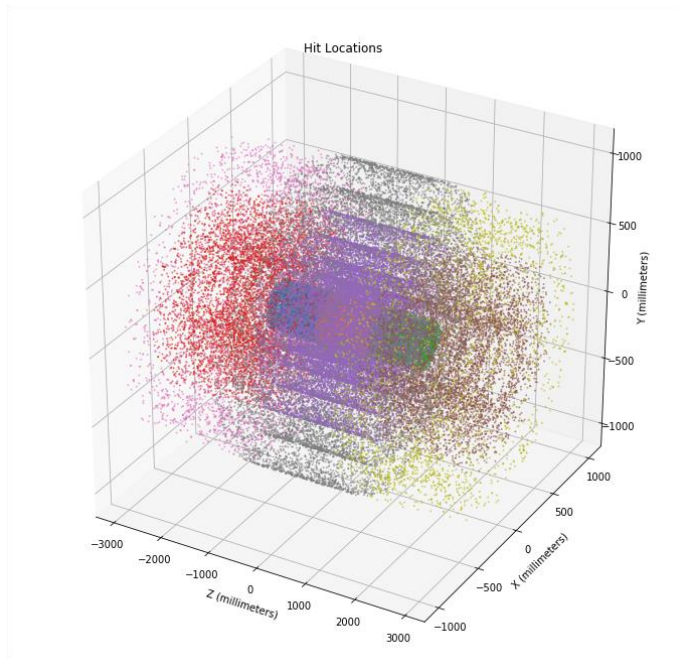


Introduction

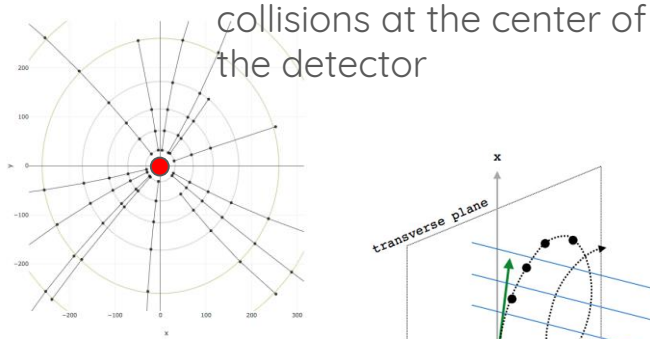


Particle tracking in HEP experiments

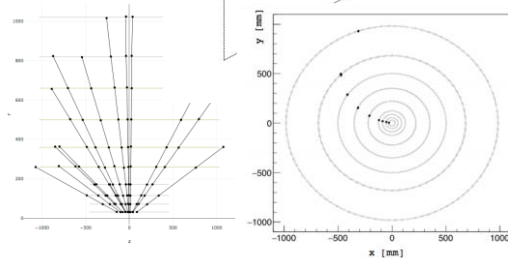
Pattern recognition from measurements (*hits*) to reconstructed particle trajectories (*tracks*).
Track fitting determine the parameters of the particle trajectory from a set of hits.



Why is tracking hard ?



perfect solenoid magnets...

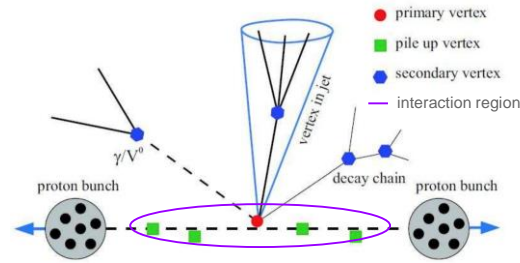


...helices in X-Y, straight lines in R-Z

complete tracks,
good measurements

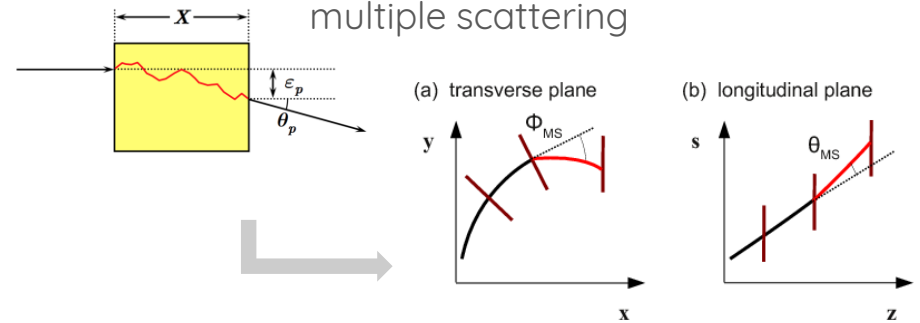
perfect world

real world

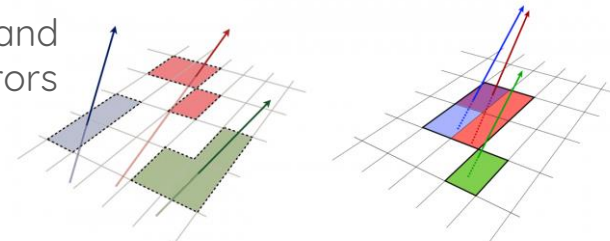


hard scattering,
secondary vertices,
decay...

multiple scattering



*holes, double hits and
measurements errors*



→ algos complex and imperfect !

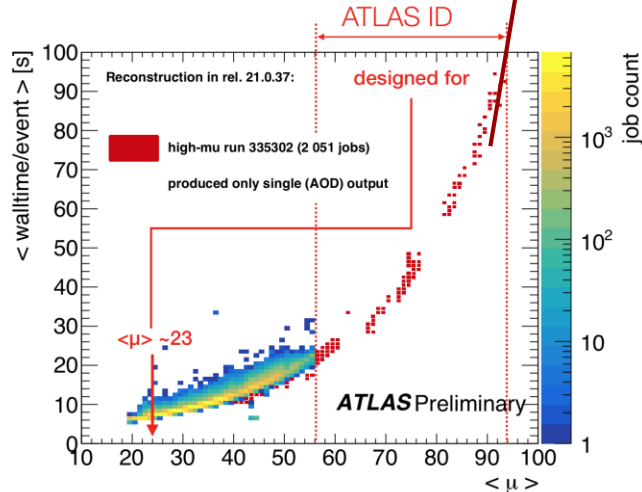


New challenge: HL-LHC

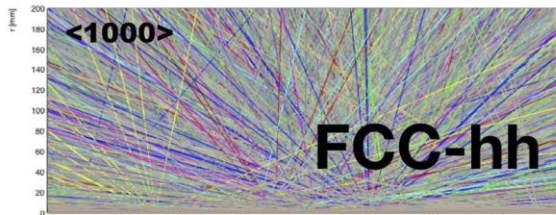
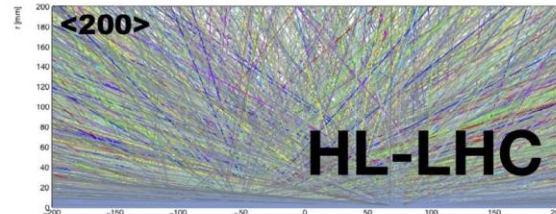
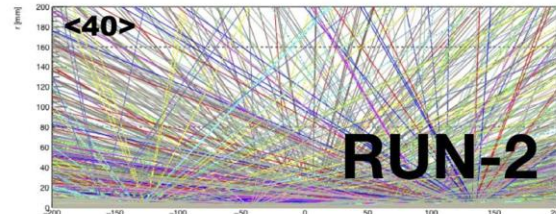
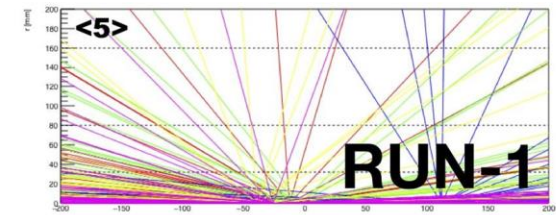
2026: HL-LHC (High-Luminosity-LHC)

- increased event rates (up to x10)
- more complex events (up to $\mu=200$, better detector)

Current algorithms of [at least] *quadratic* complexity:
 $O(N^2)$, N =hits

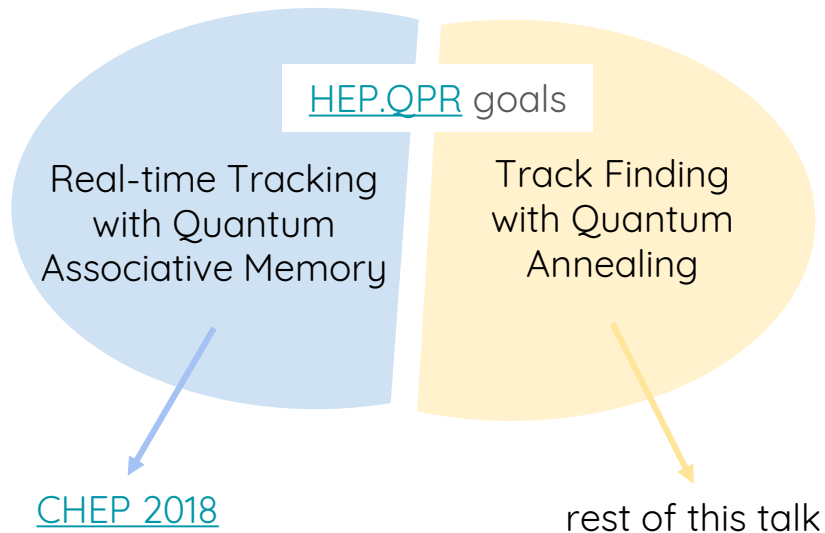


μ (μ): the average number of visible pp interactions per bunch crossing



HEP.QPR, Quantum Pattern Recognition

A DOE-HEP QuantISED pilot project



→ <https://hep-qpr.lbl.gov/> ←

Meet the team

Heather Gray (PI)

Frédéric Bapst

Wahid Bhimji

Paolo Calafiura

Steve Farrell

Wim Lavrijsen

Lucy Linder (that's me)

Illya Shapoval



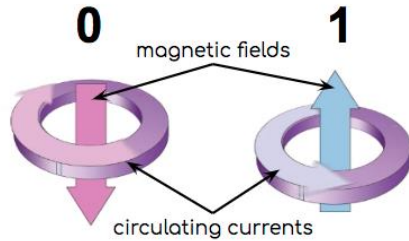


Quantum Annealing & D-Wave

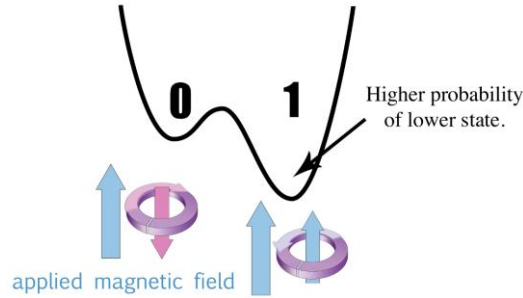


QA in D-Wave computers

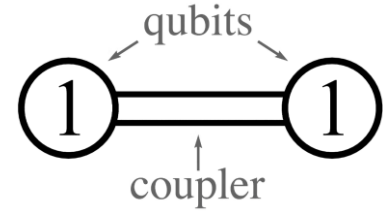
source: [dwavesys on YouTube](#)



qubits $\Rightarrow q_i$



bias weights $\Rightarrow a_i$



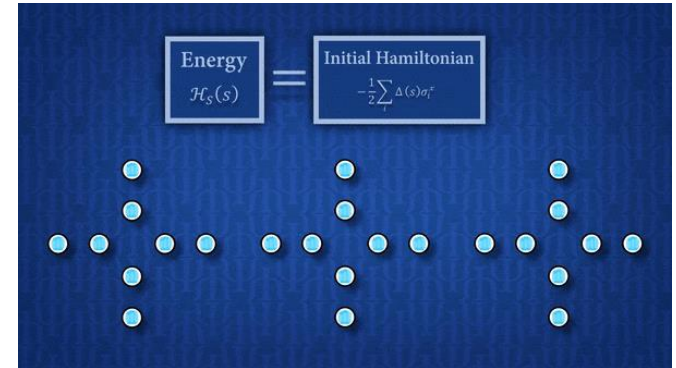
coupling strength $\Rightarrow b_{ij}$

quantum machine instruction (QMI)
objective function:

$$O(a; b; q) = \sum_{i=1}^N a_i q_i + \sum_i \sum_j b_{ij} q_i q_j \quad q_i \in \{0, 1\}$$

QUBO

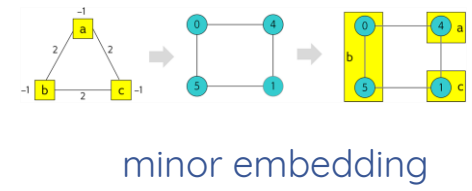
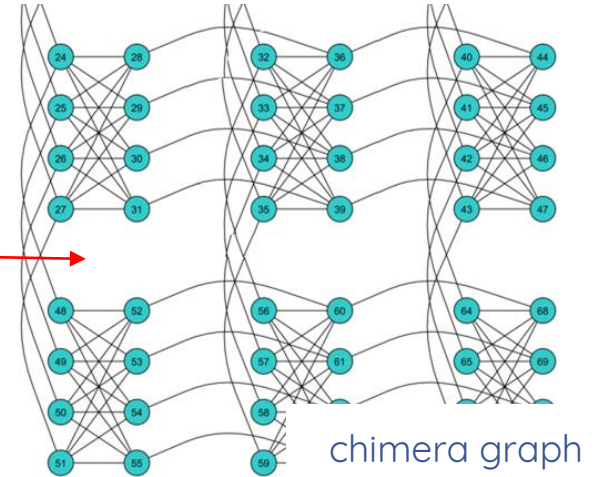
*Quadratic Unconstrained
Binary Optimisation*



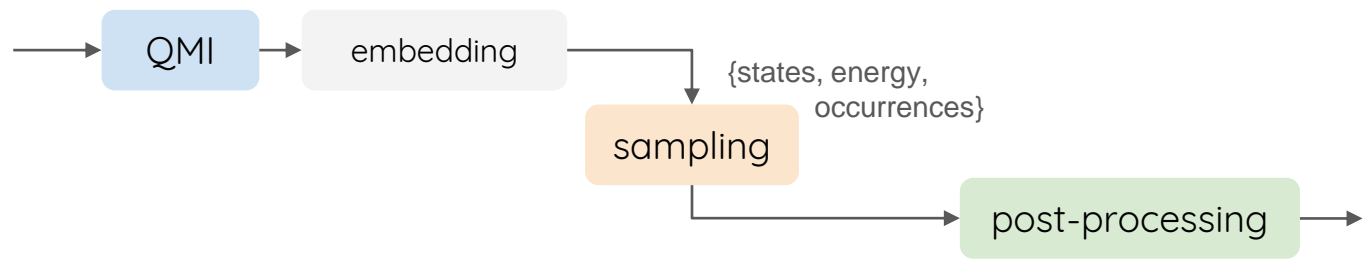
anneal time $\sim 20\mu s$

The D-Wave hardware

	D-Wave One	D-Wave Two	D-Wave 2X	D-Wave 2000Q
Release date	May 2011	May 2013	August 2015	January 2017
Code-name	Rainier	Vesuvius	?	?
Qubits	128	512	1152	2048
Couplers^[54]	352	1472	3360	6016
Josephson junctions	24,000	?	128,000	128,000
Operating temperature (K)	?	0.02	0.015	0.015
Power consumption (kW)	?	15.5	25	25
Buyers	Lockheed Martin	Lockheed Martin Google/NASA/USRA	Lockheed Martin Google/NASA/USRA Los Alamos National Laboratory	Temporal Defense Systems Google/NASA/USRA



D-Wave recap





QUBO for track reconstruction

“We can regard a *track* with n hits as a set of $n-1$ consecutive lines [doublets] with a smooth shape and without bifurcation”.

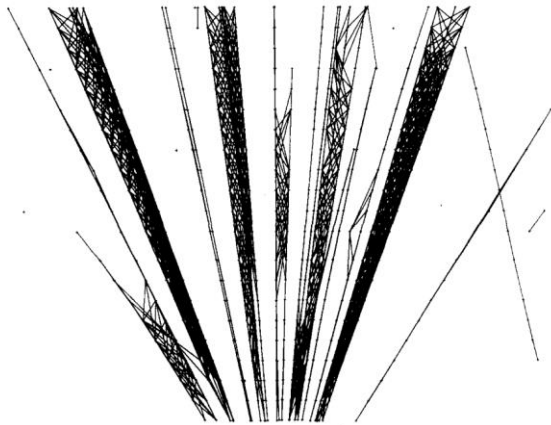


Fig. 2. Display of all generated lines for a real $Z^0 \rightarrow$ hadrons (RZ projection).

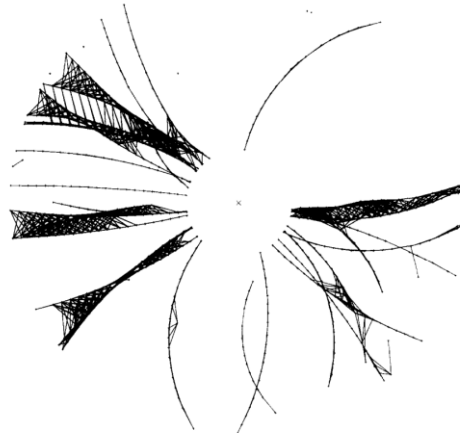


Fig. 1. Display of all generated lines for a real $Z^0 \rightarrow$ hadrons (XY projection).

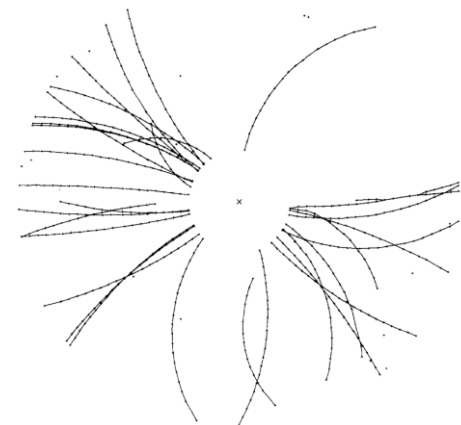


Fig. 3. Display of the activated lines after convergence for a real $Z^0 \rightarrow$ hadrons (XY projection).

source: [fast track finding with neural nets](#)

1. generate the set potential doublets (apply early cuts)
2. *binary classification task* to determine which doublets should be kept in the solution

Stimpfl-Abele & Garrido (1990) *ref*

Energy function of the Hopfield Network:

$$E = -\frac{1}{2} \left[\sum_{kln} T_{kln} V_{kl} V_{ln} \right]$$

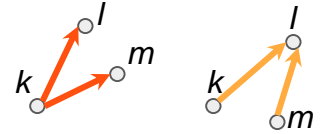
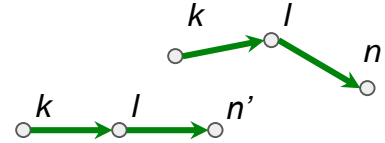
“*connection strength*”, interest of connecting two doublets

$$- \alpha \left(\sum_{klm(l \neq m)} V_{kl} V_{km} + \sum_{klm(k \neq m)} V_{kl} V_{ml} \right)$$

avoid “*conflicts*”, a hit belongs to at most one track

$$- \beta \left(\sum_{mn} V_{mn} - N_a \right), \quad V \in \{0, 1\}$$

activate only the expected number of neurons (N_a)



$$T_{kln} = \frac{\cos^\lambda \theta_{kln}}{d_{kl} + d_{ln}}$$

favour short and almost straight connections

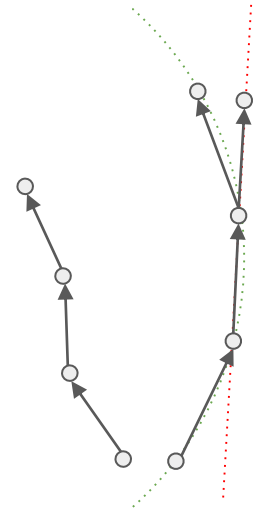
parameters: $\lambda \alpha \beta$

Stimpfl-Abele & Garrido (1990) *ref*

$$O(a; b; q) = \sum_{i=1}^N a_i q_i + \sum_i^N \sum_j^N b_{ij} q_i q_j \quad q_i \in \{0, 1\}$$

- “easy” to adapt to a QUBO / QMI
 - set the qubit bias weights a_i to 0
 - set the coupling strength b_i to either a connection strength (T) or a conflict constant (α term)
 - drop the β term
- but...
 - efficient early selection of doublets need an *origin assumption*
 - “favor straight connections” ...
 - no “continuity” between doublets → zigzag patterns

OK for high P_T tracks only,
breaks on dense datasets (> 400 particles/event)





From doublets to triplets

Focus on triplets of hits: $T_{a,b,c}$

Two triplets $T_{a,b,c}$ and $T_{d,e,f}$ can be combined:

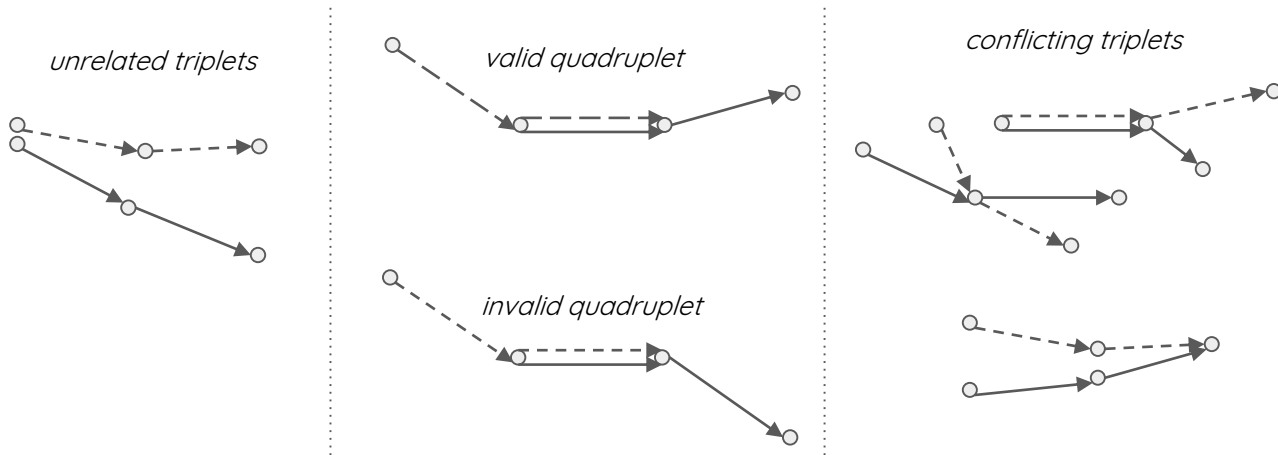
- into a **quadruplet** (qplet) if $a=d \wedge b=e$

If they share any other hit, they are in **conflict**.

New properties:

- curvature in the transverse plane
- delta angles in the R-Z plane
- ...

→ Powerful early selection,
better continuity, fewer zigzags





The new energy function

Stimpf-Abele

$$E = -\frac{1}{2} \left[\sum_{kln} T_{kln} V_{kl} V_{ln} - \alpha \left(\sum_{klm(l \neq m)} V_{kl} V_{km} + \sum_{klm(k \neq m)} V_{kl} V_{ml} \right) - \beta \left(\sum_{mn} V_{mn} - N_a \right) \right], \quad V \in \{0, 1\}$$

$$E = \alpha \left(\sum_i^N T_i \right) - \left(\sum_{i,j(\in \text{qplets})} S_{ij} T_i T_j \right) + \zeta \left(\sum_{i,k(\in \text{conflicts})} T_i T_k \right) \quad T \in \{0, 1\}$$

prior “*bias weight*”

“*connection strength*”,
interest of qplet T_i - T_j

avoid “*conflicts*”

The new energy function

$$E = \alpha \left(\sum_i^N T_i \right) - \left(\sum_{i,j(\in \text{qplets})} S_{ij} T_i T_j \right) + \zeta \left(\sum_{i,k(\in \text{conflicts})} T_i T_k \right) \quad T \in \{0, 1\}$$

As a QUBO:

$$O(a; b; T) = \alpha \left(\sum_i^N T_i \right) + \sum_i \sum_{i < j} b_{ij} T_i T_j \quad T \in \{0, 1\}$$

$$b_{i,j} = \begin{cases} -S_{ij}, & \text{if } (T_i, T_j) \text{ form a quadruplet,} \\ \zeta & \text{if } (T_i, T_j) \text{ are in conflict,} \\ 0 & \text{otherwise.} \end{cases}$$



Underlying math

a track of n hits is a set of $n-2$ triplets that can be combined into $n-3$ quadruplets
 a set of track candidates is a set of triplets with no conflict

A triplet $T_{a,b,c}$ is more interesting when:

- it has little to no hole: $\mathbf{H} = \mathbf{0}$
- the [menger curvature](#) $\text{curv}(\mathbf{a}, \mathbf{b}, \mathbf{c})$ formed by the three hits in the X-Y plane is small;
- doublets ab and bc have similar θ angles:
 $\text{drz}(T_{a,b,c}) = |\zeta(\theta_{ab}, \theta_{bc})|$ is small

A qplet (T_i, T_j) is more interesting when:

- it has few holes: $\mathbf{H}_{ij} = \mathbf{0}$
- there are similar curvatures:
 $\text{dcurv}_{ij} = |\zeta(\text{curv}(T_i), \text{curv}(T_j))|$ is small
- hits are aligned in R-Z:
 $\text{drz}_{ij} = \max(\text{drz}(T_i), \text{drz}(T_j))$ is small

The interest of connecting two triplets into a qplet can be expressed as:

$$S_{ij} = \frac{\alpha(\beta(1 - \text{dcurv}_{ij})^\gamma + (1 - \beta)(1 - \text{drz}_{ij})^\delta)}{(1 + H_{ij})^\epsilon} \xrightarrow[\text{others=1}]{\beta=0.5, \epsilon=2} S_{ij} = \frac{1 - \frac{1}{2}(\text{dcurv}_{ij} + \text{drz}_{ij})}{(1 + H_{ij})^2}$$

Experimental setup

Dataset

[TrackML dataset](#) (== HL-LHC) with events split into smaller chunks.

- select P% of particles
- select P% of noise

Set weight=0 for particles with:

- $P_T < 1$ GeV or
 - less than 5 hits
- endcaps
double hits

tune the model for that !

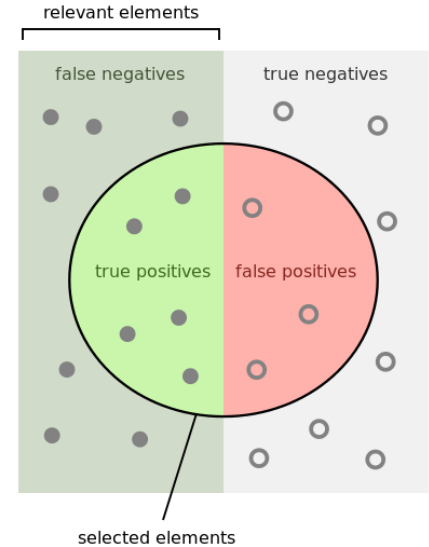
Input / output

→ set of potential doublets.
Python adaptation of the [ATLAS online seeding GPU code](#) (prototype).

← subset of the input, doublets part of track candidates.

Scoring functions

- TrackML score
- precision (~purity)
- recall (~efficiency)



How many selected items are relevant?

$$\text{Precision} = \frac{\text{true positives}}{\text{true positives} + \text{false positives}}$$

How many relevant items are selected?

$$\text{Recall} = \frac{\text{true positives}}{\text{true positives} + \text{false negatives}}$$

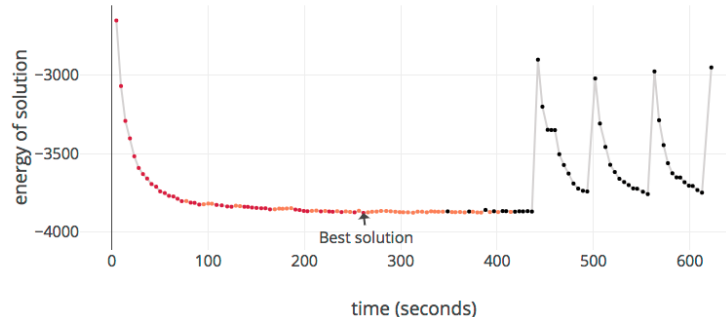
false negative = missings
false positive = fakes

iterative hybrid classical/quantum algorithm

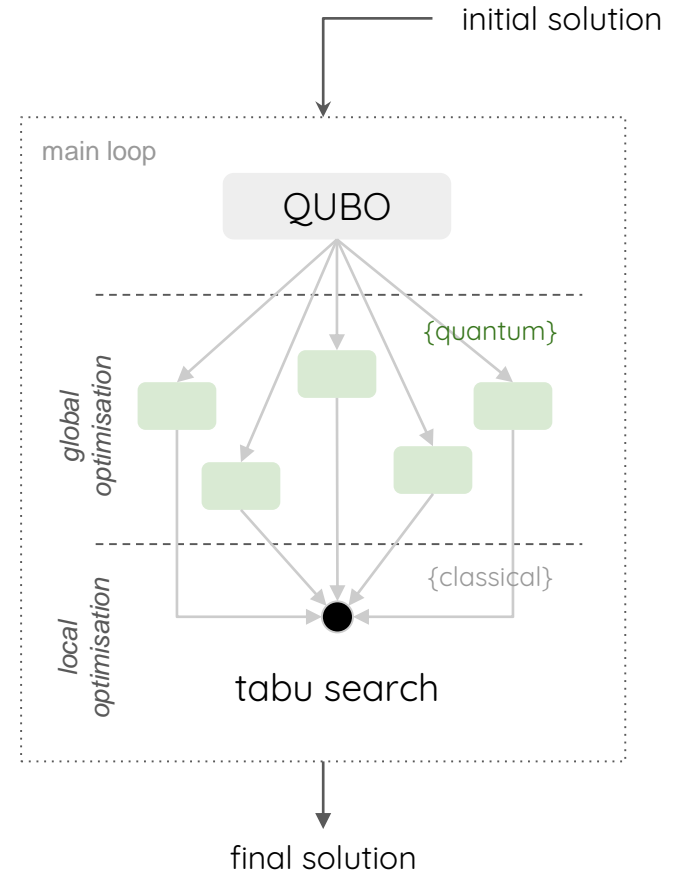
QBSOLV

large and/or densely connected QUBOs split into sub-QUBOs fitting the D-Wave hardware.

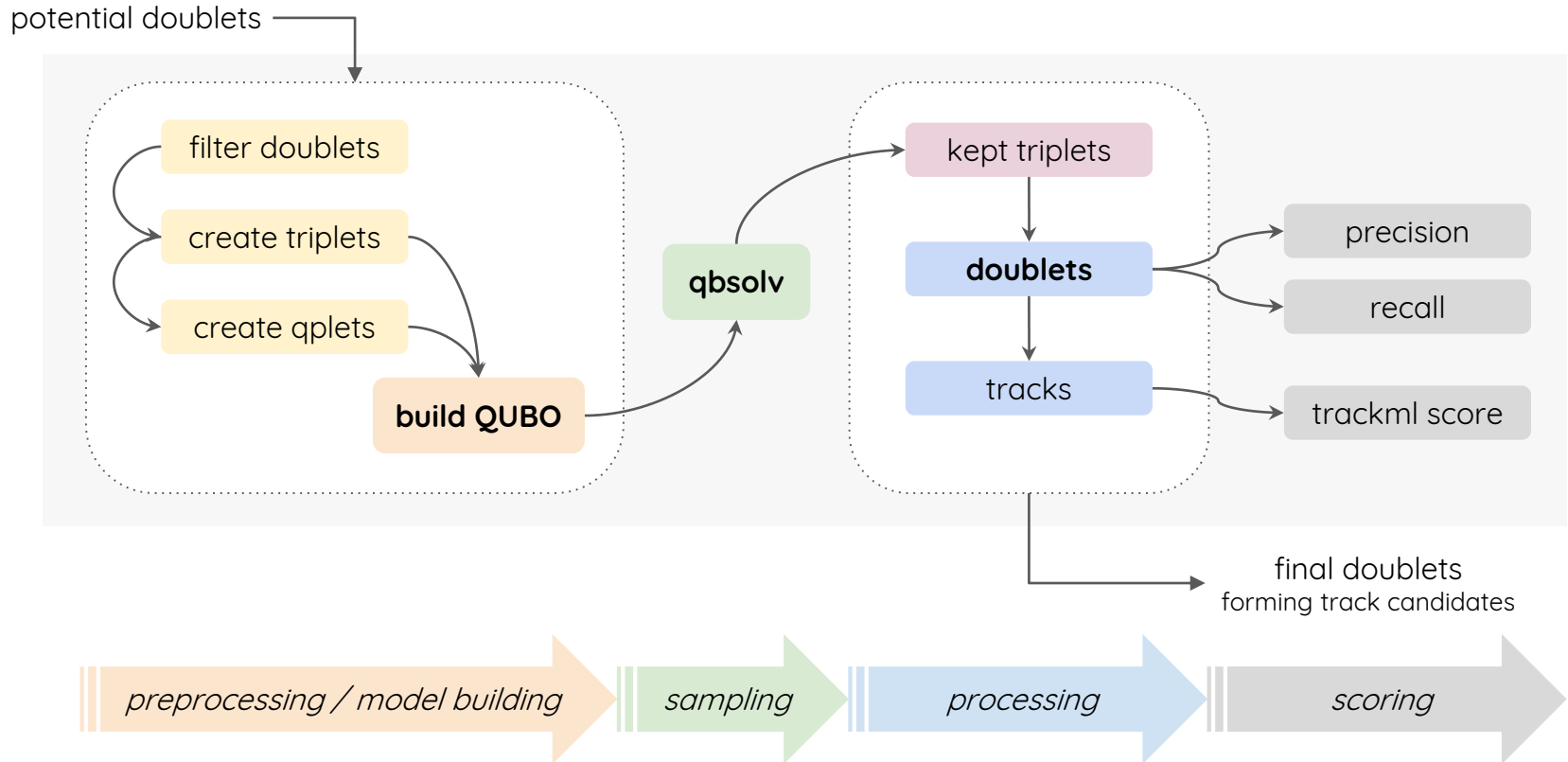
Tabu search on the recomposed solutions.



evolution of the solution in each *qbsolv* loop.
The solution is sometimes randomised to escape local minima.



Algorithm overview



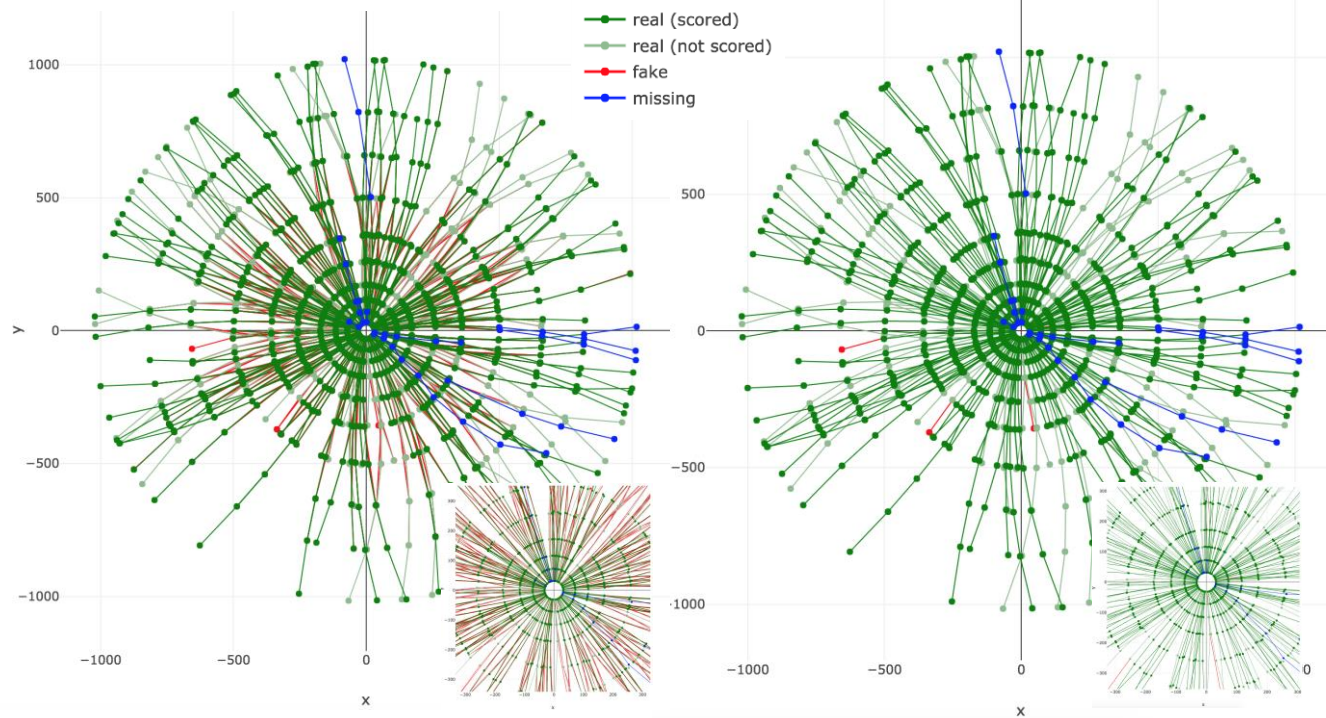


Results

Results

dataset size: ~20%
1,637 particles, 11,030 hits

*plotting error: too many
doublets 392529*



392,529 doublets
 $p=0.26\%$, $r=99.15\%$

57.3s
build
QUBO

2,546 doublets
(2,964 triplets)
QUBO size: 14,345

17.1s
sample
QUBO
running on CPU

1,512 doublets
 $p=99.13\%$, $r=97.06\%$

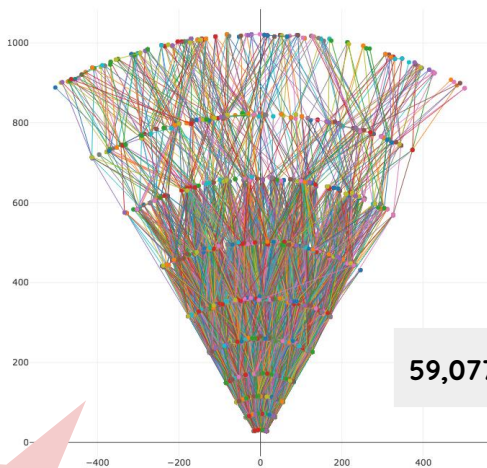
trackml score **97.55%**



Performances at low Pt

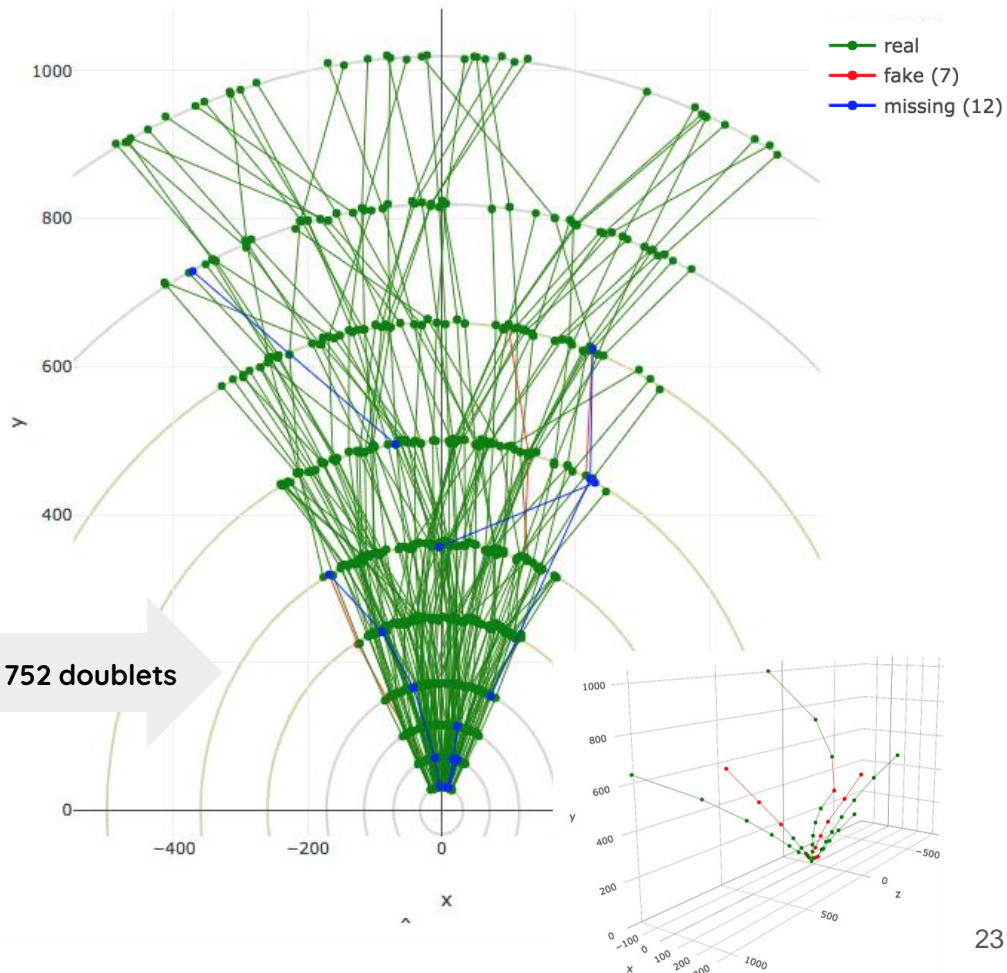
186 particles in a phi slice of $\pi/3$

precision (%): 98.5, recall (%): 98.4,
trackml score (%): **98.35**



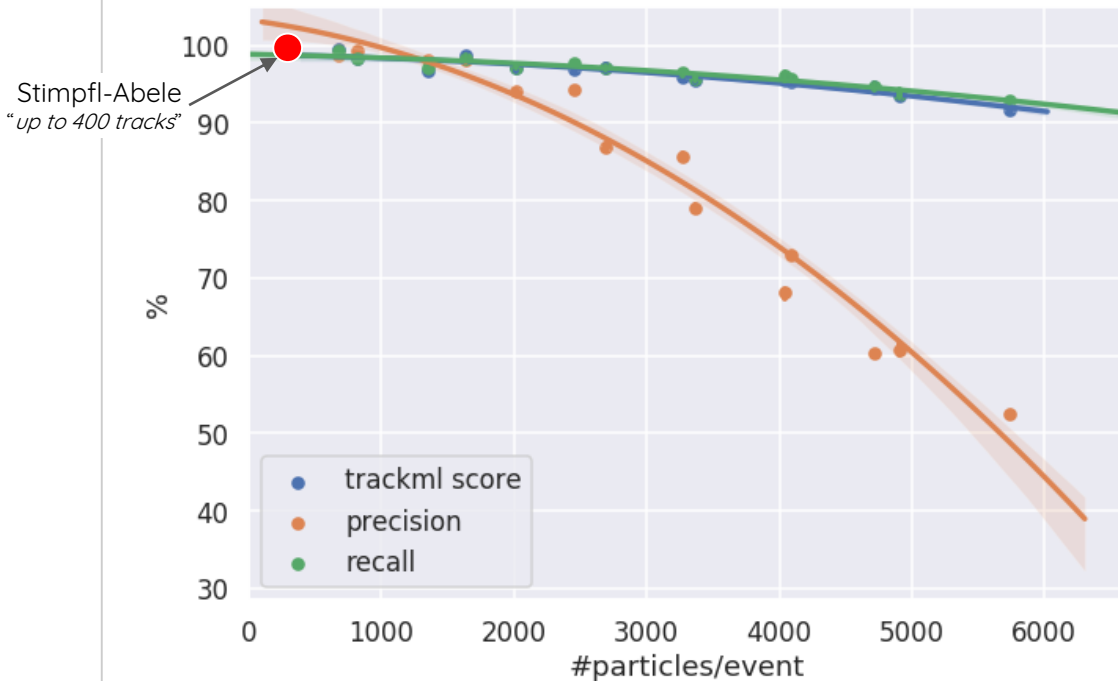
QUBO size
68,043

59,077 \Rightarrow 752 doublets

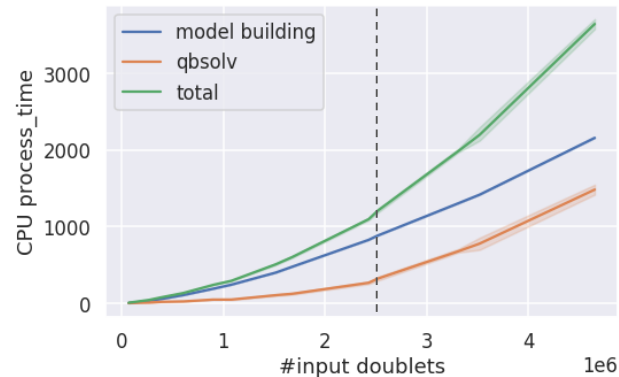


Performance

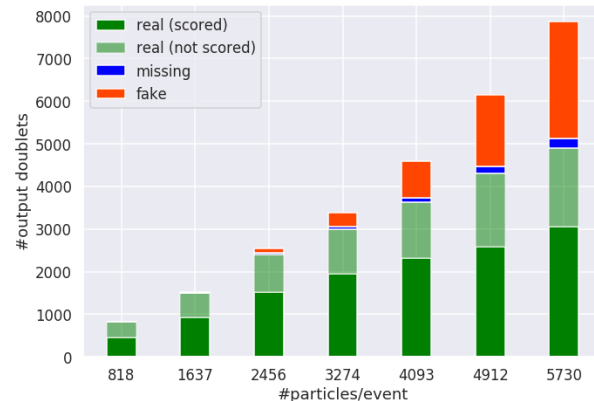
Full TrackML event 6,900 - 14,000 particles, Pt \geq 150 MeV,
 ~15% noise/lower Pt hits



time scales *?quadratically?* with the #input doublets



composition of the final doublets



> 90% recall (efficiency) and trackml score on doublets classification

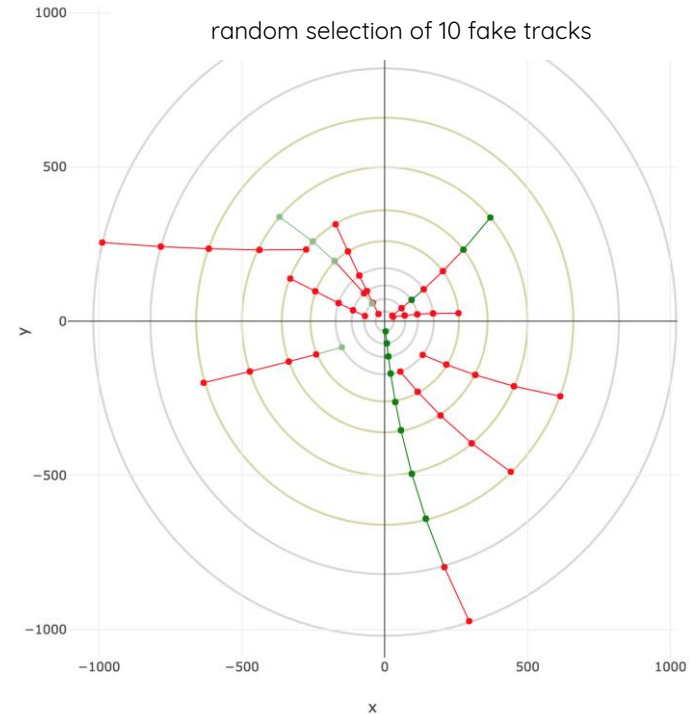
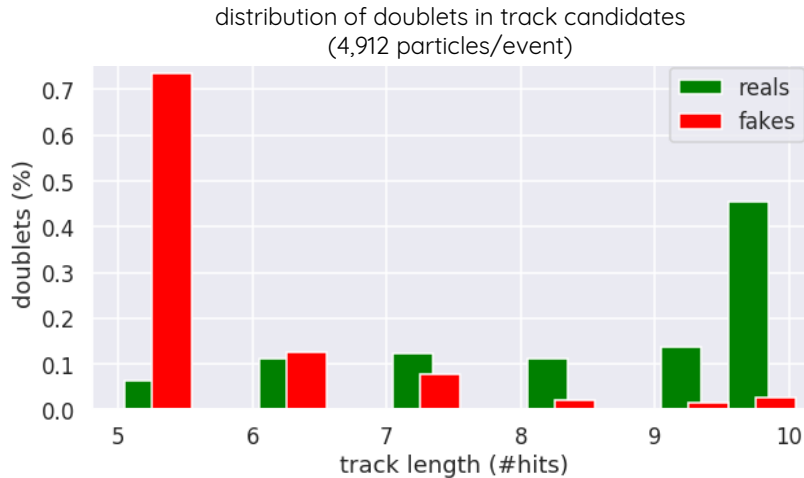


What about the fakes ?

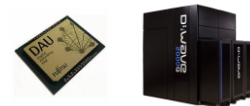
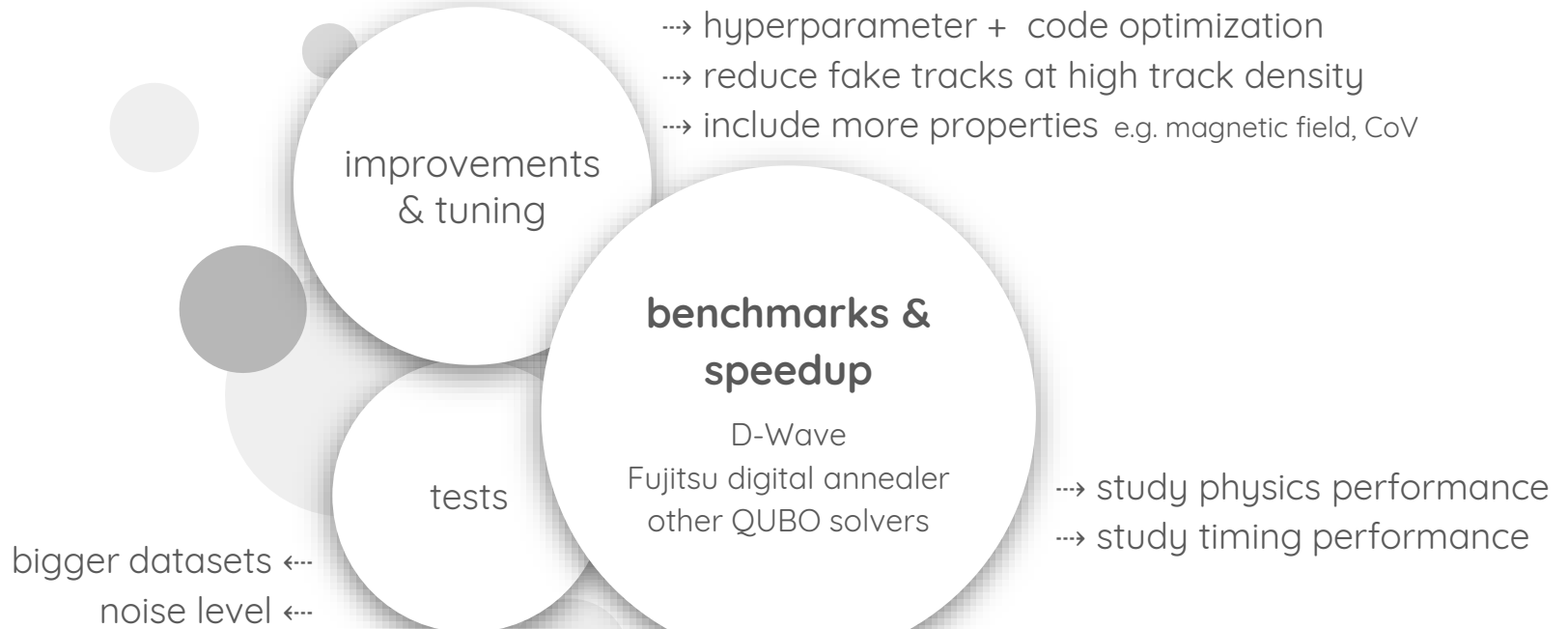
“The biggest difference [with conventional methods] is the number of wrongly associated coordinates [...]

soft constraints and very simple geometrical constants is not as good as a sophisticated algorithm based on hard constraints (fits)”.

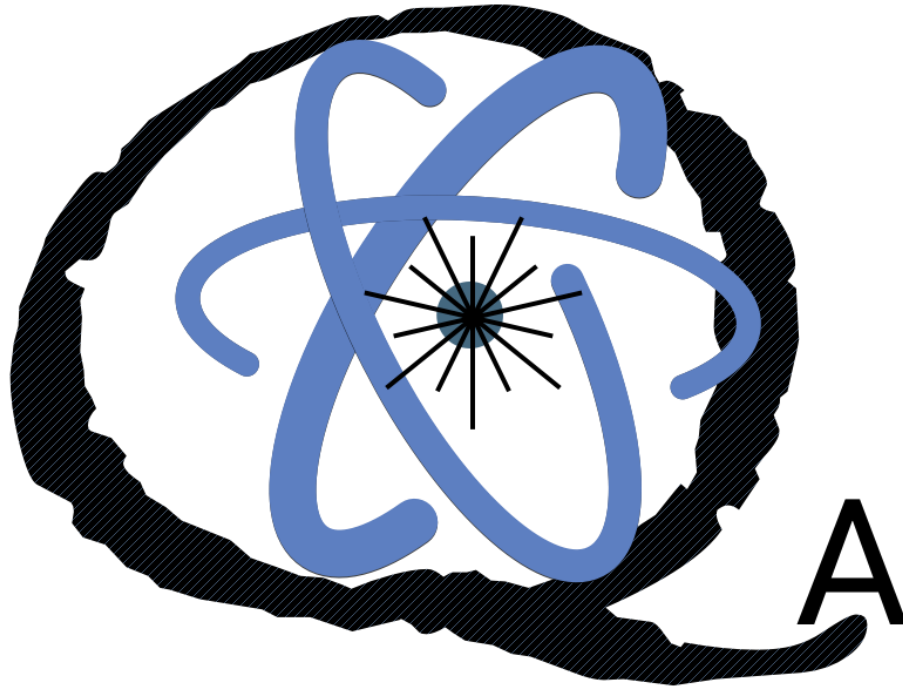
Stimpfl-Abele & Garrido, [fast track finding with neural networks](#)



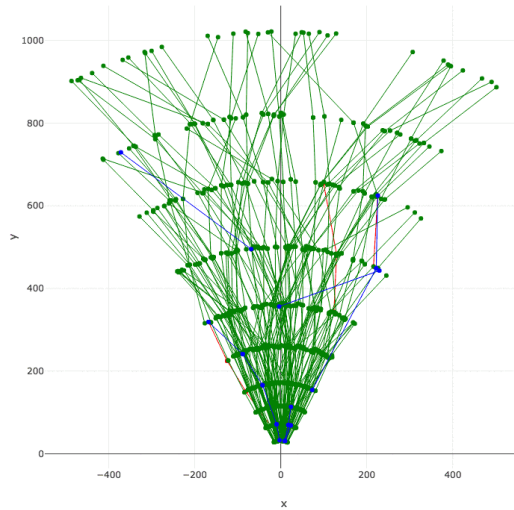
→ using track fitting methods in a post-processing step *should* let us filter many fakes



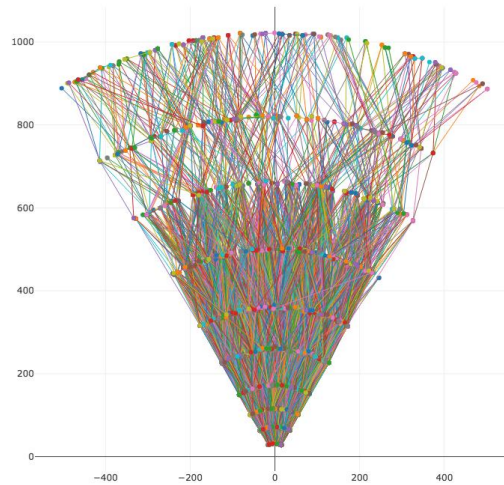
Thank you for your attention



Any questions ?



Any questions ?



backup slides

Basics of QA

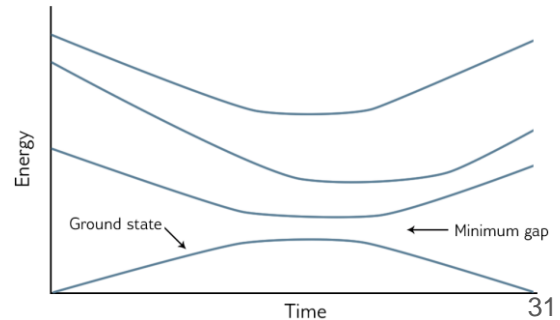
adiabatic theorem of quantum mechanics,

a physical system remains in the ground state if $\left\{ \begin{array}{l} \text{perturbations are slow} \\ \text{there is a gap} \end{array} \right\}$

QA recipe:

initial Hamiltonian H_0 \dashrightarrow $H(s) = A(s)H_0 + B(s)H_p$ with
problem Hamiltonian H_p \dashrightarrow $A(s) \searrow$ and $B(s) \nearrow$ given $s = t/t_f \lesssim 1$ ($t_f = \text{anneal time}$)

If the adiabatic conditions are respected,
the system's ground state at $t=t_f$ encodes the solution.





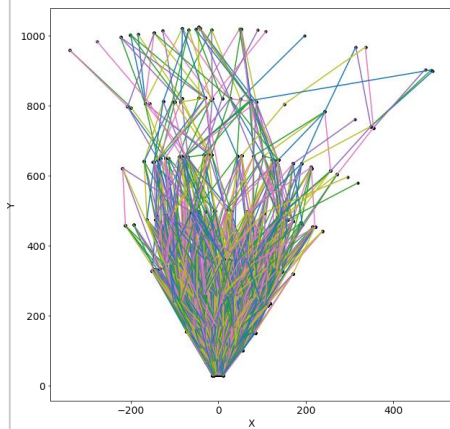
Ising, QUBO, QMI, ...

Problem Expression	Terms			
	Linear coefficient	Quadratic coefficient	Variable	States
QUBO (scalar)	a_i	b_{ij}	q_i	{0, 1}
QUBO (matrix)	$Q_{i,i}$	$Q_{i,j}$	x_i	{0, 1}
Ising	h_i	J_{ij}	s_i	{-1, 1}
Graph	Node Weight	Edge Strength	Node	
QPU	Qubit Bias	Coupling Strength	Qubit State	{Spin Up, Spin Down}

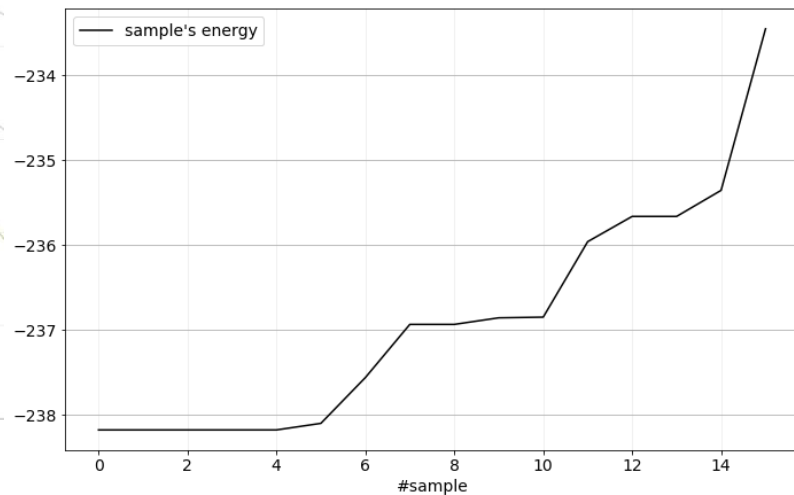
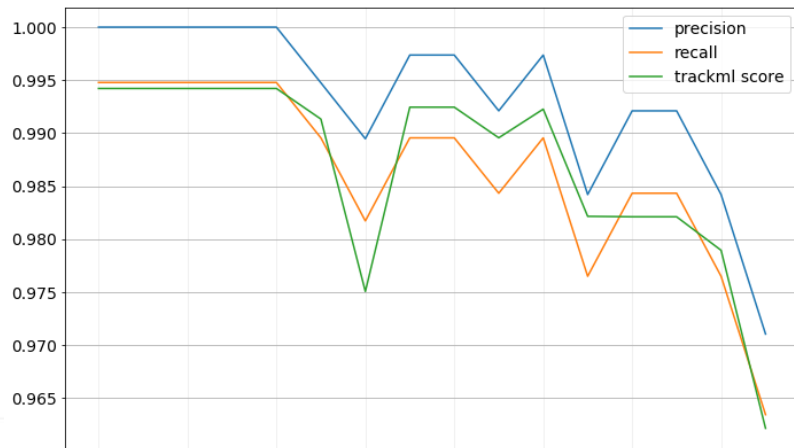
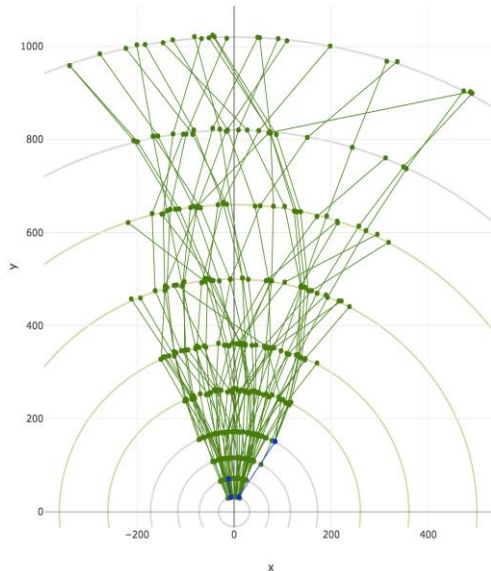
More at https://docs.dwavesys.com/docs/latest/c_gs_3.html

D-Wave run (LEAP)

trackml score of **99.42%** for 60 tracks in a phi angle of $\pi/3$ rad.



from 10,952 input doublets down to 380.



D-Wave run (LANL)

2,456 tracks/event (~30%)

Total subQUBOs submitted:
13,059

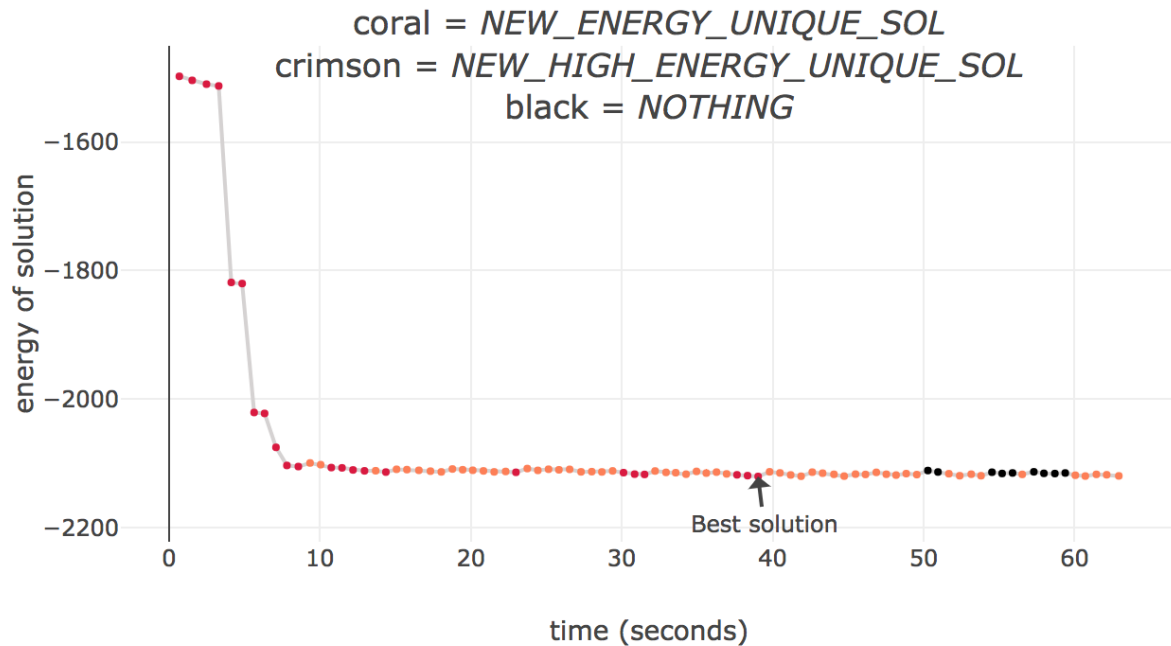
p=90.2%, r=96.5%
trackml=96%

using solver: DW2X_LANL_1

Wall time: 29:20.76

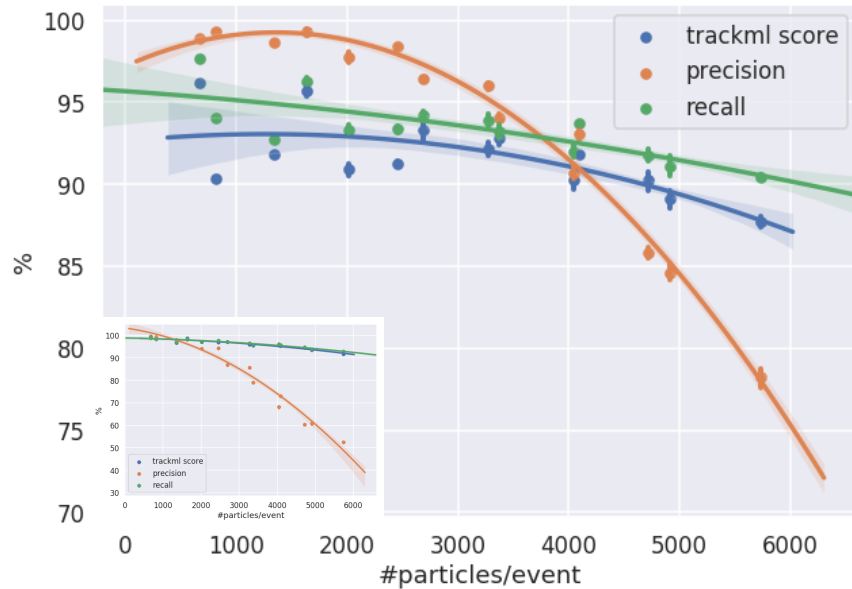
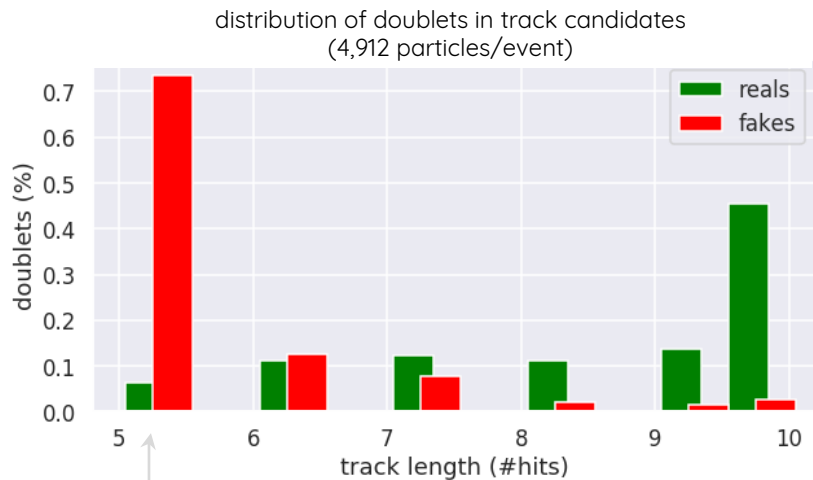
no significant difference with
the classical run.

qbsolv - solution evolution over time





Should we discard small tracks ?



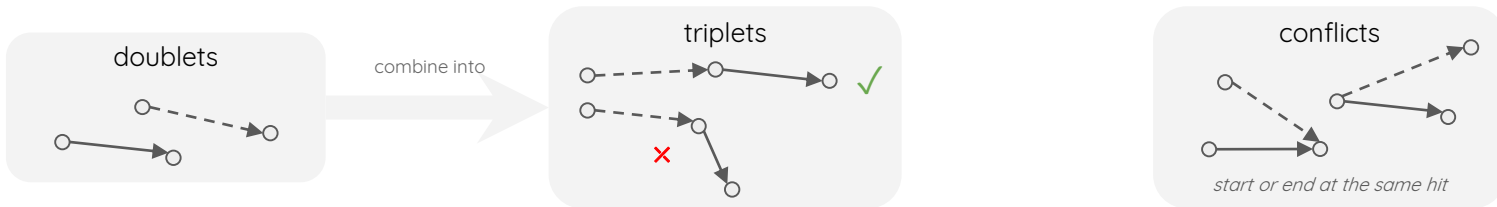
Scores after removing all tracks with 5 hits from the output.



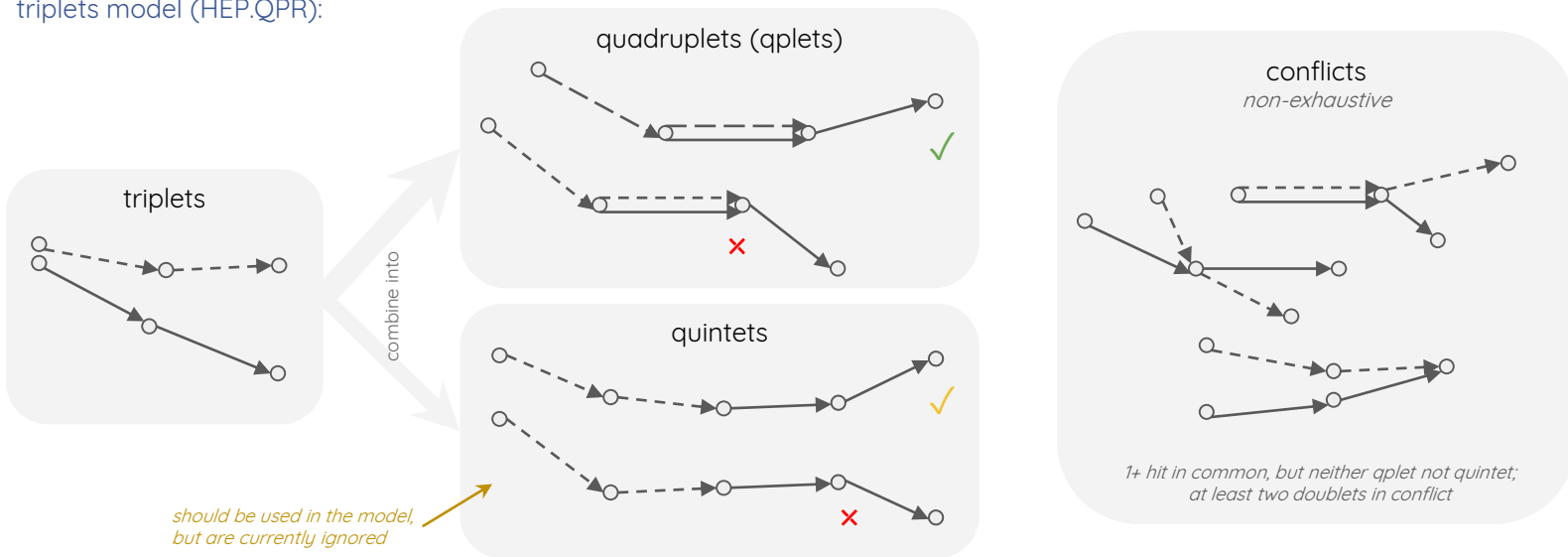
Doublets, triplets, quadruplets, quintets

✓ valid, generated by the model
✗ invalid, not generated

doublets model (Stimpfl):



triplets model (HEP.QPR):



should be used in the model,
but are currently ignored