Wake of channeling crystal
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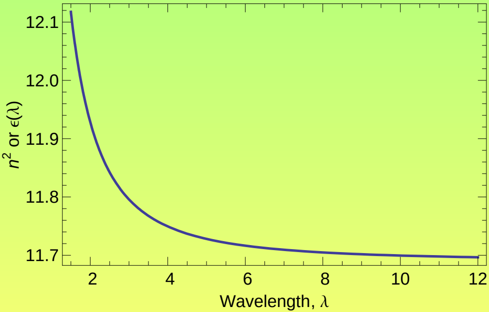
G. Stupakov, SLAC

Silicon refractive index

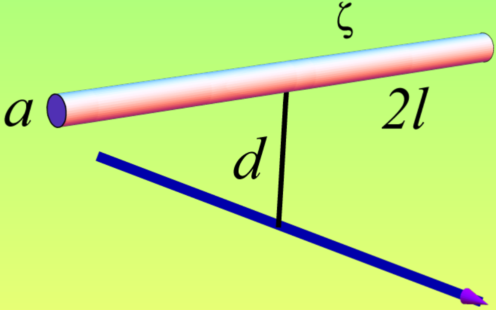
The dispersion producing the best fit to measurements at ambient room temperature (293 K) [D.F. Edwards, Silicon (Si). In: E.D. Palik, Editor, Handbook of Optical Constants of Solids, Academic Press Inc. (1985) pp. 547-569, ISBN 0-12-544420-6] with the following modified Sellmeier expression ($\epsilon = 11.7$, $\lambda_1 = 1.1\ \mu$m):

$$n^2(\lambda) = \epsilon + \frac{A}{\lambda^2} + \frac{B\lambda_1^2}{\lambda^2 - \lambda_1^2}$$



Approximations

Large value of $\epsilon \approx 12$ makes reasonably good a model of perfectly conducting metal. We assume a metal bar of radius a and length l located at a distance d from the beam orbit. If the bunch length $\sigma_z \gg d$, then the field on the bar is a slow function of time, and one can use electrostatic approximation to solve the fields.



An integral equation

ζ is coordinate measured along the bar.

The potential generated at point ζ of the bar at time t is

$$\phi_B(t,\zeta) = -2\lambda_B(t)\ln\sqrt{d^2+\zeta^2}$$

where λ_B is the charge per unit length of the bunch

$$\lambda_B(t) = \frac{Q}{\sqrt{2\pi}\sigma_z} e^{-t^2c^2/2\sigma_z^2}$$

This potential should be compensated by the image charge on the bar.

$\Lambda(t,\zeta)$ is the charge per unit length of the bar. In the limit $a \ll l, d$, the potential generated by the image charge on the surface of the bar is

$$\begin{aligned}\phi_{im}(t,\zeta) \approx & 2\Lambda(t,\zeta)\ln\left(\frac{2l}{a}\right) + \Lambda(t,l)\ln(1-\zeta/l) + \Lambda(t,-l)\ln(\zeta/l+1) \\ & - \int_\zeta^l d\xi\Lambda'(t,\xi)\ln(\xi-\zeta)/l + \int_{-l}^\zeta d\xi\Lambda'(t,\xi)\ln(\zeta-\xi)/l\,.\end{aligned}$$

The sum of two potentials does not depend on ζ

$$\phi_{im}(t,\zeta) + \phi_B(t,\zeta) = \phi_0(t)$$

Digression: charge distribution on a thin wire

This integral equation can be used to find charge distribution on a thin metallic wire of round cross section, J. Jackson, American Journal of Physics, vol. 68 (2000)

Charge density on thin straight wire, revisited

J. D. Jackson
University of California, Berkeley, California 94720

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The question of the equilibrium linear charge density on a charged straight conducting ''wire'' of finite length as its cross-sectional dimension becomes vanishingly small relative to the length is revisited in our didactic presentation. We first consider the wire as the limit of a prolate spheroidal

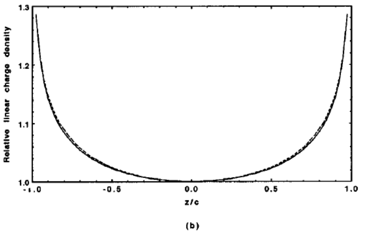


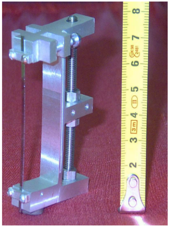
Fig. 3. Comparison of Eq. (26), normalized to unity at $z=0$, with numerical calculations. (a) Sakar and Rao (Ref. 9), $\Lambda=13.82$ (solid triangles; solid dot is mean position weighted according to the shape of (26)). (b) Waterman and Pedersen (Ref. 10), $\Lambda=15$ (dashed curve is empirical fit to their numerical results; see text). Note the suppressed zero for the ordinates.

Geometry of the experiment

One of realizations of Strip-type bent crystals

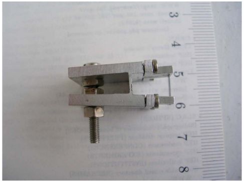
This is IHEP device N1
for efficient (85%)
extraction

Small angle - few mrad
minimal material



Device N2 - big angle, long crystal.
Bent crystal parameters are: 150 mrad bend,
100 mm length and 12 mm width

Device N3 - strong curvature.
Big angle over short length.



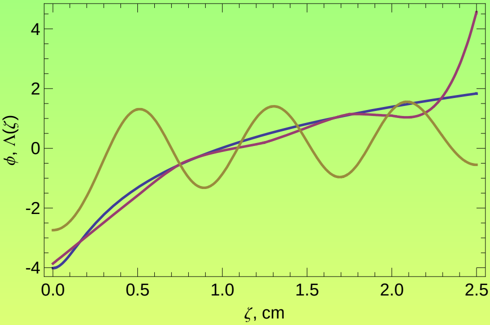
Numerical solution

To solve the integral equation, we need to specify parameters. I chose: $2l = 5$ cm, $d = 1.3$ mm, $a = 1.5$ mm.
I seek numerical solution in the form of expansion

$$\Lambda(t, \zeta) = \sum_{n=1}^{N_m} \alpha_n(t) \cos\left(\frac{\pi n \zeta}{l}\right)$$

Numerical solution

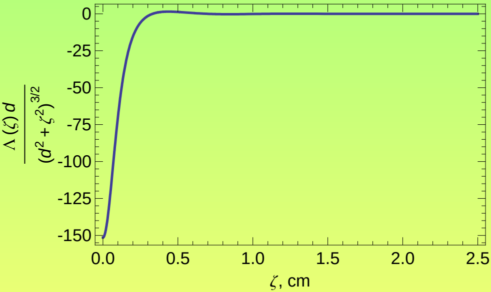
Numerical solution for $N_m = 10$.



Kick on the beam

The kick on the beam in the direction perpendicular to the orbit is due to the electric field of image charges:

$$E_\perp(t) = \int_{-l}^{l} d\zeta \, \Lambda(t,\zeta) \frac{d}{(d^2+\zeta^2)^{3/2}} = \frac{-31.2}{\mathrm{cm}} \lambda_B(t)$$



Kick on the beam

The integrated kick for the nominal number of particles in the bunch $N_b = 1.15 \times 10^{11}$ is

$$\Delta p_\perp = e \int dt E_\perp(t) = \frac{-31.2 e^2 N_b}{\mathrm{cm}} = 0.5 \frac{\mathrm{MeV}}{c}$$

The deflection angle

$$\theta = \frac{0.5\,\mathrm{MeV}}{7\,\mathrm{TeV}} = 7.3 \times 10^{-8} \mathrm{rad}$$

The angular spread in the beam ($\epsilon = 3.75 \times 10^{-6}$ m, $\beta = 100$ m)

$$\sqrt{\frac{\epsilon\gamma}{\beta}} = 2.2 \times 10^{-6}$$

Correspondingly, the deflection angle will result in the orbit bump ~ 7.4 micron.

Conclusion: it seems that the effect of the crystal on the beam is small. A better understanding of the numerical solution of the integral equation is desirable.