Workshop on future muon EDM searches at Fermilab and worldwide

University of Liverpool, 1-12 October 2018

Introduction to MDMs and EDMs

Thomas Teubner

- Motivation
- Overview EDMs and MDMs
- a_e and a_μ in the Standard Model – one more puzzle?
- Messages from BSM

Motivation

SM `too' successful, but incomplete:

- ν masses (small) and mixing point towards some high-scale (GUT) physics, so LFV in neutral sector established, but no Charged LFV & EDMs seen so far
- Need to explain dark matter & dark energy
- Not enough CP violation in the SM for matter-antimatter asymmetry
- And: $a_\mu^{EXP} - a_\mu^{SM}$ at ~ 3-4 σ plus other deviations e.g. in the flavour sector

Is there a common New Physics (NP) explanation for all these puzzles?

- Uncoloured leptons are particularly clean probes to establish and constrain/distinguish NP, complementary to high energy searches at the LHC

- No direct signals for NP from LHC so far:
  - some models like CMSSM are in trouble already when trying to accommodate LHC exclusion limits and to solve muon g-2
  - is there any TeV scale NP out there? Or unexpected new low scale physics?

The key may be provided by low energy observables incl. precision QED, EDMs, LFV.

Introduction: Lepton Dipole Moments

- **Dirac** equation (1928) combines non-relativistic Schroedinger Eq. with rel. Klein-Gordon Eq. and describes spin-1/2 particles and interaction with EM field $A_\mu(x)$:

$$(i\partial_\mu + eA_\mu(x))\,\gamma^\mu \psi(x) \;=\; m\,\psi(x)$$

with gamma matrices $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}I$ and 4-spinors $\psi(x)$.

- Great success: **Prediction** of anti-particles and magnetic moment $\vec{\mu} = g\frac{Qe}{2m}\vec{s}$ with g = 2 (and not 1) in agreement with experiment.

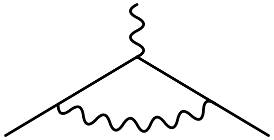
- Dirac already discussed electric dipole moment together with MDM: $\vec{\mu}\cdot\vec{H} + i\rho_1\vec{\mu}\cdot\vec{E}$ but discarded it because imaginary.

- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with $g_s = 2.00229 \pm 0.00008$.

Introduction: Lepton Dipole Moments

- 1948: Schwinger calculates the famous radiative correction: that g = 2 (1+a), with

  a = (g-2)/2 = α/(2π) = 0.001161



  This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

`` If you can't join 'em, beat 'em "

- The anomaly a (Anomalous Magnetic Moment) is from the Pauli term:

$$\delta\mathcal{L}_{\rm eff}^{\rm AMM} = -\frac{Qe}{4m} a\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)F_{\mu\nu}(x)$$

- Similarly, an EDM comes from a term $\delta\mathcal{L}_{\rm eff}^{\rm EDM} = -\frac{d}{2}\bar{\psi}(x)\, i\,\sigma^{\mu\nu}\gamma_5\psi(x)F_{\mu\nu}(x)$

  (At least) dimension 5 operator, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But can occur through radiative corrections, calculable in perturbation theory in (B)SM.

Lepton EDMs and MDMS: d_μ vs. a_μ

- Another reason why we want a direct muon EDM measurement:

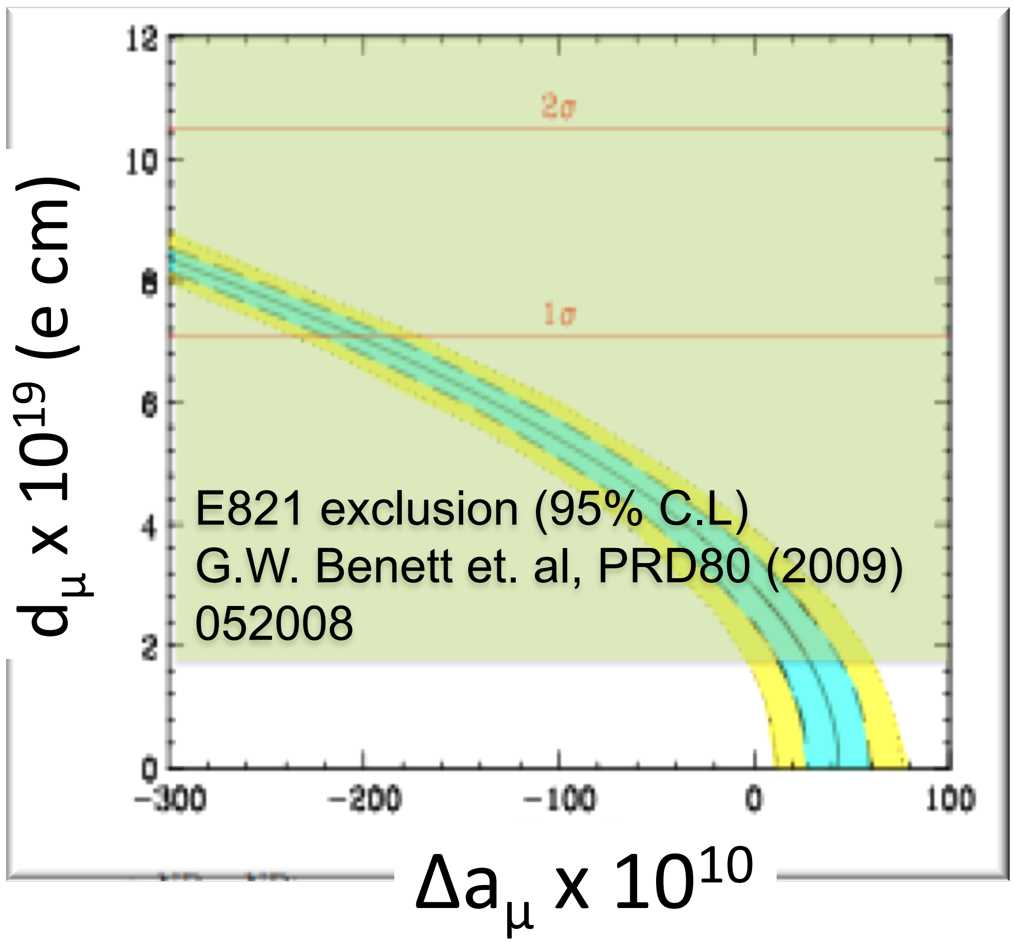
μEDM could in principle fake muon AMM `The g-2 anomaly isn't' (Feng et al. 2001)

$$\vec{\omega} = \vec{\omega}_a + \vec{\omega}_\eta$$

↓

$$\omega = \sqrt{\vec{\omega}_a^2 + \vec{\omega}_\eta^2}$$

- Less room than there was before E821 improved the limit, still want to measure



Introduction: Lepton Dipole Moments

General Lorentz decomposition of spin-1/2 electromagnetic form factor:

$$\langle f(p') \mid J_\mu^{\rm em} \mid f(p)\rangle = \bar{u}_f(p')\Gamma_\mu u_f(p)$$

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + iF_2(q^2)\sigma_{\mu\nu}q^\nu - F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + F_A(q^2)\left(\gamma_\mu q^2 - 2mq_\mu\right)\gamma_5$$

with q = p'-p the momentum transfer. In the static (classical) limit we have:

Dirac FF	$F_1(0) = Qe$	electric charge
Pauli FF	$F_2(0) = a\, Qe/(2m)$	AMM
	$F_3(0) = d\, Q$	EDM

F_2 and F_3 are finite (IR+UV) and calculable in (perturbative) QFT, though they may involve (non-perturbative) strong interaction effects.

$F_A(q^2)$ is the parity violating anapole moment, $F_A(0)=0$.
It occurs in electro-weak loop calculations and is not discussed further here.

Lepton Dipole Moments: complex formalism

- The Lagrangian for the dipole moments can be re-written in a complex formalism (Bill Marciano): $$F_D(q^2) = F_2(q^2) + iF_3(q^2)$$

  and $$\mathcal{L}_{\rm eff}^{\rm D} = -\frac{1}{2}\Big[F_D\bar{\psi}_L\sigma^{\mu\nu}\psi_R + F_D^{\star}\bar{\psi}_R\sigma^{\mu\nu}\psi_L\Big]F_{\mu\nu}$$

  with the right- and left-handed spinor projections $\psi_{R,L} = \frac{1\pm\gamma_5}{2}\psi$ and the **chirality-flip** character of the dipole interaction explicit.

- Then $$F_D(0) = \Big(a\frac{e}{2m} + id\Big)Q = |\ F_D(0)\ |\ e^{i\phi}$$ and

  the phase **Φ** parametrises the size of the EDM relative to the AMM and is **a measure for CP violation**. Useful also to parametrise NP contributions.

- Note: Dirac was wrong. The phase can in general not be rotated away as this would lead to a complex mass. The EDM is not an artifact.

Lepton Dipole Moments & CP violation

- Transformation properties under C, P and T: $\mathcal{H} = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E}$

now: $\vec{\mu},\, \vec{d} \parallel \vec{\sigma}$ and

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	$-$	$+$	$+$
C	$-$	$-$	$-$
T	$+$	$-$	$-$

so a MDM is even under C, P, T, but an EDM is odd under P and T, or,

if CPT holds, for an EDM CP must be violated.

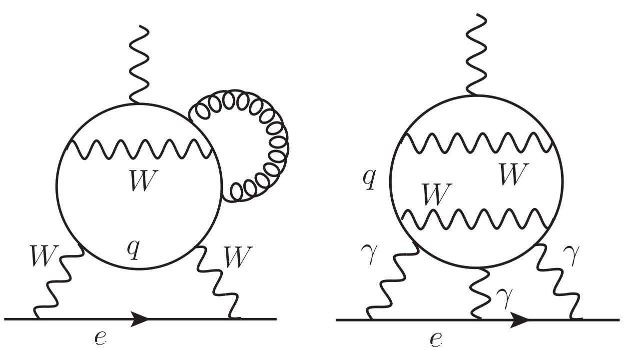
- In the SM (with CP violation only from the CKM phase), lepton EDMs are tiny.

The fundamental d_l only occur at four+ -loops:

Khriplovich+Pospelov,

FDs from Pospelov+Ritz

$d_e^{CKM} \approx O(10^{-44})$ e cm



However: …

Lepton EDMs: measurements vs. SM expectations

- Precision measurement of EDM requires control of competing effect from $\vec{\mu} \cdot \vec{B}$
  μ is large, hence need extremely good control/suppression of B field to O(fG),
  or a big enhancement of $\vec{d} \cdot \vec{E}$
  ➔ eEDM measurements done with atoms or molecules
  [operators other than d_e can dominate by orders of magnitude in SM, 2HDM, SUSY]
- Equivalent EDM of electron from the SM CKM phase is then $d_e^{equiv} \leq 10^{-38}$ e cm

- Could be larger up to ~ $O(10^{-33})$ due to Majorana ν's (d_e already at two-loop), but still way too small for (current & expected) experimental sensitivities, e.g.

- $|d_e| < 8.7 \times 10^{-29}$ e cm from ACME Collab. using ThO [Science 343(2014) 6168]

- Muon EDM: naive scaling $d_\mu \sim (m_\mu/m_e) \cdot d_e$, but can be different (bigger) w. NP
- Best limit on μEDM from E821 @ BNL: $d_\mu < 1.8 \times 10^{-19}$ e cm [PRD 80(2009) 052008]

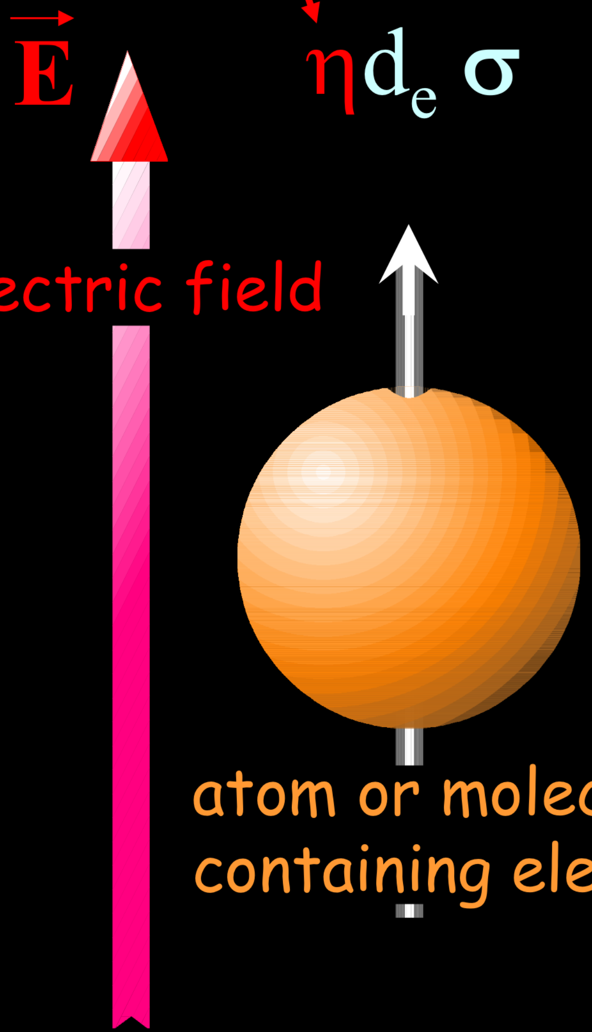
- τ EDM: $-2.2 < d_\tau < 4.5 \times 10^{-17}$ e cm [BELLE PLB 551(2003)16]

A clever solution

For more details, see E. A. H. Physica Scripta T70, 34 (1997)

amplification (Sandars)

$\eta d_e \sigma$



Interaction energy

$$-d_e \, \eta \vec{E} \cdot \vec{\sigma}$$

$\eta \vec{E} \rightarrow F \, P$

P: Polarization factor

F: Structure-dependent relativistic factor $\propto Z^3$

[From Ed Hinds' talk @ Liverpool 2013]

Overview from Rob Timmerman's talk at LM14

1st: the hunt for *discovery*

✓ Recent (and not so) measurements of EDMs:

	System	Group	Limit	C.L.	Value	Year
e	^{205}Tl	Berkeley	1.6×10^{-27}	90%	$6.9(7.4) \times 10^{-28}$	2002
e	YbF	Imperial	10.5×10^{-28}	90	$-2.4(5.7)(1.5) \times 10^{-28}$	2011
e	$Eu_{0.5}Ba_{0.5}TiO_3$	Yale	6.05×10^{-25}	90	$-1.07(3.06)(1.74) \times 10^{-25}$	2012
e	PbO	Yale	1.7×10^{-26}	90	$-4.4(9.5)(1.8) \times 10^{-27}$	2013
e	ThO	ACME	8.7×10^{-29}	90	$-2.1(3.7)(2.5) \times 10^{-29}$	2014
	n	Sussex-RAL-ILL	2.9×10^{-26}	90	$0.2(1.5)(0.7) \times 10^{-26}$	2006
	^{129}Xe	UMich	6.6×10^{-27}	95	$0.7(3.3)(0.1) \times 10^{-27}$	2001
	^{199}Hg	UWash	3.1×10^{-29}	95	$0.49(1.29)(0.76) \times 10^{-29}$	2009
	muon	E821 BNL $g-2$	1.8×10^{-19}	95	$0.0(0.2)(0.9) \times 10^{-19}$	2009

✓ Current EDM null results → probe TeV scale or $\phi_{CP} \leq O(10^{-2})$

- Next generation sensitive to 10 TeV (beyond LHC) or $\phi_{CP} \leq O(10^{-4})$

EDMs. Strong CP violation

- In principle there could be large CP violation from the `theta world' of QCD:

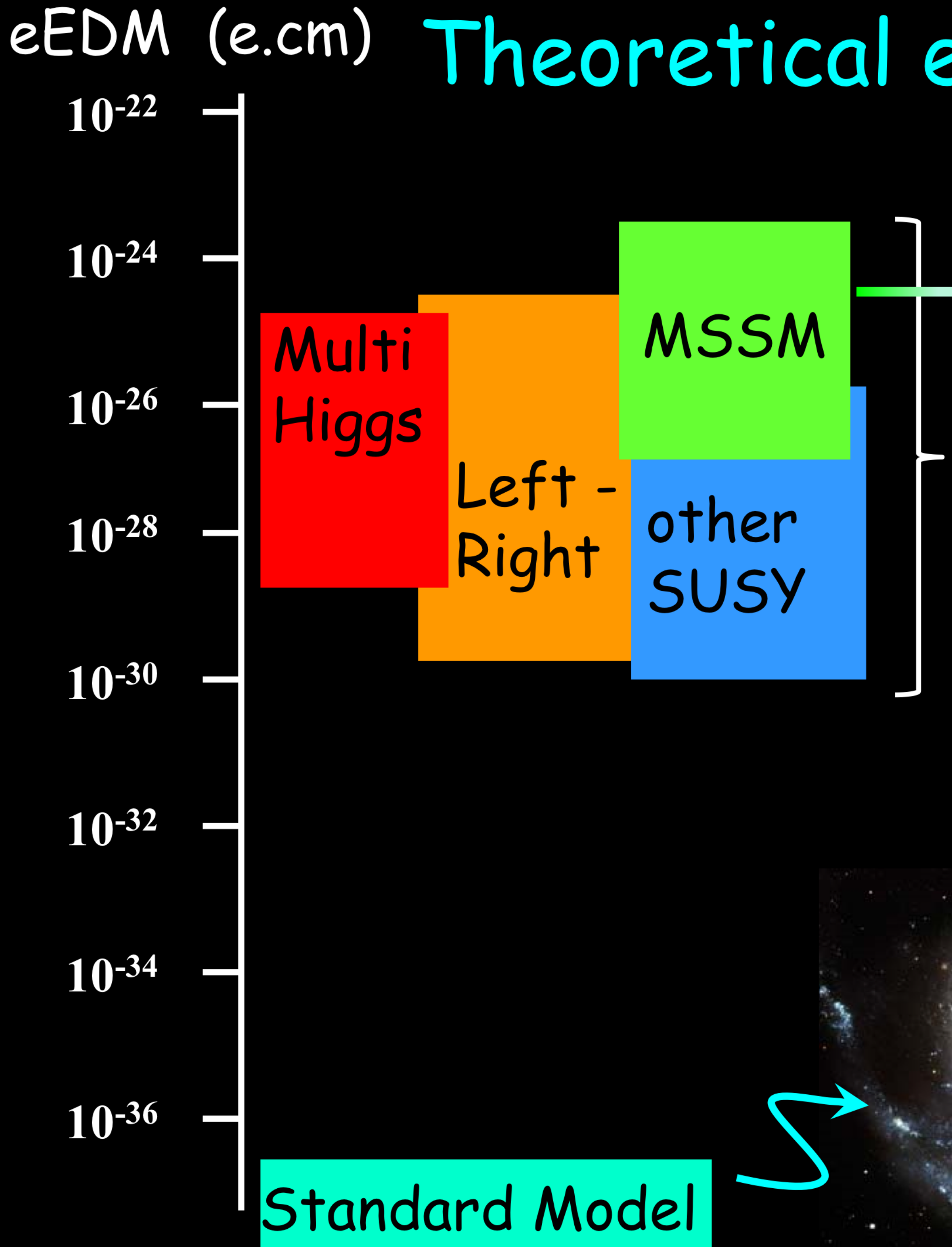
$$\mathcal{L}_{\text{QCD}}^{\text{eff}} = \mathcal{L}_{\text{QCD}} + \theta \frac{g_{\text{QCD}}^2}{32\pi^2} F^{a\mu\nu} \tilde{F}_{\mu\nu}^a , \qquad \tilde{F}_{\mu\nu}^a = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} F^{a\alpha\beta}$$

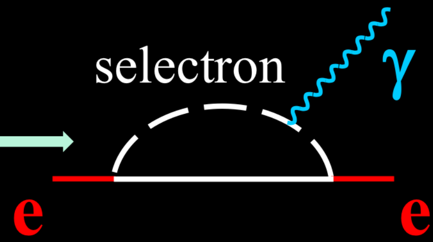
- $F\tilde{F}$ is P- and T-odd, together with non-perturbative (strong) instanton effects, $\Theta \neq 0$ could lead to **strong CP violation** and n and p EDMs, $d_n \approx 3.6\times10^{-16}\ \theta$ e cm
  - only if all quark masses ≠ 0 ✓
  - operator of θ term same as axial U(1) anomaly (from which $m_{\eta'} > m_\pi$), no fiction
- However, effective $\theta \leq 10^{-10}$ from nEDM limit: $|d_n| < 2.9\ 10^{-26}$ e cm [PRL97,131801]

- Limits on pEDM from atomic eEDM searches; in SM expect $|d_N| \approx 10^{-32}$ e cm.
  Ideally want to measure d_n and d_p to disentangle iso-vector and iso-scalar NEDM (strong CP from θ predicts iso-vector, $d_n \approx -d_p$, in leading log, but sizeable corrections)
- See Yannis Semertzidis's proposal to measure the pEDM at a storage ring

- Any non-zero measurement of a lepton or nucleon EDM would be a sign for CP violation beyond the SM and hence NP.

Theoretical estimates of eEDM



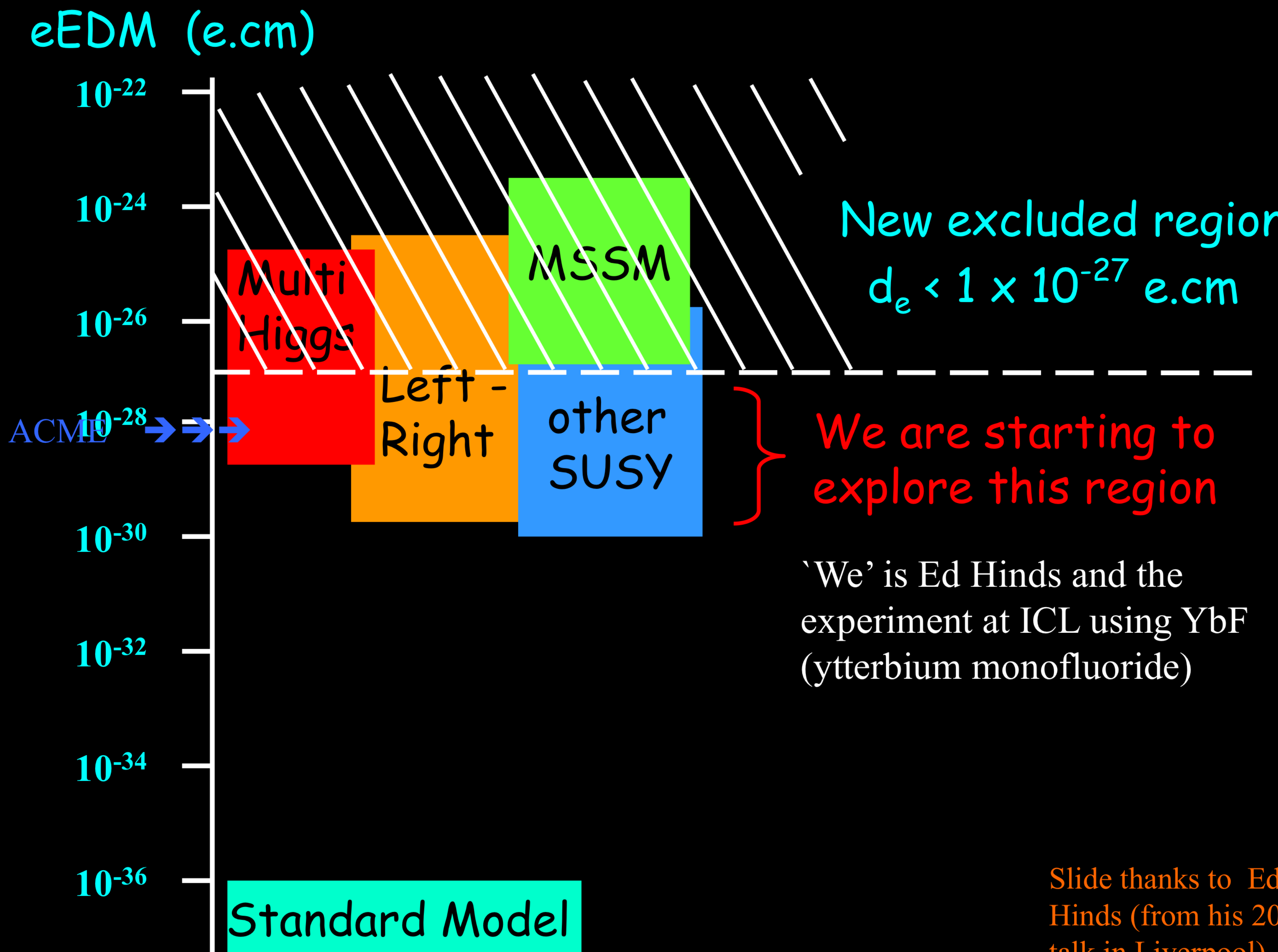


$$\frac{\alpha}{\pi}\frac{m_e}{m_s^2}\sin\phi$$

The interesting region of sensitivity

Insufficient ~~CP~~ to make universe of matter

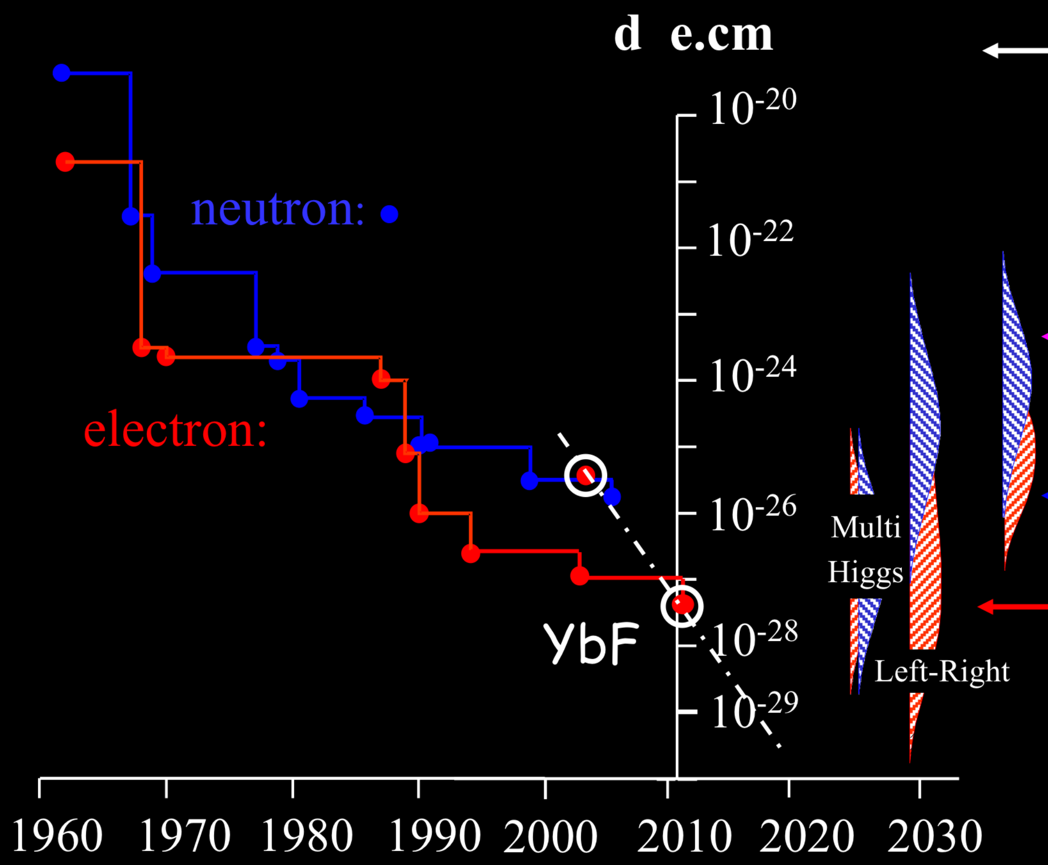
Slide thanks to Ed Hinds (from his 2013 talk in Liverpool)



`We' is Ed Hinds and the experiment at ICL using YbF (ytterbium monofluoride)

Slide thanks to Ed Hinds (from his 2013 talk in Liverpool)

Current status of EDMs



d(muon) < 1.9×10^{-19}

d(proton) < 5×10^{-24}

d(neutron) < 3×10^{-26}

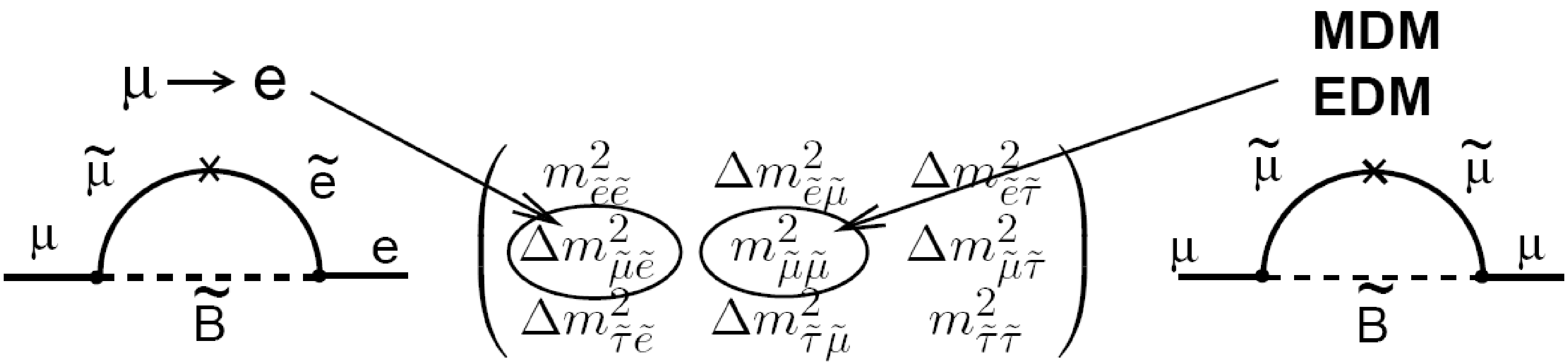
d(electron) < 1×10^{-27}

← ACME: < 8.7 10^{-29}

Slide thanks to Ed Hinds (from his 2013 talk in Liverpool)

SUSY in CLFV and dipole moments

Contributions to CLFV and DMs related to elements of slepton mixing matrix:



Large contributions to g-2 ➔ large LFV, but:

bound from MEG on μ -> eγ rules out most of the parameter space of certain SUSY models:

Motivation: SUSY in CLFV and DMs

[From Tsutomu Mibe]

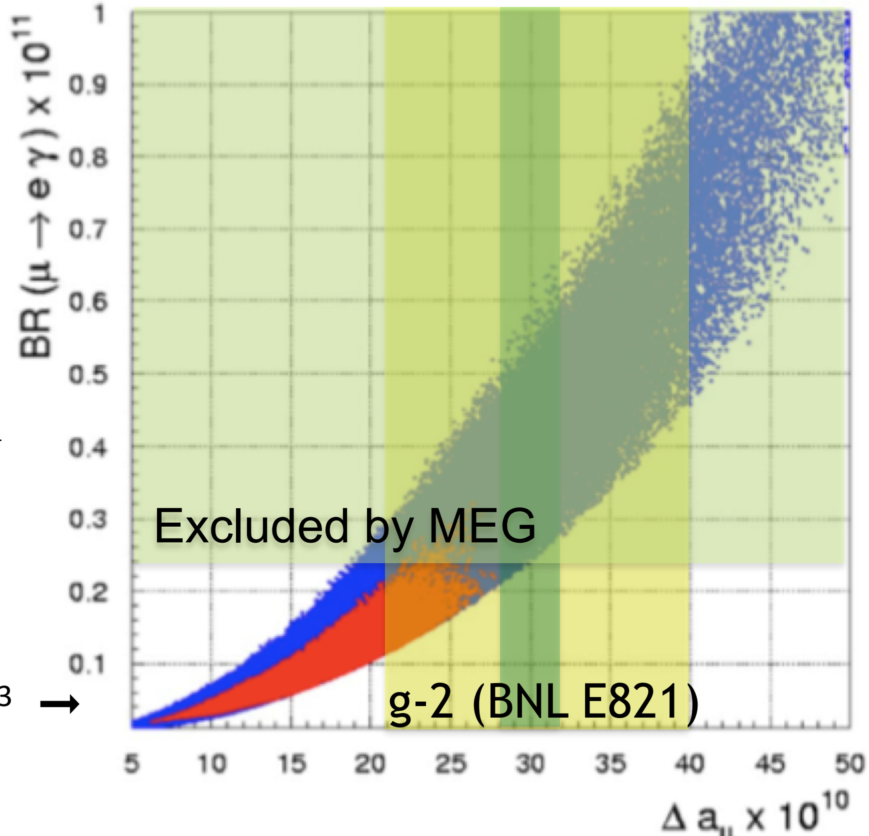
- **Large g-2 → Large CLFV**

G. Isidori, F. Mescia, P. Paradisi, and D. Temes, PRD 75 (2007) 115019
Flavour physics with large tan β with a Bino-like LSP

$$Br(\mu \to e\gamma) \times 10^{11}$$

MEG limit now even:

$< 4.2 \times 10^{-13}$ →



deviation from SM (g-2)

Magnetic Moments $\vec{\mu} = g\frac{Qe}{2m}\vec{s}$

- g-factor = 2(1+a) for spin-½ fermions

- anomaly calculable in PT for point-like leptons and is small as α/π suppressed,

$$a = \sum_i \mathcal{C}_i (\alpha/\pi)^i, \quad \mathcal{C}_1 = 1/2$$ Schwinger's leading QED contribution

- For nucleons corrections to g=2 come from sub-structure and are large, can be understood/parametrised within quark models

- Experimental g values: (g>2 → spin precession larger than cyclotron frequency)

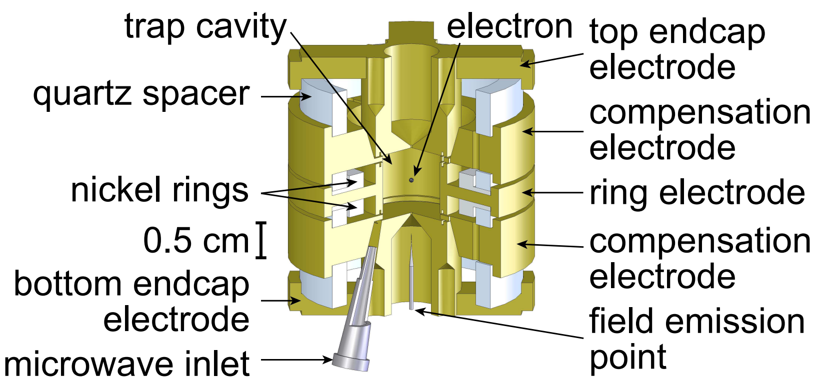
  e: 2.002 319 304 361 46(56) [Harvard 2008]
  μ: 2.002 331 841 8(13) [BNL E821]
  τ: g compatible with 2, $-0.052 < a_\tau < 0.013$ [DELPHI at LEP2, $e^+e^- \to e^+e^-\tau^+\tau^-$
  [similar results from L3 and OPAL, $e^+e^- \to \tau^+\tau^-\gamma$]
  p: 5.585 694 713(46)
  n: -3.826 085 44(90)

- Let's turn to the TH predictions for a_e and a_μ

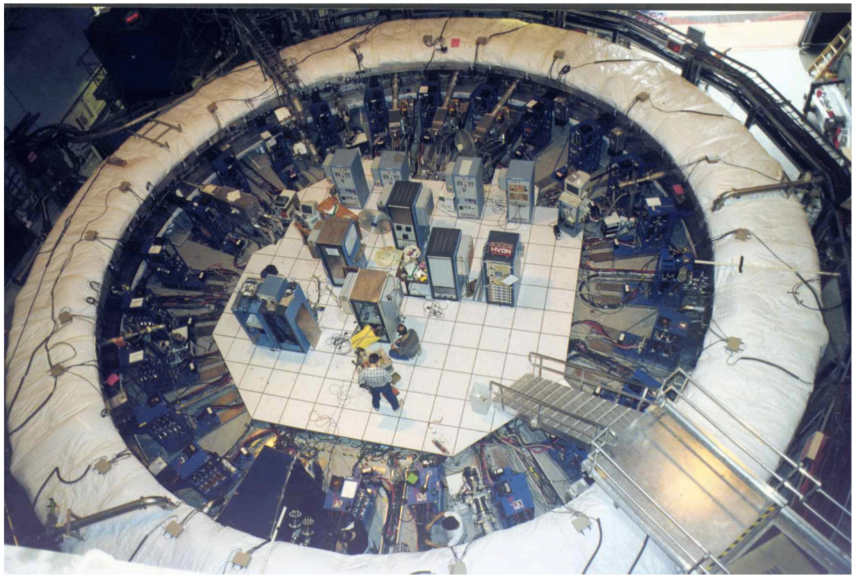
Magnetic Moments: a_e vs. a_μ

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]
Bennet et al., PRD 73(2006)072003



one electron quantum cyclotron



- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales

- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{\rm NP} \sim \mathcal{C}\, m_\ell^2/\Lambda_{\rm NP}^2$

→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, but a_e provides precise determination of α

Magnetic Moments: a_e^{SM} before very recent shift of α

- General structure: $$a_e^{\rm SM} = a_e^{\rm QED} + a_e^{\rm hadronic} + a_e^{\rm weak}$$

- Weak and hadronic contributions suppressed as induced by particles heavy compared to electron, hence a_e^{SM} dominated by QED

  $a_e^{SM} = 1\ 159\ 652\ 182.03(72) \times 10^{-12}$ [Aoyama+Kinoshita+Nio, PRD 97(2018)036001]

  small shift from ….81.78(77) after 2018 update of numerics

  including 5-loop QED and using α measured with Rubidium atoms [α to 0.66 ppb]

  [Bouchendira et al., PRL106(2011)080801; Mohr et al., CODATA, Rev Mod Phys 84(2012)1527]

  ➞ but see below for new puzzle due to recent α measurement with Cs atoms

  Of this only about

  $a_e^{\text{had, LO VP}} = 1.875(18) \times 10^{-12}$ [or our newer $1.866(11) \times 10^{-12}$]

  $a_e^{\text{had, NLO VP}} = -0.225(5) \times 10^{-12}$ [or our newer $-0.223(1) \times 10^{-12}$]

  $a_e^{\text{had, L-by-L}} = 0.035(10) \times 10^{-12}$

  $a_e^{\text{weak}} = 0.0297(5) \times 10^{-12}$,

  whose calculations are a byproduct of the μ case which I will discuss in a bit more detail.

- In turn a_e^{EXP} and a_e^{SM} can be used to get a very precise determination of α, to 0.25 ppb, consistent with Rubidium experiment and other determinations.

Magnetic Moments: a_e^{SM} with the recent shift of α

- General structure: $$a_e^{\rm SM} = a_e^{\rm QED} + a_e^{\rm hadronic} + a_e^{\rm weak}$$

- $a_e^{SM} = 1\ 159\ 652\ 182.03(72) \times 10^{-12}$ [Aoyama+Kinoshita+Nio, PRD 97(2018)036001]

  small shift from ....81.78(77) after 2018 update of numerics

  using α measured with Rubidium atoms [α to 0.66 ppb]

- is, due to a new α measurement with Cs-133 atoms [Parker et al., Science 360 (2018) 191], now more precise [α to $2{\times}10^{-10}$!] and shifted down to

  $a_e^{SM} = 1\ 159\ 652\ 181.61(23) \times 10^{-12}$

- Comparison with the experimental measurement now gives a -2.5 σ discrepancy for a_e: $\Delta a_e = a_e^{EXP} - a_e^{SM} = -\ 0.88(36) \times 10^{-12}$

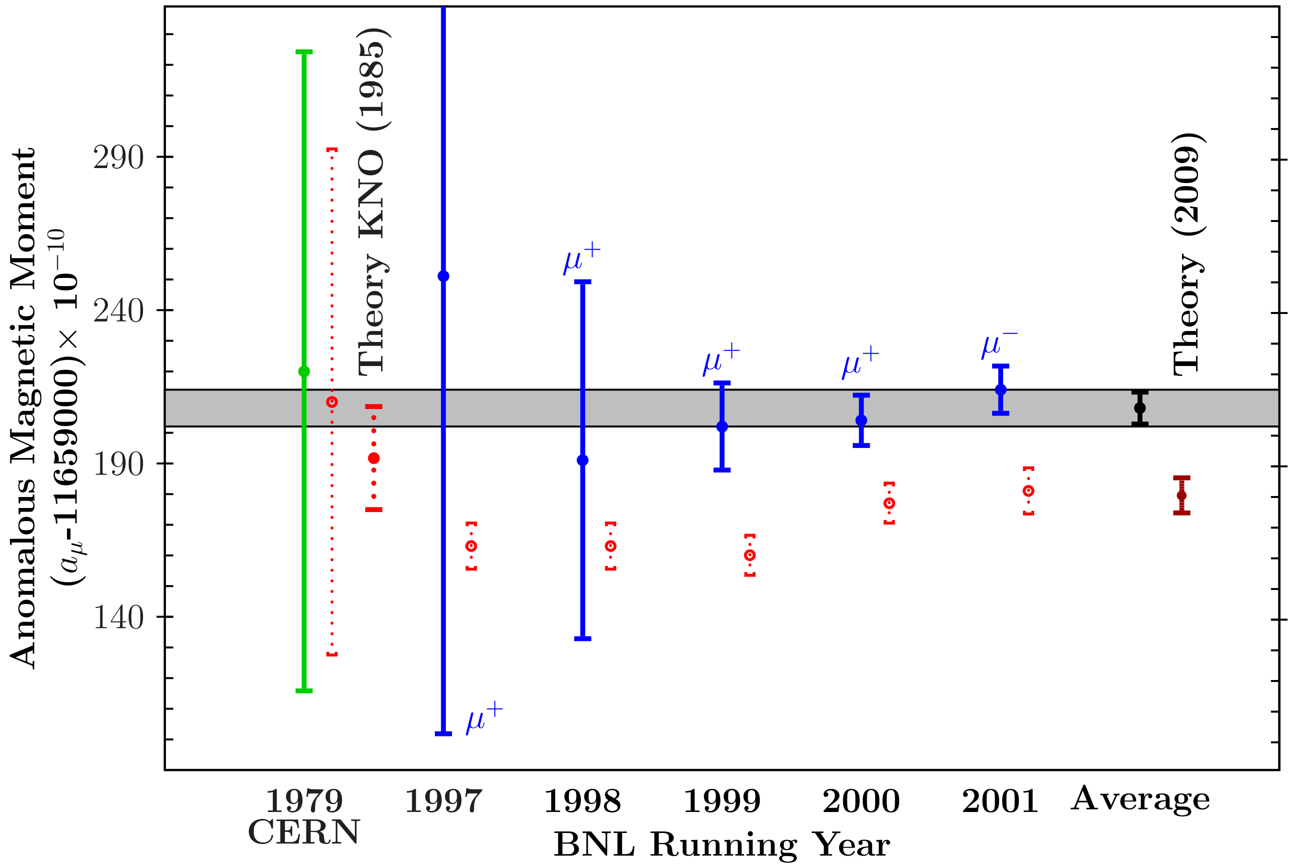
- which one may consider together with the muon g-2 discrepancy when discussing possible New Physics contributions

a_μ: back to the future

- CERN started it nearly 40 years ago
- Brookhaven delivered 0.5ppm precision
- E989 at FNAL and J-PARC's g-2/EDM experiments are happening and should give us certainty

g-2 history plot and
book motto from Fred Jegerlehner:

`The closer you look the more there is to see'



a_μ: Status and future projection → charge for SM TH

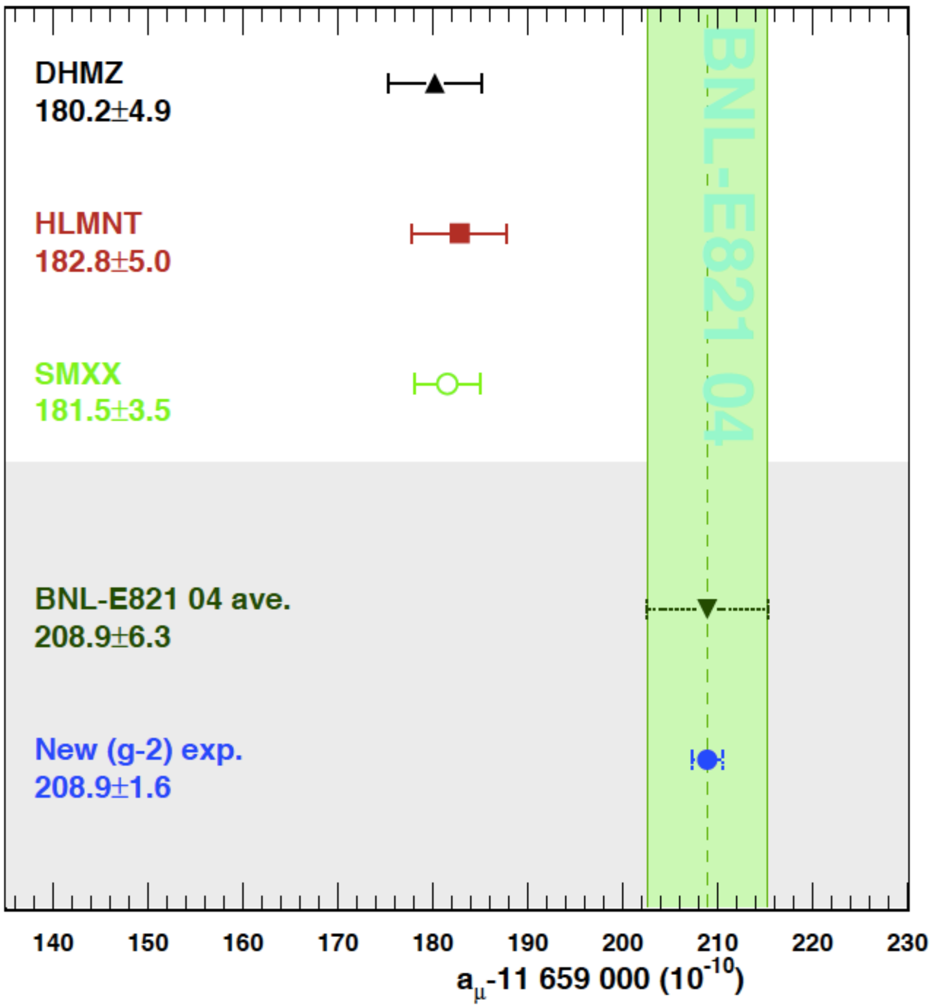
$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

From: arXiv:1311.2198
`The Muon (g-2) Theory Value: Present and Future'

- if mean values stay and with no a_μ^{SM} improvement: 5σ discrepancy

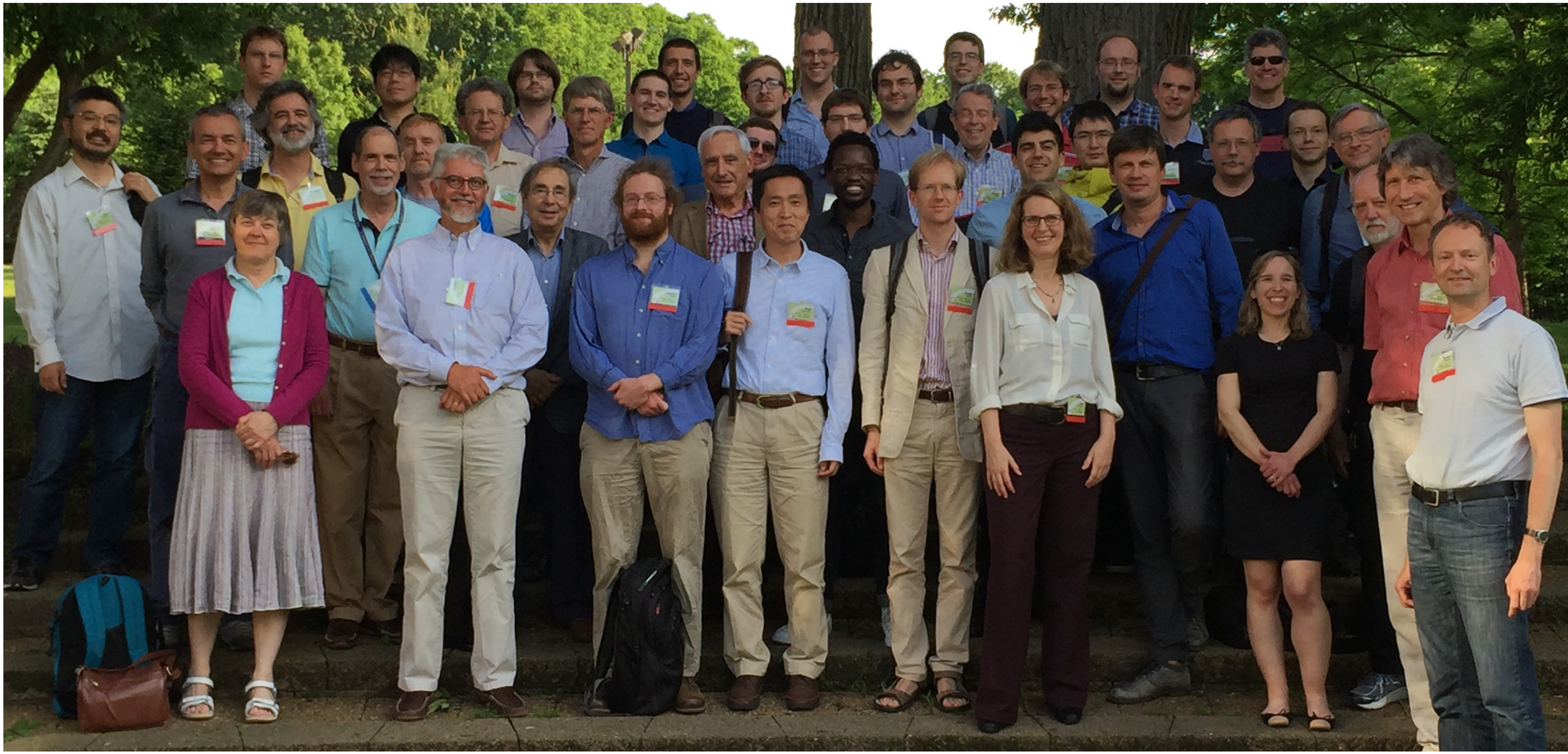
- if also EXP+TH can improve a_μ^{SM} `as expected' (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8σ

- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)…



“Muon g-2 theory initiative”, formed in June 2017

for latest June 2018 workshop see: https://indico.him.uni-mainz.de/event/11/overview



“map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results”

PRD 97, 114025
`KNT18'

The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis

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Abstract

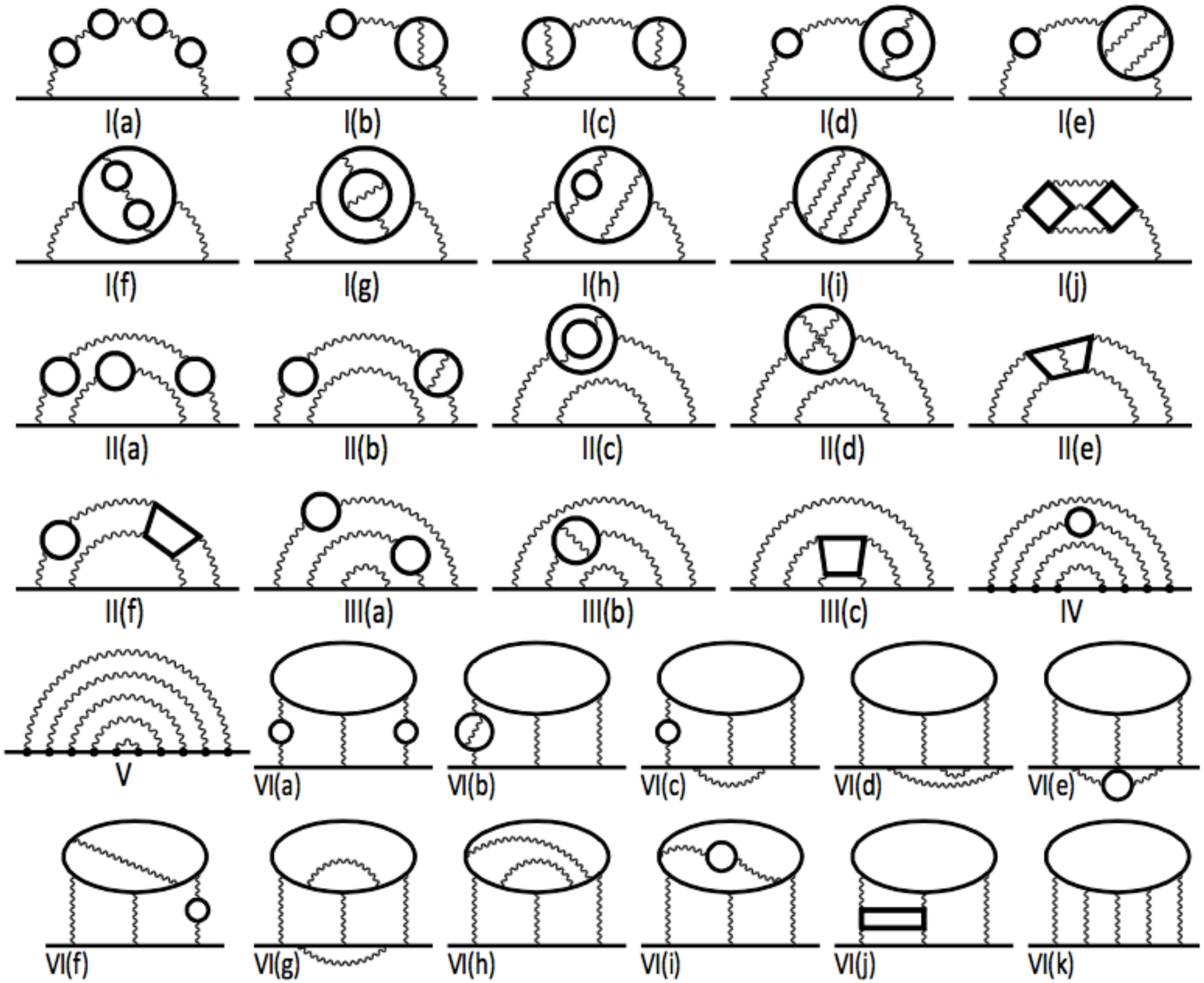
This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon, $a_\mu^{\rm had,\,VP}$ and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\rm had}(M_Z^2)$, from the combination of $e^+e^- \to$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^- \to$ hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic R-ratio and its covariance matrix in the energy range $m_\pi \leq \sqrt{s} \leq 11.2$ GeV. Using these combined data and pQCD above that range results in estimates of the hadronic vacuum polarisation contributions to $g-2$ of the muon of $a_\mu^{\rm had,\,LO\,VP} = (693.27 \pm 2.46) \times 10^{-10}$ and $a_\mu^{\rm had,\,NLO\,VP} = (-9.82 \pm 0.04) \times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\rm SM} = (11\,659\,182.05 \pm 3.56) \times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$, resulting in $\alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$. Detailed comparisons with results from similar related works are given.

a_μ^{QED} Kinoshita et al.: g-2 at 1, 2, 3, 4 & 5-loop order

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

A triumph for perturbative QFT and computing!

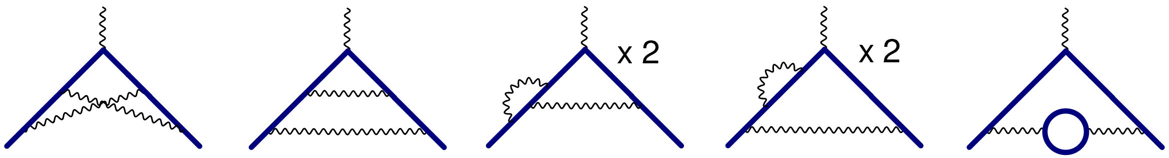
10th
12672
diagrams



- code-generating code, including
- renormalisation
- multi-dim. numerical integrations

a_μ^{QED}

- Schwinger 1948: 1-loop $a = (g-2)/2 = \alpha/(2\pi) = 116\,140\,970 \times 10^{-11}$
- 2-loop graphs:



- 72 3-loop and 891 4-loop diagrams …
- Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams):

$$a_\mu^{QED} = 116\,584\,718.951\ (0.009)\ (0.019)\ (0.007)\ (0.077) \times 10^{-11}$$

errors from: lepton masses, 4-loop,   5-loop,   α from ^{87}Rb

- QED extremely accurate, and the series is stable: $a_\mu^{\rm QED} = C_\mu^{2n} \sum_n \left(\frac{\alpha}{\pi}\right)^n$

$$C_\mu^{2,4,6,8,10} = 0.5,\ 0.765857425(17),\ 24.05050996(32),\ 130.8796(63),\ 753.29(1.04)$$

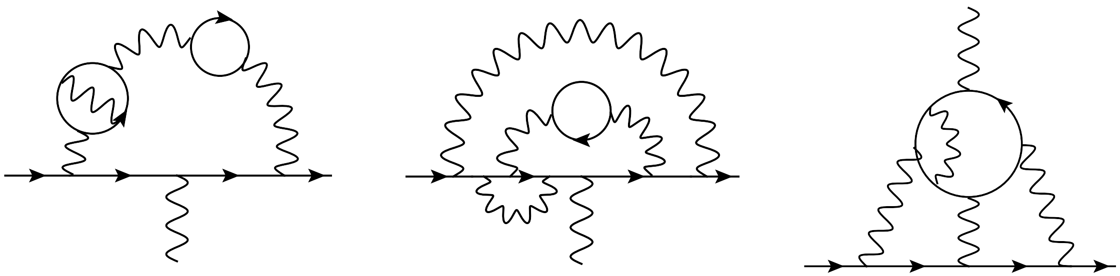
- Could a_μ^{QED} still be wrong?
  Some classes of graphs known analytically (Laporta; Aguilar, Greynat, deRafael),

a_μ^{QED}

- … but 4-loop and 5-loop rely heavily on numerical integrations

- Recently several independent checks of 4-loop and 5-loop diagrams:
  Baikov, Maier, Marquard [NPB 877 (2013) 647], Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:

- all 4-loop graphs with internal lepton loops now calculated independently, e.g.



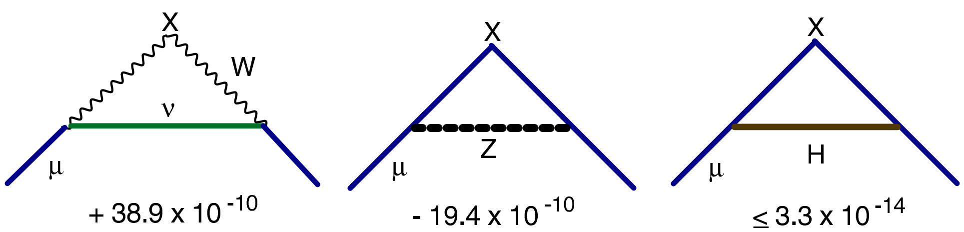
(from Steinhauser et al., PRD 93 (2016) 053017)

- 4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, arXiv:1704.06996, also new numerical results by Volkov, 1705.05800

- all agree with Kinoshita et al.'s results, so QED is on safe ground ✓

a_μ Electro-Weak

- Electro-Weak 1-loop diagrams:



$a_\mu^{EW(1)} = 195\times10^{-11}$

- known to 2-loop (1650 diagrams, the first full EW 2-loop calculation):
  Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

- agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{EW(1+2\,loop)} = (154\pm2)\times10^{-11}$

- Higgs mass now known, update by Gnendiger, Stoeckinger, S-Kim,
  PRD 88 (2013) 053005

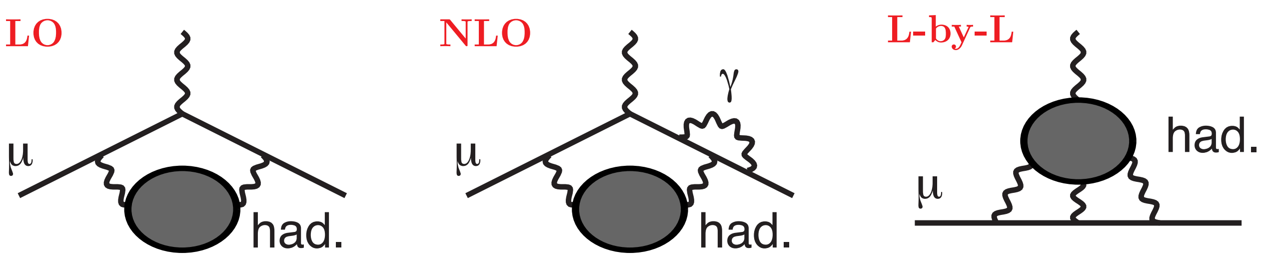
$$a_\mu^{EW(1+2\,loop)} = (153.6\pm1.0)\times10^{-11}$$ ✓

compared with $a_\mu^{QED} = 116\,584\,718.951\,(80)\times10^{-11}$

a_μ hadronic

- Hadronic: non-perturbative, the limiting factor of the SM prediction? ✗ → ✓

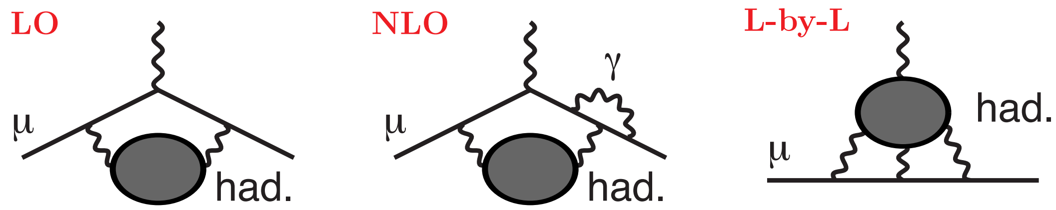
$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$



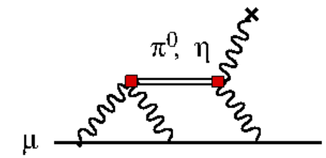
a_μ^{hadronic} : L-by-L one-page summary

- Hadronic: non-perturbative, the limiting factor of the SM prediction ✗ → ✓

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$



e.g.



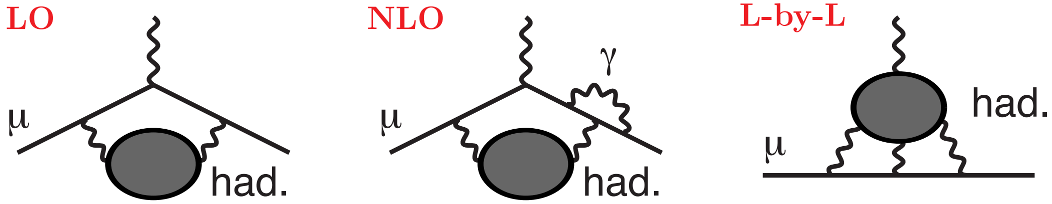
- L-by-L: - so far use of model calculations (+ form-factor data and pQCD constraints),
  - but very good news from lattice QCD, and
  - from new dispersive approaches

- For the moment, still use the \`updated Glasgow consensus':
  (original by Prades+deRafael+Vainshtein) $a_\mu^{\text{had,L-by-L}} = (98 \pm 26) \times 10^{-11}$

- But first results from new approaches confirm existing model predictions and
- indicate that L-by-L prediction will be improved further
- with new results & progress, tell politicians/sceptics: L-by-L _can_ be predicted!

$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

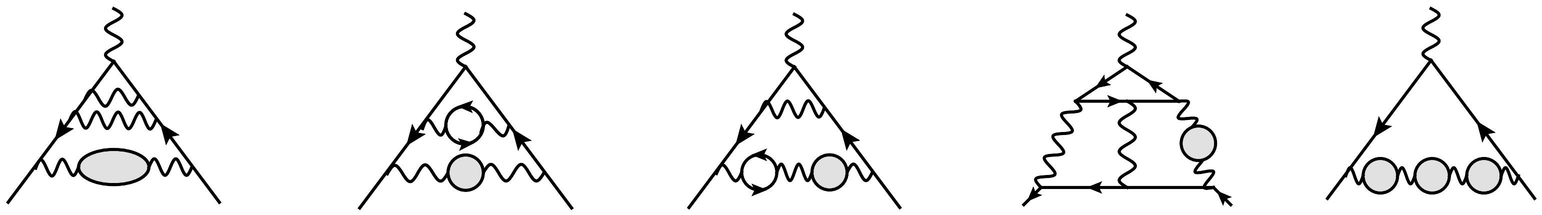
$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$



HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and well known dispersion integrals

- done at LO and NLO (see graphs)

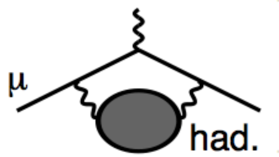
- and recently at NNLO [Steinhauser et al., PLB 734 (2014) 144, also F. Jegerlehner]
$a_\mu^{\text{HVP, NNLO}} = + 1.24 \times 10^{-10}$ not so small, from e.g.:



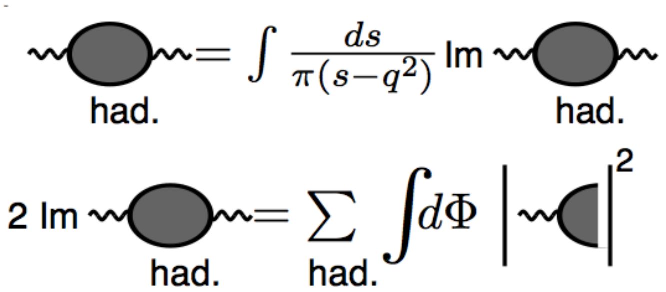
- Alternative: lattice QCD, but need QED and iso-spin breaking corrections
Lots of activity by several groups, errors coming down, QCD+QED started

Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

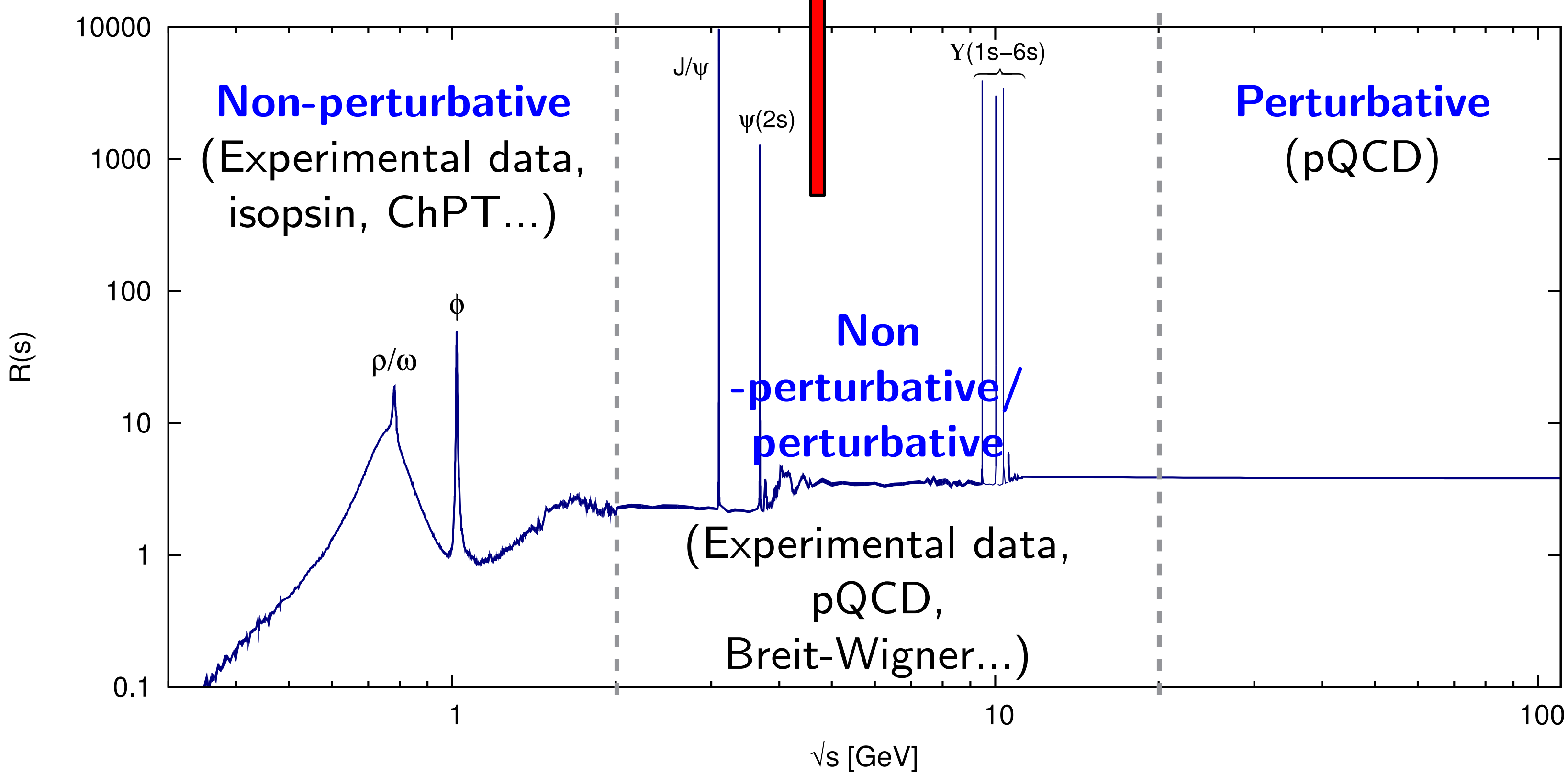
$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

How to get the most precise $\sigma^0_{\rm had}$? e^+e^- data:

- Low energies: sum ~30 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, …, use iso-spin relations for missing channels
- Above ~1.8 GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)
- Challenge of data combination (locally in √s): many experiments, different energy bins, stat+sys errors from different sources, correlations; must avoid inconsistencies/bias
- traditional `direct scan' (tunable $e^+e^-$ beams) vs. `Radiative Return' [+ τ spectral functions]
- $\sigma^0_{\rm had}$ means `bare' σ, but WITH FSR: RadCorrs [HLMNT '11: $\delta a_\mu^{\rm had,\ RadCor\ VP+FSR} = 2 \times 10^{-10}$!]

Hadronic cross section input

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\,R(s)K(s),\ \text{where}\ R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

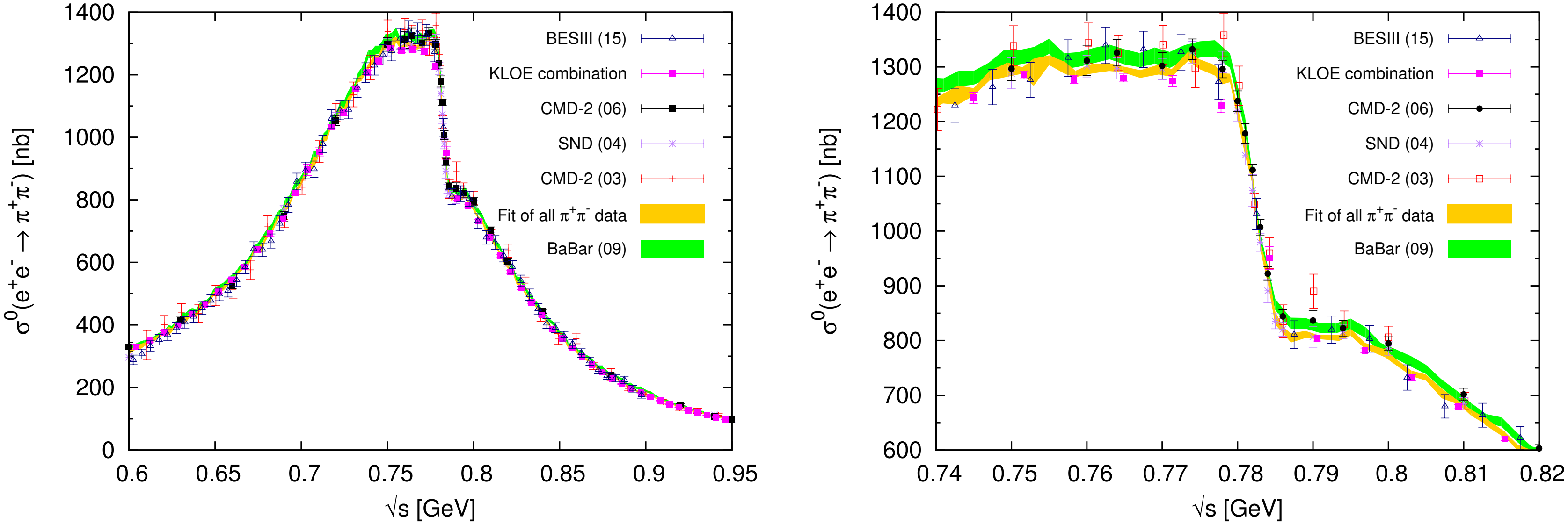


Must build full hadronic cross section/R-ratio...

$\pi^+\pi^-$ channel [KNT18: arXiv:1802.02995]

$\Rightarrow$ $\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\text{had, LO VP}}$

$\rightarrow$ Combines 30 measurements totalling nearly 1000 data points



$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \le \sqrt{s} \le 1.937 \text{ GeV}] = 502.97 \pm 1.14_{\text{stat}} \pm 1.59_{\text{sys}} \pm 0.06_{\text{vp}} \pm 0.14_{\text{fsr}}$$

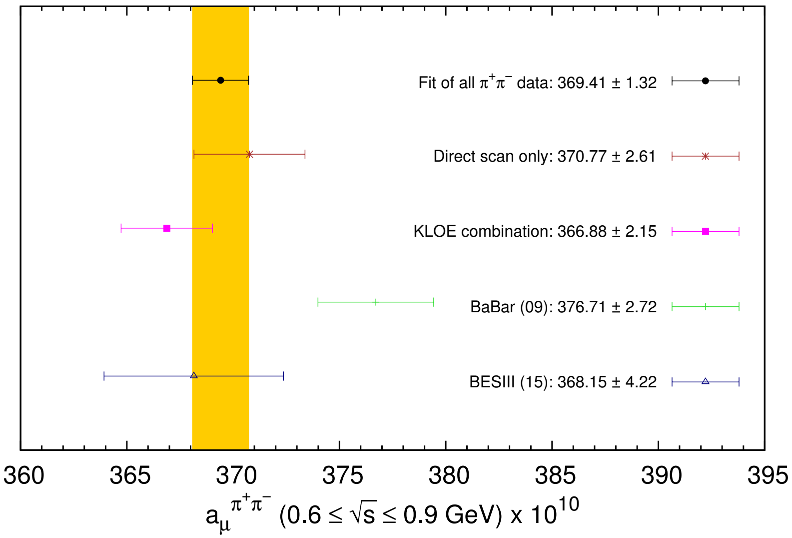
$$= 502.97 \pm 1.97_{\text{tot}} \quad \text{HLMNT11: } 505.77 \pm 3.09$$

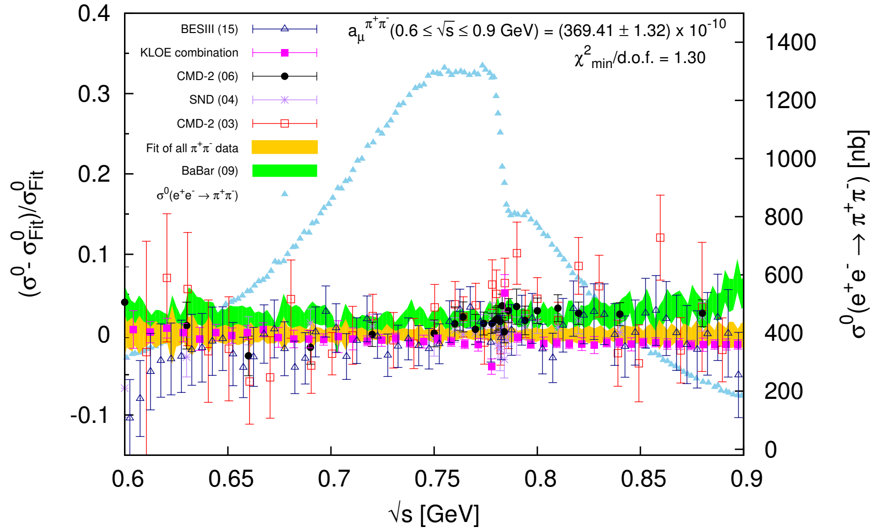
$\Rightarrow$ 15% local $\chi^2_{\text{min}}/\text{d.o.f.}$ error inflation due to tensions in clustered data

$\pi^+\pi^-$ channel [KNT18: arXiv:1802.02995]

$\Rightarrow$ Tension exists between BaBar data and all other data in the dominant ρ region.

$\rightarrow$ Agreement between other radiative return measurements and direct scan data largely compensates this.



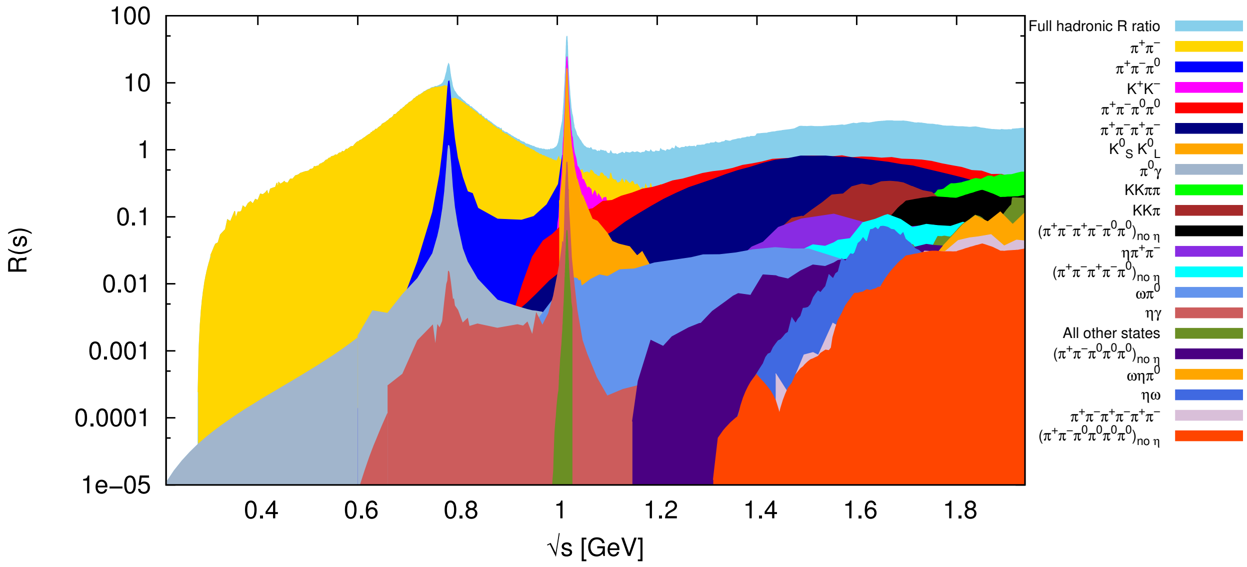


BaBar data alone $\Rightarrow a_\mu^{\pi^+\pi^-}$(BaBar data only) $= 513.2 \pm 3.8$.

Simple weighted average of all data $\Rightarrow a_\mu^{\pi^+\pi^-}$(Weighted average) $= 509.1 \pm 2.9$.
(i.e. - no correlations in determination of mean value)

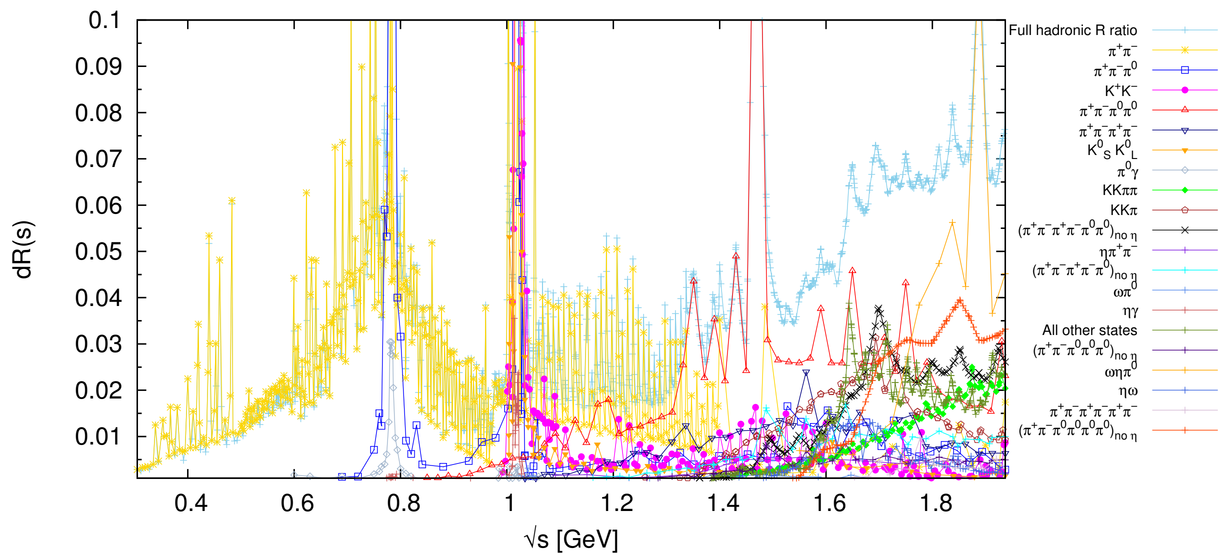
BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

Contributions below 2GeV [KNT18: arXiv:1802.02995]



→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty

→ Large improvement to cross section and uncertainty from new 4π data



KNT18 $a_\mu^{\text{had, VP}}$ update [KNT18: arXiv:1802.02995]

HLMNT(11): 694.91 ± 4.27

$\downarrow$

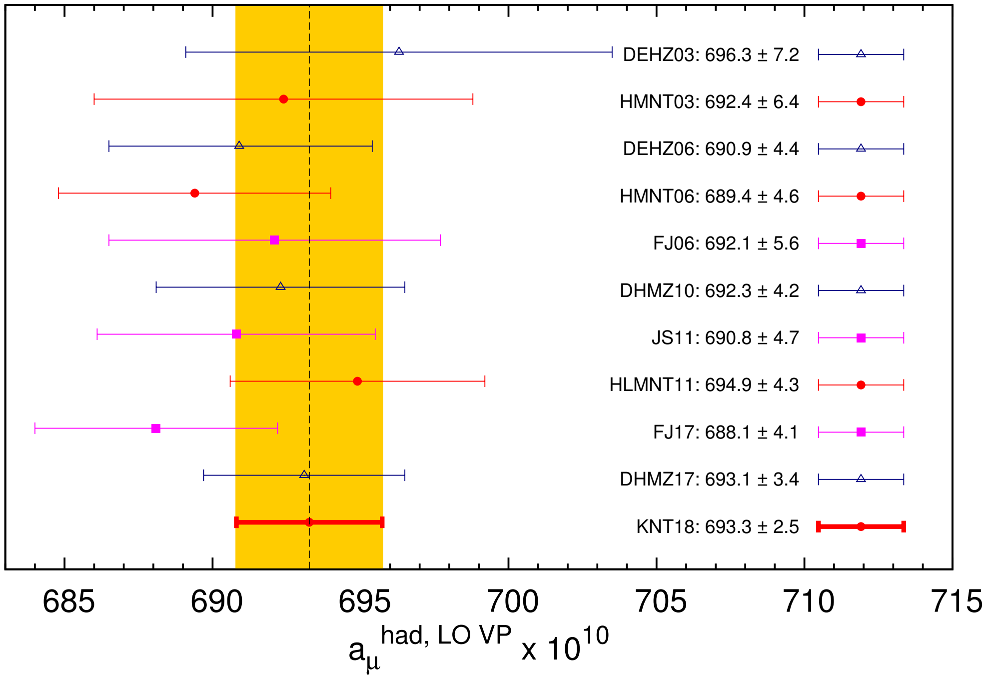
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

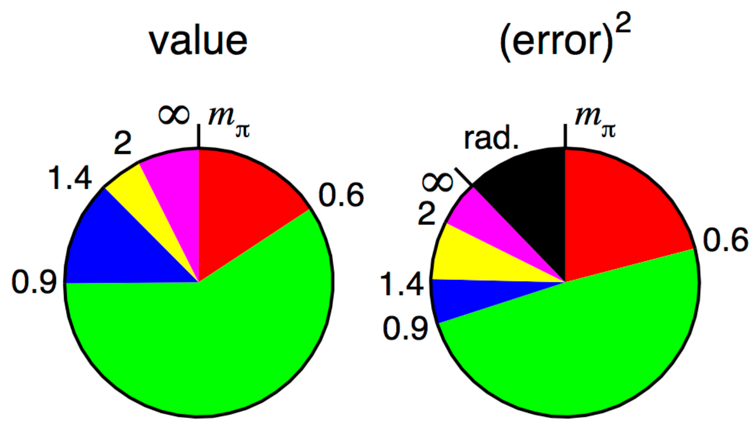
$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$

$= 693.27 \pm 2.46_{\text{tot}}$

$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$

$\Rightarrow$ Accuracy better then 0.4%
(uncertainties include all available correlations)



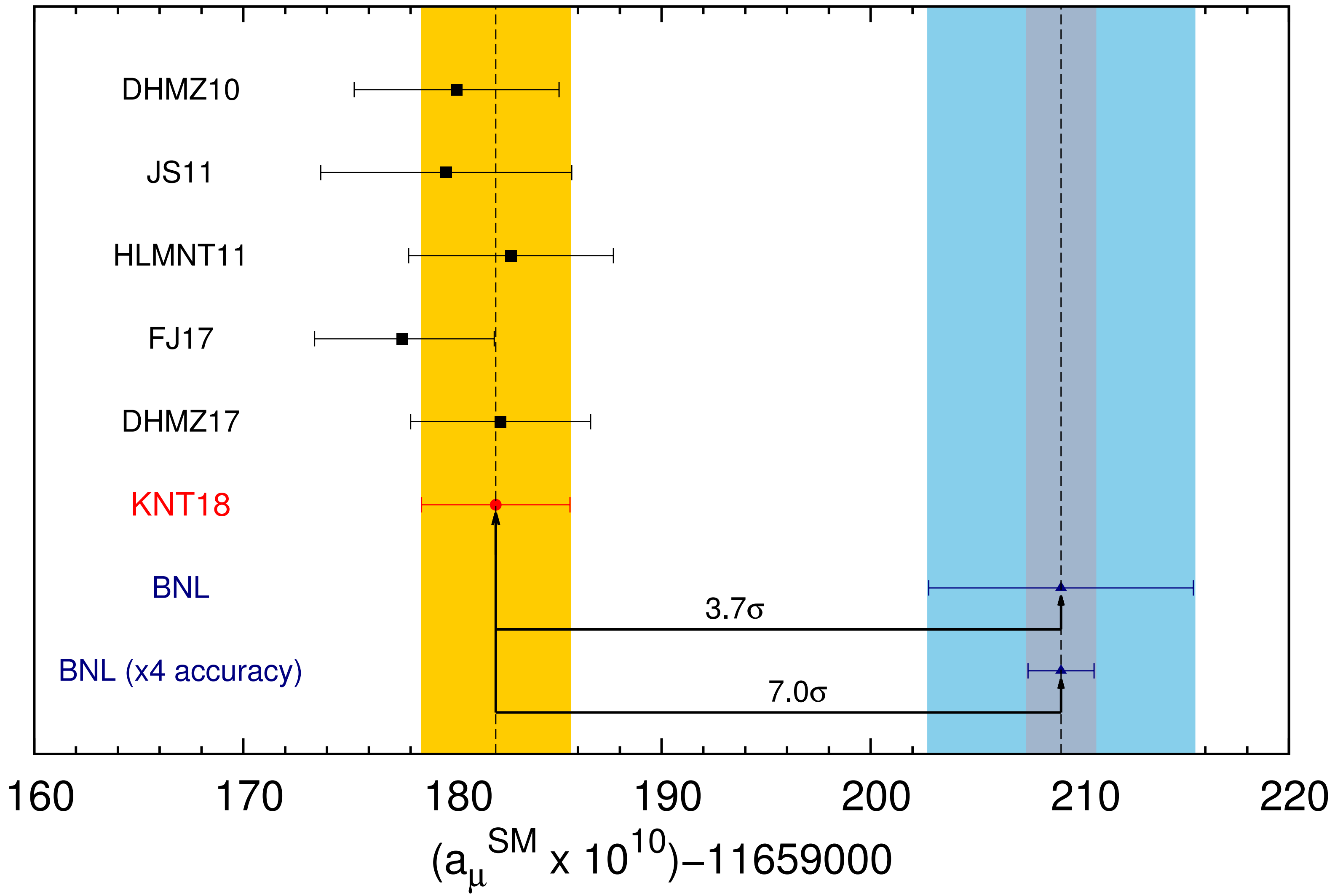


$\Rightarrow 2\pi$ dominance

KNT18 $a_\mu^{\rm SM}$ update [KNT18: arXiv:1802.02995]

	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

KNT18 $a_\mu^{\rm SM}$ update [KNT18: arXiv:1802.02995]

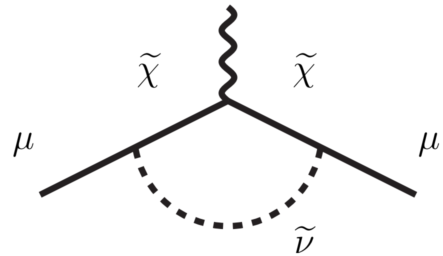


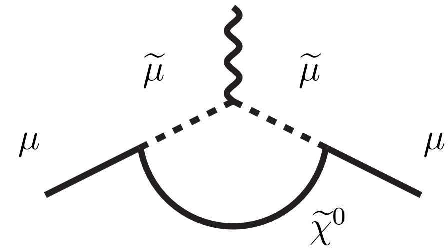
a_μ: New Physics?

- Many BSM studies use g-2 as constraint or even motivation

- SUSY could easily explain g-2

  - Main 1-loop contributions:





  - Simplest case:

$$a_\mu^{\mathrm{SUSY}} \simeq sgn(\mu)\, 130 \times 10^{-11} \tan\beta \left(\frac{100\,\mathrm{GeV}}{\Lambda_{\mathrm{SUSY}}}\right)^2$$

  - Needs $\mu>0$, \`light’ SUSY-scale Λ and/or large tan β to explain 281 x 10^{-11}

  - This is already excluded by LHC searches in the simplest SUSY scenarios (like CMSSM); causes large χ^2 in simultaneous SUSY-fits with LHC data and g-2

  - However:
    * SUSY does not have to be minimal (w.r.t. Higgs),
    * could have large mass splittings (with lighter sleptons),
    * be hadrophobic/leptophilic,
    * or not be there at all, but don’t write it off yet…

New Physics? just a few of many recent studies

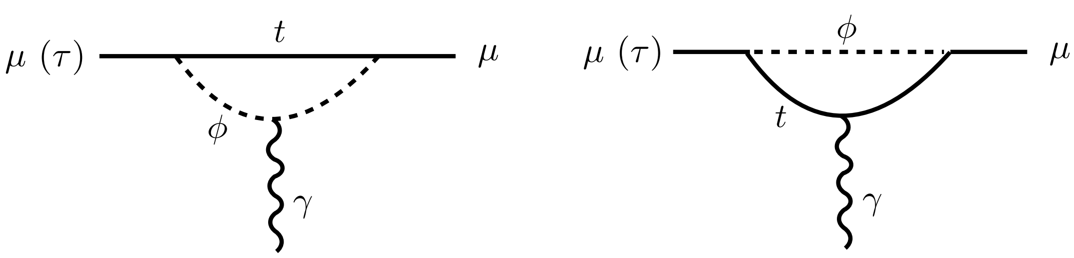
- Don't have to have full MSSM (like coded in GM2Calc [by Athron, …, Stockinger et al., EPJC 76 (2016) 62], which includes all latest two-loop contributions), and

- extended Higgs sector could do, see, e.g. Stockinger et al., JHEP 1701 (2017) 007, `The muon magnetic moment in the 2HDM: complete two-loop result'

➔ lesson: 2-loop contributions can be highly relevant in both cases; one-loop analyses can be misleading

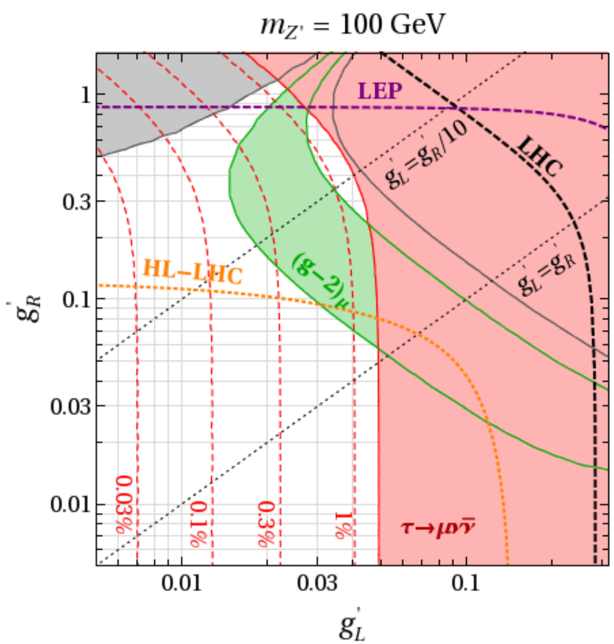
- 1 TeV Leptoquark Bauer + Neubert, PRL 116 (2016) 141802

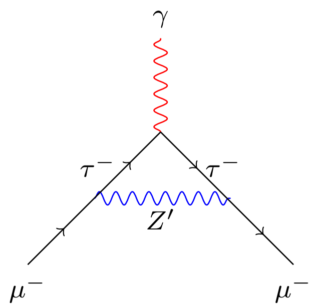
one new scalar could explain several anomalies seen by BaBar, Belle and LHC in the flavour sector (e.g. violation of lepton universality in B -> Kll, enhanced B -> Dτν) and solve g-2, while satisfying all bounds from LEP and LHC

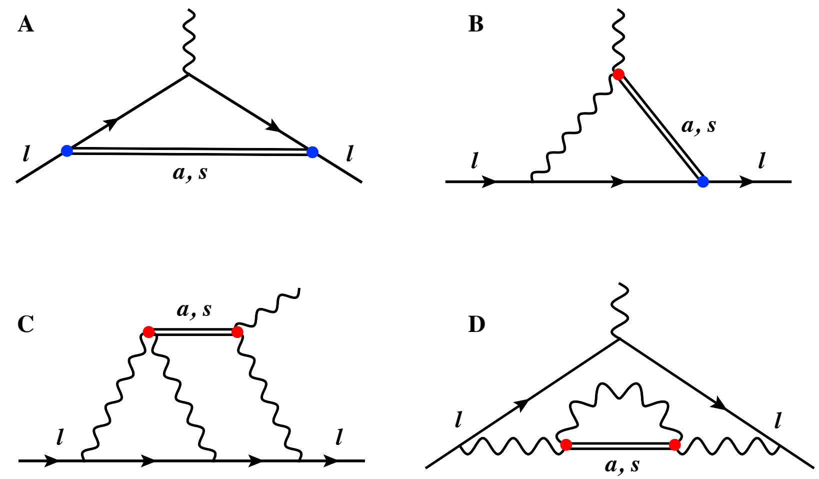


New Physics?   just a few of many recent examples

- light Z'   can evade many searches involving electrons by non-standard couplings preferring heavy leptons (but see BaBar's direct search limits in a wide mass range, PRD 94 (2016) 011102), or invoke flavour off-diagonal Z' to evade constraints [Altmannshofer et al., PLB 762 (2016) 389]





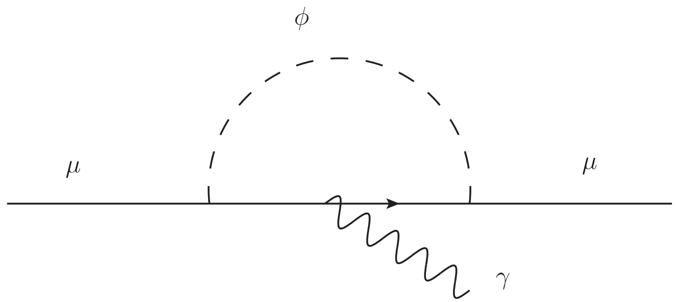


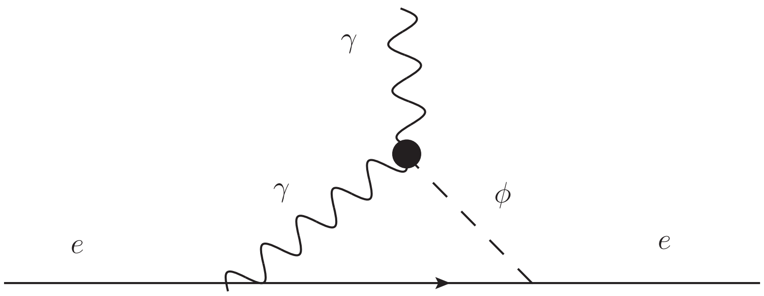
- axion-like particle (ALP), contributing like π^0 in HLbL   [Marciano et al., PRD 94 (2016) 115033]

- `dark photon' - like fifth force particle [Feng et al., PRL 117 (2016) 071803]

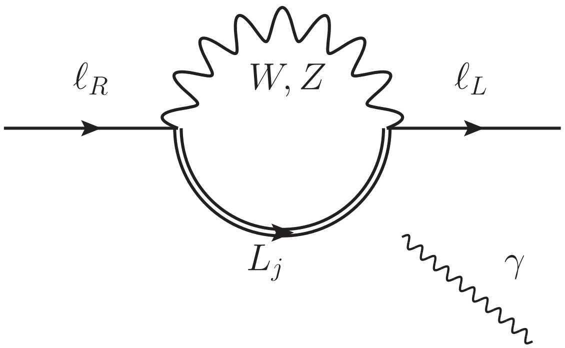
New Physics? Explaining muon and electron g-2

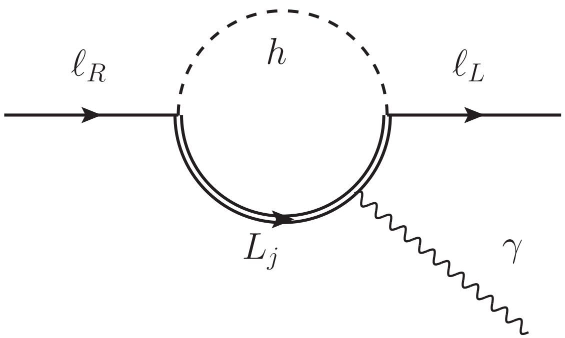
- Davoudiasl+Marciano, `A Tale of Two Anomalies', arXiv:1806.10252
  use one singlet real scalar Φ with mass ~ 250-1000 MeV and couplings ~10^{-3} and ~10^{-4} for μ and e, in one- and two-loop diagrams





- Crivellin+Hoferichter+Schmidt-Wellenburg, arXiv:1807.11484, `Combined explanation of $(g\text{-}2)_{\mu,e}$ and implications for a large muon EDM'
  discuss UV complete scenarios with vector-like fermions (**not minimally flavor violating**) which solve both puzzles and at the same time give **sizeable muon EDM contributions, $|d_\mu|$ ~10^{-23}-10^{-21},** but escaping constraints from $\mu \to e\,\gamma$.





Conclusions/Outlook:

- The still unresolved muon g-2 discrepancy, consolidated at about 3 -> 4 σ, has triggered new experiments and a lot of theory activities

- The uncertainty of the hadronic contributions will be further squeezed, with L-by-L becoming the bottleneck, but a lot of progress (lattice + new data driven approaches) is expected within the next few years
- TH will be ready for the next round
- Fermilab's g-2 experiment has started their data taking, first result planned for next year, J-PARC will take a few years longer,
  both aiming at bringing the current exp uncertainty down by a factor of 4
- with two completely different exp's, should get closure/confirmation

- We may just see the beginning of a new puzzle with a_e
- Also expect vastly improved EDM bounds. Complementarity w. LFV & MDM

- Many approaches to explain discrepancies with NP, linking g-2 with other precision observables, the flavour sector, dark matter and direct searches, but so far NP is only (con)strained.

Thank you.

Extras

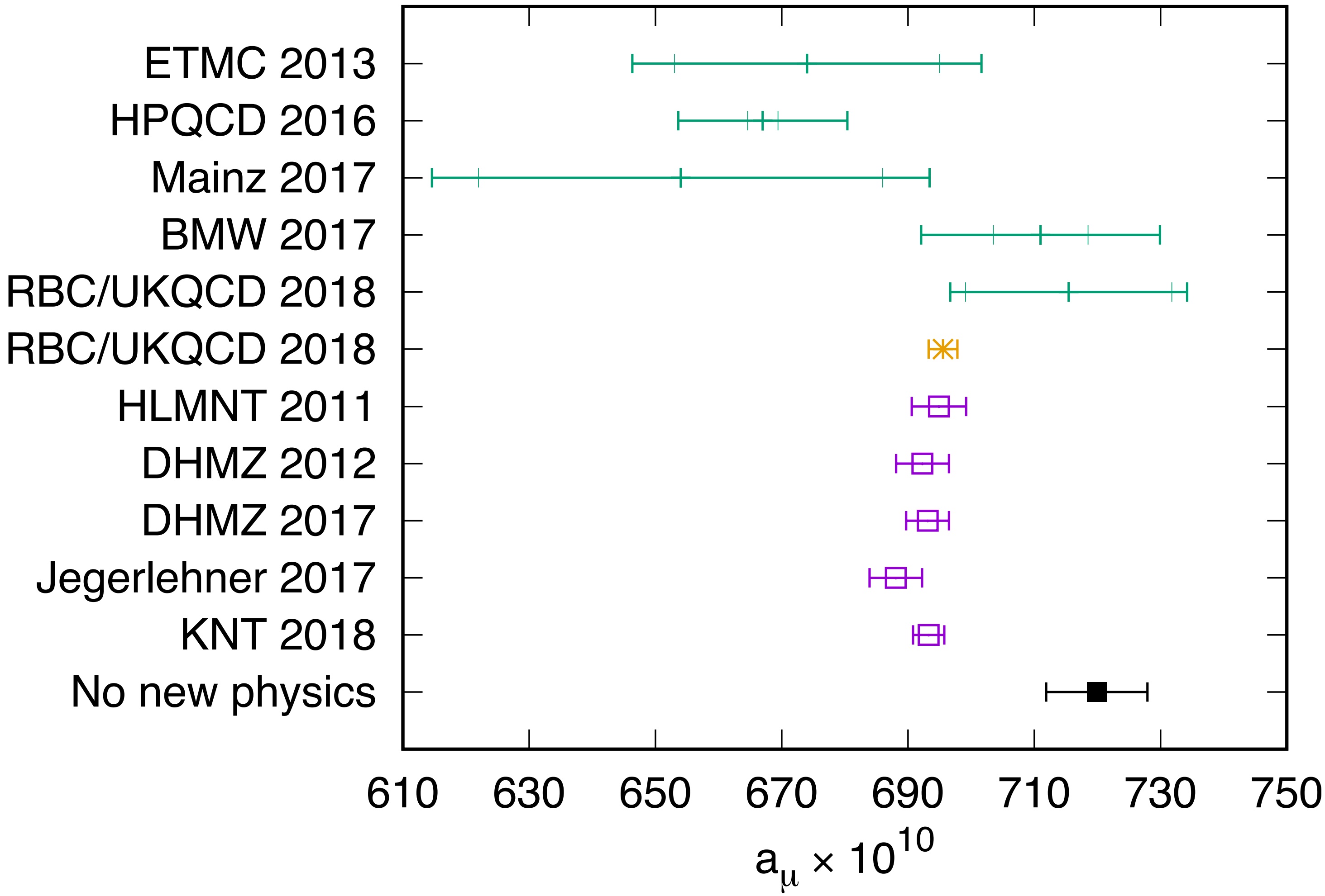
HVP from the lattice

A non-expert's re-cap of the lattice talks at the TGm2 HVP meeting at KEK in February.

- Complementary to data-driven (`pheno') DR.
- Need high statistics, and control highly non-trivial systematics:
  - need simulations at physical pion mass,
  - control continuum limit and Finite Volume effects,
  - need to include full QED and Strong Isospin Breaking effects (i.e. full QED+QCD including disconnected diagrams).

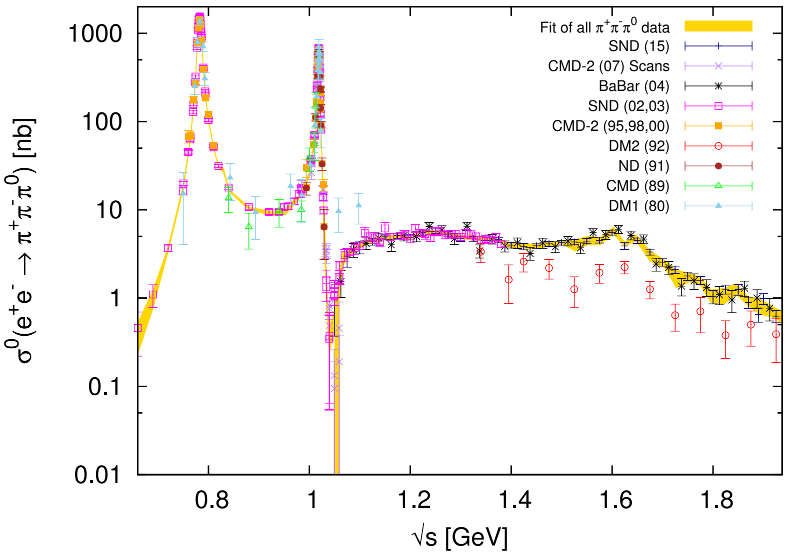
- There has been a lot of activity on the lattice, for HVP and HLbL:
  - Budapest-Marseille-Wuppertal (staggered q's, also moments)
  - RBC / UKQCD collaboration (Time-Momentum-Representation, DW fermions, window method to comb. `pheno' with lattice)
  - Mainz (CLS) group (O(a) improved Wilson fermions, TMR)
  - HPQCD & MILC collaborations (HISQ quarks, Pade fits)

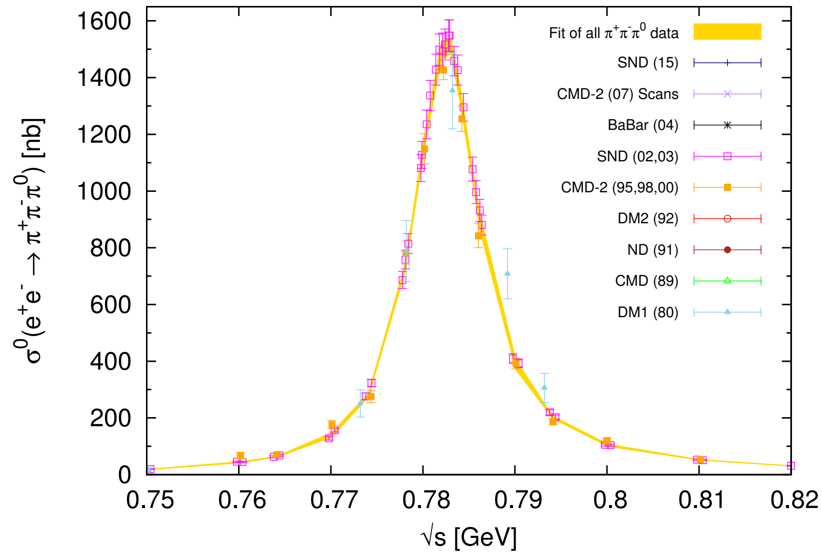
Christoph Lehner at a recent meeting of the Theory Initiative for g-2, Mainz, June 2018

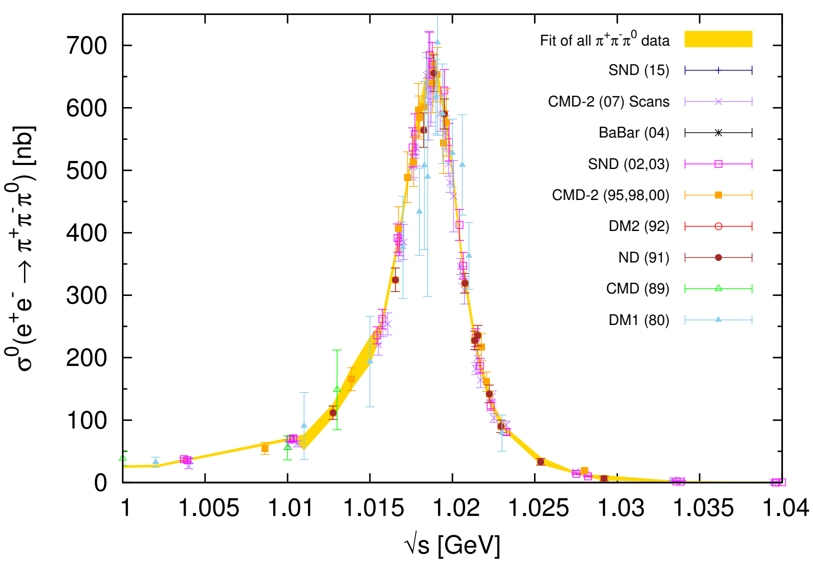


We need to improve the precision of our pure lattice result so that it can distinguish the "no new physics" results from the cluster of precise R-ratio results.

$\pi^+\pi^-\pi^0$ channel [KNT18: arXiv:1802.02995]







Improvement for 3π also

New data:

SND: [J. Exp. Theor. Phys. 121 (2015), 27.]

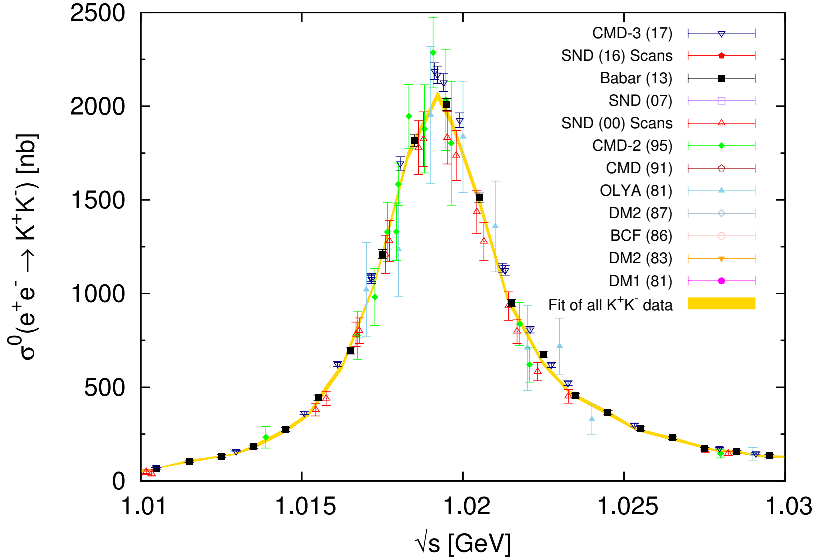
$$a_\mu^{\pi^+\pi^-\pi^0} = 47.79 \pm 0.22_{\rm stat} \pm 0.71_{\rm sys} \pm 0.13_{\rm vp} \pm 0.48_{\rm fsr}$$

$$= 47.79 \pm 0.89_{\rm tot}$$

HLMNT11: $47.51 \pm 0.99_{\rm tot}$

$K\bar{K}$ channels [KNT18: arXiv:1802.02995]

K^+K^-



New data:

BaBar: [Phys. Rev. D 88 (2013), 032013.]
SND: [Phys. Rev. D 94 (2016), 112006.]
CMD-3: [arXiv:1710.02989.]

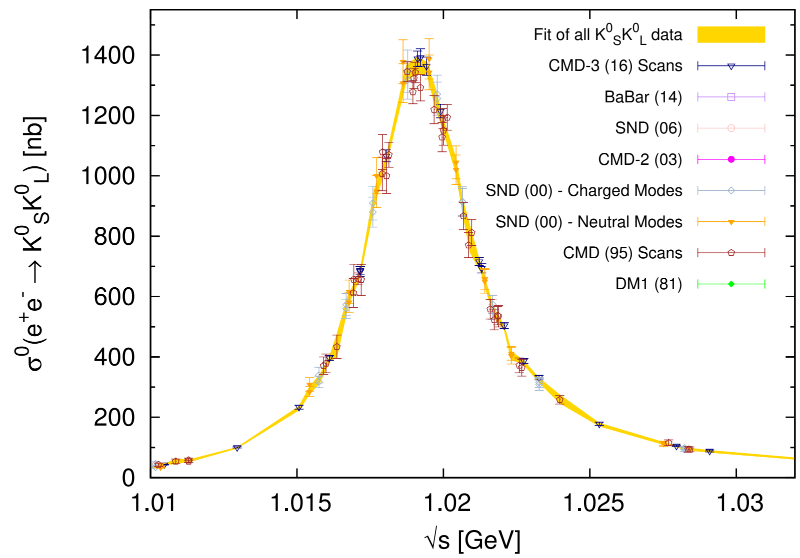
Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.] omitted as waiting reanalysis.

$$a_\mu^{K^+K^-} = 23.03 \pm 0.22_{\rm tot}$$

HLMNT11: $22.15 \pm 0.46_{\rm tot}$

Large increase in mean value

$K^0_S K^0_L$



New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.]
CMD-3: [Phys. Lett. B 760 (2016) 314.]

$$a_\mu^{K^0_S K^0_L} = 13.04 \pm 0.19_{\rm tot}$$

HLMNT11: $13.33 \pm 0.16_{\rm tot}$

Large changes due to new precise measurements on ϕ

Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19
$K^0_S K^0_L$	13.04 ± 0.19	12.82 ± 0.24	0.22
$1.8 \leq \sqrt{s} \leq 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

⇒ Total estimates from two analyses in very good agreement

⇒ Masks much larger differences in the estimates from individual channels

⇒ Unexpected tension for 2π considering the data input likely to be similar

→ Points to marked differences in way data are combined

→ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

⇒ Compensated by lower estimates in other channels

→ For example, the choice to use pQCD instead of data above 1.8 GeV

⇒ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

→ Much lower mean value, but in agreement within errors