Testing Multiple Photon Processes in the Quantum Regime at FACET-II

Fermilab workshop: “Single-electron experiments in IOTA”

November 9, 2018

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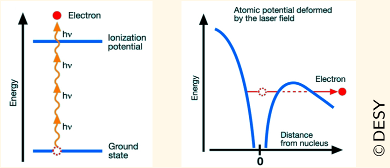
The strong-field revolution: from atoms to the Schwinger field

Atomic scales

Energy	$\mathcal{E}_{\mathrm{a}} = \alpha^2 \mathcal{E}$	$10\,\mathrm{eV}$
Length	$a_B = \lambda\!\!\!^{-}_C/\alpha$	$10^{-10}\,\mathrm{m}$
Field	$E_{\mathrm{a}} = \alpha^3 E_{\mathrm{cr}}$	$10^{11}\,\mathrm{V/m}$

$\sim 10^{16}\,\mathrm{W/cm^2}$

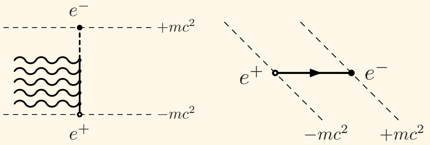


- Around $\sim 10^{16}\,\mathrm{W/cm^2}$: laser and Coulomb field are of the same order
  "Atomic strong-field revolution": Corkum, PRL **71**, 1994 (1993)

QED scales

Energy	$\mathcal{E} = mc^2$	$10^6\,\mathrm{eV}$
Length	$\lambda\!\!\!^{-}_C = \hbar/(mc)$	$10^{-13}\,\mathrm{m}$
Field	$E_{\mathrm{cr}} = m^2c^3/(e\hbar)$	$10^{18}\,\mathrm{V/m}$

$\sim 10^{29}\,\mathrm{W/cm^2}$



- Around $\sim 10^{29}\,\mathrm{W/cm^2}$: "Schwinger limit", QED vacuum becomes unstable
  "Tunnel ionization of the quantum vacuum": Schwinger PR **82**, 664 (1951)

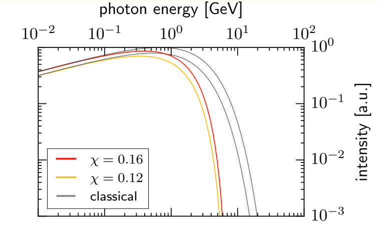
Strong-field quantum regime: novel nonperturbative effects

Quantum parameter

$$\chi = \frac{E^*}{E_{\mathrm{cr}}} = \sqrt{\frac{I^*}{I_{\mathrm{cr}}}}, \qquad \begin{cases} E_{\mathrm{cr}} = m^2c^3/(\hbar e) & \approx 1.3\times 10^{18}\,\mathrm{V/m} \\ I_{\mathrm{cr}} = c\epsilon_0 E_{\mathrm{cr}}^2 & \approx 4.6\times 10^{29}\,\mathrm{W/cm^2} \end{cases}$$

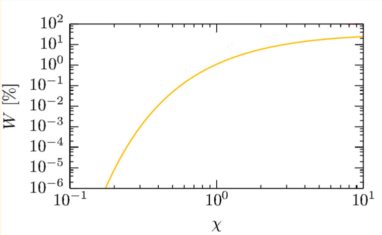
- Critical intensity I_{cr} is not reachable directly $\rightarrow$ Lorentz boost
- Lorentz invariance: rest-frame field (E^*) / intensity (I^*) decisive

Strong-field synchrotron radiation



- Critical frequency: $\hbar\omega_c = \chi\epsilon$
- $\chi \gtrsim 0.1$: strong-field regime

Photon-induced pair production

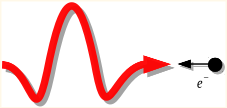


- $\chi \ll 1$: exponentially suppressed
- Tunneling: $W \sim \exp[-8/(3\chi)]$

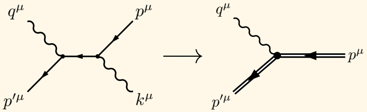
SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, Phys. Rev. D **91**, 013009 (2015)

The fundamental (strong-field) QED processes

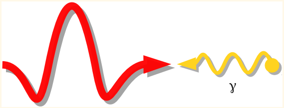
Compton scattering



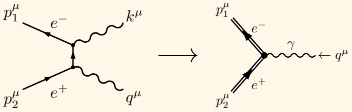
photon emission by an electron/positron



Breit-Wheeler pair production

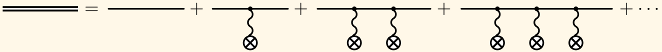


photon decay into a lepton pair



- At a fundamental QED vertex two fermions interact with one photon
- Energy-momentum conservation: we need at least two interactions
- A strong background field cannot be treated perturbatively:

Dressed states



Dressed states include the classical background field exactly

Dressed states and the classical intensity parameter

Ordinary quantum field theory

$$(\mathrm{i}\partial\!\!\!/ - m)\,\psi_p = 0,\ \psi_p = $$

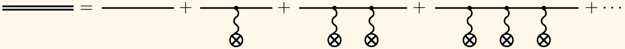
Starting point: free Dirac equation

QFT with background fields

$$(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/ - m)\Psi_p = 0,\ \Psi_p = $$

Dirac equation with classical field

Dressed states



Dressed states include the classical background field exactly

$$= \frac{p\!\!\!/ + m}{p^2 - m^2} \sim \frac{1}{m},$$

$$= -\mathrm{i}eA\!\!\!/ \sim \frac{|e|E}{\omega}$$

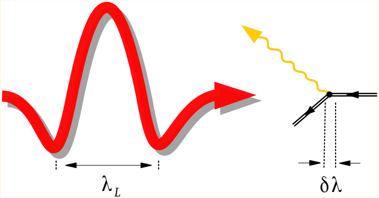
Free propagator — Coupling vertex

- The background field approximation is possible if:
  - **a)** laser modes are highly occupied (correspondence principle)
  - **b)** the interaction does not change the field configuration
- Exact solutions only known for very few field configurations, e.g., plane-wave fields
- Good theoretical descriptions for laser – parameter $\xi = a_0 = |e|E/(m\omega)$:

$\xi \ll 1$	Perturbative regime	$I < 10^{16}\,\mathrm{W/cm^2}$	
$\xi \gtrsim 1$	Nonperturbative regime	$I \gtrsim 10^{18}\,\mathrm{W/cm^2}$	$(\omega \sim 1\,\mathrm{eV})$
$\xi \gg 1$	Semiclassical regime	$I > 10^{20}\,\mathrm{W/cm^2}$	

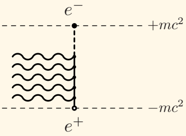
Formation region and the local constant field approximation

Formation length of SFQED processes

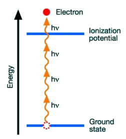


1. λ_L: laser wavelength (scale on which the field changes significantly)
2. $\delta\lambda$: formation region of a fundamental QED processes; $\delta\lambda/\lambda_L \sim 1/\xi$

Large formation length ($\xi \ll 1$)

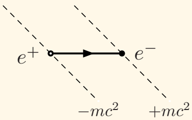


Vacuum/QED

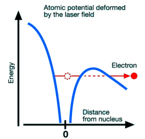


Atoms

Small formation length ($\xi \gg 1$)



Vacuum/QED

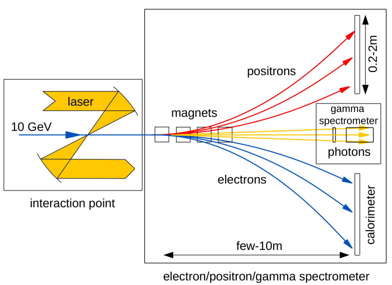


Atoms

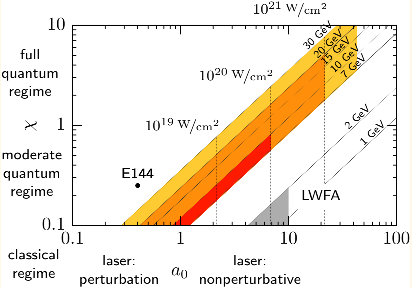
- Ionization in atomic physics – Keldysh parameter: $\gamma_K = \omega\sqrt{2mI_p}/(|e|E)$,
- Pair production in SFQED: $\gamma_K(I_p = 2mc^2) \sim 1/\xi = \omega mc/(|e|E)$

Probing strong-field QED at SLAC's FACET-II

Sketch of the setup



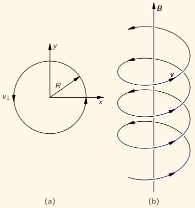
Explorable parameter space



Laser parameters	20 TW (red)	200 TW (orange)	1 PW (yellow)
Electron energy	$7-10$ GeV	20 GeV	30 GeV
Pulse energy	0.7 J	8 J	50 J
Pulse duration (FWHM)	35 fs	40 fs	50 fs
Beam waist	2.4 µm	3.0 µm	3.0 µm
Wavelength	0.8 µm	0.8 µm	0.8 µm
Intensity (average)	10^{20} W/cm^2	0.7×10^{21} W/cm^2	3.5×10^{21} W/cm^2
$\xi = a_0$ (peak)	7	18	40
χ (peak)	0.9	4	14

Synchrotron radiation: classical description

Motion in a static B field



Transverse motion:

e^-/e^+ with energy ϵ: circular orbit, radius

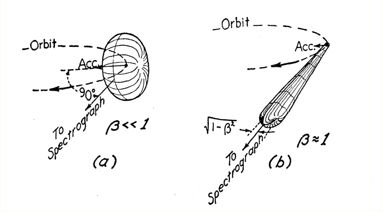
$$\rho = \frac{\epsilon}{(|e|B)} = \frac{\gamma^2\hbar}{\chi mc}$$

Longitudinal motion:

free propagation

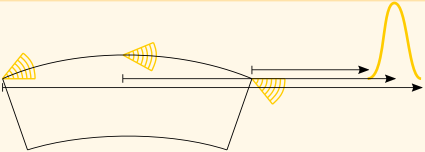
© The Feynman Lectures on Physics

Radiation pattern



© Phys. Rev. **102**, 1423 (1956)

Formation region and critical frequency



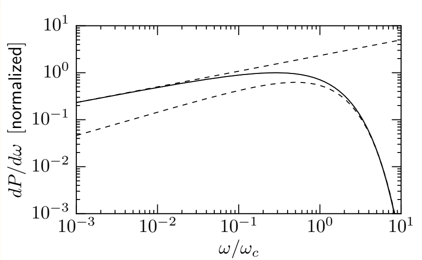
- **Formation length**: $l_f = \rho/\gamma$ (contributing circular segment)
- **Critical frequency:** $\omega_c \sim 1/T = c\gamma^3/\rho$ (typical frequency)

  [Fourier transform; burst duration: $T = \left(\frac{1}{v} - \frac{1}{c}\right) l_f \approx \rho/(c\gamma^3)$]

Jackson, *Classical Electrodynamics* (1999)

Synchrotron radiation: classical spectrum & total power

Synchrotron spectrum



Characteristic scalings

- Small frequencies:
  $dP/d\omega \sim (\omega/\omega_c)^{1/3}$
- Large frequencies:
  $dP/d\omega \sim \sqrt{\omega/\omega_c}\,\exp\left(-\omega/\omega_c\right)$

Critical frequency: $\hbar\omega_c = (2/3)\epsilon\chi$

Plot (left side): $\chi = 10^{-3}$, i.e. strong exponential suppression well before $\hbar\omega = \epsilon$

Total radiation power

Power P (energy per unit time) emitted per electron:

$$\boxed{P \sim \alpha \cdot \frac{c}{l_f} \cdot \hbar\omega_c \sim \alpha \cdot \frac{mc^2}{\hbar}\frac{\chi}{\gamma} \cdot \epsilon\chi = \alpha\chi^2 \frac{(mc^2)^2}{\hbar} \sim \frac{1}{m^4}}$$

exact: $P = \alpha\chi^2(2/3)m^2$; $m^2 = 63.56 \times 10^6\,\mathrm{W}$; $\alpha \sim 1/\hbar$; $\chi \sim \hbar$

Intuitive derivation: photon emission probability per formation time c/l_f is α; typical energy of the radiated photon: $\hbar\omega_c$

Schwinger, *On the Classical Radiation of Accelerated Electrons*, Phys. Rev. **75**, 1912 (1949)

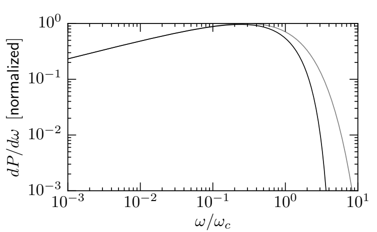
From “classical” to “quantum” synchrotron radiation

- If $\chi \gtrsim 1$ classical electrodynamics predicts $\omega_c \sim \epsilon\chi \gtrsim \epsilon$ (not possible)
- The recoil ($\hbar\omega$) induced by the emitted photon becomes important
  $\longrightarrow$ quantization of the photon field must be taken into account
- Semiclassical approach: classical trajectory + photon recoil at the vertex

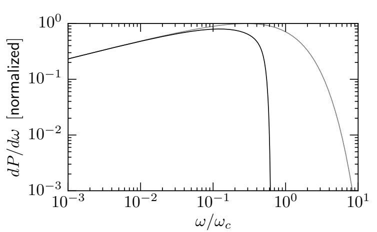
$$\frac{dP}{d\omega} = \frac{dP}{du}\frac{du}{d\omega}, \quad \frac{dP}{du} = -\alpha m^2 \frac{u}{(1+u)^3}\left\{ \int_z^\infty dt\ \mathrm{Ai}(t) + \frac{\mathrm{Ai}'(z)}{z}\left[2 + \frac{u^2}{(1+u)}\right]\right\},$$

$z = (u/\chi)^{2/3}$; $u \approx \omega/(\epsilon - \omega)$; $du/d\omega \approx (1+u)^2/\epsilon$; Classical limit: $u \approx \omega/\epsilon \sim \chi \ll 1$ ($z \sim 1$)

From classical to quantum electrodynamics



$\chi = 0.1$: onset of quantum corrections



$\chi = 1$: quantum corrections decisive

solid curve: quantum calculation; dotted curve: classical prediction

Baier, Katkov, Strakhovenko: *Electromagnetic Processes at High Energies in Oriented Single Crystals* (1998)

Multiple photons emitted by a classical current

- Exactly solvable problem: quantized photon field + classical current
- Glauber calculated the probability P_n for the emission of n photons

PHYSICAL REVIEW VOLUME 84, NUMBER 3 NOVEMBER 1, 1951

Some Notes on Multiple-Boson Processes

ROY J. GLAUBER
Institute for Advanced Study, Princeton, New Jersey
(Received July 11, 1951)

Perturbation theory: $P_n^{(\mathrm{PT})} = \dfrac{W^n}{n!}$, **Exact result:** $P_n = \dfrac{W^n}{n!} e^{-W}$,

- Problem: $\exp(-W) = \sum_n (-W)^n/n!$ contains α to all orders $(W \sim \alpha)$!
- The exponential decay factor $\exp(-W)$ becomes important if $W \gtrsim 1$
- Only the exact result respects unitarity (probability conservation):

$$\sum_{n=0}^{\infty} P_n^{(\mathrm{PT})} = e^W \quad \longleftrightarrow \quad \sum_{n=0}^{\infty} P_n = 1, \quad \langle n \rangle = \sum_{n=0}^{\infty} n P_n = W$$

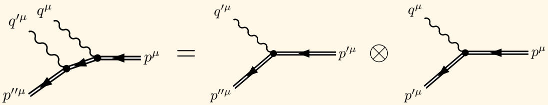
Limited applicability of the Glauber result

A classical current does not experience a recoil
$\longrightarrow$ all emissions are independent $\longrightarrow$ Poisson distribution

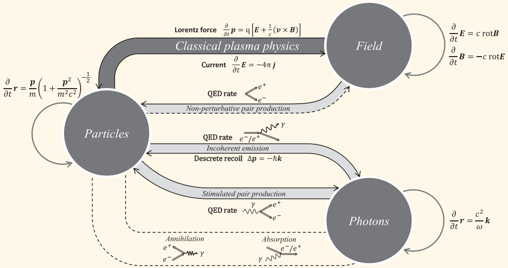
Central assumption of numerical QED simulations

Classical propagation + QED processes (Monte-Carlo)

$\xi \gg 1 \rightarrow$ small formation region $\rightarrow$ local constant field approximation (LCFA)



QED-PIC approach



A. Gonoskov, et al. Phys. Rev. E **92**, 023305 (2015)

Higher-order emission processes in the quantum regime

Double photon emission

PRL **110,** 070402 (2013) PHYSICAL REVIEW LETTERS week ending 15 FEBRUARY 2013

Nonlinear Double Compton Scattering in the Ultrarelativistic Quantum Regime

F. Mackenroth and A. Di Piazza*

A detailed analysis of the process of two-photon emission by an electron scattered from a high-intensity laser pulse is presented. We provide a semiclassical explanation for such differences, based on the possibility of assigning a trajectory to the electron in the laser field before and after each quantum photon emission.

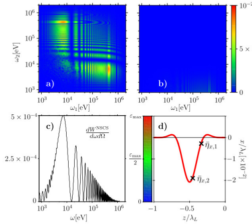


FIG. 2 (color online). Two-photon energy emission spectra $dE/\Pi_{i=1}^{2} d\omega_i d\Omega_i[\mathrm{eV}^{-1}\,\mathrm{sr}^{-2}]$ at $\chi = 5\times10^{-3}$ observed at $\theta_1 = \pi - \theta_0/2$, and at $\theta_2 = \theta_1$ (a) and at $\theta_2 = \pi - 1.1\theta_0$ (b), with $\theta_0 = 0.19$ rad. The other numerical parameters are given in the text. (c): NSCS emission probability $dW^{\mathrm{NSCS}}/d\omega d\Omega[\mathrm{eV}^{-1}\,\mathrm{sr}^{-1}]$ at (θ_1, ϕ_1). (d): Classical electron trajectories (the instantaneous electron energy is color encoded) with initial four-momenta p_i^μ and p_t^μ joined at $\bar{\eta}_{x,1}$. Since as a typical photon emission energy, the value 10^5 eV has been chosen [see (a) and (c)], then $p_i^\mu \approx p_t^\mu$ and the two trajectories are indistinguishable. $\varepsilon_{\max}$ is the maximum electron energy in the laser field. The crosses mark the points of the trajectory where the classical electron's velocity is along (θ_1, ϕ_1).

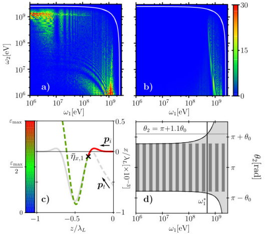
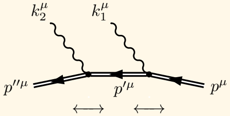


FIG. 3 (color online). Two-photon energy emission spectra $dE/\Pi_{i=1}^{2} d\omega_i d\Omega_i[\mathrm{eV}^{-1}\mathrm{sr}^{-2}]$ at $\chi \approx 1.1$ observed at $\theta_1 = \pi - \theta_0/2$, and at $\theta_2 = \theta_1$ (a) and at $\theta_2 = \pi - 1.1\theta_0$ (b), with $\theta_0 = 7.6\times10^{-3}$ rad. The other numerical parameters are given in the text. The solid white lines correspond to the cutoff-energy equation $\omega_1 + \omega_2 = \varepsilon_i$. (c): The two classical electron trajectories with initial electron momentum $\boldsymbol{p}_i$ (solid line) and $\boldsymbol{p}_t$ (dashed line). The color-encoded line shows the actual electron trajectory for a photon with energy $\omega_1 = 0.8$ GeV and momentum along (θ_1, ϕ_1) emitted at $\bar{\eta}_{x,1}$. (d): Emission opening angle for the second emitted photon as a function of ω_1 (light shaded area) compared with the emission cone for NSCS with initial electron momentum $\boldsymbol{p}_i$ (dark stripes).

see also: D. Seipt and B. Kämpfer, Phys. Rev. D **85**, 101701 (2012),
E. Lötstedt and U. D. Jentschura, Phys. Rev. Lett. **103**, 110404 (2009)

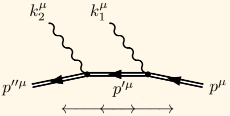
Emission of multiple photons: radiation reaction

Incoherent emissions



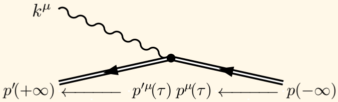
Emission vertices are well separated, standard approx. in numerical codes

Coherent emissions



Formation regions overlap, emission processes cannot be separated

Semiclassical description of photon emissions



- Classical motion between subsequent emissions
- Photon recoil changes the trajectory discontinuously

Classical radiation reaction ($\chi \ll 1$)

- Each emitted photon carries only a very small fraction of the electron energy
- The electron energy is changed *adiabatically* over many emissions

Quantum radiation reaction ($\chi \gtrsim 1$)

- The recoil of a *single* photon changes energy and trajectory significantly
- The changed electron trajectory strongly modifies the subsequent emissions

A. Di Piazza et al., Rev. Mod. Phys. **84**, 1177 (2012)

Breakdown of the LCFA for "small" photon energies

Formation length

$$l_f = \frac{\gamma}{\chi_e}(1+\chi_e/u)^{1/3}\frac{1}{m}, \quad u = \frac{\omega_\gamma}{\epsilon-\omega_\gamma}$$

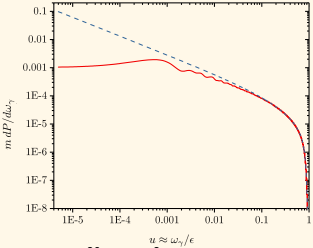
For small frequencies ($\omega_\gamma \to 0$):

$$\delta\phi_f \sim \omega l_f \sim \frac{1}{\xi}\left(\frac{\chi_e}{\omega_\gamma/\epsilon}\right)^{1/3}, \quad u \approx \frac{\omega_\gamma}{\epsilon}$$

Threshold photon energy:

$$\omega_\gamma \lesssim \omega_{\mathrm{IR}} = \epsilon\chi_e/\xi^3$$

Probability to emit a photon



$I = 4.4\times 10^{20}$ W/cm^2 $\omega = 1.55$ eV ($\xi \approx 10$)
$\epsilon = 10$ GeV ($\chi \approx 1.2$) pulse length: 5 fs

- The LCFA predicts an IR divergence in the photon spectrum (blue dashed curve)
- A full numerical calculation reveals a finite probability for $u \to 0$ (red solid curve)
  $\longrightarrow$ For photon energies $\omega_\gamma \lesssim \omega_{\mathrm{IR}} = (\chi/\xi^3)\,\epsilon$ the LCFA breaks down
- Plot parameters: breakdown at $\omega_{\mathrm{IR}} \approx 10\,\mathrm{MeV} = 10^{-3}\,10\,\mathrm{GeV}$ ($\xi = 10$, $\chi \approx 1$)

The IR divergence of the LCFA is unphysical,
as any realistic field has a finite extend

A. Di Piazza, M. Tamburini, SM and C. H. Keitel, Phys. Rev. A **98**, 012134 (2018)

To be continued...

Thank you for your attention and your questions!

The strong-field QED (SFQED) collaboration:

SFQED theory & simulation	A. Di Piazza, F. Fiuza, T. Grismayer, C. H. Keitel, SM, L. O. Silva, D. Del Sorbo, M. Tamburini, M. Vranic
SLAC E144 experiment	D. A. Reis (SF AMO/xray), T. Koffas (HEP)
LWFA SFQED experiments	G. Sarri, M. Zepf
Crystal SFQED experiments	R. Holtzapple, U. I. Uggerhøj, T. N. Wistisen
Strong-field AMO/xray science	P. H. Bucksbaum, M. Fuchs, C. Rödel
Laser-plasma interaction, HEDP	F. Albert, S. Corde, S. Glenzer, C. Joshi, M. Litos, W. Mori
Accelerator physics	G. White
Detectors	A. Dragone, C. J. Kenney
High intensity lasers	A. Fry

Collaborating Institutions: Carleton University (Canada), Aarhus University (Denmark). École Polytechnique (France) Max-Planck-Institut für Kernphysik (Germany), Helmholtz-Institut Jena (Germany), Friedrich-Schiller-Universität Jena (Germany), Universidade de Lisboa (Portugal), Queen's University Belfast (UK), California Polytechnic State University (CA USA), Lawrence Livermore National Laboratory (CA USA), SLAC National Accelerator Laboratory (CA USA), University of California Los Angeles (CA USA), University of Colorado Boulder (CO USA), University of Nebraska - Lincoln (NE USA), Princeton University (NJ USA)