Quantum Monte Carlo calculations of Neutrino-Nucleus Interactions

PONDD Physics Opportunities in the Near DUNE Detector Hall

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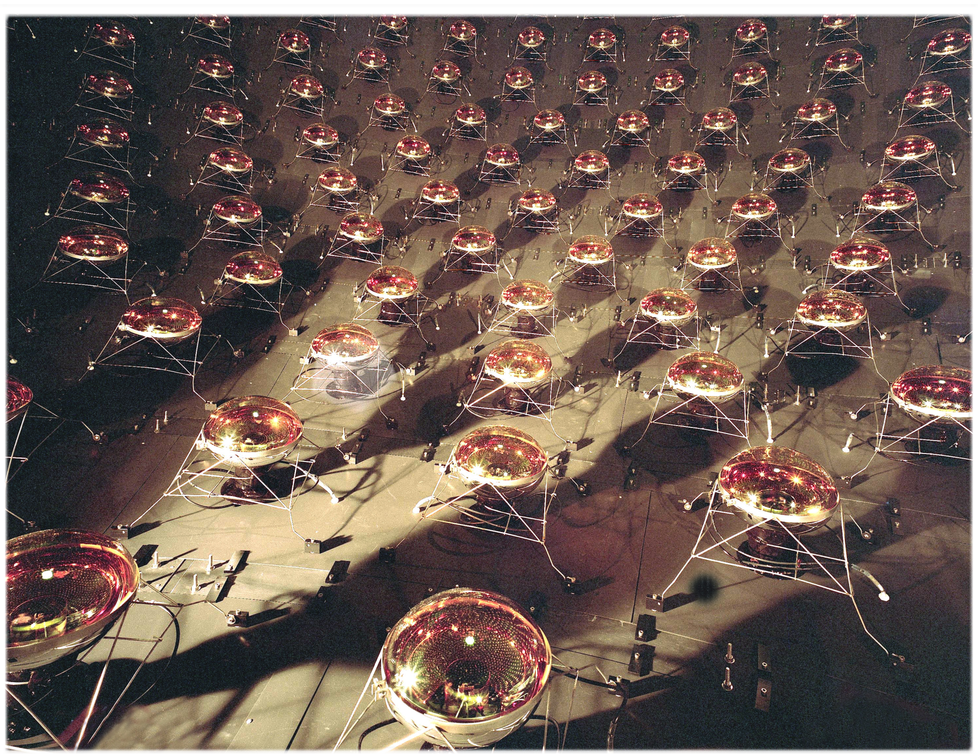
The Physics case

Neutrino-oscillation and 0νββ experiments

- Accurately measure neutrino-oscillation parameters
- Determine whether the neutrino is a Majorana or a Dirac particle
- Need for including nuclear dynamics; mean-field models inadequate to describe neutrino-nucleus interaction



Multi-messenger era for nuclear astrophysics

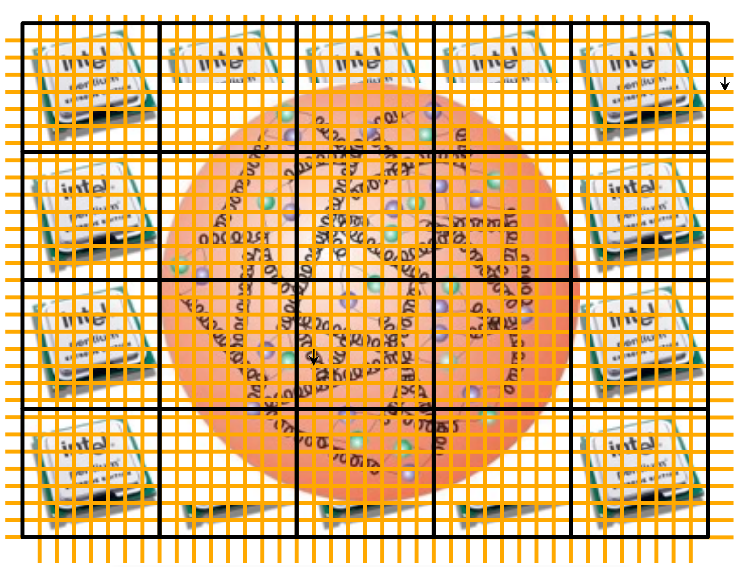
- Gravitational waves have been detected!
- Supernovae neutrinos will be detected by the current and next generation neutrino experiments
- Nuclear dynamics determines the structure and the cooling of neutron stars

The *basic model*

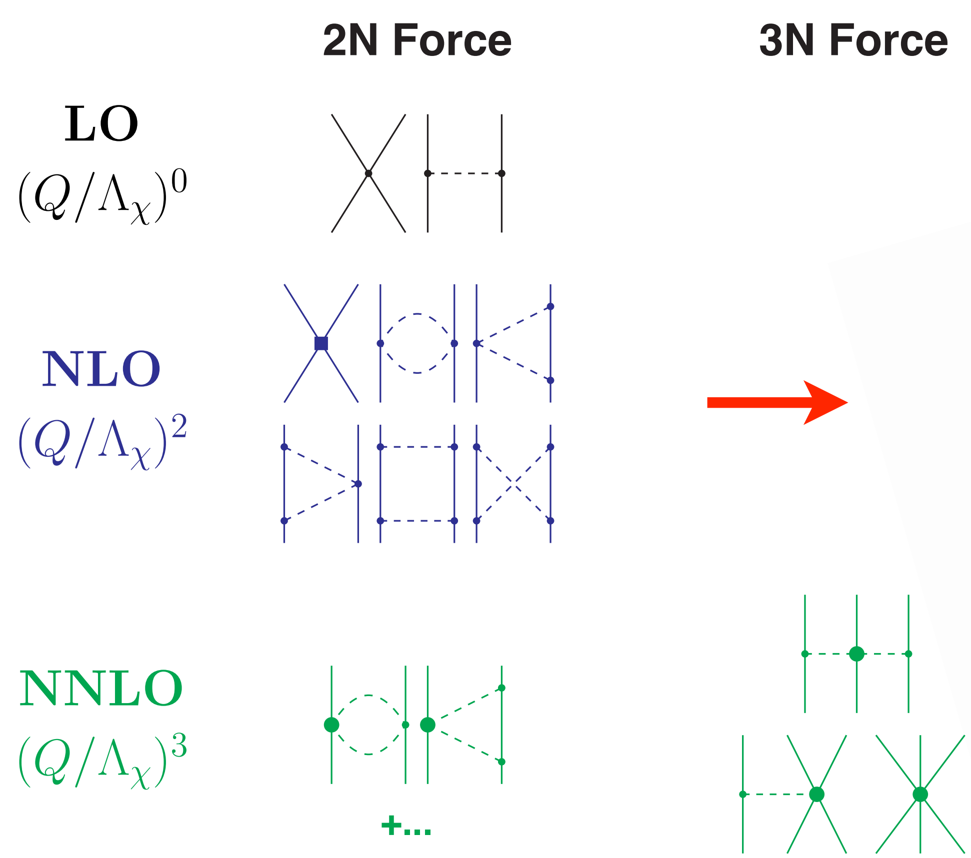
- In the low-energy regime, quark and gluons are confined inside hadrons. Nucleons can treated as point-like particles interacting through the Hamiltonian

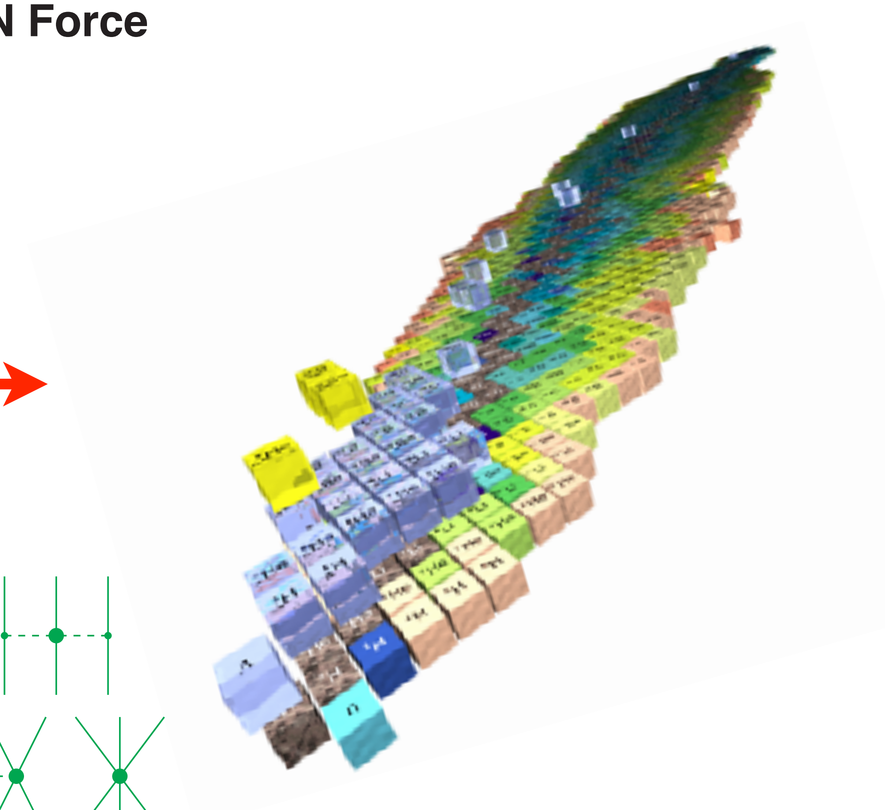
$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- Effective field theories are the link between QCD and nuclear observables. They exploit the separation between the “hard” (M~nucleon mass) and “soft” (Q ~ exchanged momentum) scales



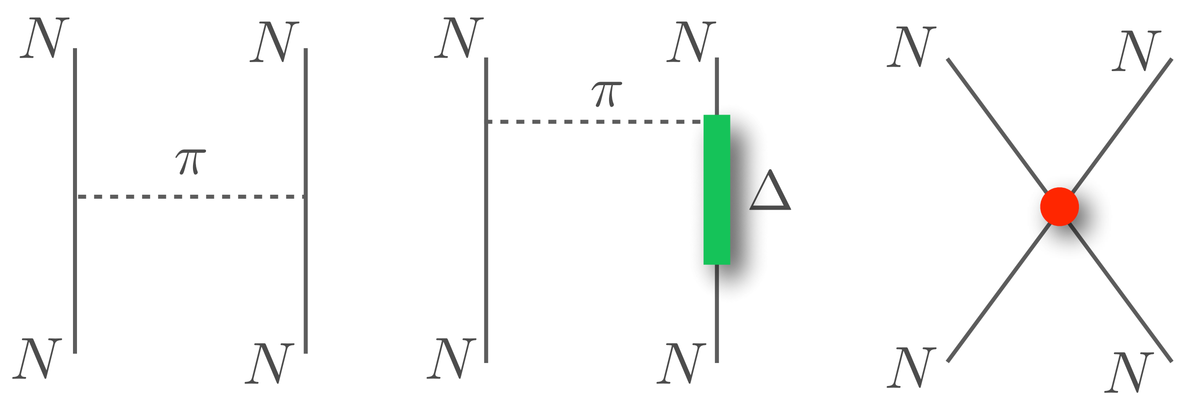
Courtesy of M. Savage



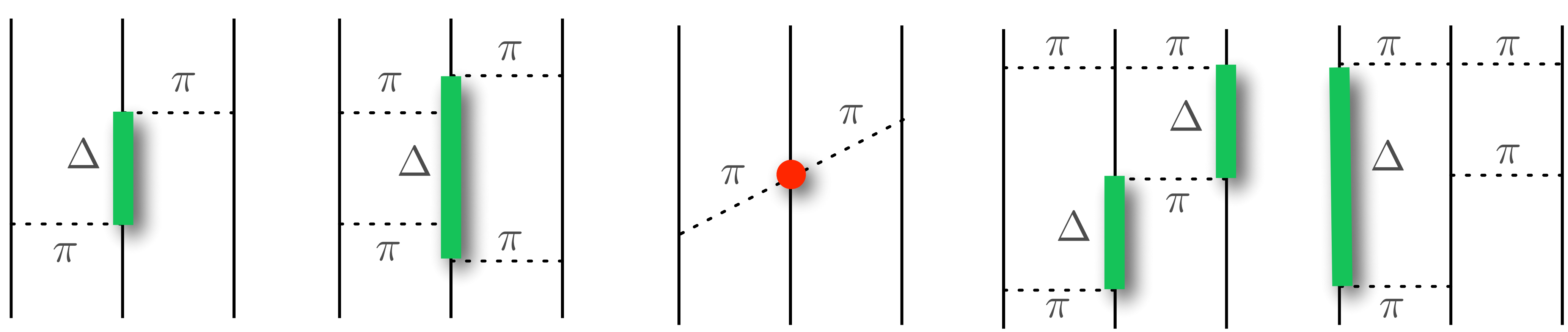


Nuclear (phenomenological) Hamiltonian

The Argonne v_{18} is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database



Three-nucleon interactions effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

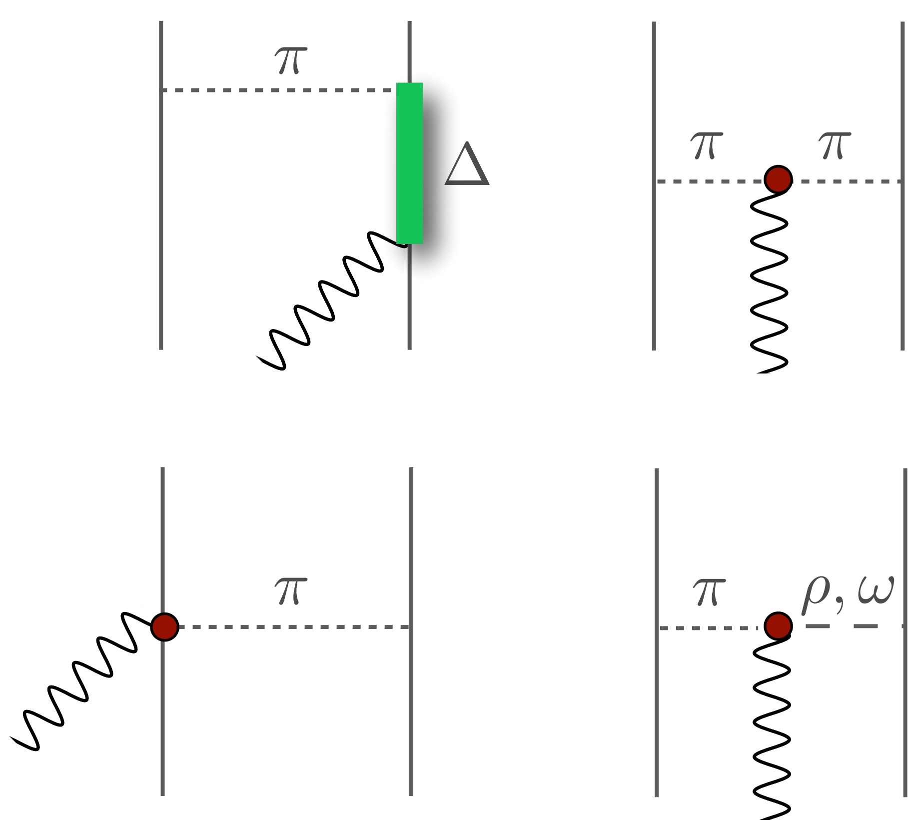


Nuclear electroweak currents

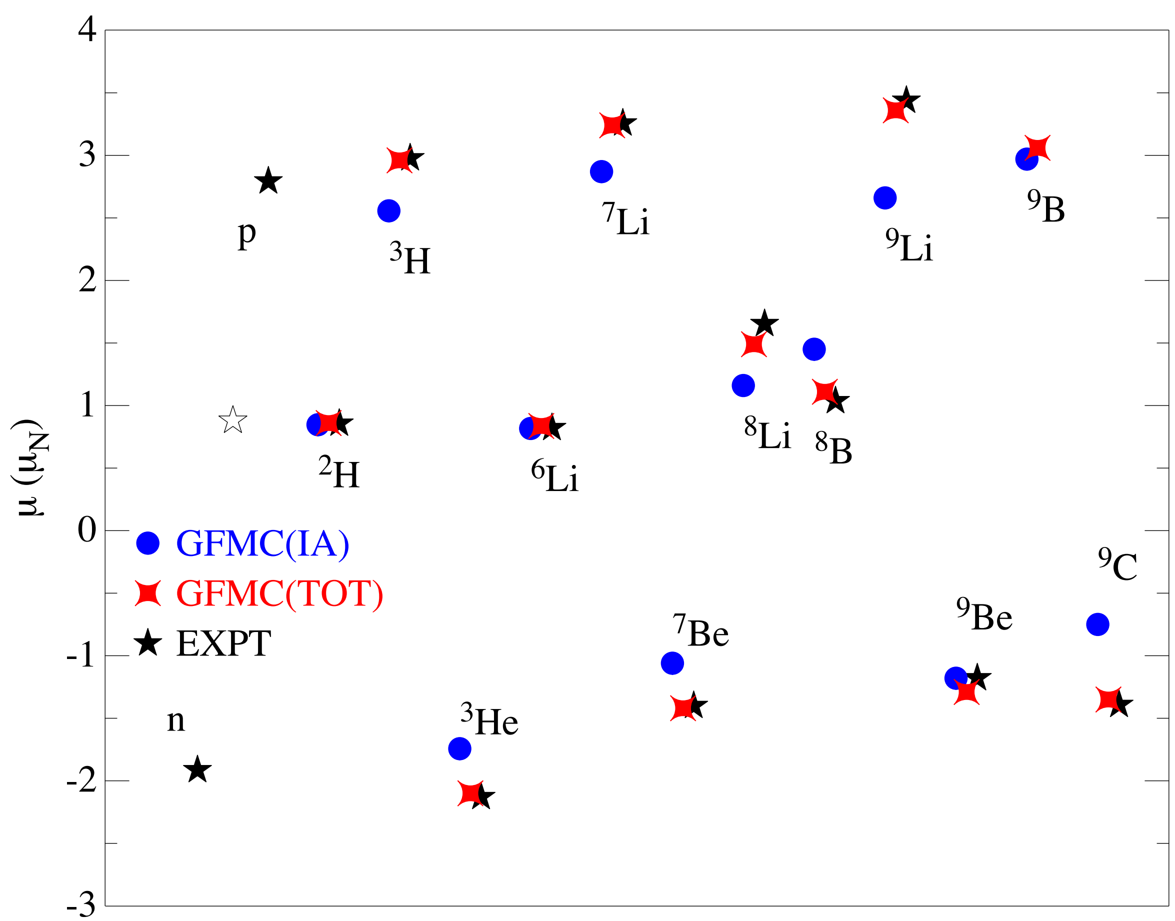
The nuclear electromagnetic current is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.



- They are essential for low-momentum and low-energy transfer transitions.



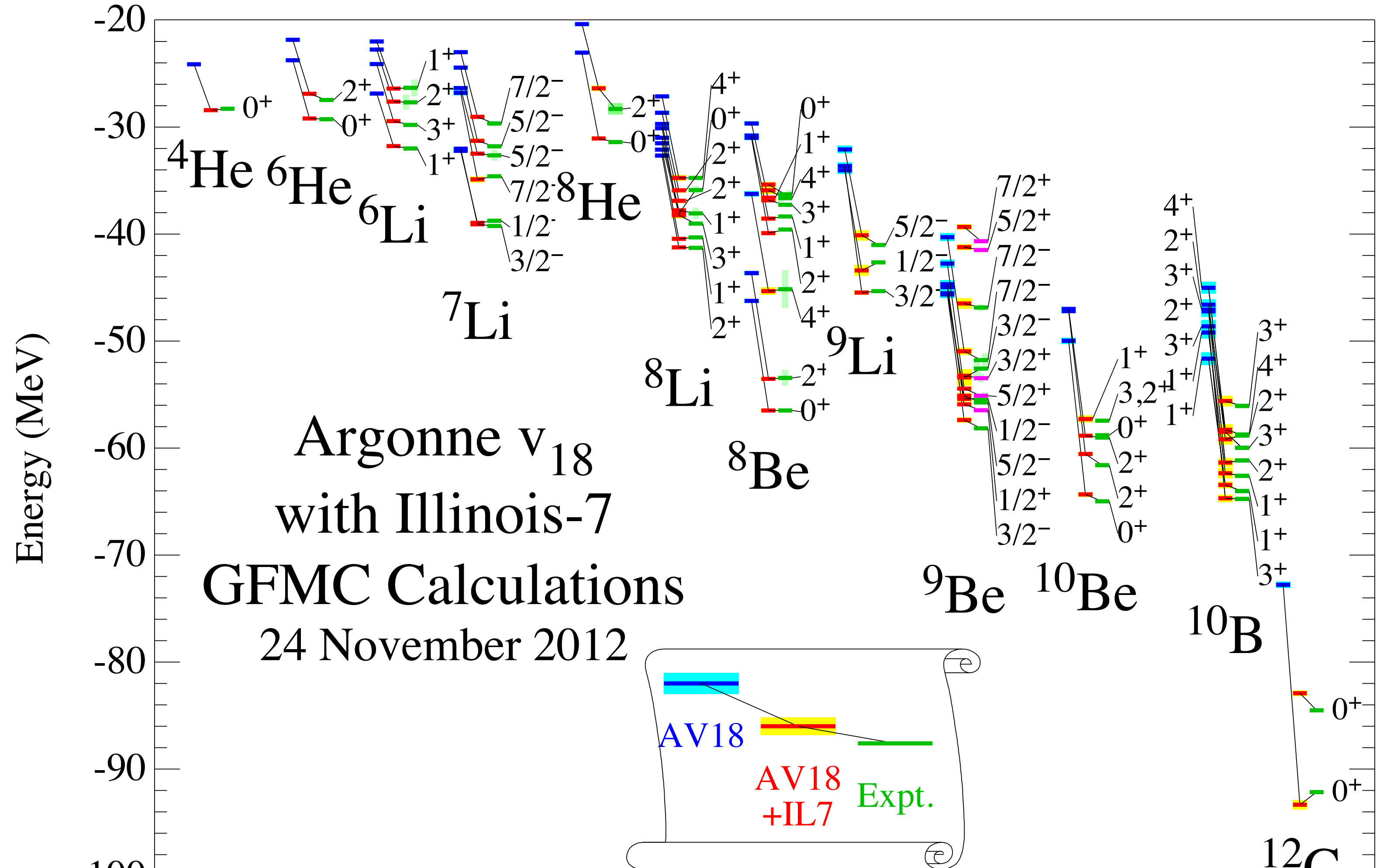
S. Pastore at al., PRC 87, 035503 (2013)

Quantum Monte Carlo

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting trial wave function.

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

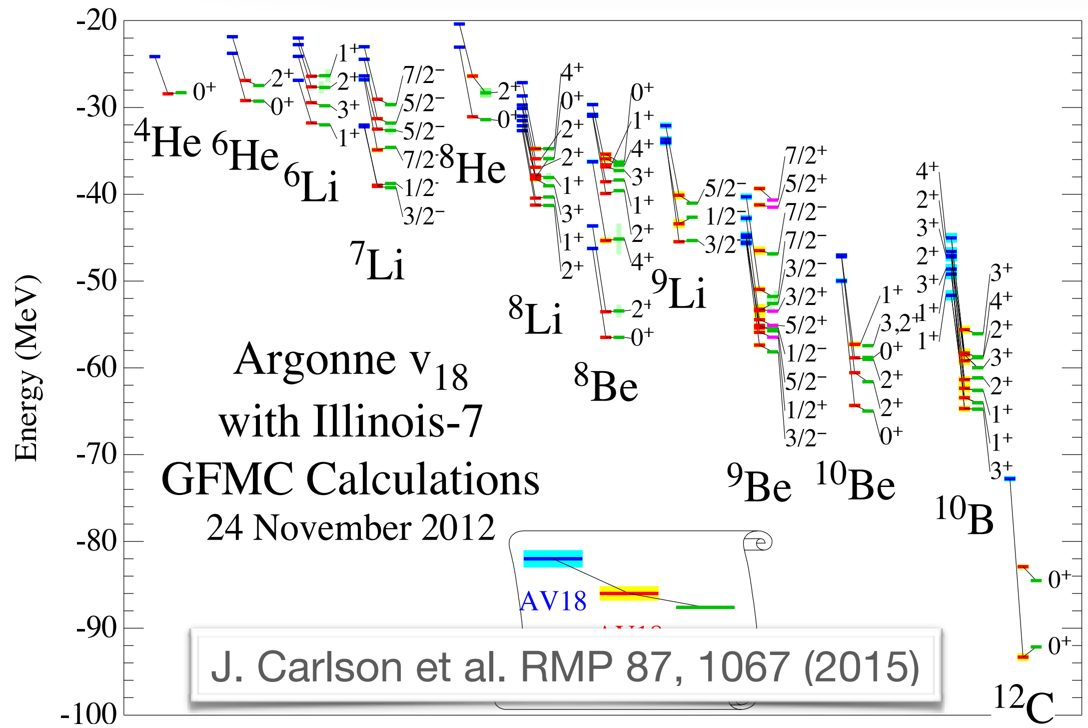
- Suitable to solve of A ≤ 12 nuclei with ~1% accuracy



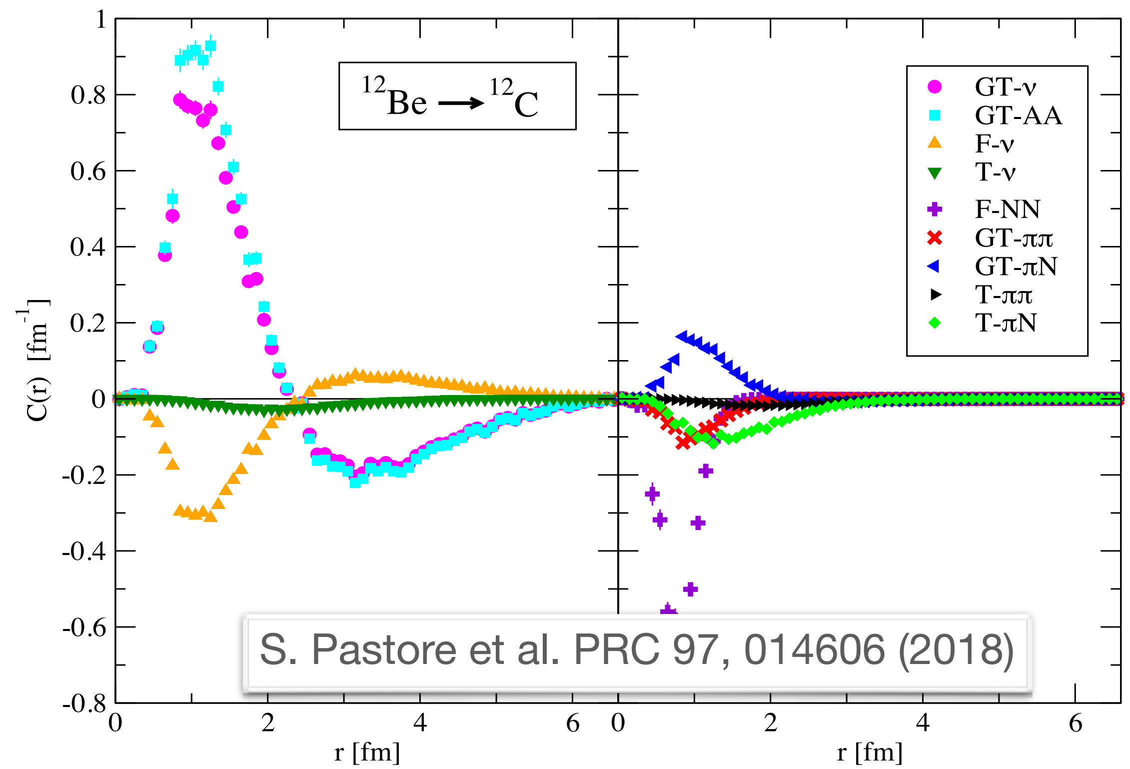
J. Carlson et al. RMP 87, 1067 (2015)

The *basic model* of nuclear Physics

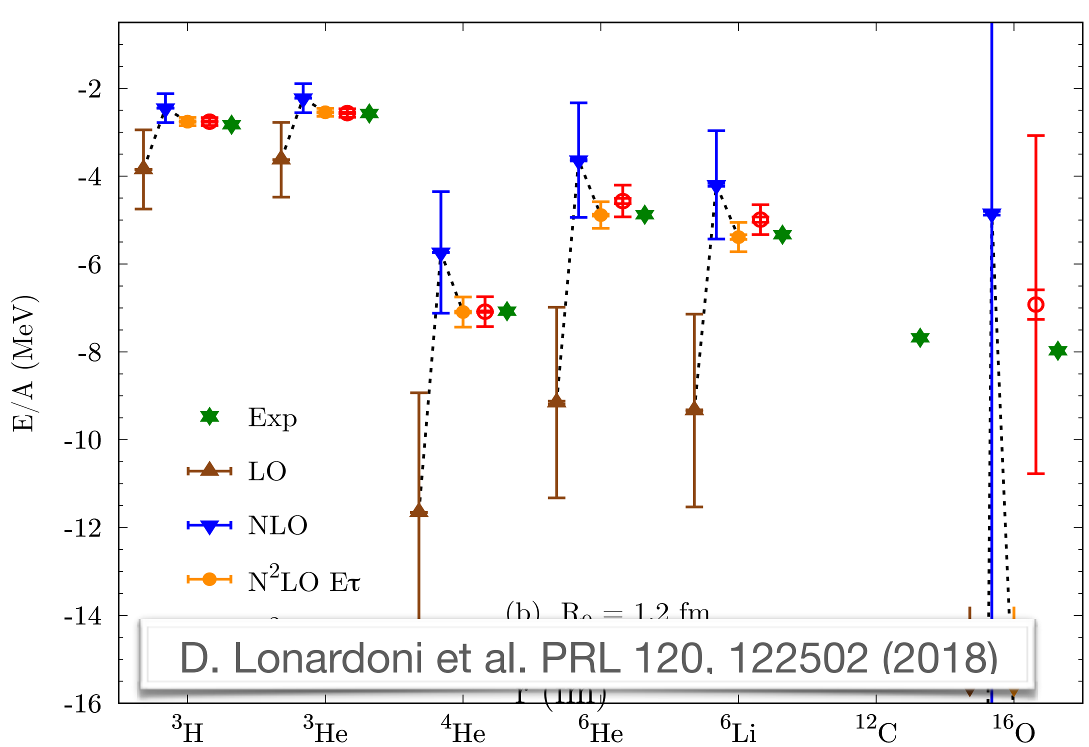
Realistic nuclear interactions + Nuclear ab-initio methods



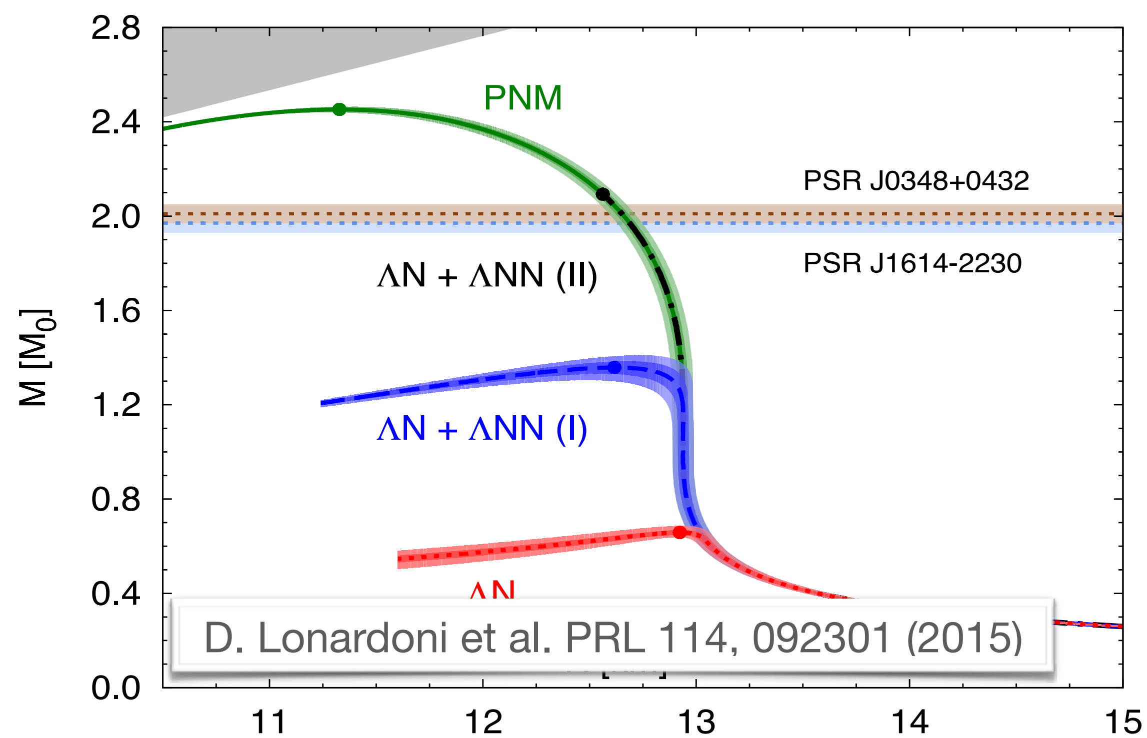
J. Carlson et al. RMP 87, 1067 (2015)



S. Pastore et al. PRC 97, 014606 (2018)



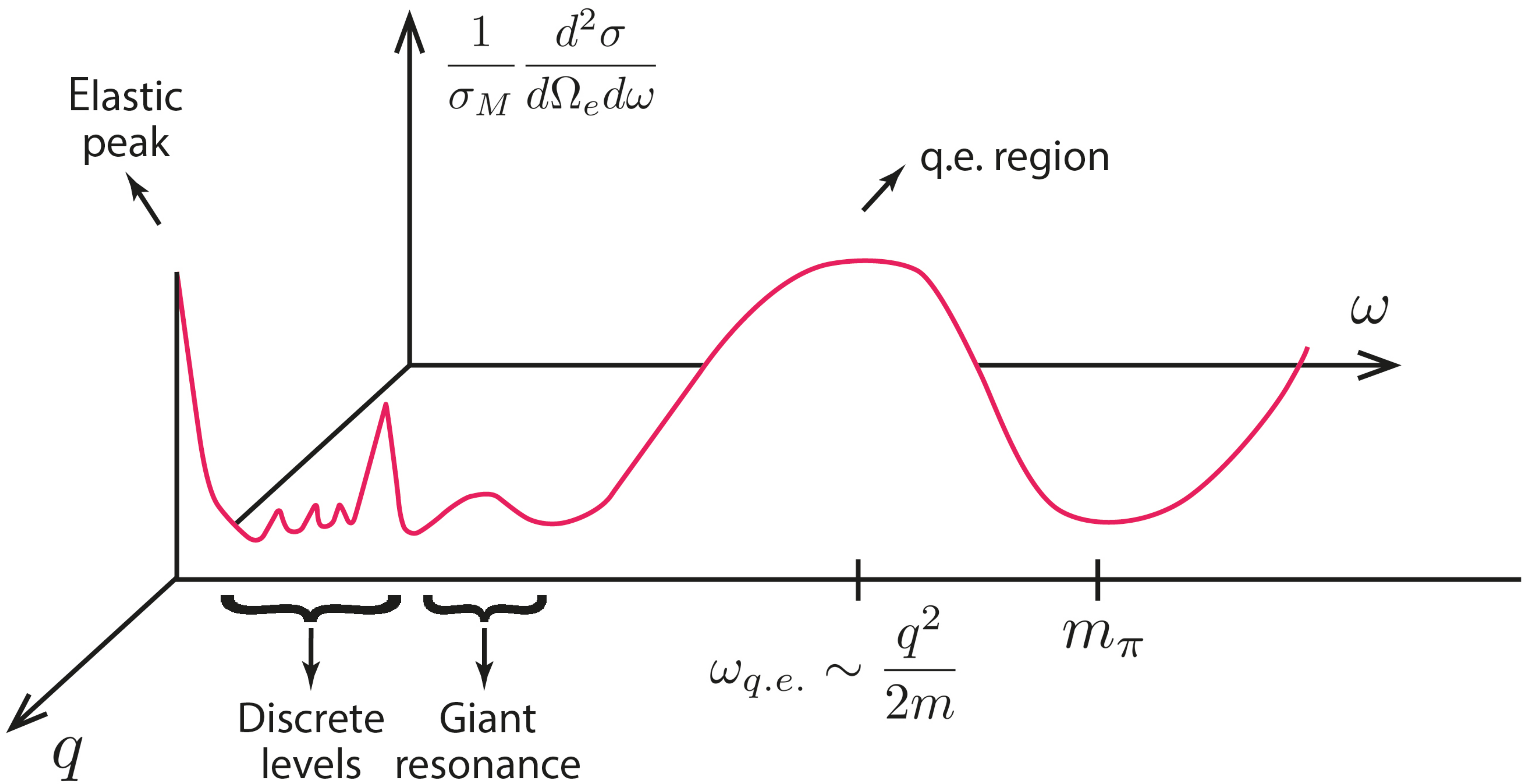
D. Lonardoni et al. PRL 120, 122502 (2018)



D. Lonardoni et al. PRL 114, 092301 (2015)

Lepton-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.

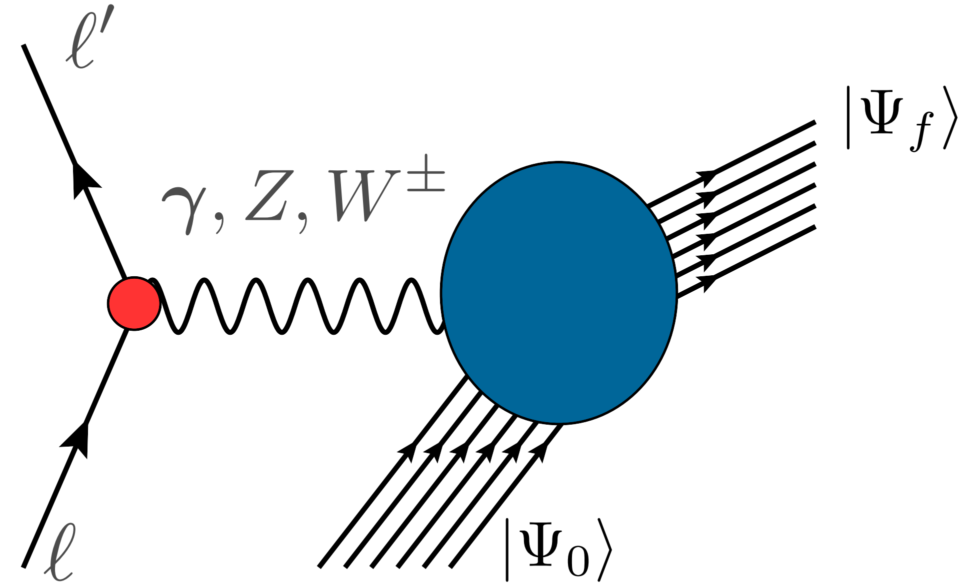


Courtesy of Saori Pastore

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'}d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$

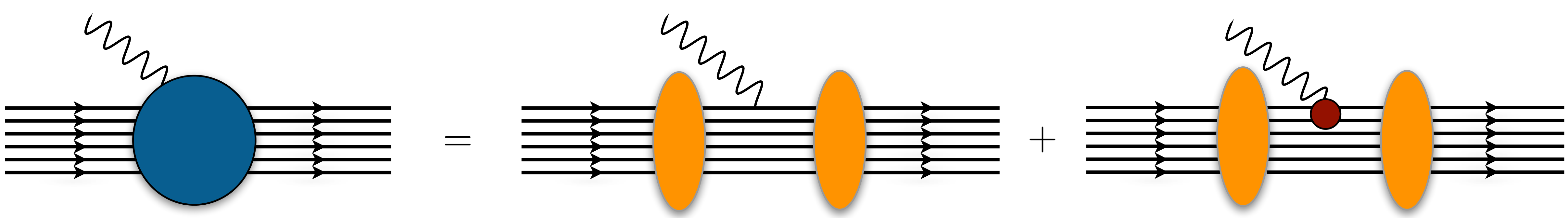


- In the electromagnetic case only the longitudinal and the transverse response functions contribute

- The response functions contain all the information on target structure and dynamics

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

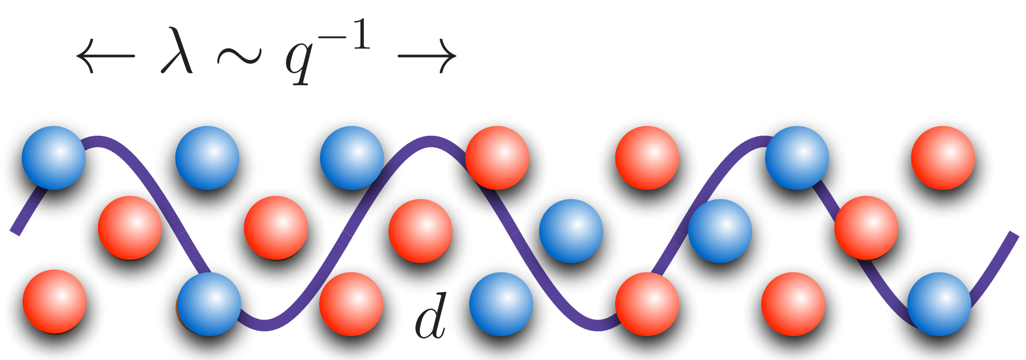
- They account for initial state correlations, final state correlations and two-body currents



Lepton-nucleus scattering

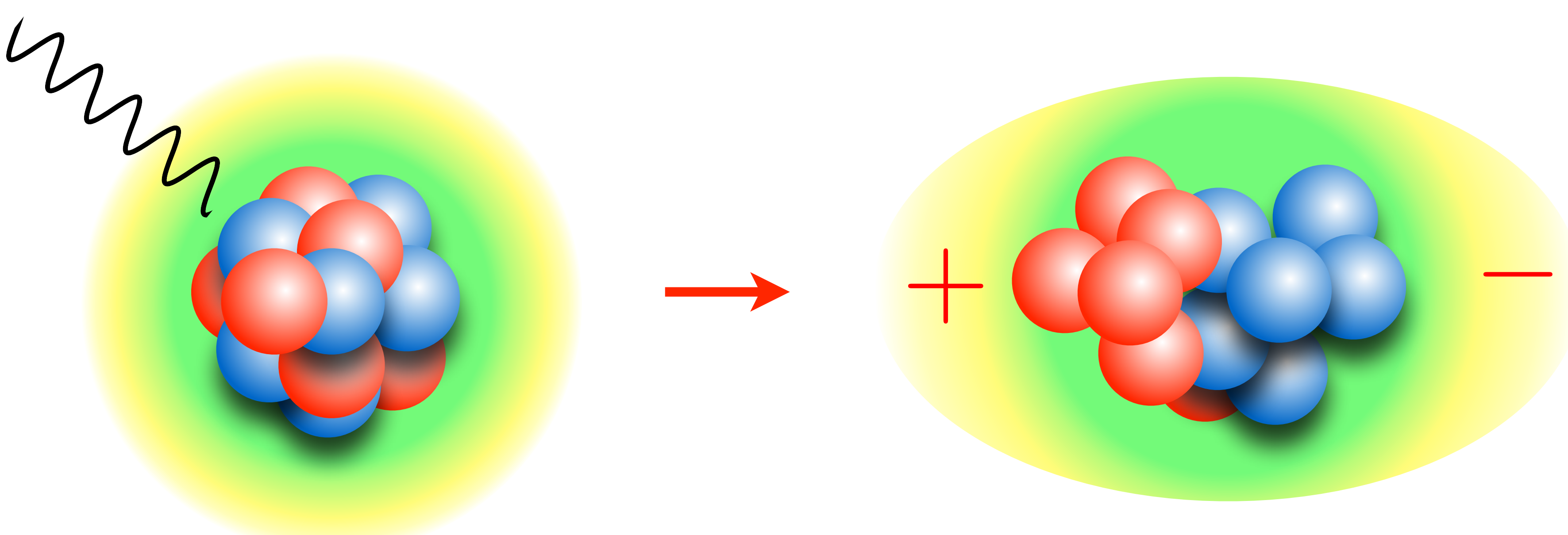
- At low momentum transfer the space resolution of the lepton becomes much larger than the average NN separation distance (~ 1.5 fm).

- In this regime the interaction involves many nucleons → long-range correlations



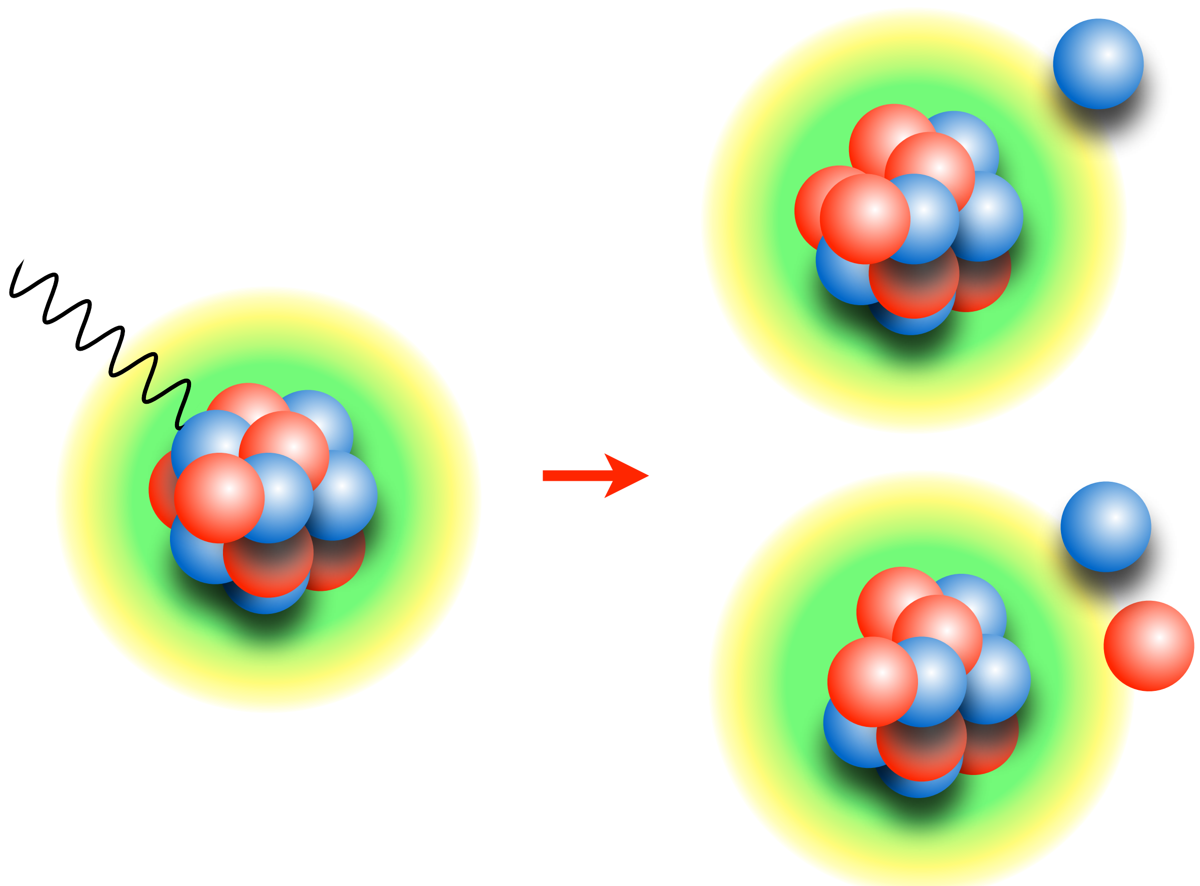
$$|\Psi_f\rangle = \sum c^f_{1p,1h} |\Psi_{1p1h}\rangle$$

- The giant dipole resonance is a manifestation of long-range correlations



Lepton-nucleus scattering

- At (very) large momentum transfer, scattering off a nuclear target reduces to the sum of scattering processes involving bound nucleons → short-range correlations.



$$|\Psi_f\rangle \simeq |p_1\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$|\Psi_f\rangle \simeq |p_1, p_2\rangle \otimes |\Psi_f\rangle_{A-2}$$

- Relativistic effects play a major role and need to be accounted for along with nuclear correlations (Non trivial interplay between them)

- Resonance production and deep inelastic scattering also need to be accounted for

Moderate momentum-transfer regime

- At moderate momentum transfer, the inclusive cross section can be written in terms of the response functions

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

- Both initial and final states are eigenstates of the nuclear Hamiltonian

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \qquad\qquad H|\Psi_f\rangle = E_f|\Psi_f\rangle$$

- As for the electron scattering on ^{12}C

$$|^{12}C^*\rangle, |^{11}\mathrm{B}, p\rangle,\ |^{11}\mathrm{C}, n\rangle,\ |^{10}\mathrm{B}, pn\rangle,\ |^{10}\mathrm{Be}, pp\rangle$$

- Relativistic corrections are included in the current operators and in the nucleon form factors

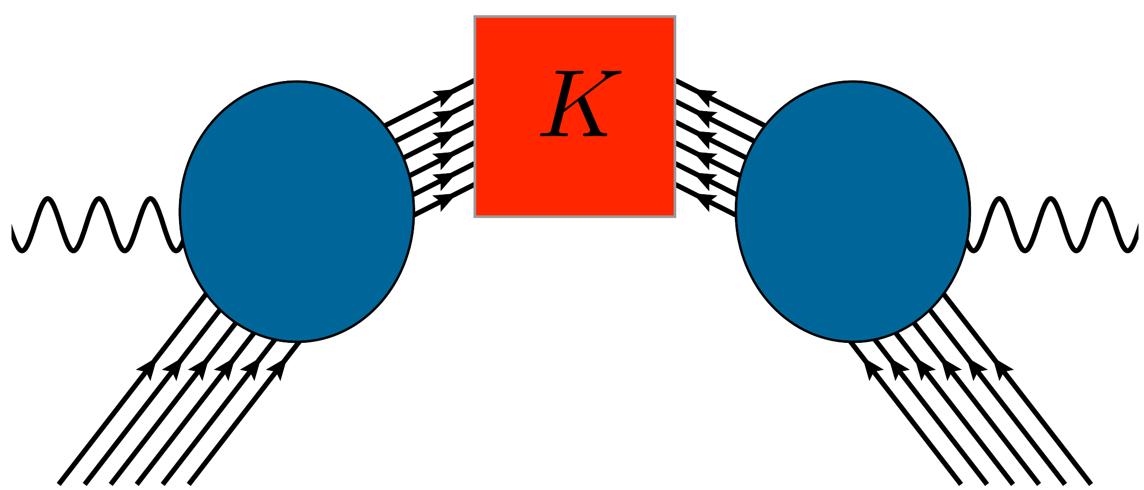
Integral transform techniques

- The integral transform of the response function are generally defined as

$$E_{\alpha\beta}(\sigma,\mathbf{q}) \equiv \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega,\mathbf{q})$$

- Using the completeness of the final states, they can be expressed in terms of ground-state expectation values

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \langle \Psi_0 | J^\dagger_\alpha(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \Psi_0 \rangle$$

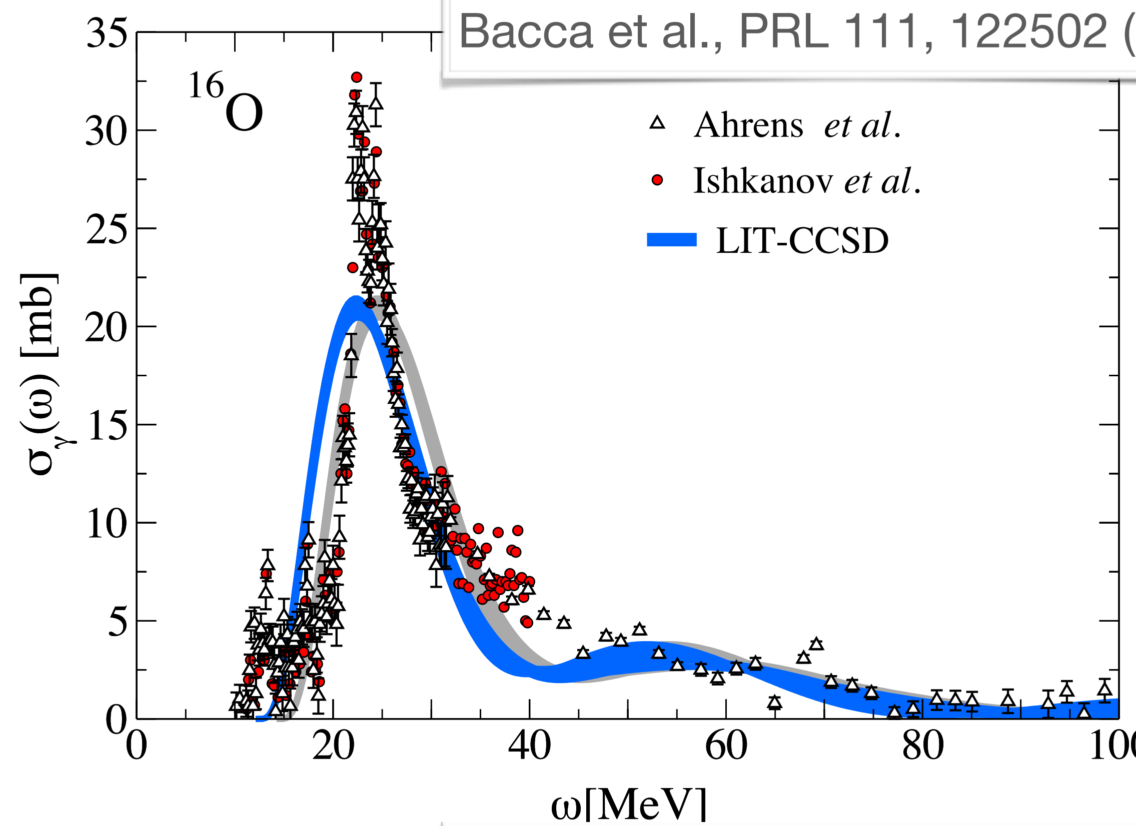


Lorentz integral transform (LIT)

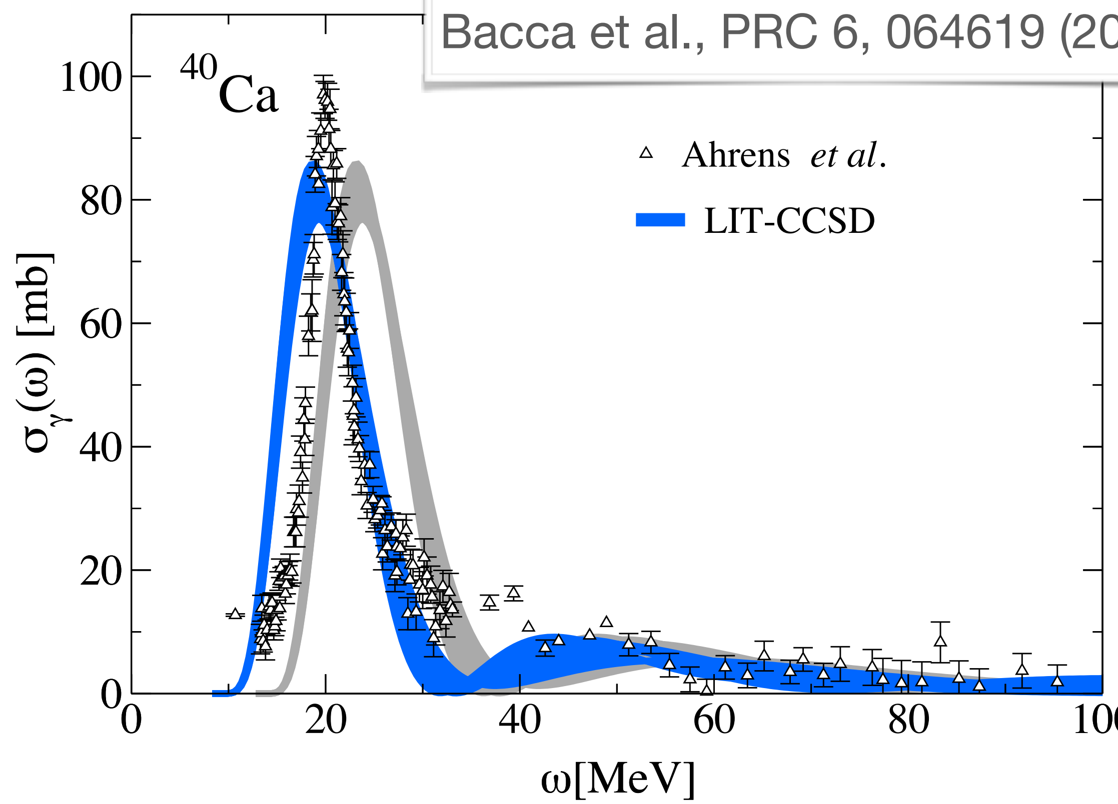
- The Lorentz integral transform

$$K(\sigma,\omega) = \frac{1}{(\omega-\sigma_R)^2+\sigma_I^2}$$

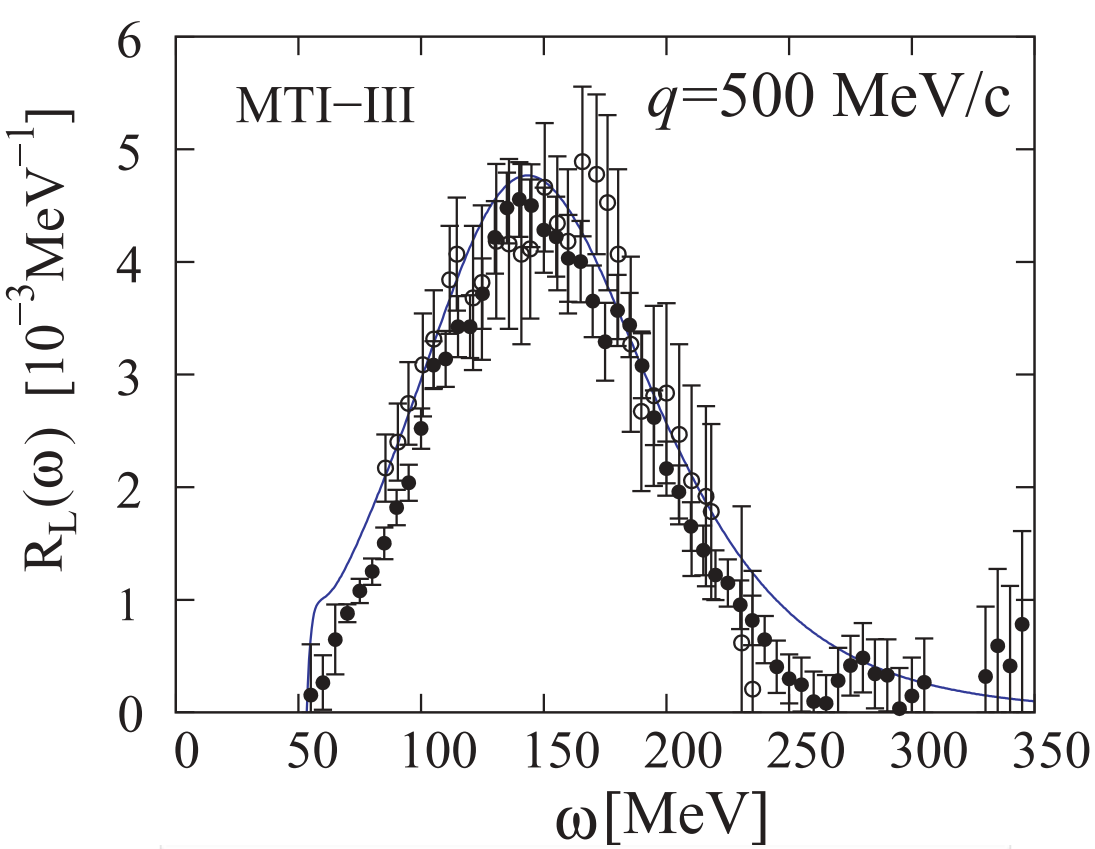
has been successfully exploited in the calculation of electromagnetic and neutral-weak responses



Bacca et al., PRL 111, 122502 (2013)



Bacca et al., PRC 6, 064619 (2014)



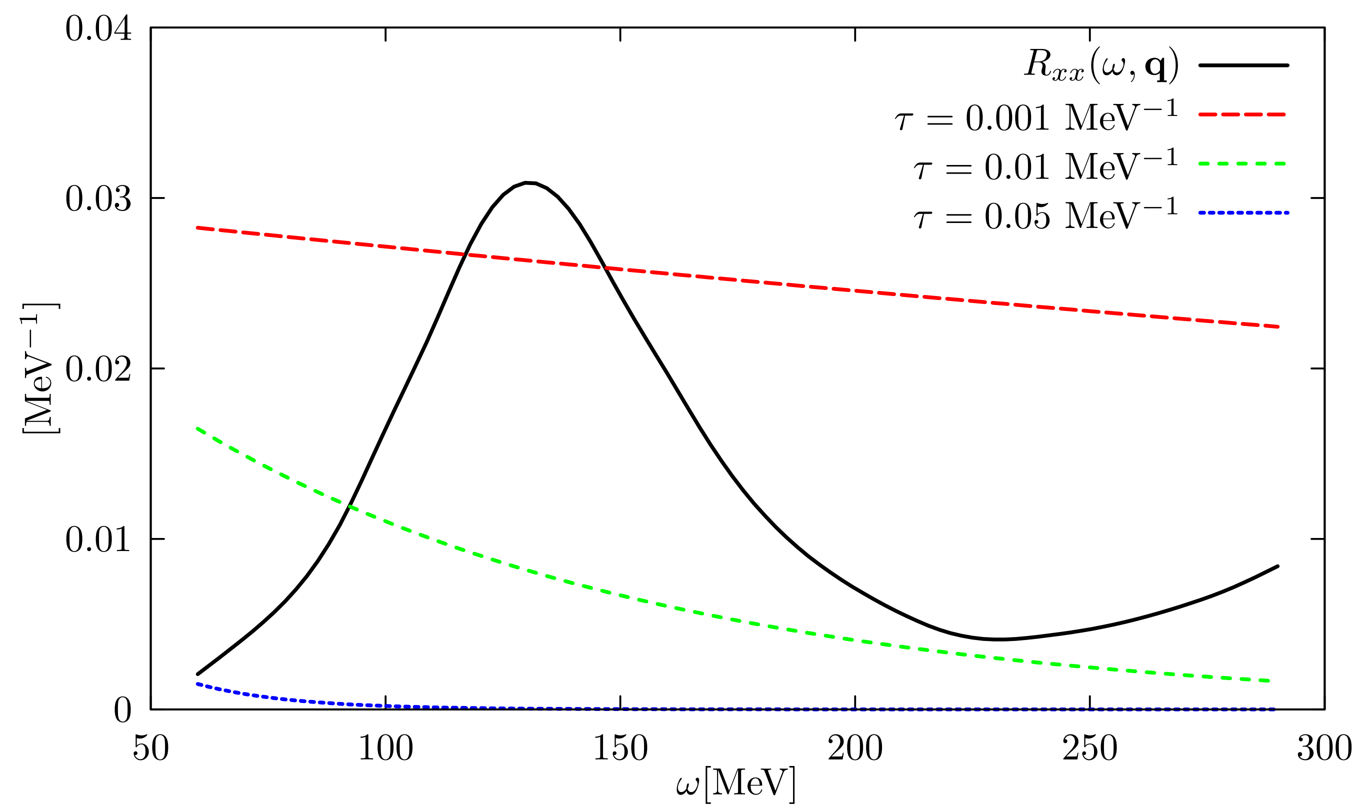
Bacca et al., PRC 76, 014003 (2007)

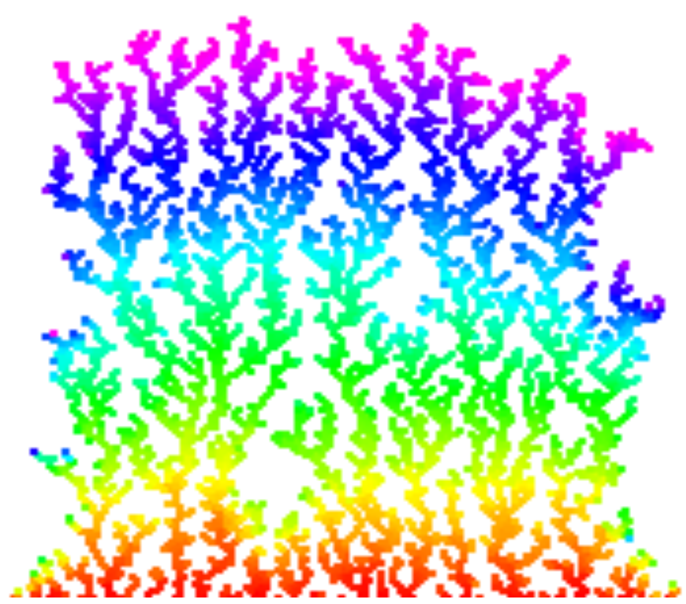
Euclidean response function

Valuable information on the energy dependence of the response functions can be inferred from their Laplace transforms

$$E_{\alpha\beta}(\tau, \mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega, \mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed





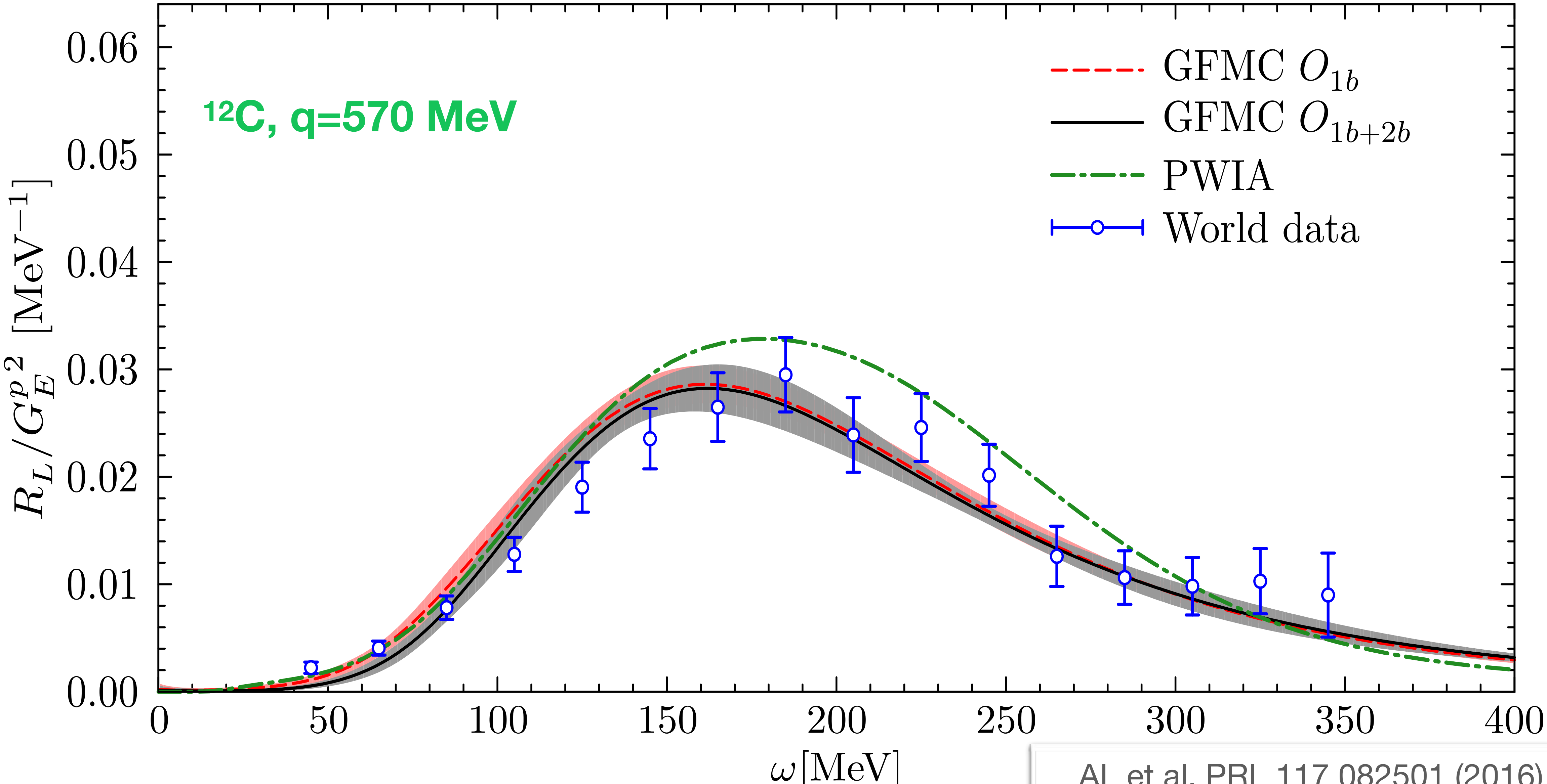
The system is first heated up by the transition operator.
Its cooling determines the Euclidean response of the system

$$E_{\alpha\beta}(\tau, \mathbf{q}) = \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) e^{-(H-E_0)\tau} J_\beta(\mathbf{q}) | \Psi_0 \rangle$$

Same technique used in Lattice QCD, condensed matter physics…

^{12}C electromagnetic response

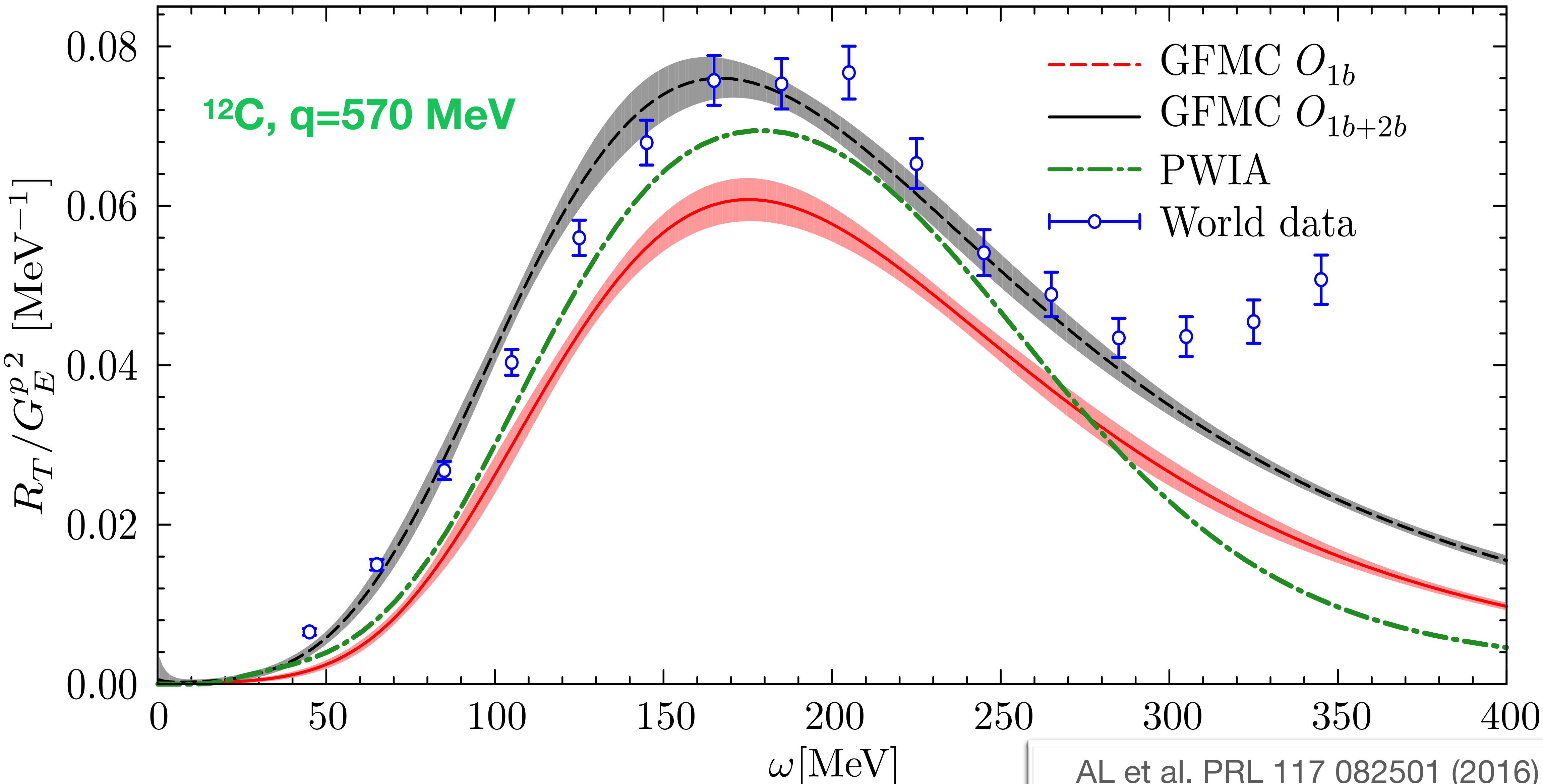
- We inverted the electromagnetic Euclidean response of ^{12}C
- Good agreement with data without in-medium modifications of the nucleon form factors
- Small contribution from two-body currents.



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

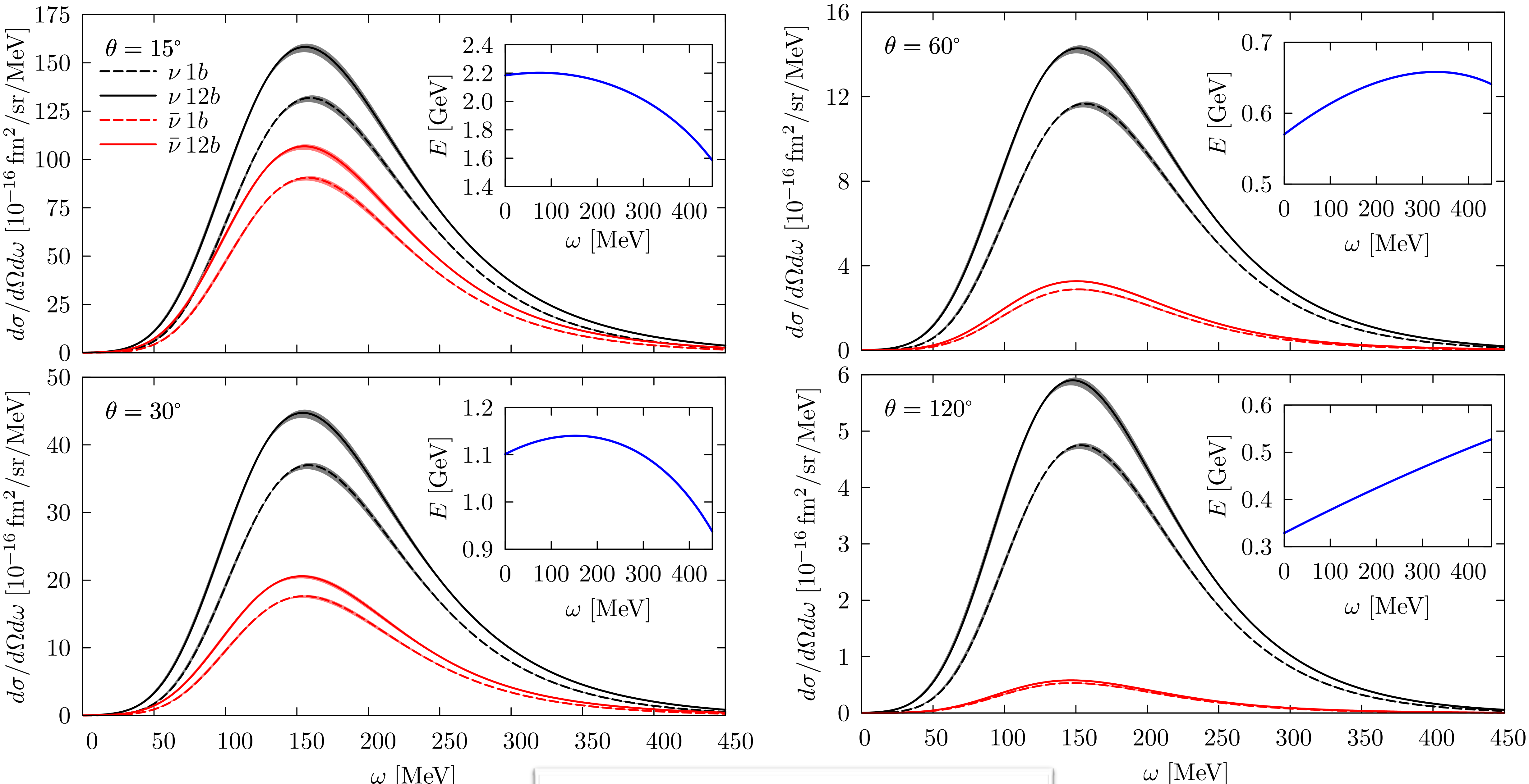
- We inverted the electromagnetic Euclidean response of ^{12}C
- Good agreement with the experimental data once two-body currents are accounted for
- Need to include relativistic corrections in the kinematics



AL et al. PRL 117 082501 (2016)

^{12}C neutral-current cross-section

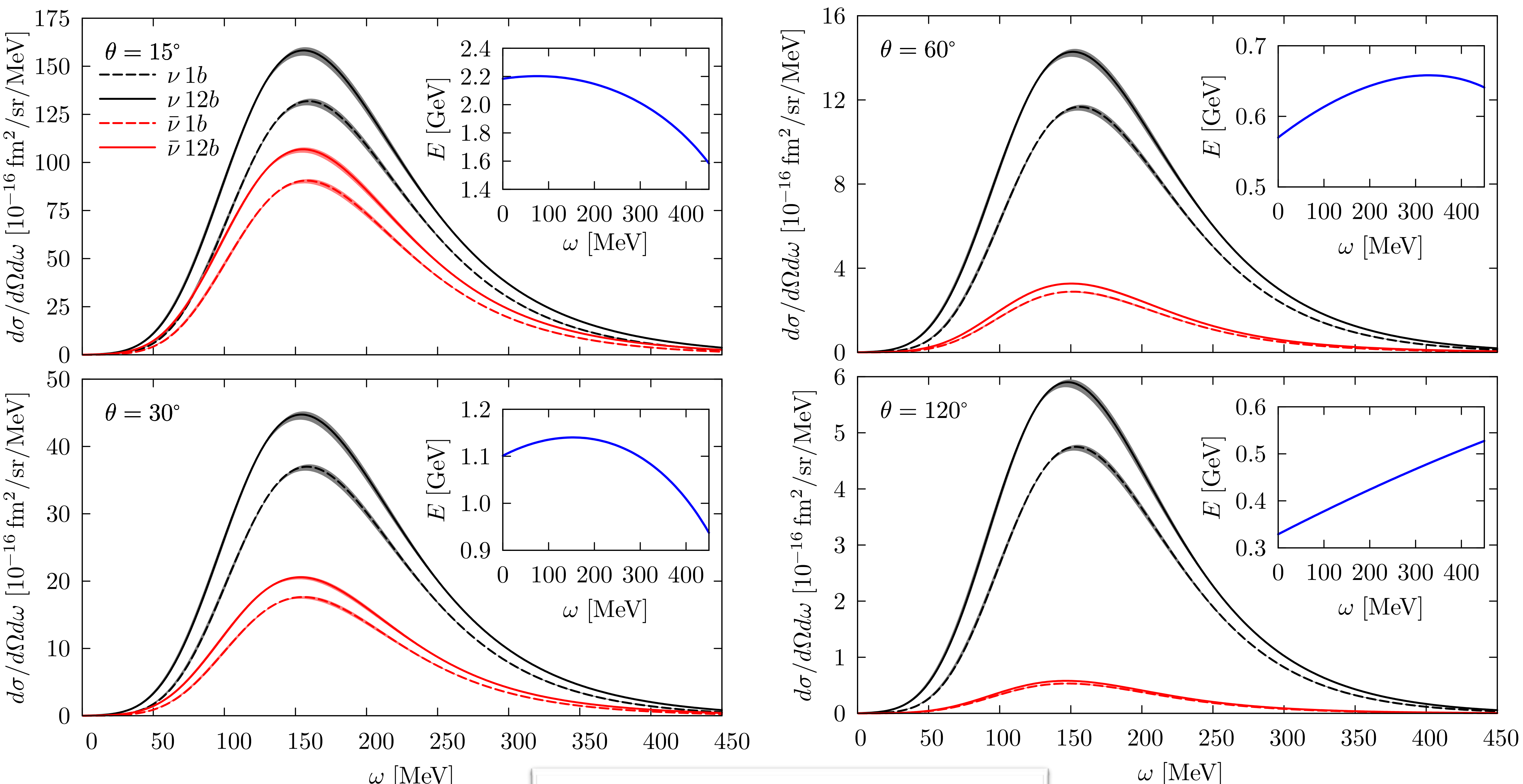
- We computed the neutrino and anti-neutrino differential cross sections for a fixed value of the three-momentum transfer as function of the energy transfer for a number of scattering angles



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

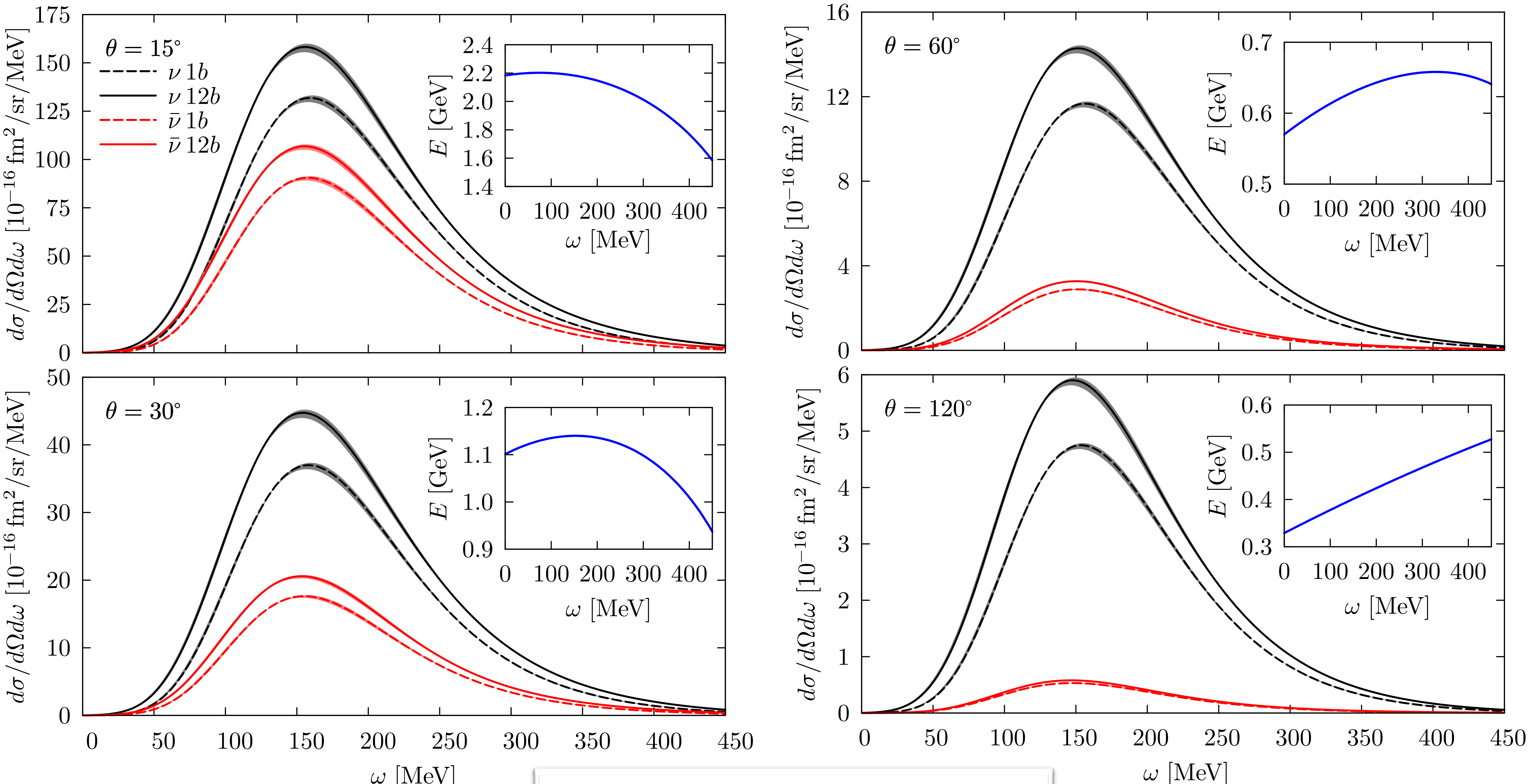
- The anti-neutrino cross section decreases rapidly relative to the neutrino cross section as the scattering angle changes from the forward to the backward hemisphere



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

- For this same reason, two-body current contributions are smaller for the antineutrino than for the neutrino cross section

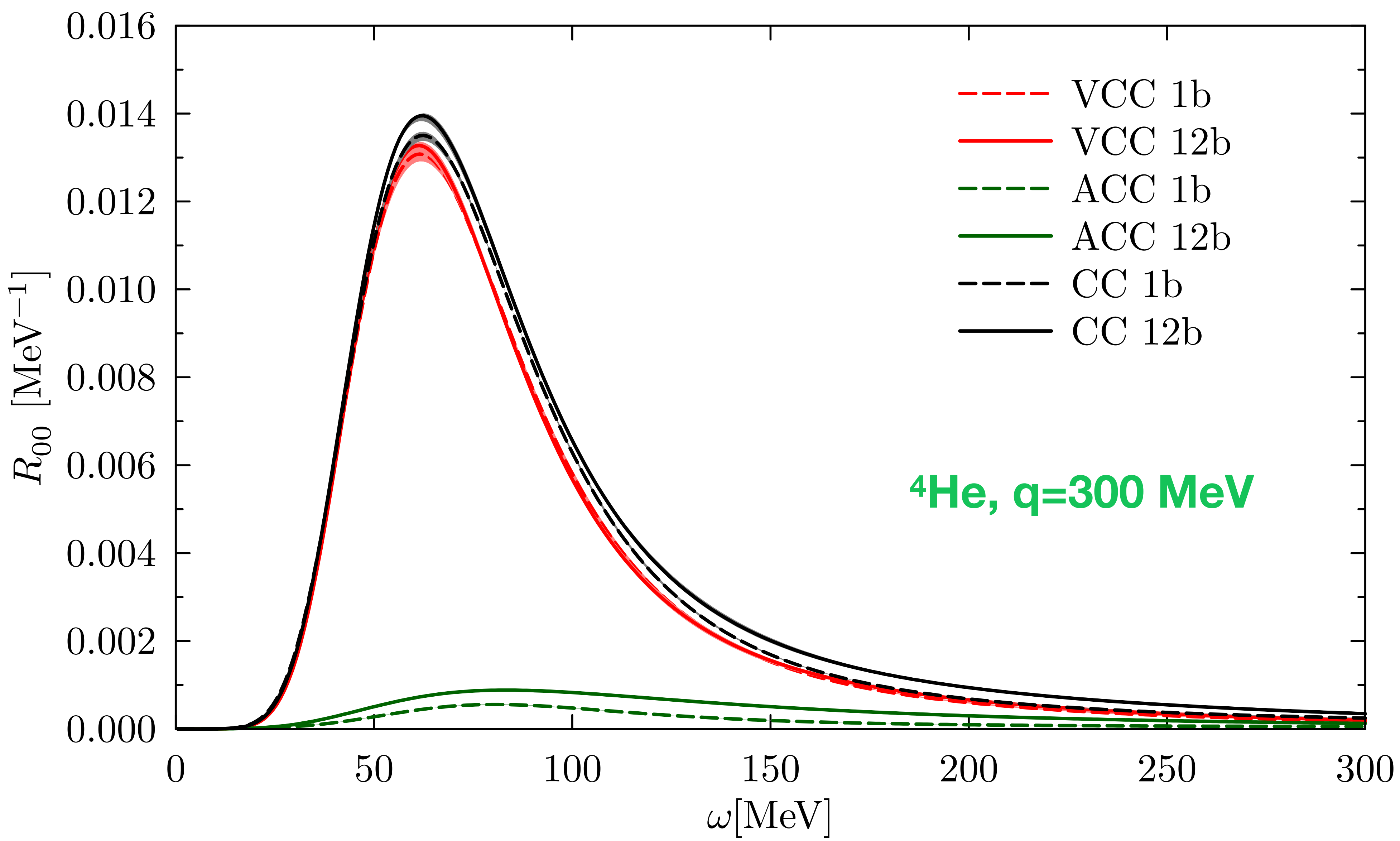


AL et al. PRC 97 022502 (2018)

Charged-current results

- We computed the charged-current response function of ^{4}He

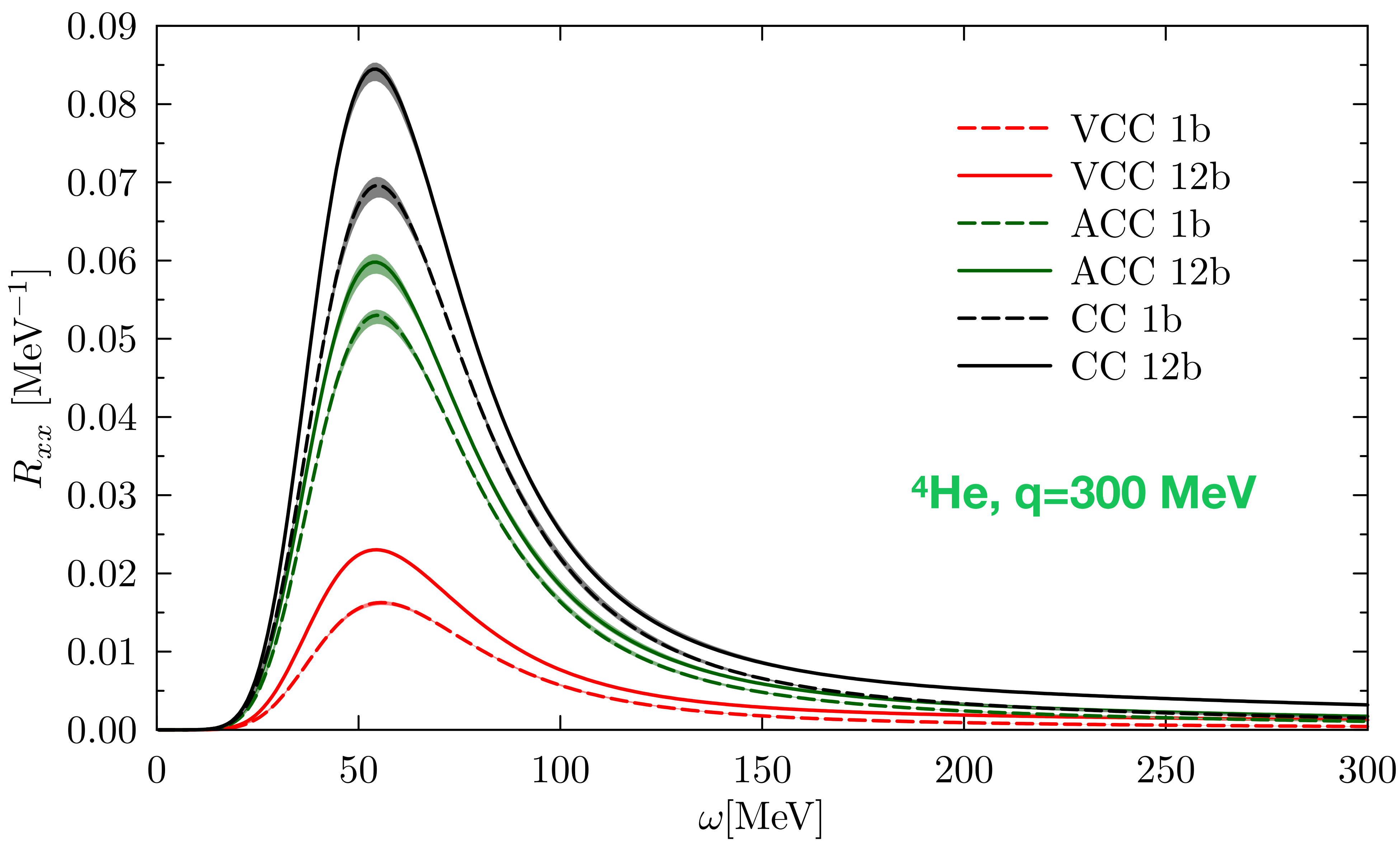
- Two-body currents have little effect in the vector term, but enhance the axial contribution at energy larger than quasi-elastic kinematics



Charged-current results

- We computed the charged-current response function of ^{4}He

- Two-body currents have a sizable effect in the transverse response, both in the vector and in the axial contributions



Relativistic effects in a correlated system

- Non relativistic approaches are limited to moderate momentum transfers

- In a generic reference frame the longitudinal response reads

$$R_L^{fr} = \sum_f \left| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \right|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

- The response in the LAB frame is given by the Lorentz transform

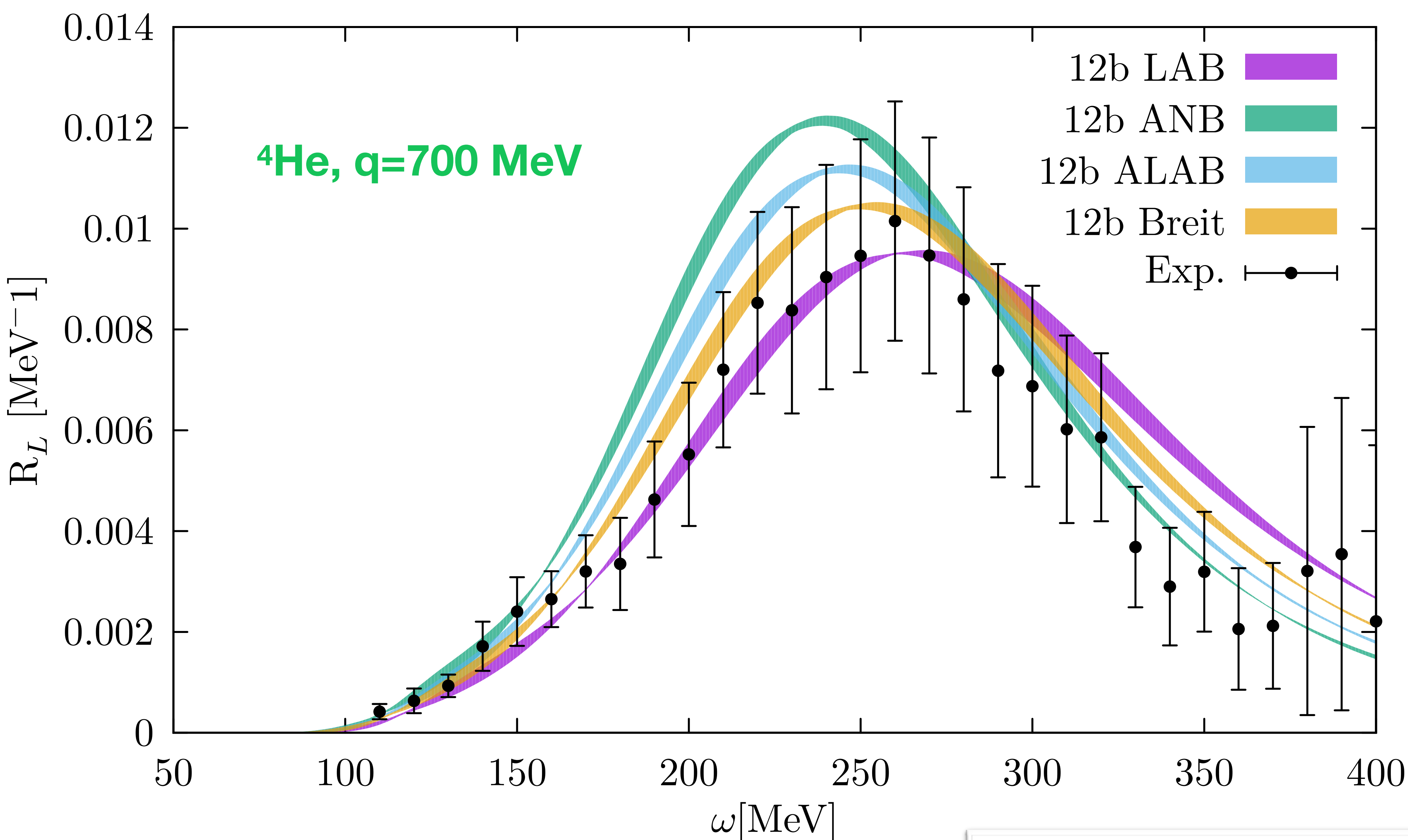
$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega), \; \omega^{fr} = \gamma(\omega - \beta q), \; P_i^{fr} = -\beta\gamma M_0, \; E_i^{fr} = \gamma M_0$$

Relativistic effects in a correlated system

- The ^4He longitudinal response at q=700 MeV **strongly** depends on the original reference frame



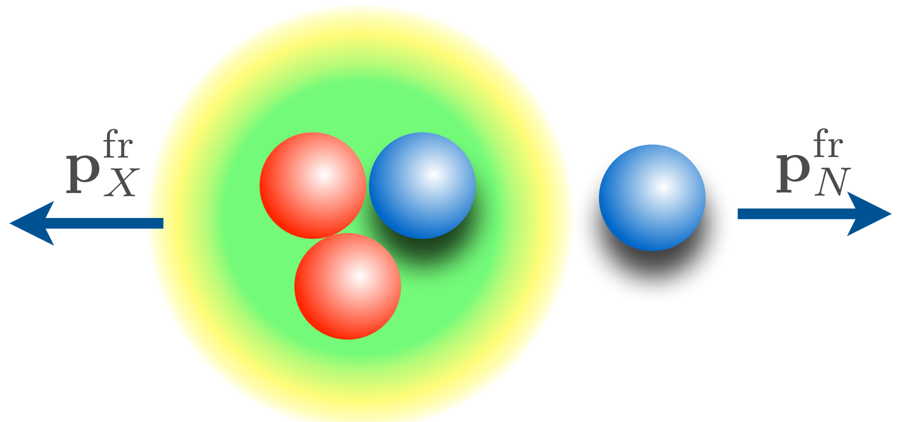
N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system

- To determine the relativistic corrections, we consider a two-body breakup model

$$\mathbf{p}^{\rm fr} = \mu(\frac{\mathbf{p}_N^{\rm fr}}{m} - \frac{\mathbf{p}_X^{\rm fr}}{M_X})$$
$$\mathbf{P}_f^{\rm fr} = \mathbf{p}_N^{\rm fr} + \mathbf{p}_X^{\rm fr}$$

$$\mu = \frac{mM_X}{m + M_X}$$
$$M_X = (A-1)m + \epsilon_0^{A-1}$$



- The relative momentum is derived in a relativistic fashion

$$\omega^{\rm fr} = E_f^{\rm fr} - E_i^{\rm fr}$$
$$E_f^{\rm fr} = \sqrt{m^2 + (\mathbf{p}^{\rm fr} + (\mu/M_{A-1})\mathbf{P}_f^{\rm fr})^2} + \sqrt{M_{A-1}^2 + (\mathbf{p}^{\rm fr} - (\mu/m)\mathbf{P}_f^{\rm fr})^2}$$

- And it is used as input in the non relativistic kinetic energy

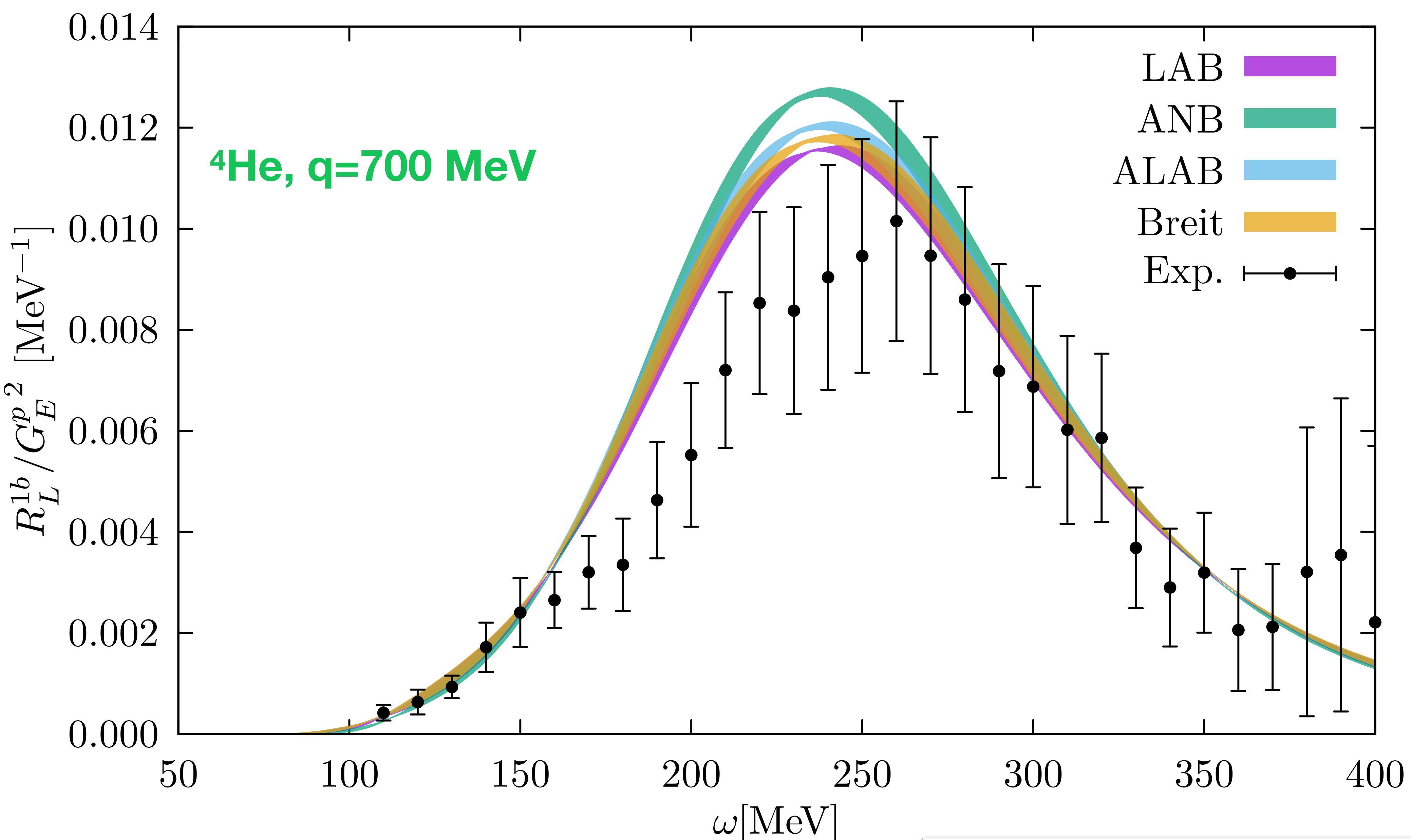
$$\epsilon_f = \frac{p_f^2}{2\mu} + \epsilon_0^{A-1}$$

- The energy-conserving delta function reads

$$\delta(\omega^{fr} - E_f^{\rm fr}(\epsilon_f) + E_0^{fr}) = \left(\frac{\partial E_f^{\rm fr}(\epsilon_f)}{\partial \epsilon_f^{\rm fr}}\right)^{-1} \delta\left(\epsilon_f - \frac{p_f^2(\omega^{\rm fr}, |\mathbf{q}^{\rm fr})}{2\mu} - \epsilon_0^{A-1}\right)$$

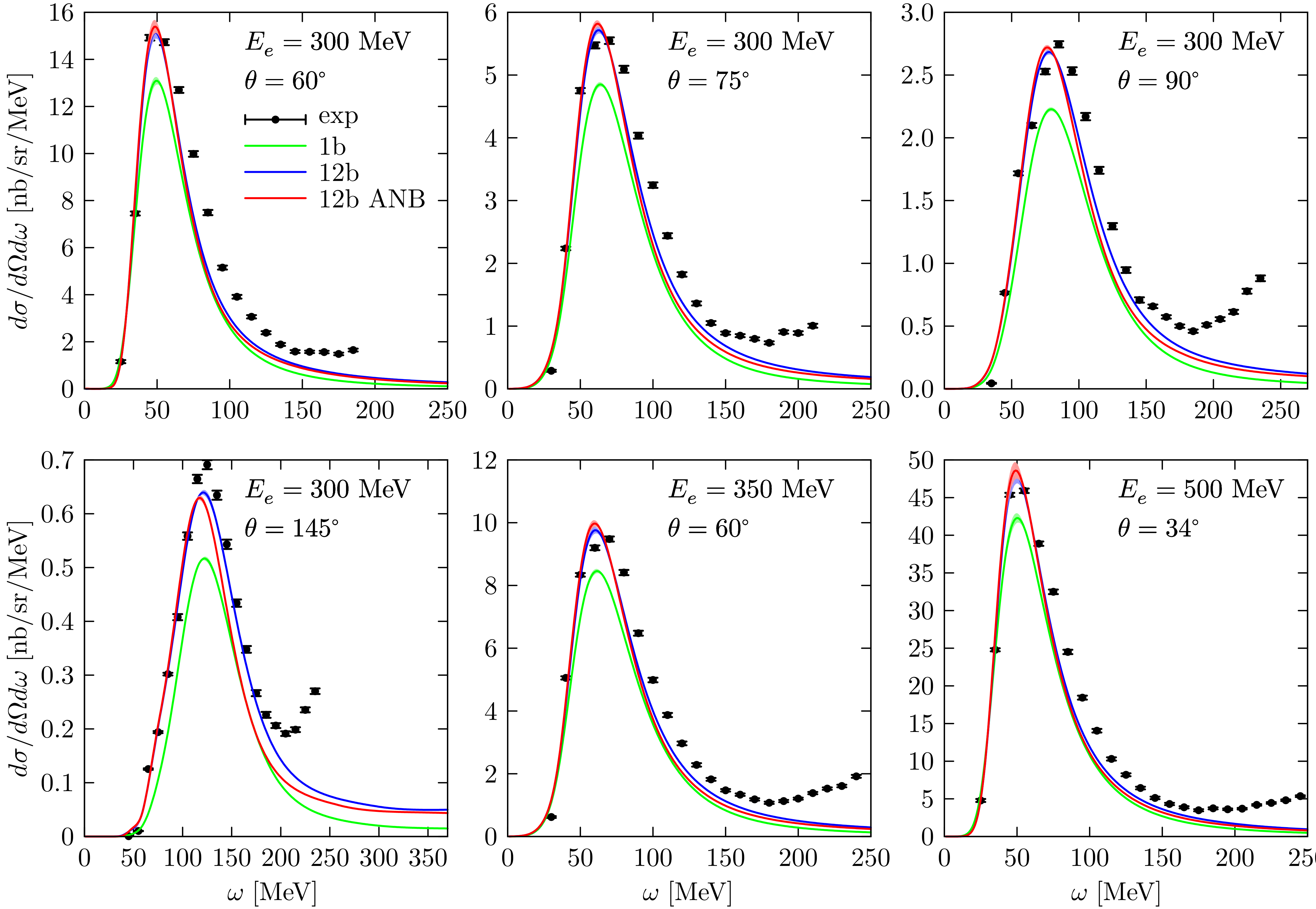
Relativistic effects in a correlated system

- The ^4He longitudinal response at q=700 MeV **mildly** depends on the original reference frame



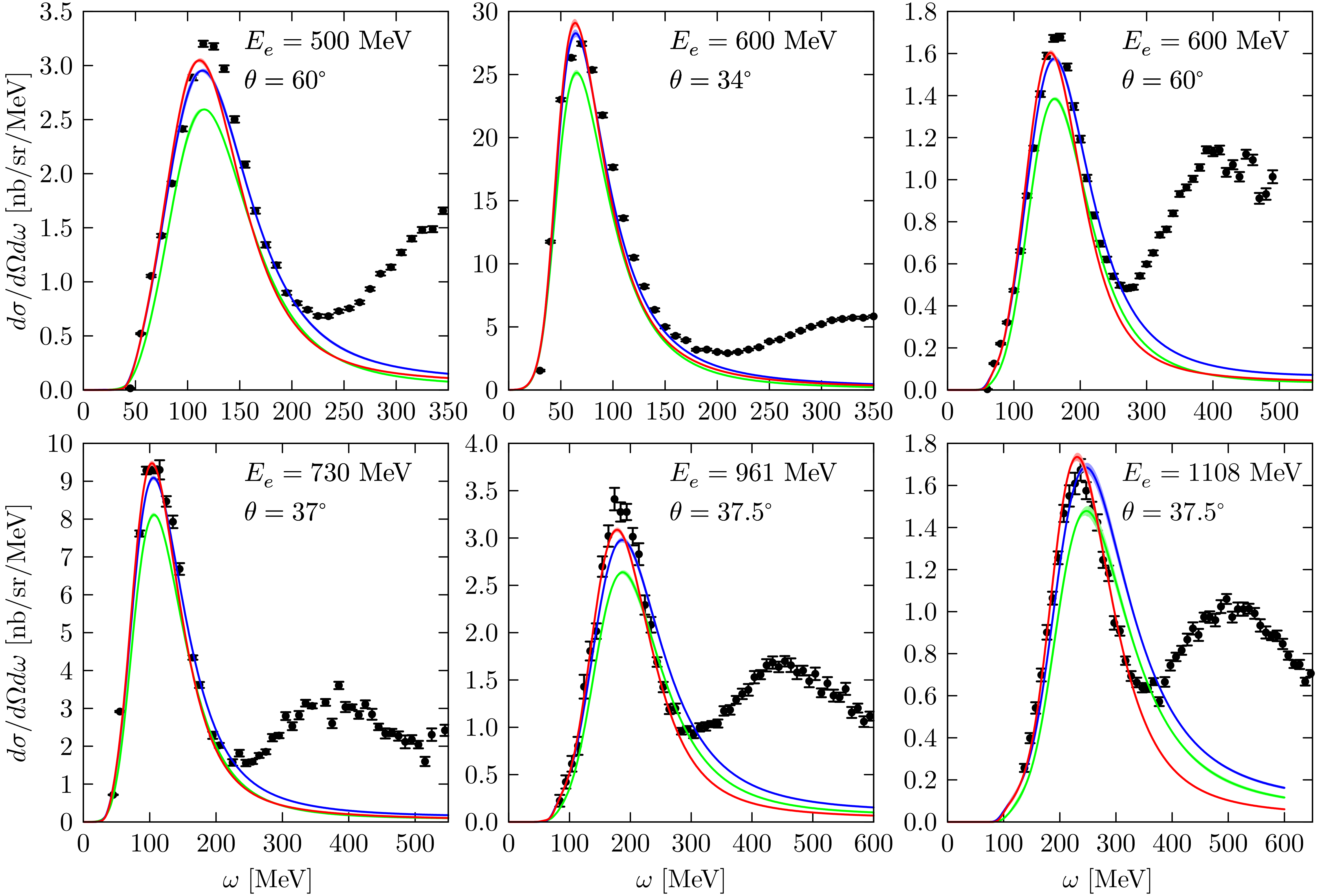
N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system



N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system



N. Rocco et al. PRC 97 055501(2018)

Spectral function approach

Neglecting (for now) two-body currents and assuming the factorization of the final state

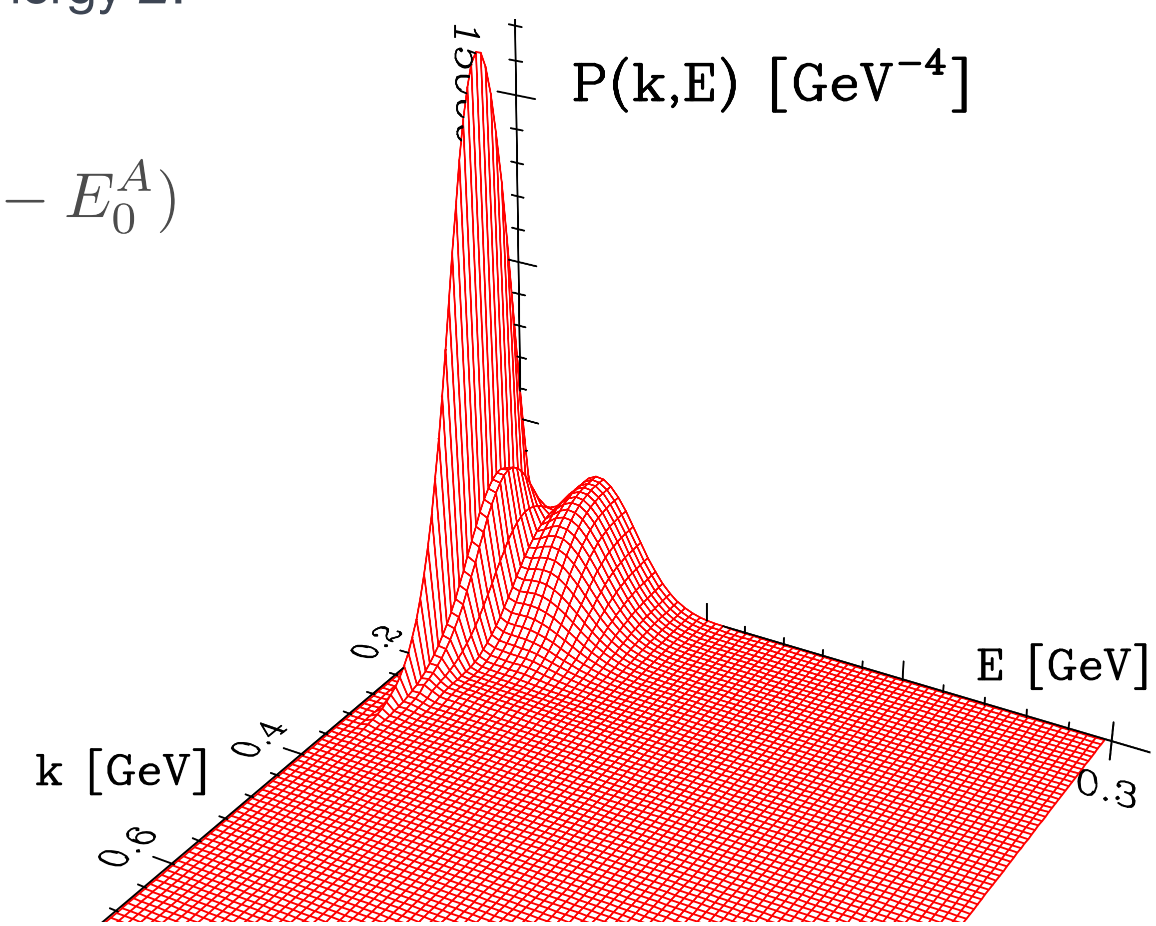
$$J^\mu \to \sum_i j_i^\mu \qquad\qquad |\Psi_f\rangle \to |\mathbf{p}\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-1}$$

The response function is sum of scattering processes involving individual bound nucleons

$$R_{\alpha\beta} = \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k},E) \sum_i \langle k|j_\alpha^{i\,\dagger}|k+q\rangle\langle k+q|j_\beta^i|k\rangle\delta(\omega+E-e_{\mathbf{k+q}})$$

The **spectral function** yields the probability of removing a nucleon with momentum **k** from the ground state leaving the residual system with excitation energy *E*.

$$P_h(\mathbf{k},E) = \sum_f |\langle\psi_0^A|[|k\rangle\otimes|\psi_f^{A-1}\rangle]|^2\delta(E+E_f^{A-1}-E_0^A)$$

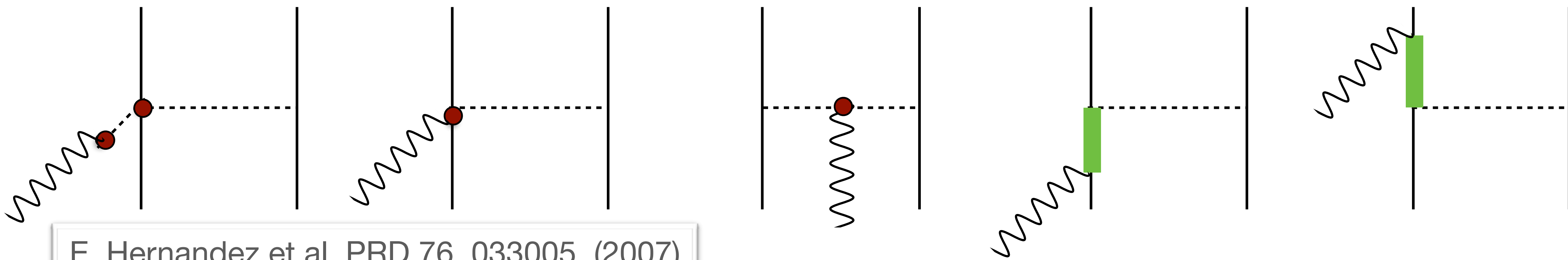


Approximate spectral functions are based on electron scattering data and on the local-density approximation

$$P_h(\mathbf{k},E) = P_h^{1h}(\mathbf{k},E) + P_h^{\rm corr}(\mathbf{k},E)$$

Neutrino-nucleus scattering

- We implemented vector and axial vector relativistic two-body currents in the factorization scheme



E. Hernandez et al. PRD 76, 033005 (2007)

We developed an highly-parallel Monte Carlo integration code

The calculation of the MEC current matrix elements is carried our automatically

No need to use approximations such that of the “frozen nucleons”

Simplifies the use of a different version of the MEC

- We employ the factorization of the two-body spectral function, related to

$$n(\mathbf{k}_1, \mathbf{k}_2) = n(\mathbf{k}_1) n(\mathbf{k}_2) + \mathcal{O}\left(\frac{1}{A}\right)$$

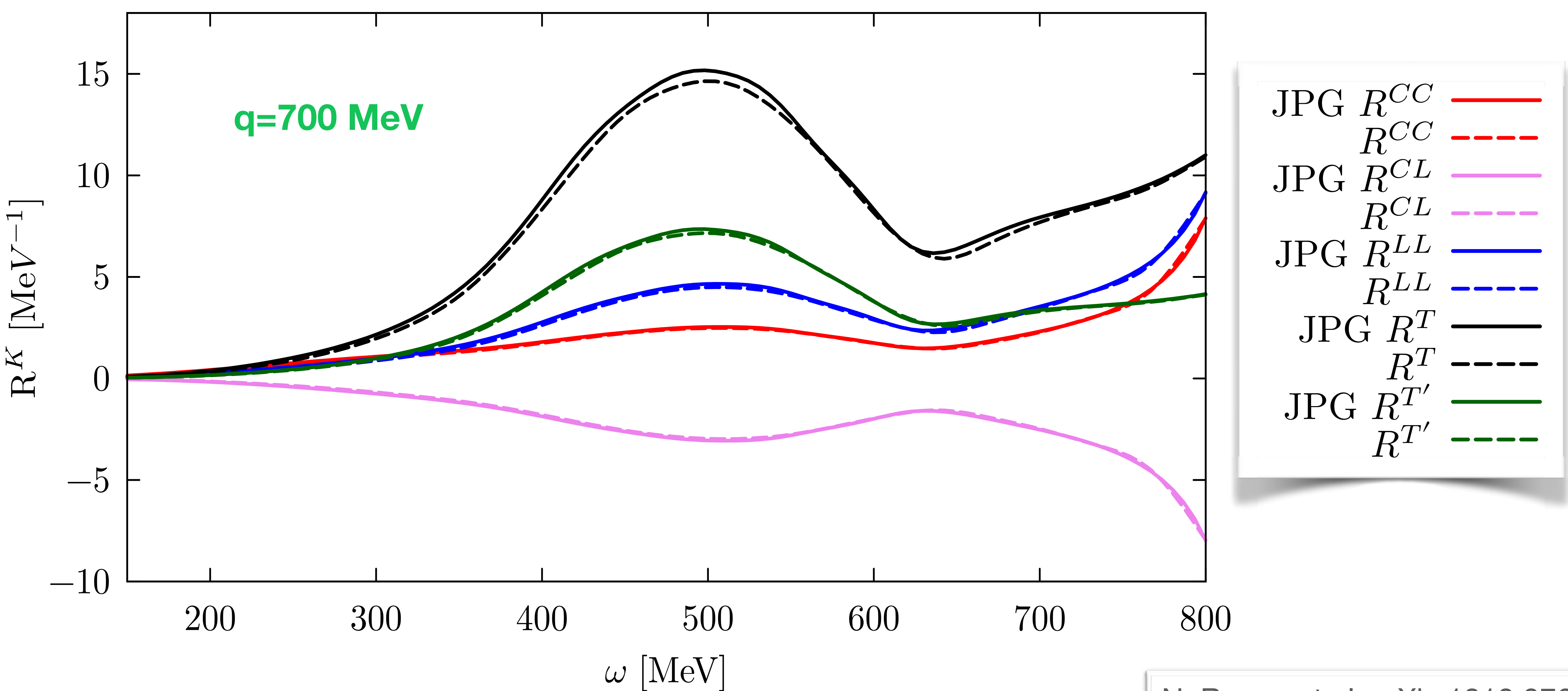
We are improving this approximation using the cluster-expansion formalism

Analogy with the “short-time approximation” and the “contact formalism”

Charged responses

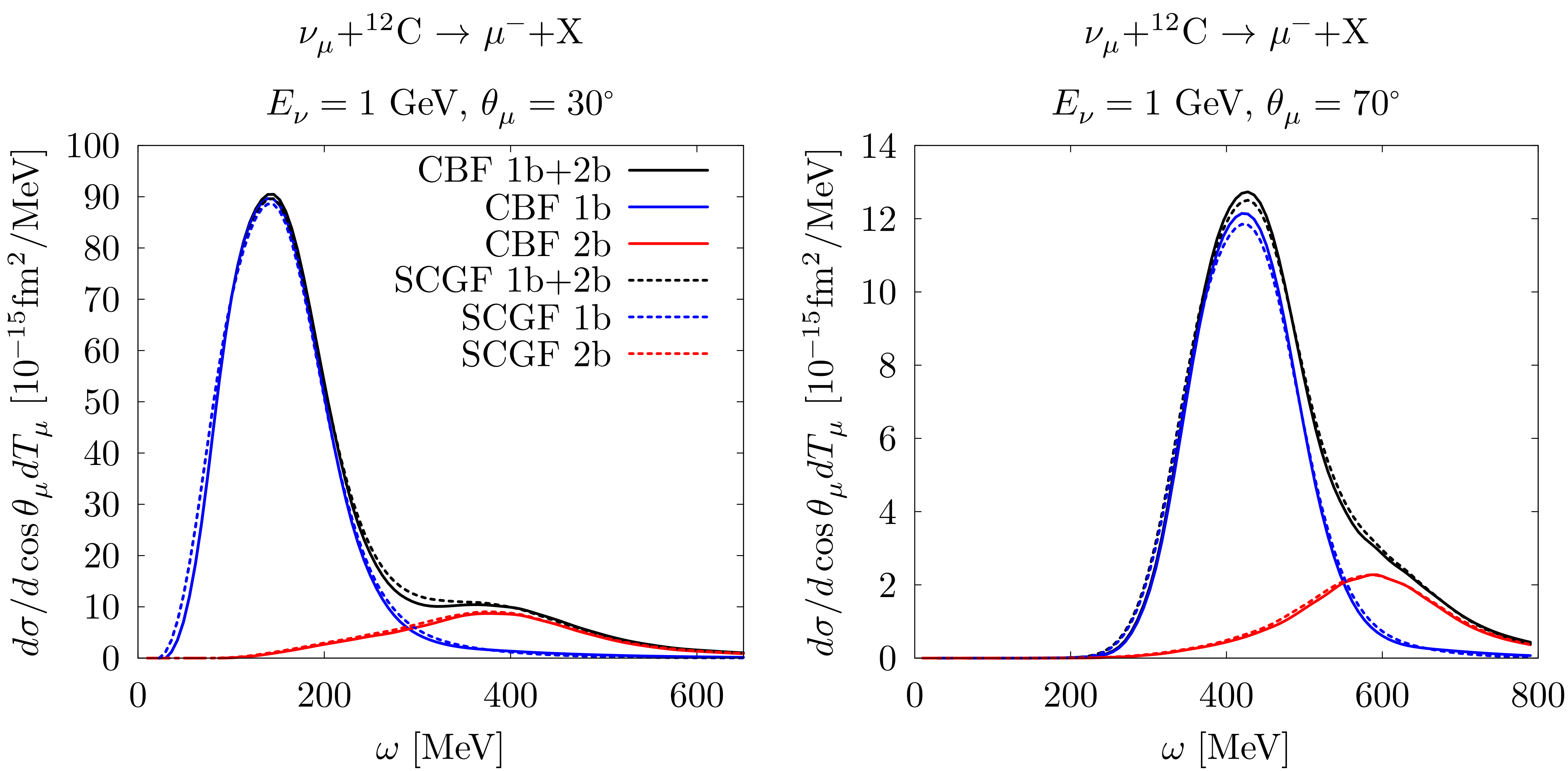
- We successfully compared the charged-current response functions of ^{12}C with the results of I. Ruiz Simo, et. al, Journal of Phys. G 44, no. 6 (2017)

- To this aim we approximated the two-body spectral function with that of the global relativistic Fermi gas model



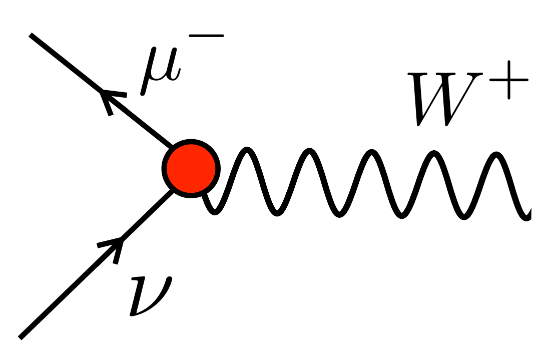
N. Rocco et al. arXiv:1810.07647

Neutrino-^{12}C charged-current scattering



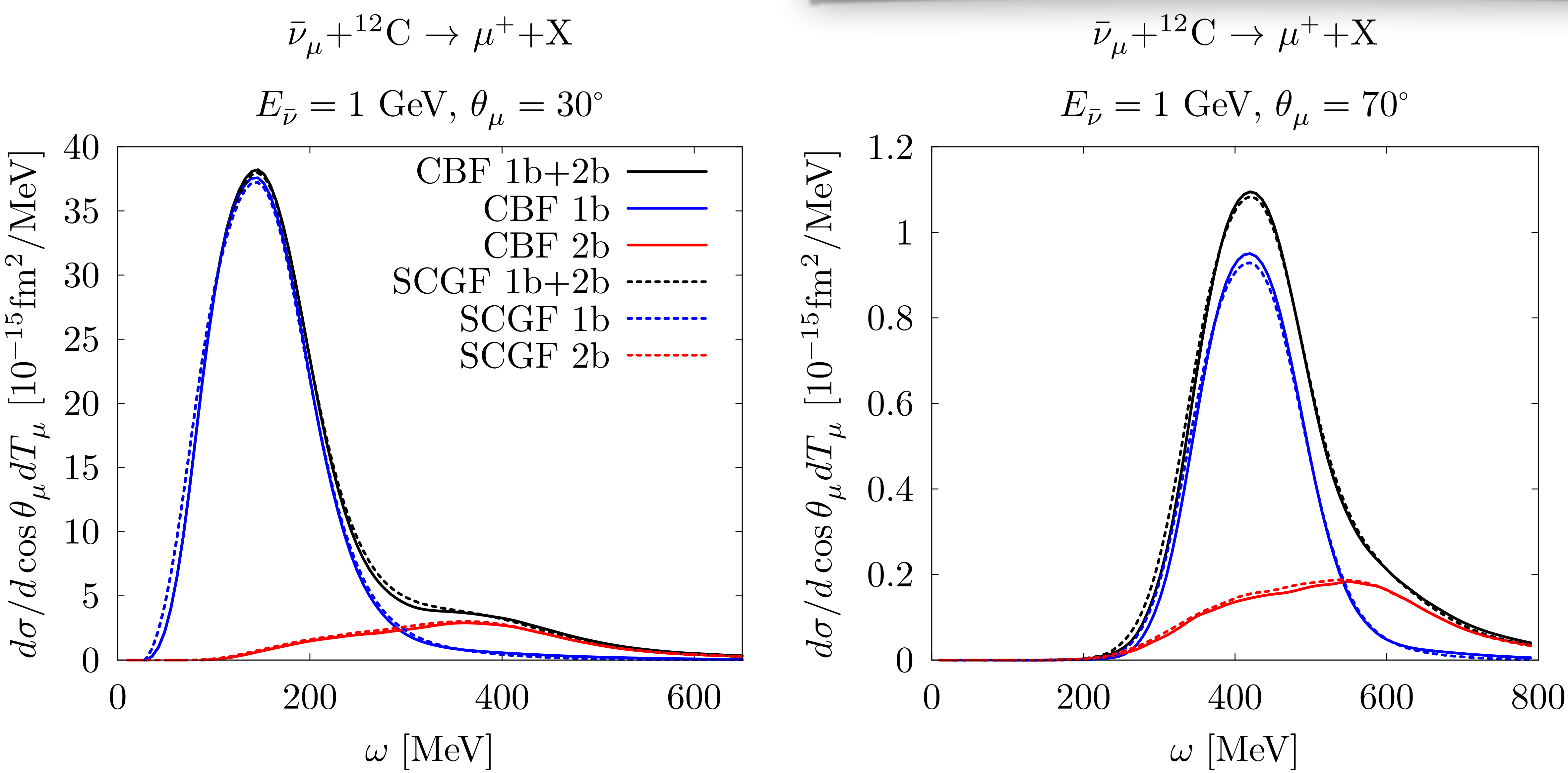
- Two contributions mostly affect the ‘dip’ region

- Meson exchange currents strongly enhance the cross section for large values of the scattering angle



Neutrino-^{12}C charged-current scattering

NR, C.Barbieri, O. Benhar, A. De Pace, A. Lovato, arXiv:1810.07647



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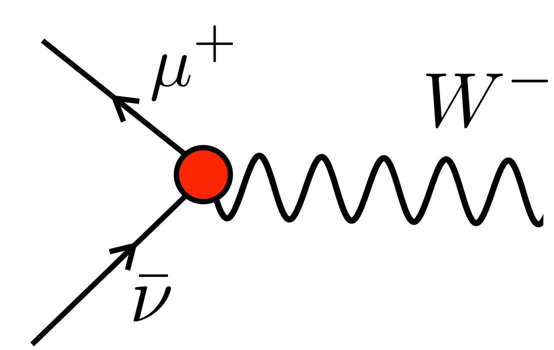

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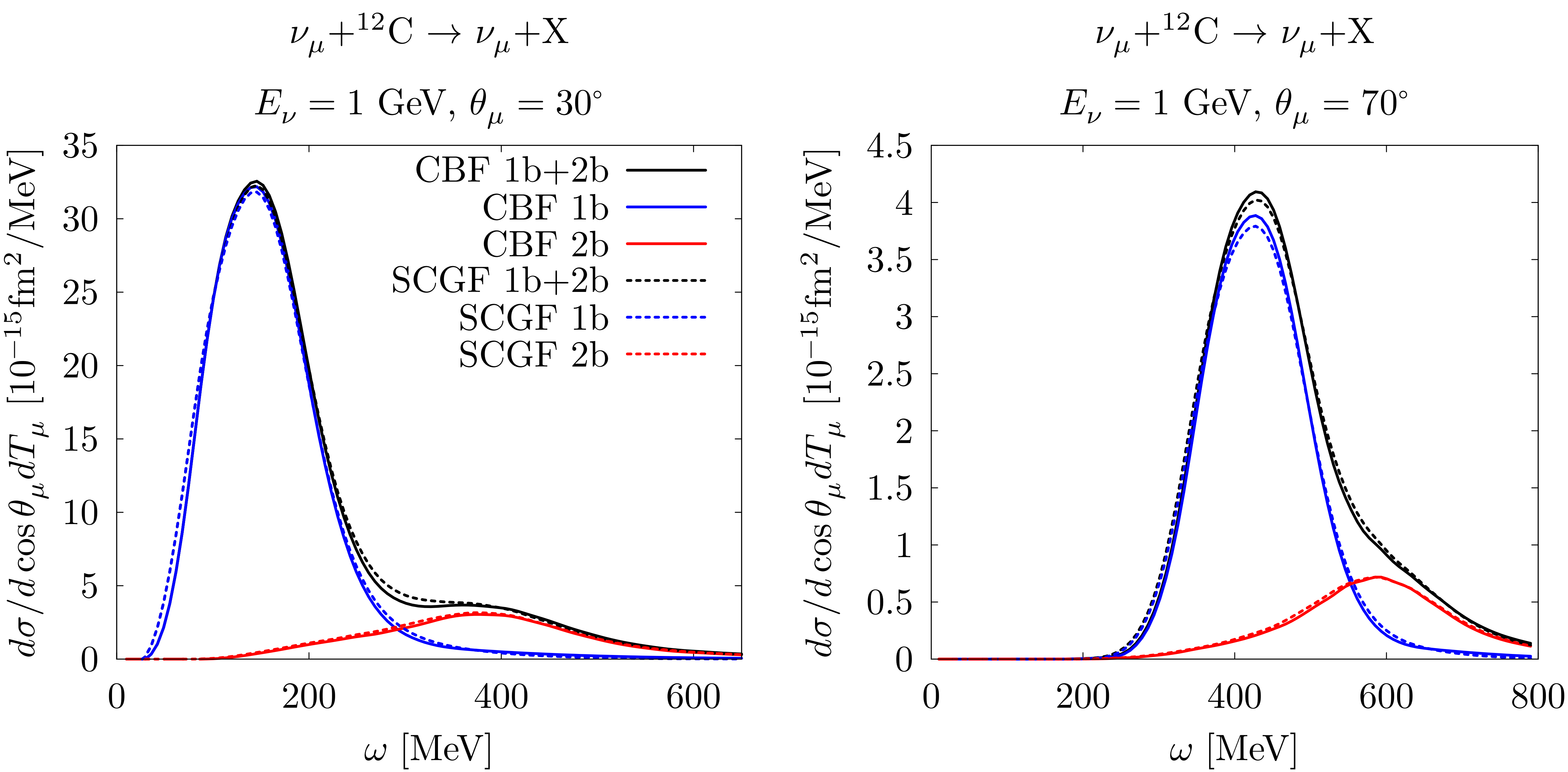

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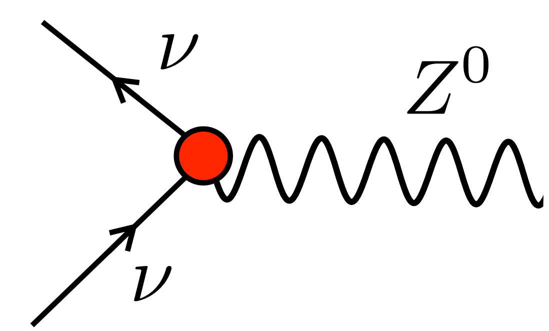

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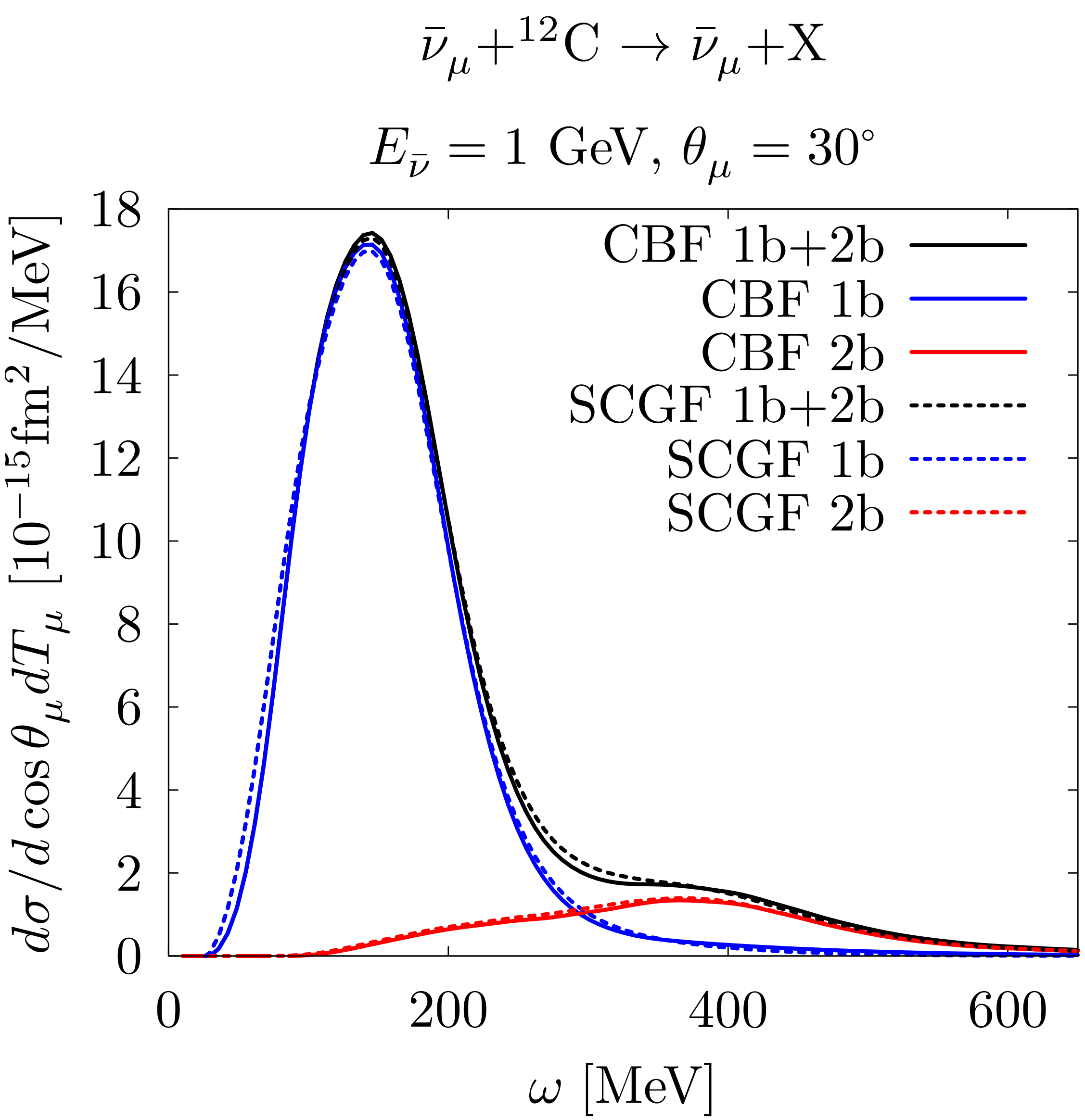

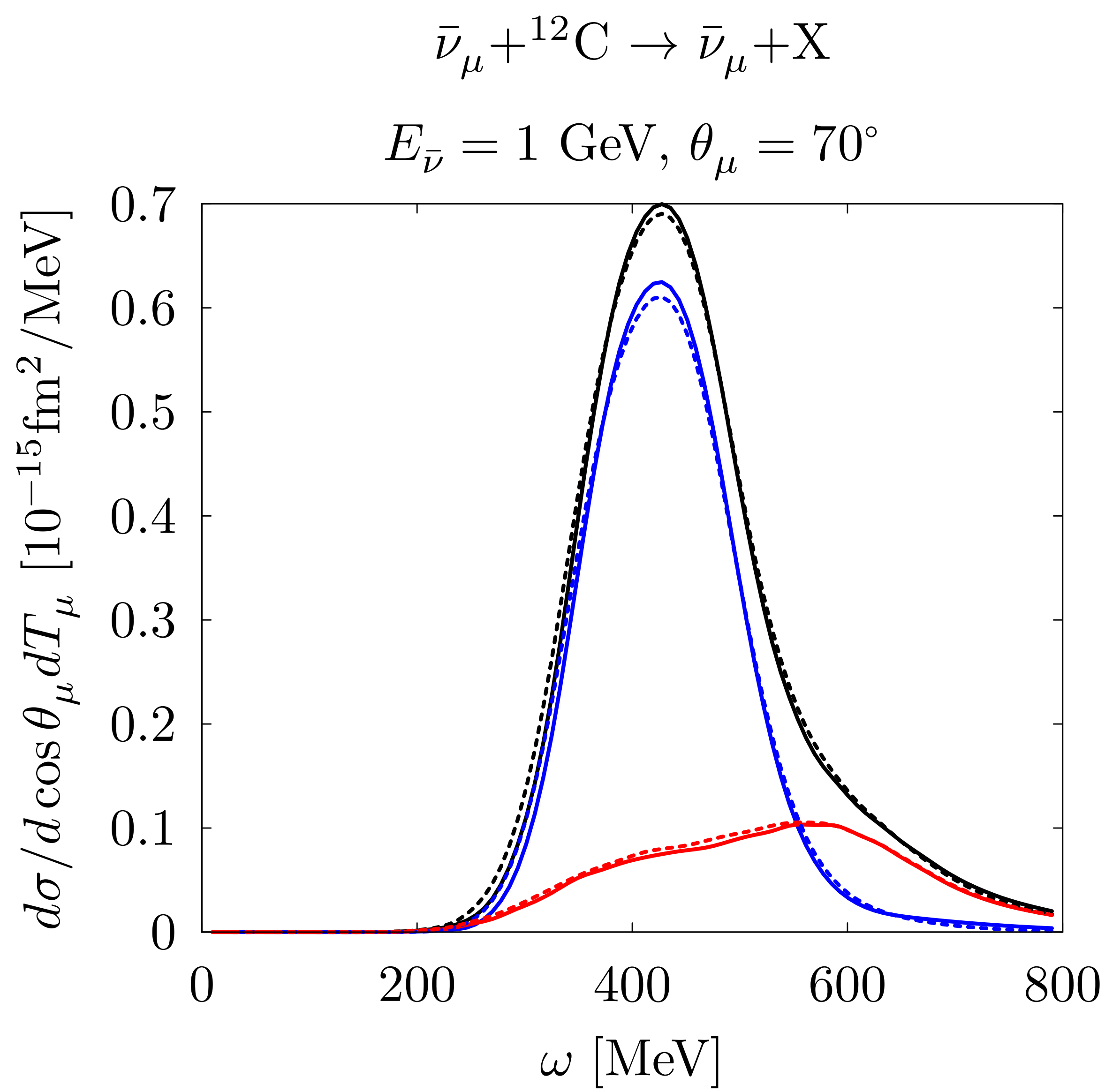

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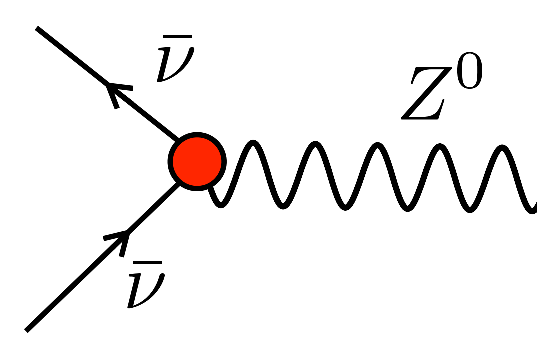
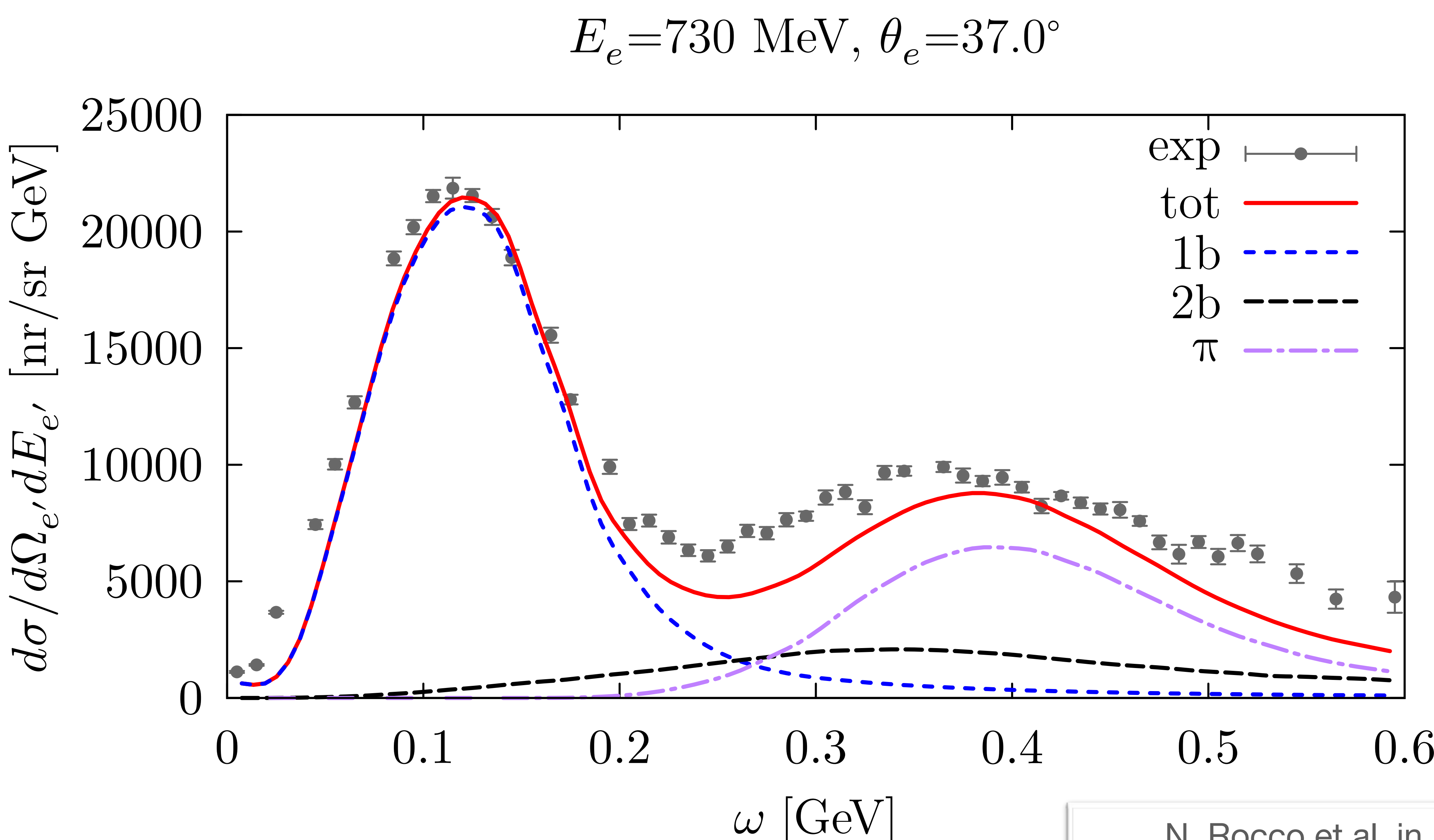


Spectral function approach

We extended the spectral function approach to include pion-production mechanisms

$$|\Psi_f\rangle \rightarrow |\mathbf{p}\rangle \otimes |\mathbf{p}_\pi\rangle \otimes |\Psi_f\rangle_{A-1}$$

Good agreement with experimental data, although some strength is missing in the Delta region



N. Rocco et al. in preparation

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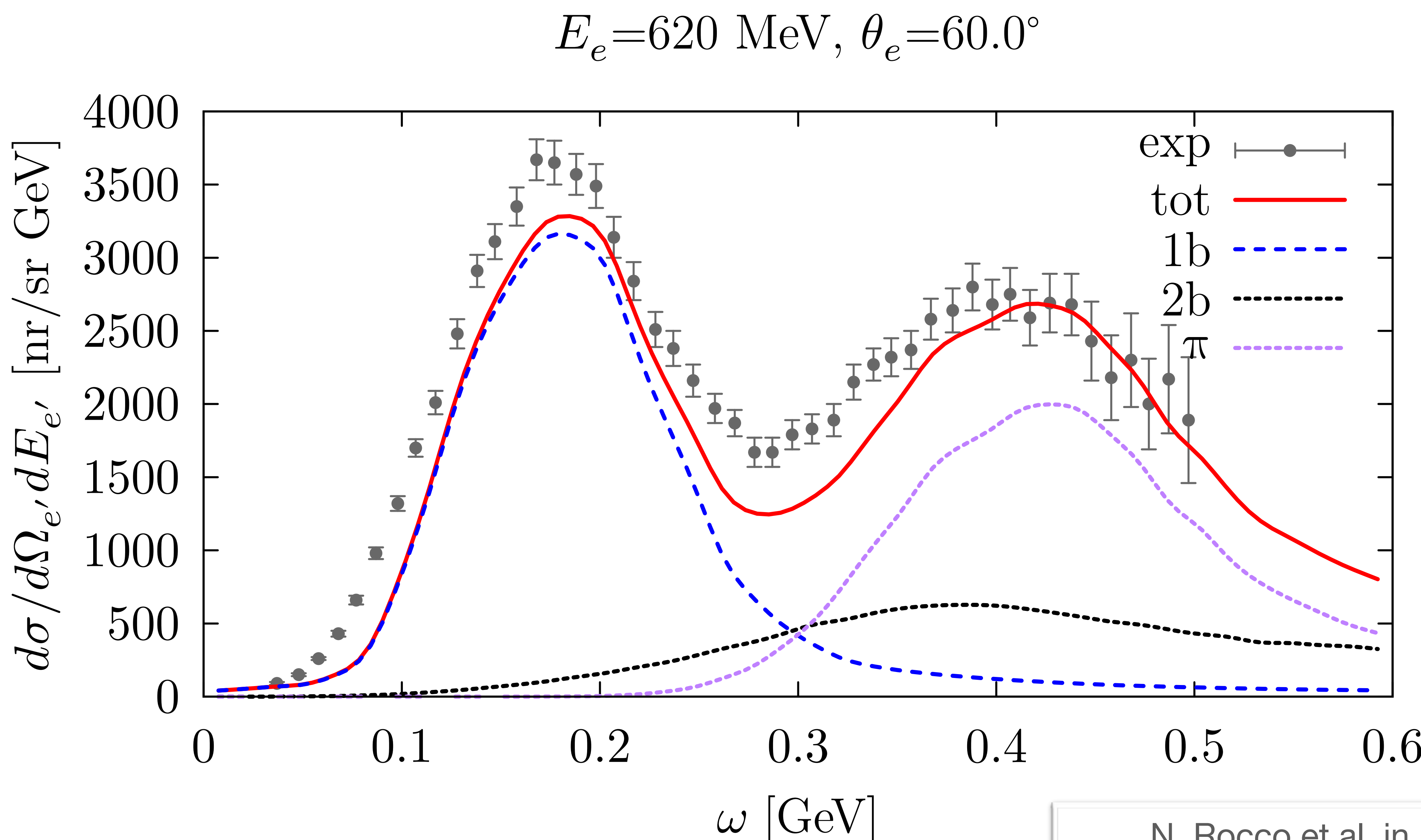

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Summary and plans

Current status

- GFMC calculations of ^{12}C electromagnetic responses in good agreement with experiments.
- Two-body currents enhance the electromagnetic, neutral- and charged-current responses
- We devised a scheme to account for relativistic kinematics in the GFMC
- We extended the factorization scheme to include relativistic two-body currents and (some) pion-production mechanisms

GFMC Plans

- GFMC calculations of the charged-current neutrino and anti-neutrino scattering off ^{12}C
- GFMC calculations of the spectral function of light nuclei

$$\int dE e^{-E\tau} P_h(\mathbf{k}, E) \sim \frac{\langle \Psi_0 | a^\dagger_{\mathbf{k}} e^{-(H-E_0)\tau} a_{\mathbf{k}} | \Psi_0 \rangle}{\langle \Psi_0 | e^{-(H-E_0)\tau} | \Psi_0 \rangle}$$

- Interference term in the factorization ansatz within the cluster expansion formalism
- Extend the spectral function approach to account for the resonance production mechanism