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Quantum effects in undulator radiation: Proposed experiments and results of theoretical analysis

Ihar Lobach THE UNIVERSITY OF CHICAGO
Budker Seminar
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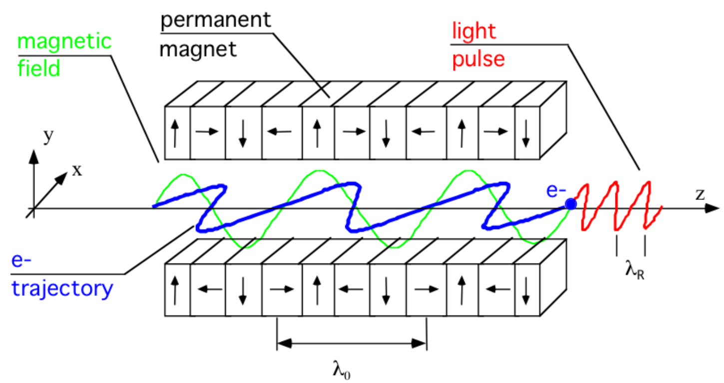
Outline

- Introduction
- How to get a single electron in a storage ring
- Theoretical predictions for undulator radiation produced by a single electron
- Experiment ideas for IOTA

Introduction

Undulator radiation:



Quantum effects in undulator radiation:

1) quantized radiation

(more than one photon can be emitted per pass)

2) quantum nature of electron

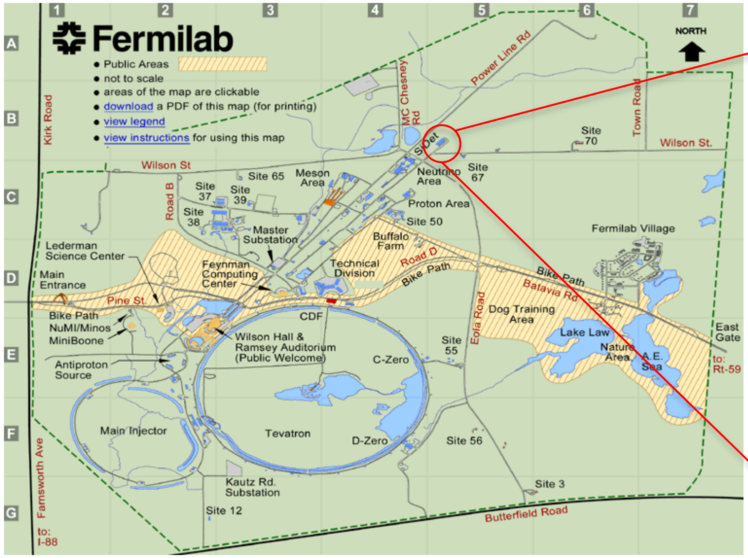
(electron wavefunction’s size may be considerable)

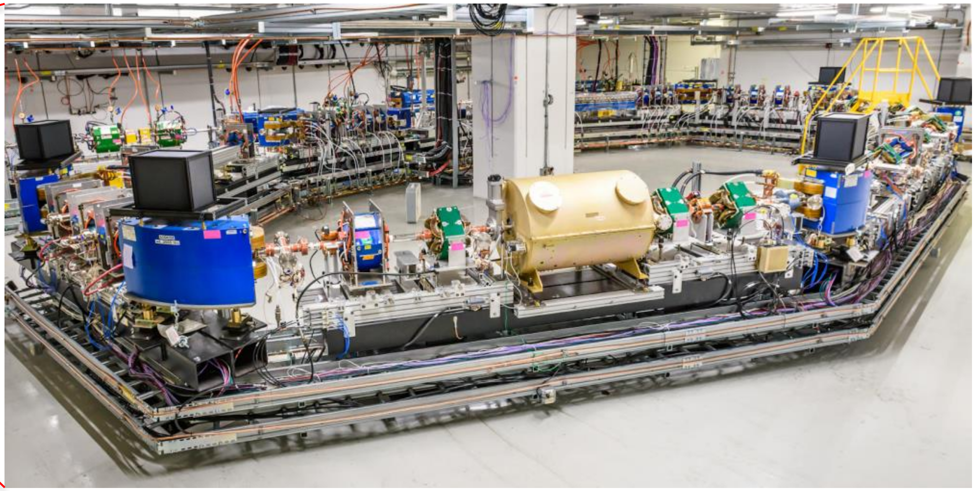
When we detect two photons we want to be sure that they were produced by the same electron

→

We need to keep a single electron in the ring

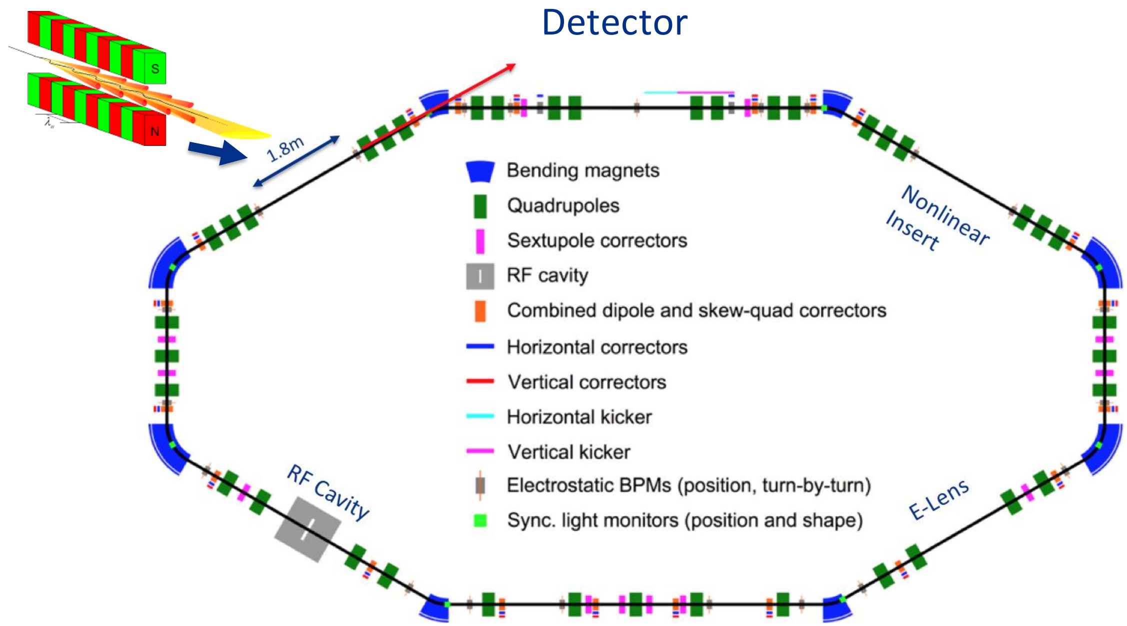
IOTA ring: first beam Aug 21, 2018





- A 40-m ring (electrons and protons)
- Design beam energy: 150 MeV (electrons)

A proposed experimental setup in IOTA



We have two undulators that we can borrow, from SLAC and JLab
Both have 55mm period.
(N=11 and N = 30) Variable K.

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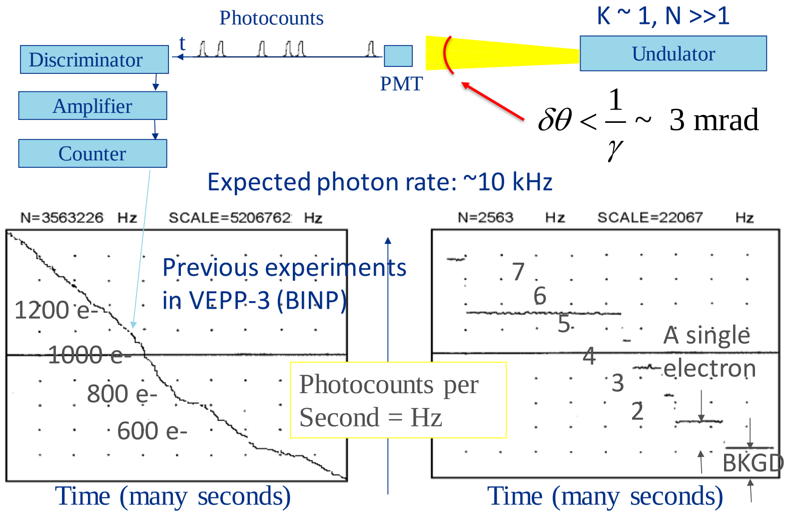
Single electron in a storage ring

- Experiments in VEPP-3 in Novosibirsk (1993):



- Reducing RF voltage for a moment
- SR intensity measurement by PMTs

- Metrology Light Source (MLS) in Germany (2008):

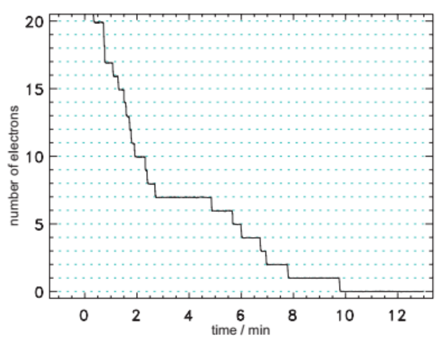
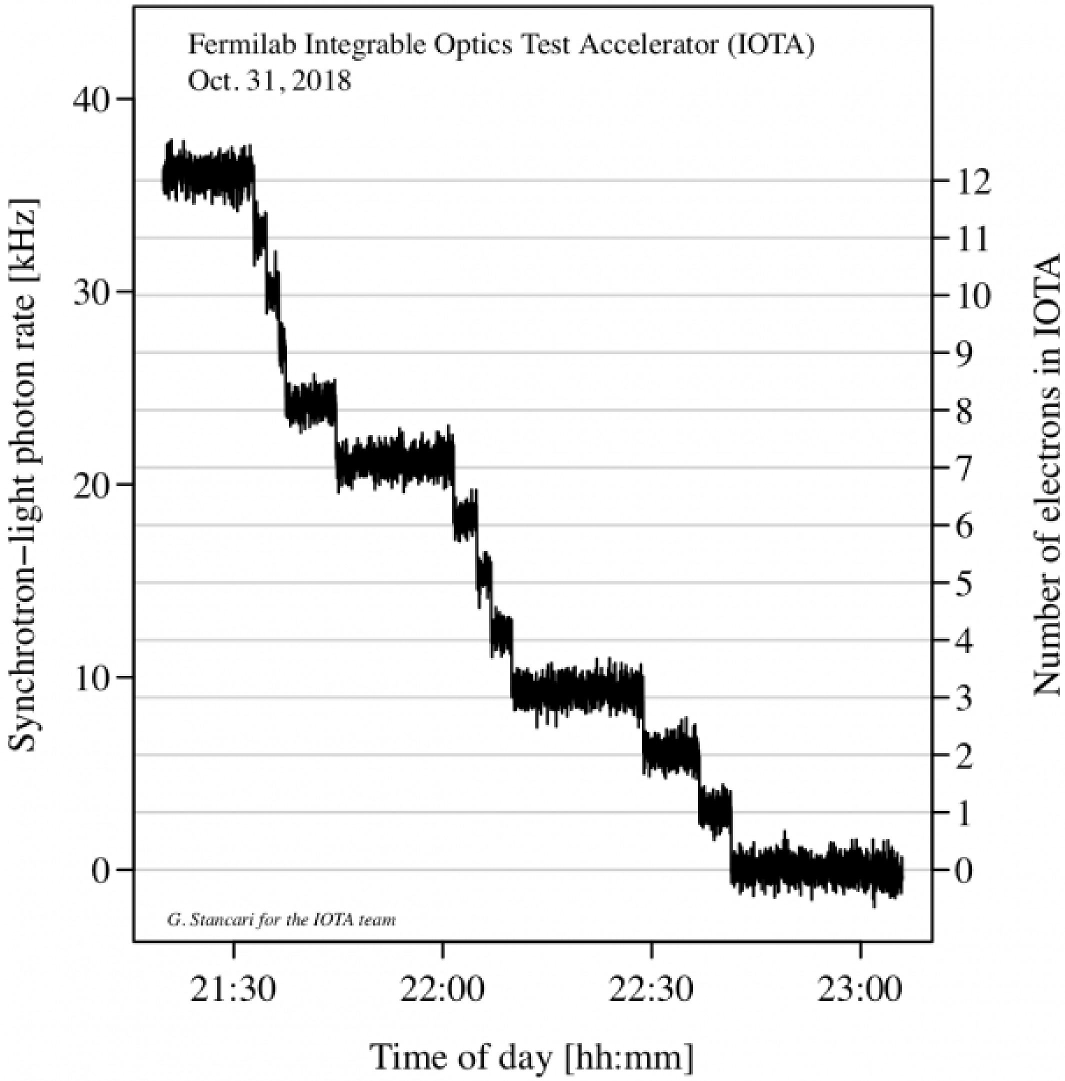


Figure 3: Single electrons stored in the MLS.

- Mechanical scraper
- SR intensity measurement by cooled photodiodes

Single electron in IOTA 100 MeV (2018)



- In this specific case they were simply waiting as electrons were lost due to residual gas.
- Beam current was measured through synchrotron radiation detected by a PMT. Also by cameras.

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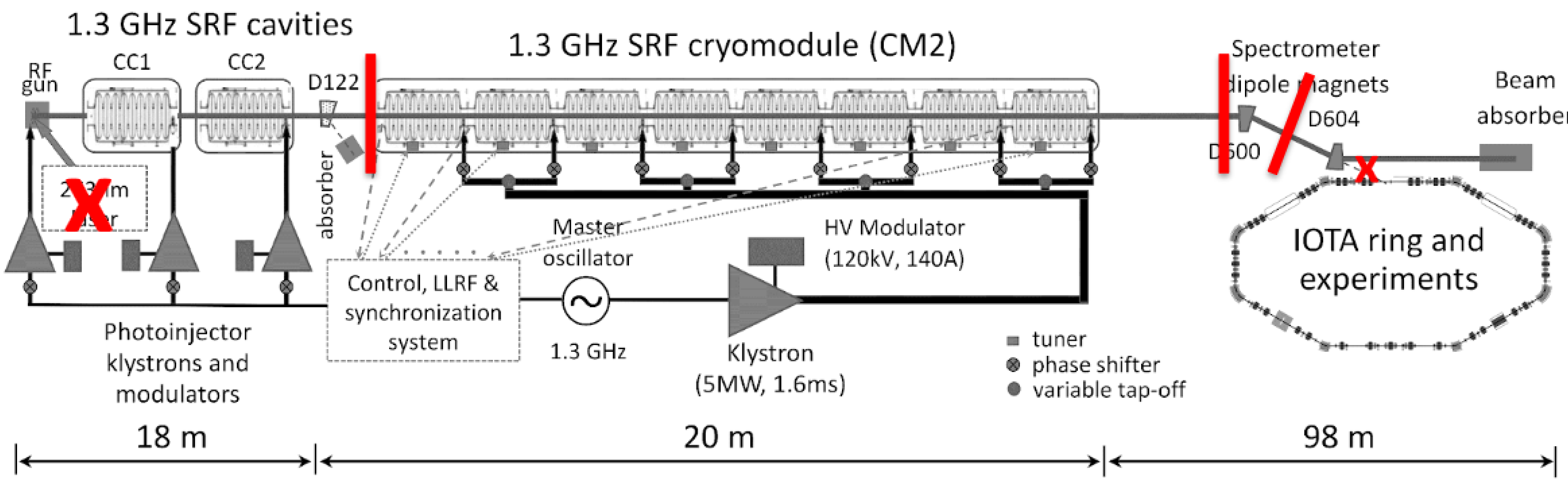
Single electron injection

*Sasha Romanov's slide

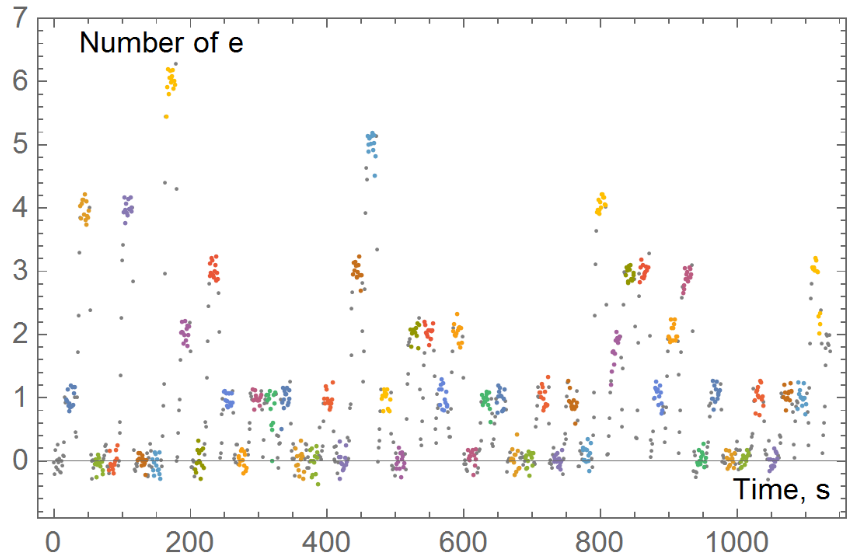


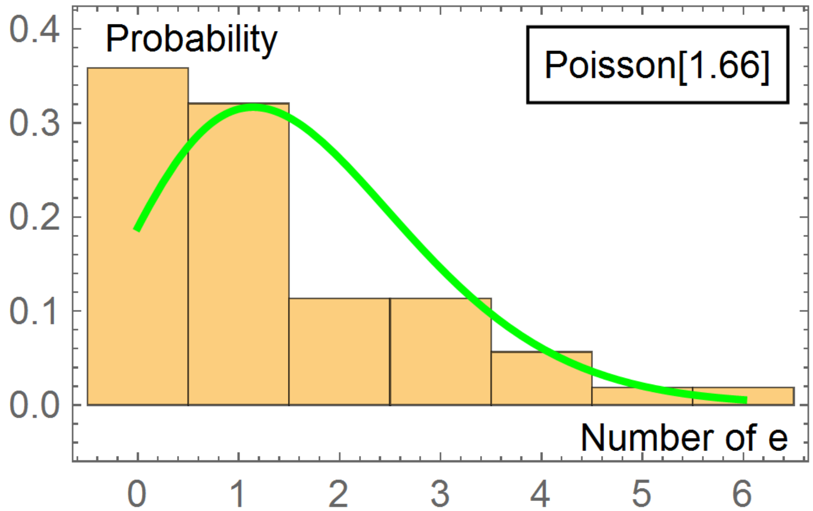
- Block laser with shutter to get only dark current
- Insert several OTR foils in LE and HE lines
- Decrease last injection quadrupole to stretch phase volume and distort incoming trajectory

Single electron capture probability

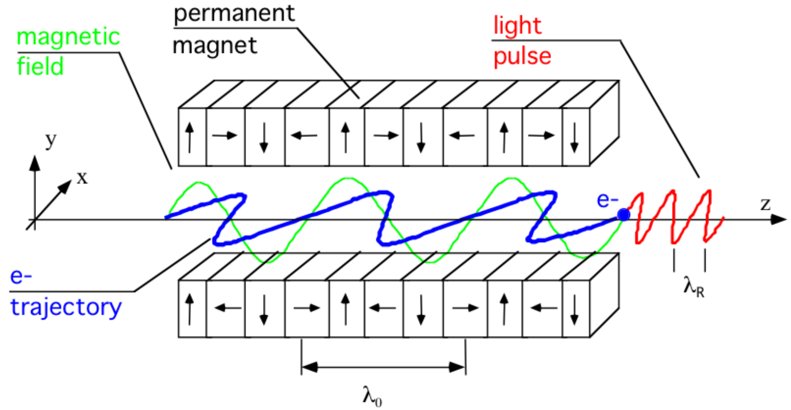
*Sasha Romanov's slide

- To test selected method of intensity attenuation 53 injections were done with interval of 21 seconds
- Resulting probability of single electron injection: 32%
  - For purely Poisson distribution maximum probability is 36.8%





Theoretical predictions for undulator radiation produced by a single electron



* figure from http://old.clio.lcp.u-psud.fr/clio_eng/FELrad.html

- Multi-photon emission

- Differential rates?
- Photons' arrival times?

- Two models were considered:
  - QED approach with Dirac-Volkov solution
    (classical undulator field + quantum electron + quantized radiation)
  - Glauber's approach
    (classical current + quantized radiation)

Dirac-Volkov approach has already been used for electron in constant uniform magnetic field

TWO-PHOTON SYNCHROTRON EMISSION

A. A. Sokolov, A. M. Voloshchenko, V. Ch. Zhukovskii, and Yu. G. Pavlenko

A second-order effect of perturbation theory — two-photon
cussed for an ultrarelativistic electron in a constant and un
$H \ll H_0 = 4.41 \cdot 10^{13}$ Oe on the basis of the exact solutions of

Formation length in uniform field $\sim R/\gamma$

For undulator, formation length will be the entire length of undulator

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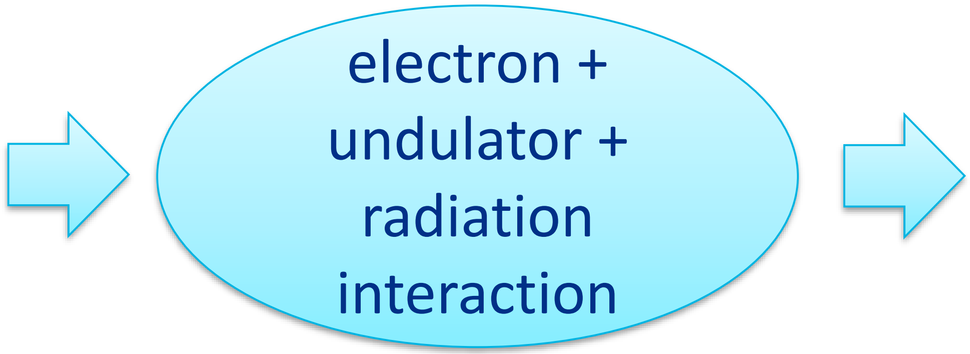
Theoretical part; General remarks

$$\hat{\rho}_{\mathrm{in}} = |\mathrm{vac}\rangle \langle \mathrm{vac}|$$



$$\hat{\rho}_{\mathrm{fin}}$$

The probability to detect a single photon of any energy at location $\boldsymbol{r}$ at time $\boldsymbol{t}$ is given by correlation function of first order $G^{(1)}_{\mu\mu}(\boldsymbol{r},t;\boldsymbol{r},t) = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{fin}} \hat{E}^{(-)}_{\mu}(\boldsymbol{r},t)\, \hat{E}^{(+)}_{\mu}(\boldsymbol{r},t)\right]$

*introduced by R.J. Glauber

The probability to detect two photons at location $\boldsymbol{r}$ at times t_1 and t_2 is given by correlation function of second order

$$G^{(2)}_{\mu\nu\nu\mu}(\boldsymbol{r},t_1;\boldsymbol{r},t_2;\boldsymbol{r},t_2;\boldsymbol{r},t_1) = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{fin}} \hat{E}^{(-)}_{\mu}(\boldsymbol{r},t_1)\, \hat{E}^{(-)}_{\nu}(\boldsymbol{r},t_2)\, \hat{E}^{(+)}_{\nu}(\boldsymbol{r},t_2)\, \hat{E}^{(+)}_{\mu}(\boldsymbol{r},t_1)\right]$$

If there is a filter, then only allowed components of electric field operator should be left with corresponding weights. If there is a filter with infinitesimal band, then the time dependence is lost (plane waves occupy all space) and for single photon and classical current we get $\mathrm{Tr}\left[\hat{\rho}_{\mathrm{fin}} \hat{a}^{+}_{k} \hat{a}_{k}\right] \Longrightarrow |\langle k|\mathrm{fin}\rangle|^2 = |\langle k|\hat{S}|\mathrm{vac}\rangle|^2$ $\quad \hat{\rho}_{\mathrm{fin}} = |\mathrm{fin}\rangle \langle \mathrm{fin}|$

If the electron is quantum the trace is also calculated over electron's states and the usual QED matrix element will emerge in the calculation $|\langle p', k|\hat{S}|p, \mathrm{vac}\rangle|^2$

One can obtain similar results for two-photon differential rate.

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Dirac-Volkov model

Volkov states are exact solutions of the Dirac equation for electron in plane electromagnetic wave

$$(i\partial\!\!\!/ - eA\!\!\!/(x) - m)\Psi_\alpha(x) = 0$$

Positive and negative energy solutions:

$$\Psi^{(+)}_{p,r}(x) = E_p(x)u_{p,r} = e^{-ip\cdot x}\Omega_p(x)u_{p,r}\ ,$$
$$\Psi^{(-)}_{p,r}(x) = E_{-p}(x)v_{p,r} = e^{ip\cdot x}\Omega_{-p}(x)v_{p,r}\ ,$$

*these are spinor functions

Where

$$\Omega_p(\phi) = \left[1 + \frac{e}{2k\cdot p}k\!\!\!/A\!\!\!/(\phi)\right]\exp\left\{-\frac{i}{2k\cdot p}\int^{\phi} d\phi'\left[2ep\cdot A(\phi') - e^2A^2(\phi')\right]\right\}$$

ϕ is the phase in the undulator's field (plane wave) $\phi = k\cdot x$

How is it related to an electron in an undulator?

Weizsäcker-Williams approximation → In electron's rest frame undulator's field looks like a plane wave

*this problem has been considered in dissertation of Daniel Seipt

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Dirac-Volkov model

Takes into account:

- **Quantum nature of radiated field**
- **Quantum nature of electron, i.e.**
  - Finite size of electron's wavefunction
  - Electron's spin

- Furry picture

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{ext}} + \mathcal{V} = \mathcal{H}_B + \mathcal{V}$$

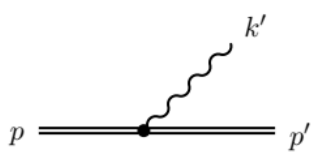
$$\hat{S}[A] = \mathsf{T}\exp\left\{-i\int d^4x\,\mathscr{H}_{\text{int}}(x)\right\}$$

$$= \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int d^4x_1\cdots d^4x_n\,\mathsf{T}\Big(\mathscr{H}_{\text{int}}(x_1)\cdots\mathscr{H}_{\text{int}}(x_n)\Big)$$

$$\mathscr{H}_{\text{int}}(x) \equiv e : \hat{\bar{\Psi}}(x)\gamma^{\mu}\hat{A}_{\mu}(x)\hat{\Psi}(x) :$$

Single-photon emission

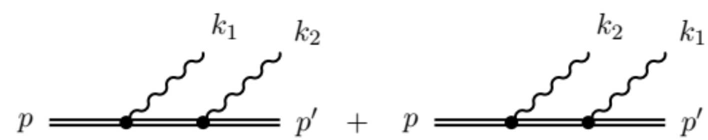
$$S^{(1)} = \langle \boldsymbol{p}'r';\boldsymbol{k}'\lambda'|\hat{S}[A]|\boldsymbol{p}r\rangle = \langle \boldsymbol{p}'r';\boldsymbol{k}'\lambda'|\left(-i\int d^4x\,\mathscr{H}_{\text{int}}(x)\right)|\boldsymbol{p}r\rangle$$



$$\frac{d^2W}{d\omega' d\Omega} = \frac{e^2\omega'}{64\pi^3\,(k_u\cdot p)\,(k_u\cdot p')}|M_1|^2$$

Two-photon emission

$$S^{(2)} = \frac{(-i)^2}{2!}\langle \boldsymbol{p}'r';\boldsymbol{k}_1\lambda_1;\boldsymbol{k}_2\lambda_2|\mathsf{T}\int d^4x d^4y\,\mathscr{H}_{\text{int}}(x)\mathscr{H}_{\text{int}}(y)|\boldsymbol{p}r\rangle$$



$$\frac{d^4W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2} = \frac{\alpha^2\omega_1\omega_2}{64\pi^4\,(k_u\cdot p)\,(k_u\cdot p')}|M_2|^2$$

*see Daniel Seipt's dissertation

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Crucial parameters

- Field strength parameter (undulator parameter)

$$K = \frac{eB\lambda_u}{2\pi m_e c}$$

$$K \sim 1$$

- Quantum parameter (electron recoil parameter)

$$\chi = K \frac{\hbar\omega}{\gamma m_e c^2}$$

*see E. Lötstedt and U.D. Jentschura Phys Rev A 80, 053419 (2009)

$$\hbar\omega \sim 2\,\mathrm{eV} \qquad \gamma m_e c^2 \sim 150\,\mathrm{MeV}$$

$$\chi \sim 10^{-8}$$

Scattering amplitude can be conveniently decomposed as a series in powers of χ

$$M(\chi) = M|_{\chi=0} + \chi M'_{\chi}|_{\chi=0} + \frac{\chi^2}{2} M''_{\chi\chi}|_{\chi=0} + \ldots = M|_{\chi=0} + O(\chi)$$

Differential rates in Dirac-Volkov model

Single-photon rate

$$\frac{d^2W}{d\omega_1 d\Omega_1} = \frac{\alpha}{16\pi^2}\frac{\omega_1}{\gamma^2 k_u^2}|\tilde{M}(1)|^2\left(\delta_{-1,s}\delta_{-1,s'} + \delta_{+1,s}\delta_{+1,s'}\right) + O(\chi)$$

Two-photon rate

$$\frac{d^4W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2} = \left(\frac{\alpha}{16\pi^2}\frac{\omega_1}{\gamma^2 k_u^2}|\tilde{M}(1)|^2\right)\left(\frac{\alpha}{16\pi^2}\frac{\omega_2}{\gamma^2 k_u^2}|\tilde{M}(2)|^2\right)\left(\delta_{-1,s}\delta_{-1,s'} + \delta_{+1,s}\delta_{+1,s'}\right) + O(\chi)$$

*a factor of ½ will emerge after integration over a detector

where

$$\tilde{M}(1) = \int d\phi\left(Ka(\phi)\epsilon_x^{(1)} + \gamma\epsilon_z^{(1)}\right)\exp\left[i\frac{\omega_1}{4k_u\gamma^2}\left(1+\gamma^2\theta_1^2\right)\phi - i\frac{K\omega_1 n_x^{(1)}}{2k_u\gamma}\int_0^\phi a(\phi')\,d\phi' + i\frac{K^2\omega_1}{4\gamma^2 k_u}\int_0^\phi a^2(\phi')\,d\phi'\right]$$

Basically this is a classical result for $K \sim 1$. See for example V.I. Ritus, Journal of Soviet Laser Research 6.5 (1985): 497-617.

For $K \ll 1$, I obtained

$$\frac{d^2W}{d\omega_1 d\Omega_1} = \frac{e^2\omega_1}{64\pi^3 4k_u^2 p_0^2}|M_1|^2 = \frac{\alpha}{16\pi^2}\frac{K^2\omega_1 L_u^2}{\gamma^2}\left(\left[1 - \frac{\theta_x^2\omega_1}{k_w}\right]^2 + \left[\frac{\theta_x\theta_y\omega_1}{k_w}\right]^2\right)\mathrm{sinc}^2\left[\pi N_u\left(\frac{\omega_1}{\omega(\theta)} - 1\right)\right]$$

which agrees with classical results from Jackson and, for example, V. Kocharyan and E. Saldin, arXiv:1202.0691v1

Differential rates in Dirac-Volkov model

Single-photon rate

$$\frac{d^2W}{d\omega_1 d\Omega_1} = \frac{\alpha}{16\pi^2}\frac{\omega_1}{\gamma^2 k_u^2}|\tilde{M}\left(1\right)|^2\left(\delta_{-1,s}\delta_{-1,s'} + \delta_{+1,s}\delta_{+1,s'}\right) + O\left(\chi\right)$$

Two-photon rate

$$\frac{d^4W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2} = \left(\frac{\alpha}{16\pi^2}\frac{\omega_1}{\gamma^2 k_u^2}|\tilde{M}\left(1\right)|^2\right)\left(\frac{\alpha}{16\pi^2}\frac{\omega_2}{\gamma^2 k_u^2}|\tilde{M}\left(2\right)|^2\right)\left(\delta_{-1,s}\delta_{-1,s'} + \delta_{+1,s}\delta_{+1,s'}\right) + O\left(\chi\right)$$

Spin does not change. Essentially electron can be regarded as spinless

*a factor of ½ will emerge after integration over a detector

Factorization of two-photon differential rate means absence of correlation between the two photons. To increase correlation one has to increase $O(\chi)$. Is it at all possible to see correlation on experiment? Yes:

In D. Seipt and B. Kampfer, Phys Rev D 85, 101701(R) (2012):

FACET-II parameters:

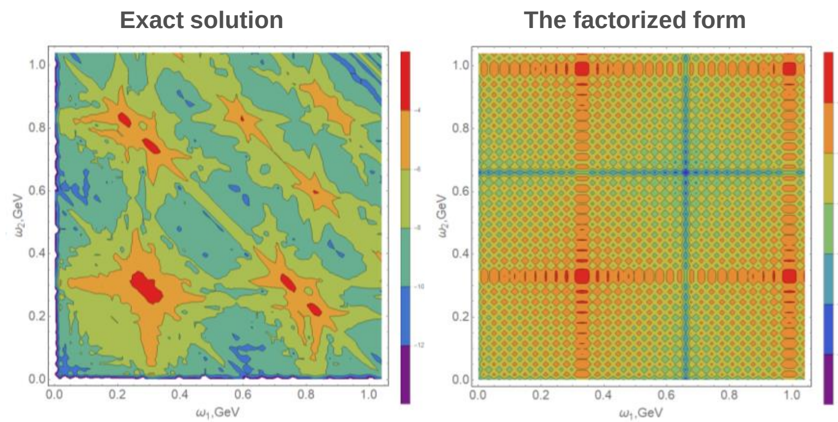
$$\gamma \sim 10^4$$

Optical undulator:

$$\frac{hc}{\lambda_u} \sim 2.5\,\mathrm{eV}$$

$$\chi \sim 0.04$$

Energy spectrum for two-photon emission for on axis photons



Color represents differential rate: $\frac{d^4W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2}$

*some difference can also be seen at 150 MeV and optical undulator

Glauber’s model

PHYSICAL REVIEW VOLUME 131, NUMBER 6 15 SEPTEMBER 1963

Coherent and Incoherent States of the Radiation Field*

Roy J. Glauber
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 29 April 1963)

Methods are developed for discussing the photon statistics of arbitrary radiation fields in fully quantum-mechanical terms. In order to keep the classical limit of quantum electrodynamics plainly in view, extensive use is made of the coherent states of the field. These states, which reduce the field correlation functions to factorized forms, are shown to offer a convenient basis for the description of fields of all types. Although they are not orthogonal to one another, the coherent states form a complete set. It is shown that any quantum state of the field may be expanded in terms of them in a unique way. Expansions are also developed for arbitrary operators in terms of products of the coherent state vectors. These expansions are discussed as a general method of representing the density operator for the field. A particular form is exhibited for the

Takes into account:

- **Quantum nature of radiated field**

Assumptions:

- **Negligible electron recoil (classical current)**

$$\widehat{H}_I(t) = -\frac{1}{c}\int \boldsymbol{j}(\boldsymbol{r},t)\cdot\widehat{\boldsymbol{A}}(\boldsymbol{r},t)\,d^3\boldsymbol{r}$$

Classical current

Operator

$$\boldsymbol{u}_k(\boldsymbol{r}) = L^{-\frac{3}{2}}\widehat{e}^{(\lambda)}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$\widehat{\boldsymbol{A}}(\boldsymbol{r},t) = c\sum_k\left(\frac{\hbar}{2\omega_k}\right)^{\frac{1}{2}}\left(\widehat{a}_k\boldsymbol{u}_k(\boldsymbol{r})e^{-i\omega_k t} + \widehat{a}_k^{+}\boldsymbol{u}_k^{*}(\boldsymbol{r})e^{i\omega_k t}\right)$$

$$i\hbar\frac{\partial}{\partial t}|\rangle = \widehat{H}_I|\rangle \;\Rightarrow\; |\psi(t)\rangle = \prod_k \exp\left(\alpha_k a_k^{+} - \alpha_k^{*}a_k\right)|\psi(0)\rangle$$

$$\alpha_k(t) = \frac{i}{(2\hbar\omega_k)^{\frac{1}{2}}}\int_{-\infty}^{t}dt'\int d^3\boldsymbol{r}\,\boldsymbol{u}_k^{*}(\boldsymbol{r})\cdot\boldsymbol{j}(\boldsymbol{r},t')\,e^{i\omega_k t'}$$

$$D(\alpha_k) = e^{\alpha_k\widehat{a}_k^{+} - \alpha_k^{*}\widehat{a}_k}$$

Displacement operator (creates coherent state):

$$D(\alpha_k)|\text{vac}\rangle = |\alpha_k\rangle$$

Final state is a coherent state:

$$|\psi(t)\rangle = |\{\alpha_k\}\rangle = \prod_k|\alpha_k\rangle$$

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Glauber's model: results for correlation function

Some noteworthy properties:

$$\widehat{a}_k \left|\alpha_k\right\rangle = \alpha_k \left|\alpha_k\right\rangle$$

$$\left\langle \{\alpha_k\} \right| \widehat{\boldsymbol{E}} \left(\boldsymbol{r}, t\right) \left|\{\alpha_k\}\right\rangle = \boldsymbol{E}_{\text{classical}} \left(\boldsymbol{r}, t\right)$$

$$\widehat{\rho}_{\text{fin}} = \left|\{\alpha_k\}\right\rangle \left\langle \{\alpha_k\}\right|$$

Definition of correlation function from the beginning of presentation:

$$G^{(1)}_{\mu\mu} \left(\boldsymbol{r}, t; \boldsymbol{r}, t\right) = \mathrm{Tr}\left[\widehat{\rho}_{\text{fin}} \widehat{E}^{(-)}_{\mu} \left(\boldsymbol{r}, t\right) \widehat{E}^{(+)}_{\mu} \left(\boldsymbol{r}, t\right)\right] = \left\langle \{\alpha_k\} \right| \widehat{E}^{(-)}_{\mu} \left(\boldsymbol{r}, t\right) \widehat{E}^{(+)}_{\mu} \left(\boldsymbol{r}, t\right) \left|\{\alpha_k\}\right\rangle$$

$$\widehat{\boldsymbol{E}}^{(+)} \left(\boldsymbol{r}, t\right) = i \sum_k \left(\frac{1}{2}\hbar\omega_k\right)^{\frac{1}{2}} \widehat{a}_k \boldsymbol{u}_k \left(\boldsymbol{r}\right) e^{-i\omega_k t}$$

$$\boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t\right) = i \sum_k \left(\frac{1}{2}\hbar\omega_k\right)^{\frac{1}{2}} \alpha_k \boldsymbol{u}_k \left(\boldsymbol{r}\right) e^{-i\omega_k t}$$

$$\widehat{\boldsymbol{E}}^{(+)} \left(\boldsymbol{r}, t\right) \left|\{\alpha_k\}\right\rangle = \boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t\right) \left|\{\alpha_k\}\right\rangle$$

$$G^{(1)}_{\mu\mu} \left(\boldsymbol{r}, t; \boldsymbol{r}, t\right) = \left(\boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t\right)\right)^* \boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t\right) = \left|\boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t\right)\right|^2$$

$$G^{(2)}_{\mu\nu\nu\mu} \left(\boldsymbol{r}, t_1; \boldsymbol{r}, t_2; \boldsymbol{r}, t_2; \boldsymbol{r}, t_1\right) = \left|\boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t_1\right)\right|^2 \left|\boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{r}, t_2\right)\right|^2$$

For infinitesimally thin filter (spectral correlation function):

$$\mathrm{Tr}\left[\widehat{\rho}_{\text{fin}} \widehat{a}^{+}_k \widehat{a}_k\right] \Longrightarrow \left|\boldsymbol{E}^{(+)}_{\text{classical}} \left(\boldsymbol{k}\right)\right|^2 \sim \left|\alpha_k\right|^2 \sim \left|\boldsymbol{j} \left(\boldsymbol{k}\right)\right|^2$$

- classical result. See for example V.I. Ritus, Journal of Soviet Laser Research 6.5 (1985): 497-617.

Part 2: Ideas for experiment in IOTA

- **Measurement of difference of arrival times of two photons in two-photon emission**
- Is photon statistics Poissonian? (for number of emitted photons/in time)
  - Experiment with two PMTs with non-overlapping filters. “Violation” of Poisson statistics
- Experiments with a 2D array of single photon detectors (Correlation/entanglement in emitted photon pairs?)
- Experiments with undulators of different lengths (peak intensity $\sim L^2$ if the formation length is equal to undulator’s length)
- Other vague (for now) ideas

Two-photon events: time spread

Experiment idea #1

$$G^{(2)}_{\mu\nu\nu\mu}(\boldsymbol{r},t_1;\boldsymbol{r},t_2;\boldsymbol{r},t_2;\boldsymbol{r},t_1) = |\boldsymbol{E}^{(+)}_{\text{classical}}(\boldsymbol{r},t_1)|^2|\boldsymbol{E}^{(+)}_{\text{classical}}(\boldsymbol{r},t_2)|^2$$

Classical formula from Jackson:

$$\mathbf{B} = [\mathbf{n}\times\mathbf{E}]_{\text{ret}}$$

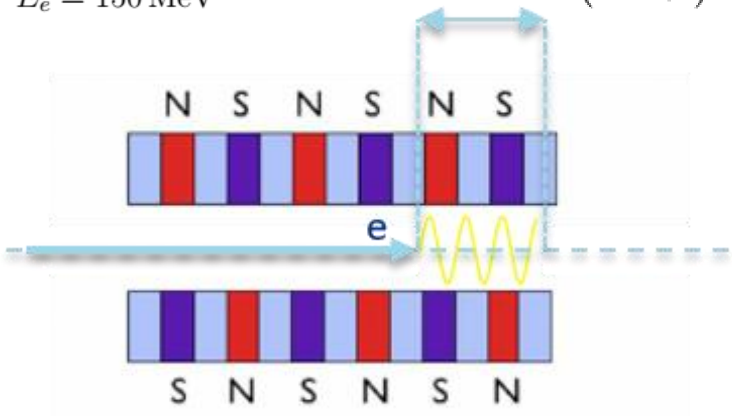
$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\mathbf{n})^3R^2}\right]_{\text{ret}} + \frac{e}{c}\left[\frac{\mathbf{n}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^3R}\right]_{\text{ret}}$$

It's important to have two photons, because it is easier to measure Δt, then absolute time of arrival of a photon

$$K = 1$$
$$L_u = 1\,\text{m}$$
$$E_e = 150\,\text{MeV}$$

$$\Delta t = (1-\bar{\beta})\frac{L_u}{c} \sim N_u\lambda_{\text{rad}}/c \sim 30\,\text{fs}$$

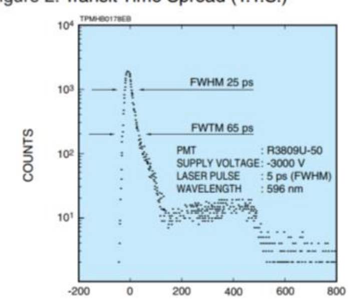
*S.V. Faleev in arXiv:hep-ph/9706372v1 found $\Delta t \sim \lambda_{\text{rad}}/c$ for dipole radiation

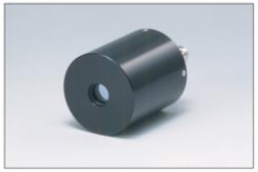


Capabilities of presently available PMTs:

HAMAMATSU
PHOTON IS OUR BUSINESS
MICROCHANNEL PLATE-PHOTOMULTIPLIER TUBES (MCP-PMT)
R3809U-50 SERIES

Figure 2: Transit Time Spread (T.T.S.)



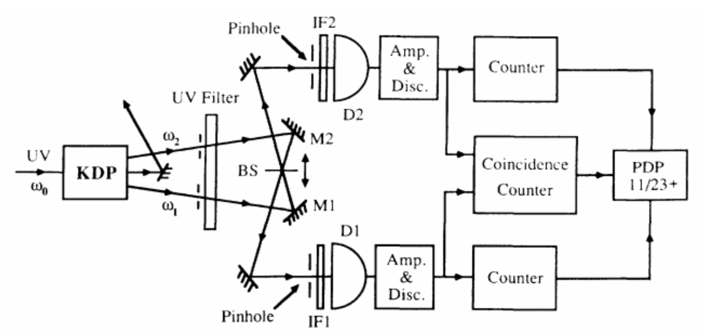


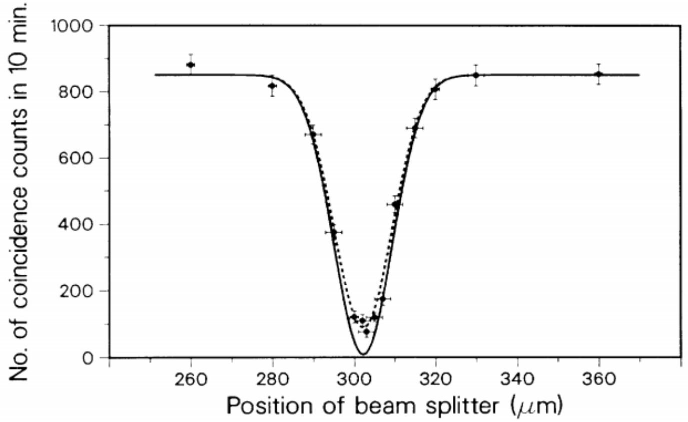
TTS=25ps

Hong-Ou-Mandel interferometer

Never used for synchrotron radiation before!

The original HOM interferometer (1987):





Measured $\Delta t \sim 100$ fs.
Accuracy < 1 fs

Later papers (2018), Attosecond-Resolution HOM interferometer:

Attosecond-Resolution Hong-Ou-Mandel Interferometry

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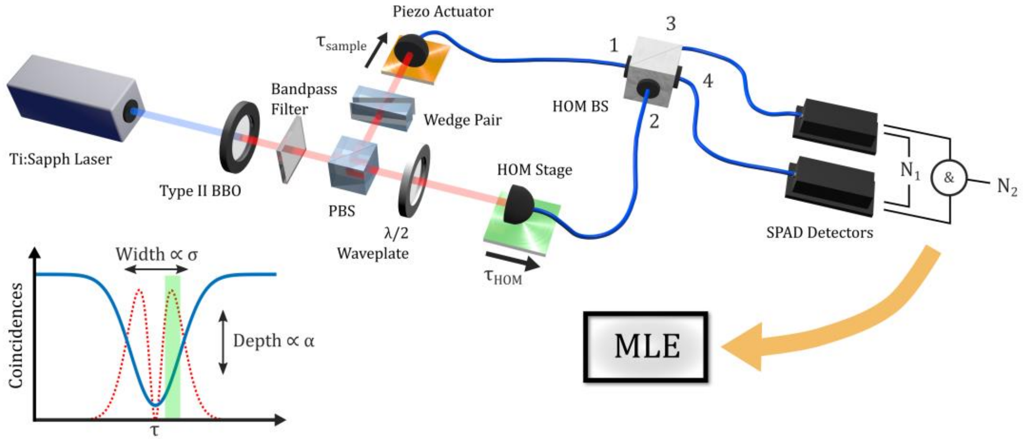
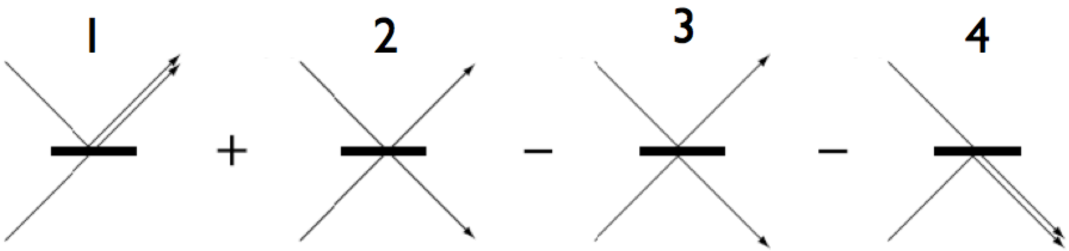
(Dated: November 5, 2018)

When two indistinguishable photons are each incident on separate input ports of a beamsplitter they 'bunch' deterministically, exiting via the same port as a direct consequence of their bosonic nature. This two-photon interference effect has long-held the potential for application in precision measurement of time delays, such as those induced by transparent specimens with unknown thickness profiles. However, the technique has never achieved resolutions significantly better than the few femtosecond (micron)-scale other than in a common-path geometry that severely limits applications. Here we develop the precision of HOM interferometry towards the ultimate limits dictated by statistical estimation theory, achieving few-attosecond (or nanometre path length) scale resolutions in a dual-arm geometry, thus providing access to length scales pertinent to cell biology and mono-atomic layer 2D materials.



Hong-Ou-Mandel interferometer: theory

One photon at each port of a beam splitter:



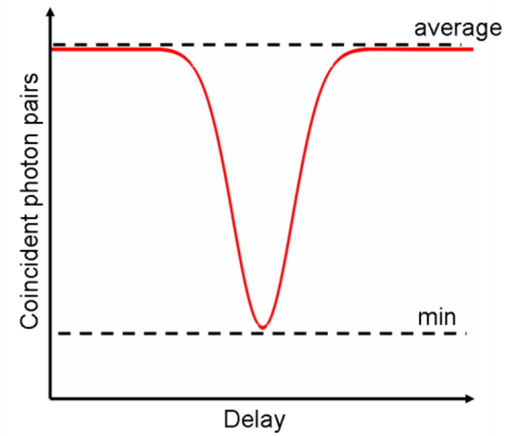
For two indistinguishable photons:

$$\hat{a}^\dagger \hat{b}^\dagger |0,0\rangle_{ab} = |1,1\rangle_{ab},$$

$$\hat{a}^\dagger \to \frac{\hat{c}^\dagger + \hat{d}^\dagger}{\sqrt{2}} \quad \text{and} \quad \hat{b}^\dagger \to \frac{\hat{c}^\dagger - \hat{d}^\dagger}{\sqrt{2}}.$$

$$|1,1\rangle_{ab} = \hat{a}^\dagger \hat{b}^\dagger |0,0\rangle_{ab} \to \frac{1}{2}\left(\hat{c}^\dagger + \hat{d}^\dagger\right)\left(\hat{c}^\dagger - \hat{d}^\dagger\right)|0,0\rangle_{cd} = \frac{1}{2}\left(\hat{c}^{\dagger 2} - \hat{d}^{\dagger 2}\right)|0,0\rangle_{cd} = \boxed{\frac{|2,0\rangle_{cd} - |0,2\rangle_{cd}}{\sqrt{2}}}.$$

HOM signature:

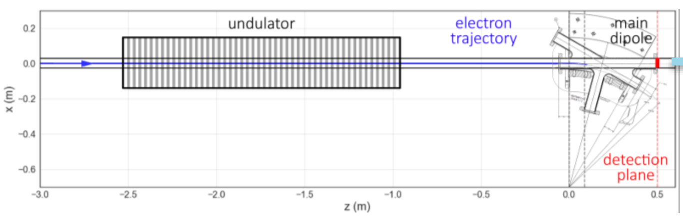


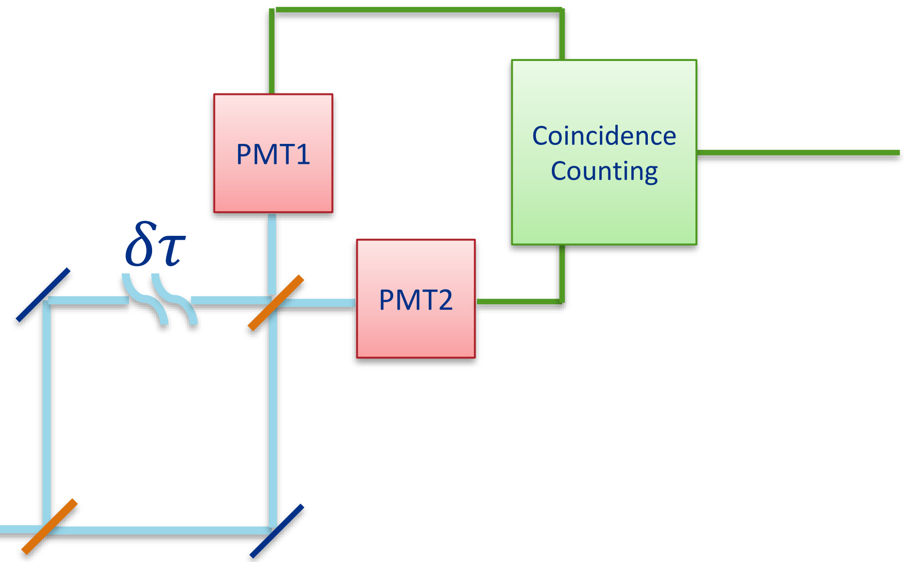
Fermilab

HOM interferometer for undulator radiation

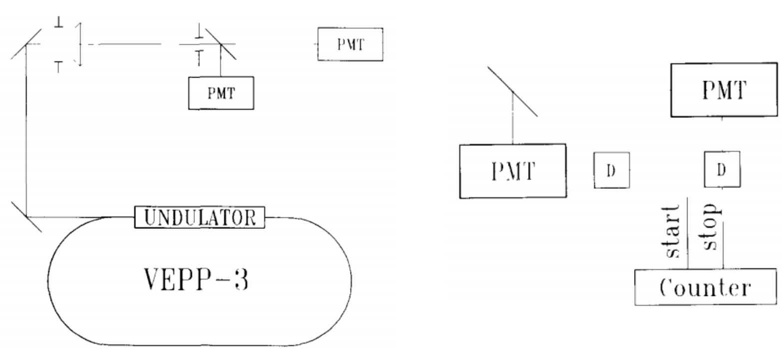
- Attosecond time resolution
- We will be able to see what is longer: photon or electron?

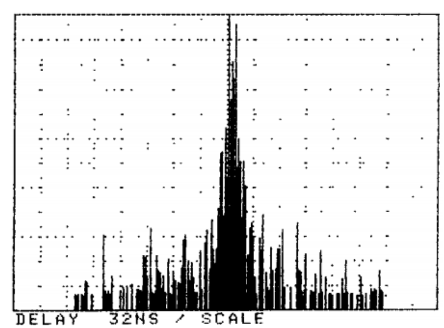
$$\Delta t = (1-\bar{\beta})\,\frac{L_u}{c} \sim N_u \lambda_{\mathrm{rad}}/c \sim 30\,\mathrm{fs}$$





Older Novosibirsk's experiment in VEPP-3:



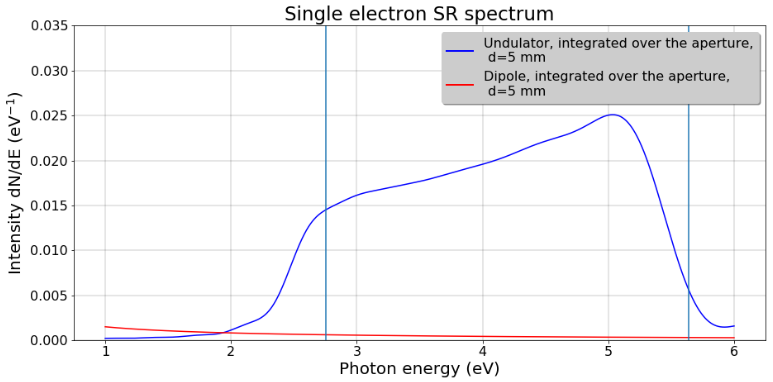


1 ns time resolution.

Fermilab

Photon counts estimates

SRW simulation for 200 MeV electron and 60 cm undulator:

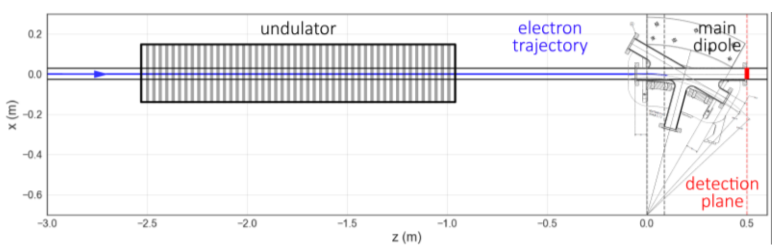


Parameter	Value
IOTA circumference	40 m
Turns per second	7.5 MHz
Electron energy	150 MeV (up to 200 maybe)
Undulator length	60 cm (SLAC)/1.8 m (JLab)
Undulator period	5.5 cm
Photon energy	2.6 eV
Photon wavelength	475 nm

PMT's
QE~ 25%

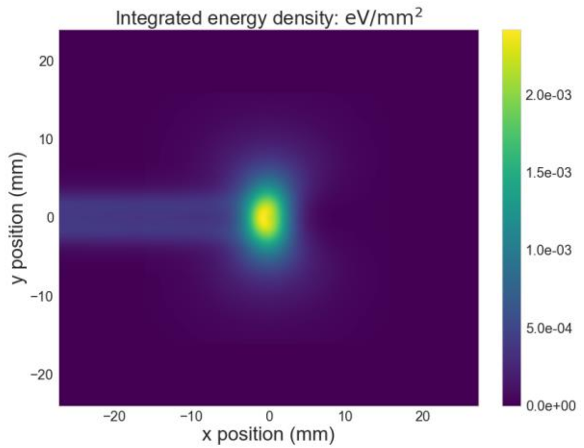
Expected
Single-photon counts: ~ 50 KHz
Two-photon counts: ~ 180 Hz

But we need to exclude dipole radiation:

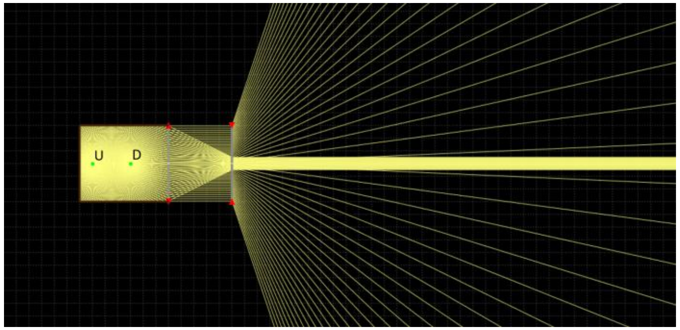


Suppressing dipole magnet radiation

- Small aperture detector:

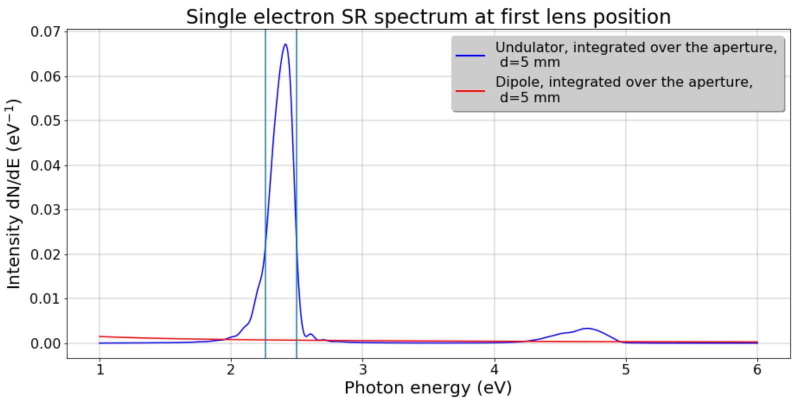


- System of lenses:

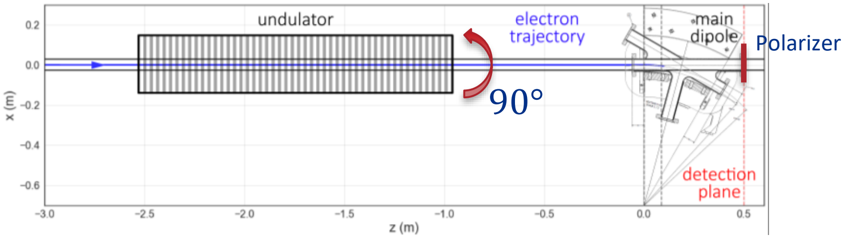


Dipole radiation is defocused.
Undulator radiation is focused.

- Narrow filter:

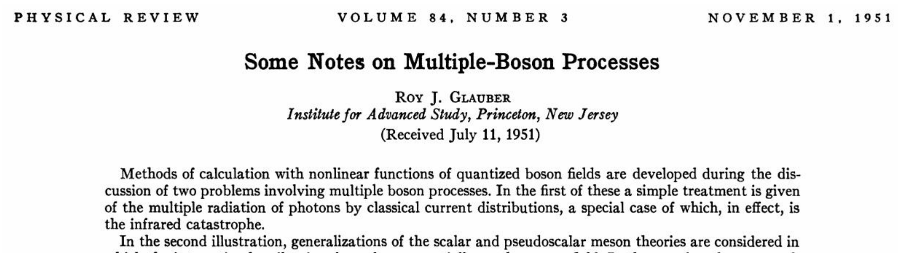


- Vertical orientation of undulator + polarizer



*Also, with HOM interferometer we might actually be able to resolve dipole photons and undulator photons in time. So we do not really need to suppress dipole radiation.

Photon statistics



PHYSICAL REVIEW VOLUME 84, NUMBER 3 NOVEMBER 1, 1951

Some Notes on Multiple-Boson Processes

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(Received July 11, 1951)

Methods of calculation with nonlinear functions of quantized boson fields are developed during the discussion of two problems involving multiple boson processes. In the first of these a simple treatment is given of the multiple radiation of photons by classical current distributions, a special case of which, in effect, is the infrared catastrophe.
In the second illustration, generalizations of the scalar and pseudoscalar meson theories are considered in

Emission probabilities form Poisson distribution:

$$w_n = W^n e^{-W}/n!$$

$$\omega_1 = W e^{-W}$$

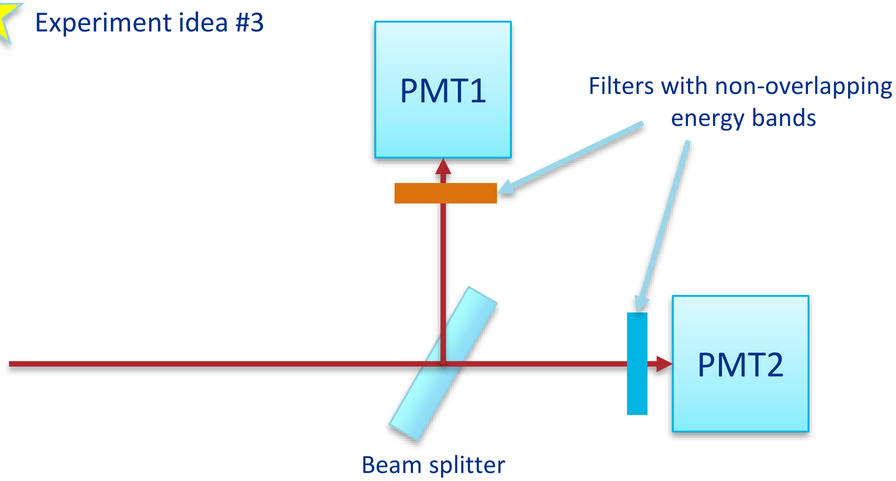
$$\omega_2 = \frac{W^2}{2} e^{-W}$$

Experiment idea #2

Also does distribution of emissions in time correspond to Poissonian? (for single-photon and for two-photon events)

An idea on how to “violate” Poisson statistics:

Experiment idea #3



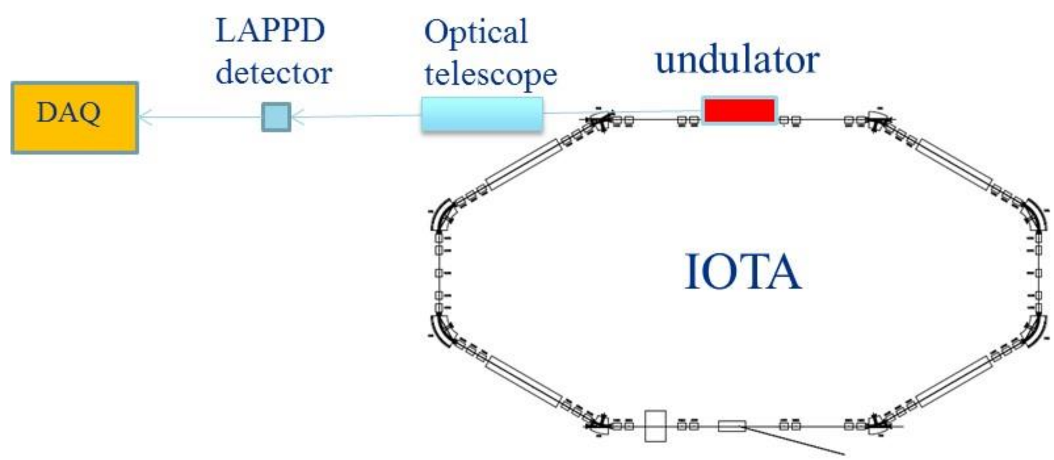
If probability to detect a photon in PMT1 is P_1 and probability to detect a photon in PMT2 is P_2, then the probability to detect one photon in each PMT will be $P_1 P_2$, not $\frac{1}{2} P_1 P_2$ (which would be true if the filters were identical)

Experiment with 2D array of single photon sensors

Experiment idea #4

- It will allow us to measure the angle at which a photon is detected
- It will be possible to see if there is any correlation between these angles in emitted photon pairs
- Also experiments aimed at polarization correlation in photon pairs may be done

Example: Large Area Picosecond Photon Detector



There is some correlation and entanglement for optical undulator (much bigger χ):

PHYSICAL REVIEW A **80**, 053419 (2009)

Correlated two-photon emission by transitions of Dirac-Volkov states in intense laser fields: QED predictions

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IV. ANGULAR CORRELATION AND ENTANGLEMENT

We now turn our attention to the important questions regarding the quantum-mechanical correlation, i.e., entanglement, of the two final photons. The theory we apply in this section have been previously used extensively to characterize the final-state correlation in bound states transitions [56–59]. The idea is to use the information contained in the matrix elements (26) and (31), to obtain an expression for the *density matrix* ρ_f of the polarizations of the final system "electron+two photons." Given an expression for ρ_f, it is then straightforward to calculate the concurrence [19], which is a measurement of how much the two photons are en-

$$\rho_i = \sum_{r_i=1}^{2} |r_i,0,0\rangle\langle r_i,0,0|, \qquad (51)$$

where r_i is the spin of the initial electron and the zeros denote the absence of photons (other than laser photons of course) in the initial state. The initial electron is thus assumed to be unpolarized. Note also that all dependencies on energies and angles, etc, of the state vectors are not written out. Next, due to the interaction R, the density matrix ρ_i evolves into the final-state density matrix ρ_f,

$$\rho_f = R\rho_i R^\dagger = \sum_{r_i=1}^{2} R|r_i,0,0\rangle\langle r_i,0,0|R^\dagger. \qquad (52)$$

Fermilab

Length of formation of radiation

Experiment idea #5

A bunch of electrons can be used for this experiment

$K \ll 1$ here

$$\frac{d^2W}{d\omega_1 d\Omega_1} = \frac{e^2\omega_1}{64\pi^3 4k_u^2 p_0^2}|M_1|^2 = \frac{\alpha}{16\pi^2}\frac{K^2\omega_1 L_u^2}{\gamma^2}\left(\left[1-\frac{\theta_x^2\omega_1}{k_w}\right]^2+\left[\frac{\theta_x\theta_y\omega_1}{k_w}\right]^2\right)\mathrm{sinc}^2\left[\pi N_u\left(\frac{\omega_1}{\omega(\theta)}-1\right)\right]$$

Small aperture/small energy band detector:

$$\Delta\omega \ll \frac{\omega_0}{N_u} \qquad \theta \ll \frac{1}{\sqrt{N_u}\gamma}$$

$$\omega_0 = 2\gamma^2 k_u c$$

$$W = \frac{\alpha}{16\pi^2}\frac{K^2\omega_0 L_u^2}{\gamma^2}\Delta\omega\frac{\theta^2}{2}$$

If formation length is shorter than undulator:

$$L_u^2 \Longrightarrow nL_{\mathrm{form}}^2 = \frac{L_u}{L_{\mathrm{form}}}L_{\mathrm{form}}^2 = L_{\mathrm{form}}L_u$$

We might have two undulators (SLAC/JLab) of different lengths to check the square law.

One cannot determine formation length with big detector because after integration over the detector the dependence on length is linear:

$$\frac{dW}{dw} = \frac{\alpha\omega K^2 L_u}{4c\gamma^2}\left[1+\left(\frac{\omega}{ck_u\gamma^2}-1\right)^2\right]$$

If formation length is shorter than undulator:

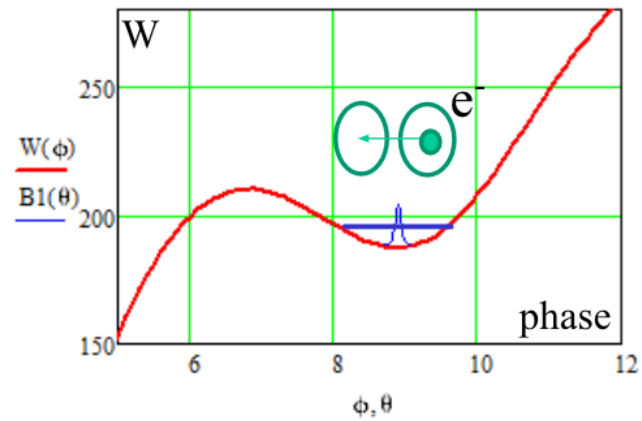
$$L_u \Longrightarrow nL_{\mathrm{form}} = \frac{L_u}{L_{\mathrm{form}}}L_{\mathrm{form}} = L_u$$

Has it been done before?

Other ideas for experiment

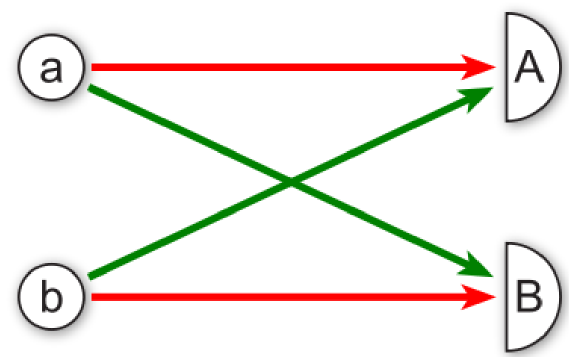
Experiment idea #6

- Develop a barrier across separatrix. Split separatrix into two islands. Control width of the RF barrier.
  What is the probability of a single electron tunneling through the barrier into the 2nd separatrix?
  (Timur Shaftan's idea)



Experiment idea #7

- We can do something like Hanbury-Brown and Twiss experiment for interference of light coming from far away double-stars, but for two electrons in an undulator.
  (Bernhard Adams' idea)



Fermilab

Optical stochastic cooling with single electron

Experiment idea #8

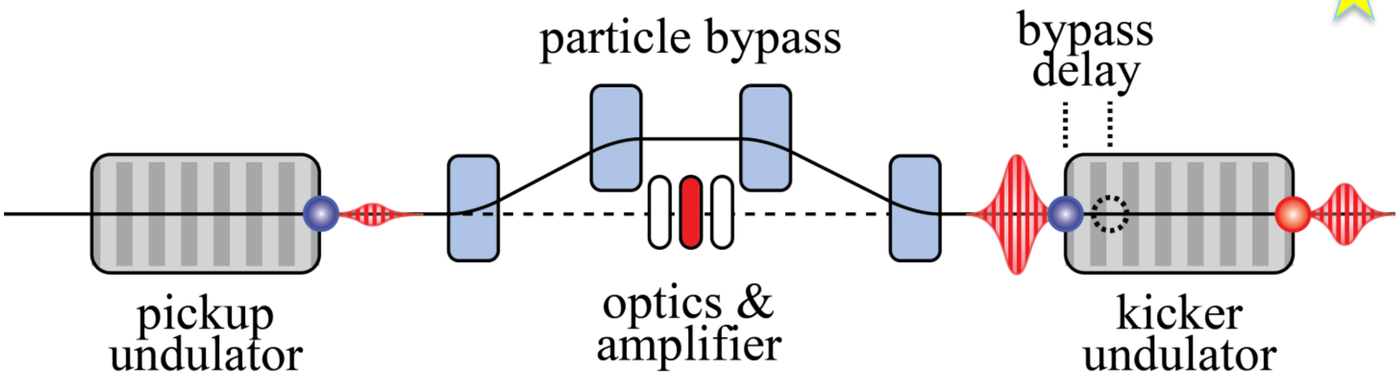
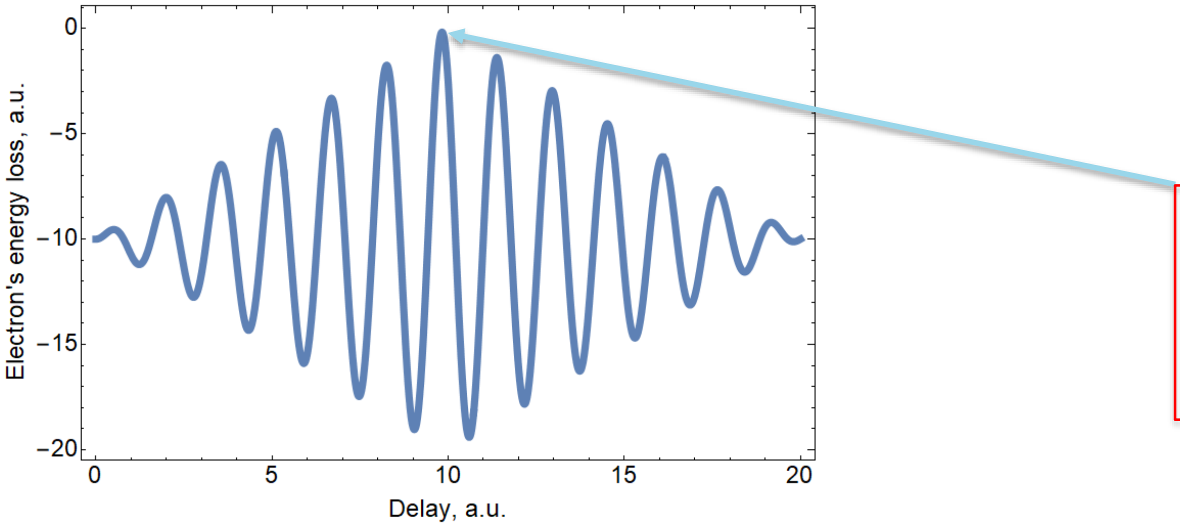


*figure from Andorf, Matthew *et al.* Phys. Rev. Accel. Beams 21 (2018) no. 10



At certain delay the probability to emit a photon will be zero!

Conclusions

- It is possible to keep a single electron in IOTA

Theoretical predictions:

- Electron recoil and spin effects are negligible
- Glauber's model with classical current is sufficient
- Still, electron wavefunction's size may be measurable

Ideas for experiment:

- Difference in time of arrival of photons in a photon pair can be measured with unprecedented accuracy (attosecond). We can determine what is longer: photon or electron
- Photon statistics (number distribution/independence in time)
- Transverse correlations can be tested with 2D array of single photon detectors
- Experiments with small aperture/small energy band detector for formation length of radiation
- RF Barrier tunneling/ Hanbury-Brown and Twiss interferometer for two electrons

Thank you for your attention!