



Combined explanations of $(g-2)$ and implications for a large muon EDM

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Muon Department Journal Club

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Overview

- Paper for discussion today:
 - “Combined explanations of $(g-2)$ and implications for a large muon EDM” — Andreas Crivellin, Martin Hoferichter, Philipp Schmidt-Wellenburg, [arXiv:1807.11484](https://arxiv.org/abs/1807.11484)
- Summary of main points in the paper and background
- Key arguments
- Conclusions
- Further reading

Combined explanations of $(g - 2)_{\mu,e}$ and implications for a large muon EDM

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With the long-standing tension between experiment and Standard-Model (SM) prediction in the anomalous magnetic moment of the muon, $a_\mu = (g-2)_\mu/2$, at the level of $3-4\sigma$, it is natural to ask if there could be a sizable effect in the electric dipole moment (EDM) d_μ as well. In this context it has often been argued that in UV complete models the electron EDM, which is very precisely measured, excludes a large effect in d_μ . However, the recently observed 2.5σ tension in $a_e = (g-2)_e/2$, if confirmed, requires that the muon and electron sectors effectively decouple to avoid constraints from $\mu \rightarrow e\gamma$. We briefly discuss UV complete models that possess such a decoupling, which can be enforced by an Abelian flavor symmetry $L_\mu - L_\tau$. We show that, in such scenarios, there is no reason to expect a correlation between the electron and muon EDM, so that the latter can be sizable. New limits on d_μ improved by up to two orders of magnitude are expected from the upcoming $(g-2)_\mu$ experiments at Fermilab and J-PARC. Beyond, a proposed dedicated muon EDM experiment at PSI could further advance the limit. In this way, future improved measurements of a_e , a_μ , as well as the fine-structure constant α are not only set to provide exciting precision tests of the SM, but, in combination with EDMs, to reveal crucial insights into the flavor structure of physics beyond the SM.

Background to the paper

- We are all familiar with the muon magnetic dipole moment anomaly:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = 270(85) \times 10^{-11}$$

- and the limit from E821 on the muon electric dipole moment (EDM):

$$|d_\mu| < 1.9 \times 10^{-19} \text{ e.cm}$$

- As well as confirming/denying the a_μ discrepancy, the FNAL g-2 experiment hopes to reduce the limit by factor of 100.
- The paper explores the question of whether the same BSM scenario could contribute both the muon magnetic dipole anomaly and a large muon electric dipole moment.

Muon EDM and $(g-2)_\mu$

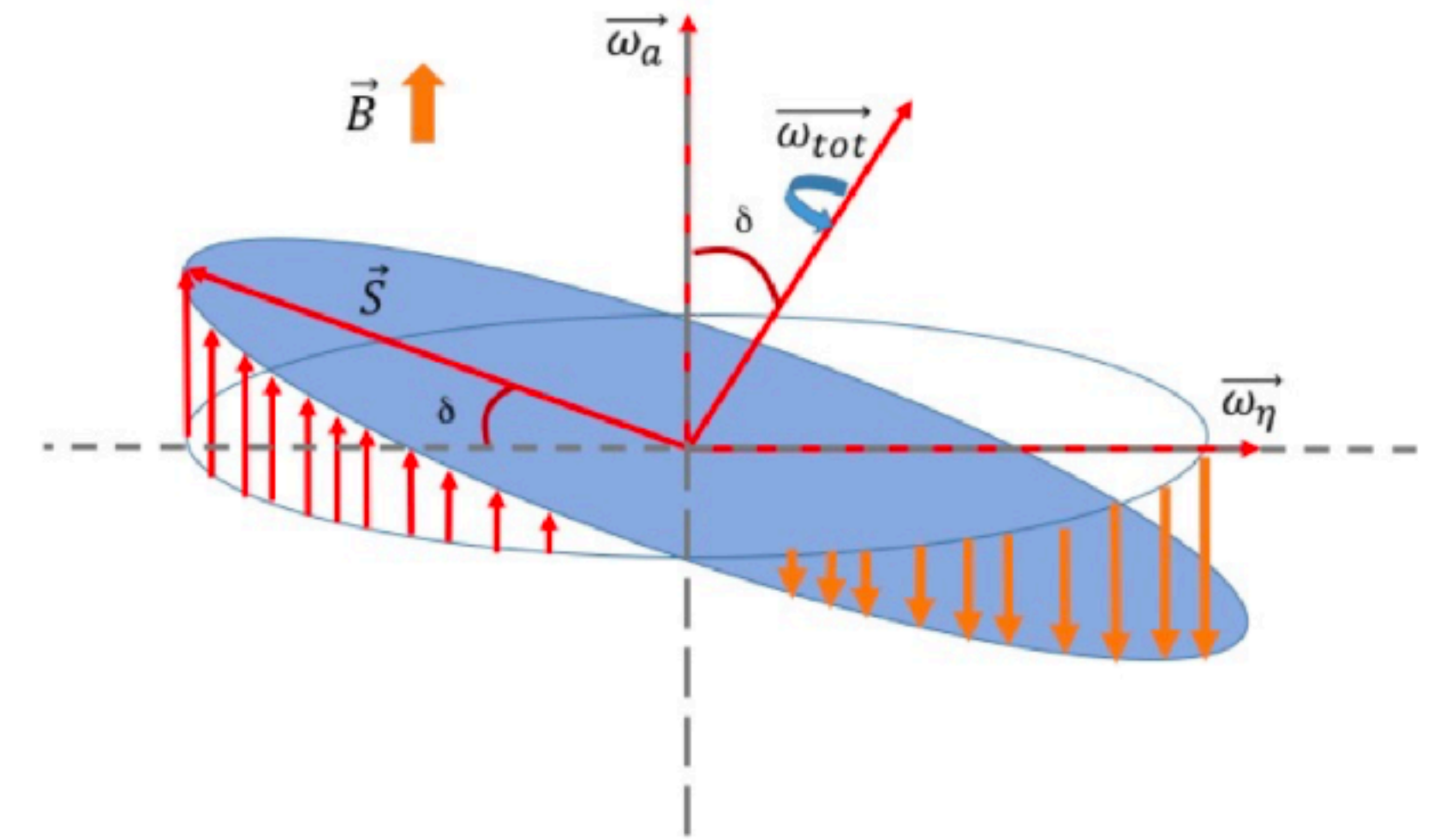
- What is the maximum possible size for the muon EDM?
- In the paper, they claim that the limit $|d_\mu| < 1.9 \times 10^{-19}$ e.cm is “600 times larger than than expected from the central value of a_μ assuming that the imaginary part of the corresponding BSM contribution is as large as the real one”
- Where does this number come from?

Muon EDM and $(g-2)_\mu$

$$\omega_{a\eta} = \omega_a + \omega_\eta = \frac{e}{m} \left[a_\mu \mathbf{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\boldsymbol{\beta} \times \mathbf{E}}{c} \right] + \eta \frac{e}{2m} \left[\frac{\mathbf{E}}{c} + \boldsymbol{\beta} \times \mathbf{B} \right]$$

This is ω_η

$$\eta = \frac{4d_\mu + m_\mu c}{\hbar}$$



Tilt angle due to muon EDM:

$$\delta = \tan^{-1} \left(\frac{\omega_\eta}{\omega_a} \right) = \tan^{-1} \left(\frac{\eta \beta}{2a_\mu} \right)$$

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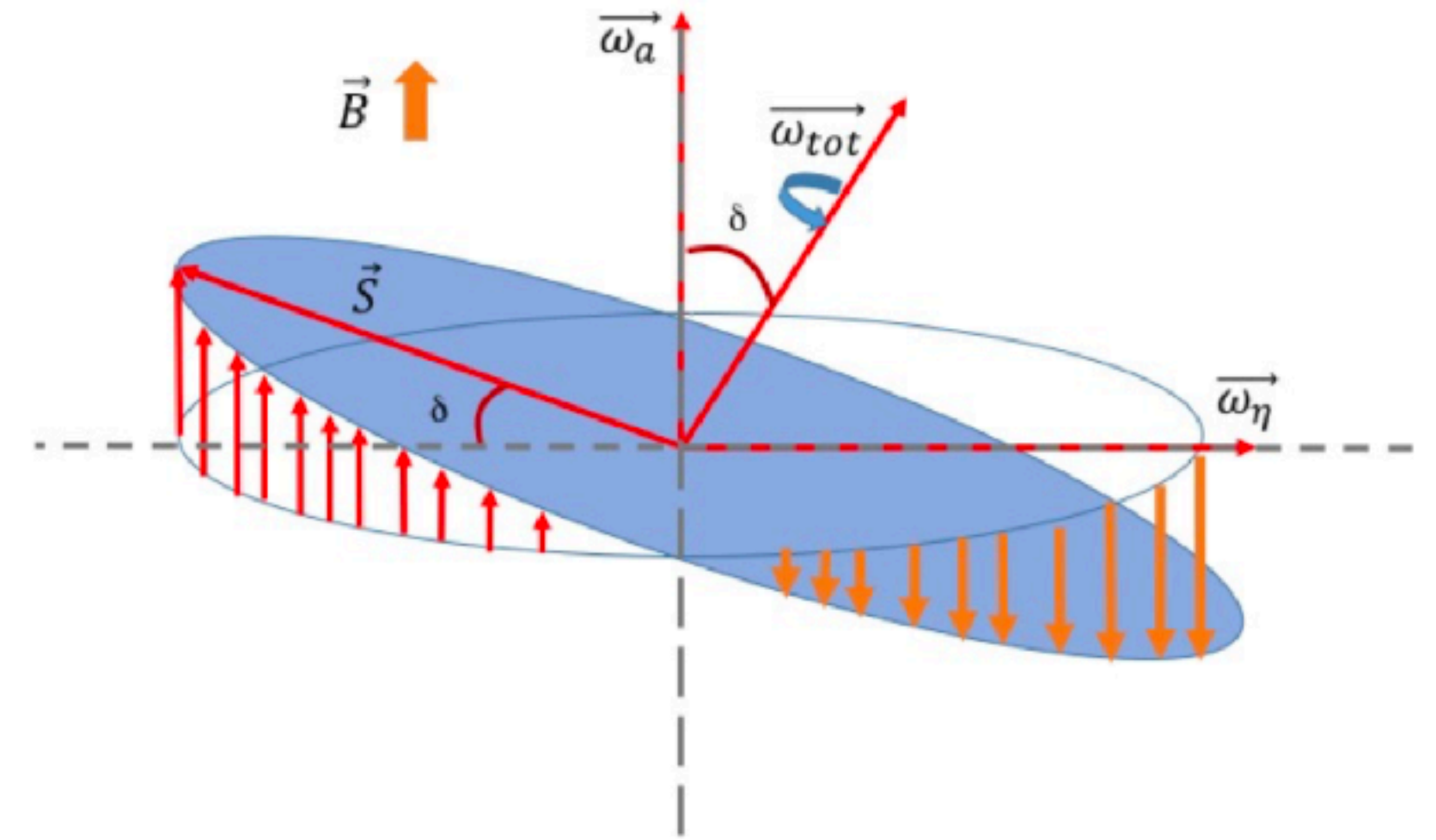
$$\eta = \frac{4d_\mu + m_\mu c}{\hbar}$$

We can also say that:

$$\begin{aligned} \frac{\omega_{a\eta}}{\omega_a} &= \sqrt{1 + \frac{\omega_\eta}{\omega_a}} \\ &= \sqrt{1 + \delta^2} \\ &\approx 1 + \frac{\delta^2}{2} \\ &= 1 + \frac{\eta^2 \beta^2}{8a_\mu^2} \end{aligned}$$

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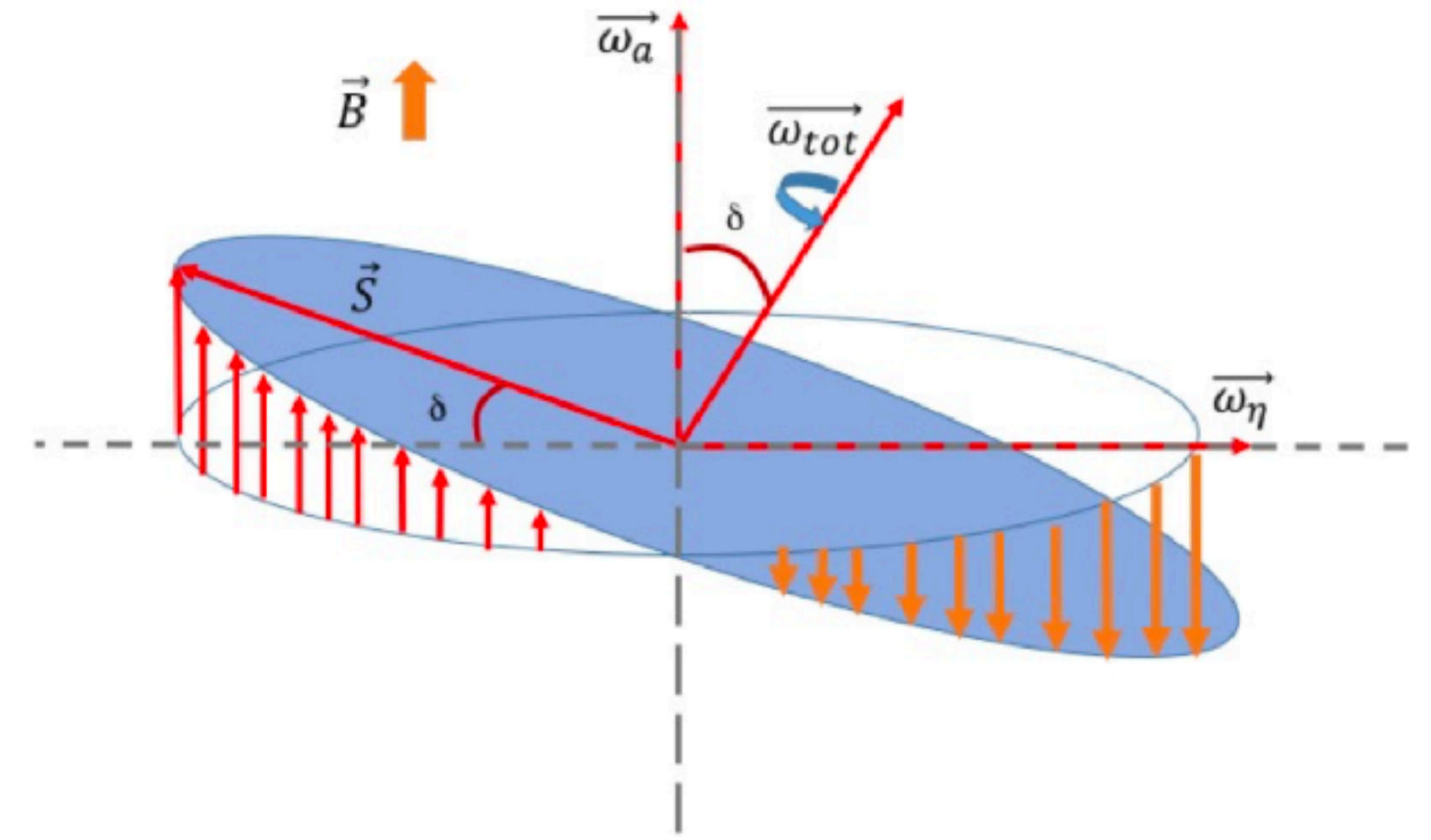
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Rearrange to get

$$\Delta a_\mu = \omega_a \frac{\eta^2 \beta^2}{8a_\mu^2}$$

Putting this together we get:

$$d_\mu^{\text{BNL}} \sim 600 \times d_\mu^{\text{CALCULATED}}$$

$$d_\mu^{\text{CALCULATED}} = \mathcal{O}(10^{-22} \text{ e.cm})$$

Main points of the paper

- A value of d_μ greater than 3.7×10^{-24} e.cm is ruled out in minimally-flavor-violating (MFV) scenarios since the limit on the EDM of the electron, d_e , is tiny (from quadratic mass scaling):
 $|d_e| < 1.1 \times 10^{-29}$ e.cm¹
- A recent precise measurement of the fine structure constant α suggests a discrepancy in a_e at the 2.5σ level of the opposite sign to Δa_μ .
- A scenario that allows an electron $g-2$ anomaly in the opposite direction to the $g-2$ anomaly must contain flavor violation.

¹Nature volume 562, pages 355–360 (2018)

²Science volume 360, pages 191–195 (2018)

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- Recent measurements in semileptonic B-decays also strongly challenge the MFV assumption.

These discrepancies with the SM predictions are most pronounced in semi-leptonic B decays. Here, we have two classes of processes:

- $b \rightarrow c\tau\nu$: In these processes, mediated at tree-level in the SM, several measurements like

$$R_\tau(X) \equiv \frac{\mathcal{B}(B \rightarrow X \tau \nu_\tau)}{\mathcal{B}(B \rightarrow X \ell \nu_\ell)} \quad \text{with } X = D, D^*, \quad (1)$$

$$R_\tau(J/\psi) \equiv \frac{\mathcal{B}(B_c \rightarrow J/\psi \tau \nu_\tau)}{\mathcal{B}(B_c \rightarrow J/\psi \ell \nu_\ell)}$$

with $\ell = e, \mu$ point towards lepton flavour universality violation (LFUV) in $\tau - \mu, e$ at the $\approx 4\sigma$ level [1].

- $b \rightarrow s\ell^+\ell^-$: This flavour changing neutral current process is loop suppressed and is proportional to the CKM element V_{ts} . Here the measurements of $R_\mu(K)$ [2] and $R_\mu(K^*)$ [3], defined as

$$R_\mu(X) \equiv \frac{\mathcal{B}(B \rightarrow X \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow X e^+ e^-)}, \quad (2)$$

are supported by other $b \rightarrow s\mu^+\mu^-$ observables (like $P_5^{\prime\mu} \equiv P_5^{\prime}$ as defined in [4]) which also show deviations from the SM predictions.

ArXiv 1803.10097

The paper proposes and compares non-MFV scenarios that account for the following conditions:

Δa_μ and Δa_e of opposite sign

$$|d_\mu| \gg |d_e|$$



Criteria for BSM scenarios that fit

- A BSM scenario that has Δa_μ in the opposite direction to Δa_e would have to **violate quadratic mass scaling**
- Must include **effective decoupling of the μ and e BSM sectors** in order to satisfy limit on $\mu \rightarrow e\gamma$ from MEG
- Such a scenario would allow large d_μ and small d_e

ArXiv 1605.05081

$$\text{Br}[\mu \rightarrow e\gamma] < 4.2 \times 10^{-13} \quad 90\% \text{ C.L.}$$

What scenarios could work?

Criteria for BSM scenarios that fit

- Some form of **enhancement** required to the BSM mechanism that allows all this; either:
 - It must be **light**
 - It must have $\mathcal{O}(1)$ **couplings for TeV-scale masses**
 - It must have **large ($> SM$) coupling to Higgs field** (chiral enhancement)
 - e.g. $\tan\beta$ in MSSM, m_q/m_l in leptoquark models
- **Light (pseudo-) vector particles (dark photons) ruled out**

As mentioned in the introduction, light (pseudo-) vector particles (“dark photons”) are problematic. Neutral vectors give a necessarily positive effect and can therefore only account for a_μ , while neutral axial vectors give a negative effect and are therefore only compatible with a_e .

Criteria for BSM scenarios that fit

- A model that introduces a **single light scalar** to resolve both anomalies is proposed in **ArXiv 1806.10252** (“A tale of two anomalies” H. Davoudiasl and W. J. Marciano)
- Crivellin et. al.’s paper says that this model would require heavy BSM degrees of freedom to make it UV complete
—> not as simple as it appears
- Instead, proposes models **above the EW breaking scale with chiral enhancement**

Specific scenarios

- The paper considers the following simplified models:
 - (1) Leptoquark (LQ) models
 - (2) MSSM
 - (3) Little-Higgs inspired models / extra-dimensions
 - (4) Model with new heavy leptons and possibly a new scalar
- It concludes that, of these, the only plausible scenario is (4)

What is wrong in the first 3?

Specific scenarios

- **Leptoquark (LQ) models**
 - Minimal LQ models add only one new scalar or vector particle to the SM → minimal chiral enhancement
 - Can only account for a_μ by decoupling the electron sector completely → **can't explain both Δa_μ and Δa_e at the same time**
- **Extra-Dimension and little-Higgs models**
 - e.g. Randall-Sundrum scenario, littlest-Higgs model
 - Provide massive fermions and vectors that are resonances of SM particles that do not mix with the SM
 - Small effect on a_μ since couplings are mainly LH
→ not enough chiral enhancement
 - **Vector resonances are not flavor-specific and violate the MEG limit**

Specific scenarios

- MSSM
 - Usually discuss **constrained MSSM**
 - Assume flavor-universal SUSY breaking terms that respect naive MFV (which we already found out has to be rejected)
 - Although the MSSM has 3 generations of sleptons so it is **technically** possible to decouple effects in electrons and muons...
 - ... but introduces unnatural flavor dependence e.g. fine-tuning

We are left with scenario (4): model with a new scalar and fermions

Model with a new scalar and fermions

- Vector-like generations of leptons introduced
- Same requirements for maximal chiral enhancement
- Models with vector-like fermions could account for such a case, using an Abelian flavor symmetry to ensure the decoupling of e and μ
 - This could also be relevant to the anomalies seen in $b \rightarrow s\mu^+\mu^-$ decays
 - Would allow large d_μ and small d_e
 - Would remain viable even if the tension in a_e vanished

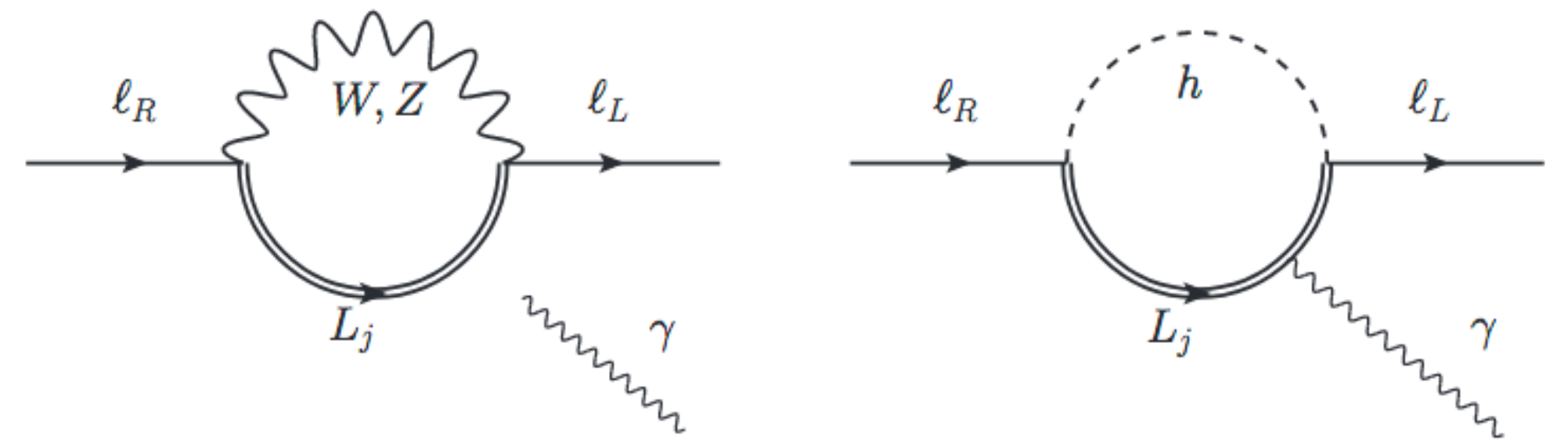


FIG. 1: Generic diagrams contributing to the dipole operator in Model I.

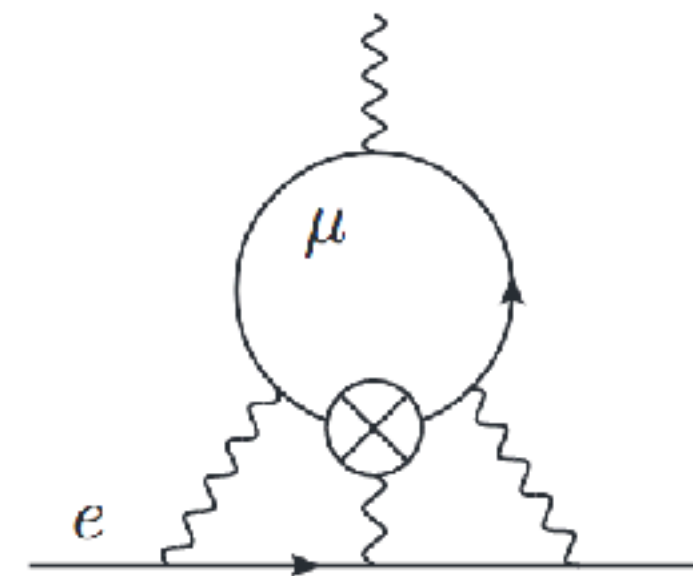


FIG. 3: Three-loop diagram that produces a contribution to the electron EDM by an insertion of the muon EDM operator indicated by the cross. The other diagrams with insertions at the remaining muon-photon vertices as well as the permutations at the electron line are not shown.

Use limit on α to constrain muon EDM
Gives 7.5×10^{-19} e.cm

Model with a new scalar and fermions

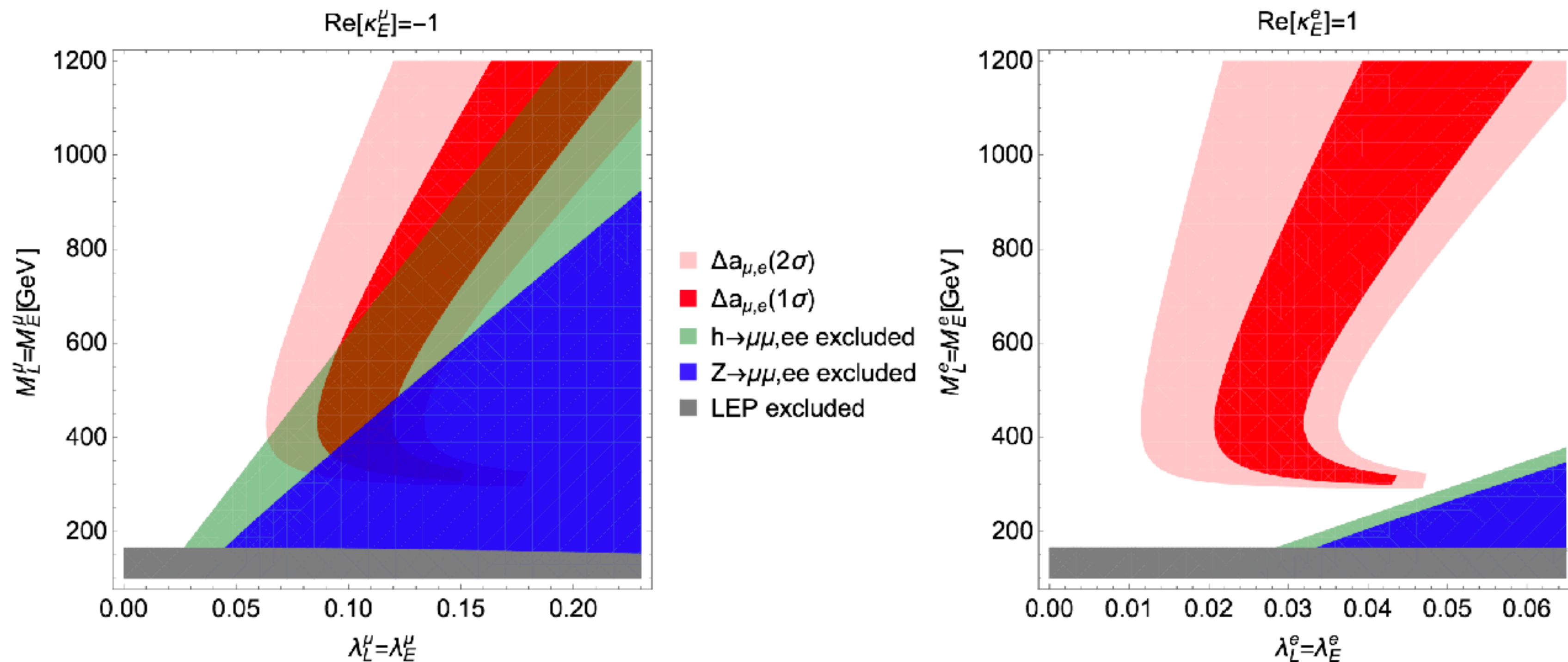


FIG. 2: Allowed regions of a_μ in the $\lambda_E = \lambda_L - M_E = M_L$ plane for $\kappa_L = 0$ and $\kappa_E = \mp 1$ for muon (left) and electron (right). The bounds are derived from $\sigma(h \rightarrow \mu^+ \mu^-) / \sigma(h \rightarrow \mu^+ \mu^-)_{\text{SM}} = 0 \pm 1.3$ [79–81], $\sigma(h \rightarrow e^+ e^-) / \sigma(h \rightarrow e^+ e^-)_{\text{SM}} < 3.7 \times 10^5$ [82], $Z \rightarrow \ell\ell$ [79, 83], and direct searches for new heavy charged leptons [84]. The $h \rightarrow \ell\ell$ limits are implemented at 2σ , the ones for $Z \rightarrow \ell\ell$ at 3σ , as explained in the main text.

Support slides

Transformation properties of MDM and EDM

	P	T	CP
μ	\times	\checkmark	\checkmark
d	\times	\checkmark	\checkmark
B	\times	\checkmark	\checkmark
E	\checkmark	\times	\times
$\mu \cdot B$	\times	\checkmark	\checkmark
$d \cdot E$	\checkmark	\times	\times

Table 1: The transformation properties of the magnetic and electric dipole moments, and their respective terms in the interaction Hamiltonian in equation 2.3.

fields B and E is given by:

$$\mathcal{H} = -\mu \cdot B - d \cdot E \tag{2.3}$$