

Implementation of the SuSAv2-MEC model in event generators

2p2h: Done in GENIE. NuWro in progress

1p1h: in progress

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NuSTEC Meeting, December 11, 2018

Challenges and open questions for theoretical nuclear models

- 1 Are current theoretical models (CRPA, Valencia LFG+2p2h, Benhar's SF, SuSAv2-MEC, RGF, etc.) good enough to analyze 1p1h and 2p2h channels in CC inclusive neutrino interactions?

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- 4 Is it possible to introduce sophisticated microscopic models in generators in a fully consistent way?
- 5 What are the physics behind these models?

Contents

- 1 Theoretical description and Results
 - Theoretical models and Description of 2p2h channels
 - Comparison with CC ν_μ -nucleus experimental data
- 2 Conclusions and Further Work
 - Conclusions and Further Work

SuperScaling Approach (SuSA)

(see [G.D. Megias' Thesis](#) for details)

► The analysis of the large amount of existing (e, e') data at different kinematics is a solid benchmark to **test** the validity of theoretical models for neutrino reactions as well as to study the nuclear dynamics. The **SuperScaling Approach** exploits **universal features** of lepton-nucleus scattering to connect the two processes.

$$f(\psi) \equiv f(q, \omega) \sim \frac{\sigma_{QE}(\text{nuclear effects})}{\sigma_{\text{single nucleon}}(\text{no nuclear effects})} ; \quad f_L = k_F R_L / G_L ; \quad f_T = k_F R_T / G_T$$

In inclusive QE scattering we can observe:

- ☆ Scaling of 1st kind (independence on q)
- ☆ Scaling of 2nd kind (independence on Z)

⇒

SuperScaling

$$f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega} \right)_{exp}}{\sigma_{Mott} (v_L G_L^{ee'} + v_T G_T^{ee'})}$$

SuperScaling Approach (SuSA)

(see [G.D. Megias' Thesis](#) for details)

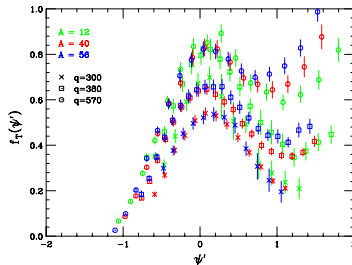
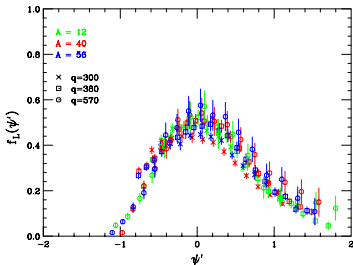
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SuperScaling

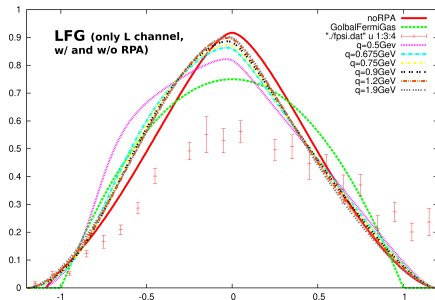
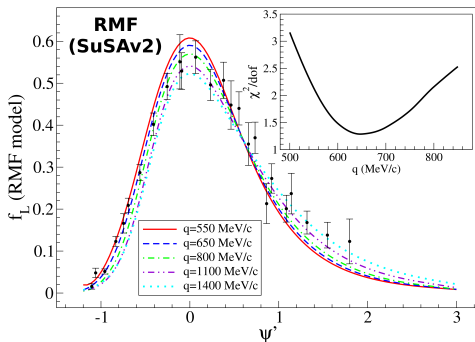


$$f_L = k_F R_L / G_L$$

$$f_T = k_F R_T / G_T$$

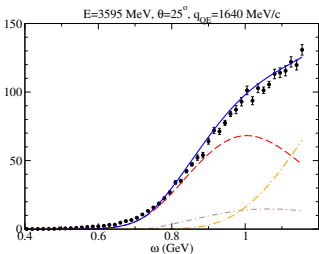
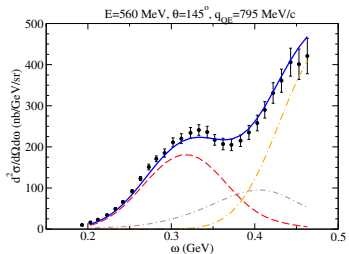
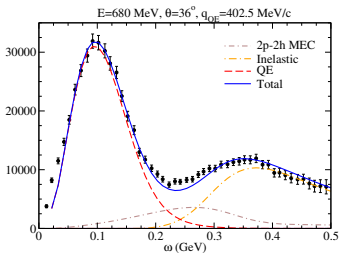
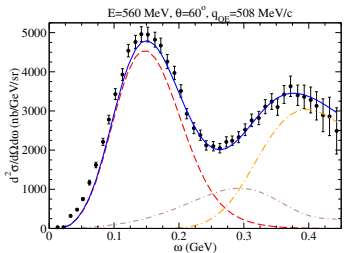
Scaling violations in the T channel ⇒ 2p-2h MEC, correlations, Δ -resonance ⇒ Mainly transverse

Testing SuperScaling for $^{12}\text{C}(e, e')$ in different nuclear models

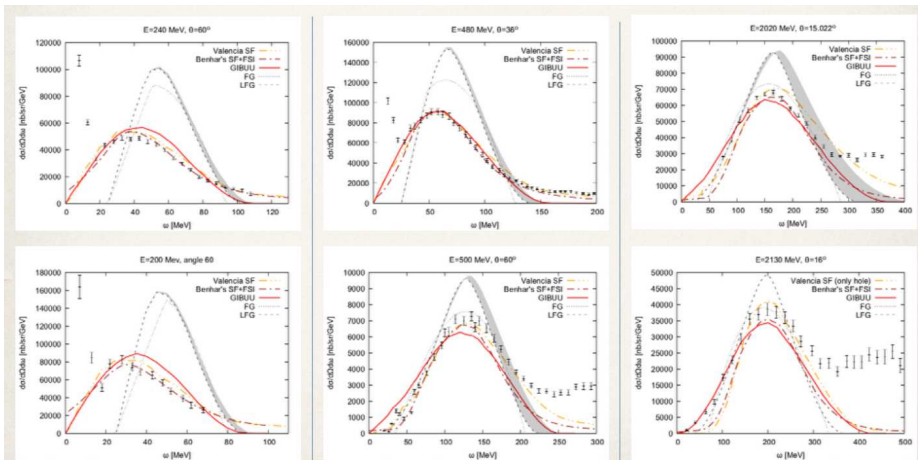


Inclusive $^{12}\text{C}(e, e')$ cross sections

PRD 94, 013012 (2016)



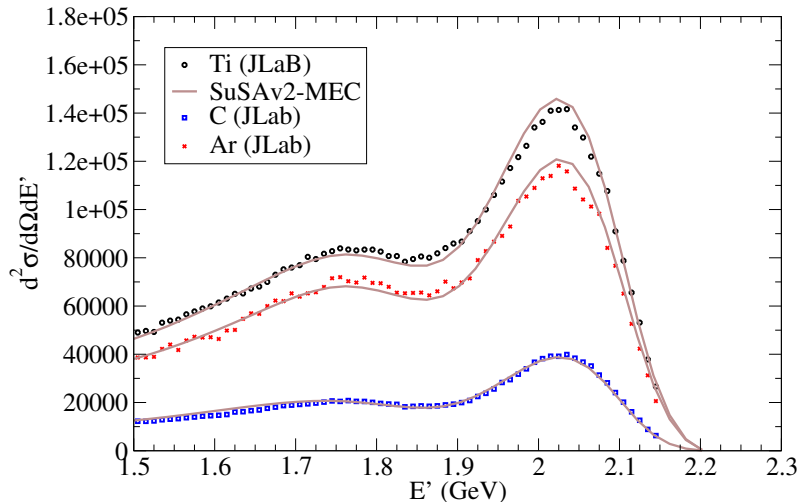
Inclusive $^{12}\text{C}(e, e')$ cross sections with different models (J.Sobczyk's talk at NUINT18)



We plot INCLUSIVE electron scattering data and theoretical models for the QE mechanism.

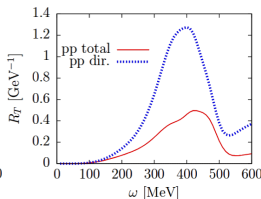
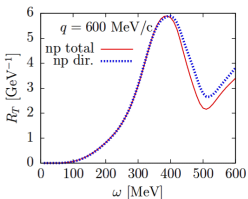
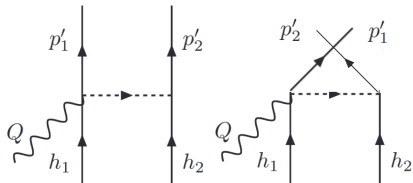
Inclusive (e, e') JLab data (PRELIMINARY)

$E=2.222$ GeV, $\theta=15.541^\circ$



2p-2h MEC for (e, e') and CC ν reactions PRD91, 073004 (2015)

Other 2p2h models neglect direct/exchange interference terms \Rightarrow strongly affects np/pp ratio by a factor ~ 2 (PRC94:054610,2016) \Rightarrow Implications in nucleon multiplicity and hadron E_{reco}



- ★ The 2p-2h model is based on the calculation performed by De Pace et al., (2003) for (e, e') scattering and extended to the weak sector by Amaro, Ruiz Simo et al. [PRD 90, 033012 (2014); PRD 90, 053010 (2014); JPG 44, 065105 (2017); PLB 762, 124 (2016)].
- ★ The numerical evaluation of the hadronic tensor $W_{2p2h}^{\mu\nu}(R_K^{2p2h})$ is performed in the RFG model in a fully relativistic way without any approximation.
- ★ Separation into pp , nn and np pairs in the FS \Rightarrow also valid for $N \neq Z$ (^{40}Ar , ^{56}Fe , ^{208}Pb)
- ★ It is computationally non-trivial and involves 7D integrals of thousands of terms (+1 for ν -flux) \Rightarrow High increase of the computing time of $R_K^{2p2h} \Rightarrow$ Parametrization/Implementation

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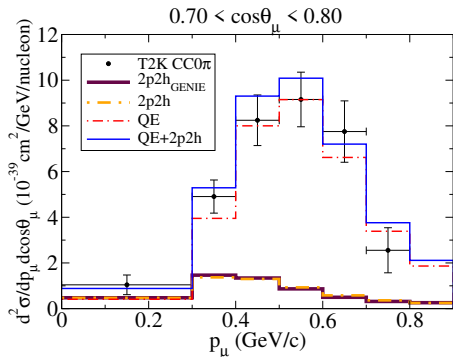
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SuSA-2p2h implementation in GENIE (PRELIMINARY)

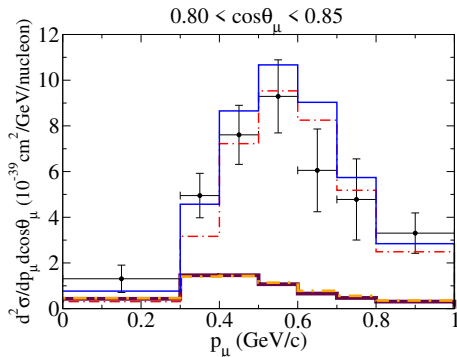
Implementation of the $W^{\mu\nu}$ hadron tensor components in GENIE using the SuSA formalism (L and T components, Rosenbluth-like decomposition).

Validation of the SuSA-2p2h implementation in GENIE (maroon thick lines) with the microscopic calculations (orange dot-dashed lines).

SuSA-2p2h: no FSI, 2p2h_{GENIE}: SuSA-2p2h implementation on GENIEv3 hN FSI



T2K CC0 π data on ^{12}C

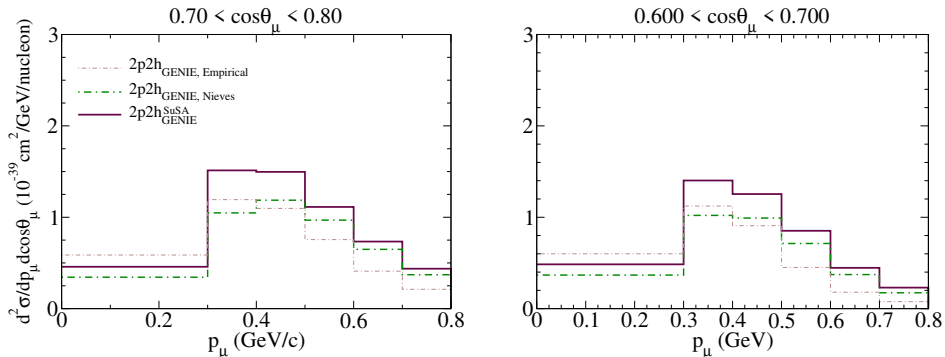


SuSA-2p2h implementation in GENIE (PRELIMINARY)

Implementation of the $W^{\mu\nu}$ hadron tensor components in GENIE using the SuSA formalism (L and T components, Rosenbluth-like decomposition).

Comparison with other 2p2h models (Nieves, Empirical) in GENIE.

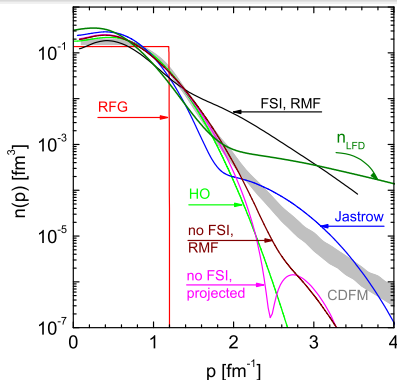
SuSA-2p2h: no FSI, 2p2h_{GENIE}: SuSA-2p2h implementation on GENIEv3 hN FSI



T2K CC0 π data on ^{12}C

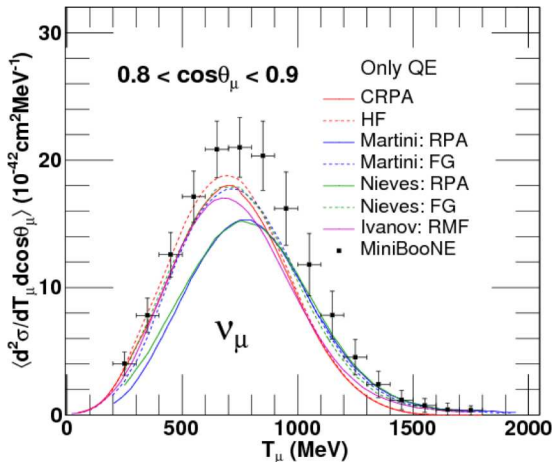
1p1h implementation: RMF and SuSAv2

- ▶ **1st step:** Implementing SuSAv2 hadron tensor $W^{\mu\nu}(q, \omega)$ + RFG/LFG on the top and comparison with original SuSAv2 model (**short term**)
- ▶ **2nd step:** Adding SuSAv2 formulas, parameters and parametrization of scaling functions into GENIE to speed up simulations (**mid term**)
- ▶ **3rd step:** Introducing RMF nucleon momentum distribution and spectral function in GENIE to reproduce the nuclear dynamics (**mid term**)



CCQE ν with different models (Pandey's talk at NUINT18)

Different models can give similar inclusive CS but different semi-inclusive ones.



Characterization of nuclear effects at T2K ($E_\nu \sim 0.8$ GeV) (PRELIMINARY)

T2K CC0 π Np data \Rightarrow novel probe of nuclear-medium effects through p kinematics, nucleon multiplicity and p and μ kinematic imbalances.

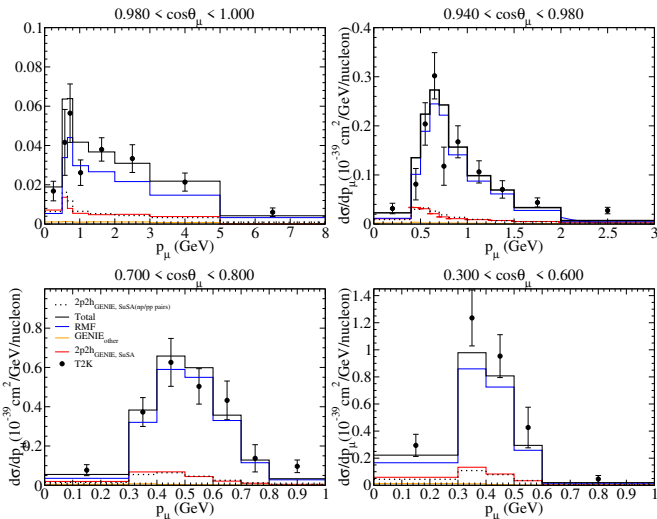
THE FUTURE IS SEMI-INCLUSIVE \Rightarrow Best way to produce consistent theory-vs-data comparison. Less dependency on simulations and deeper analysis of model nuclear effects.

PROBLEM: Current lack of semi-inclusive models and proper implementation in generators.

Different models can give similar inclusive CS but different semi-inclusive ones.

Semi-inclusive \Rightarrow Inclusive (but not viceversa).

2p2h_{GENIE}: SuSA-2p2h implementation on GENIEv3 hN FSI



“Semi-semi-inclusive” T2K CC0 π Np data on ^{12}C with 0_p above 500 MeV/c

Semi-Inclusive CC0 π Np T2K data using SuSA-2p2h^{GENIE} (PRELIMINARY)

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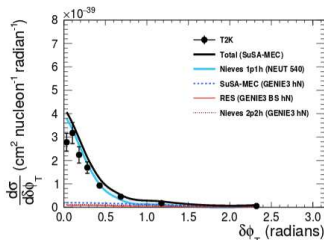
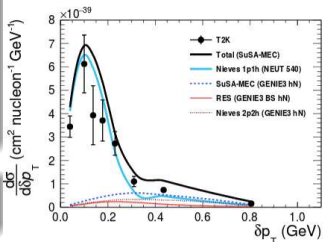
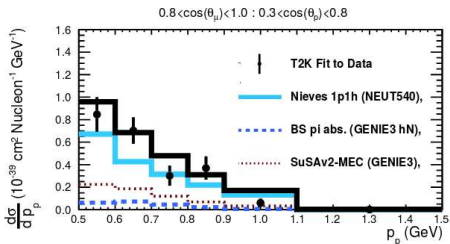
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Inclusive and Semi-inclusive formalism

Different models can give similar inclusive CS but different semi-inclusive ones.

Semi-inclusive \Rightarrow Inclusive (but not viceversa).

Double differential **inclusive** cross section

$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu} \right]_\chi = \frac{|\vec{k}_l|}{|\vec{k}_{\nu_l}|} \frac{G_F^2}{4\pi^2} \tilde{\eta}_{\mu\nu} \tilde{W}^{\mu\nu} = \sigma_0 \mathcal{F}_\chi^2 \quad ; \quad \sigma_0 = \frac{(G_F^2 \cos^2 \theta_c)^2}{2\pi^2} \left(k_\mu \cos \frac{\tilde{\theta}}{2} \right)^2$$

$$\mathcal{F}_\chi^2 = V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T + \chi \left[2V_{T'} R_{T'} \right]$$

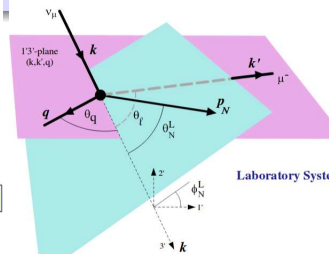
$$R_{CC} = W^{00}; R_{CL} = -(W^{03} + W^{30})/2; R_{LL} = W^{33}; R_T = W^{11} + W^{22}; R_{T'} = -i(W^{12} - W^{21})/2$$

Double differential **semi-inclusive** cross section

$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\frac{d\sigma}{dk'_l d\Omega_{k'} dp_N^l d\Omega_N^l} = \frac{G^2 \cos^2 \theta_c m_N k'^2 \varepsilon p_N^l W_{A-1} v_0}{2(2\pi)^5 k \varepsilon' E_N \sqrt{\lambda_B^2 + m^2 a_B}} \mathcal{F}_\chi^2 \delta(k - k_0),$$

$$\begin{aligned} \mathcal{F}_\chi^2 = & \hat{V}_{CC}(w_{CC}^{VV(I)} + w_{CC}^{AA(I)}) + 2\hat{V}_{CL}(w_{CL}^{VV(I)} + w_{CL}^{AA(I)}) + \hat{V}_{LL}(w_{LL}^{VV(I)} + w_{LL}^{AA(I)}) \\ & + \hat{V}_T(w_T^{VV(I)} + w_T^{AA(I)}) + \hat{V}_{TT} \left[(w_{TT}^{VV(I)} + w_{TT}^{AA(I)}) \cos 2\phi_N + (w_{TT}^{VV(II)} + w_{TT}^{AA(II)}) \sin 2\phi_N \right] \\ & + \hat{V}_{TC} \left[(w_{TC}^{VV(I)} + w_{TC}^{AA(I)}) \cos \phi_N + (w_{TC}^{VV(II)} + w_{TC}^{AA(II)}) \sin \phi_N \right] \\ & + \hat{V}_{TL} \left[(w_{TL}^{VV(I)} + w_{TL}^{AA(I)}) \cos \phi_N + (w_{TL}^{VV(II)} + w_{TL}^{AA(II)}) \sin \phi_N \right] \\ & + \chi \left[\hat{V}_{TV} w_T^{VA(I)} + \hat{V}_{TC} (w_{TC}^{VA(I)} \sin \phi_N + w_{TC}^{VA(II)} \cos \phi_N) + \hat{V}_{TL} (w_{TL}^{VA(I)} \sin \phi_N + w_{TL}^{VA(II)} \cos \phi_N) \right] \end{aligned}$$



➤ **Implementation of semi-inclusive models.** Difficulties: redefinition of the “semi-inclusive formalism” in generators [inclusive (5 responses) \Rightarrow semi-inclusive (15 responses)]. Current 1p1h and 2p2h semi-inclusive CS from generators “must be treated carefully”, but there are not good theoretical semi-inclusive models at present. Work in progress with RMF (SuSAv2).

- QE and 2p2h inclusive: We only need $W^{\mu\nu}(q, \omega)$ or, equivalently, $W^{\mu\nu}(p_\mu, \cos \theta_\mu)$
- QE semi-inclusive : 5D diff. CS $(\theta_\mu, p_\mu, p_N, \theta_N, \phi_N)$ - 2p2h semi-inclusive: 9D diff. CS.

Double differential inclusive cross section

$$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu} \right]_\chi = \frac{|\vec{k}_f|}{|\vec{k}_{\nu_f}|} \frac{G_F^2}{4\pi^2} \sim \eta_{\mu\nu} \tilde{W}^{\mu\nu} = \sigma_0 \mathcal{F}_\chi^2 \quad ; \quad \sigma_0 = \frac{(G_F^2 \cos \theta_c)^2}{2\pi^2} \left(k_\mu \cos \frac{\tilde{\theta}}{2} \right)^2$$

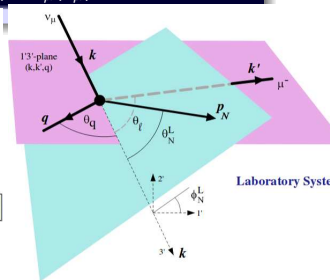
$$\mathcal{F}_\chi^2 = V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_{TR}R_T + \chi \left[2V_{T'}R_{T'} \right]$$

Double differential semi-inclusive cross section

$$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$$

$$\frac{d\sigma}{dk'_d d\Omega_k dp_N^2 d\Omega_N^2} = \frac{G^2 \cos^2 \theta_{cm} k'^2 \varepsilon p_N^2 W_{A-1} v_0}{2(2\pi)^5 k_e' E_N \sqrt{X_B^2 + m^2} a_B} \mathcal{F}_\chi^2 \delta(k - k_0),$$

$$\begin{aligned} \mathcal{F}_\chi^2 = & \hat{V}_{CC}(w_{CC}^{VV(I)} + w_{CC}^{AA(I)}) + 2\hat{V}_{CL}(w_{CL}^{VV(I)} + w_{CL}^{AA(I)}) + \hat{V}_{LL}(w_{LL}^{VV(I)} + w_{LL}^{AA(I)}) \\ & + \hat{V}_T(w_T^{VV(I)} + w_T^{AA(I)}) + \hat{V}_{TT} \left[(w_{TT}^{VV(I)} + w_{TT}^{AA(I)}) \cos 2\phi_N + (w_{TT}^{VV(II)} + w_{TT}^{AA(II)}) \sin 2\phi_N \right] \\ & + \hat{V}_{TC} \left[(w_{TC}^{VV(I)} + w_{TC}^{AA(I)}) \cos \phi_N + (w_{TC}^{VV(II)} + w_{TC}^{AA(II)}) \sin \phi_N \right] \\ & + \hat{V}_{TL} \left[(w_{TL}^{VV(I)} + w_{TL}^{AA(I)}) \cos \phi_N + (w_{TL}^{VV(II)} + w_{TL}^{AA(II)}) \sin \phi_N \right] \\ & + \chi \left[\hat{V}_T w_T^{VA(I)} + \hat{V}_{TC} (w_{TC}^{VA(I)} \sin \phi_N + w_{TC}^{VA(II)} \cos \phi_N) + \hat{V}_{TL} (w_{TL}^{VA(I)} \sin \phi_N + w_{TL}^{VA(II)} \cos \phi_N) \right] \end{aligned}$$



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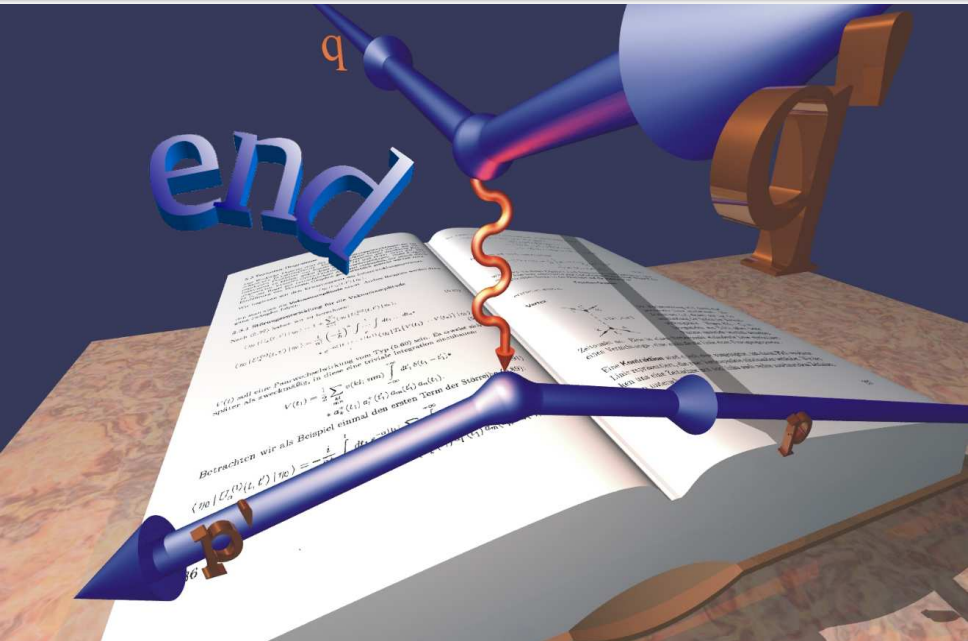
- 2 Conclusions and Further Work
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Collaborators

- Stephen Dolan (CEA-Irfu, University of Paris-Saclay, France)
- Sara Bolognesi (CEA-Irfu, University of Paris-Saclay, France)
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- Juan A. Caballero (University of Seville, Spain)
- Maria B. Barbaro (INFN and University of Turin, Italy)
- Raúl González-Jiménez (University Complutense of Madrid, Spain)
- Jose E. Amaro (University of Granada, Spain)
- I. Ruiz-Simó (University of Granada, Spain)
- Martin Ivanov (Bulgarian Academy of Sciences, Bulgaria)
- Anton Antonov (Bulgarian Academy of Sciences, Bulgaria)
- W. Van Orden (Old Dominion University, JLab, USA)

Thanks for your attention!





BACKUP SLIDES

Theoretical description: ν -nucleus cross section

Double differential cross section

$$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu} \right]_\chi = \frac{|\vec{k}_l|}{|\vec{k}_{\nu_l}|} \frac{G_F^2}{4\pi^2} \tilde{\eta}_{\mu\nu} \tilde{W}^{\mu\nu} = \sigma_0 \mathcal{F}_\chi^2 \quad ; \quad \sigma_0 = \frac{(G_F^2 \cos^2 \theta_c)^2}{2\pi^2} \left(k_\mu \cos \frac{\tilde{\theta}}{2} \right)^2$$

Nuclear structure information

$$\mathcal{F}_\chi^2 = V_L R_L + V_T R_T + \chi [2V_{T'} R_{T'}]$$

$$V_L R_L = V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL}$$

$$R_L = R_L^{VV} + R_L^{AA} \quad ; \quad R_T = R_T^{VV} + R_T^{AA} \quad ; \quad R_{T'} = R_{T'}^{VA}$$

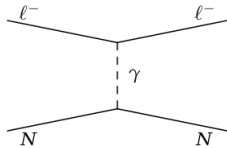
Nuclear responses R_K can be calculated in terms of the single nucleon ones G_K and the nuclear dependence of the model $\Rightarrow R_K \approx F(\text{nuclear}) \cdot G_K$

$$\begin{aligned} R_{CC} &= W^{00} \\ R_{CL} &= -\frac{1}{2} (W^{03} + W^{30}) \\ R_{LL} &= W^{33} \\ R_T &= W^{11} + W^{22} \\ R_{T'} &= -\frac{i}{2} (W^{12} - W^{21}) \end{aligned}$$

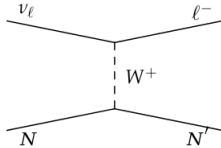
Comparison with (e, e') reactions

$$\left[\frac{d\sigma}{dk_\mu d\Omega} \right] = \sigma_{Mott} \left(v_L R_L^{VV} + v_T R_T^{VV} \right) \quad ; \quad \sigma_{Mott} = \frac{\alpha^2 \cos^2 \theta/2}{4E_i \sin^4 \theta/2}$$

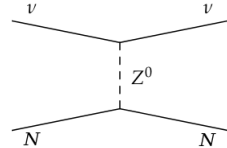
Connection between ν -A and e-A reactions



(a) Electromagnetic scattering



(b) Charged-current scattering



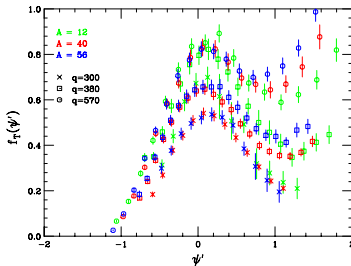
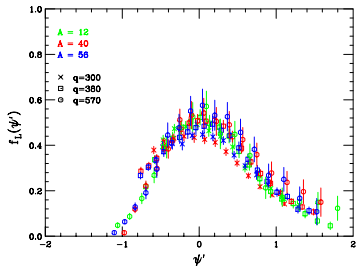
(c) Neutral-current scattering

$l = e, \mu, \tau$

- Experimental conditions are different:
 - ➔ (e, e') : E_e is well determined and different channels can be clearly identified by knowing the energy and momentum transfer
 - ➔ $CC(\nu_l, l)$: E_ν is broadly distributed in the neutrino beam and different channels and nuclear effects can contribute to the same kinematics of the outgoing lepton
- From a theoretical framework, neutrino- and electron-nucleus scattering are obviously connected (CVC) to each other and a **reliable model** must be able to describe both processes.
- Neutrinos can probe both the **vector** and **axial** nuclear responses, unlike electrons which are only sensitive to the vector response.

⇒ Although not sufficient to fully constrain neutrino cross sections, electron scattering constitutes a **necessary test and a solid benchmark for nuclear models**.

Separate L/T scaling functions



$$f_L = k_F R_L / G_L$$

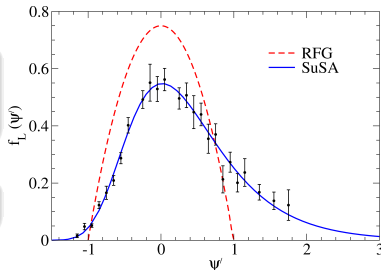
$$f_T = k_F R_T / G_T$$

Scaling violations in the T channel \Rightarrow 2p-2h MEC, correlations, Δ -resonance \Rightarrow Mainly transverse

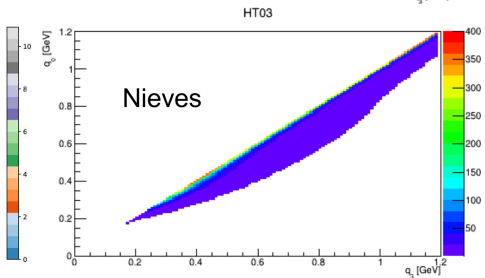
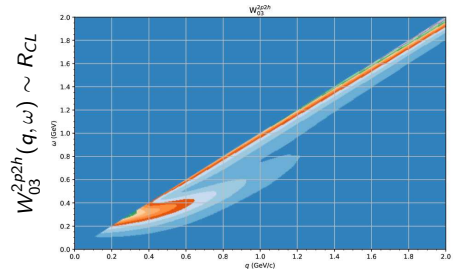
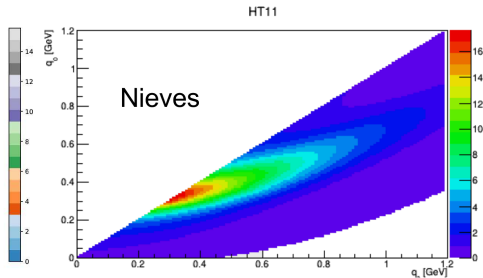
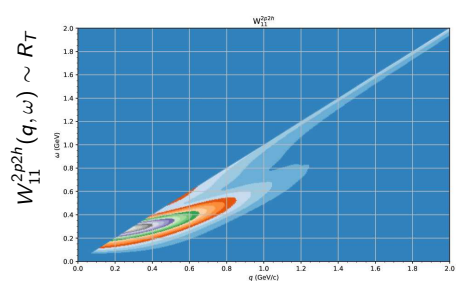
SuSA model: a semiphenomenological approach

- ★ Extracted from the (e, e') longitudinal scaling data
- ★ Assumption $f_L(\psi) = f_T(\psi)$ (as in most IA models)

★ It is experimentally observed $f_{T, \text{exp}}^{ee'} > f_{L, \text{exp}}^{ee'}$ (15-20%)



Comparison with other models implemented in GENIE



The SuSAv2 model

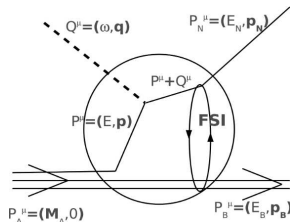
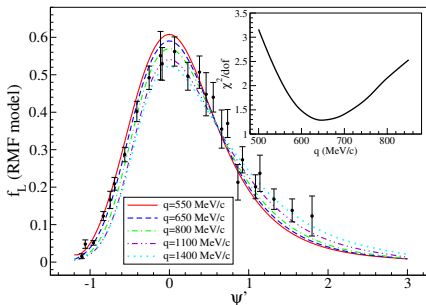
PRC90, 035501 (2014)

PRD94, 013012 (2016)

★ **SuSAv2 model:** lepton-nucleus reactions addressed within the **SuperScaling Approach** and the sophisticated **Relativistic Mean Field (RMF)** theory (FSI) to determine theoretical scaling functions that reproduce nuclear dynamics. Complete set of scaling functions for all lepton-nucleus reaction channels (EM, weak, L/T, isovector/isoscalar, V/A).

★ RMF: Good description of the QE (e, e') data and **superscaling properties** ($f_{L,exp}^{ee'}$).

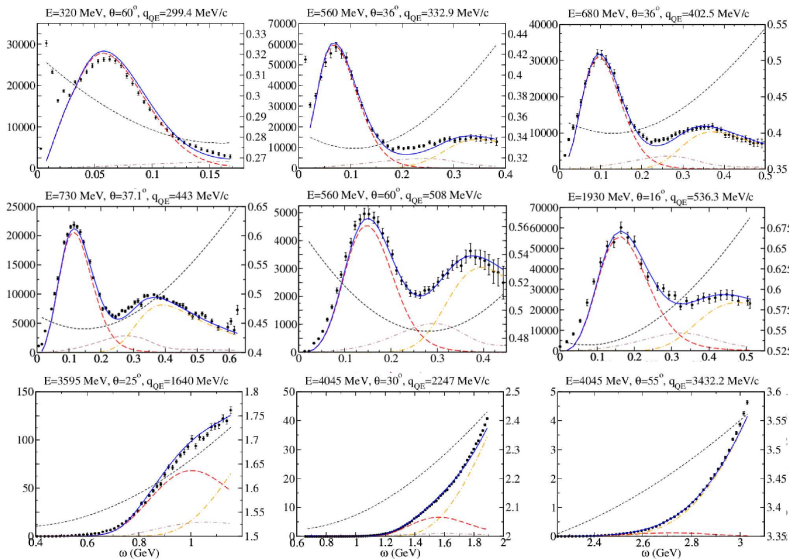
RMF predicts $f_T > f_L$ ($\sim 20\%$) as a pure relativistic effect (distortion of the lower components of the outgoing Ψ_N by the FSI with the residual nucleus).



RMF-FSI: Scattered nucleon w.f. is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon w.f.

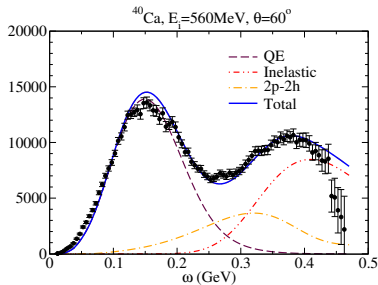
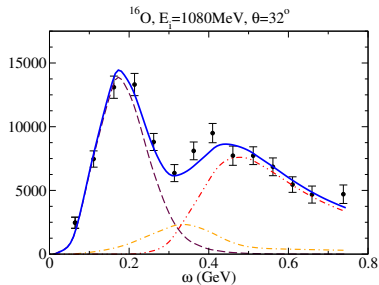
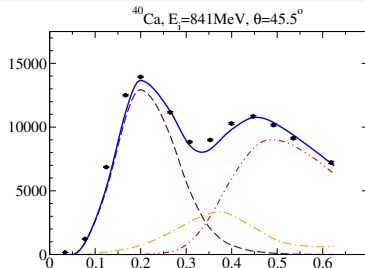
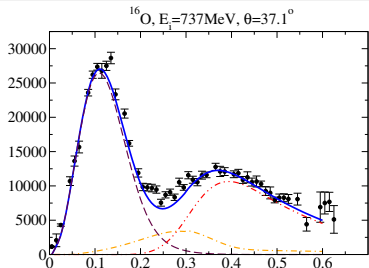
Inclusive $^{12}\text{C}(e, e')$ cross sections

PRD 94, 013012 (2016)



Inclusive $^{16}\text{O}(e, e')$ and $^{40}\text{Ca}(e, e')$ cross sections

arXiv:1711.00771 [nucl-th] (2018)

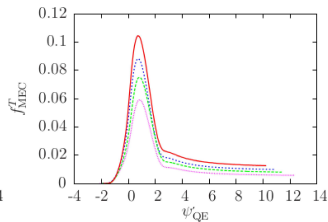
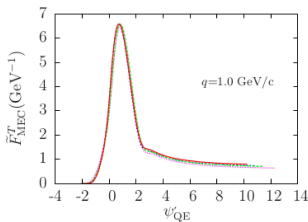
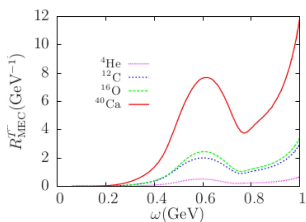


Density dependence of 2p-2h MEC [PRC95, 065502 (2017)]

☆ Most existing calculations of 2p-2h MEC refer to ^{12}C , but other nuclei are interesting for oscillation experiments (^{16}O , ^{40}Ar) \Rightarrow Extension of the 2p-2h MEC analysis to other nuclei

☆ A-scaling (2^{nd} kind) on 2p-2h MEC responses? \Rightarrow A description of 2p-2h MEC responses in terms of k_F allow to extend easily 2p2h calculation to other nuclei reducing significantly the computational time.

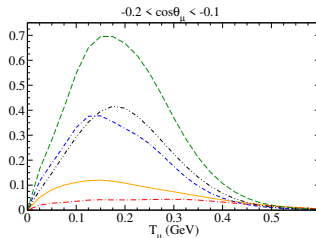
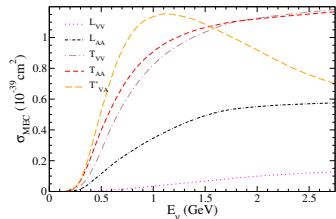
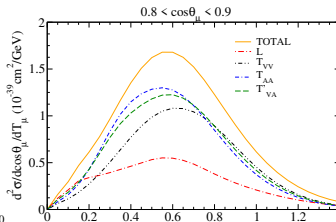
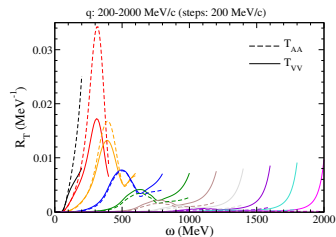
$$\tilde{F}_T^{\text{MEC}}(q, \omega) \equiv \frac{m_N^2}{k_F^2} \frac{R_T^{\text{MEC}}(q, \omega)}{G_T(\tau)} \quad ; \quad f_T^{\text{MEC}}(q, \omega) \equiv \frac{k_F}{m_N} \frac{R_T^{\text{MEC}}(q, \omega)}{G_T(\tau)}$$



\Rightarrow 2p-2h responses scales as $A \cdot k_F^2$ whereas the QE one scales as A/k_F :

$$R_T^{\text{MEC}} \sim k_F^2 \tilde{F}_T^{\text{MEC}} G_T \quad R_T^{\text{QE}} = \frac{1}{k_F} f_T^{\text{QE}} G_T$$

2p2h for (e, e') processes \nRightarrow 2p2h in CC (ν, l) reactions

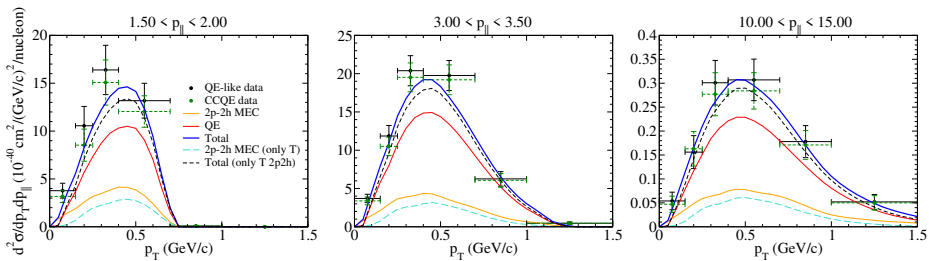


$R_T^{AA} > R_T^{VV}$ at $q < 800$ MeV/c
 $R_T^{AA} < R_T^{VV}$ at $q > 800$ MeV/c
 $\Rightarrow \sigma(T_{AA}) \sim \sigma(T_{VV})$ but not
 for all lepton kinematics (see Mini-BooNE $\bar{\nu}_{\mu}$ $d^2\sigma$, right panels).

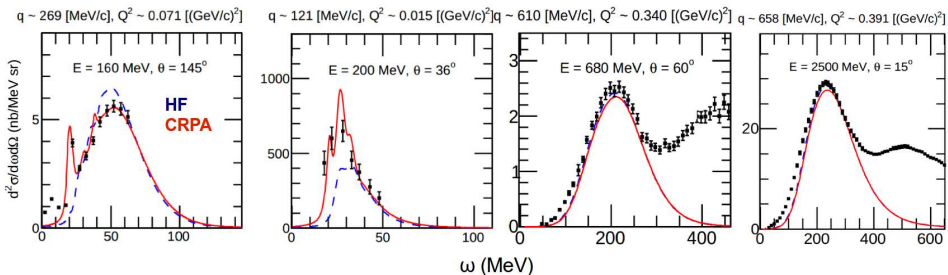
$R_T^{VV}(e, e') \rightarrow R_T^{VV}(\nu, l)$
 $R_T^{VV}(e, e') \nrightarrow R_T^{AA}, R_T^{VA}(\nu, l)$

$R_L^{VV}(e, e')$ negligible but not
 $R_L(\nu, l)$ because of $R_L^{AA}(\nu, l)$.
 Highly relevant for antineutrino
 reactions ($\sigma_{T'}^{VA} < 0$).

Relevance of 2p2h longitudinal channel on MINER ν A $\bar{\nu}_\mu$ -CH data

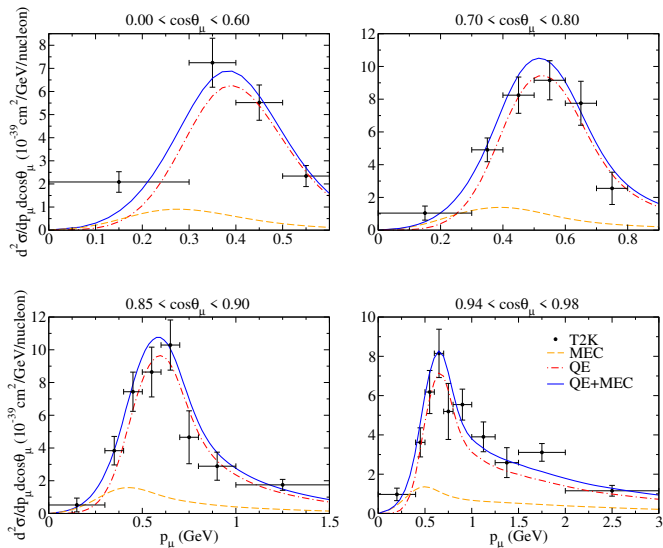


Inclusive $^{12}\text{C}(e, e')$ cross sections with different models (Pandey's talk at NUINT18)



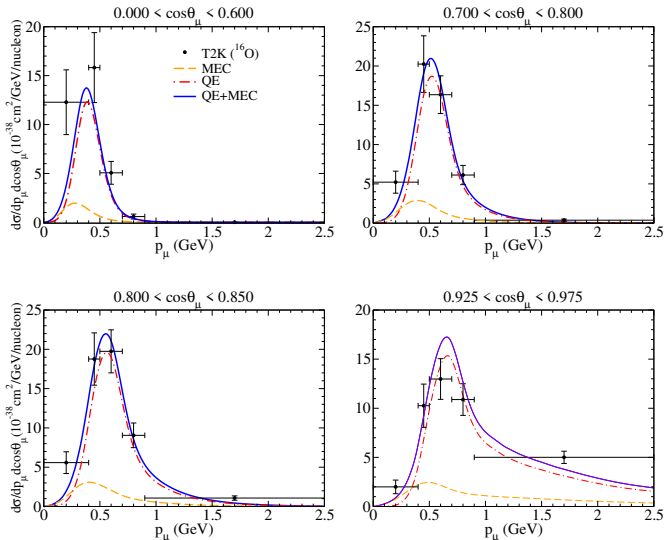
T2K CC0 π ν_μ -C₈H₈ cross sections

PRD 94, 093004 (2016)



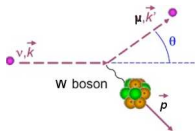
T2K CC0 π ν_μ -H₂O cross sections

arXiv:1711.00771 [nucl-th] (2017)



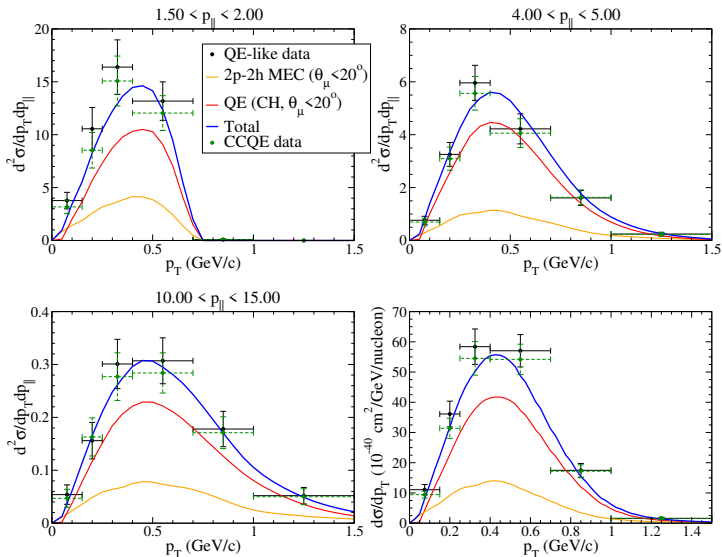
MINERνA $\bar{\nu}_\mu$ -CH reactions at $E_\nu \sim 3$ GeV

arXiv:1807.10532 [nucl-th]



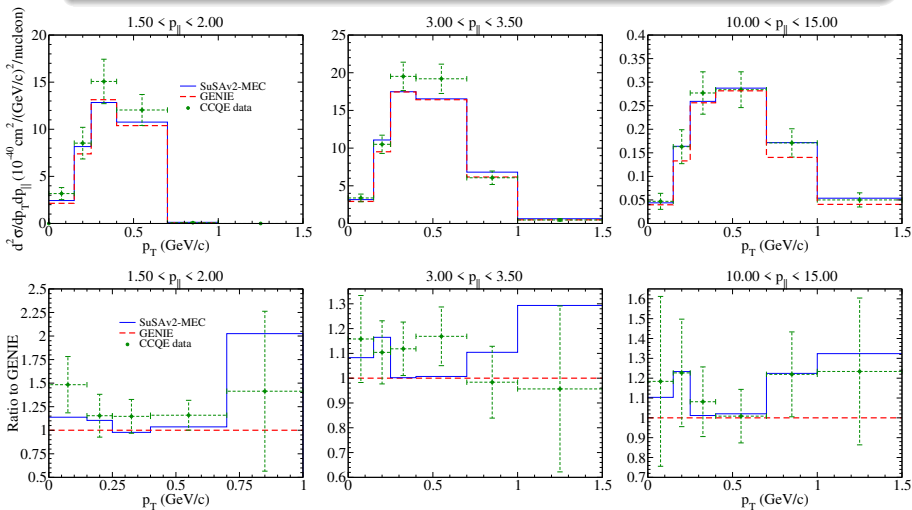
$$p_{||} = p_\mu \cos \theta_\mu$$

$$p_T = p_\mu \sin \theta_\mu$$



GENIE and SuSAv2-MEC vs. MINERvA $\bar{\nu}_\mu$ -CH data (χ^2 analysis)

$\chi^2/d.o.f. = 1.79$ (SuSAv2-MEC) ; $\chi^2/d.o.f. = 1.58$ (GENIE)



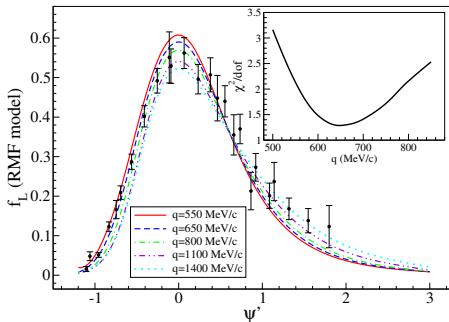
Theoretical description: RMF and SuSAv2 models

The SuSAv2 model

PRC90, 035501 (2014)

PRD94, 013012 (2016)

- ★ In the **SuSAv2**, the scaling functions are **calculated within the Relativistic Mean Field model (FSI)**, which predicts, for instance, different scaling functions in the L and T channels and for the different isospin channels (CCν reactions are purely isovector).
- ★ RMF: Good description of the QE (e,e') data and **superscaling properties** ($f_{L,exp}^{ee'}$)
- ★ RMF predicts $f_T > f_L$ ($\sim 20\%$) as a pure relativistic effect (distortion of the lower components of the outgoing Ψ_N by the FSI with the residual nucleus)



$$\begin{aligned}
 R_{L,T}^{ee'} &= \frac{1}{k_F} \left[f_{L,T}^{T=1,ee'}(\psi') G_{L,T}^{T=1} \right. \\
 &\quad \left. + f_{L,T}^{T=0,ee'}(\psi') G_{L,T}^{T=0} \right] \\
 R_L^{VV,\nu(\bar{\nu})} &= \frac{1}{k_F} f_L^{VV,\nu(\bar{\nu})}(\psi') G_L^{VV} \\
 R_{CC,CL,LL}^{AA,\nu(\bar{\nu})} &= \frac{1}{k_F} f_{CC,CL,LL}^{AA,\nu(\bar{\nu})}(\psi') G_{CC,CL,LL}^{AA} \\
 R_T^{VV(AA),\nu(\bar{\nu})} &= \frac{1}{k_F} f_T^{VV(AA),\nu(\bar{\nu})}(\psi') G_T^{VV(AA)} \\
 R_{T'}^{\nu(\bar{\nu})} &= \frac{1}{k_F} f_{T'}^{VA,\nu(\bar{\nu})}(\psi') G_{T'}^{VA}.
 \end{aligned}$$

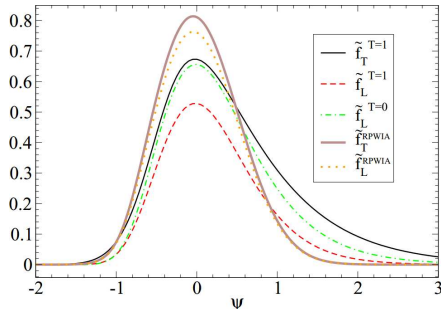
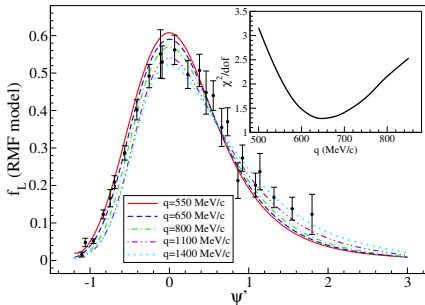
Theoretical description: RMF and SuSAv2 models

The SuSAv2 model

PRC90, 035501 (2014)

PRD94, 013012 (2016)

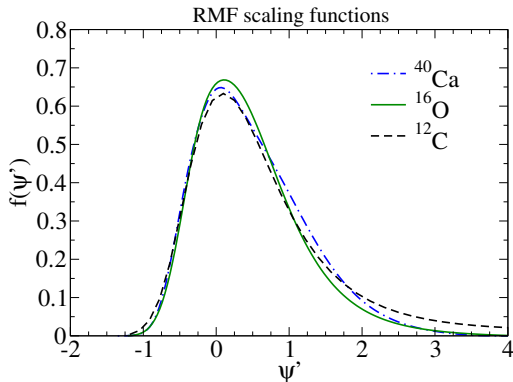
- ★ **SuSAv2 model:** lepton-nucleus reactions addressed within the **SuperScaling Approach** and the sophisticated **Relativistic Mean Field (RMF)** theory (FSI) to determine theoretical scaling functions that reproduce nuclear dynamics. Complete set of scaling functions for all lepton-nucleus reaction channels (EM, weak, L/T, isovector/isoscalar, V/A).
- ★ RMF: Good description of the QE (e, e') data and **superscaling properties** ($f_{L,exp}^{ee'}$)



Extension of the SuSAv2-MEC model to other nuclei

SuSAv2 scaling functions for different nuclei

- ➔ 2-nd kind scaling within the RMF and RPWIA models.
- ➔ k_F and E_{shift} are the only different parameters.



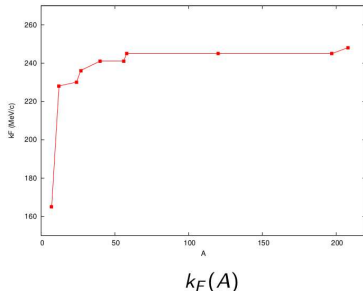
Density dependence of the 2p-2h MEC responses

☆ Most existing calculations of 2p-2h MEC refer to ^{12}C , but other nuclei are interesting for oscillation experiments (^{16}O , ^{40}Ar) \Rightarrow [Extension of the 2p-2h MEC analysis to other nuclei](#)

☆ In the RFG and in the SuSAv2-MEC model, each nucleus is characterized by two parameters: k_F and E_{shift} , fitted to reproduce the width and position of the QEP in inclusive electron scattering.

TABLE I. Adjusted parameters.

Nucleus	k_F (MeV/c)	E_{shift} (MeV)
Lithium	165	15
Carbon	228	20
Magnesium	230	25
Aluminum	236	18
Calcium	241	28
Iron	241	23
Nickel	245	30
Tin	245	28
Gold	245	25
Lead	248	31



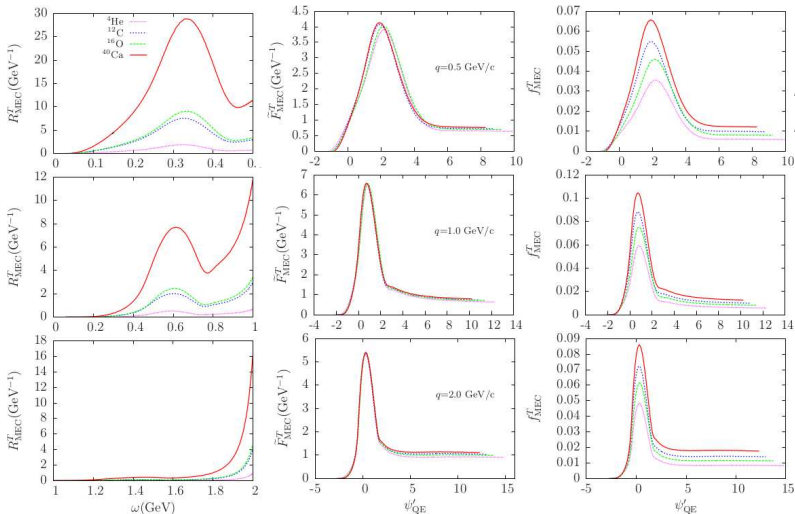
Maieron, Donnelly, Sick, PRC65 (2002)

☆ A-scaling (2^{nd} kind) on 2p-2h MEC responses? \Rightarrow A description of 2p-2h MEC responses in terms of k_F allow to extend easily our calculation to other nuclei reducing significantly the computational time.

Density dependence of 2p-2h MEC [PRC95, 065502 (2017)]

$$\tilde{R}_T^{MEC}(q, \omega) \equiv \frac{m_N^2 R_T^{MEC}(q, \omega)}{k_F^2 G_T(\tau)} ; f_T^{MEC}(q, \omega) \equiv \frac{k_F R_T^{MEC}(q, \omega)}{m_N G_T(\tau)}$$

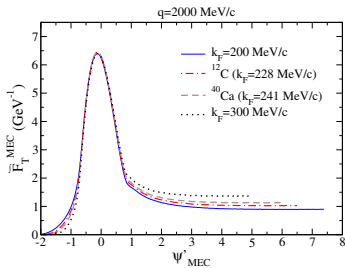
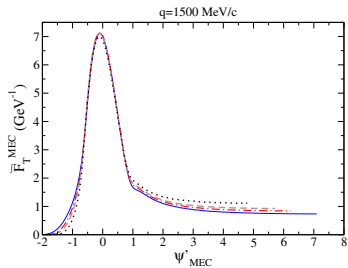
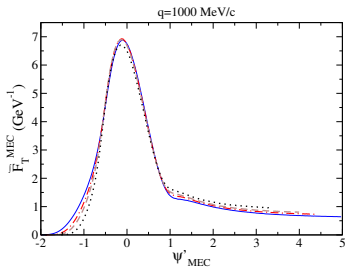
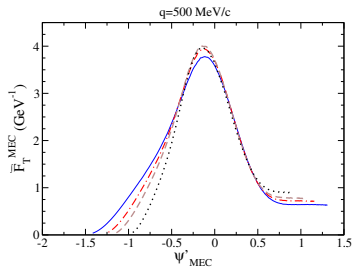
\Rightarrow 2p-2h responses scales as $A \cdot k_F^2$
 whereas the QE one scales as A/k_F



$$R_T^{MEC} \sim k_F^2 \tilde{R}_T^{MEC} G_T$$

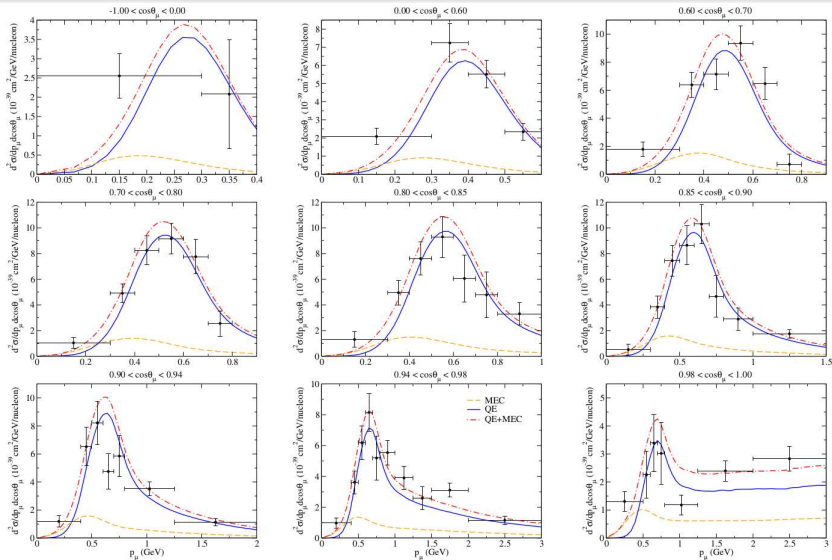
$$R_T^{QE} = \frac{1}{k_F} f_T^{QE} G_T$$

Density dependence of 2p-2h MEC [PRC95, 065502 (2017)]

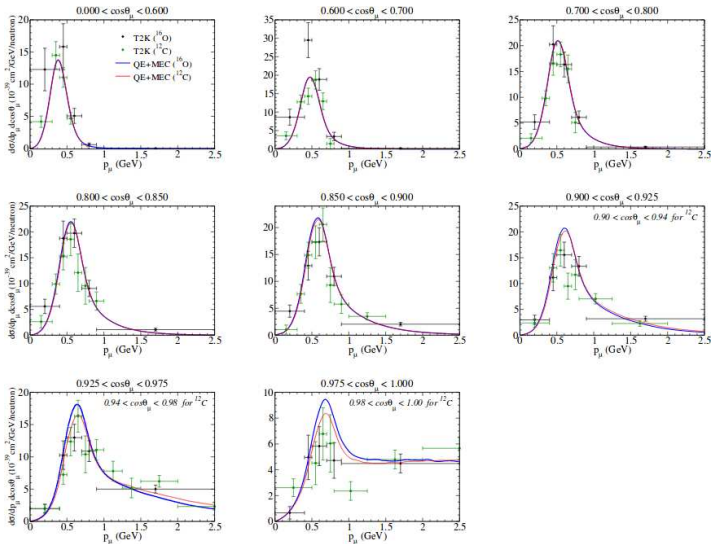


$$\psi'_{MEC}(q, \omega, k_F)$$

T2K $\nu_\mu - {}^{12}\text{C}$ cross sections

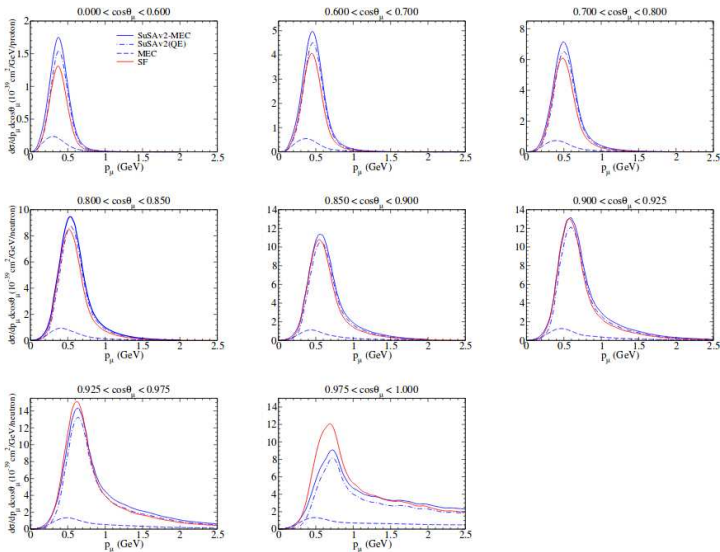


T2K $\nu_\mu - \text{C}_8\text{H}_8$ versus $\nu_\mu - \text{H}_2\text{O}$ cross sections

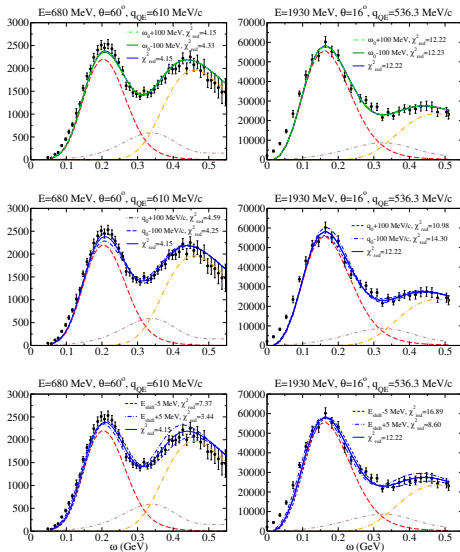


T2K $\bar{\nu}_\mu - \text{H}_2\text{O}$ cross sections

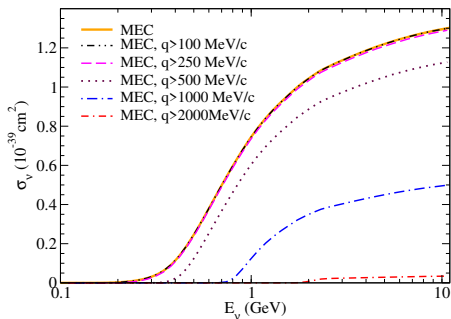
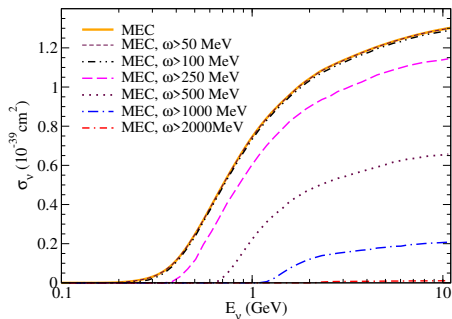
arXiv:1711.00771 [nucl-th] (2017)



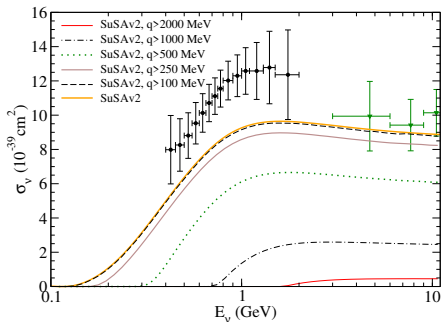
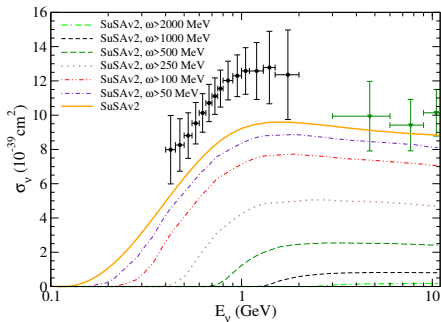
Sensitivity of the SuSAv2-MEC model



Relevant kinematic regions in the 2p2h cross section

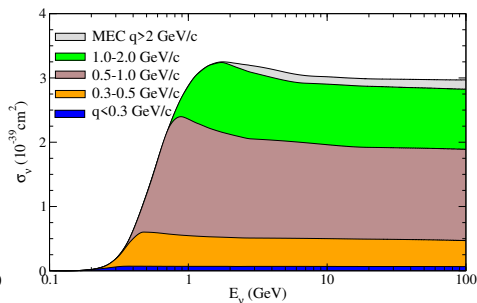
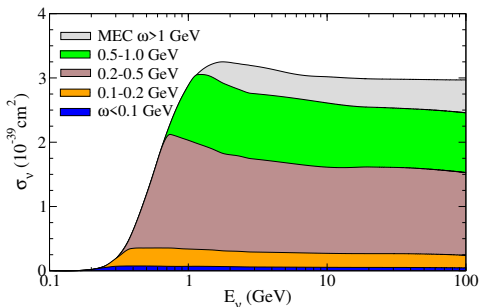


Relevant kinematic regions in the QE cross section



The main contribution to the total QE CS comes from $q < 1 \text{ GeV}/c$ and $\omega < 0.5 \text{ GeV}$, even at high neutrino energies.

Relevant kinematic regions in the 2p-2h MEC cross section



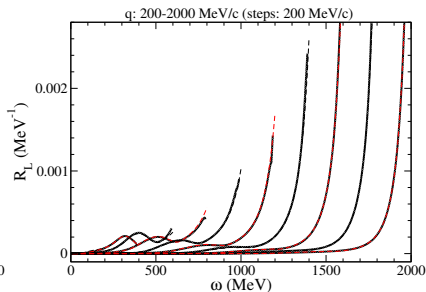
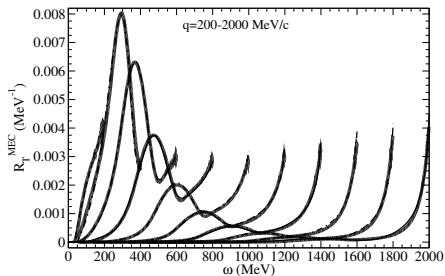
Although very similar to the QE case, the relevance of 2p-2h MEC contributions extends slightly to higher kinematics.

2p-2h MEC parametrization *PRD91, 073004 (2015) PRD94, 093004 (2016)*

$$R_X^{2p-2hMEC}(\psi', q) = \frac{2a_{3,X} e^{-\frac{(\psi' - a_{4,X})^2}{a_{5,X}}} + \sum_{k=0}^2 b_{k,X} \cdot (\psi')^k}{1 + e^{-\frac{(\psi' - a_{1,X})^2}{a_{2,X}}}}$$

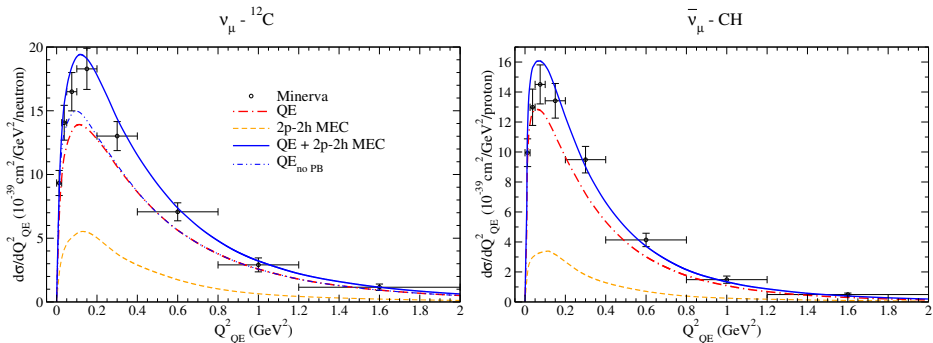
$X = CC, CL, LL, T(= T_{VV} + T_{AA}), T'_{VA}$

$a_{i,X}(q), b_{k,X}(q)$



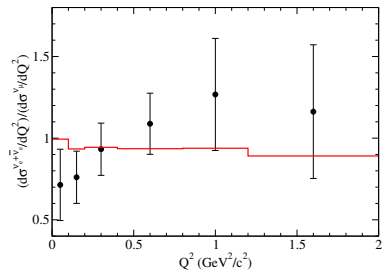
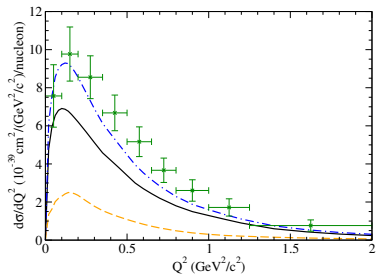
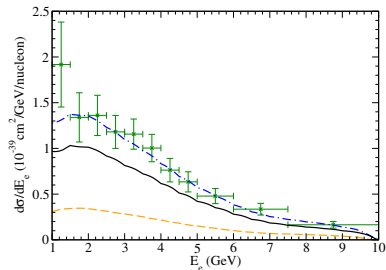
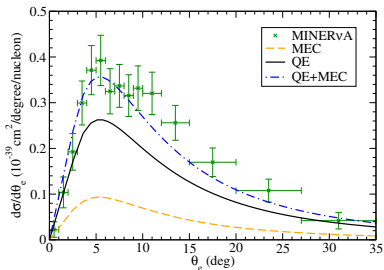
MINER ν A $\nu_{\mu}(\bar{\nu}_{\mu})$ -CH cross sections

PRD 94, 093004 (2016)

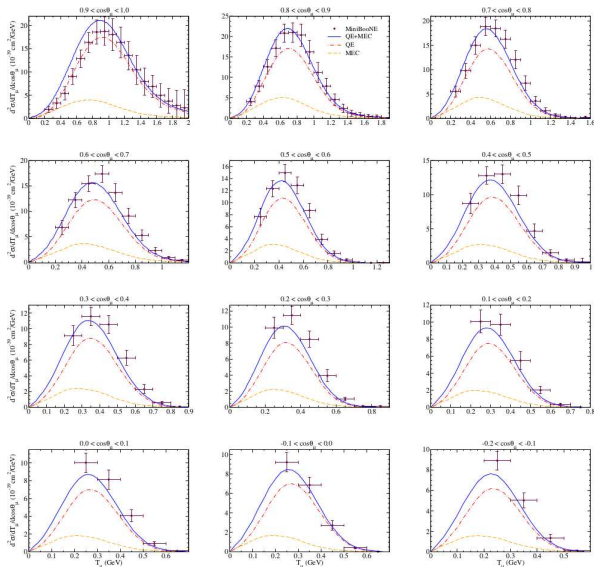
T2K, MiniBooNE: $\langle E_{\nu} \rangle \sim 0.8$ GeV \implies MINER ν A: $\langle E_{\nu} \rangle \sim 3.0$ GeV*More prominent 2p-2h MEC effects*

MINERvA $\nu_e - {}^{12}\text{C}$ cross sections

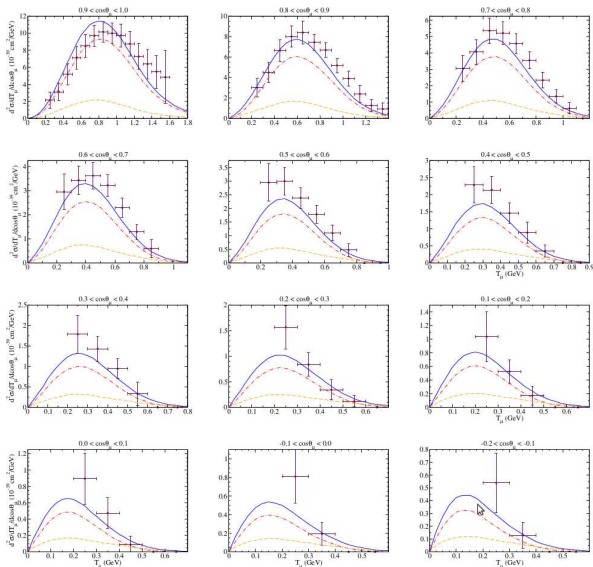
PRD 94, 093004 (2016)



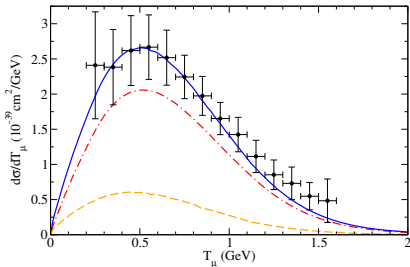
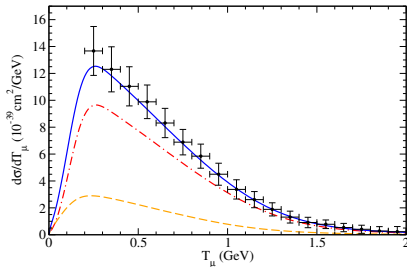
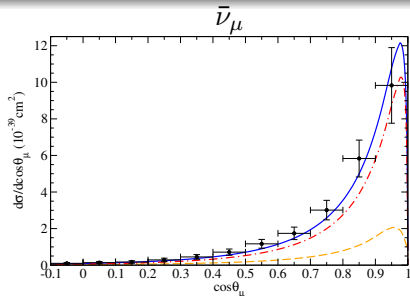
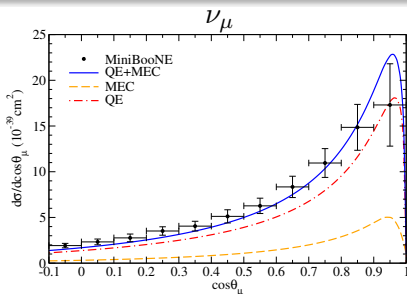
MiniBooNE $\nu_\mu - {}^{12}\text{C}$ double differential cross sections



MiniBooNE $\bar{\nu}_\mu - ^{12}\text{C}$ double differential cross sections



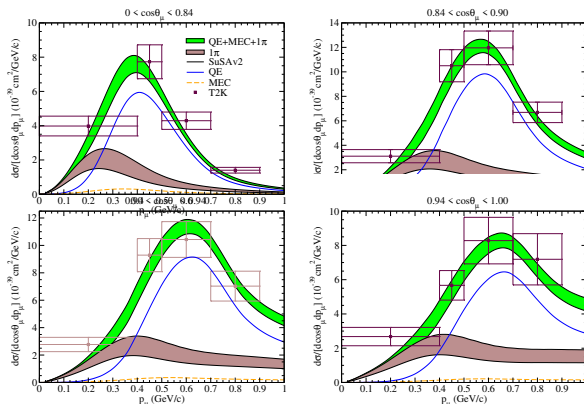
MiniBooNE $\nu_\mu - {}^{12}\text{C}$ single differential cross sections



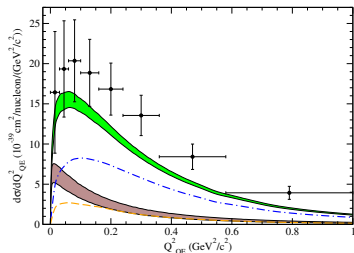
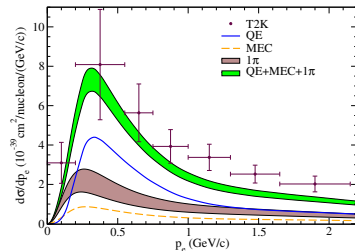
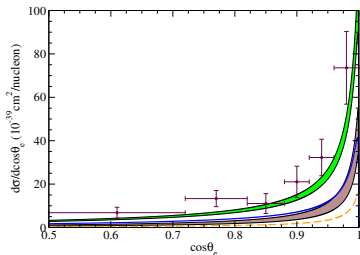
QE+MEC+ Δ contributions in $\nu_\mu - {}^{12}\text{C}$ scatteringAnalysis of T2K ν_μ data ($\langle E_{\nu_\mu} \rangle \sim 0.8$ GeV)

JPG 43, 045101 (2016)

- Deep Inelastic Scattering contributions are not relevant at T2K kinematics.
- Work in progress to include the DIS description \Rightarrow analysis of higher-energy data.



T2K $\nu_e - {}^{12}\text{C}$ cross sections

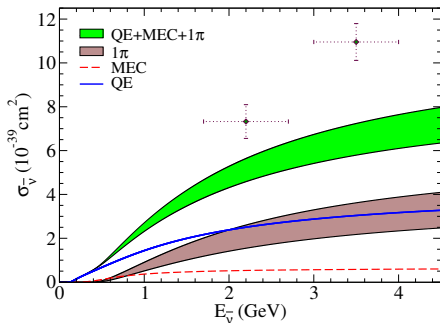
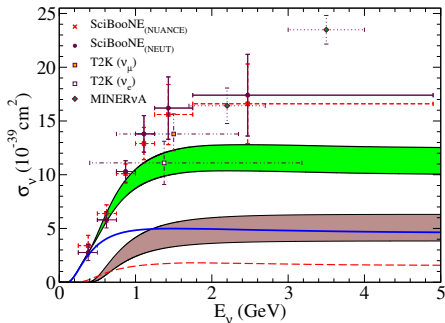


Analysis of T2K ν_e data ($\langle E_{\nu_e} \rangle \sim 1.3$ GeV)
J.Phys.G 43, 045101 (2016)

➡ Agreement with data is slightly worse as for
 $E_\nu \gtrsim 1$ GeV DIS starts to be relevant.

Inclusive total cross section \Rightarrow Δ -scaling model

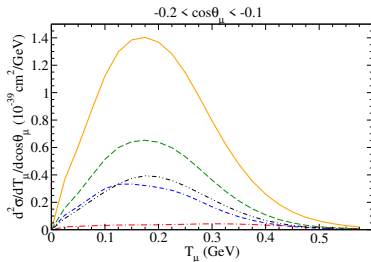
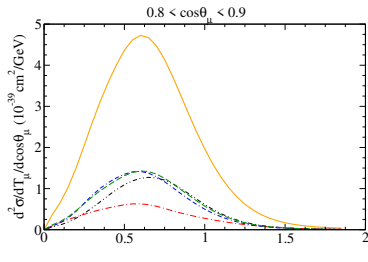
Extension of the SuSA into the non-QE region assuming Δ -resonance dominance [*J.Phys.G* 43, 045101 (2016)]. Substraction of the QE + 2p-2h MEC contributions from the total CS.



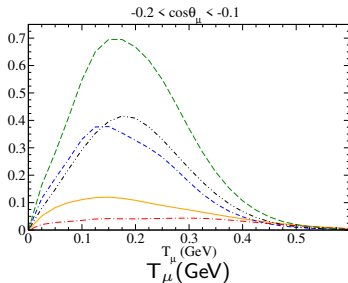
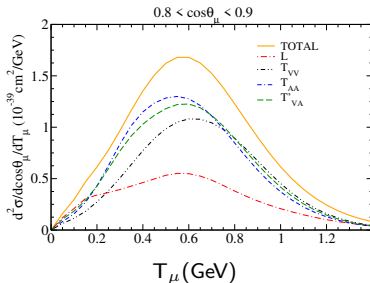
QE+MEC+ Δ contributions are not enough to describe inclusive cross section at $E_\nu \gtrsim 1$ GeV \Rightarrow Work in progress to include DIS in the ν interaction model.

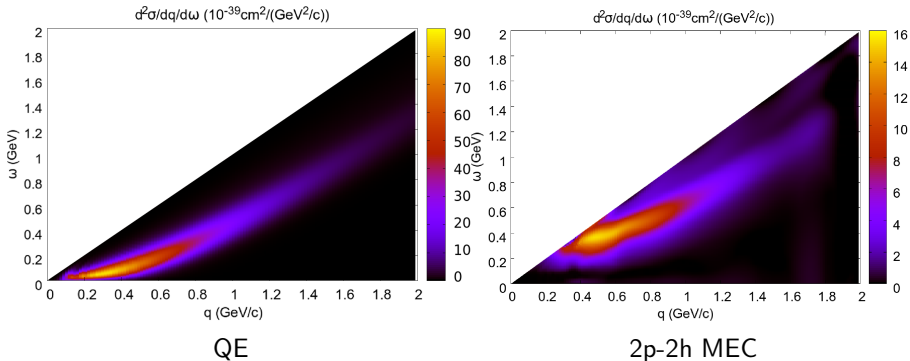
2p-2h MEC channels at MiniBooNE kinematics

$\nu_\mu \Rightarrow$



$\bar{\nu}_\mu \Rightarrow$



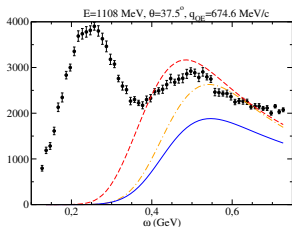
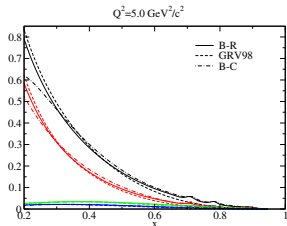
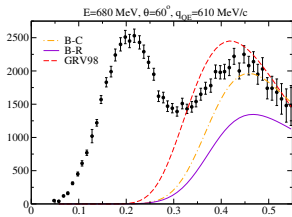
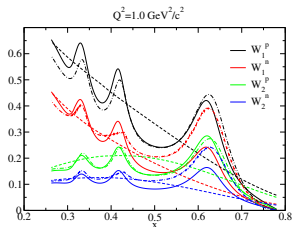
Relevant kinematic regions at $E_\nu = 3$ GeV

Although very similar to the QE case, the relevance of 2p-2h MEC contributions extends slightly to higher kinematics.

Inelastic Nuclear Responses & SuSAv2-inelastic model

Inelastic structure functions

Inclusive $^{12}\text{C}(e, e')$ double differential cross section

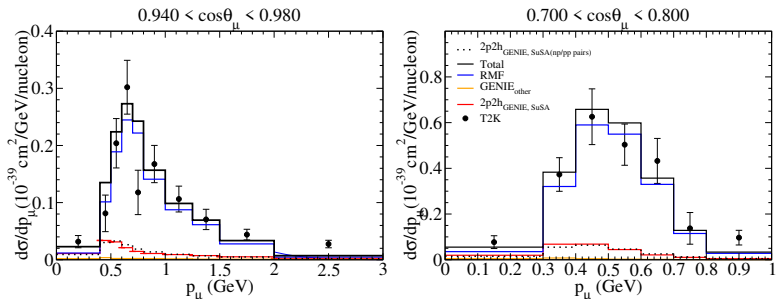


Bodek-Ritchie: poor description of the resonance region.

Bosted-Christy: Good description of the resonant structures observed in (e, e') reactions.

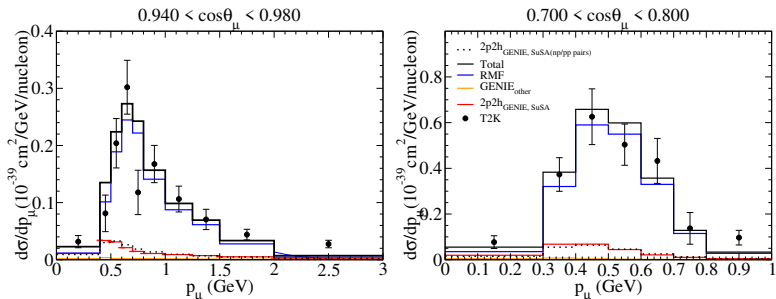
GRV98: No resonant structures (average) and poor description at $Q^2 \lesssim 1 \text{ GeV}^2$.

Characterization of nuclear effects at T2K experiment (PRELIMINARY)

T2K CC0 π Np data on ^{12}C with $0p$ above 500 MeV/c

- **Next step:** Analysis of **transverse variables** (T2K, MINER ν A) within the SuSAv2-MEC model.
- **Transverse variables:** more sensitive to different nuclear effects and useful to disentangle initial state (initial momentum distribution, in medium modifications) from final state (rescattering) effects.

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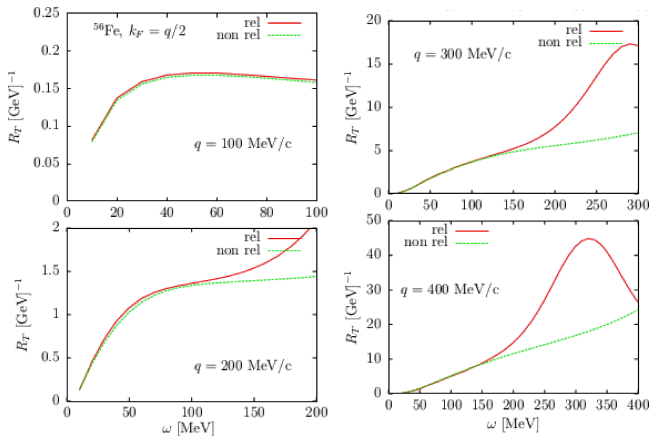
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➤ **Transverse variables:** more sensitive to different nuclear effects and useful to disentangle initial state (initial momentum distribution, in medium modifications) from final state (rescattering) effects.

➤ **THE FUTURE IS SEMI-INCLUSIVE** \Rightarrow Less dependency on simulations and deeper analysis of model nuclear effects. **PROBLEM:** Current lack of semi-inclusive models and proper implementation in generators.

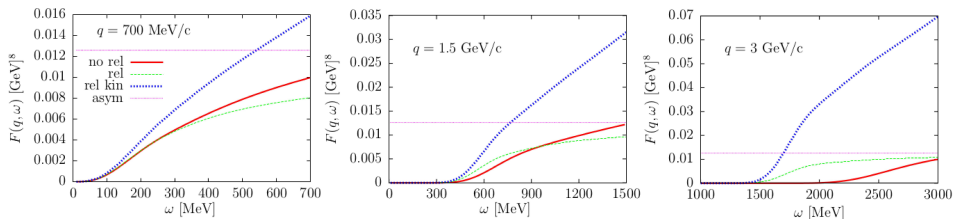
Relativity is essential in 2p2h models

JPG 44, 065105 (2017)



Relativity is essential in 2p2h models

PRD90, 033012 (2014)



★ Effect of implementing relativistic kinematics in a non-relativistic calculation of the phase-space function $F(q, \omega)$ can be delicate and misleading. Differences at high kinematics can be even larger than the ones related to a non-relativistic approach.

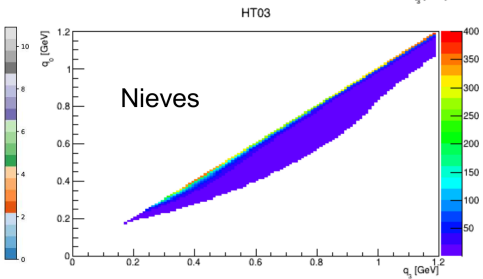
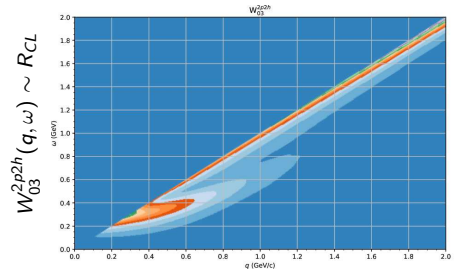
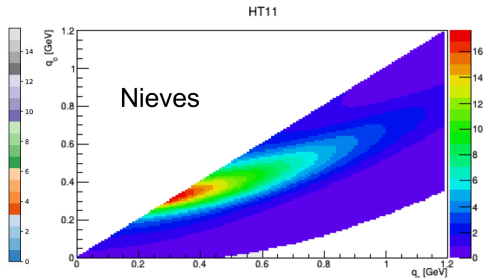
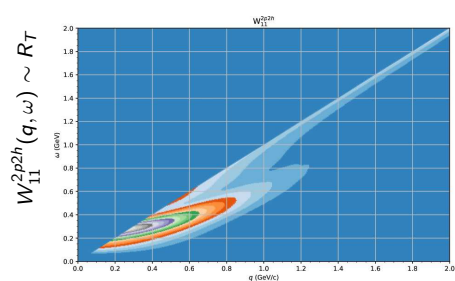
★ All 2p-2h nuclear models should “agree” at the level of $F(q, \omega)$. Good starting point for model comparison.

2p-2h MEC hadronic tensor ($W_{2p-2h}^{\mu\nu}$) and elementary hadronic tensor ($r_{2p-2h}^{\mu\nu}$) in the RFG model

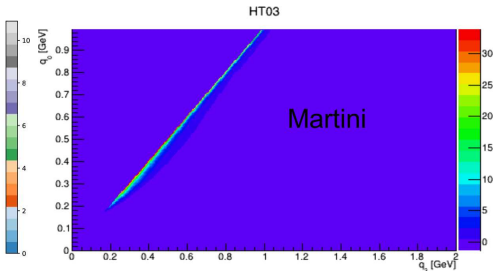
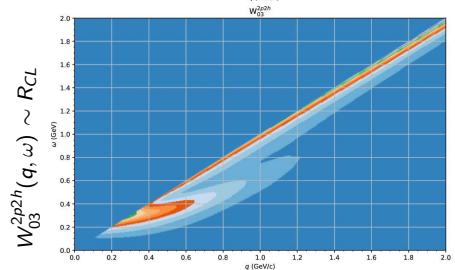
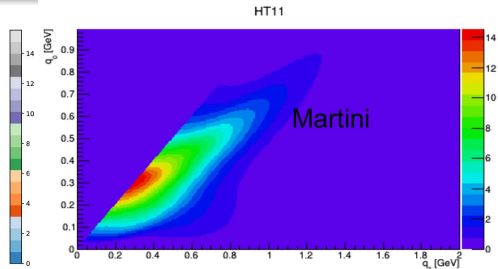
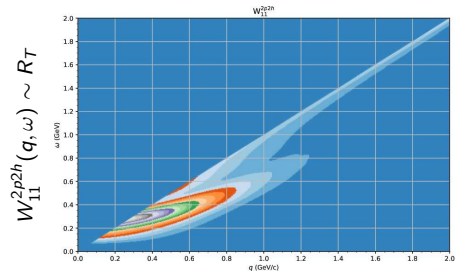
$$W_{2p-2h}^{\mu\nu} = \frac{V}{(2\pi)^9} \int d^3 p'_1 d^3 h_1 d^3 h_2 \frac{M^4}{E_1 E_2 E'_1 E'_2} \Theta(p'_1, p'_2, h_1, h_2) r^{\mu\nu}(p'_1, p'_2, h_1, h_2) \delta(E'_1 + E'_2 - E_1 - E_2 - \omega)$$

$$F(q, \omega) = \int d^3 p'_1 d^3 h_1 d^3 h_2 \frac{M^4}{E_1 E_2 E'_1 E'_2} \Theta(p'_1, p'_2, h_1, h_2) \delta(E'_1 + E'_2 - E_1 - E_2 - \omega)$$

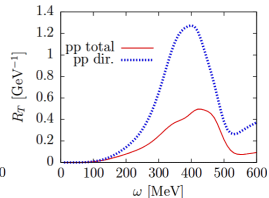
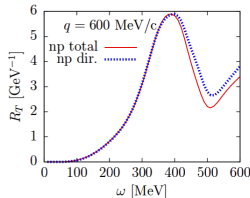
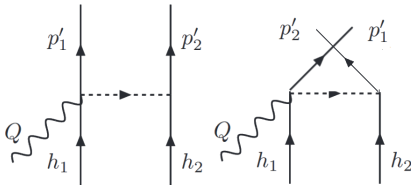
Comparison with other models implemented in GENIE



Comparison with other models implemented in GENIE



Relevance of direct/exchange interference in np/pp ratio



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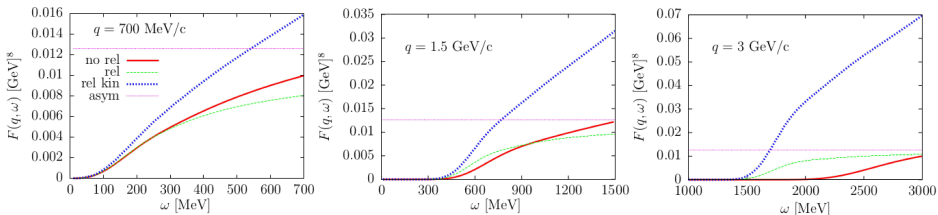
Other Fermi Gas based 2p2h models

Martini model: Based on a non-relativistic treatment of MEC and correlations with relativistic corrections added and axial 2p2h estimated from vector one.

Nieves model: Relativistic with some approximations to compute the momentum-space integrals.

Both models neglect direct/exchange interference terms \Rightarrow strongly affects np/pp ratio by a factor ~ 2 (PRC94:054610, 2016). **Relevant implications in nucleon multiplicity and hadronic**

E_{reco}

2p-2h MEC for (e, e') and CC ν reactions PRD91, 073004 (2015)

✦ The 2p-2h model is based on the calculation performed by De Pace et al., (2003) for (e, e') scattering and extended to the weak sector by Amaro, Ruiz Simo et al. [PRD 90, 033012 (2014); PRD 90, 053010 (2014); JPG 44, 065105 (2017); PLB 762, 124 (2016)].

✦ The numerical evaluation of the hadronic tensor $W_{2p2h}^{\mu\nu}$ is performed in the RFG model in a fully relativistic way without any approximation.

SuperScaling Approach (SuSA)

(see [G.D. Megias' Thesis](#) for details)

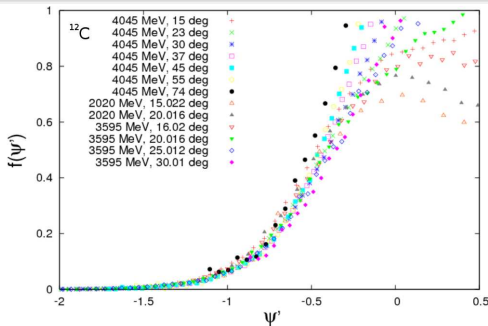
► The analysis of the large amount of existing (e, e') data at different kinematics is a solid benchmark to **test** the validity of theoretical models for neutrino reactions as well as to study the nuclear dynamics. The **SuperScaling Approach** exploits **universal features** of lepton-nucleus scattering to connect the two processes.

In inclusive QE scattering we can observe:

- ☆ Scaling of 1st kind (independence on q)
- ☆ Scaling of 2nd kind (independence on Z)

⇒

SuperScaling



$$f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega} \right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

Good superscaling behavior at $\psi' < 0$ (below QE peak). At higher kinematics (ψ'), other contributions beyond QE and IA (2p2h, Δ , etc.) can play an important role and scaling is broken.

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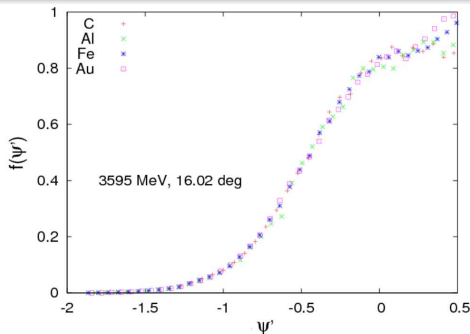
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