Neutrino Cross section

‘12th International Neutrino Summer School 2019’
Fermilab, August 13, 2019

Noemi Rocco

Argonne NATIONAL LABORATORY

Fermilab

Introduction

? Which are the topics that have been in the previous lectures?

- Neutrinos are extremely elusive particles, many open questions: how they oscillate, what is their mass hierarchy…
- Neutrinos can be produced by different sources, different energies and very different physics
- Neutrinos can not be directly measured: we study the interactions that take place in the detectors and “extract” neutrino properties

! Understanding neutrino interactions means understanding their cross sections

- What is a cross section?
- What did we learn from electron scattering that can be used to better understand neutrinos?
- How do we compute a cross section and compare with the data collected by accelerator-based neutrino experiments?

Disclaimer: We will discuss the problem mainly from a theoretical perspective.

Practical Info

- I will do a couple of calculations using the black-board. If you prefer not to write, the notes will be posted on the website

- Some of the Figures have been taken from papers published in peer reviews. The references are reported as:

  ✐ Authors, Journal's name and # of the paper

  If there is anything you find interesting, I strongly encourage you to download the paper and read it!

- I also included some suggestions for more 'pedagogical' readings. The references are indicated as

Author, Title of the Book/Journal

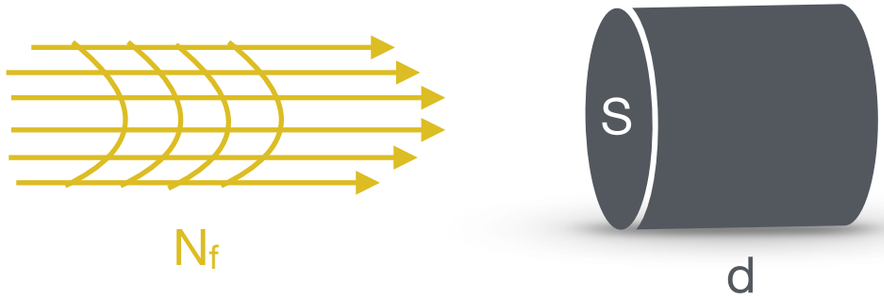
- Please, ask question! Now or later: nrocco@anl.gov

What is a cross section?

- Definition: The Cross Section is a measure for the probability of a process to happen

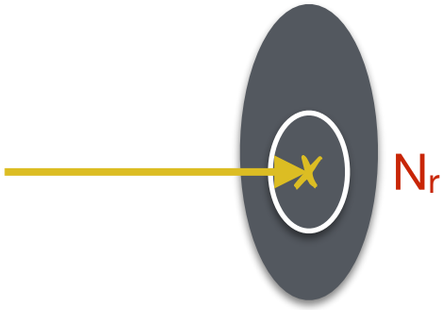
? How does this probability translate into visible scatterings? To answer this question let's try to connect the cross section σ to the event rate

Fixed-target experiment:



- We define: $n_b \cdot d$, number of targets per surface unit

- Beam (composed of N_f particles) hits a target (of width d): and we want to measure the (# of reactions) dNr/ dt



$$\frac{dN_r}{dt} = \sigma_r \cdot \frac{dN_f}{dt} \cdot n_b \cdot d \xrightarrow[\text{Integrating over t}]{} \sigma_r = \frac{N_r}{N_f} \frac{1}{n_b \cdot d}$$

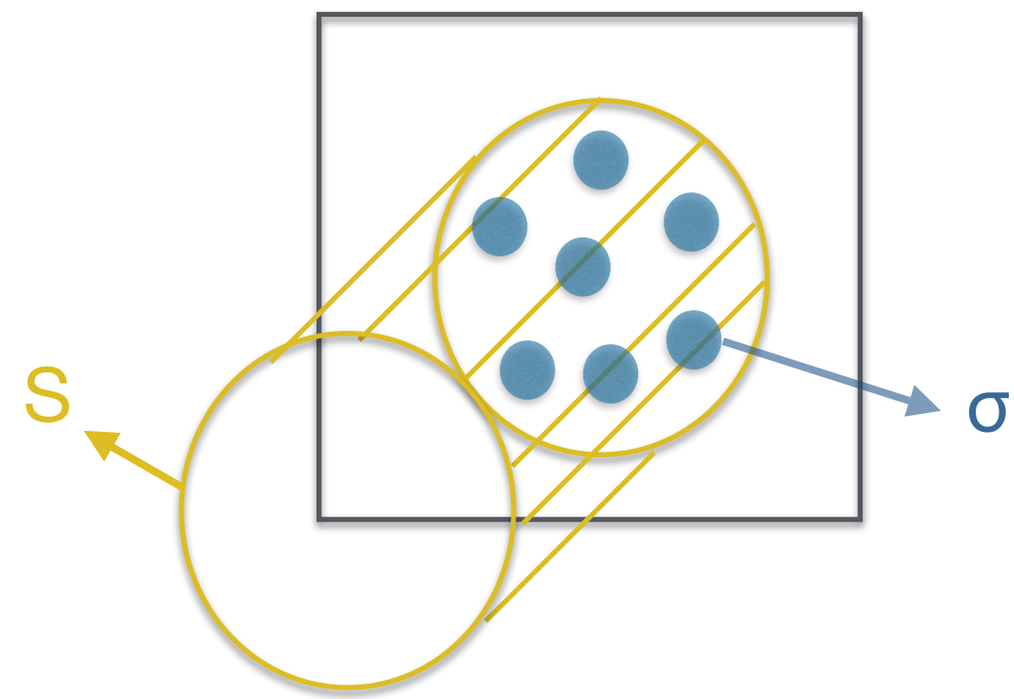
- The unit of measure of σ is: 1 barn = 1 b = 10^{-28} m^2

Geometrical interpretation of the cross section

? Why does a probability have units of area

- We rewrite: $n_b \cdot d = N_b / S$

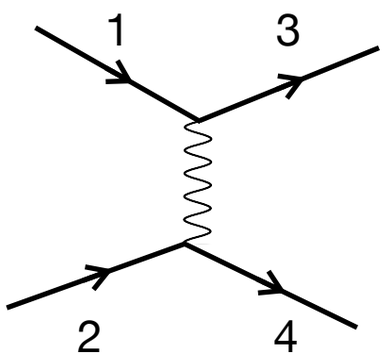
- We observe that N_r / N_f is the probability that a beam particle hits the target



- σ can be interpreted as the effective surface area of a target, S is the surface area of the beam hitting the target

$$\frac{N_r}{N_f} = \frac{\sigma_r}{S} N_b \quad \longleftrightarrow \quad \text{Probability of interaction} = \frac{\text{Area of the target} \times N_b}{\text{Area of the beam}}$$

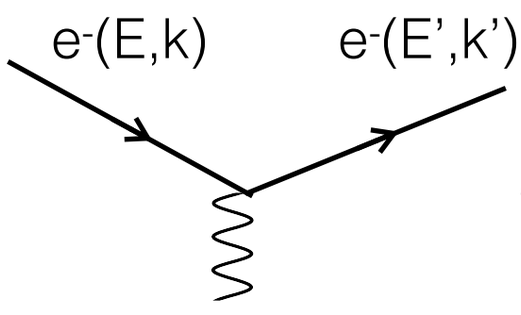
How do we compute a cross section?



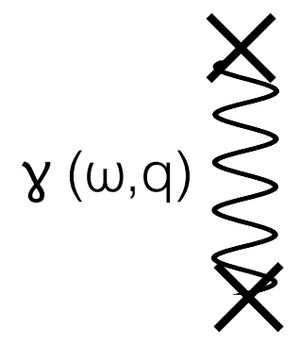
- We start from a generic process: 1+2 ➜ 3+4
- The cross section can be written as

$$d\sigma = \frac{1}{\text{flux}} \frac{1}{2E_1} \frac{1}{2E_2} |\mathcal{A}|^2 \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)$$

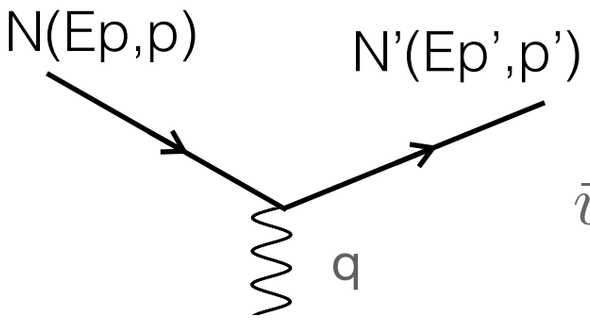
F. Close, An Introduction to quark and partons



$$\bar{u}(\mathbf{k}', \lambda')\gamma^\mu u(\mathbf{k}, \lambda)$$

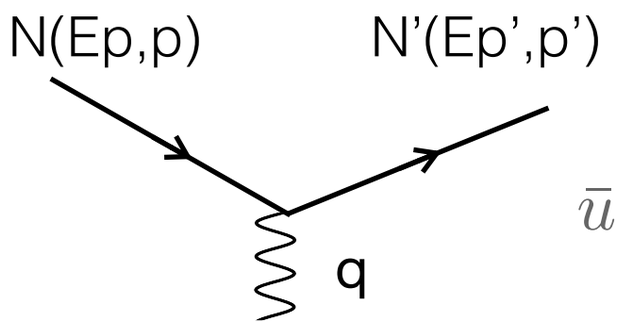


$$\frac{e^2 g^{\mu\nu}}{q^2}$$



$$\bar{u}(\mathbf{p}', \alpha')\Gamma^\mu(q)u(\mathbf{p}, \alpha) = \bar{u}(\mathbf{p}', \alpha')\left[F_1\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2\right]u(\mathbf{p}, \alpha)$$

How do we compute a cross section?



$$\bar{u}(\mathbf{p}',\alpha')\Gamma^{\mu}(q)u(\mathbf{p},\alpha) = \bar{u}(\mathbf{p}',\alpha')\left[F_1\gamma^{\mu} + i\frac{\sigma^{\mu\nu}q_{\nu}}{2M}F_2\right]u(\mathbf{p},\alpha)$$

- The Dirac and Pauli form factors are corrections to "point-like coupling" which comes from the fact that the nucleon has an internal structure

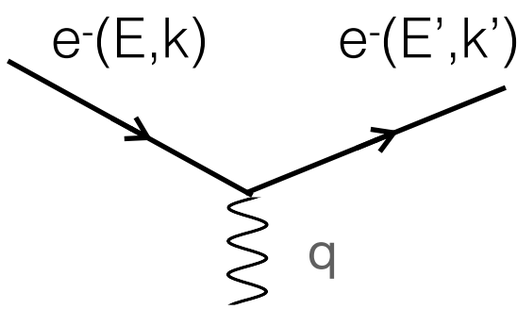
- Alternatively, F_1 and F_2 are written as a combination of the electric and magnetic form factors:

$$F_1^{p,n} = \frac{G_E^{p,n} - q^2/(4M^2)G_M^{p,n}}{1 - q^2/(4M^2)} \qquad F_2^{p,n} = \frac{G_M^{p,n} - G_E^{p,n}}{1 - q^2/(4M^2)}$$

- The form factors are related to the spatial distributions of the charge and magnetization in the proton, and in the non relativistic limit are simply the Fourier transforms of these distributions.

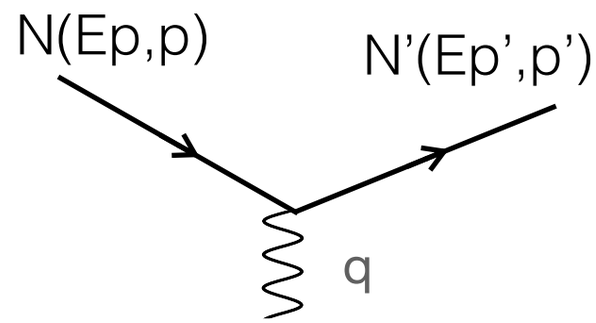
- The accurate determination of G_E and G_M is an important focus of both experimental and theory programs

How do we compute a cross section?

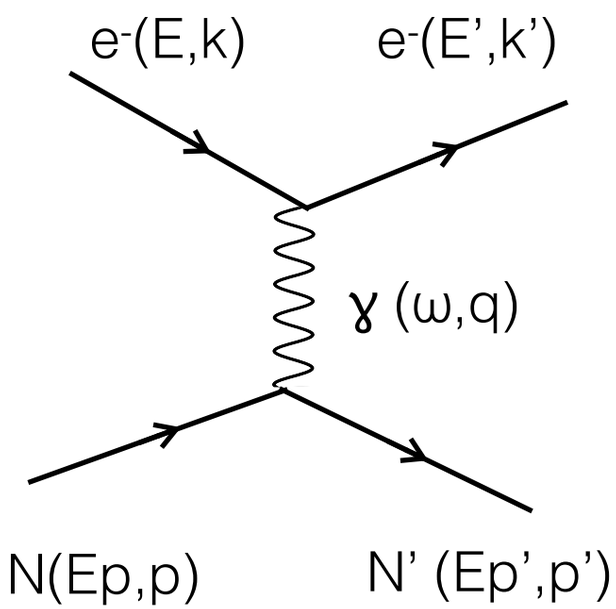


$$\bar{u}(\mathbf{k}', \lambda')\gamma^\mu u(\mathbf{k}, \lambda)$$

$$\frac{e^2\, g^{\mu\nu}}{q^2}$$



$$\bar{u}(\mathbf{p}', \alpha')\Gamma^\mu(q)u(\mathbf{p}, \alpha)$$



- We can put together what we learned and rewrite the squared amplitude as:

$$|\mathcal{A}|^2 = \frac{1}{4}\sum_{\lambda,\lambda',\alpha,\alpha'}\left|\bar{u}(\mathbf{k}', \lambda')\gamma^\mu u(\mathbf{k}, \lambda)\frac{e^2}{q^2}\bar{u}(\mathbf{p}', \alpha')\Gamma_\mu u(\mathbf{p}, \alpha)\right|^2$$

- The cross section per unit of energy of the scattered electron and solid angle

$$\frac{d\sigma}{d\Omega dE'} = \frac{1}{\text{flux}}\frac{\alpha^2}{q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}$$

- How can we define L and W? Let's write it up on the blackboard !!

Summary of electron-nucleon scattering

- We consider the process:

$$\ell^-(k) + N(p) \to \ell^-(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E_k^2 \sin^2\frac{\theta}{2}} \frac{E_{k'}}{E_k} \cos^2\frac{\theta}{2}$$

← Scattering on a point-like spinless target

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

← Scattering on a point-like 1/2 spin target

- Protons and neutrons have an internal structure: described by electric and magnetic form factors

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2 \tan^2\frac{\theta}{2}\right]$$

Rosenbluth separation

Determination of the nucleon form factors

✐ Qattan et al.,PRL 94, 142301 (2005)

- A reduced cross section can be defined as

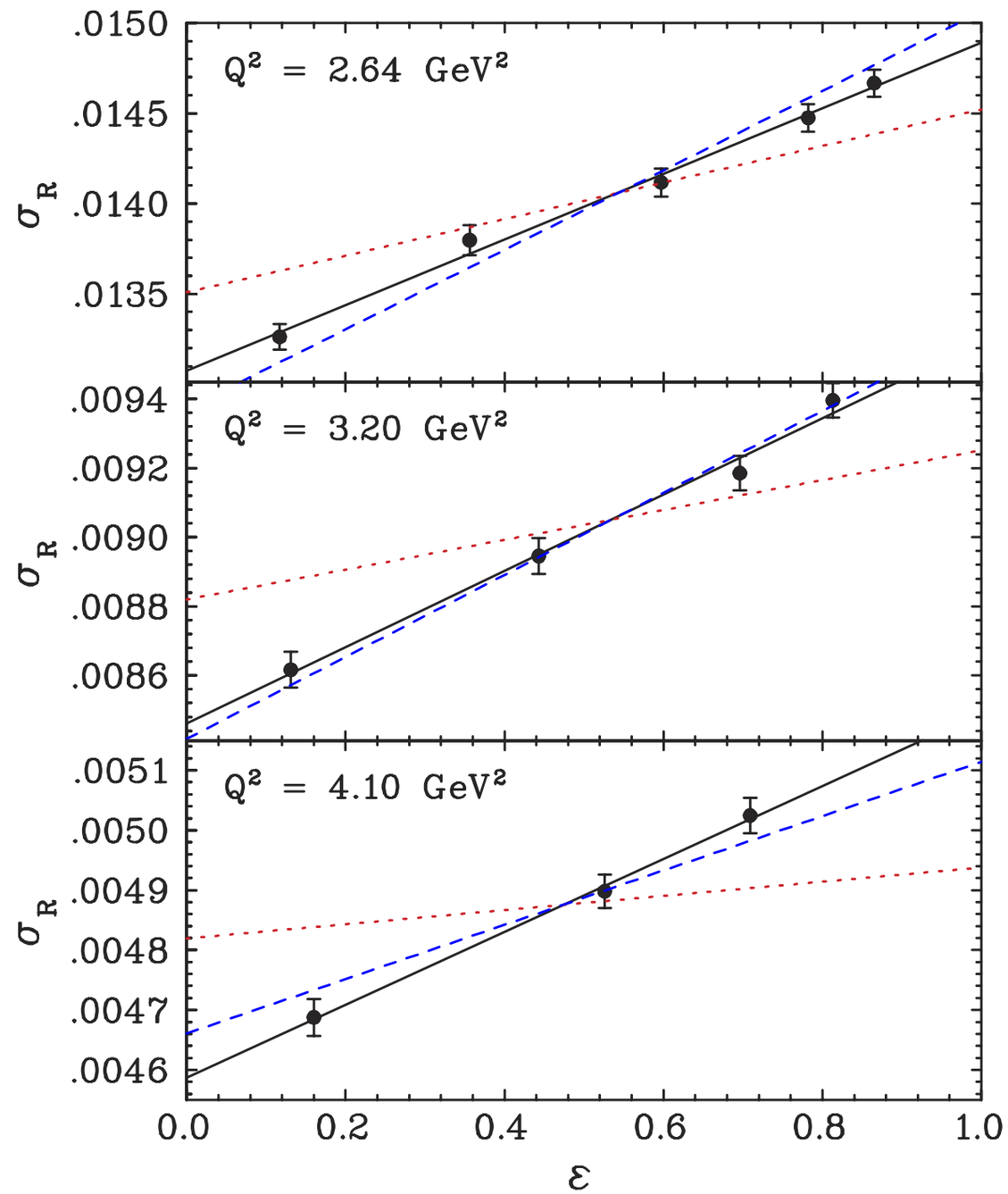
$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \times \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}$$

- The virtual photon parametrization parameter is

$$\epsilon = \left[1 + 2(\tau+1)\tan^2\frac{\theta}{2}\right]^1$$

- Measuring angular dependence of the cross section at fixed Q^2

$$\sigma_R = \epsilon(1+\tau)\frac{\sigma}{\sigma_{\text{Mott}}} = \epsilon G_E^2 + \tau G_M^2$$



··· polarization transfer experiment

- - - previous Rosenbluth

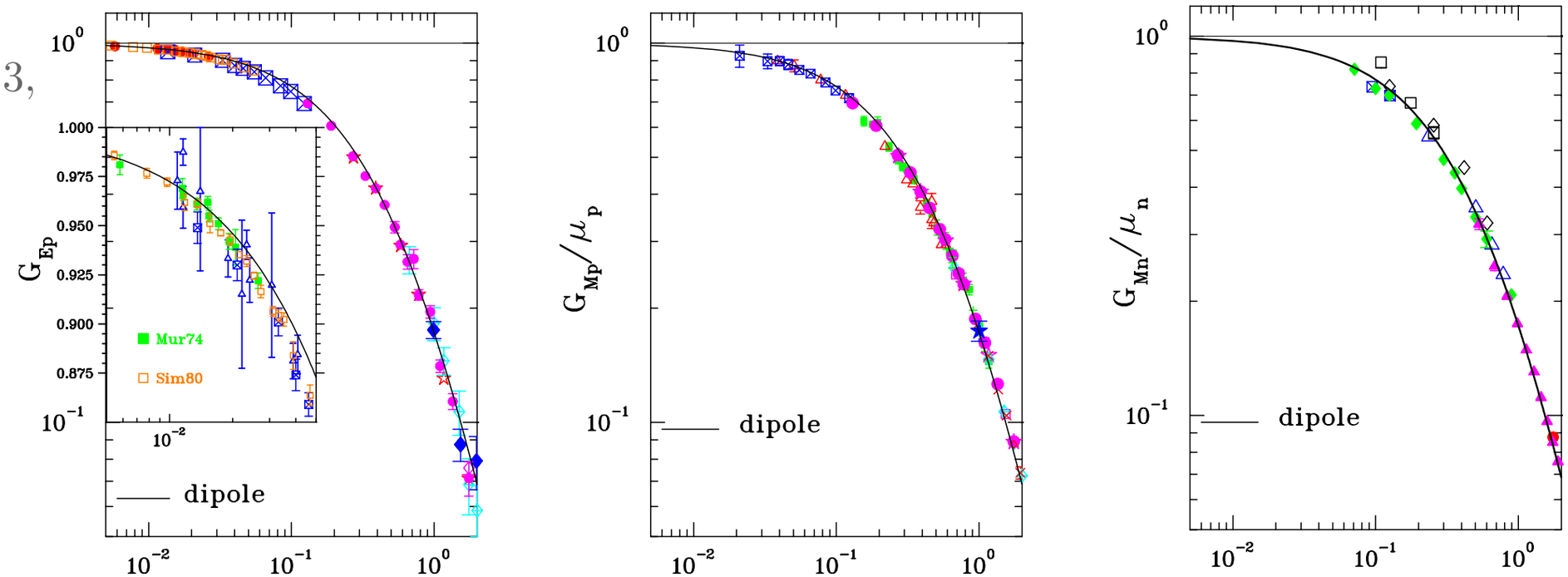
- In Born approximation: G_E^2 is the slope and the intercept is τG_M^2

Determination of the nucleon form factors

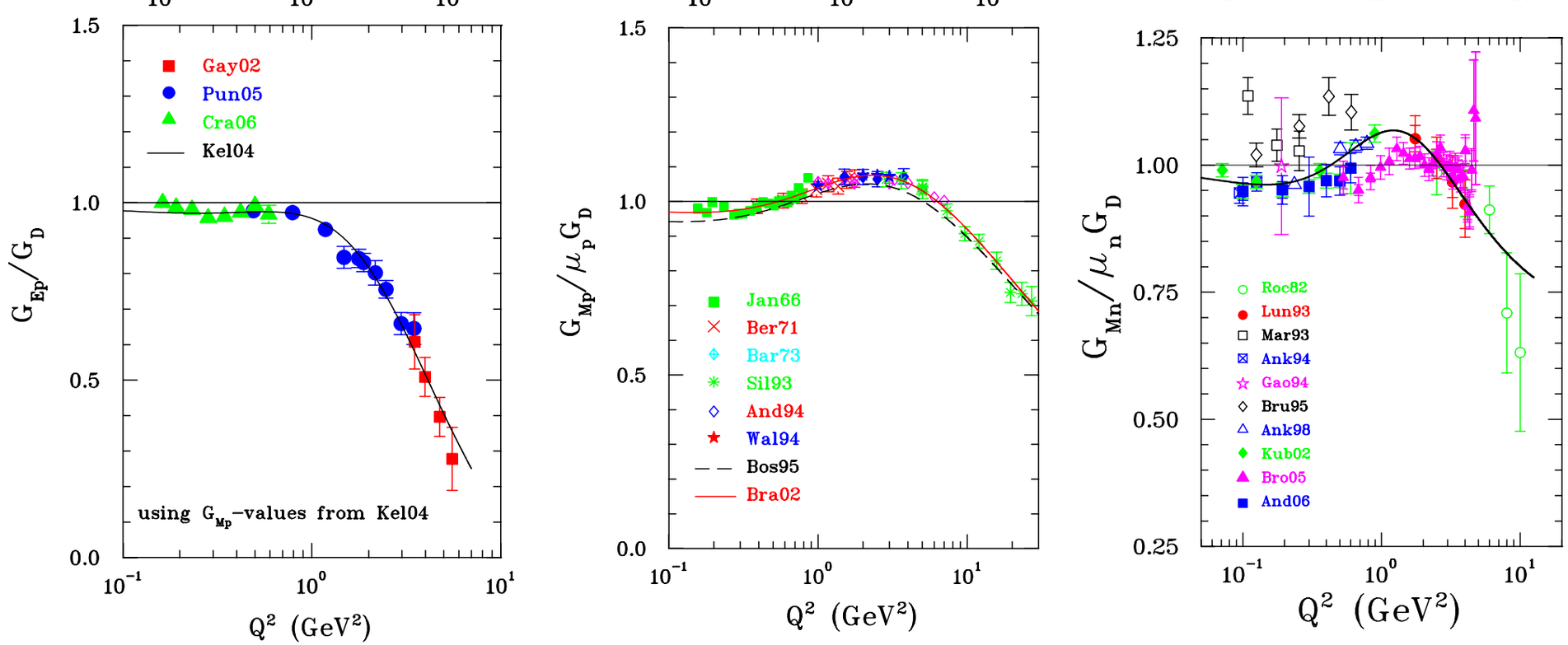
$$G_E^p(q^2) = G_D(q^2)\ ,\ G_M^p(q^2) = \mu_p G_D(q^2)\ ,\ G_M^n(q^2) = \mu_n G_D(q^2)\ ,\ G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$\mu_p = 2.793,\ \mu_n = -1.913,$
$M_V^2 = 0.71\ \mathrm{GeV}^2$

✐ Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694–764



Scaling did not survive to the emergence of double polarization results $Q^2 > 1\ \mathrm{GeV}^2$



Lattice QCD in a Nutshell

- Lattice QCD is a technique in which space-time is discretized into a four-dimensional grid
- The QCD path integral over the quark and gluon fields at each point in the grid is performed in Euclidean space-time using Monte Carlo methods
- The quark mass is a parameter of the calculation : m_π
- Space time is discretized, step a: UV cutoff
- Finite volume L : infrared cutoff
- The physical limit is recovered by imposing that

$$m_\pi \to m_\pi^{phys}, \; a \to 0, \; L \to \infty$$

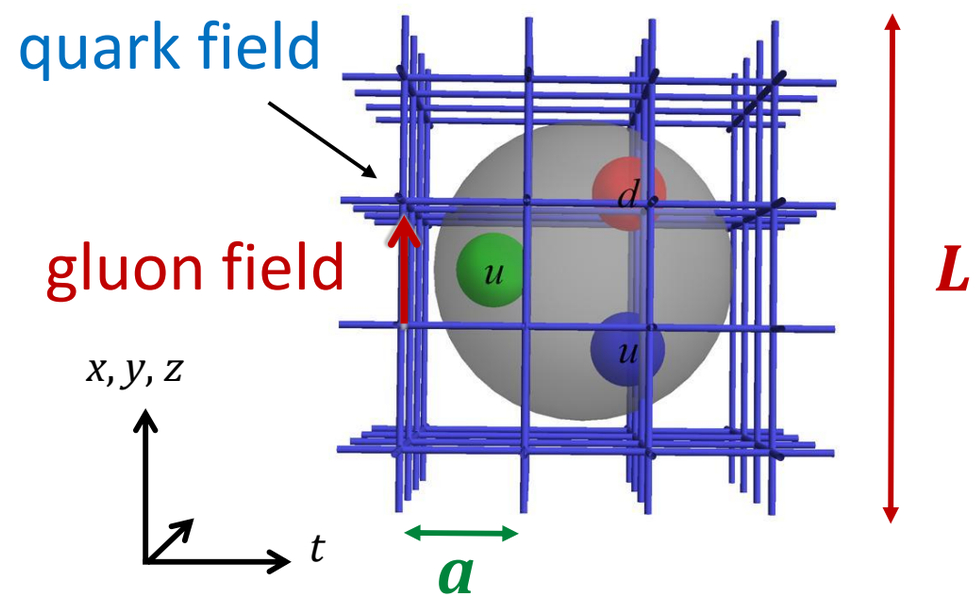
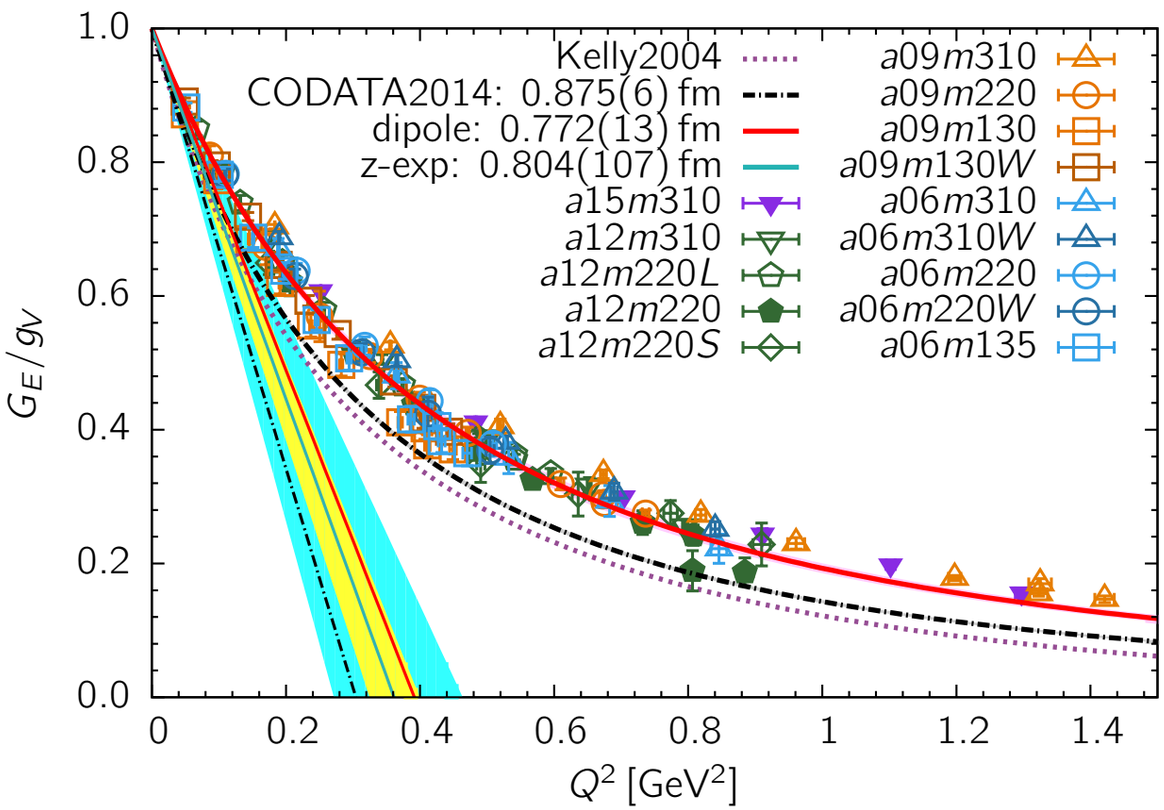
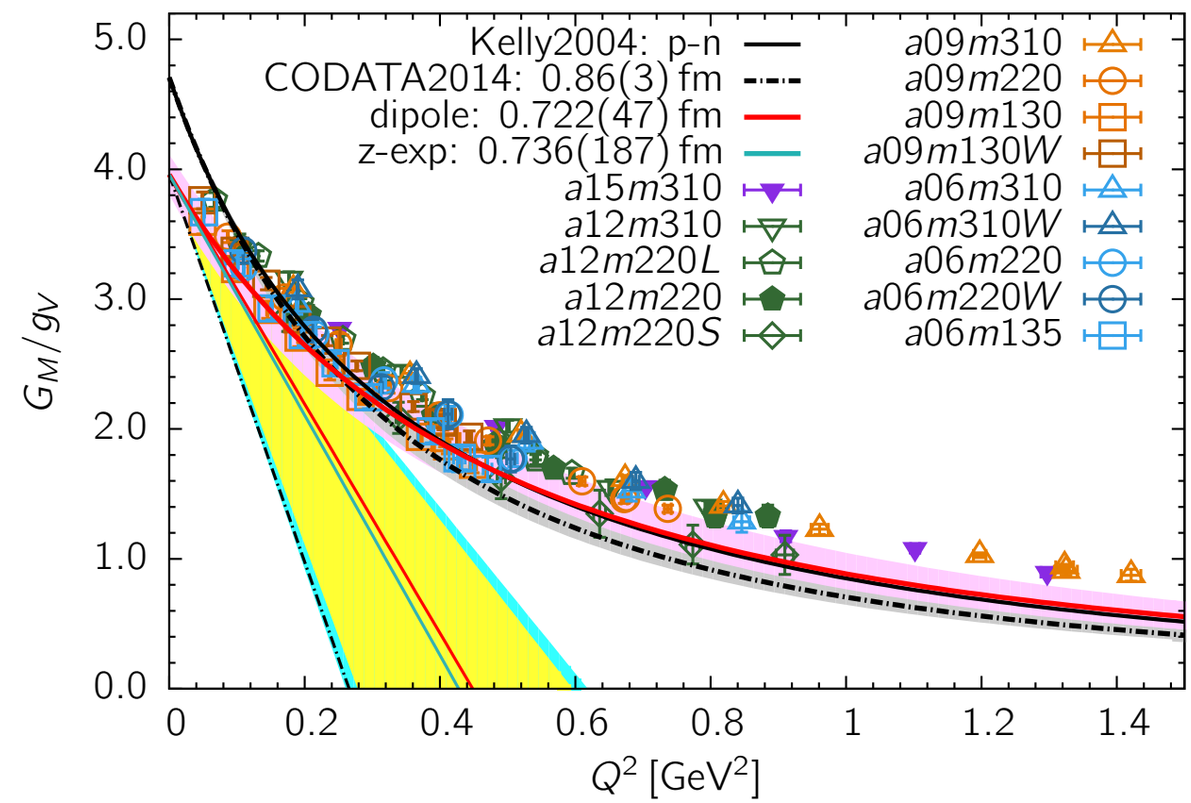


Figure by Huey-Wen Lin

Lattice QCD Form Factor





✐ Y-C Jang et al,PoS LATTICE2018 (2018) 123

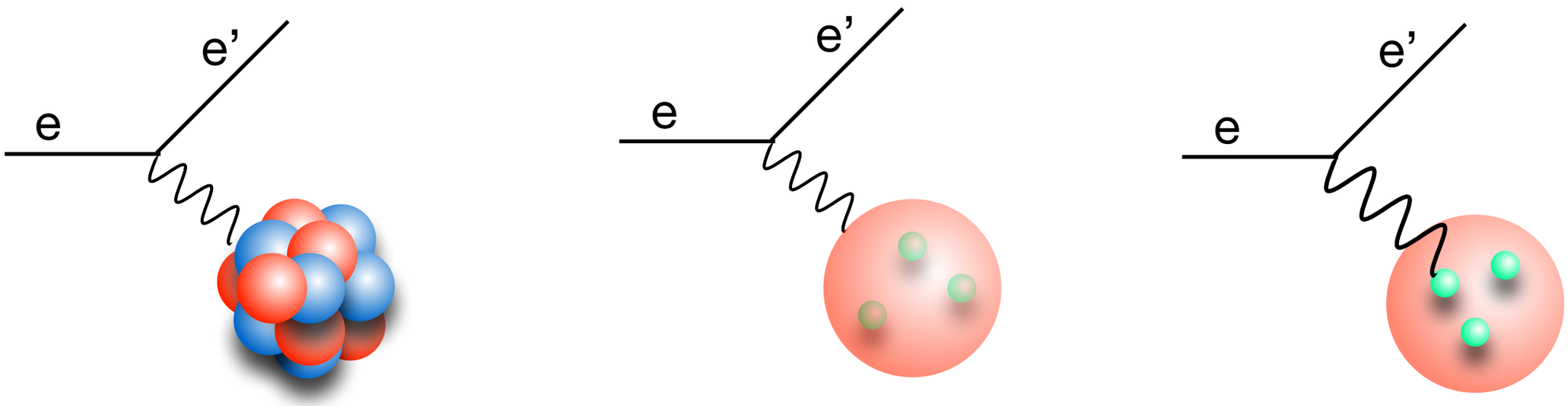
- Isovector electric and magnetic form factors
- The data cover the range 0.057 fm < a < 0.15 fm, 135 MeV < m_π < 320 MeV and 3.3 < $m_\pi L$ < 5.5
- The straight line corresponds to the slope fit at Q2=0

R. Gupta, Introduction to Lattice QCD, arXiv:hep-lat/9807028

Electron scattering: from nuclei to quarks

- The most general expression for the hadron tensor is given by

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle\delta^{(4)}(p_0 + q - p_f)$$



- Nuclear level
- Nucleon level
- Quark level

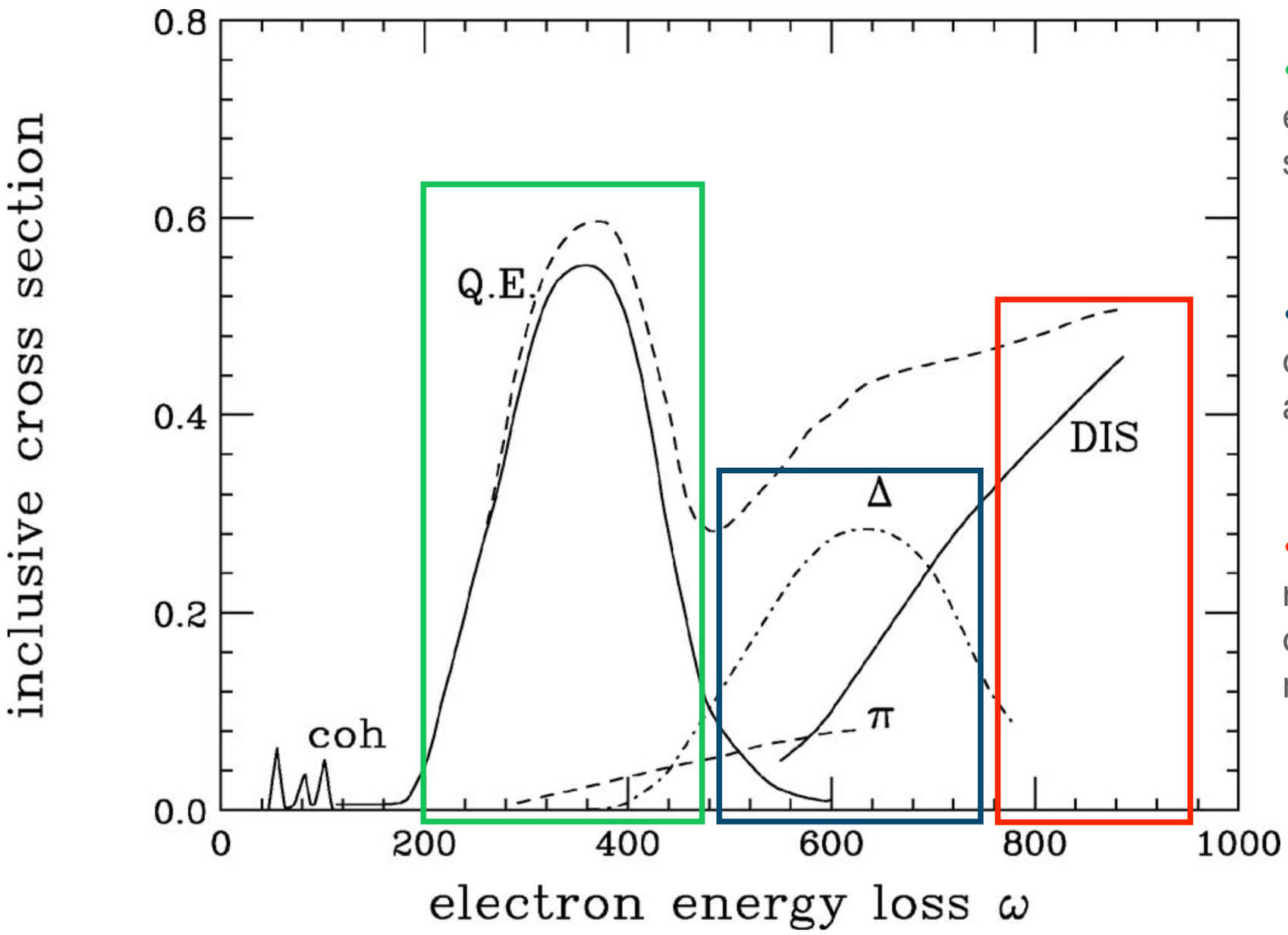
$$d\sigma \propto \frac{\alpha^2}{q^4} L_{\mu\nu} W_{\mu\nu}$$

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k\,k')$$

- Universal structure maintained @ different energy scales

Electron-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.



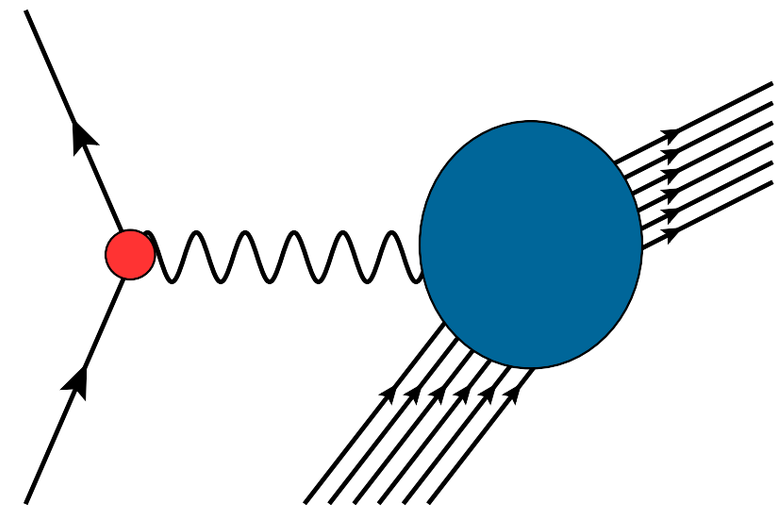
- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.
- Deep Inelastic Scattering region, productions of hadrons other than protons and neutrons

O. Benhar, et al Review of Modern Physics **80**, 189 (2008)

Electron-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus and the hadronic final state is undetected can be written as

$$\frac{d^2\sigma}{d\Omega_\ell dE_{\ell'}} = L_{\mu\nu}W^{\mu\nu}$$



- The Hadronic tensor contains all the information on target response

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle\delta^{(4)}(p_0 + q - p_f)$$

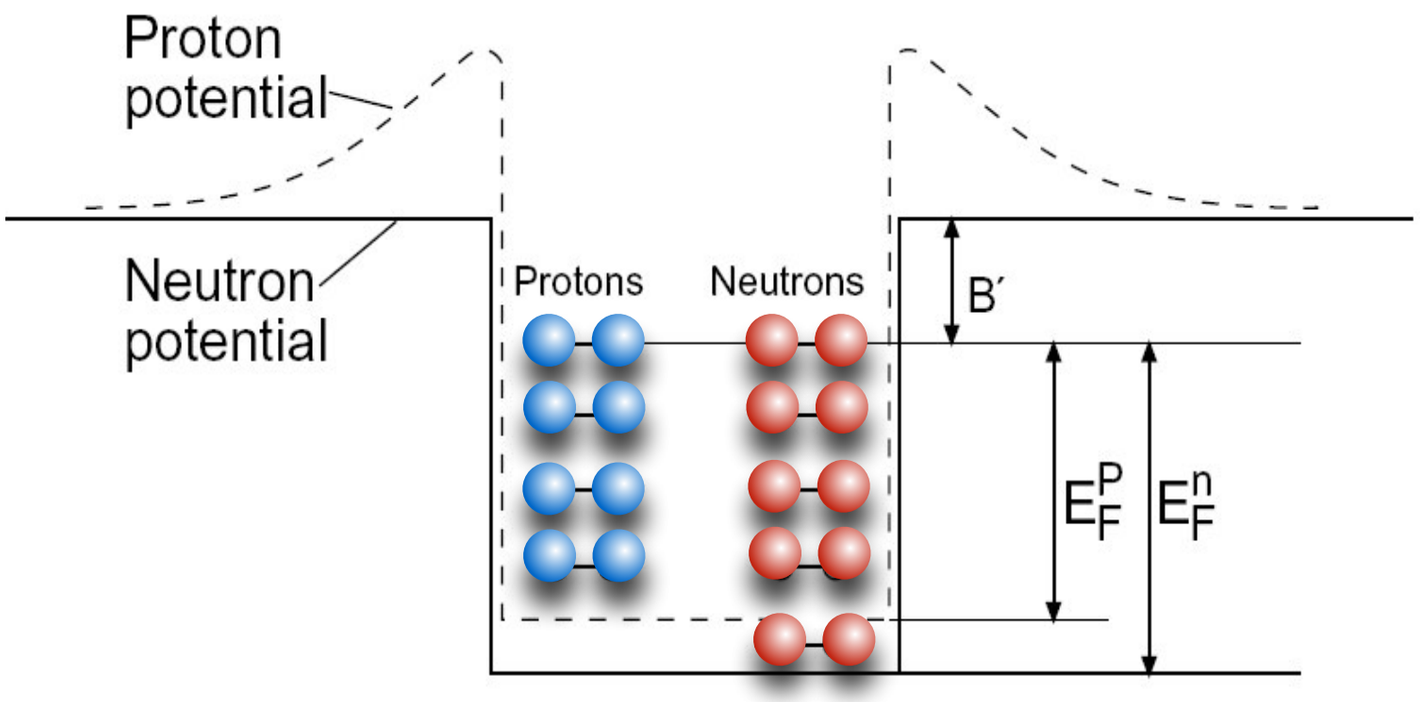
- The initial and final state

$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

- The cross section can be separated in the longitudinal and transverse channel

$$\frac{d\sigma}{dE'd\Omega'} = \sigma_{\text{Mott}}\left[\left(\frac{q^2}{\mathbf{q}^2}\right)^2 R_L + \left(\frac{-q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2}\right)R_T\right] \quad \text{where} \quad \begin{aligned} R_L &= W^{00} \\ R_T &= W^{xx} + W^{yy} \end{aligned}$$

Independent particles: global Fermi gas



- Simple picture of the nucleus: only statistical correlations are retained (Pauli exclusion principle)

- Protons and neutrons are considered as moving freely within the nuclear volume

- The nuclear potential wells are rectangular: constant inside the nucleus and goes sharply to zero at its edge

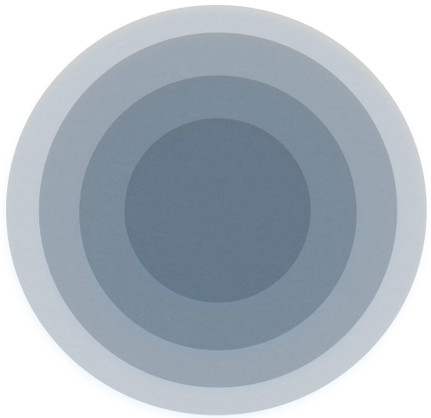
- The energy of the highest occupied state is the Fermi energy: E_F

- The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A ~ 7-8 MeV

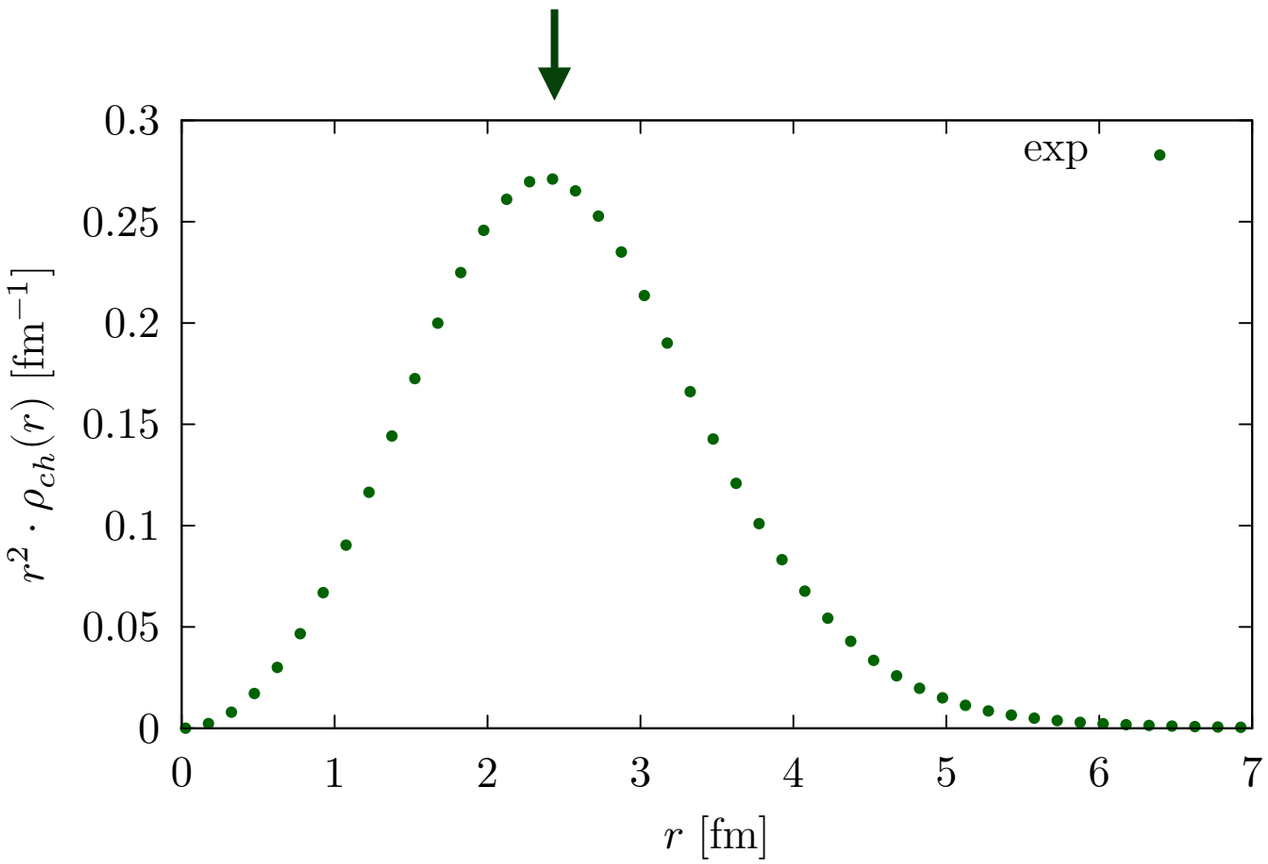
C. Bertulani, Nuclear Physics in a Nutshell

Independent particles: Local Fermi gas

- A spherically symmetric nucleus can be approximated by concentric spheres of a constant density.



More likely to find a particle r ~ r_{ch}~ 2.5 fm



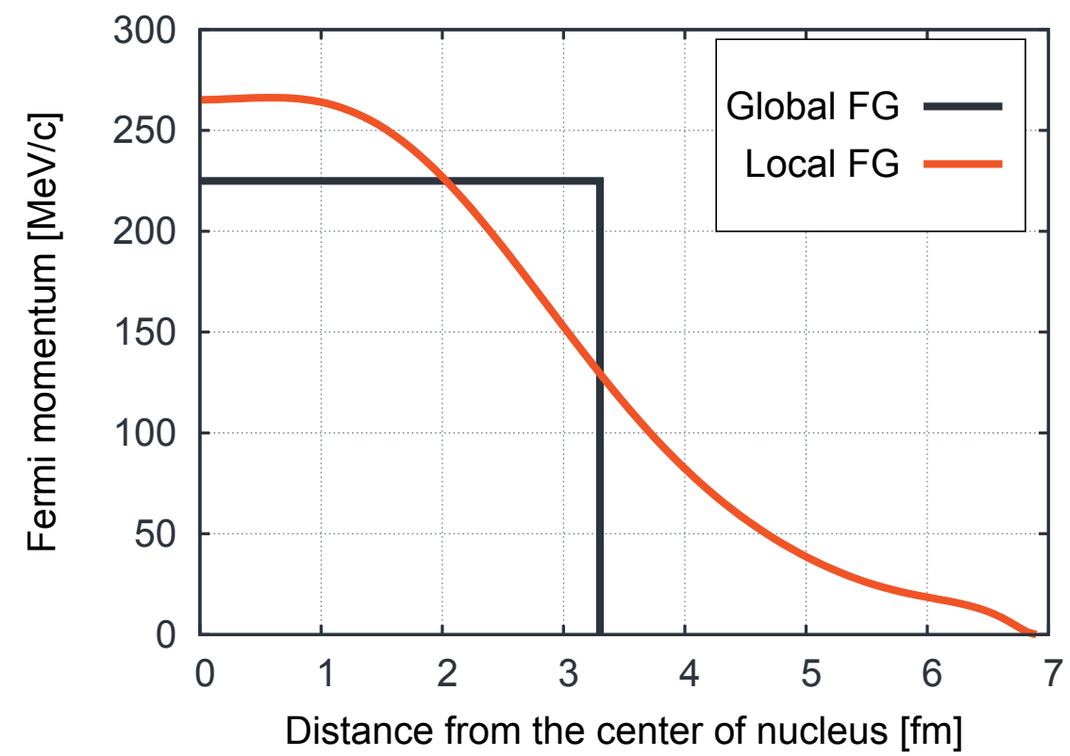


Figure by T. Golan

- **Global Fermi Gas**

$$p_F = \left(\frac{9\pi \cdot n}{4A}\right)^{1/3} \cdot \frac{\hbar}{R_0}$$

- **Local Fermi Gas**

$$p_F = \hbar\left(3\pi^2 \rho(r) \frac{n}{A}\right)^{1/3}$$

Independent particles: Shell Model

- As in the Fermi Gas model: the nucleons move within the nucleus independently of each other
- Difference: the nucleons are not free: subject to a central potential

- Each nucleon moves in an average potential created by the other nucleons, the potential should be chosen to best reproduce the experimental results

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \dots \longrightarrow H = \sum_i \frac{p_i^2}{2m} + \sum_i^A U_i + H_{\text{res}}$$

- We solve the Schrödinger Equation:

$$H\,\psi = E\,\psi \qquad \begin{cases} E = E_1 + E_2 + \dots + E_A \\ \psi(1,\dots,A) = \mathcal{A}[\phi_1(1)\dots\phi_A(A)] \end{cases}$$

Independent particles: Shell Model

- Example: Particles are subject to an harmonic oscillator potential

$$U(r) = \frac{1}{2} m \omega^2 r^2$$

The frequency should be adapted to the mass number A

- We will seek solutions of the type $\psi(r) = \frac{u(r)}{r} Y_l^m(\theta, \phi)$ ← Spherical Harmonics

- Solving the Schrödinger Equation reduces to a solution of u:

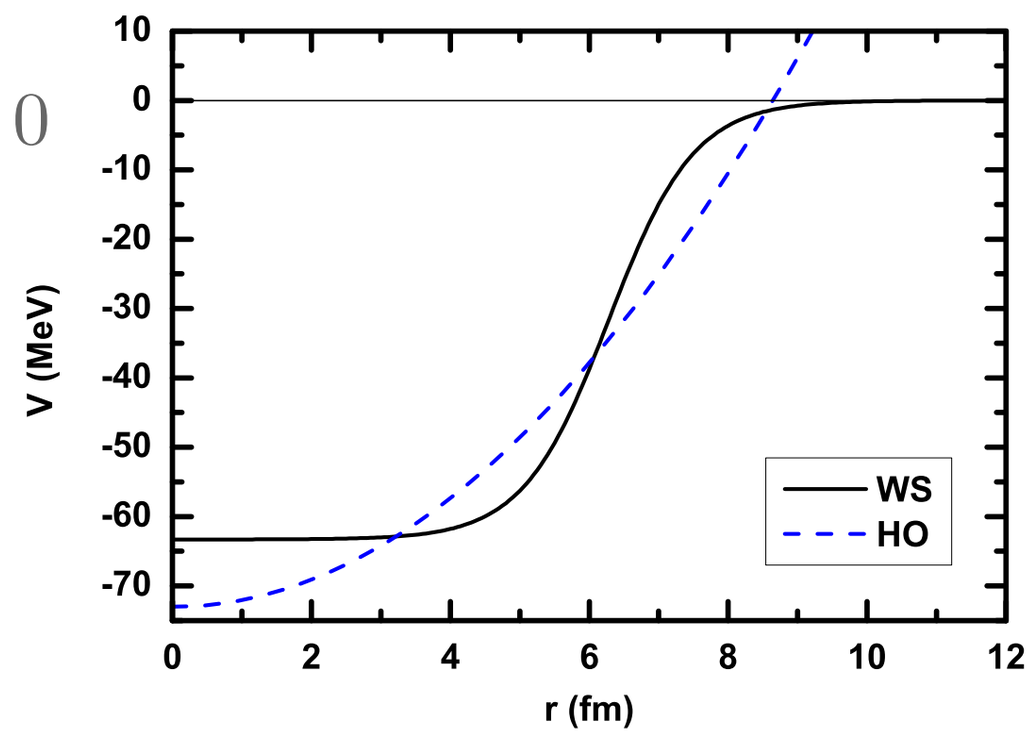
$$\frac{d^2u}{dr^2} + \left\{ \frac{2m}{\hbar^2}[E - U(r)] - \frac{l(l+1)}{r^2} \right\} u(r) = 0$$

$$E_{nl} = \hbar\omega\left(2n + l - \frac{1}{2}\right)$$ Eigenvalues

- A more realistic potential is the Wood Saxon:

$$V = V_0/[1 + exp[(r - R)/a]$$

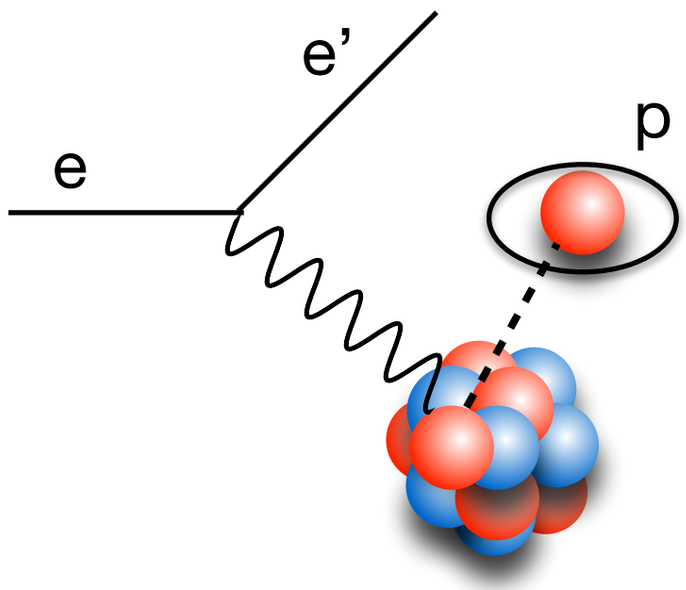
V_0, r, a, are adjustable parameters



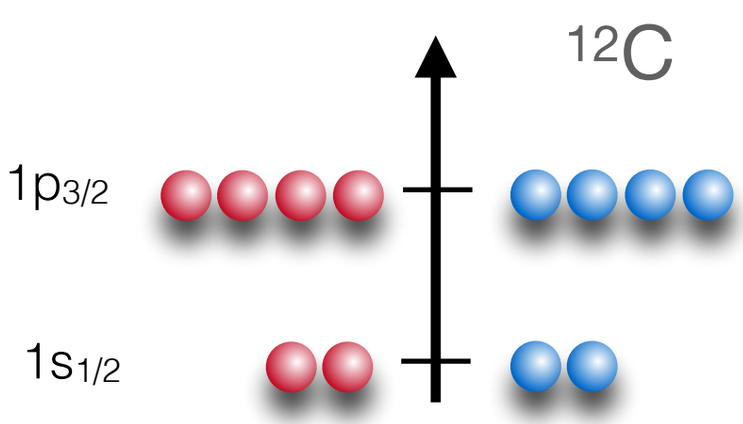
✐ Physical Review C 87(1):014334

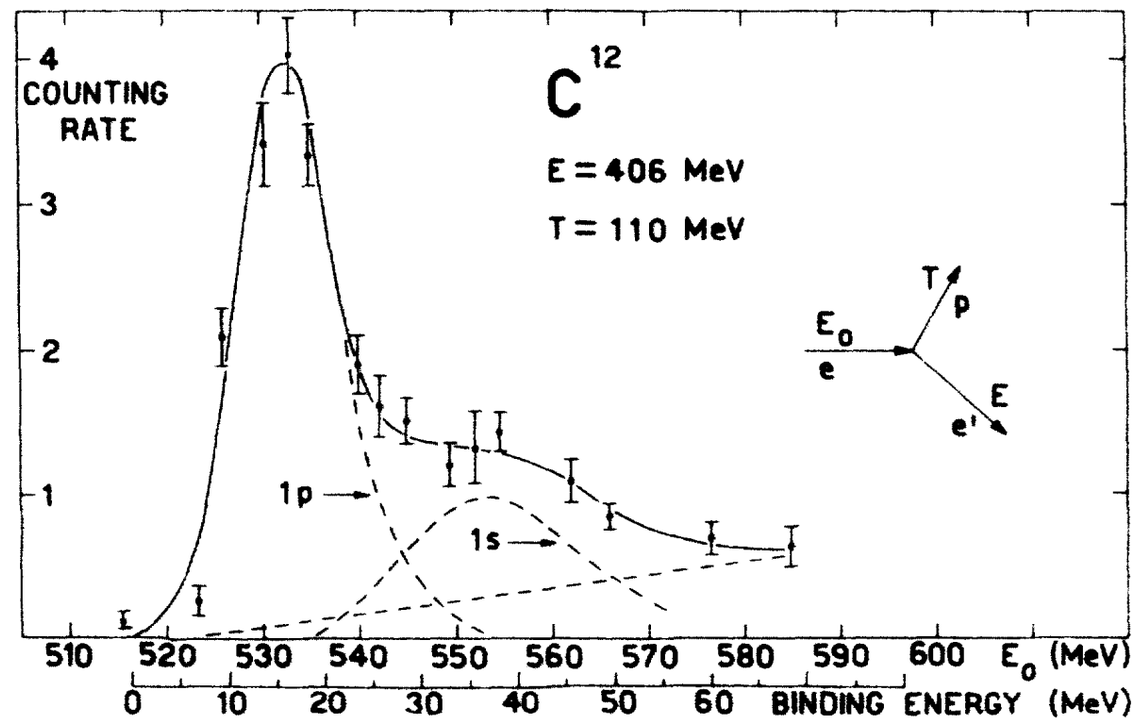
(e,e'p) scattering experiments

- (e,e'p) experiments are extremely important to investigate the internal structure of the nucleus



- Assuming NO FSI the energy and momentum of the initial nucleon can be identified with the measured p_{miss} and E_{miss}





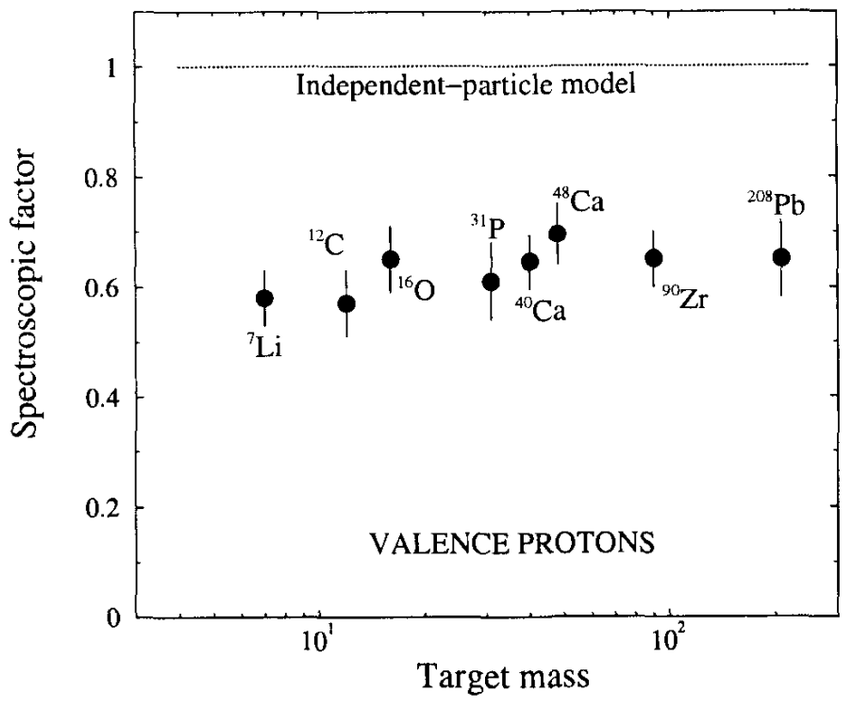
✐ U.Amaldi et al, Phys. Rev. Lett. 13, 10 (1964)

- The peak coming from four 1p protons is visible

- The contribution of the two 1s protons is not clearly separated with this resolution

Beyond mean field: Short Range Correlations

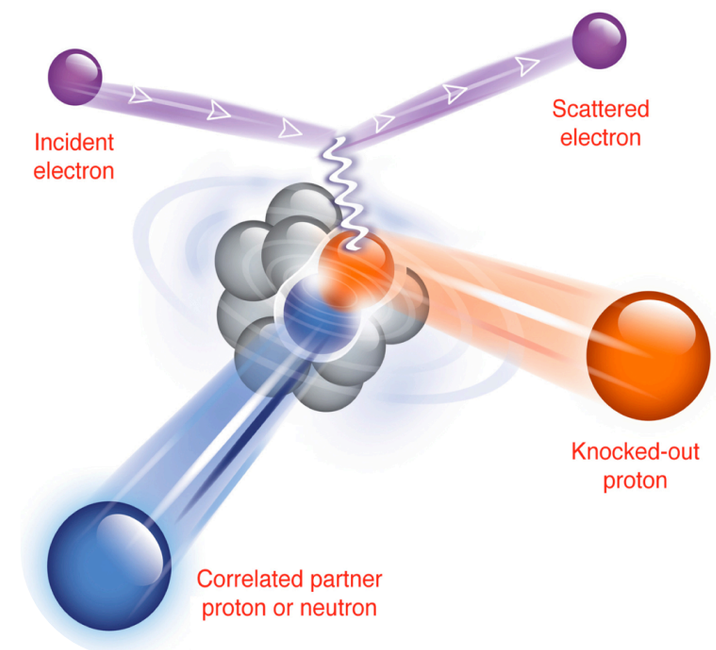
- Electron and proton experiments also pinned down the limitations of MF approaches



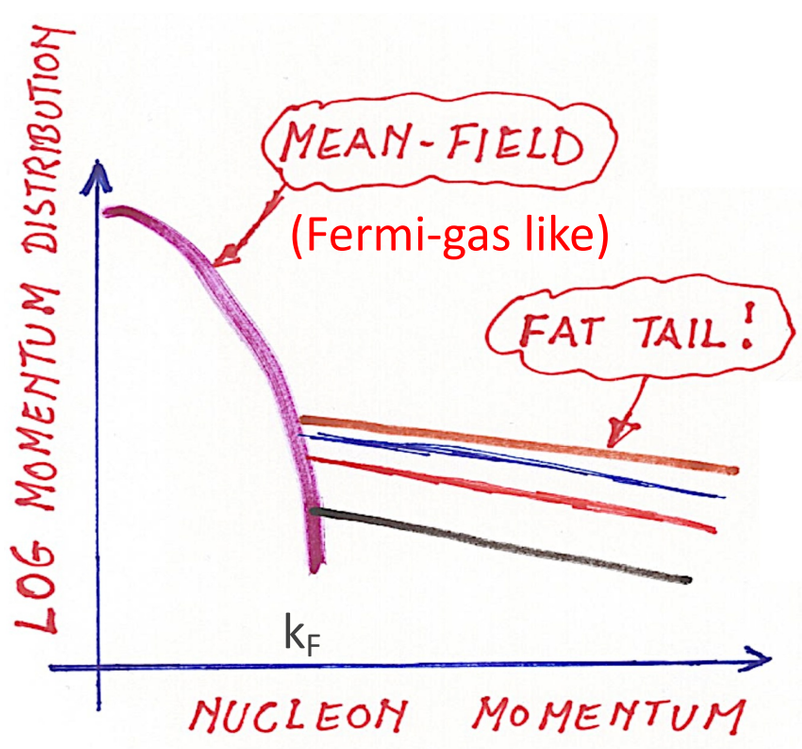
- Quenching of the spectroscopic factors of valence states has been confirmed by a number of high resolution (e,e'p) experiments

✐ Subedi et al., Science 320, 1476 (2008)

- Semi-exclusive 2N-SRC experiments at x>1 allows to detect both nucleons and reconstruct the initial state



- Confirmed that the high momentum tail of the nuclear wave function consists mainly of 2N-SRC

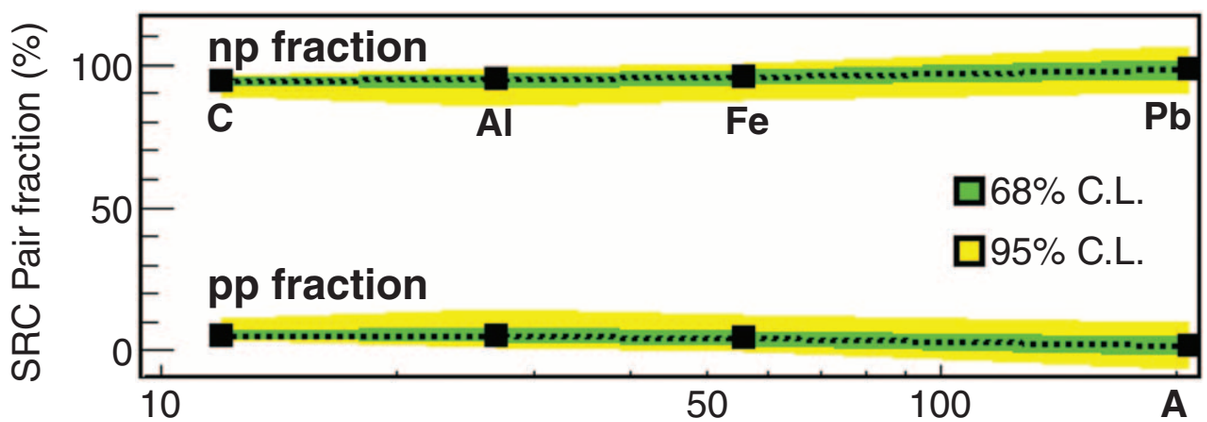


- The large-momentum (short-range) component of the wave function is dominated by the presence of Short Range Correlated (SRC) pairs of nucleons

Figure by Or Hen

Beyond mean field: Short Range Correlations

- Observed dominance of np-over-pp pairs for a variety of nuclei



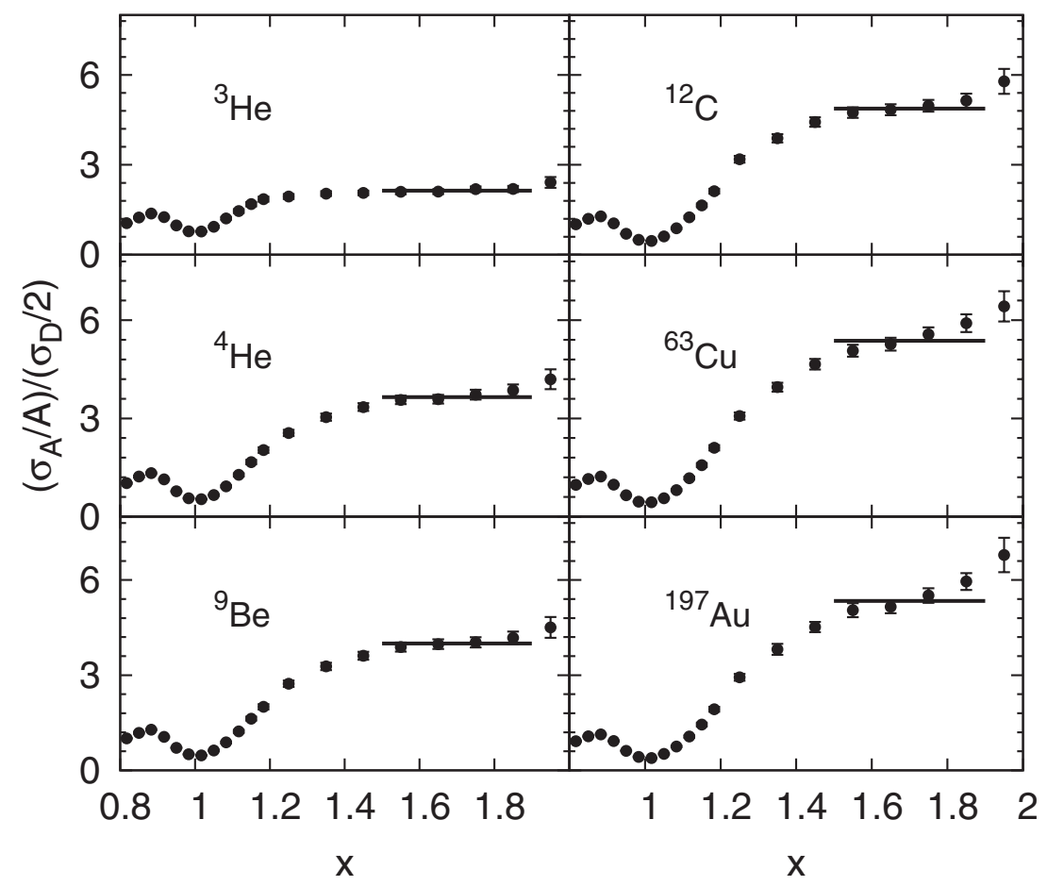
✐ Subedi et al., Science 320, 1476 (2008)

- SRC pairs are in spin-triplet state, a consequence of the tensor part of the nucleon-nucleon interaction

Bottom Line

- Two-body Physics can not be neglected:
  - ~20% of the nucleons in nuclei
  - ~100% of the high k (>pF) nucleons
- Have large relative momentum and low center of mass momentum

- Universality of high-momentum component



✐ N. Fomin et al., PRL **108**, 092502 (2012)

- The cross section ratio: A/d, sensitive to $n_A(k)/n_d(k)$

- Observed scaling for x>1.5

$$n_A(k > p_F) = a_2(A) \times n_d(k)$$

Scaling in the Fermi gas model

- Scaling of the first kind: the nuclear electromagnetic responses divided by an appropriate function describing the single-nucleon physics no longer depend on two variables ω and **q**, but only upon $\psi(\mathbf{q},\omega)$

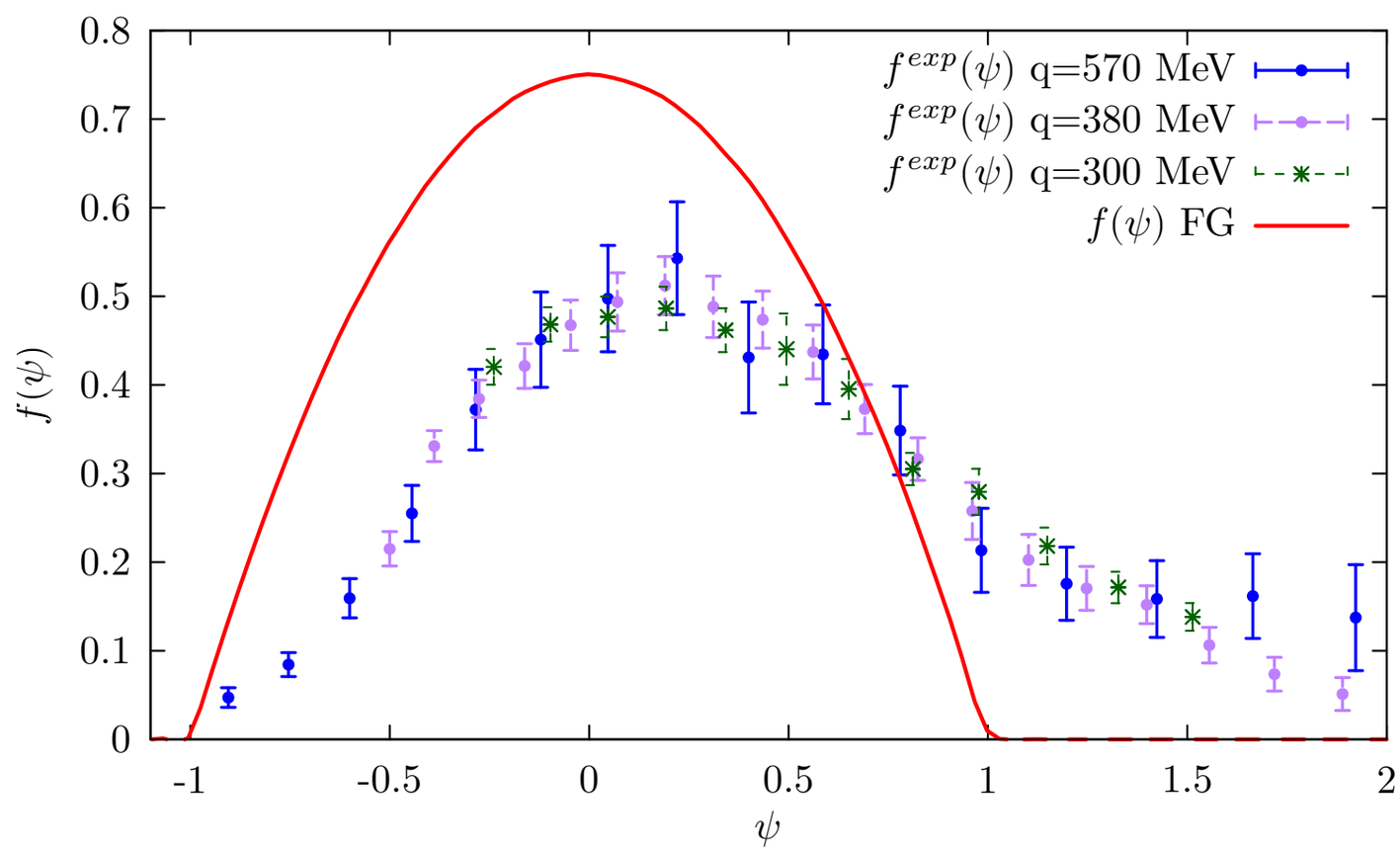
$$f_{L,T}(\mathbf{q},\omega) = p_F \frac{R_{L,T}}{G_{L,T}} = \frac{\text{nuclear response function}}{\text{elem single nucleon function}}$$

- For sufficiently large values of **q**

$$f(\mathbf{q},\omega) \to f(\psi)$$

- Within the Relativistic Fermi gas scaling of the first kind is exactly fulfilled

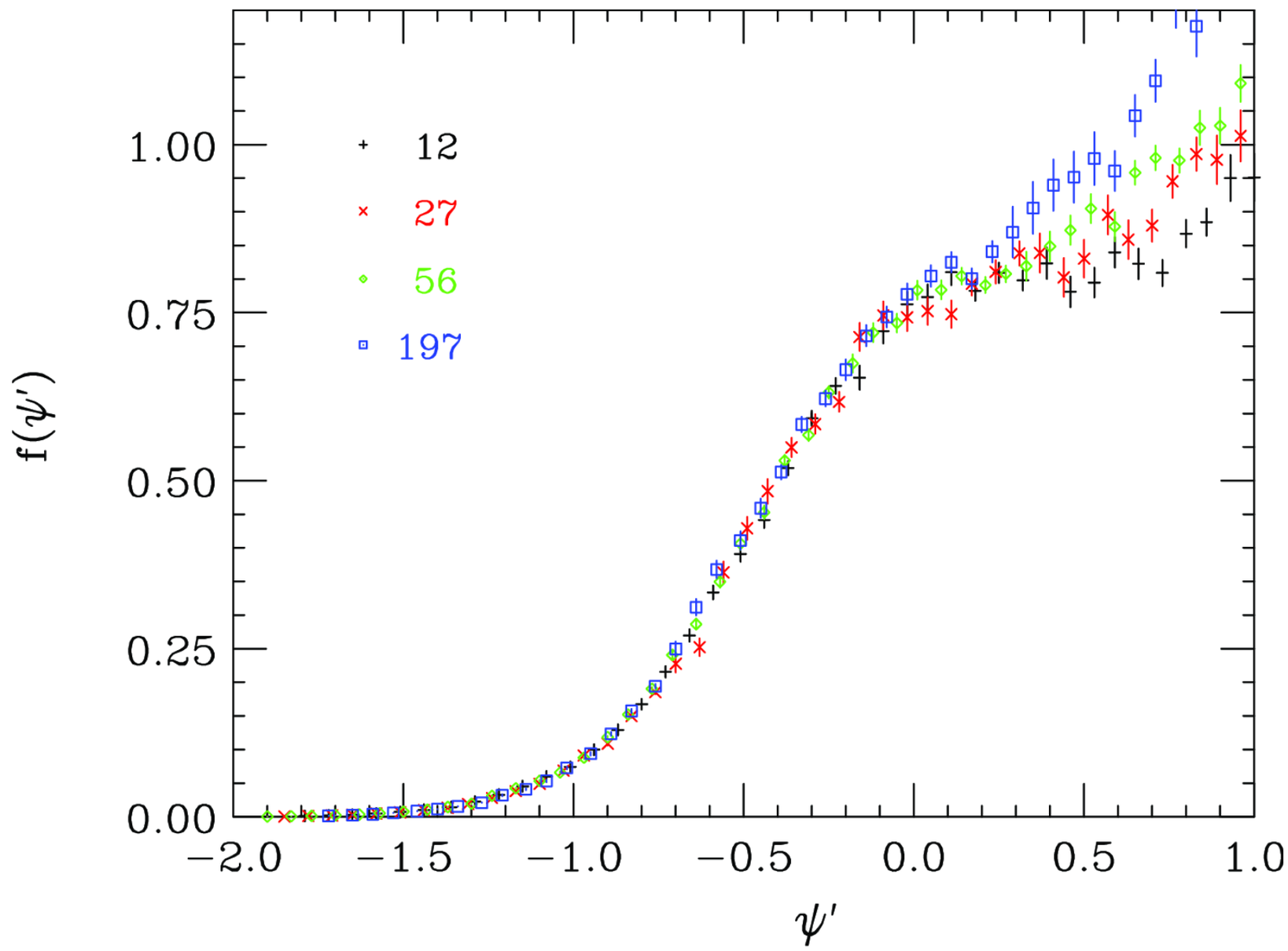
$$f_{\mathrm{RFG}} = \frac{3}{4}(1-\psi^2)$$



- Scaling function of ^{12}C, obtained dividing the R_L by the single nucleon functions for different **q**

Scaling in the Fermi gas model

- Scaling of the second kind: independence of the nuclear species A

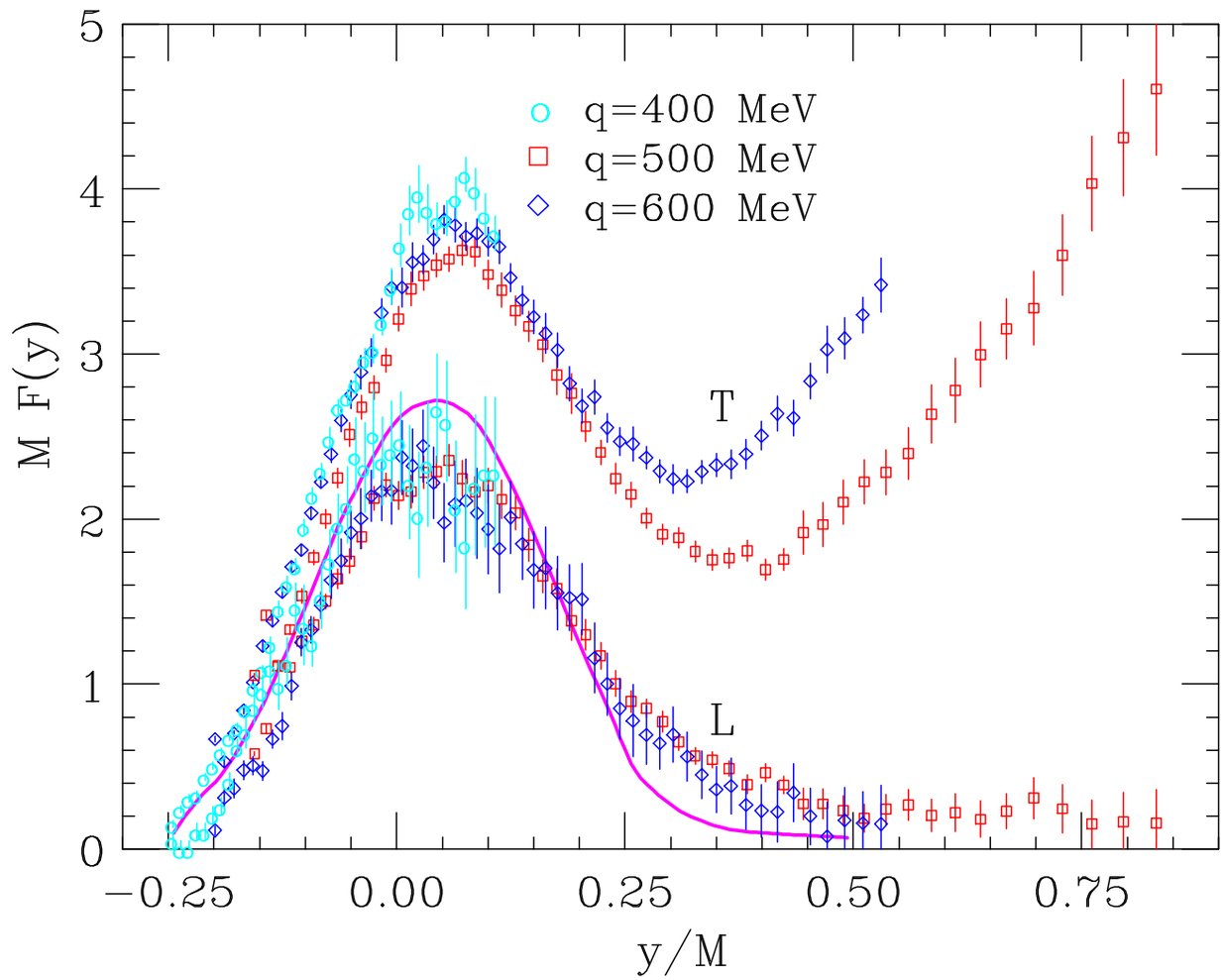


Two-body currents violate scaling

- Within the FG model we have this simple expression:

$$R_{L,T} = (1-\psi^2)\theta(1-\psi^2) \times G_{L,T}$$

- Scaling functions associated with the longitudinal (L) and transverse (T) response of Carbon,



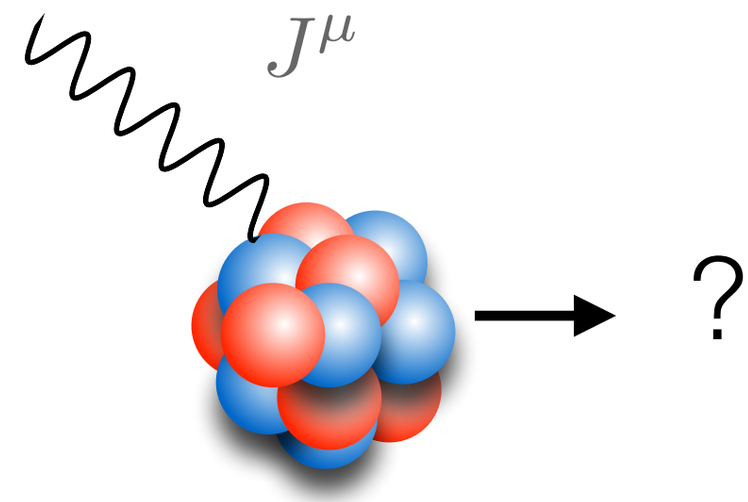
- The onset of scaling is clearly visible in the quasi-elastic peak

- Large scaling violations appear in $F_T(y)$ at $y>0$

Another way to introduce two-body currents

Nuclear dynamics is described by:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$



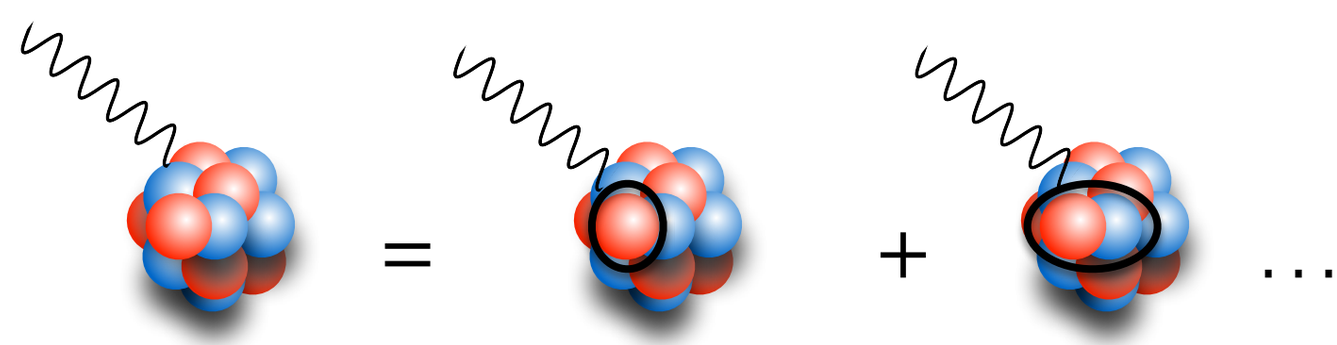
Solve the Schrödinger Equation for the initial and final states

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \ , \ H|\Psi_f\rangle = E_f|\Psi_f\rangle$$

$|\Psi_0\rangle$ $|\Psi_f\rangle$

The nuclear current operator is given by a sum of different terms

$$J^\mu(q) = \sum_i j_i^\mu + \sum_{i<j} j_{ij}^\mu + \dots$$



Another way to introduce two-body currents

- In electromagnetic processes the current operator has to be conserved:

$$\partial_\mu J^\mu_{\rm EM} = 0 \quad \longleftrightarrow \quad \nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

Two-body interactions in the Hamiltonian leads to a violation of the current conservation if only one-body terms are retained

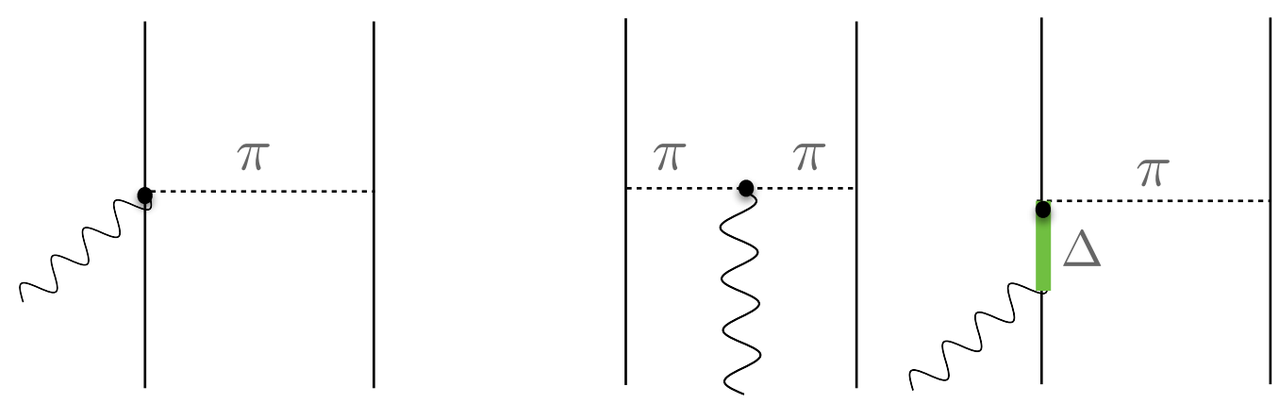
$$[v_{ij}, j_i^0] \neq 0$$

The inclusion of two-body currents is essential to satisfy the continuity equation

- Chiral effective field theory tells us how to go from a given Hamiltonian to current operators in a consistent way

? What do we mean by two-body currents

These are some of the diagrams that have to be included in the calculations:



R. Machleidt , D. R. Entem, Chiral effective field theory and nuclear forces, Phys.Rept.503:1-75,2011

E. Epelbaum, Nuclear forces from chiral effective field theory: a primer, arXiv:1001.3229

Introduction to electroweak interactions

Interaction	Current Theory	Mediators	Relative Strength	Range (m)
Strong	Quantum chromodynamics(QCD)	gluons	10^{38}	10^{-15}
Electromagnetic	Quantum electrodynamics(QED)	photons	10^{36}	∞
Weak	Electroweak Theory	W and Z bosons	10^{25}	10^{-18}

~order of 10^{11} of difference between electron and neutrino interactions

Number of Interactions = $\sigma \times \Phi \times N$

Cross Section

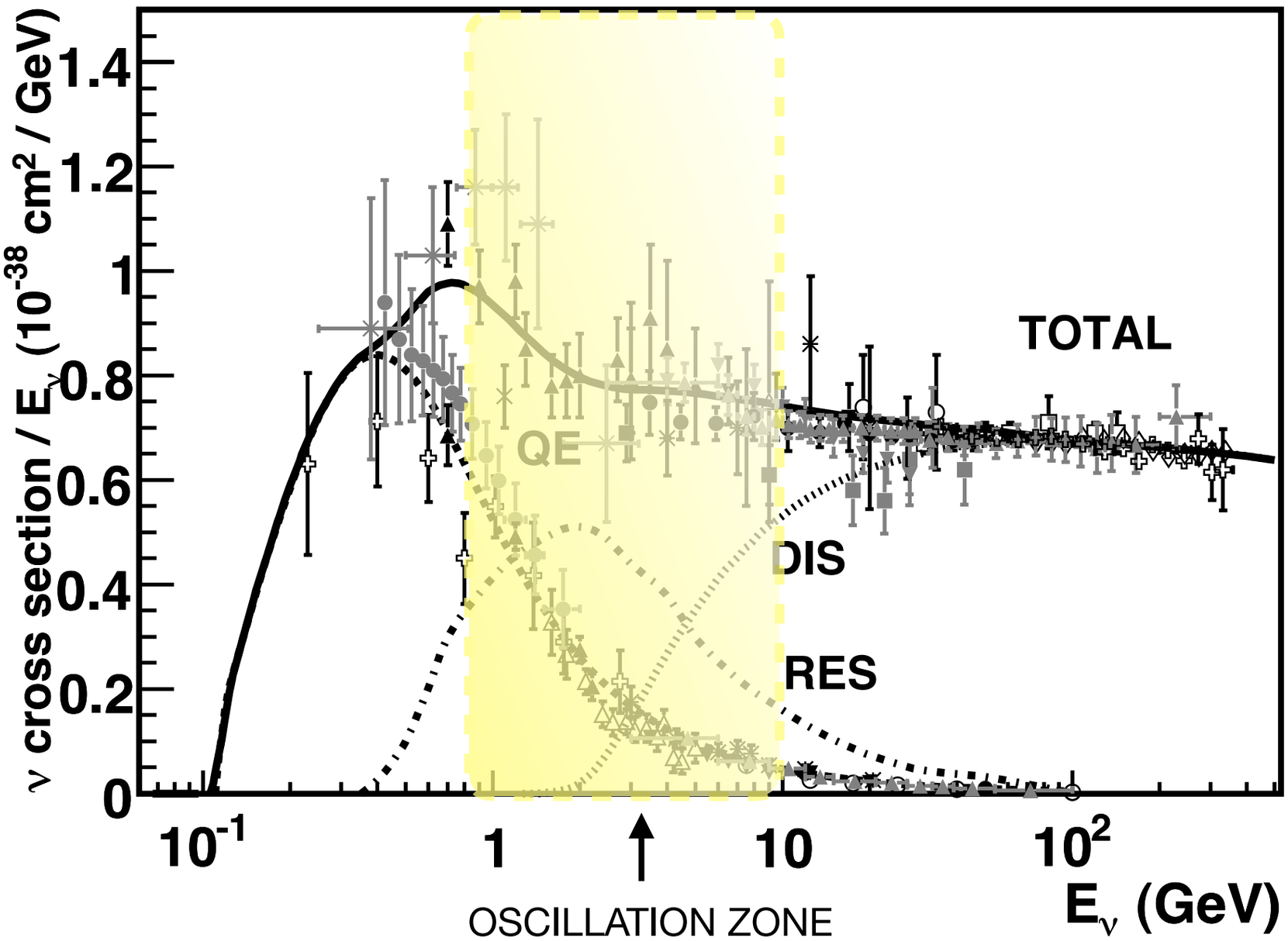
Neutrino Flux

Number of Targets (# of nucleons in the detector)

- Use nuclei as a detector to increase the event rate
- Incoming neutrino energy reconstruction needs accurate neutrino-nucleus cross sections

Addressing Neutrino-Oscillation Physics

✐ J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (2012)

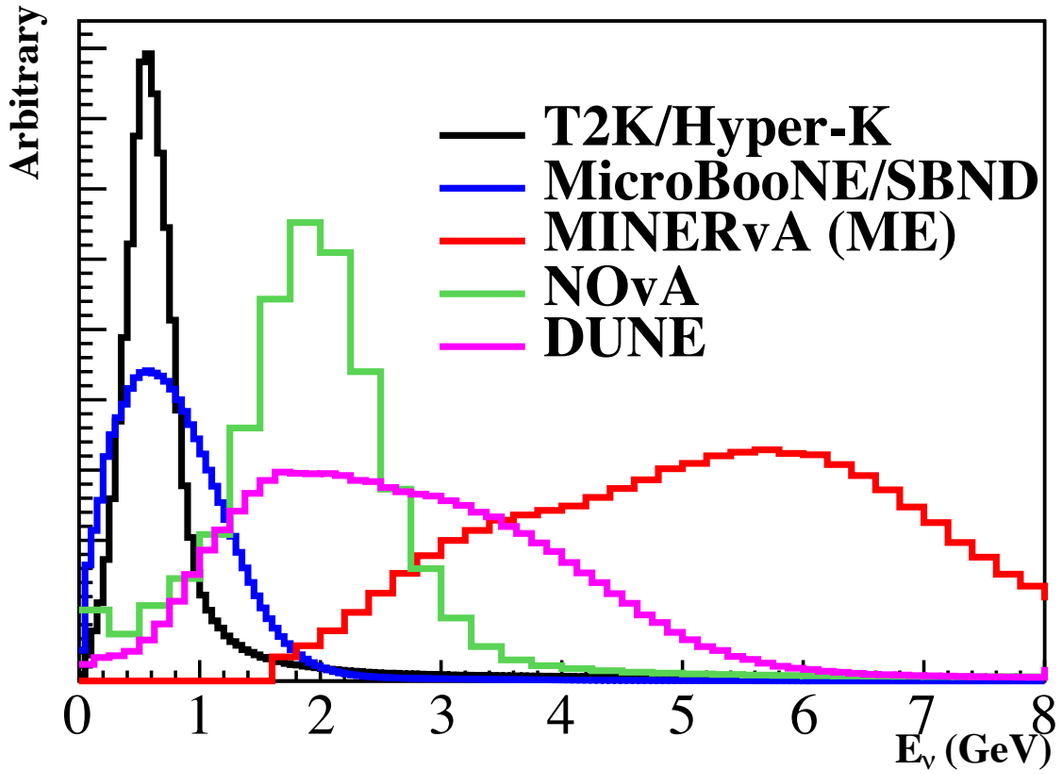


- **QE:** Broad peak due to quasi-elastic electron-nucleon scattering.
- **RES:** Excitation of the nucleon to distinct resonances (like the Δ) and pion production.
- **DIS:** Deep Inelastic Scattering region, productions of hadrons other than protons and neutrons

- The dominant reaction mechanism changes dramatically over the region of interest to oscillation experiment

Addressing Neutrino-Oscillation Physics

✐ T. Katori and M. Martini, J.Phys. G45 (2018) no.1, 013001

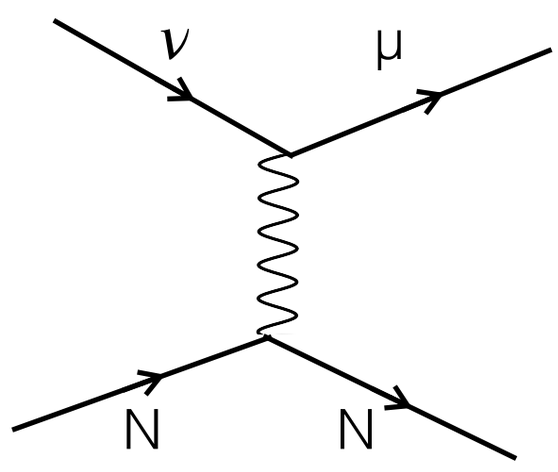


- Neutrino beams are a secondary decay product (π and K decay): their energy it is not sharply defined (as in electron scattering experiments) but broadly distributed

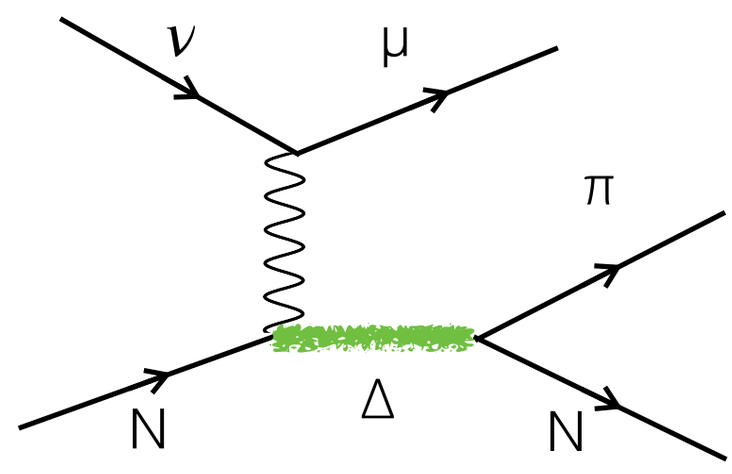
- Energy distribution of neutrino fluxes of different experiments

- Many-different reaction mechanisms come into play

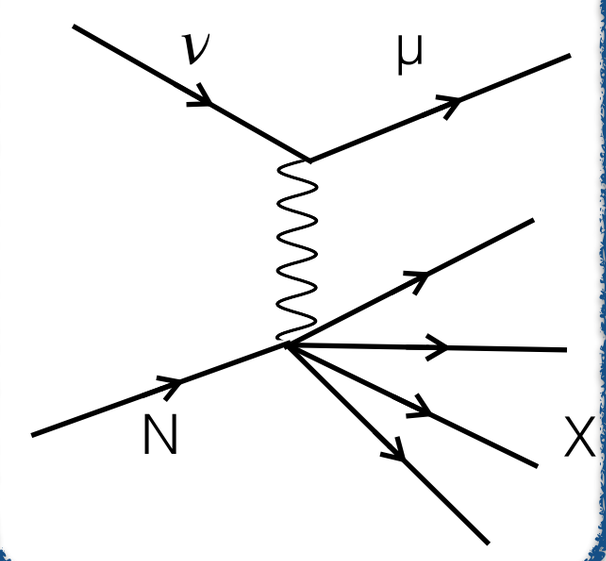
- CCQE



- CC 1π



- DIS



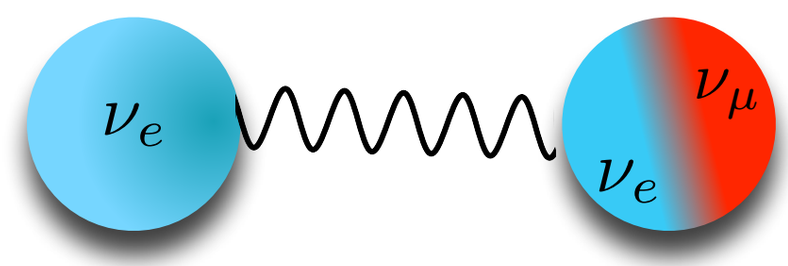
How do neutrino oscillate?

- Simple case of two-flavor neutrino oscillation
- The neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian

$$|\nu_i\rangle = e^{-E_i t}|\nu_i\rangle$$

- The neutrino flavor eigenstates can be written as a combination of the mass ones

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$



- The ν_e propagation in vacuum is written as

$$|\nu(t,\mathbf{x})\rangle = \cos\theta e^{-i(E_1 t - \mathbf{p}_1\cdot\mathbf{x})}|\nu_1\rangle + \sin\theta e^{-i(E_2 t - \mathbf{p}_2\cdot\mathbf{x})}|\nu_2\rangle$$

- The oscillation probability in the ultra relativistic limit ($t \simeq L$, $E_i - |p_i|^2 \simeq m_i^2/2E_i$)

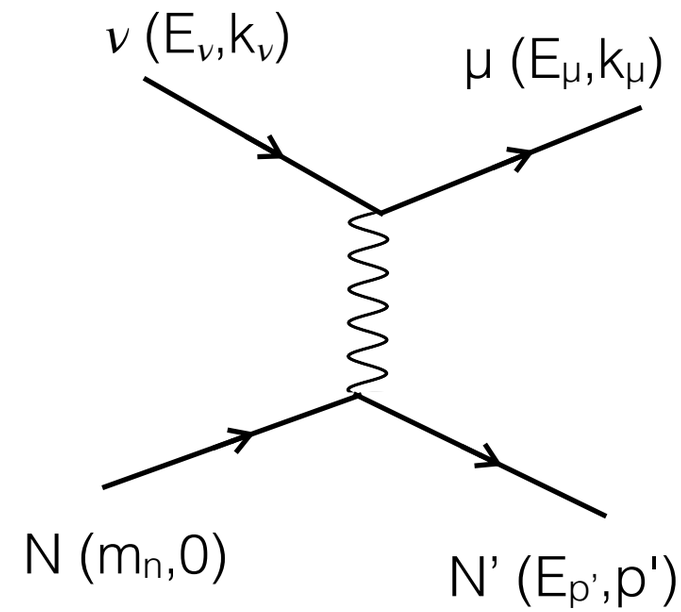
$$P_{e\mu}(L) = |\langle\nu_\mu|\nu(t, z=L)\rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

Dependence on the energy distribution of the neutrino source

Neutrino energy reconstruction

- Kinematic Reconstruction

$$E_\nu = \frac{2m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos\theta)}$$



- Main problems: not exact on nuclear targets, lack of knowledge of the hadronic undetected hadrons in the final state.

For example, when a produced pion is absorbed or undetected the reconstructed neutrino energy is underestimated at least by ~ 300 – 350 MeV

- Calorimetric Reconstruction

$$E_\nu = E_\ell + E_{\rm had}$$

- The ability to reconstruct the hadronic final state in the most accurate way is a huge experimental challenge and <u>relies on simulations</u>

For example, neutrons typically escape detection and any undetected meson will result as an energy underestimation equal, at least, to the pion mass.

Neutrino-Nucleon scattering

- Differential cross section for CC and NC processes

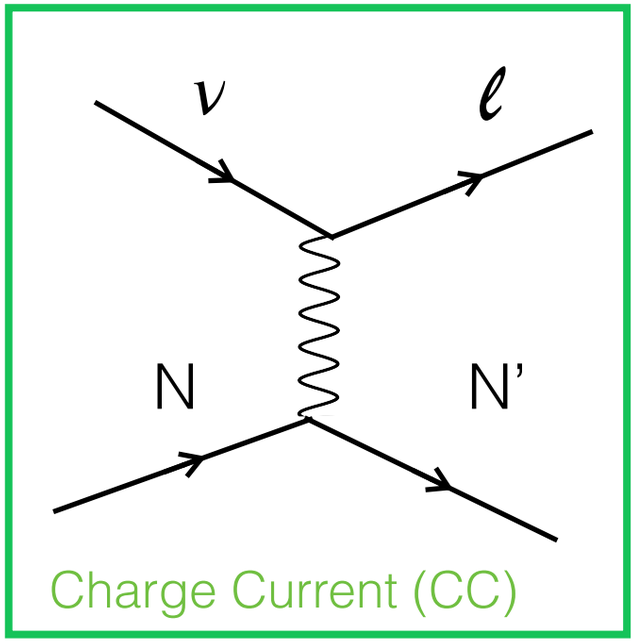
$$\frac{d^2\sigma}{dE' d\Omega'} = \frac{1}{16\pi^2}\frac{G^2}{2} L_{\mu\nu} W^{\mu\nu}$$

- For NC

$$G = G_F$$

- For CC

$$G = G_F \cos\theta_c$$

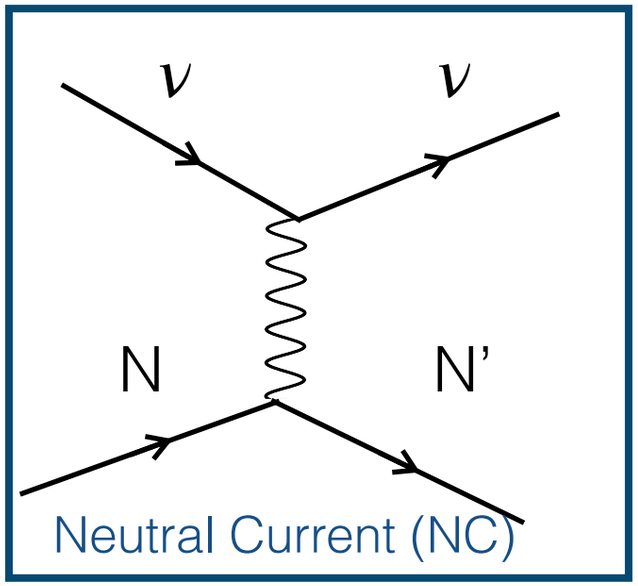


$$G_F = 1.1803 \times 10^{-5}\ \mathrm{GeV}^{-2}\ ,\ \cos\theta_c = 0.97425$$

- Leptonic Tensor:

$$L_{\mu\nu} = 8[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' \pm i\epsilon_{\mu\nu\alpha\beta} k'^{\alpha} k^{\beta}]$$

$\nu/\bar{\nu}$



- Hadronic Tensor:

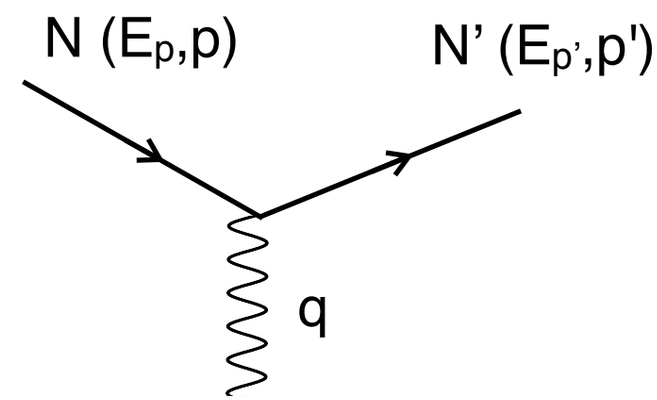
$$W_{\mu\nu} = \sum_{\sigma_i\sigma_f} \frac{1}{2E_p} \int \frac{d^3p'}{2E_{p'}} \langle N(p)|J^\mu|N'(p')\rangle\langle N'(p')|J^\nu|N(p)\rangle$$
$$\times\, \delta^{(4)}(p' + k' - p - k)$$

Neutrino-Nucleon scattering

- Electroweak current operator:

$$\langle N'|J^\mu|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = J_V^\mu + J_A^\mu$$

Vector Axial



- The vector contribution is given by:

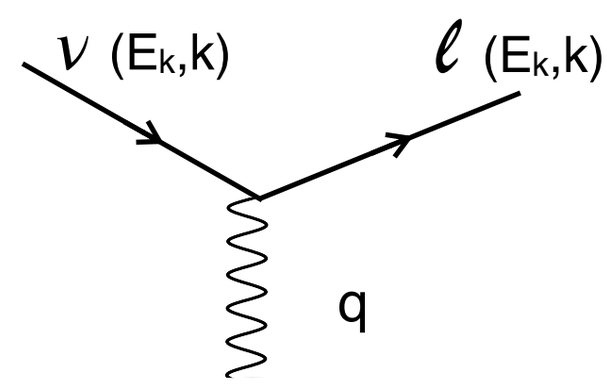
$$J_V^\mu = \mathcal{F}_1\gamma^\mu + i\sigma^{\mu\nu}q_\nu\frac{\mathcal{F}_2}{2M}$$

- The axial contribution is given by:

$$J_A^\mu = -\gamma^\mu\gamma_5\mathcal{F}_A - q^\mu\gamma_5\frac{\mathcal{F}_P}{M}$$

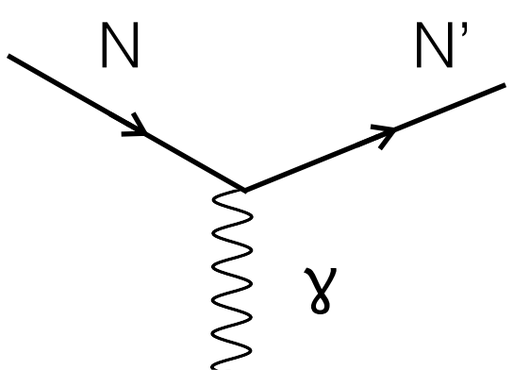
- In the Leptonic vertex:

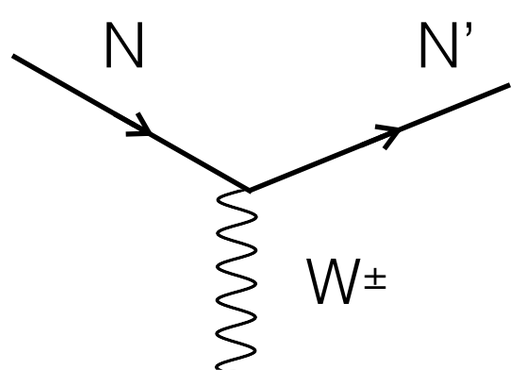
$$\langle \ell^-|j^\mu|\nu_\ell\rangle = \bar{u}_\ell(k')\gamma^\mu(1-\gamma_5)u_{\nu_\ell}(k)$$

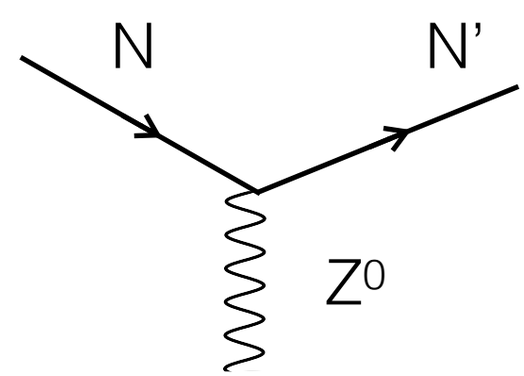


T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Electron- and neutrino-nucleus scattering from the quasielastic to the resonance region, Phys. Rev. C 79, 034601 (2009).

Neutrino-Nucleon scattering







- EM

$$\mathcal{F}_1 = \frac{1}{2}[F_1^S + F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[F_2^S + F_2^V \tau_z]$$

- CC

$$\mathcal{F}_1 = F_1^V \tau_\pm$$

$$\mathcal{F}_2 = F_2^V \tau_\pm$$

$$\mathcal{F}_A = F_A \tau_\pm$$

$$\mathcal{F}_P = F_P \tau_\pm$$

- NC

$$\mathcal{F}_1 = \frac{1}{2}[-2sin^2\theta_W F_1^S + (1 - 2sin^2\theta_W) F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[-2sin^2\theta_W F_2^S + (1 - 2sin^2\theta_W) F_2^V \tau_z]$$

$$\mathcal{F}_A = \frac{1}{2} F_A \tau_z$$

$$\mathcal{F}_P = \frac{1}{2} F_p \tau_z$$

- where:

$$F_{1,2}^S = F_{1,2}^p + F_{1,2}^n$$

$$F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$$

- We used the Conserved Vector Current hypothesis:

$$F_1^V \tau_z \rightarrow F_1^V \tau_\pm \ , \ F_2^V \tau_z \rightarrow F_2^V \tau_\pm$$

Neutrino-Nucleon scattering

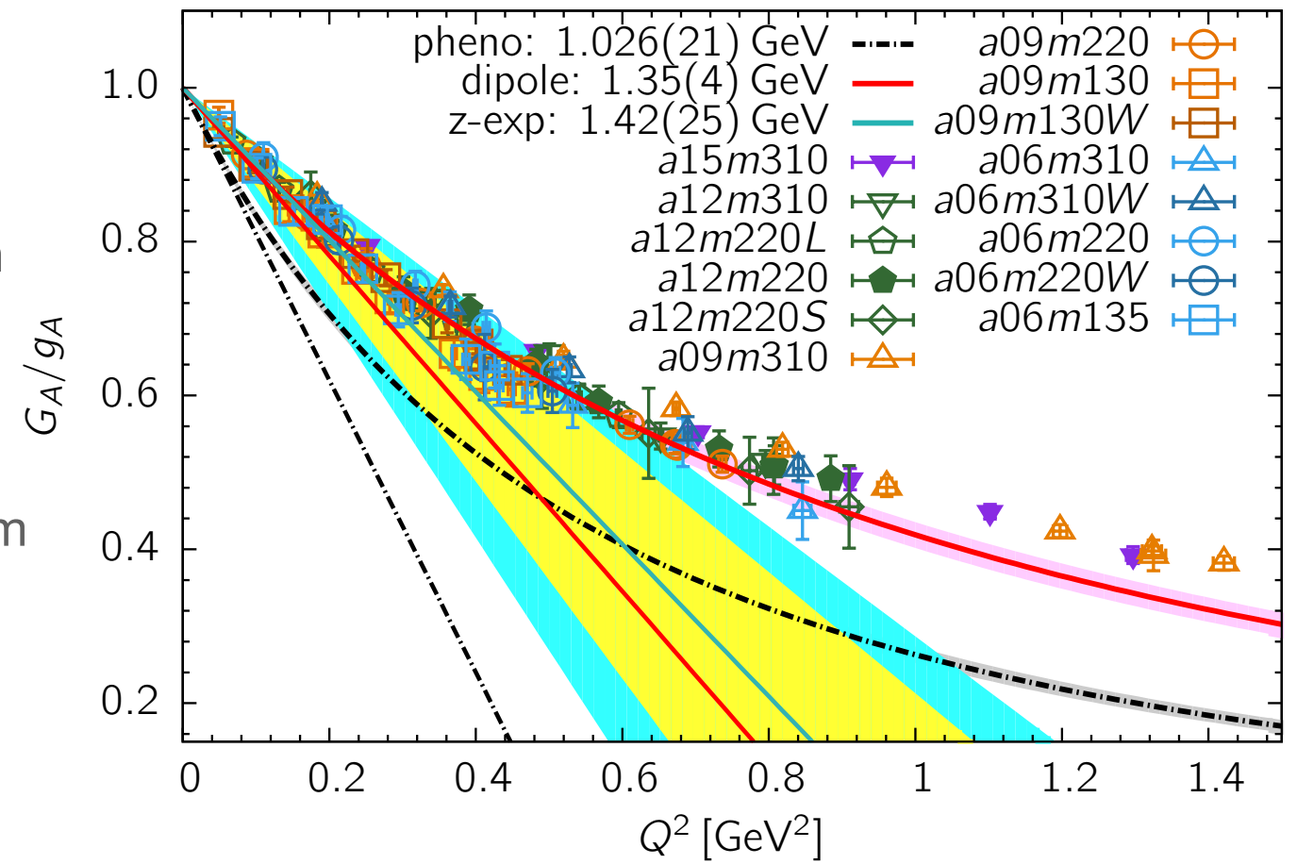
✐ Y-C Jang et al,PoS LATTICE2018 (2018) 123

- The axial form-factor has been fit to the dipole form

$$F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2}$$

- The intercept g_A=-1.2723 is known from neutron β decay

- Different values of m_A from experiments
  - m_A =1.02 GeV q.e. scattering from deuterium
  - m_A =1.08 GeV pion electroproduction
  - m_A =1.20 GeV @ K2K
  - m_A =1.35 GeV @ MiniBooNE
- The last two determinations of m_A assume a nuclear model for the target material



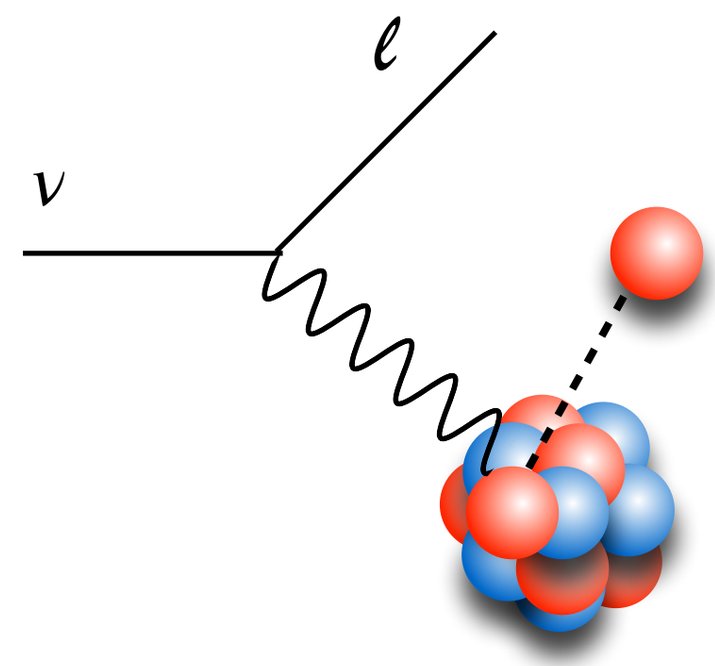
- Different Lattice QCD predictions for the axial form factor: correspond to different lattice spacings in the range 0.06-0.15 fm and pion masses 130-310 MeV

- A full and robust prediction accounting of all systematics involved in the lattice-QCD calculations has not been achieved, yet.

Neutrino-Nucleus scattering

- General expression of the lepton-nucleus inclusive cross section

$$\frac{d^2\sigma}{dE' d\Omega'} = \frac{1}{16\pi^2}\frac{G^2}{2} L_{\mu\nu} W^{\mu\nu}$$



- The hadron tensor is

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle \delta^{(4)}(p_0 + q - p_f)$$

- The initial and final state

$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

- The most general expression for the hadron tensor that you may find in many textbooks is:

$$W^{\mu\nu} = -g_{\mu\nu}W_1 + \frac{W_2}{M^2}p_\mu p_\nu - i\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \frac{W_3}{2M^2} + q_\mu q_\nu \frac{W_4}{M^2} + (p_\mu q_\nu + q_\mu p_\nu)\frac{W_5}{M^2}$$

- For one-body current processes, one can derive the explicit expressions of W_i as functions of F_1, F_2, F_A, and F_p

Electron- and neutrino-nucleus scattering

- We start by defining the nuclear response functions, for a given value of **q** and ω

$$W^{\mu\nu}(\mathbf{q},\omega) = \sum_f \langle 0|(J^\mu)^\dagger(\mathbf{q},\omega)|f\rangle\langle f|J^\nu(\mathbf{q},\omega)|0\rangle\delta^{(4)}(p_0+\omega-p_f)$$

- Electron case we write the double differential cross section as:

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}}\left[\left(\frac{q^2}{\mathbf{q}^2}\right)^2 R_L + \left(\frac{-q^2}{2\mathbf{q}^2} + tan^2\frac{\theta}{2}\right)R_T\right]$$

where: $R_L = W_{00} \quad , \quad R_T = W_{xx} + W_{yy}$

- Neutrino case:

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{\nu/\bar{\nu}} = \frac{G^2}{4\pi^2}\frac{k'}{2E_\nu}\left[\hat{L}_{CC}R_{CC} + 2\hat{L}_{CL}R_{CL} + \hat{L}_{LL}R_{LL} + \hat{L}_T R_T \pm 2\hat{L}_{T'}R_{T'}\right] ,$$

- Where the nuclear responses are given by

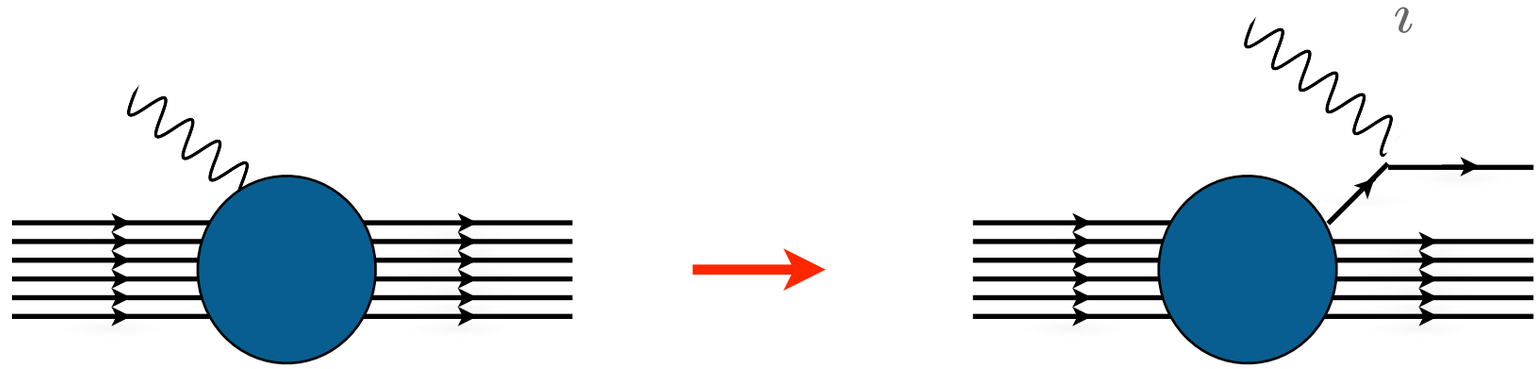
$$R_{CC} = W^{00} \qquad R_{LL} = W^{33} \qquad , \quad R_{T'} = -\frac{i}{2}(W^{12} - W^{21})$$
$$R_{CL} = -\frac{1}{2}(W^{03} + W^{30}) \qquad , \qquad R_T = W^{11} + W^{22}$$

Impulse Approximation and Spectral Function

- For sufficiently large values of |**q**|, the IA can be applied under the assumptions

$$|f\rangle \longrightarrow |p\rangle \otimes |f\rangle_{A-1}$$

$$J_\alpha = \sum_i j_\alpha^i$$



- The matrix element of the current can be written in the factorized form

$$\langle 0|J_\alpha|f\rangle \to \sum_k \langle 0|[|k\rangle \otimes |f\rangle_{A-1}]\langle k| \sum_i j_\alpha^i |p\rangle$$

- The nuclear cross section is given in terms of the one describing the interaction with individual bound nucleons

$$\boxed{d\sigma_A = \int dE\, d^3k d\sigma_N P(\mathbf{k}, E)}$$

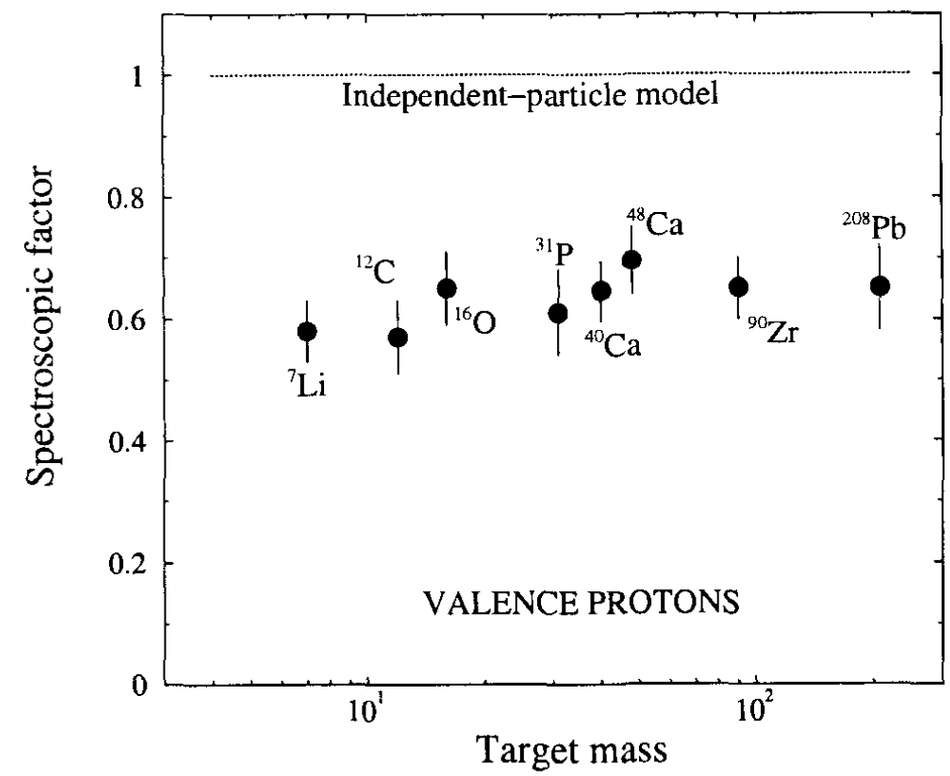
- The intrinsic properties of the nucleus are described by the hole spectral function

The CBF Spectral Function of finite nuclei

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)$$

$$\boxed{\sum_n Z_n |\phi_n(\mathbf{k})|^2 F_n(E - E_n)}$$



✐ O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

✐ O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl. Phys. A579, 493 (1994)

The CBF Spectral Function of finite nuclei

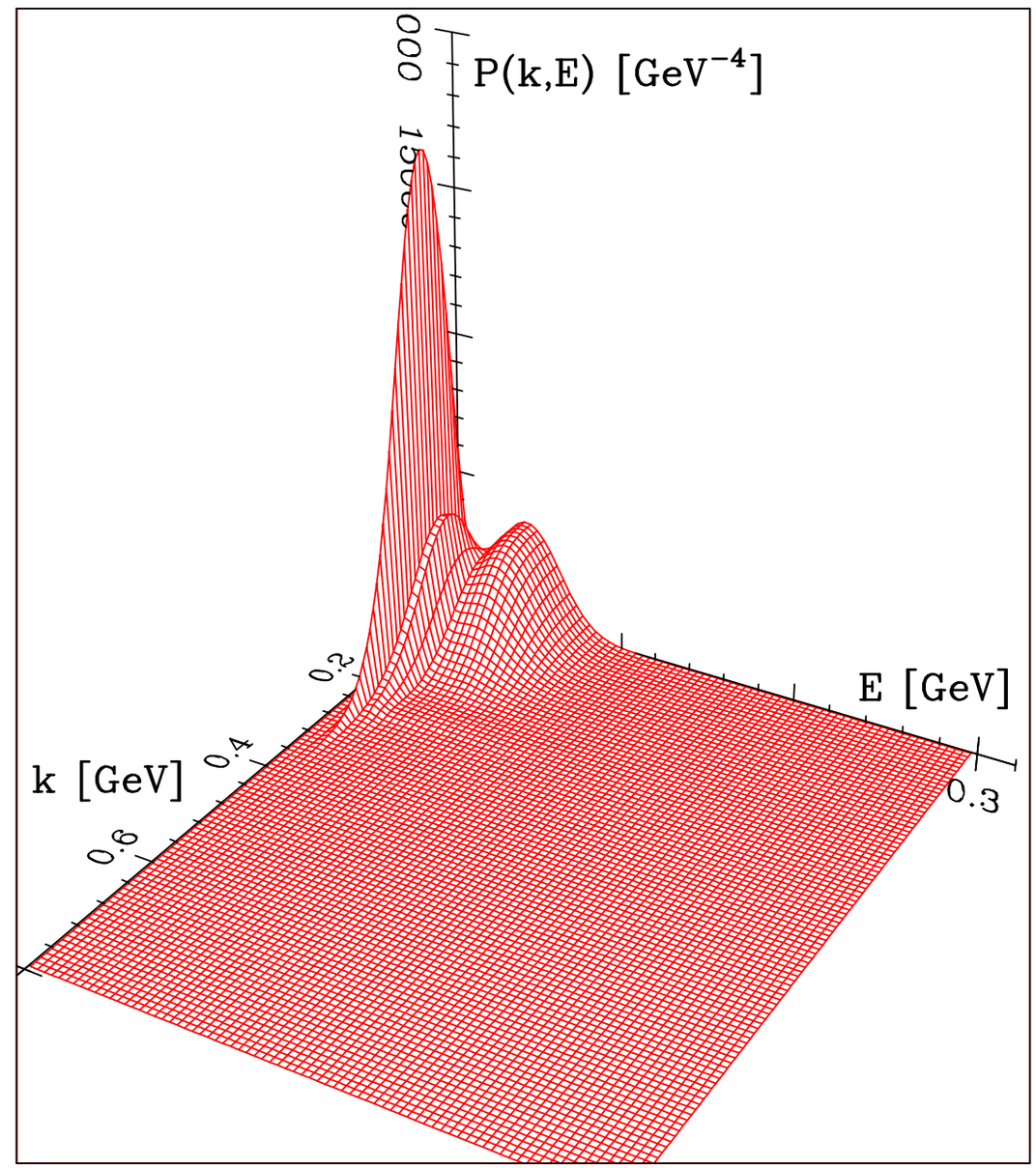
- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E) \longrightarrow \int d^3r P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

- The one-body Spectral function of nuclear matter:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

Argonne v18 — UIX, IL7



- The Correlated Basis Function approach accounts for correlations induced by the nuclear interactions

$$\Phi_n(x_1 \dots x_A) \longrightarrow \mathcal{F}\,\Phi_n(x_1 \dots x_A)$$

✐ O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

The CBF Spectral Function of finite nuclei

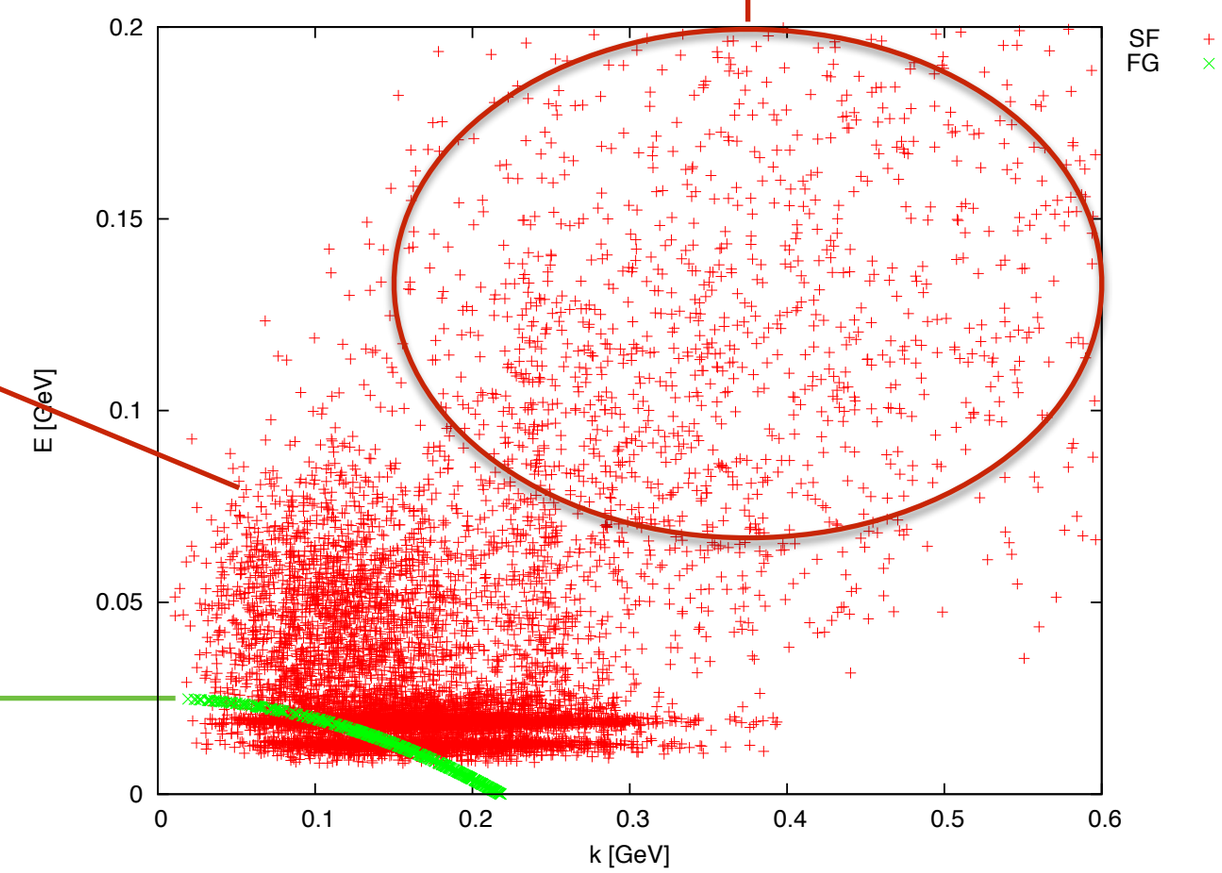
- Within the Fermi Gas model we can define the SF as:

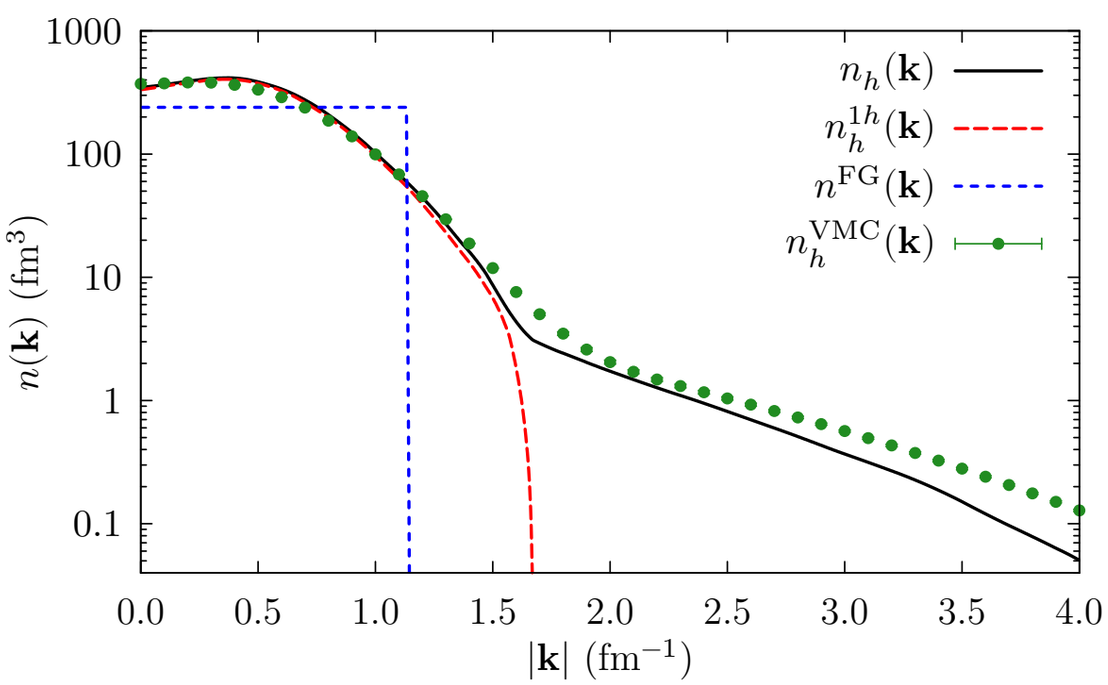
$$P^{\mathrm{FG}}(\mathbf{k}, E) = \delta(E - \epsilon_B)\theta(p_F - |\mathbf{k}|)$$

- High energy and momentum correlated pairs

Realistic SF: 80% shell model picture, 20% SRC

Fermi gas contribution

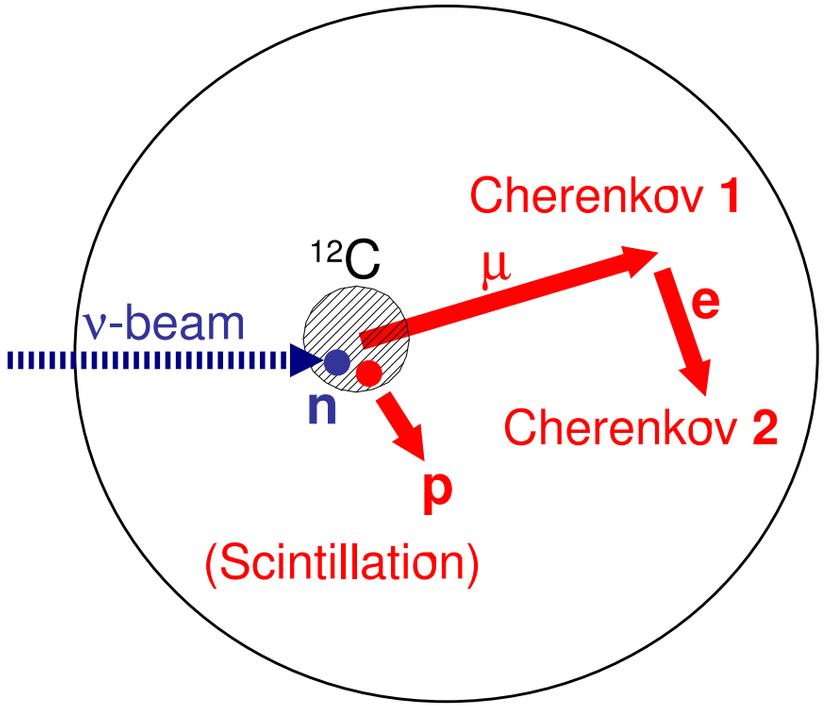


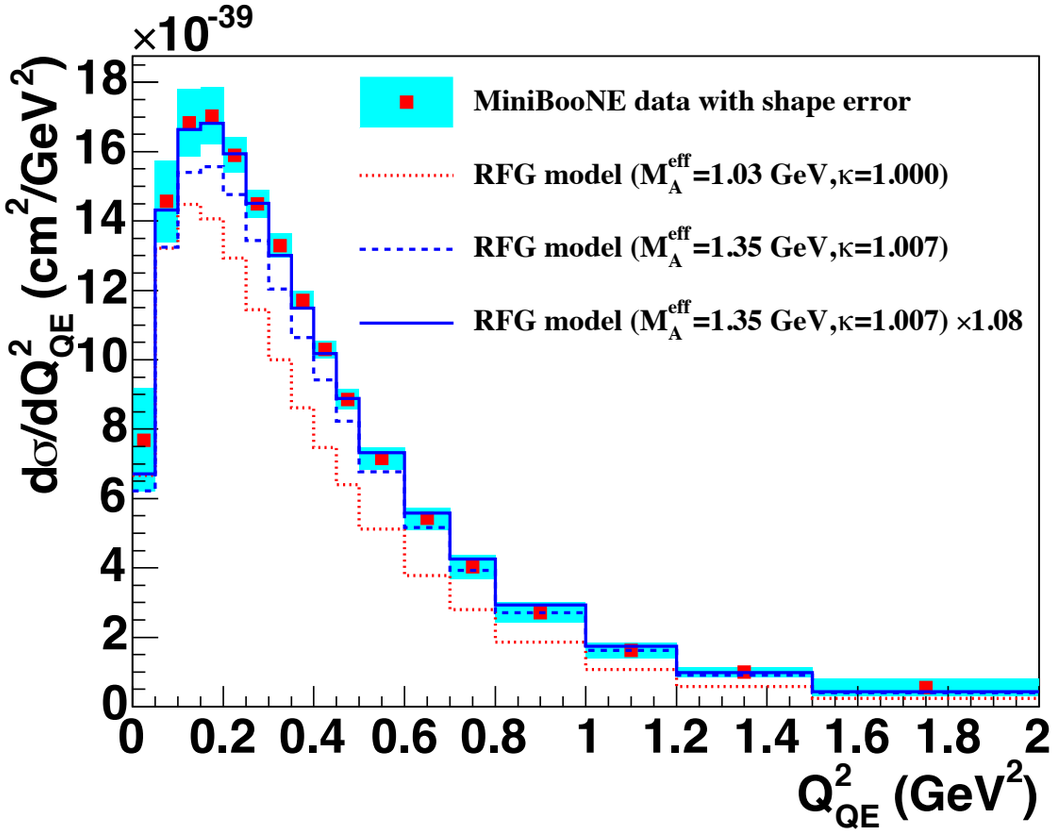


- VMC: exact calculation of the momentum distribution including SRC pairs
- CBF: calculation
- 1h corresponds to the MF, rapidly drops
- FG: unrealistic momentum distribution, totally missing the high momentum component

MiniBooNE: the axial mass puzzle

- In MiniBooNE data analysis, an event is labeled as CCQE if no final state pions are detected in addition to the outgoing muon.
- The simplest reaction mechanism compatible with this definition is single nucleon knockout
- The detector measures the Cherenkov and scintillation light.

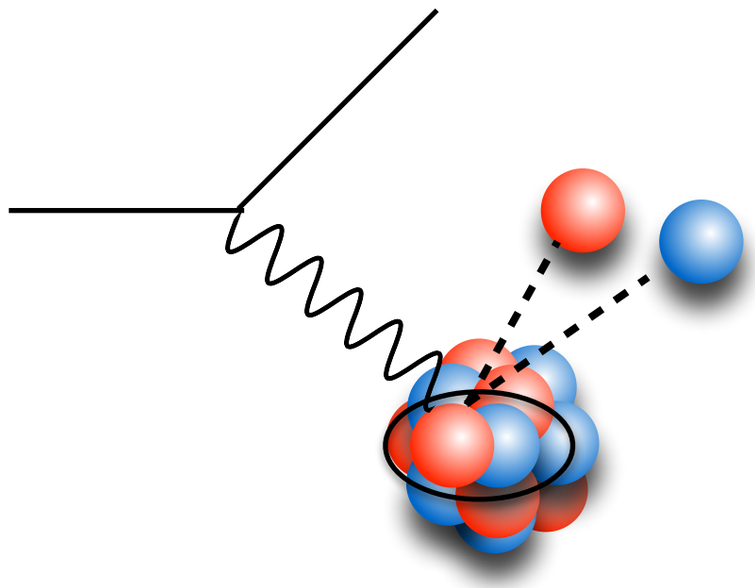




✐ MiniBooNE collaboration, PRDD81 (2010) 092005

- The M_A is extracted from the measurement assuming a Fermi Gas description of the nucleus
- The value of M_A~1.35 GeV is incompatible with former measurements: M_A~1.03 GeV
- The observed excess of CCQE cross-section may be traced back to the occurrence of events with two nucleon emission, which are often referred to CCQE-like.

MiniBooNE: the axial mass puzzle



- Presence of CCQE-like events with two-nucleon emission processes

✐ M. Martini et al, PRC 80, 065501 (2009) ; PRC 81, 045502 (2010).

⚠ We discussed the need of introducing two-body currents to preserve the continuity equation in the electromagnetic sector

- The two-body current contribution has been included by a number of groups relying on different nuclear models. Different expressions for the currents have been used, general tension on the results obtained for this contribution

✐ A. Depace, et al, Nuclear Physics A 726, 303 (2003); G. Megias et al, Phys. Rev. D94, 093004 (2016)

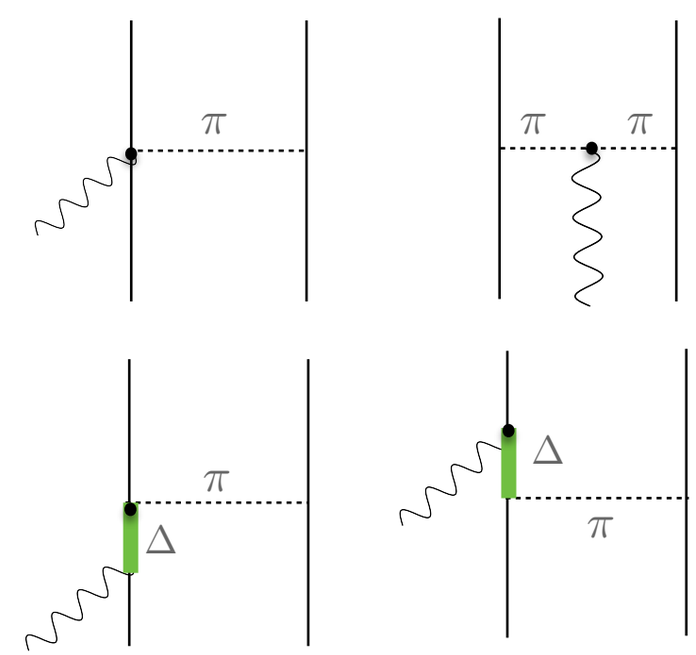
✐ J. Nieves, et al, Phys. Rev. D85 113008 (2012)

✐ T. Van Cuyck et al, Phys. Rev. C95, 054611 (2017)

✐ A. Lovato et al, Phys. Rev. Lett. 117, 082501 (2016)

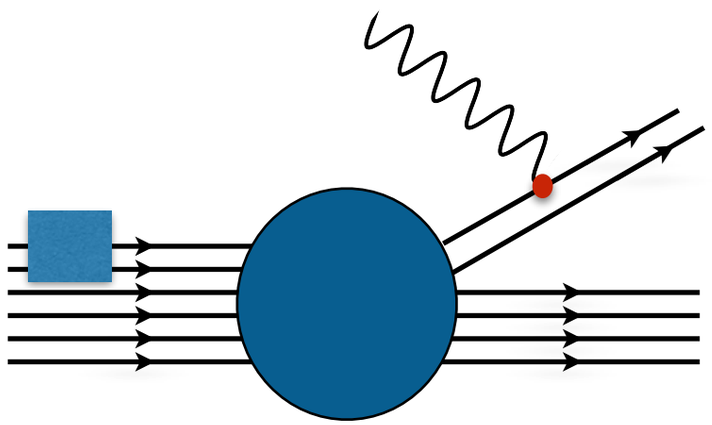
✐ N. Rocco et al, Phys. Rev. Lett. 116, 192501 (2016)

Some of the diagrams representing two-body currents

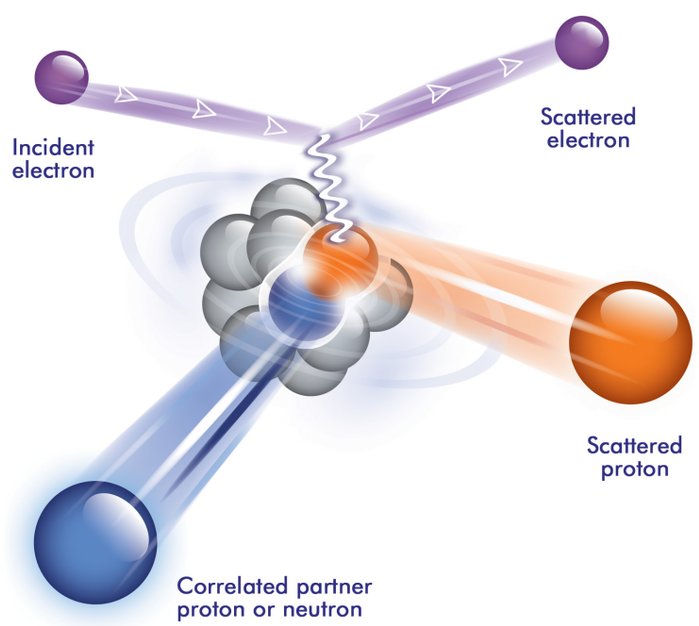


Two-nucleon emission in correlated systems

- Initial State Correlations



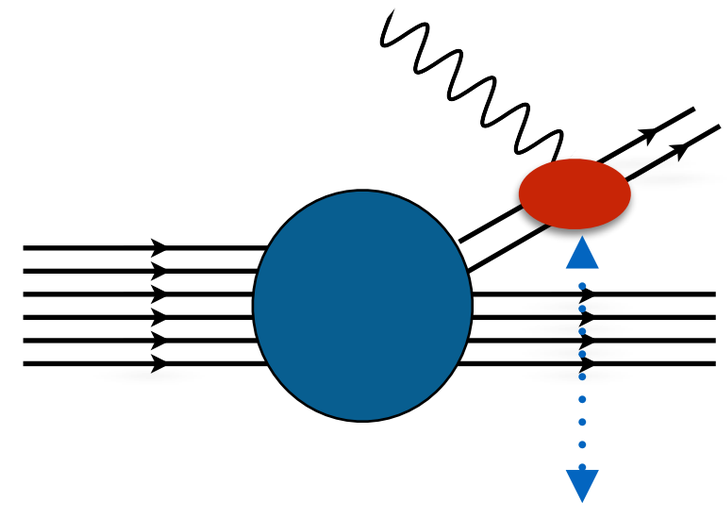
Subedi et al., Science 320, 1476 (2008)

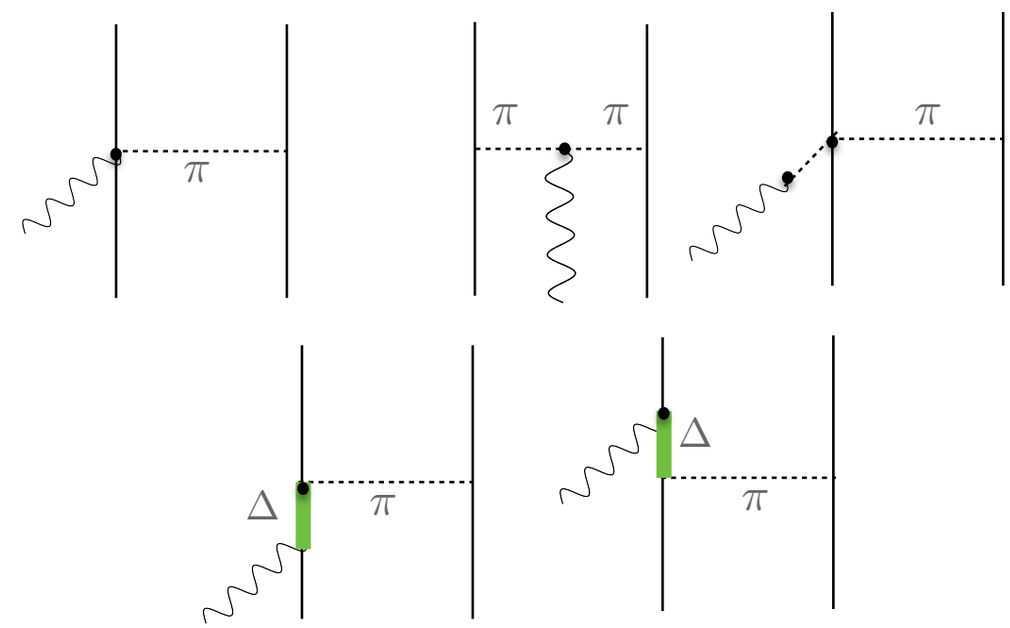


- $P_{corr}(\mathbf{k},E)$ accounts for the presence of strongly correlated pairs: appearance of a tail in the large energy transfer region in the cross section

+

- Meson Exchange currents





- The Impulse Approximation can be generalized:

$$W^{\mu\nu}_{2p2h} = W^{\mu\nu}_{ISC} + W^{\mu\nu}_{MEC}$$

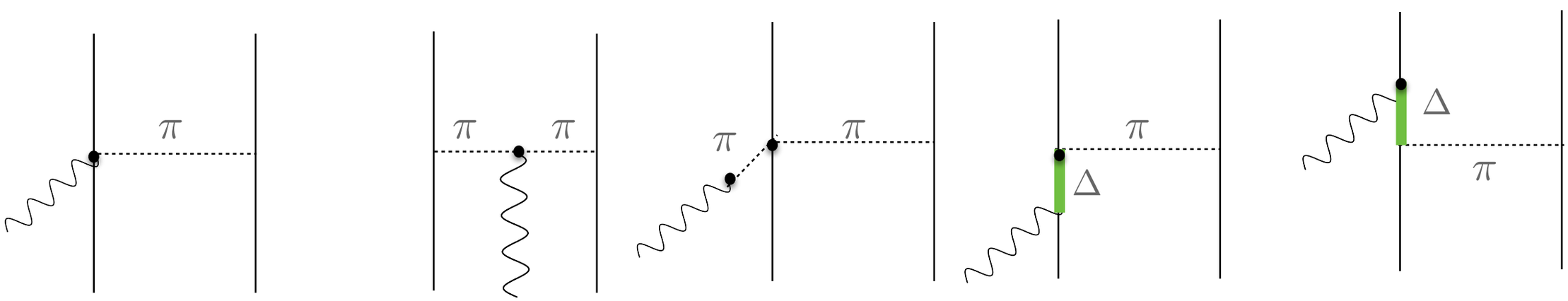
(Anti)neutrino -^{12}C scattering cross sections

- The spectral function formalism generalized to include relativistic two-body currents

$$W^{\mu\nu}_{2\mathrm{b}}(\mathbf{q},\omega) \propto \int d\tilde{E}\frac{d^3k}{(2\pi)^3}d\tilde{E}'\frac{d^3k'}{(2\pi)^3}P_h(\mathbf{k},\tilde{E})P_h(\mathbf{k}',\tilde{E}')\sum_{ij}\langle k\,k'|j^{\mu\,\dagger}_{ij}|p\,p'\rangle\langle p\,p'|j^{\nu}_{ij}|k\,k'\rangle$$

- NN correlations are accounted for by the including the product of two one-hole spectral functions

- Two-body currents have an axial and vector terms, yielding the appearance of new diagrams



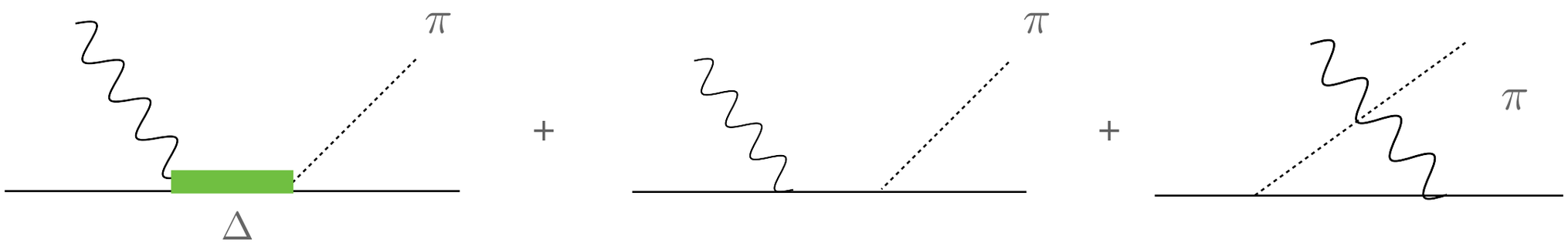
- The calculation of the meson exchange current matrix is carried out automatically: thousands of terms; can not be computed analytically

- The phase space consists on a 14-dim integral which can be reduced to a 8-dim one

Pion production mechanisms

- For the description of the pion production reaction mechanisms we adopted the DCC (dynamical couple-channel) model; H. Kamano, S.X. Nakamura, T.-S.H. Lee, and T. Sato, Phys.Rev. C 88, 035209 (2013),Phys. Rev. C 94, 015201 (2016)

- The diagrams considered are resonant and non resonant



- The parameters entering the Hamiltonian have been obtained by fitting about 26,000 data points of the πN and γN reactions up to the invariant mass W ≃ 2 GeV;

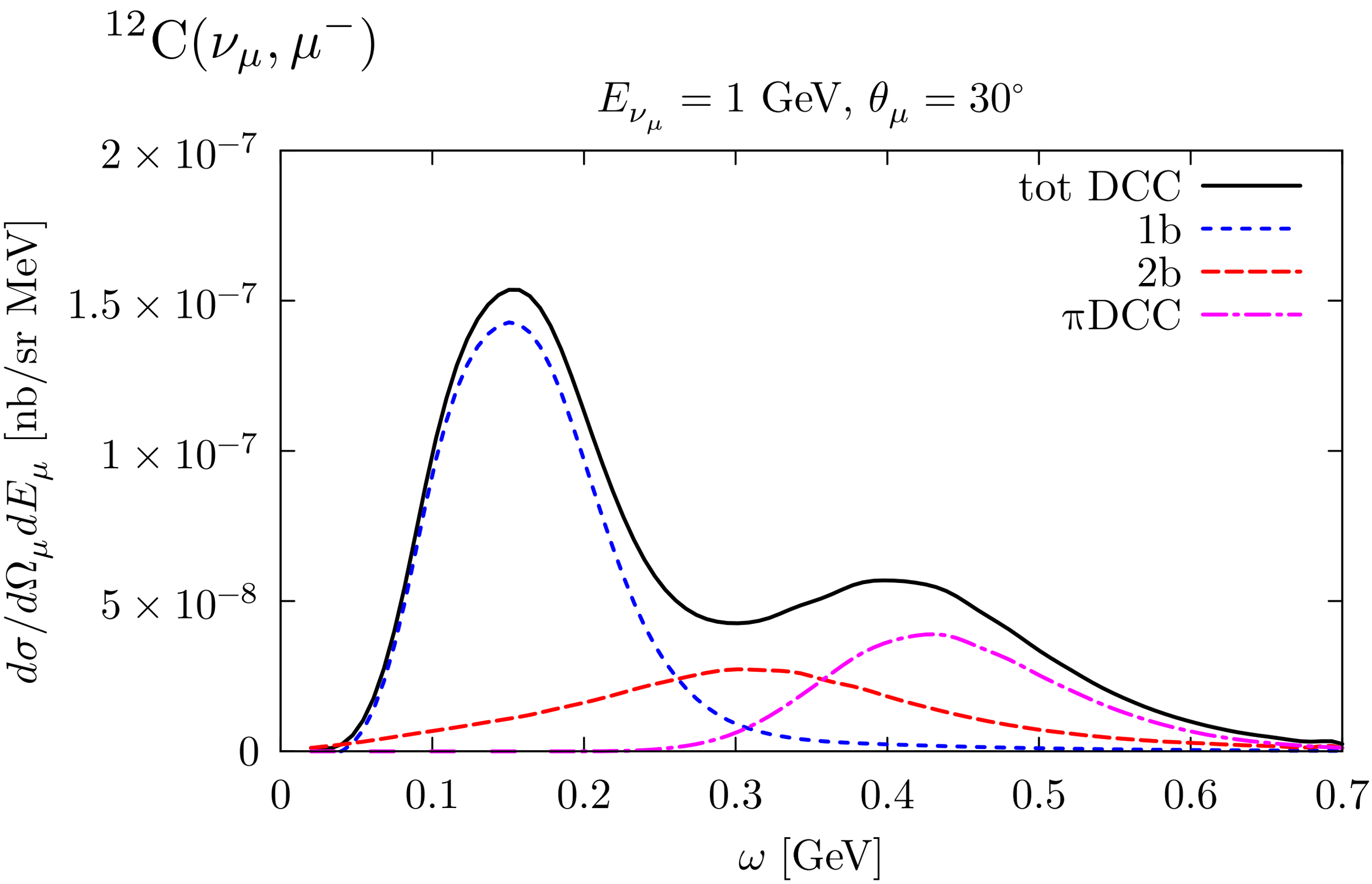
- The pion production mechanism can be described within the IA as

$$d\sigma_A = \int dE d^3k d^3k_\pi \, P(k,E) d\sigma_{\gamma N \to N\pi}$$

- One-body current operator for pion production is defined by its matrix element in momentum space

$$\langle pk_\pi | j_1^\mu | k \rangle = \langle pk_\pi | [j_1^\mu]_{\rm Born} | k \rangle + \langle pk_\pi | [j_1^\mu]_\Delta | k \rangle$$

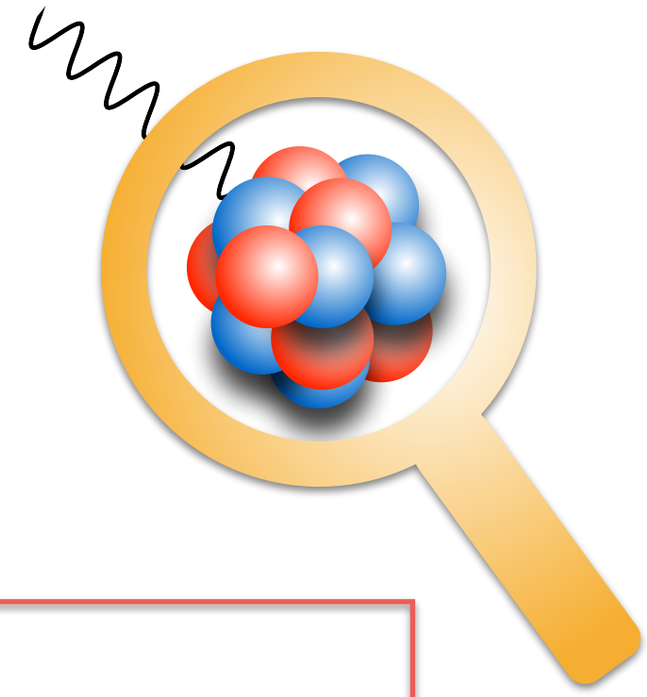
Example of THEO calculation using SF



From the Quasi-elastic to the Deep Inelastic Scattering

- Within the Spectral Function and Impulse Approximation Formalism

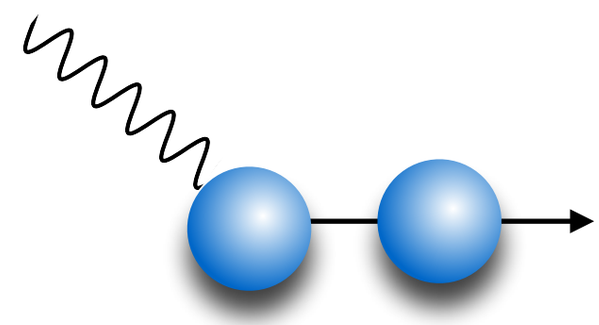
$$d\sigma_A = \int dE\, d^3k\, d\sigma_N\, P(\mathbf{k}, E)$$



$$d\sigma_N \propto W_2(\omega, q^2) cos^2\frac{\theta}{2} + 2W_1(\omega, q^2) sin^2\frac{\theta}{2}$$

$$W_1^p = -\frac{q^2}{4m^2} G_{M_p}^2 \delta(\omega + \frac{q^2}{2m})$$

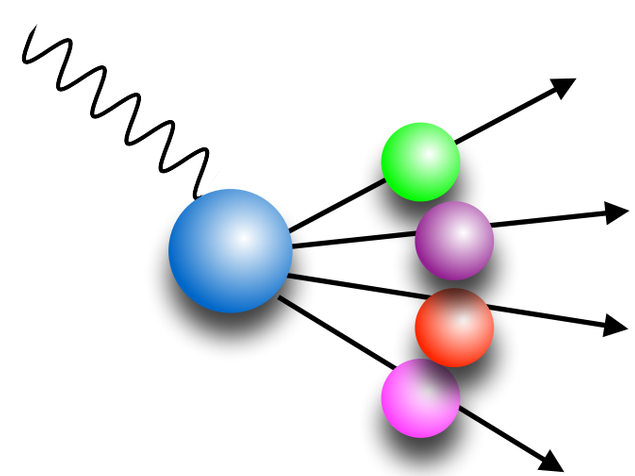
$$W_2^p = \frac{G_{E_p}^2 - q^2/(4m^2) G_{M_p}^2}{1 - q^2/(4m^2)} \delta(\omega + \frac{q^2}{2m})$$



$Q^2 \to \infty,\ \omega \to \infty$

$$\omega W_2(\omega, q^2) \to F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$mW_1(\omega, q^2) \to F_1(x) = \frac{1}{2x} F_2(x)$$



Final State Interactions

- Many theoretical models utilized to describe neutrino-nucleus interactions are based on the Impulse Approximation
- However: the applicability of these approaches has to be extended to regions in which the produced particles are not free
- Describe the propagation of the particles produced at the interaction vertex through the nucleus
- Final State Interactions include both elastic and inelastic reactions: elastic scattering with energy change, charge exchange, production of new particles, absorption
- Monte Carlo event generators includes different models of intra-nuclear cascades

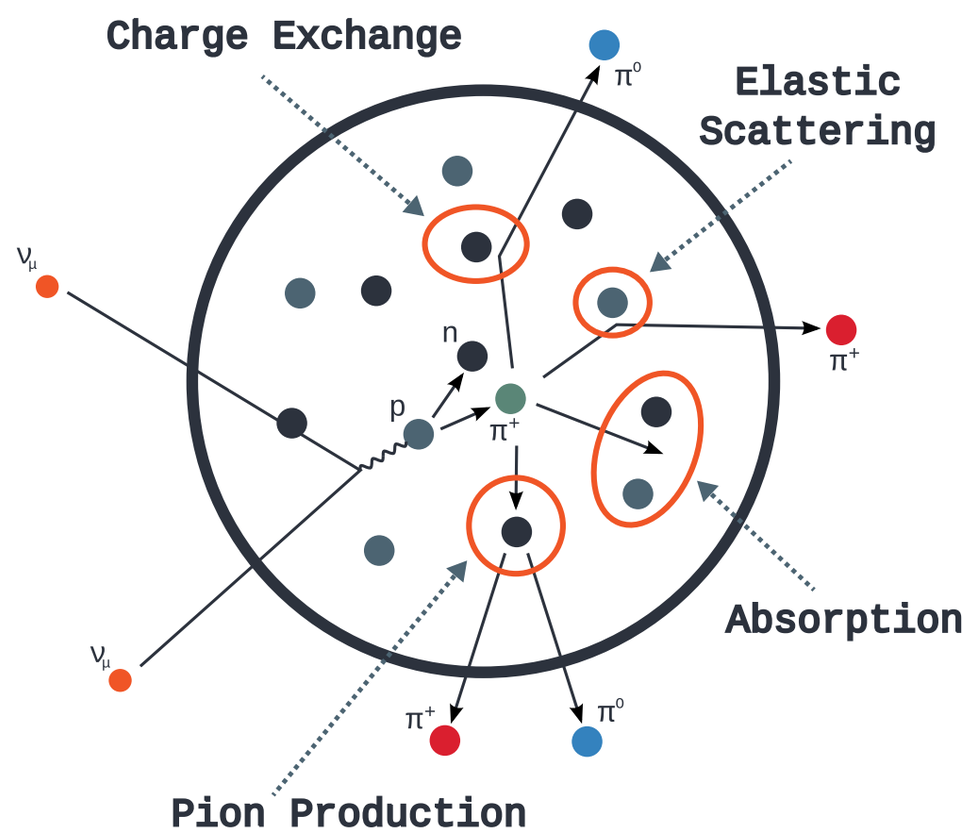
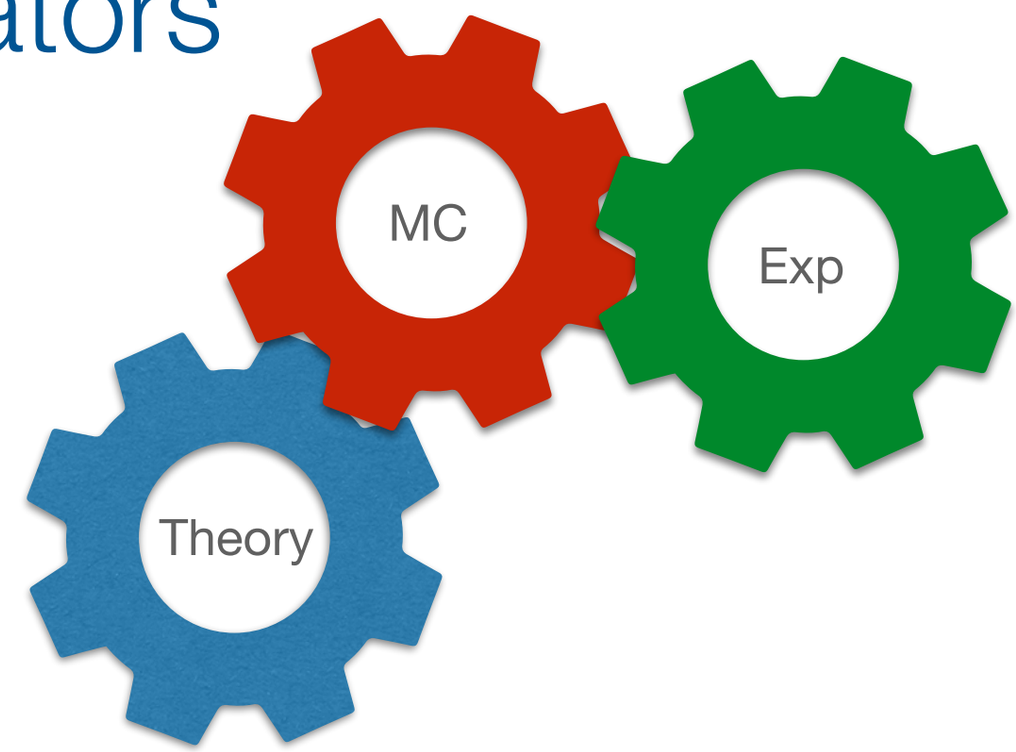


Figure by T. Golan

Monte Carlo Event Generators



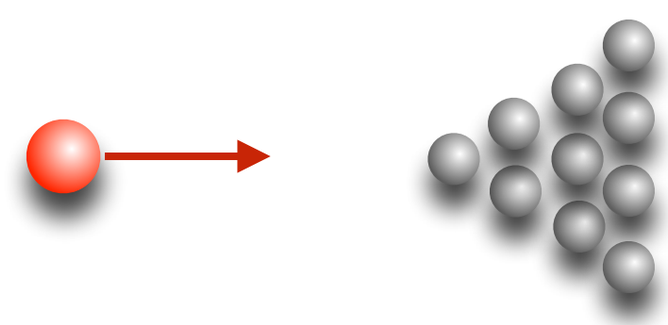
- Monte Carlo event generators connects theoretical predictions to experimental measurements

- Main features: integrations of cross sections, and event generation using acceptance and rejection techniques

- Main Event Generators:
  - Neutrino Interactions: GENIE, NEUT, GIBUU, NuWro
  - Particle transport in matter: Geant, FLUKA

- In the Intra-Nuclear Cascade: particles are assumed to be classical and move along a straight line

  This implies that INCLUSIVE quantities are not modified by the cascade



Comparison with electron-scattering data

✐ http://faculty.virginia.edu/qes-archive/data/index.html

Home Page
2H
3H
3He
4He
6Li
12C
16O
27Al
40Ca
48Ca
56Fe
208Pb
238U
Other
Nuclear Matter

QES Archive Data Page

Welcome to Quasielastic Electron Nucleus Scattering Archive Data page.

Click on the item to the left and you will be directed to a page where you can download the data

---

Data file structure

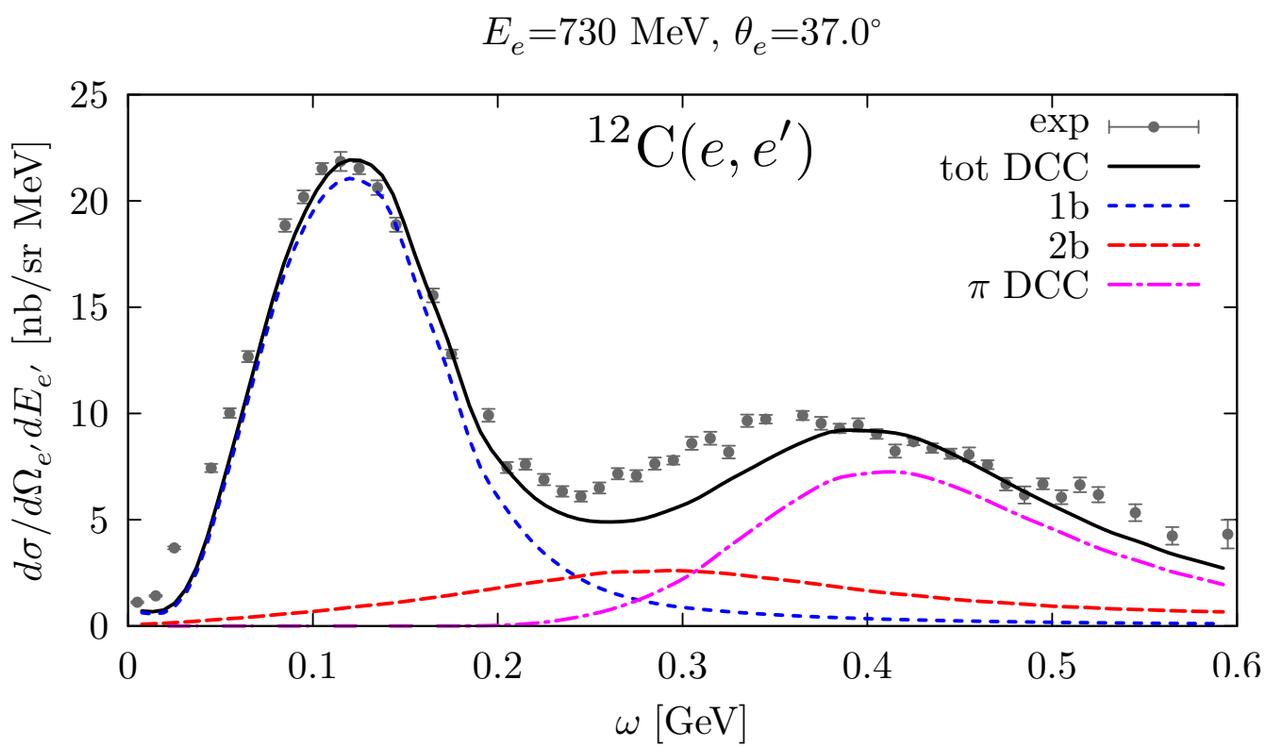
The data files consists of many lines, each with 8 (space delimited) columns as follows:

Z	A	E (GeV)	Theta (degrees)	energy loss (GeV)	sigma (nb/sr/GeV)	error (random)	citation (Spires notation)

This structure allows one to keep all the data (even all nuclei and all energies and angles) in a single file and extract particular data files with fortan, C, or even a simple awk command in a terminal (see the Utilities section).

- Comparing your theoretical calculation with electron scattering data is very simple!!

Comparison with electron-scattering data

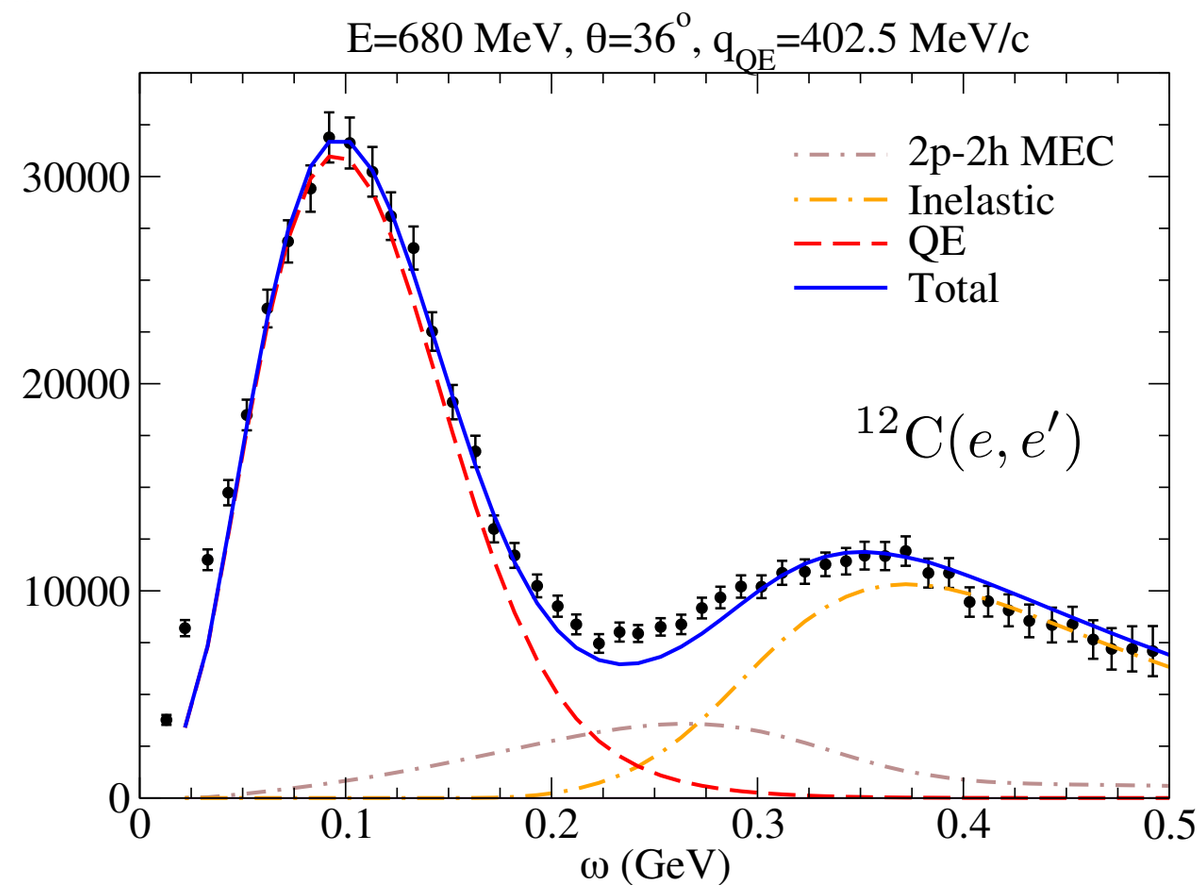


- Calculation performed using the Impulse Approximation and a realistic Spectral Function

✐ H. Lee, A. Lovato, S. Nakamura, and NR, arXiv:1907.01093

- Calculation performed using the SuSa v2 model and Global Fermi gas for the Meson Exchange Currents

✐ G.D. Megias PRD 94, 013012 (2016)



Comparison with MINERνA data

✐ MINERνA collaboration, Phys.Rev. D99 (2019) no.1, 012004

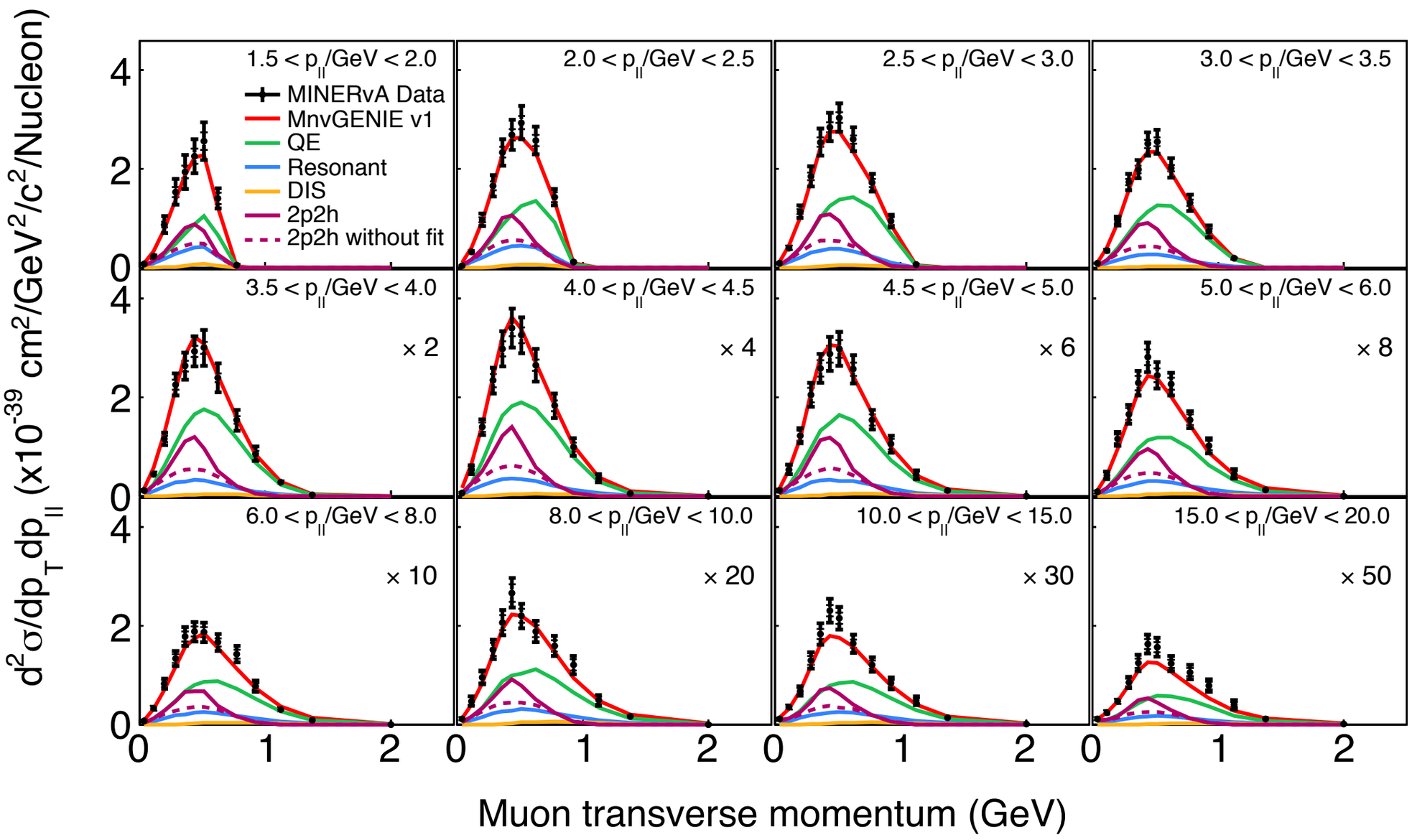
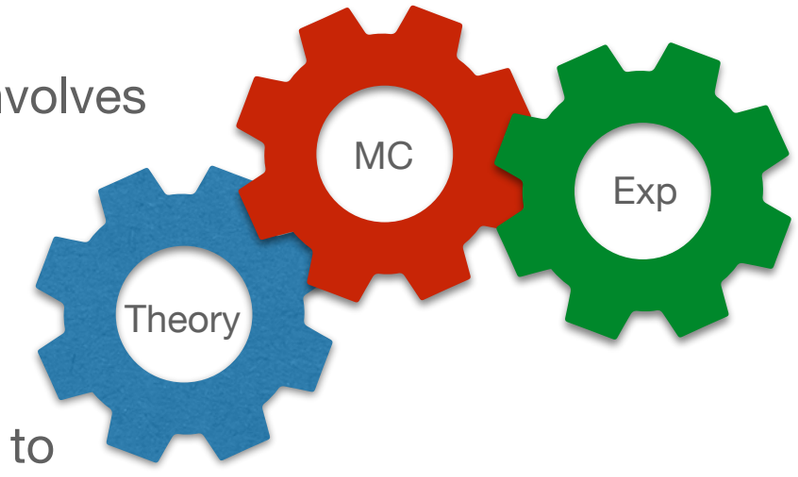


FIG. 27: Full double differential results projected as a function of p_t and also a function of $p_{||}$. Unstacked curves show the various components of the MINERvA GENIE tune v1. The dotted line denotes original strength and shape of the 2p2h contribution.

Conclusions

- Understanding neutrino-nucleus interactions is crucial for neutrino oscillation experiments

- To do this we rely on cross-sections: large important topic that involves



- What did we learn today about the Theory ?

  - Electron and Neutrino scattering are connected, we need to start from the electromagnetic case first and then extend to the electroweak sector

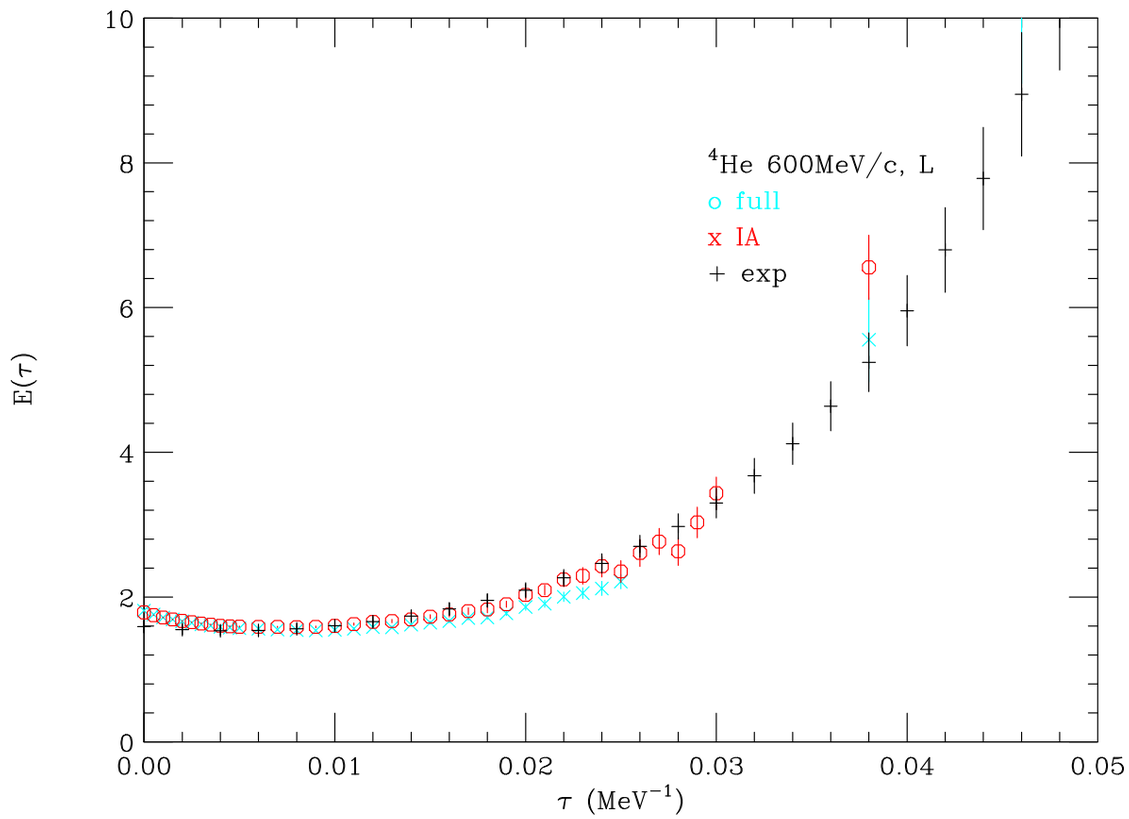
  - Nuclei are complicated objects, we need a good model to describe them

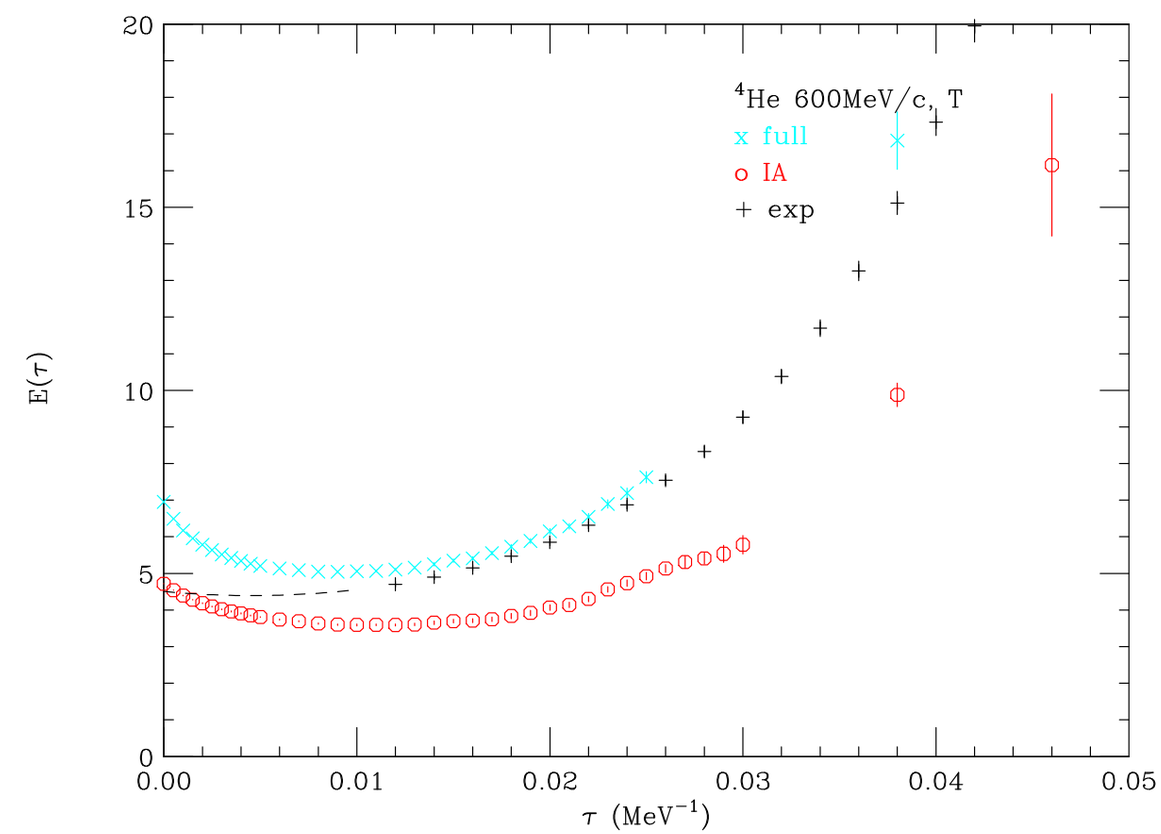
  - There are different reactions mechanisms at play: we need to account for all of them

- The study of neutrino properties has the unique feature of connecting: Lattice-QCD, Nuclear, and Particle Physics

Back up slides

Two-body currents violate scaling





✐ J.Carlson, J. Jourdan, R. Schiavilla, and I. Sick, Phys. Rev. C 65, 024002 (2002)

- Euclidean Response: Laplace transform of the electromagnetic response

$$E(\tau, \mathbf{q}) = \int d\omega e^{-\omega\tau} R(\omega, \mathbf{q})$$

- The inclusion of two-body currents leads to a significant enhancement in the transverse channel: needed to correctly reproduce the experimental data

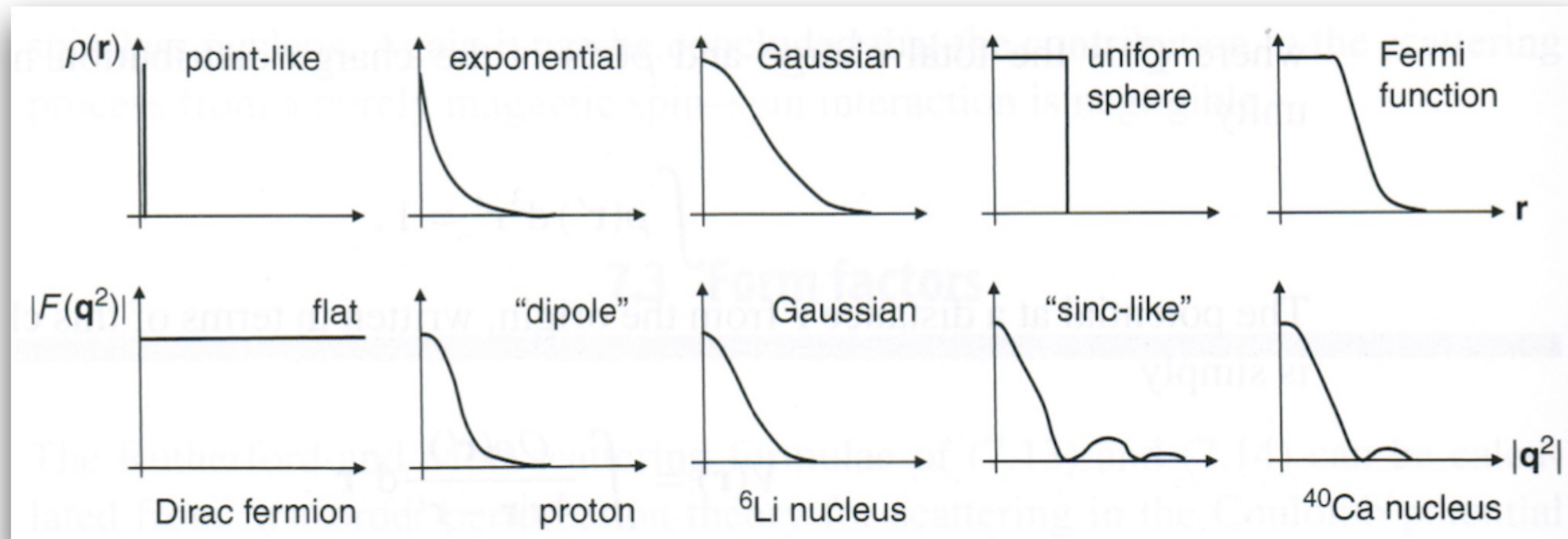
Nucleon Form Factor

- Accounts for the finite size of the nucleon

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot |F(\mathbf{q}^2)|^2$$

- Form factor and electric charge distribution are Fourier pairs

$$F(\mathbf{q}^2) = \int d^3\mathbf{r}\ \rho(\mathbf{r}) \cdot e^{i\mathbf{q}\cdot\mathbf{r}}$$



✐ Thomson, M.: Modern Particle Physics, Cambridge University Press, 2013