INTRODUCTION TO THE PHYSICS OF MASSIVE NEUTRINOS I

Concha Gonzalez-Garcia

(YITP-Stony Brook & ICREA-University of Barcelona)

INSS2019, Fermilab, August 2019

Global fit to neutrino oscillation data

http://www.nu-fit.org

elusives neutrinos, dark matter & dark energy physics

Plan of Lecture I

Historic Introduction

Neutrinos in the SM

Neutrino Properties:
Helicity versus Chirality, Majorana versus Dirac

Neutrino Mass Terms Beyond the SM:
Dirac, Majorana, the See-Saw Mechanisms, Lepton Mixing

Probes of Neutrino Mass Scale

Plan of Lecture II

Neutrino Oscillations in Vacuum

Matter Potentials

Flavour Transitions in Matter: MSW

Present Status

Discovery of ν's

- At end of 1800's radioactivity was discovered and three types identified: α, β, γ

  β : an electron comes out of the radioactive nucleus.

- Energy conservation $\Rightarrow$ e^- should have had a fixed energy

$$(A, Z) \to (A, Z+1) + e^- \;\Rightarrow\; E_e = M(A, Z+1) - M(A, Z)$$

Discovery of ν's

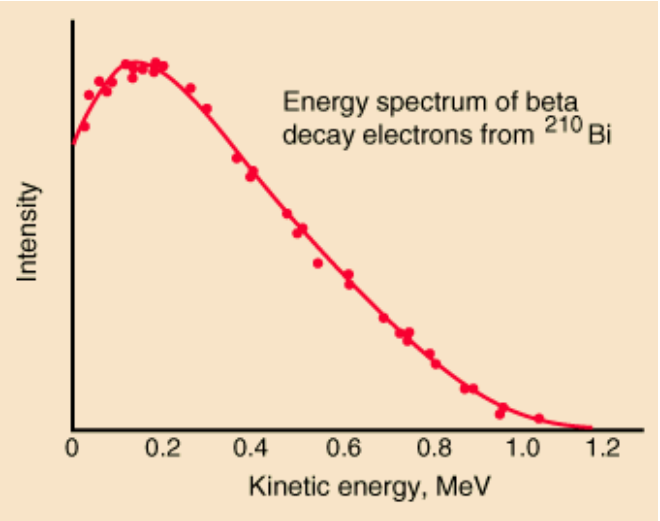
- At end of 1800's radioactivity was discovered and three types identified: α, β, γ

  β : an electron comes out of the radioactive nucleus.

- Energy conservation $\Rightarrow$ e^- should have had a fixed energy

$$(A, Z) \to (A, Z+1) + e^- \;\Rightarrow\; E_e = M(A, Z+1) - M(A, Z)$$

But 1914 James Chadwick showed that the electron energy spectrum is continuous



Discovery of ν's

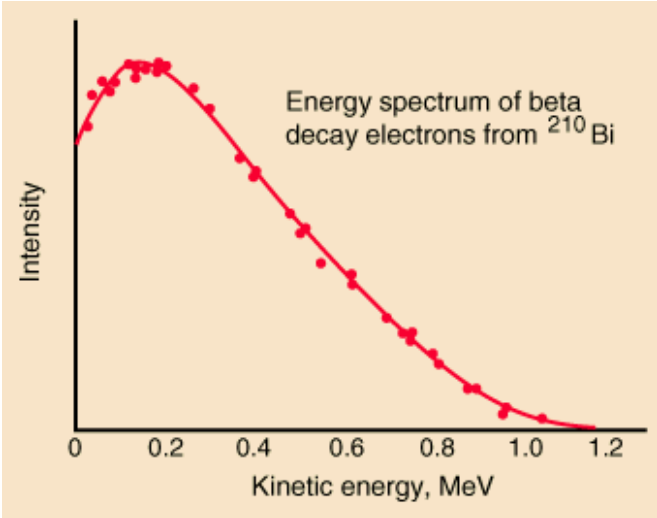
- At end of 1800's radioactivity was discovered and three types identified: α, β, γ

  β : an electron comes out of the radioactive nucleus.

- Energy conservation $\Rightarrow$ e^- should have had a fixed energy

$$(A, Z) \to (A, Z+1) + e^- \;\Rightarrow\; E_e = M(A, Z+1) - M(A, Z)$$

But 1914 James Chadwick showed that the electron energy spectrum is continuous



Do we throw away the energy conservation?

Bohr: *we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β ray disintegrations*

Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of "the energy conservation principle".

In his letter addressed to the *Liebe Radioaktive Damen und Herren* (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as *constituent of nuclei, the neutron* ν, able to explain the continuous spectrum of nuclear beta decay

$$(A, Z) \to (A, Z{+}1){+}e^- {+}\nu$$

Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of "the energy conservation principle".

In his letter addressed to the *Liebe Radioaktive Damen und Herren* (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as *constituent of nuclei, the neutron* ν, able to explain the continuous spectrum of nuclear beta decay

$$(A, Z) \to (A, Z{+}1){+}e^- {+}\nu$$

• The ν is light (in Pauli's words:
m_ν *should be of the same order as the* m_e),
neutral and has spin 1/2

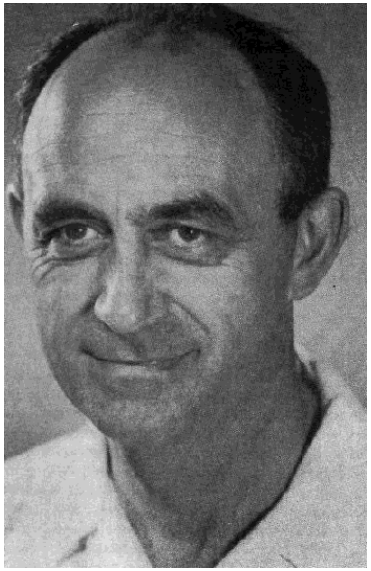
Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of "the energy conservation principle".

In his letter addressed to the *Liebe Radioaktive Damen und Herren* (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as *constituent of nuclei, the neutron* ν, able to explain the continuous spectrum of nuclear beta decay

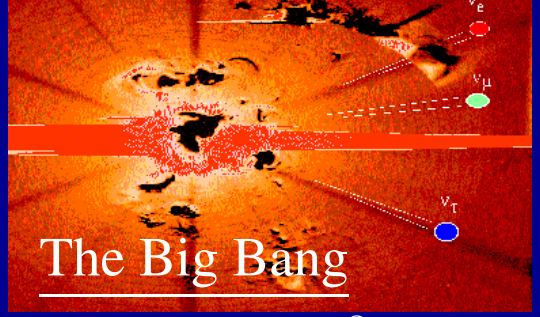
$$(A, Z) \to (A, Z{+}1){+}e^- {+}\nu$$

• The ν is light (in Pauli's words: m_ν *should be of the same order as the* m_e), neutral and has spin 1/2



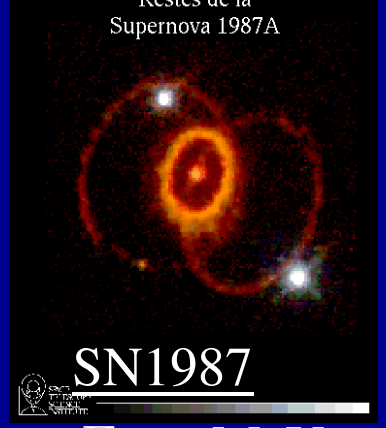
• In order to distinguish them from heavy neutrons, Fermi proposed to name them neutrinos.

Sources of ν's

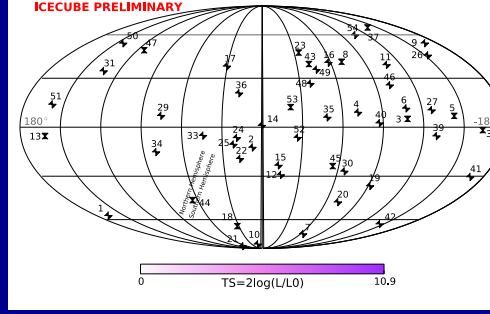


$\rho_\nu = 330/\text{cm}^3$

$p_\nu = 0.0004$ eV



$E_\nu \sim$ MeV



ExtraGalactic

$E_\nu \gtrsim 30$ TeV

The Sun

ν_e

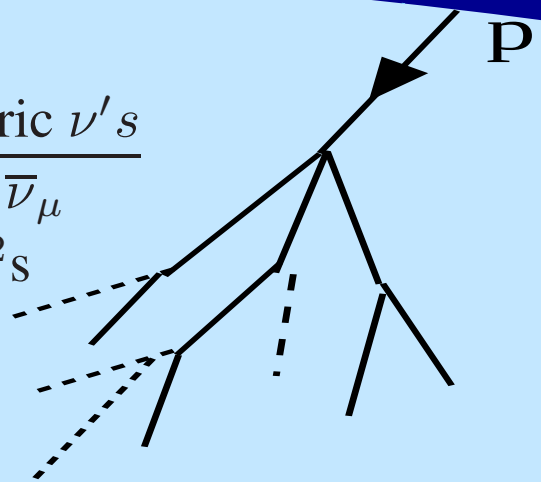
$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim 0.1$–20 MeV

Atmospheric $\nu's$

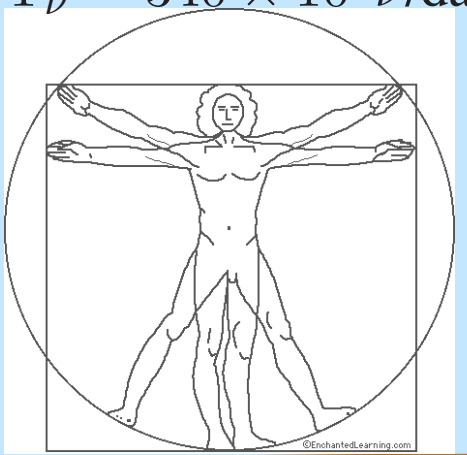
$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1\nu/\text{cm}^2\text{s}$



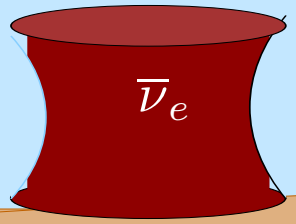
Human Body

$\Phi_\nu = 340 \times 10^6 \nu/\text{day}$



Nuclear Reactors

$E_\nu \sim$ few MeV



Earth's radioactivity

$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators

$E_\nu \simeq 0.3$–30 GeV

Neutrino Detection

Fighting Pauli's "Curse":

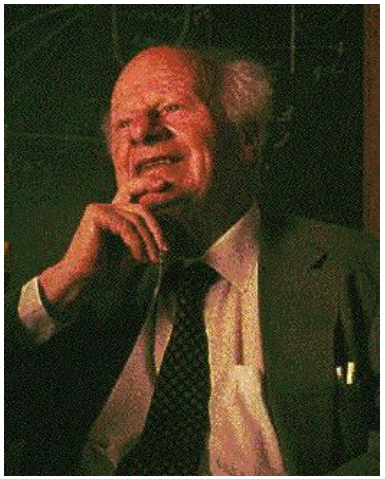
I have done a terrible thing, I have postulated a particle that cannot be detected.

Neutrino Detection

But in principle seems easy!: If β decay $n \rightarrow p + e^- + \nu$

Then $\nu + p \rightarrow e^+ + n$

Neutrino Detection

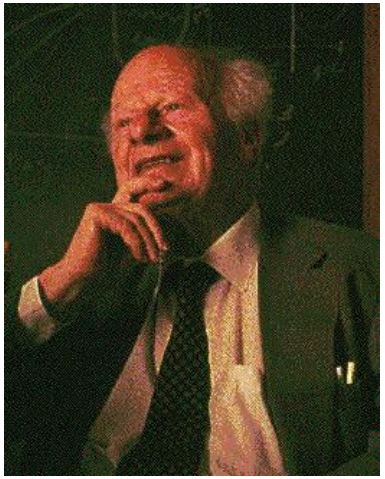


But in principle seems easy!: If β decay $n \to p + e^- + \nu$

Then $\nu + p \to e^+ + n$

Problem: Already in 1934, Hans Bethe showed that the probability of this interaction was so small that a solar ν could cross the whole Earth without ever interacting with it

Neutrino Detection



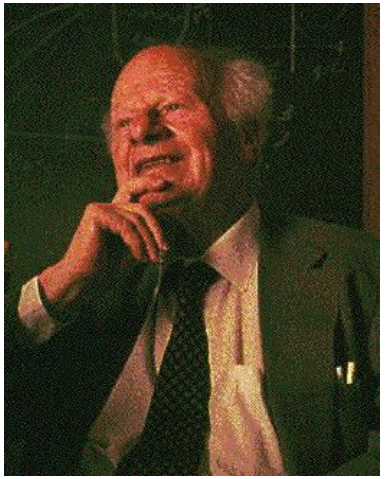
But in principle seems easy!: If β decay $n \to p + e^- + \nu$

Then $\nu + p \to e^+ + n$

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38} \mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

Neutrino Detection



But in principle seems easy!: If β decay $n \to p + e^- + \nu$

Then $\nu + p \to e^+ + n$

Problem: Quantitatively: a ν sees a proton of area:

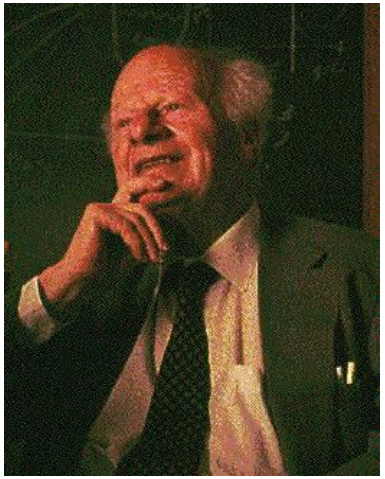
$$\sigma^{\nu p} \sim 10^{-38} \mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

- So let's consider the atmospheric ν's:

$$\Phi_\nu^{\mathrm{ATM}} = 1\, \nu/(\,\mathrm{cm}^2\ \mathrm{second}) \ \ \mathrm{y} \ \ \langle E_\nu \rangle = 1\ \mathrm{GeV}$$

- How many interact?

Neutrino Detection



But in principle seems easy!: If β decay $n \to p + e^- + \nu$

Then $\nu + p \to e^+ + n$

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38}\mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

- So let's consider the atmospheric ν's:

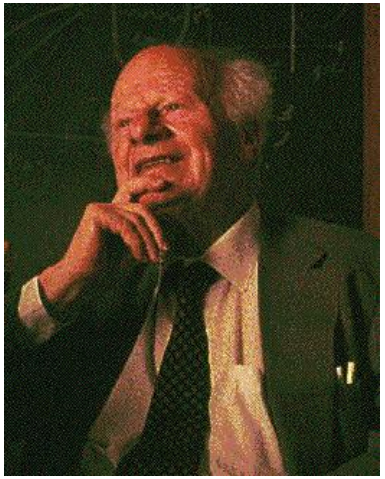
$$\Phi_\nu^{\mathrm{ATM}} = 1\,\nu/(\,\mathrm{cm}^2\ \mathrm{second}) \ \ \mathrm{y} \ \ \langle E_\nu \rangle = 1\ \mathrm{GeV}$$

- How many interact? In a human body

$$N_{\mathrm{int}} = \Phi_\nu \times \sigma^{\nu p} \times N_{\mathrm{prot}}^{\mathrm{human}} \times T_{\mathrm{life}}^{\mathrm{human}}$$

$$\left.\begin{array}{l} N_{\mathrm{protons}}^{\mathrm{human}} = \frac{M^{\mathrm{human}}}{gr} \times N_A = 80\mathrm{kg} \times N_A \sim 5 \times 10^{28}\mathrm{protons} \\ T^{\mathrm{human}} = 80\,\mathrm{years} = 2 \times 10^9\,\mathrm{sec} \end{array}\right\} \begin{array}{l} (M \times T \equiv \mathrm{Exposure}) \\ \mathrm{Exposure}_{\mathrm{human}} \\ \sim \mathrm{Ton} \times \mathrm{year} \end{array}$$

Neutrino Detection



But in principle seems easy!: If β decay $n \to p + e^- + \nu$

Then $\nu + p \to e^+ + n$

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38}\mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

- So let's consider the atmospheric ν's:

$$\Phi_\nu^{\mathrm{ATM}} = 1\,\nu/(\,\mathrm{cm}^2\ \mathrm{second}) \ \ \mathrm{y} \ \ \langle E_\nu \rangle = 1\ \mathrm{GeV}$$

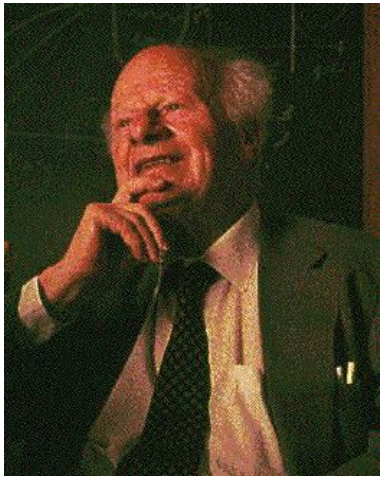
- How many interact? In a human body

$$N_{\mathrm{int}} = \Phi_\nu \times \sigma^{\nu p} \times N_{\mathrm{prot}}^{\mathrm{human}} \times T_{\mathrm{life}}^{\mathrm{human}}$$

$$\left.\begin{array}{l} N_{\mathrm{protons}}^{\mathrm{human}} = \frac{M^{\mathrm{human}}}{gr} \times N_A = 80\mathrm{kg} \times N_A \sim 5 \times 10^{28}\mathrm{protons} \\ T^{\mathrm{human}} = 80\,\mathrm{years} = 2 \times 10^9\ \mathrm{sec} \end{array}\right\} \begin{array}{l} (M \times T \equiv \mathrm{Exposure}) \\ \mathrm{Exposure}_{\mathrm{human}} \\ \sim \mathrm{Ton} \times \mathrm{year} \end{array}$$

$$N_{\mathrm{int}} = (5 \times 10^{28})\,(2 \times 10^9) \times 10^{-38} \sim 1\,\mathrm{interaction\ in\ life}$$

Neutrino Detection



But in principle seems easy!: If β decay $n \to p + e^- + \nu$

Then $\nu + p \to e^+ + n$

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38}\mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

- So let's consider the atmospheric ν's:

$$\Phi_\nu^{\mathrm{ATM}} = 1\,\nu/(\,\mathrm{cm}^2\ \mathrm{second}) \ \ \mathrm{y} \ \ \langle E_\nu \rangle = 1\ \mathrm{GeV}$$

- How many interact? In a human body

$$N_{\mathrm{int}} = \Phi_\nu \times \sigma^{\nu p} \times N_{\mathrm{prot}}^{\mathrm{human}} \times T_{\mathrm{life}}^{\mathrm{human}}$$

$$N_{\mathrm{protons}}^{\mathrm{human}} = \frac{M^{\mathrm{human}}}{gr} \times N_A = 80\mathrm{kg} \times N_A \sim 5 \times 10^{28}\mathrm{protons}$$

$$T^{\mathrm{human}} = 80\,\mathrm{years} = 2 \times 10^9\ \mathrm{sec}$$

$(M \times T \equiv \mathrm{Exposure})$

$\mathrm{Exposure}_{\mathrm{human}}$

$\sim \mathrm{Ton} \times \mathrm{year}$

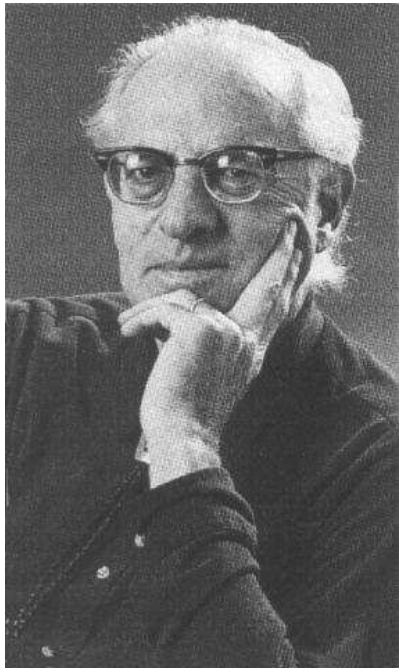
$$N_{\mathrm{int}} = (5 \times 10^{28})\,(2 \times 10^9) \times 10^{-38} \sim 1\,\mathrm{interaction\ in\ life}$$

To detect neutrinos we need very intense source and/or a hugh detector with Exposure $\sim$ KTon $\times$ year

Lectures M. Messier

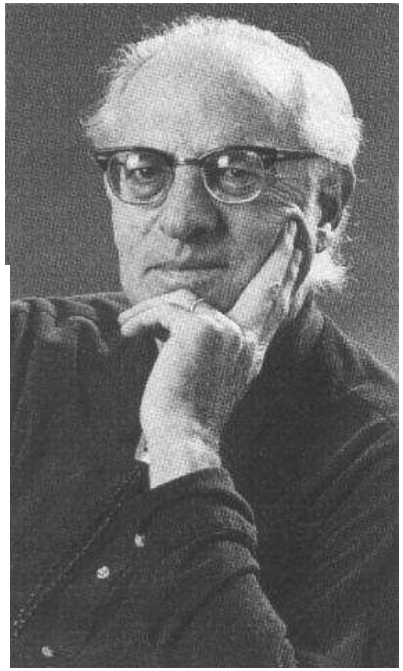
First Neutrino Detection

In 1953 Frederick Reines and Clyde Cowan put a detector near a nuclear reactor (the most intense source available)

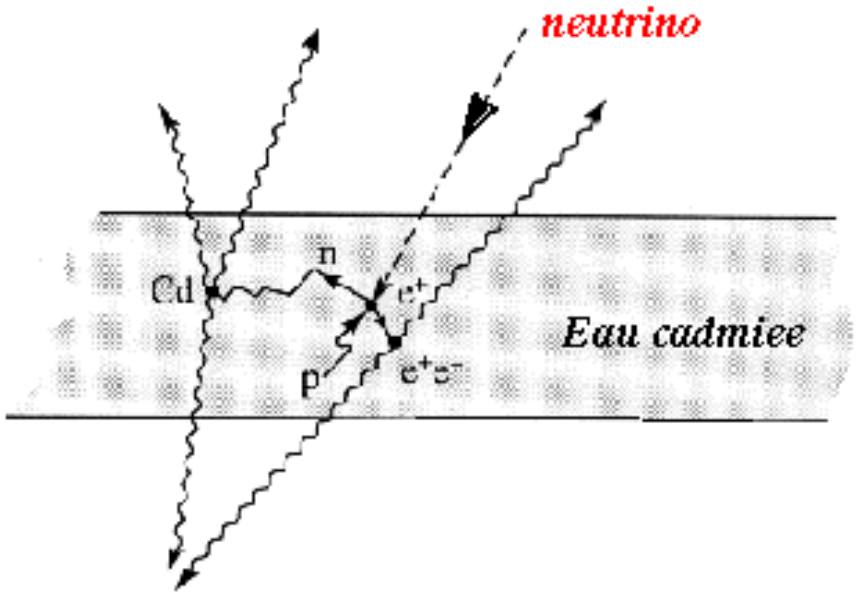


First Neutrino Detection

In 1953 Frederick Reines and Clyde Cowan put a detector near a nuclear reactor (the most intense source available)



400 l of water
and Cadmium Chloride.



e^+ annihilates with e^- in the water and produces two γ's simultaneouoly.
neutron is captured by por the cadmium and a γ's is emitted 15 msec latter

Reines y Clyde saw clearly this signature: the first neutrino had been detected

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

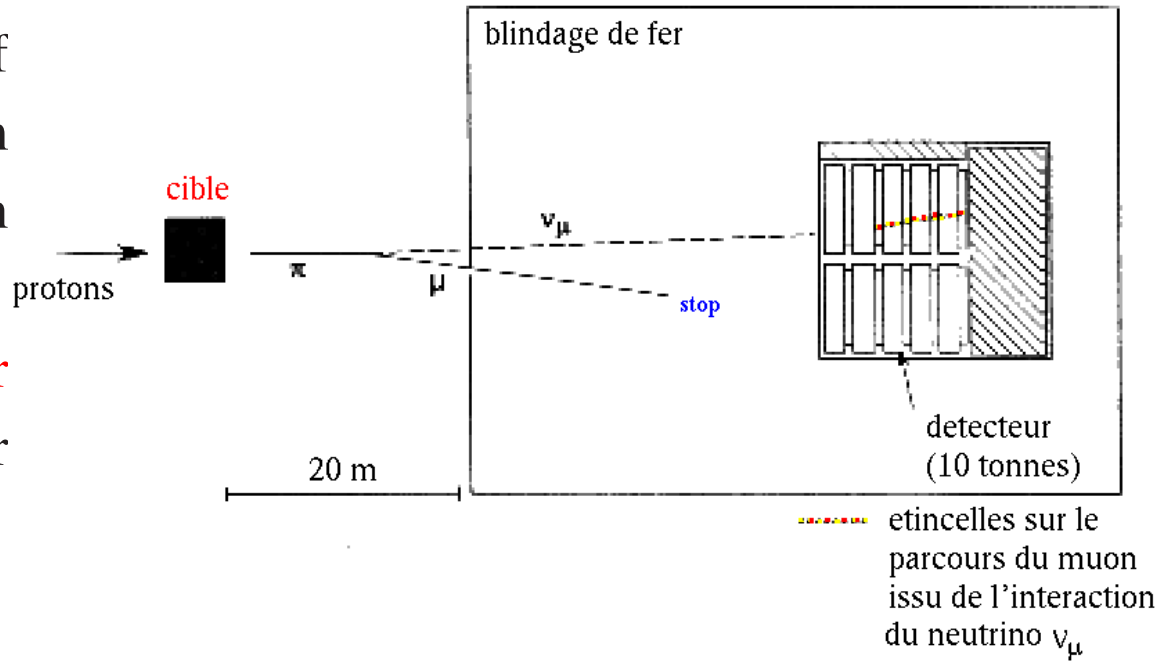
Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

In 1959 M. Schwartz thought of producing an intense ν beam from π's decay (produced when a proton beam of GeV energy hits matter) Schwartz, Lederman, Steinberger and Gaillard built a spark chamber (a 10 tons of neon gas) to detect ν_μ

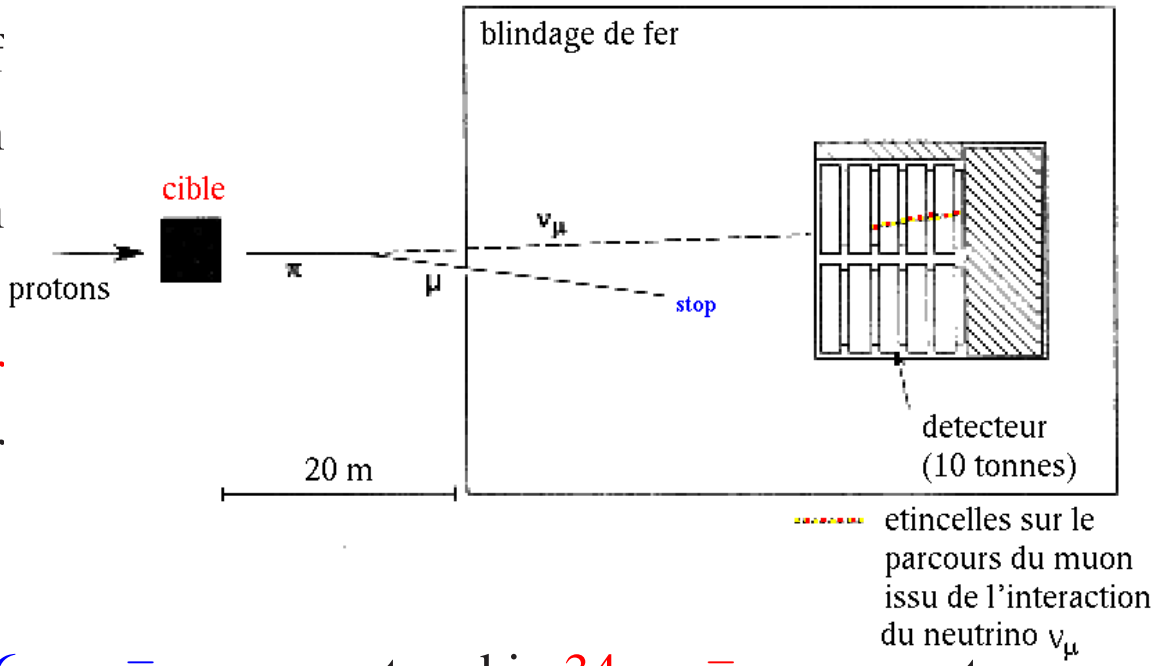


The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

In 1959 M. Schwartz thought of producing an intense ν beam from π's decay (produced when a proton beam of GeV energy hits matter) Schwartz, Lederman, Steinberger and Gaillard built a spark chamber (a 10 tons of neon gas) to detect ν_μ



They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

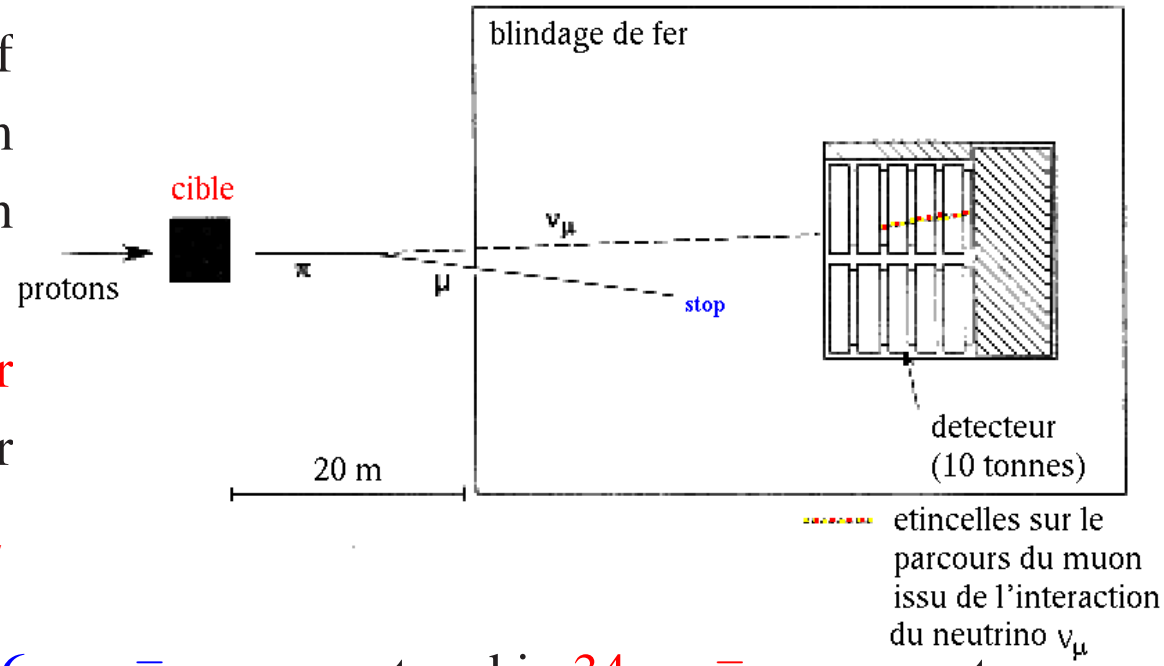
If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and $e-$

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

In 1959 M. Schwartz thought of producing an intense ν beam from π's decay (produced when a proton beam of GeV energy hits matter) Schwartz, Lederman, Steinberger and Gaillard built a spark chamber (a 10 tons of neon gas) to detect ν_μ



They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

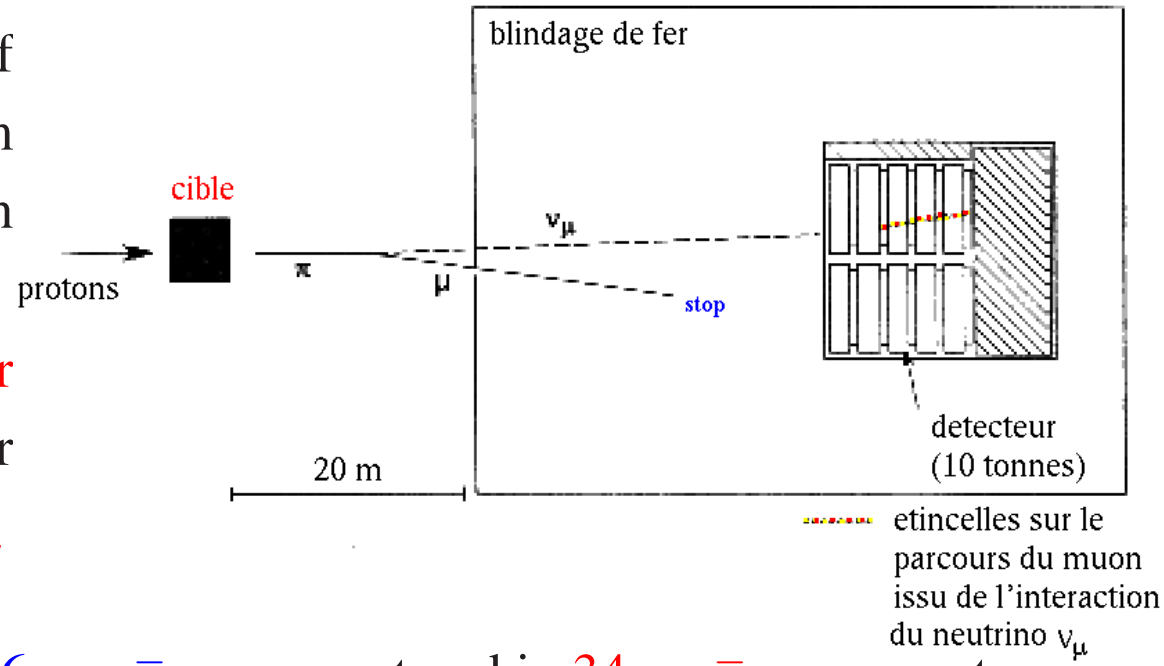
If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and $e-$ $\Rightarrow$ Conclusion: ν_μ is a different particle

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

In 1959 M. Schwartz thought of producing an intense ν beam from π's decay (produced when a proton beam of GeV energy hits matter) Schwartz, Lederman, Steinberger and Gaillard built a spark chamber (a 10 tons of neon gas) to detect ν_μ



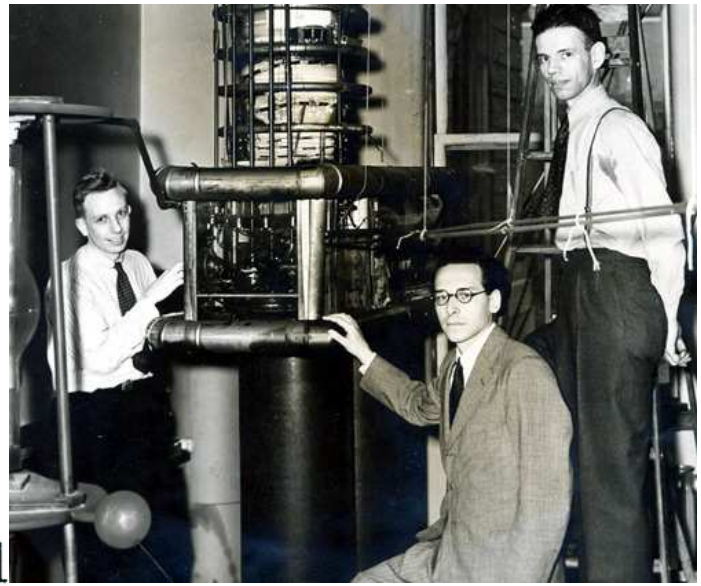
They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and $e-$ $\Rightarrow$ Conclusion: ν_μ is a different particle

In 1977 Martin Perl discovers the particle tau $\equiv$ the third lepton family.

The ν_τ was observed by DONUT experiment at FNAL in 1998 (*officially* in Dec. 2000).

Neutrinos = "Left-handed"



Helicity of Neutrinos*

M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR
Brookhaven National Laboratory, Upton, New York
(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

ν in the SM

ν in the SM

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

ν in the SM

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

- 3 Generations of Fermions:

$(1, 2, -\frac{1}{2})$	$(3, 2, \frac{1}{6})$	$(1, 1, -1)$	$(3, 1, \frac{2}{3})$	$(3, 1, -\frac{1}{3})$
L_L	Q^i_L	E_R	U^i_R	D^i_R
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

ν in the SM

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

- 3 Generations of Fermions:

$(1, 2, -\frac{1}{2})$	$(3, 2, \frac{1}{6})$	$(1, 1, -1)$	$(3, 1, \frac{2}{3})$	$(3, 1, -\frac{1}{3})$
L_L	Q^i_L	E_R	U^i_R	D^i_R
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

- Spin-0 particle ϕ: $(1, 2, \frac{1}{2})$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \overset{SSB}{\rightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

ν in the SM

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

- 3 Generations of Fermions:

$(1, 2, -\frac{1}{2})$	$(3, 2, \frac{1}{6})$	$(1, 1, -1)$	$(3, 1, \frac{2}{3})$	$(3, 1, -\frac{1}{3})$
L_L	Q_L^i	E_R	U_R^i	D_R^i
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u_R^i	d_R^i
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c_R^i	s_R^i
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t_R^i	b_R^i

$$Q_{EM} = T_{L3} + Y$$

- ν's are $T_{L3} = \frac{1}{2}$ components lepton doublet L_L
- ν's have no strong or EM interactions
- No ν_R (they are singlets of gauge group)

- Spin-0 particle ϕ: $(1, 2, \frac{1}{2})$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \overset{SSB}{\rightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

Helicity versus Chirality

Helicity versus Chirality

- We define the chiral projections $P_{R,L} = \frac{1 \pm \gamma_5}{2}$

$$\psi = \psi_L + \psi_R \qquad \psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

Helicity versus Chirality

- We define the chiral projections $P_{R,L} = \frac{1 \pm \gamma_5}{2}$

$$\psi = \psi_L + \psi_R \qquad \psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

- In the SM the neutrino interaction terms

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}}[j^+_\mu W^-_\mu + j^-_\mu W^+_\mu] + \frac{i\,g}{\sqrt{2}\cos\theta_W} j^Z_\mu Z_\mu$$

$$j^-_\mu = \bar{l}_\alpha \gamma_\mu P_L \nu_\alpha \quad \alpha = e, \mu, \tau \quad j^+_\mu = j^{-\dagger}_\mu \quad j^Z_\mu = \bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha$$

Helicity versus Chirality

- We define the chiral projections $P_{R,L} = \frac{1 \pm \gamma_5}{2}$

$$\psi = \psi_L + \psi_R \qquad \psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

- In the SM the neutrino interaction terms

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}}[j_\mu^+ W_\mu^- + j_\mu^- W_\mu^+] + \frac{i\,g}{\sqrt{2}\cos\theta_W} j_\mu^Z Z_\mu$$

$$j_\mu^- = \bar{l}_\alpha \gamma_\mu P_L \nu_\alpha \quad \alpha = e, \mu, \tau \quad j_\mu^+ = j_\mu^{-\dagger} \quad j_\mu^Z = \bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha$$

$\Rightarrow$ ν_L interact and ν_R do not interact

$\Rightarrow$ chirality states are physical states for weak interactions

Helicity versus Chirality

- We define the chiral projections $P_{R,L} = \frac{1 \pm \gamma_5}{2}$

$$\psi = \psi_L + \psi_R \qquad \psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

- In the SM the neutrino interaction terms

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}}[j^+_\mu W^-_\mu + j^-_\mu W^+_\mu] + \frac{i\,g}{\sqrt{2}\cos\theta_W} j^Z_\mu Z_\mu$$

$$j^-_\mu = \bar{l}_\alpha \gamma_\mu P_L \nu_\alpha \quad \alpha = e, \mu, \tau \quad j^+_\mu = j^{-\dagger}_\mu \quad j^Z_\mu = \bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha$$

$\Rightarrow$ ν_L interact and ν_R do not interact

$\Rightarrow$ chirality states are physical states for weak interactions

- However what Goldhaber measured was the helicity not the chirality of ν

Neutrino Helicity

Neutrino Helicity

- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^* \to {}^{152}Sm + \gamma$$

with $J({}^{152}Eu) = J({}^{152}Sm) = 0$ and $L(e^-) = 0$

Neutrino Helicity

- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^* \to {}^{152}Sm + \gamma$$

with $J({}^{152}Eu) = J({}^{152}Sm) = 0$ and $L(e^-) = 0$

- Angular momentum conservation $\Rightarrow$

$$\begin{cases} J_z(e^-) = J_z(\nu) + J_z(Sm^*) \\ \qquad\quad = J_z(\nu) + J_z(\gamma) \\ \pm\frac{1}{2} = \pm\frac{1}{2} \quad \pm 1 \Rightarrow J_z(\nu) = -\frac{1}{2} J_z(\gamma) \end{cases}$$

Neutrino Helicity

- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^* \to {}^{152}Sm + \gamma$$

with $J({}^{152}Eu) = J({}^{152}Sm) = 0$ and $L(e^-) = 0$

- Angular momentum conservation $\Rightarrow$

$$\begin{cases} J_z(e^-) = J_z(\nu) + J_z(Sm^*) \\ \quad\quad\quad = J_z(\nu) + J_z(\gamma) \\ \pm\frac{1}{2} = \pm\frac{1}{2} \quad \pm 1 \Rightarrow J_z(\nu) = -\frac{1}{2} J_z(\gamma) \end{cases}$$

- Nuclei are heavy $\Rightarrow \vec{p}({}^{152}Eu) \simeq \vec{p}({}^{152}Sm) \simeq \vec{p}({}^{152}Sm^*) = 0$

So momentum conservation $\Rightarrow \vec{p}(\nu) = -\vec{p}(\gamma) \Rightarrow$ ν helicity= γ helicity

Neutrino Helicity

- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^* \to {}^{152}Sm + \gamma$$

with $J({}^{152}Eu) = J({}^{152}Sm) = 0$ and $L(e^-) = 0$

- Angular momentum conservation $\Rightarrow$

$$\begin{cases} J_z(e^-) = J_z(\nu) + J_z(Sm^*) \\ \quad\quad\quad = J_z(\nu) + J_z(\gamma) \\ \pm\frac{1}{2} = \pm\frac{1}{2} \quad \pm 1 \Rightarrow J_z(\nu) = -\frac{1}{2} J_z(\gamma) \end{cases}$$

- Nuclei are heavy $\Rightarrow \vec{p}({}^{152}Eu) \simeq \vec{p}({}^{152}Sm) \simeq \vec{p}({}^{152}Sm^*) = 0$

  So momentum conservation $\Rightarrow \vec{p}(\nu) = -\vec{p}(\gamma) \Rightarrow$ ν helicity= γ helicity

- Goldhaber et al found γ had negative helicity $\Rightarrow$ ν has helicity -1

Helicity versus Chirality

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma \cdot \partial - m)\psi(x)$

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma \cdot \partial - m)\psi(x)$

- The Equation of Motion is: $i\dfrac{\partial}{\partial t}\psi = H\psi = \gamma^0\left(\vec{\gamma}.\vec{p} + m\right)\psi$

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma \cdot \partial - m)\psi(x)$

- The Equation of Motion is: $i\dfrac{\partial}{\partial t}\psi = H\psi = \gamma^0\left(\vec{\gamma}.\vec{p} + m\right)\psi$

- has 4 possible solutions $(\gamma \cdot p - m)u_s(\vec{p}) = 0$ $\qquad$ $(\gamma \cdot p + m)v_s(\vec{p}) = 0$

$s = \pm\frac{1}{2}$ and $u_s(\vec{p})$ and $v_s(\vec{p})$ are the four component Dirac spinors.

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma \cdot \partial - m)\psi(x)$

- The Equation of Motion is: $i\dfrac{\partial}{\partial t}\psi = H\psi = \gamma^0\left(\vec{\gamma}.\vec{p} + m\right)\psi$

- has 4 possible solutions $(\gamma \cdot p - m)u_s(\vec{p}) = 0$ $\qquad$ $(\gamma \cdot p + m)v_s(\vec{p}) = 0$

$s = \pm\frac{1}{2}$ and $u_s(\vec{p})$ and $v_s(\vec{p})$ are the four component Dirac spinors.

- For this free fermion $[H, \vec{J}]$=0 and $[\vec{p}, \vec{J}.\vec{p}]$=0 with $\vec{J} = \vec{L} + \dfrac{\vec{\sigma}}{2}$ $\qquad$ $(\sigma^i = -\gamma^0\gamma^5\gamma^i)$

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma \cdot \partial - m)\psi(x)$

- The Equation of Motion is: $i\frac{\partial}{\partial t}\psi = H\psi = \gamma^0 \left(\vec{\gamma}.\vec{p} + m\right)\psi$

- has 4 possible solutions $(\gamma \cdot p - m)u_s(\vec{p}) = 0$ $\qquad$ $(\gamma \cdot p + m)v_s(\vec{p}) = 0$

$s = \pm\frac{1}{2}$ and $u_s(\vec{p})$ and $v_s(\vec{p})$ are the four component Dirac spinors.

- For this free fermion $[H, \vec{J}]$=0 and $[\vec{p}, \vec{J}.\vec{p}]$=0 with $\vec{J} = \vec{L} + \frac{\vec{\sigma}}{2}$ $\qquad$ $(\sigma^i = -\gamma^0\gamma^5\gamma^i)$

  $\Rightarrow$ we can chose $u_s(\vec{p})$ and $v_s(\vec{p})$ to be eigenstates also of the helicity projector

$$P_\pm = \frac{1}{2}\left(1 \pm 2\vec{J}\frac{\vec{p}}{|p|}\right) = \frac{1}{2}\left(1 \pm \vec{\sigma}\frac{\vec{p}}{|p|}\right)$$

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma\cdot\partial - m)\psi(x)$

- The Equation of Motion is: $i\frac{\partial}{\partial t}\psi = H\psi = \gamma^0\left(\vec{\gamma}.\vec{p} + m\right)\psi$

- has 4 possible solutions $(\gamma\cdot p - m)u_s(\vec{p}) = 0$ $\qquad$ $(\gamma\cdot p + m)v_s(\vec{p}) = 0$

$s = \pm\frac{1}{2}$ and $u_s(\vec{p})$ and $v_s(\vec{p})$ are the four component Dirac spinors.

- For this free fermion $[H, \vec{J}]$=0 and $[\vec{p}, \vec{J}.\vec{p}]$=0 with $\vec{J} = \vec{L} + \frac{\vec{\sigma}}{2}$ $\qquad$ $(\sigma^i = -\gamma^0\gamma^5\gamma^i)$

  $\Rightarrow$ we can chose $u_s(\vec{p})$ and $v_s(\vec{p})$ to be eigenstates also of the helicity projector

$$P_\pm = \frac{1}{2}\left(1 \pm 2\vec{J}\frac{\vec{p}}{|p|}\right) = \frac{1}{2}\left(1 \pm \vec{\sigma}\frac{\vec{p}}{|p|}\right)$$

- For massless fermions the Dirac equation can be written

$$\vec{\sigma}\,\vec{p}\,\psi = -\gamma^0\gamma^5\vec{\gamma}\,\vec{p}\,\psi = -\gamma^0\gamma^5\gamma^0 E\,\psi = \gamma^5 E\psi \Rightarrow \text{For } m = 0\ P_\pm = P_{R,L}$$

Only for massless fermions Helicity and chirality states are the same.

Helicity versus Chirality

- The Lagrangian of a massive free fermion ψ is $\mathcal{L} = \overline{\psi}(x)(i\gamma\cdot\partial - m)\psi(x)$

- The Equation of Motion is: $i\frac{\partial}{\partial t}\psi = H\psi = \gamma^0\left(\vec{\gamma}.\vec{p} + m\right)\psi$

- has 4 possible solutions $(\gamma\cdot p - m)u_s(\vec{p}) = 0$ $\qquad$ $(\gamma\cdot p + m)v_s(\vec{p}) = 0$

$s = \pm\frac{1}{2}$ and $u_s(\vec{p})$ and $v_s(\vec{p})$ are the four component Dirac spinors.

- For this free fermion $[H, \vec{J}]$=0 and $[\vec{p}, \vec{J}.\vec{p}]$=0 with $\vec{J} = \vec{L} + \frac{\vec{\sigma}}{2}$ $\qquad$ $(\sigma^i = -\gamma^0\gamma^5\gamma^i)$

  $\Rightarrow$ we can chose $u_s(\vec{p})$ and $v_s(\vec{p})$ to be eigenstates also of the helicity projector

$$P_\pm = \frac{1}{2}\left(1 \pm 2\vec{J}\frac{\vec{p}}{|p|}\right) = \frac{1}{2}\left(1 \pm \vec{\sigma}\frac{\vec{p}}{|p|}\right) = P_{L,R} + \mathcal{O}(\tfrac{m}{p})$$

- For massless fermions the Dirac equation can be written

$$\vec{\sigma}\,\vec{p}\,\psi = -\gamma^0\gamma^5\vec{\gamma}\,\vec{p}\,\psi = -\gamma^0\gamma^5\gamma^0 E\,\psi = \gamma^5 E\psi \Rightarrow \text{For } m = 0\ P_\pm = P_{R,L}$$

Only for massless fermions Helicity and chirality states are the same.

SM Fermion Lagrangian

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{Q^i_{L,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g\frac{\tau_a}{2}\delta_{ij}W^a_\mu - g'\frac{Y}{2}\delta_{ij}B_\mu\right) Q^j_{L,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} +\overline{U^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{Y}{2}\delta_{ij}B_\mu\right) U^j_{R,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} +\overline{D^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{Y}{2}\delta_{ij}B_\mu\right) D^j_{R,k} \\
& \sum_{k=1}^{3} +\overline{L_{L,k}}\gamma^\mu \left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu - g'\frac{Y}{2}B_\mu\right) L_{L,k} + \overline{E_{R,k}}\gamma^\mu \left(i\partial_\mu - g'\frac{Y}{2}B_\mu\right) E_{R,k} \\
& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
$$

SM Fermion Lagrangian

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{Q^i_{L,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g\frac{\tau_a}{2}\delta_{ij}W^a_\mu - g'\frac{Y}{2}\delta_{ij}B_\mu\right) Q^j_{L,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} +\overline{U^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{Y}{2}\delta_{ij}B_\mu\right) U^j_{R,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} +\overline{D^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{Y}{2}\delta_{ij}B_\mu\right) D^j_{R,k} \\
& \sum_{k=1}^{3} +\overline{L_{L,k}}\gamma^\mu \left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu - g'\frac{Y}{2}B_\mu\right) L_{L,k} + \overline{E_{R,k}}\gamma^\mu \left(i\partial_\mu - g'\frac{Y}{2}B_\mu\right) E_{R,k} \\
& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
$$

- Invariant under global rotations

$$q_i \to e^{i\alpha_B/3}q_i \qquad l_i \to e^{i\alpha_{L_i}/3}l_i \qquad \nu_i \to e^{i\alpha_{L_i}/3}\nu_i$$

SM Fermion Lagrangian

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{Q^i_{L,k}}\gamma^\mu \left(i\partial_\mu - g_s\tfrac{\lambda_{a,ij}}{2}G^a_\mu - g\tfrac{\tau_a}{2}\delta_{ij}W^a_\mu - g'\tfrac{Y}{2}\delta_{ij}B_\mu\right) Q^j_{L,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} +\overline{U^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\tfrac{\lambda_{a,ij}}{2}G^a_\mu - g'\tfrac{Y}{2}\delta_{ij}B_\mu\right) U^j_{R,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} +\overline{D^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\tfrac{\lambda_{a,ij}}{2}G^a_\mu - g'\tfrac{Y}{2}\delta_{ij}B_\mu\right) D^j_{R,k} \\
& \sum_{k=1}^{3} +\overline{L_{L,k}}\gamma^\mu \left(i\partial_\mu - g\tfrac{\tau_i}{2}W^i_\mu - g'\tfrac{Y}{2}B_\mu\right) L_{L,k} + \overline{E_{R,k}}\gamma^\mu \left(i\partial_\mu - g'\tfrac{Y}{2}B_\mu\right) E_{R,k} \\
& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
$$

- Invariant under global rotations

$$q_i \to e^{i\alpha_B/3}q_i \qquad l_i \to e^{i\alpha_{L_i}/3}l_i \qquad \nu_i \to e^{i\alpha_{L_i}/3}\nu_i$$

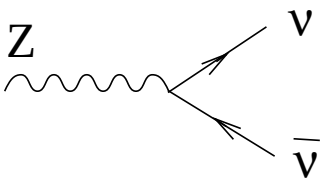
$\Rightarrow$ *Accidental ($\equiv$ not imposed)* global symmetry: $B \times L_e \times L_\mu \times L_\tau$

$\Rightarrow$ Each lepton flavour, L_i, is conserved

$\Rightarrow$ Total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

Number of Neutrinos

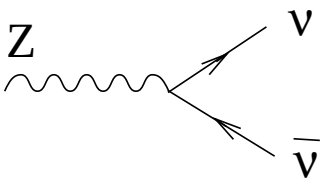
• The counting of light active left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}$$

Number of Neutrinos

• The counting of light active left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}$$

• For $m_{\nu_i} < m_Z/2$ one can use the total Z-width Γ_Z to extract N_ν

$$\begin{aligned} N_\nu &= \frac{\Gamma_{\rm inv}}{\Gamma_\nu} \equiv \frac{1}{\Gamma_\nu}(\Gamma_Z - \Gamma_h - 3\Gamma_\ell) \\ &= \frac{\Gamma_\ell}{\Gamma_\nu}\left[\sqrt{\frac{12\pi R_{h\ell}}{\sigma_h^0 m_Z^2}} - R_{h\ell} - 3\right] \end{aligned}$$

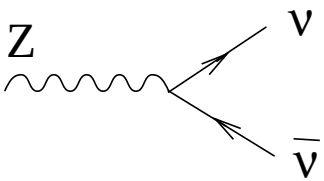
Γ_{inv}= the invisible width

Γ_h= the total hadronic width

Γ_l= width to charged lepton

Number of Neutrinos

• The counting of light active left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}$$

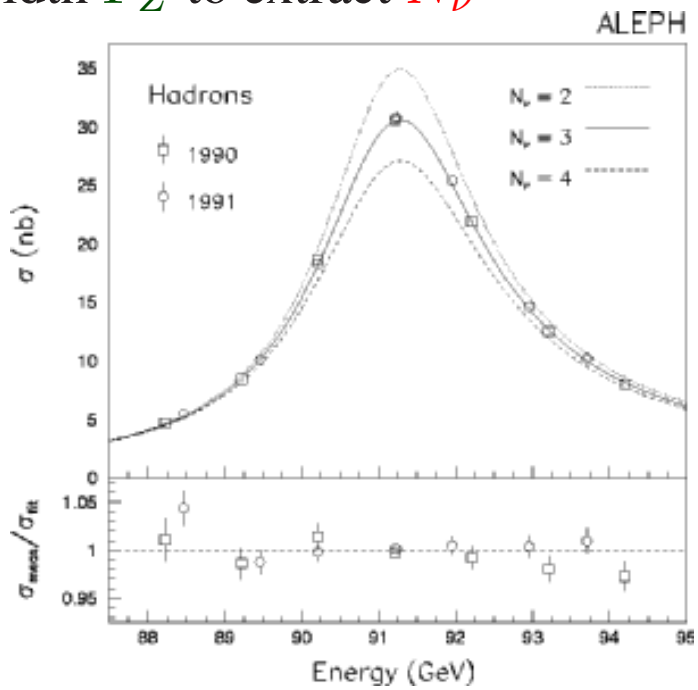
• For $m_{\nu_i} < m_Z/2$ one can use the total Z-width Γ_Z to extract N_ν

$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} \equiv \frac{1}{\Gamma_\nu}(\Gamma_Z - \Gamma_h - 3\Gamma_\ell)$$
$$= \frac{\Gamma_\ell}{\Gamma_\nu}\left[\sqrt{\frac{12\pi R_{h\ell}}{\sigma_h^0 m_Z^2}} - R_{h\ell} - 3\right]$$

Γ_{inv}= the invisible width

Γ_h= the total hadronic width

Γ_l= width to charged lepton



Leads $N_\nu = 2.9840 \pm 0.0082$

Fermion Masses in SM

- A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c.$$ (this is not $SU(2)_L$ gauge invariant)

Fermion Masses in SM

• A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

• In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

Fermion Masses in SM

- A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

- In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

- After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^l = -\bar{E}_L M^\ell E_R + \text{h.c. with } M^\ell = \frac{1}{\sqrt{2}} \lambda^l v$$

Fermion Masses in SM

• A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

• In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

• After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^l = -\bar{E}_L M^\ell E_R + \text{h.c. with } M^\ell = \frac{1}{\sqrt{2}} \lambda^l v$$

In the SM:

– There are no right-handed neutrinos

$\Rightarrow$ No renormalizable (ie dim$\leq$ 4) gauge-invariant operator for tree level ν mass

– SM gauge invariance also implies an accidental symmetry

$$G_{\text{SM}}^{\text{global}} = U(1)_{\text{B}} \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

Fermion Masses in SM

• A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

• In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

• After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^l = -\bar{E}_L M^\ell E_R + \text{h.c. with } M^\ell = \frac{1}{\sqrt{2}} \lambda^l v$$

In the SM:

– There are no right-handed neutrinos

$\Rightarrow$ No renormalizable (ie dim$\leq$ 4) gauge-invariant operator for tree level ν mass

– SM gauge invariance also implies an accidental symmetry

$$G_{\text{SM}}^{\text{global}} = U(1)_{\text{B}} \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

In SM Neutrinos are *Strictly* Massless

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

$\Rightarrow$ OPEN QUESTION: are neutrino and antineutrino the same or different particles?

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

$\Rightarrow$ OPEN QUESTION: are neutrino and antineutrino the same or different particles?

$*$ ANSWER 1: ν different from anti-ν $\Rightarrow$ ν is a *Dirac* particle (like e)

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

$\Rightarrow$ OPEN QUESTION: are neutrino and antineutrino the same or different particles?

$*$ ANSWER 1: ν different from anti-ν $\Rightarrow$ ν is a *Dirac* particle (like e)

$\Rightarrow$ It is described by a *Dirac* field $\nu(x) = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

$\Rightarrow$ OPEN QUESTION: are neutrino and antineutrino the same or different particles?

$*$ ANSWER 1: ν different from anti-ν $\Rightarrow$ ν is a *Dirac* particle (like e)

$\Rightarrow$ It is described by a *Dirac* field $\nu(x) = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

$\Rightarrow$ And the charged conjugate neutrino field $\equiv$ the antineutrino field

$$\nu^C = \mathcal{C}\, \nu\, \mathcal{C}^{-1} = \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = -\eta_C^* \, C\, \overline{\nu}^T$$

$$(C = i\gamma^2\gamma^0)$$

which contain two sets of creation–annihilation operators

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

$\Rightarrow$ OPEN QUESTION: are neutrino and antineutrino the same or different particles?

$*$ ANSWER 1: ν different from anti-ν $\Rightarrow$ ν is a *Dirac* particle (like e)

$\Rightarrow$ It is described by a *Dirac* field $\nu(x) = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

$\Rightarrow$ And the charged conjugate neutrino field $\equiv$ the antineutrino field

$$\nu^C = \mathcal{C}\, \nu\, \mathcal{C}^{-1} = \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = -\eta_C^* \, C\, \overline{\nu}^T$$

$$(C = i\gamma^2\gamma^0)$$

which contain two sets of creation–annihilation operators

$\Rightarrow$ These two fields can rewritten in terms of 4 chiral fields

$$\nu_L \ , \ \nu_R \ , \ (\nu_L)^C \ , \ (\nu_R)^C \quad \text{with} \quad \nu = \nu_L + \nu_R \quad \text{and} \quad \nu^C = (\nu_L)^C + (\nu_R)^C$$

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

$\Rightarrow$ A Majorana particle can be described with only 2 independent chiral fields:

ν_L and $(\nu_L)^C$ which verify $\nu_L = (\nu_R)^C$ $(\nu_L)^C = \nu_R$

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

$\Rightarrow$ A Majorana particle can be described with only 2 independent chiral fields:

ν_L and $(\nu_L)^C$ which verify $\nu_L = (\nu_R)^C$ $(\nu_L)^C = \nu_R$

- In the SM the interaction term for neutrinos

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}} [(\bar{l}_\alpha \gamma_\mu P_L \nu_\alpha) W_\mu^- + (\bar{\nu}_\alpha \gamma_\mu P_L l_\alpha) W_\mu^+] + \frac{i\,g}{\sqrt{2}\cos\theta_W} (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) Z_\mu$$

Only involves two chiral fields $P_L\,\nu = \nu_L$ and $\overline{\nu}\,P_R = \eta_C (\nu_L)^{C^T} C^\dagger$

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

$\Rightarrow$ A Majorana particle can be described with only 2 independent chiral fields:

ν_L and $(\nu_L)^C$ which verify $\nu_L = (\nu_R)^C$ $(\nu_L)^C = \nu_R$

- In the SM the interaction term for neutrinos

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}} [(\bar{l}_\alpha \gamma_\mu P_L \nu_\alpha) W_\mu^- + (\bar{\nu}_\alpha \gamma_\mu P_L l_\alpha) W_\mu^+] + \frac{i\,g}{\sqrt{2}\cos\theta_W} (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) Z_\mu$$

Only involves two chiral fields $P_L\,\nu = \nu_L$ and $\overline{\nu}\,P_R = \eta_C (\nu_L)^{C^T} C^\dagger$

$\Rightarrow$ Weak interaction cannot distinguish if neutrinos are Dirac or Majorana

Dirac versus Majorana Neutrinos

$*$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

$\Rightarrow$ A Majorana particle can be described with only 2 independent chiral fields:

ν_L and $(\nu_L)^C$ which verify $\nu_L = (\nu_R)^C$ $(\nu_L)^C = \nu_R$

- In the SM the interaction term for neutrinos

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}} [(\bar{l}_\alpha \gamma_\mu P_L \nu_\alpha) W_\mu^- + (\bar{\nu}_\alpha \gamma_\mu P_L l_\alpha) W_\mu^+] + \frac{i\,g}{\sqrt{2}\cos\theta_W} (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) Z_\mu$$

Only involves two chiral fields $P_L\,\nu = \nu_L$ and $\overline{\nu}\,P_R = \eta_C (\nu_L)^{C^T} C^\dagger$

$\Rightarrow$ Weak interaction cannot distinguish if neutrinos are Dirac or Majorana

The difference arises from *the mass term*

ν Mass Terms

- A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

ν Mass Terms

- A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

- In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

ν Mass Terms

- A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

- In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

- After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^l = -\bar{L}_L M^{(\ell)} E_R + \text{h.c.}$$

$M^\ell = \frac{1}{\sqrt{2}} \lambda^l \, v \equiv$ Dirac mass matrix for charged leptons

ν Mass Terms

• A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

• In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

• After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^l = -\bar{L}_L M^{(\ell)} E_R + \text{h.c.}$$

$M^\ell = \frac{1}{\sqrt{2}} \lambda^l \, v \equiv$ Dirac mass matrix for charged leptons

• ν's do not participate in QED or QCD and only ν_L is relevant for weak interactions $\Rightarrow$ there is no *dynamical* reason for introducing ν_R, so

ν Mass Terms

• A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

• In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet E_R:

$$\mathcal{L}_Y^l = -\lambda_{ij}^l \overline{L}_{Li} E_{Rj} \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

• After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^l = -\bar{L}_L M^{(\ell)} E_R + \text{h.c.}$$

$M^\ell = \frac{1}{\sqrt{2}} \lambda^l \, v \equiv$ Dirac mass matrix for charged leptons

• ν's do not participate in QED or QCD and only ν_L is relevant for weak interactions $\Rightarrow$ there is no *dynamical* reason for introducing ν_R, so

How can we generate a mass for the neutrino?

ν Mass Terms: Dirac Mass

ν Mass Terms: Dirac Mass

OPTION 1:

- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\lambda_{ij}^{\nu} \overline{\nu_{Ri}} L_{Lj} \widetilde{\phi}^\dagger + \text{h.c.} \qquad (\widetilde{\phi} = i\tau_2 \phi^*)$$

ν Mass Terms: Dirac Mass

OPTION 1:

- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\lambda_{ij}^{\nu}\overline{\nu_{Ri}}L_{Lj}\widetilde{\phi}^{\dagger} + \text{h.c.} \qquad (\widetilde{\phi} = i\tau_2\phi^*)$$

- Under spontaneous symmetry-breaking $\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$

$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\overline{\nu_R}M_D^{\nu}\nu_L + \text{h.c.} \equiv -\frac{1}{2}(\overline{\nu}_R M_D^{\nu}\nu_L + \overline{(\nu_L)^c}M_D^{\nu\,T}(\nu_R)^c) + \text{h.c.} \equiv -\sum_k m_k \overline{\nu}_k^D \nu_k^D$$

$M_D^{\nu} = \frac{1}{\sqrt{2}}\lambda^{\nu}\, v$ =Dirac mass for neutrinos

$V_R^{\nu\dagger} M_D V^{\nu} = \text{diag}(m_1, m_2, m_3)$

ν Mass Terms: Dirac Mass

OPTION 1:

- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\lambda^\nu_{ij}\overline{\nu_{Ri}}L_{Lj}\widetilde{\phi}^\dagger + \text{h.c.} \qquad (\widetilde{\phi} = i\tau_2\phi^*)$$

- Under spontaneous symmetry-breaking $\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$

$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\overline{\nu_R}M_D^\nu\nu_L + \text{h.c.} \equiv -\frac{1}{2}(\overline{\nu}_R M_D^\nu \nu_L + \overline{(\nu_L)^c}M_D^{\nu\,T}(\nu_R)^c) + \text{h.c.} \equiv -\sum_k m_k \overline{\nu}_k^D \nu_k^D$$

$M_D^\nu = \frac{1}{\sqrt{2}}\lambda^\nu\, v$ =Dirac mass for neutrinos

$V_R^{\nu\dagger} M_D V^\nu = \text{diag}(m_1, m_2, m_3)$

- $\mathcal{L}_{\text{mass}}^{(\text{Dirac})}$ involves the four chiral fields ν_L , ν_R , $(\nu_L)^C$, $(\nu_R)^C$

ν Mass Terms: Dirac Mass

OPTION 1:

- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\lambda^\nu_{ij}\overline{\nu_{Ri}}L_{Lj}\widetilde{\phi}^\dagger + \text{h.c.} \qquad (\widetilde{\phi} = i\tau_2\phi^*)$$

- Under spontaneous symmetry-breaking $\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$

$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\overline{\nu_R}M_D^\nu\nu_L + \text{h.c.} \equiv -\frac{1}{2}(\overline{\nu}_R M_D^\nu \nu_L + \overline{(\nu_L)^c}M_D^{\nu\,T}(\nu_R)^c) + \text{h.c.} \equiv -\sum_k m_k \overline{\nu}_k^D \nu_k^D$$

$M_D^\nu = \frac{1}{\sqrt{2}}\lambda^\nu\, v$ =Dirac mass for neutrinos

$V_R^{\nu\dagger} M_D V^\nu = \text{diag}(m_1, m_2, m_3)$

- $\mathcal{L}_{\text{mass}}^{(\text{Dirac})}$ involves the four chiral fields ν_L , ν_R , $(\nu_L)^C$, $(\nu_R)^C$

$\Rightarrow$ The eigenstates of M_D^ν are Dirac particles (same as quarks and charged leptons)

$$\nu^D = V^{\nu\dagger}\nu_L + V_R^{\nu\dagger}\nu_R$$

ν Mass Terms: Dirac Mass

OPTION 1:

- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\lambda^\nu_{ij}\overline{\nu_{Ri}}L_{Lj}\widetilde{\phi}^\dagger + \text{h.c.} \qquad (\widetilde{\phi} = i\tau_2\phi^*)$$

- Under spontaneous symmetry-breaking $\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$

$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\overline{\nu_R}M_D^\nu\nu_L + \text{h.c.} \equiv -\frac{1}{2}(\overline{\nu}_R M_D^\nu \nu_L + \overline{(\nu_L)^c}M_D^{\nu\,T}(\nu_R)^c) + \text{h.c.} \equiv -\sum_k m_k \overline{\nu}_k^D \nu_k^D$$

$M_D^\nu = \frac{1}{\sqrt{2}}\lambda^\nu\, v$ =Dirac mass for neutrinos

$V_R^{\nu\dagger} M_D V^\nu = \text{diag}(m_1, m_2, m_3)$

- $\mathcal{L}_{\text{mass}}^{(\text{Dirac})}$ involves the four chiral fields ν_L , ν_R , $(\nu_L)^C$, $(\nu_R)^C$

$\Rightarrow$ The eigenstates of M_D^ν are Dirac particles (same as quarks and charged leptons)

$$\nu^D = V^{\nu\dagger}\nu_L + V_R^{\nu\dagger}\nu_R$$

$\Rightarrow$ Total Lepton number is conserved *by construction* (not *accidentally*):

$$U(1)_L\,\nu = e^{i\alpha}\,\nu \quad \text{and} \quad U(1)_L\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

$$U(1)_L\,\nu^C = e^{-i\alpha}\,\nu^C \quad \text{and} \quad U(1)_L\,\overline{\nu^C} = e^{i\alpha}\,\overline{\nu^C}$$

ν Mass Terms: Majorana Mass

ν Mass Terms: Majorana Mass

OPTION 2:

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\rm mass}^{\rm (Maj)} = -\frac{1}{2}\overline{\nu_L^c} M_M^\nu \nu_L + \text{h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}_i^M \nu_i^M$$

M_M^ν =Majorana mass for neutrinos is *symmetric*

$V^{\nu T} M_M V^\nu = \text{diag}(m_1, m_2, m_3)$

ν Mass Terms: Majorana Mass

OPTION 2:

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\rm mass}^{\rm (Maj)} = -\frac{1}{2}\overline{\nu_L^c} M_M^\nu \nu_L + \text{h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}_i^M \nu_i^M$$

M_M^ν =Majorana mass for neutrinos is *symmetric*

$V^{\nu T} M_M V^\nu = \text{diag}(m_1, m_2, m_3)$

• But under any $U(1)$ symmetry

$$U(1)\,\nu^c = e^{-i\alpha}\,\nu^c \quad \text{and} \quad U(1)\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

ν Mass Terms: Majorana Mass

OPTION 2:

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\rm mass}^{\rm (Maj)} = -\frac{1}{2}\overline{\nu_L^c} M_M^\nu \nu_L + \text{h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}_i^M \nu_i^M$$

M_M^ν =Majorana mass for neutrinos is *symmetric*

$V^{\nu T} M_M V^\nu = \text{diag}(m_1, m_2, m_3)$

• But under any $U(1)$ symmetry

$$U(1)\,\nu^c = e^{-i\alpha}\,\nu^c \quad \text{and} \quad U(1)\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

$\Rightarrow$ it can only appear for particles without electric charge

$\Rightarrow$ Total Lepton Number is not conserved

$\Rightarrow$ The eigenstates of M_M^ν are Majorana particles

$\nu^M = V^{\nu\dagger}\nu_L + (V^{\nu\dagger}\nu_L)^c$ (verify $\nu^{M\,c}_i = \nu_i^M$)

ν Mass Terms: Majorana Mass

OPTION 2:

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\rm mass}^{\rm (Maj)} = -\frac{1}{2}\overline{\nu_L^c} M_M^\nu \nu_L + \text{h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}_i^M \nu_i^M$$

M_M^ν =Majorana mass for neutrinos is *symmetric*

$V^{\nu T} M_M V^\nu = \text{diag}(m_1, m_2, m_3)$

• But under any $U(1)$ symmetry

$$U(1)\,\nu^c = e^{-i\alpha}\,\nu^c \quad \text{and} \quad U(1)\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

$\Rightarrow$ it can only appear for particles without electric charge

$\Rightarrow$ Total Lepton Number is not conserved

$\Rightarrow$ The eigenstates of M_M^ν are Majorana particles

$\nu^M = V^{\nu\dagger}\nu_L + (V^{\nu\dagger}\nu_L)^c$ (verify $\nu^{M\,c}_i = \nu_i^M$)

$\Rightarrow$ But $SU(2)_L$ gauge invariance is broken!!!

General $SU(2)_L$ invariant ν Mass Terms

General $SU(2)_L$ invariant ν Mass Terms

- Introduce ν_{R_i} $(i = 1, m)$ and write all Lorentz and $SU(2)_L$ invariant mass term

$$\mathcal{L}_Y^{(\nu)} = -\lambda_{ij}^{\nu} \overline{\nu_{R,i}} L_{L,j} \widetilde{\phi}^{\dagger} - \frac{1}{2} \overline{\nu_{R,i}} M_{N,ij}^{\nu} \nu_{R,j}^{c} + \text{h.c.}$$

General $SU(2)_L$ invariant ν Mass Terms

- Introduce ν_{R_i} $(i = 1, m)$ and write all Lorentz and $SU(2)_L$ invariant mass term

$$\mathcal{L}_Y^{(\nu)} = -\lambda_{ij}^{\nu} \overline{\nu_{R,i}} L_{L,j} \widetilde{\phi}^{\dagger} - \frac{1}{2} \overline{\nu_{R,i}} M_{N,ij}^{\nu} \nu_{R,j}^{c} + \text{h.c.}$$

- After spontaneous symmetry-breaking

$$\mathcal{L}_{\text{mass}}^{(\nu)} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2} \overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2} \overline{\vec{\nu}^c} M^{\nu} \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $M^{\nu} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

General $SU(2)_L$ invariant ν Mass Terms

- Introduce ν_{R_i} $(i = 1, m)$ and write all Lorentz and $SU(2)_L$ invariant mass term

$$\mathcal{L}_Y^{(\nu)} = -\lambda_{ij}^{\nu} \overline{\nu_{R,i}} L_{L,j} \widetilde{\phi}^{\dagger} - \frac{1}{2} \overline{\nu_{R,i}} M_{N,ij}^{\nu} \nu_{R,j}^{c} + \text{h.c.}$$

- After spontaneous symmetry-breaking

$$\mathcal{L}_{\text{mass}}^{(\nu)} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2} \overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2} \overline{\vec{\nu}^c} M^{\nu} \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $M^{\nu} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

- $\mathcal{L}_{\text{mass}}^{(\nu)} = -\sum_k \frac{1}{2} m_k \overline{\nu}_k^M \nu_k^M$ where $V^{\nu T} M^{\nu} V^{\nu} = \text{diag}(m_1, m_2, \ldots, m_{3+m})$

General $SU(2)_L$ invariant ν Mass Terms

- Introduce ν_{R_i} $(i = 1, m)$ and write all Lorentz and $SU(2)_L$ invariant mass term

$$\mathcal{L}_Y^{(\nu)} = -\lambda^{\nu}_{ij}\overline{\nu_{R,i}}L_{L,j}\widetilde{\phi}^{\dagger} - \frac{1}{2}\overline{\nu_{R,i}}M^{\nu}_{N,ij}\nu^c_{R,j} + \text{h.c.}$$

- After spontaneous symmetry-breaking

$$\mathcal{L}^{(\nu)}_{\text{mass}} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu^c_R + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^{\nu} \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}$ and $M^{\nu} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

- $\mathcal{L}^{(\nu)}_{\text{mass}} = -\sum_k \frac{1}{2} m_k \overline{\nu}^M_k \nu^M_k$ where $V^{\nu T} M^{\nu} V^{\nu} = \text{diag}(m_1, m_2, \ldots, m_{3+m})$

- In general if $M_N \neq 0 \Rightarrow$ 3+m Majorana neutrino states

$$\nu^M = V^{\nu\dagger}\nu_L + (V^{\nu\dagger}\nu_L)^c \quad \text{(verify } \nu^{M\,c}_i = \nu^M_i \text{)}$$

$\Rightarrow$ Total Lepton Number is not conserved

The See-Saw Mechanism

The See-Saw Mechanism

- A particular realization: Add m ν_{R_i} so

$$\mathcal{L}^{(\nu)}_{\rm mass} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

The See-Saw Mechanism

- A particular realization: Add m ν_{R_i} so

$$\mathcal{L}^{(\nu)}_{\rm mass} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

- Assume $M_N \gg m_D \Rightarrow$

The See-Saw Mechanism

- A particular realization: Add m ν_{R_i} so

$$\mathcal{L}^{(\nu)}_{\rm mass} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

- Assume $M_N \gg m_D \Rightarrow$

  – 3 light neutrinos ν's of mass $m_{\nu_l} \simeq M_D^T M_N^{-1} M_D$

  – m Heavy ν's of mass $m_{\nu_H} \simeq M_N$

The See-Saw Mechanism

- A particular realization: Add m ν_{R_i} so

$$\mathcal{L}^{(\nu)}_{\rm mass} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}$$

$$\text{with } \vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \text{ and } M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

- Assume $M_N \gg m_D \Rightarrow$

  – 3 light neutrinos ν's of mass $m_{\nu_l} \simeq M_D^T M_N^{-1} M_D$

  – m Heavy ν's of mass $m_{\nu_H} \simeq M_N$

  – The heavier ν_H the lighter $\nu_l \Rightarrow$ See-Saw Mechanism

  – Natural explanation to $m_\nu \ll m_l, m_q$

The See-Saw Mechanism

- A particular realization: Add m ν_{R_i} so

$$\mathcal{L}^{(\nu)}_{\rm mass} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu_R^c + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}$$

$$\text{with } \vec{\nu} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \text{ and } M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

- Assume $M_N \gg m_D \Rightarrow$

  – 3 light neutrinos ν's of mass $m_{\nu_l} \simeq M_D^T M_N^{-1} M_D$

  – m Heavy ν's of mass $m_{\nu_H} \simeq M_N$

  – The heavier ν_H the lighter $\nu_l \Rightarrow$ See-Saw Mechanism

  – Natural explanation to $m_\nu \ll m_l, m_q$

  – Arises in many extensions of the SM: SO(10) GUTS, Left-right...

Lectures of S. Senjanovic

ν Mass from Non-Renormalizable Operator

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim$>$ 4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim>4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect $\Rightarrow$ dim=5 operator

There is only one!

$$\mathcal{L}_5 = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi}\right) \left(\tilde{\phi}^T L^C_{L,j}\right)$$

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim>4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect $\Rightarrow$ dim=5 operator

There is only one!

$$\mathcal{L}_5 = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi}\right) \left(\tilde{\phi}^T L^C_{L,j}\right)$$

which after symmetry breaking

induces a ν Majorana mass

$$(M_\nu)_{ij} = Z^\nu_{ij} \frac{v^2}{\Lambda_{\rm NP}}$$

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim>4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect $\Rightarrow$ dim=5 operator

There is only one!

$$\mathcal{L}_5 = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi}\right) \left(\tilde{\phi}^T L^C_{L,j}\right)$$

which after symmetry breaking

induces a ν Majorana mass

$$(M_\nu)_{ij} = Z^\nu_{ij} \frac{v^2}{\Lambda_{\rm NP}}$$

Implications:

– It is natural that ν mass is the first evidence of NP

– Naturally $m_\nu \ll$ other fermions masses $\sim \lambda^f v$ if $\Lambda_{\rm NP} >> v$

– See-saw with heavy fermions or scalar integrated out is a particular example of this

Lectures of S. Senjanovic

Lepton Mixing

- Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j in weak basis

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu\nu_i^W W^+_\mu - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}}M_{\ell ij}\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu_i^{cW}}M_{\nu ij}\nu_j^W + \text{h.c.}$$

Lepton Mixing

- Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j in weak basis

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu\nu_i^W W^+_\mu - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}}M_{\ell ij}\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu_i^{cW}}M_{\nu ij}\nu_j^W + \text{h.c.}$$

- Changing to mass basis by rotations

$$\ell^W_{L,i} = V^\ell_{Lij}\ell_{L,j} \qquad \ell^W_{R,i} = V^\ell_{Rij}\ell_{R,j} \qquad \nu_i^W = V^\nu_{ij}\nu_j$$

$$V_L^{\ell\dagger}M_\ell V_R^\ell = \text{diag}(m_e, m_\mu, m_\tau) \qquad V^{\nu T}M_\nu V^\nu = \text{diag}(m_1^2, m_2^2, m_3^2, \ldots, m_N^2)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices $\qquad V^\nu \equiv$ Unitary $N\times N$ matrix.

Lepton Mixing

- Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j in weak basis

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu\nu_i^W W^+_\mu - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}}M_{\ell ij}\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu_i^{cW}}M_{\nu ij}\nu_j^W + \text{h.c.}$$

- Changing to mass basis by rotations

$$\ell^W_{L,i} = V^\ell_{Lij}\ell_{L,j} \qquad \ell^W_{R,i} = V^\ell_{Rij}\ell_{R,j} \qquad \nu_i^W = V^\nu_{ij}\nu_j$$

$$V_L^{\ell\dagger}M_\ell V_R^\ell = \text{diag}(m_e, m_\mu, m_\tau) \qquad V^{\nu T}M_\nu V^\nu = \text{diag}(m_1^2, m_2^2, m_3^2, \ldots, m_N^2)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices $\qquad V^\nu \equiv$ Unitary $N\times N$ matrix.

- The charged current in the mass basis

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\overline{\ell^i_L}\,\gamma^\mu\, U^{ij}_{\text{LEP}}\,\nu_j\, W^+_\mu$$

Lepton Mixing

- Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j in weak basis

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu\nu^W_i W^+_\mu - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}}M_{\ell ij}\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu^{cW}_i}M_{\nu ij}\nu^W_j + \text{h.c.}$$

- Changing to mass basis by rotations

$$\ell^W_{L,i} = V^\ell_{Lij}\ell_{L,j} \qquad \ell^W_{R,i} = V^\ell_{Rij}\ell_{R,j} \qquad \nu^W_i = V^\nu_{ij}\nu_j$$

$$V^{\ell\dagger}_L M_\ell V^\ell_R = \text{diag}(m_e, m_\mu, m_\tau) \qquad V^{\nu T} M_\nu V^\nu = \text{diag}(m_1^2, m_2^2, m_3^2, \ldots, m_N^2)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices $\qquad V^\nu \equiv$ Unitary $N\times N$ matrix.

- The charged current in the mass basis

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\overline{\ell^i_L}\,\gamma^\mu\, U^{ij}_{\text{LEP}}\,\nu_j\, W^+_\mu$$

$U_{\text{LEP}} \equiv 3\times N$ matrix $\qquad U^{ij}_{\text{LEP}} = \sum_{k=1}^{3} P^\ell_{ii} V^{\ell\dagger\, ik}_L\, V^{\nu\, kj}\, P^\nu_{jj}$

Lepton Mixing

$U_{\rm LEP} \equiv 3 \times N$ matrix

$$U^{ij}_{\rm LEP} = \sum_{k=1}^{3} P^{\ell}_{ii} V_L^{\ell\,\dagger\,ik} \, V^{\nu\,kj} \, P^{\nu}_{jj}$$

Lepton Mixing

$U_{\rm LEP} \equiv 3 \times N$ matrix

$$U^{ij}_{\rm LEP} = \sum_{k=1}^{3} P^{\ell}_{ii} V_L^{\ell\dagger\, ik} \, V^{\nu\, kj} \, P^{\nu}_{jj}$$

- P^{ℓ}_{ii} phase absorbed in l_i
- P^{ν}_{kk} phase absorbed in ν_i (only possible if ν_i is Dirac)

Lepton Mixing

$U_{\rm LEP} \equiv 3 \times N$ matrix

$$U_{\rm LEP}^{ij} = \sum_{k=1}^{3} P_{ii}^{\ell} V_L^{\ell\dagger\, ik} V^{\nu\, kj} P_{jj}^{\nu}$$

- P_{ii}^{ℓ} phase absorbed in l_i
- P_{kk}^{ν} phase absorbed in ν_i (only possible if ν_i is Dirac)
- $U_{\rm LEP} U_{\rm LEP}^{\dagger} = I_{3\times 3}$ but in general $U_{\rm LEP}^{\dagger} U_{\rm LEP} \neq I_{N\times N}$

Lepton Mixing

$U_{\rm LEP} \equiv 3 \times N$ matrix

$$U^{ij}_{\rm LEP} = \sum_{k=1}^{3} P^{\ell}_{ii} V_L^{\ell\dagger\, ik} V^{\nu\, kj} P^{\nu}_{jj}$$

- P^{ℓ}_{ii} phase absorbed in l_i
- P^{ν}_{kk} phase absorbed in ν_i (only possible if ν_i is Dirac)
- $U_{\rm LEP} U^{\dagger}_{\rm LEP} = I_{3\times 3}$ but in general $U^{\dagger}_{\rm LEP} U_{\rm LEP} \neq I_{N\times N}$
- For example for 3 Dirac ν's : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Lepton Mixing

$U_{\rm LEP} \equiv 3 \times N$ matrix

$$U^{ij}_{\rm LEP} = \sum_{k=1}^{3} P^{\ell}_{ii} V_L^{\ell\dagger\, ik} V^{\nu\, kj} P^{\nu}_{jj}$$

- P^{ℓ}_{ii} phase absorbed in l_i
- P^{ν}_{kk} phase absorbed in ν_i (only possible if ν_i is Dirac)
- $U_{\rm LEP} U^{\dagger}_{\rm LEP} = I_{3\times 3}$ but in general $U^{\dagger}_{\rm LEP} U_{\rm LEP} \neq I_{N\times N}$
- For example for 3 Dirac ν's : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- For 3 Majorana ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}$$

Effects of ν Mass

Effects of ν Mass

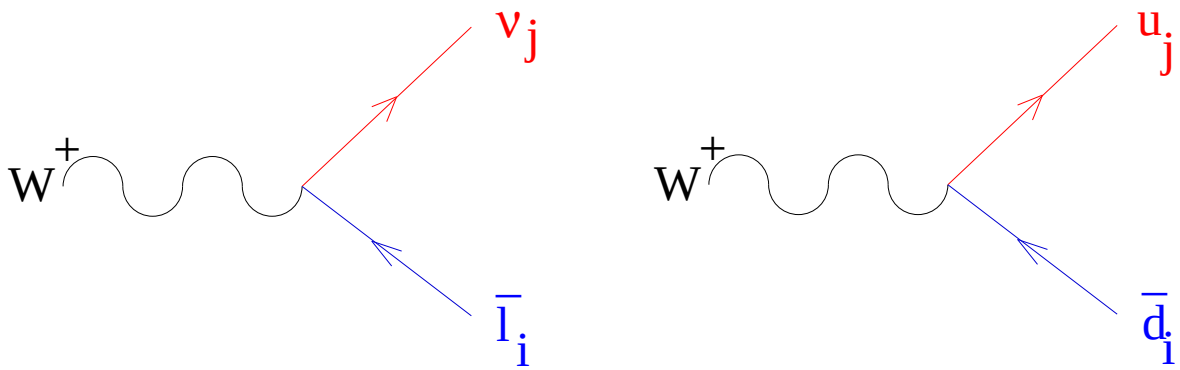
- Neutrino masses can have kinematic effects

Effects of ν Mass

- Neutrino masses can have kinematic effects

- Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W^+_\mu \sum_{ij} \left(U^{ij}_{LEP} \, \overline{\ell^i} \, \gamma^\mu \, L \, \nu^j + U^{ij}_{CKM} \, \overline{U^i} \, \gamma^\mu \, L \, D^j \right) + h.c.$$

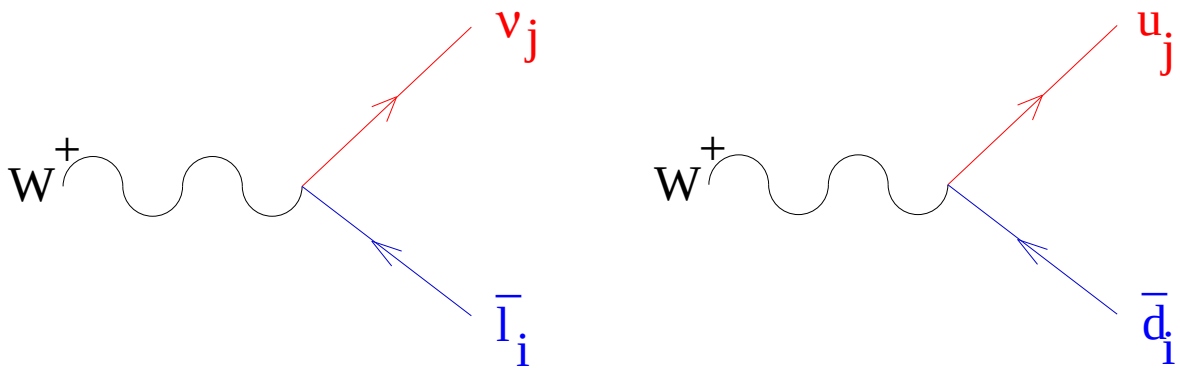


Effects of ν Mass

- Neutrino masses can have kinematic effects

- Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W^+_\mu \sum_{ij} \left(U^{ij}_{LEP} \, \overline{\ell^i} \, \gamma^\mu \, L \, \nu^j + U^{ij}_{CKM} \, \overline{U^i} \, \gamma^\mu \, L \, D^j \right) + h.c.$$



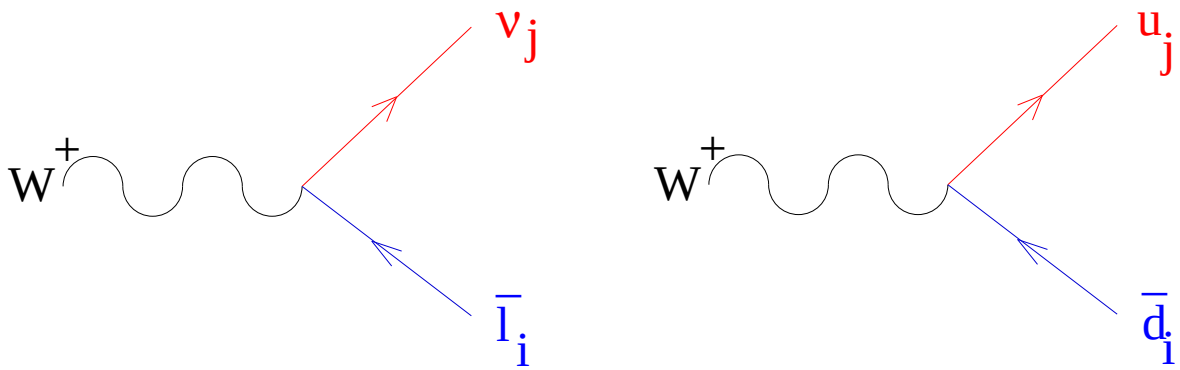
- SM gauge invariance *does not imply* $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ symmetry

Effects of ν Mass

- Neutrino masses can have kinematic effects

- Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W^+_\mu \sum_{ij} \left(U^{ij}_{LEP} \, \overline{\ell^i} \, \gamma^\mu \, L \, \nu^j + U^{ij}_{CKM} \, \overline{U^i} \, \gamma^\mu \, L \, D^j \right) + h.c.$$



- SM gauge invariance *does not imply* $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ symmetry

- Total lepton number $U(1)_L = U(1)_{Le+L_\mu+L_\tau}$ can be or cannot be still a symmetry depending on whether neutrinos are Dirac or Majorana

Neutrino Mass Scale: Tritium β Decay

Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3H \rightarrow ^3He + e + \overline{\nu}_e$$

Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3H \rightarrow ^3He + e + \overline{\nu}_e$$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{Cp\,E\,F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}}$$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ^3H beta decay $Q = 18.6$ KeV)

Taking into account mixing $m_{\nu_e}^{\rm eff} \equiv \sqrt{\sum m_{\nu_j}^2 |U_{ej}|^2}$

Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3H \rightarrow ^3He + e + \overline{\nu}_e$$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

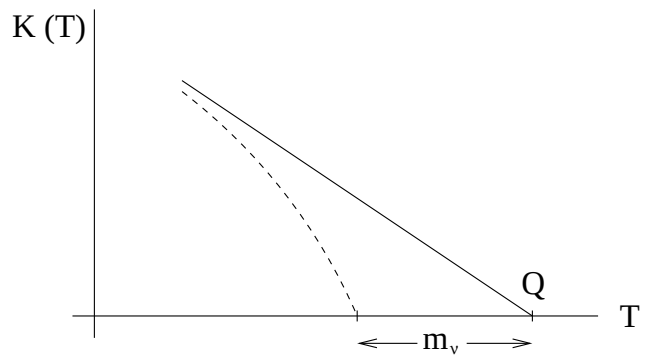
$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{Cp\,E\,F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}}$$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ^3H beta decay $Q = 18.6$ KeV)

Taking into account mixing $m_{\nu_e}^{\rm eff} \equiv \sqrt{\sum m_{\nu_j}^2 |U_{ej}|^2}$

- $m_\nu \neq 0 \Rightarrow$ distortion from the straight-line at the end point of the spectrum

$$m_\nu = 0 \;\Rightarrow\; T_{\rm max} = Q$$
$$m_\nu \neq 0 \;\Rightarrow\; T_{\rm max} = Q - m_\nu$$



Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3H \rightarrow ^3He + e + \overline{\nu}_e$$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

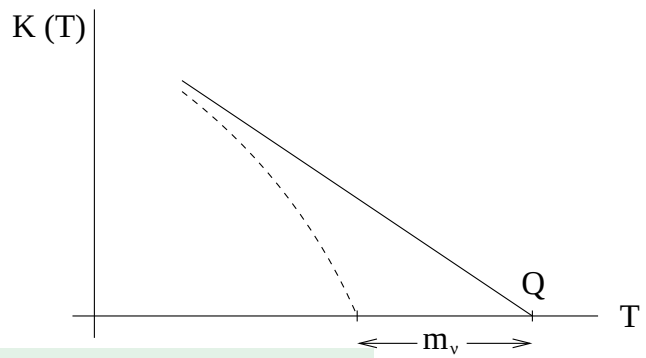
$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{Cp\,E\,F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}}$$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ^3H beta decay $Q = 18.6$ KeV)

Taking into account mixing $m_{\nu_e}^{\rm eff} \equiv \sqrt{\sum m_{\nu_j}^2 |U_{ej}|^2}$

- $m_\nu \neq 0 \Rightarrow$ distortion from the straight-line at the end point of the spectrum

$$m_\nu = 0 \;\Rightarrow\; T_{\rm max} = Q$$
$$m_\nu \neq 0 \;\Rightarrow\; T_{\rm max} = Q - m_\nu$$



– At present only a bound: $m_{\nu_e}^{\rm eff} < 2.2$ eV (at 95 % CL) (Mainz & Troisk experiments)

– Katrin operating to improve present sensitivity to $m_{\nu_e}^{\rm eff} \sim 0.3$ eV

Neutrino Mass Scale: Other Channels

Neutrino Mass Scale: Other Channels

Muon neutrino mass

- From the two body decay at rest

$$\pi^- \to \mu^- + \overline{\nu}_\mu$$

- Energy momentum conservation:

$$m_\pi = \sqrt{p_\mu^2 + m_\mu^2} + \sqrt{p_\mu^2 + m_\nu^2}$$

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2 + m_\mu \sqrt{p^2 + m_\pi^2}$$

- Measurement of p_μ plus the precise knowledge of m_π and $m_\mu \Rightarrow m_\nu$
- The present experimental result bound:

$$\boxed{m_{\nu_\mu}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\mu j}|^2} < 190 \text{ KeV}}$$

Neutrino Mass Scale: Other Channels

Muon neutrino mass

- From the two body decay at rest

$$\pi^- \to \mu^- + \overline{\nu}_\mu$$

- Energy momentum conservation:

$$m_\pi = \sqrt{p_\mu^2 + m_\mu^2} + \sqrt{p_\mu^2 + m_\nu^2}$$

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2 + m_\mu \sqrt{p^2 + m_\pi^2}$$

- Measurement of p_μ plus the precise knowledge of m_π and $m_\mu \Rightarrow m_\nu$
- The present experimental result bound:

$$m_{\nu_\mu}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\mu j}|^2} < 190 \text{ KeV}$$

Tau neutrino mass

- The τ is much heavier $m_\tau = 1.776$ GeV

$\Rightarrow$ Large phase space $\Rightarrow$ difficult precision for m_ν

- The best precision is obtained from hadronic final states

$$\tau \to n\pi + \nu_\tau \quad \text{with } n \geq 3$$

- Lep I experiments obtain:

$$m_{\nu_\tau}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\tau j}|^2} < 18.2 \text{ MeV}$$

Neutrino Mass Scale: Other Channels

Muon neutrino mass

- From the two body decay at rest

$$\pi^- \to \mu^- + \overline{\nu}_\mu$$

- Energy momentum conservation:

$$m_\pi = \sqrt{p_\mu^2 + m_\mu^2} + \sqrt{p_\mu^2 + m_\nu^2}$$

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2 + m_\mu \sqrt{p^2 + m_\pi^2}$$

- Measurement of p_μ plus the precise knowledge of m_π and $m_\mu \Rightarrow m_\nu$
- The present experimental result bound:

$$m_{\nu_\mu}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\mu j}|^2} < 190 \text{ KeV}$$

Tau neutrino mass

- The τ is much heavier $m_\tau = 1.776$ GeV

$\Rightarrow$ Large phase space $\Rightarrow$ difficult precision for m_ν

- The best precision is obtained from hadronic final states

$$\tau \to n\pi + \nu_\tau \quad \text{with } n \geq 3$$

- Lep I experiments obtain:

$$m_{\nu_\tau}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\tau j}|^2} < 18.2 \text{ MeV}$$

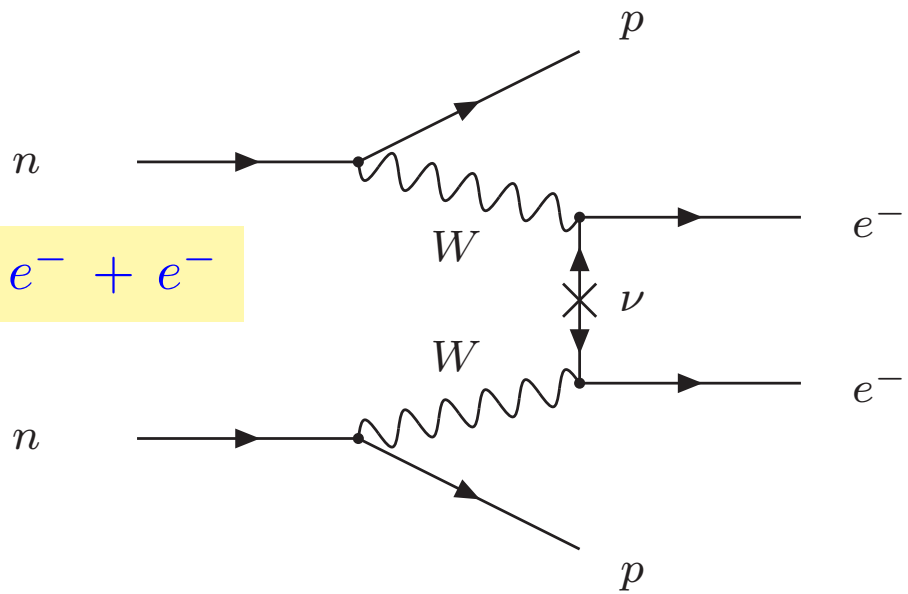
$\Rightarrow$ If mixing angles U_{ej} are not negligible Lectures of C. Patrick

Best kinematic limit on Neutrino Mass Scale comes from Tritium Beta Decay

Dirac or Majorana? ν-less Double-β Decay

Dirac or Majorana? ν-less Double-β Decay

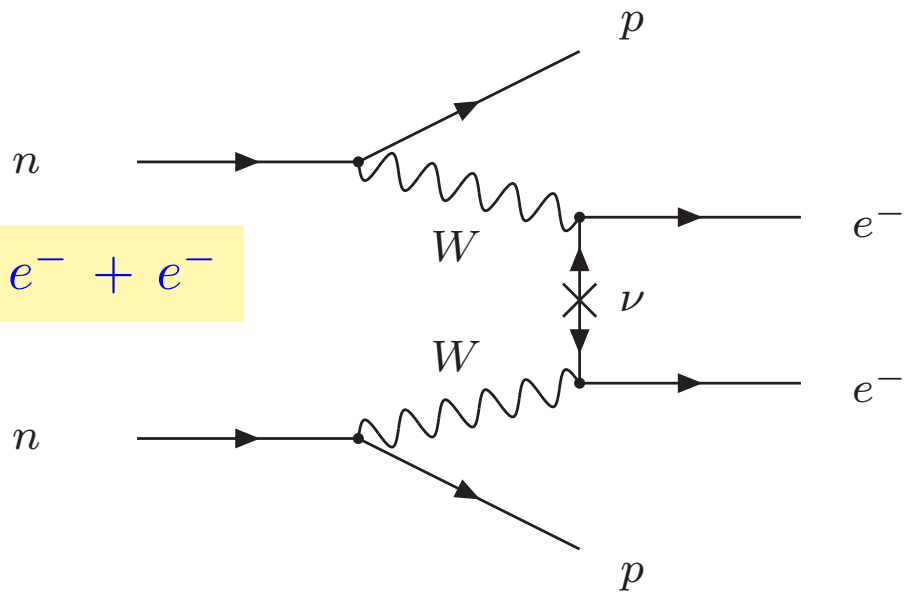
$$(A, Z) \to (A, Z+2) + e^- + e^-$$



- Amplitude involves the product of two leptonic currents: $\left[\overline{e}\gamma^\mu L\nu\right]\left[\overline{e}\gamma^\mu L\nu\right]$

Dirac or Majorana? ν-less Double-β Decay

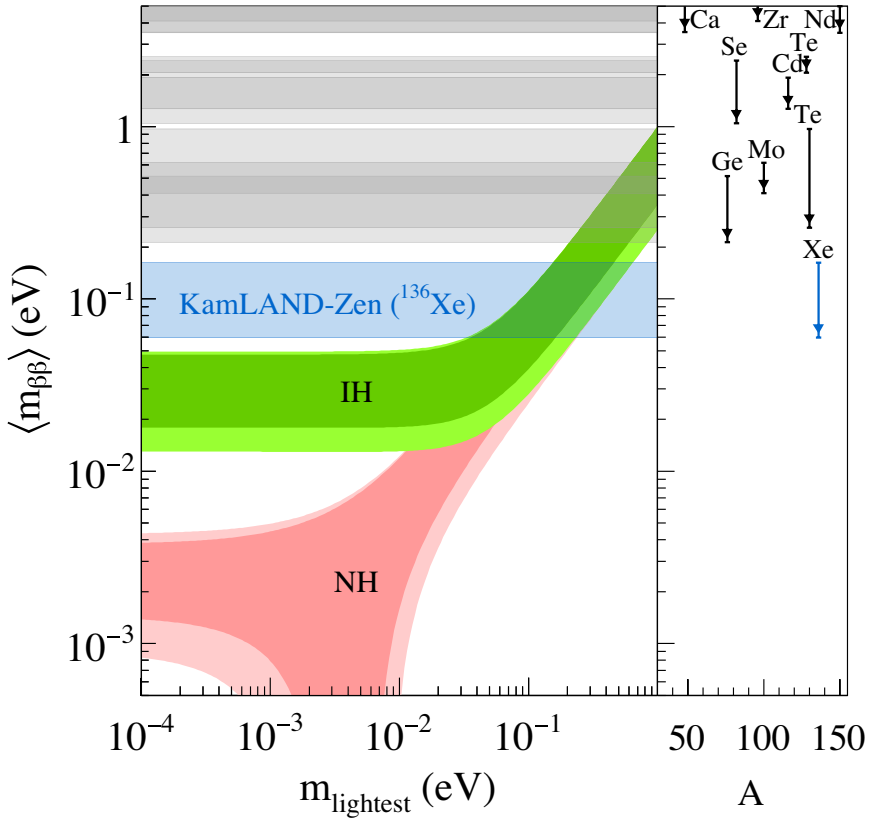
$$(A, Z) \to (A, Z+2) + e^- + e^-$$



- Amplitude involves the product of two leptonic currents: $\left[\overline{e}\gamma^\mu L\nu\right]\left[\overline{e}\gamma^\mu L\nu\right]$
  - If ν Dirac $\Rightarrow$ ν *annihilates* a *neutrino* and *creates* an *antineutrino*
    $\Rightarrow$ no same state $\Rightarrow$ Amplitude = 0
  - If ν Majorana $\Rightarrow$ $\nu = \nu^c$ *annihilates and creates* a *neutrino=antineutrino*
    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto \overline{\nu\ (\nu)}^T \neq 0$
- Amplitude of ν-less-$\beta\beta$ decay is proportional to $|\langle m_{\beta\beta}\rangle| = |\sum U_{ej}^2 m_j|$

$0\nu\beta\beta$ Decay: Present

Only bounds from ^{136}Xe (EXO and KamLAND-Zen), ^{76}Ge (Gerda) and ^{130}Te (Cuore-0)

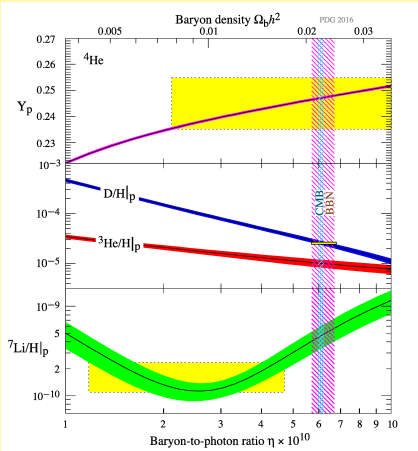
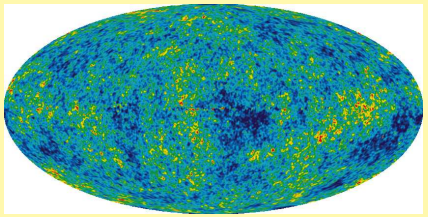
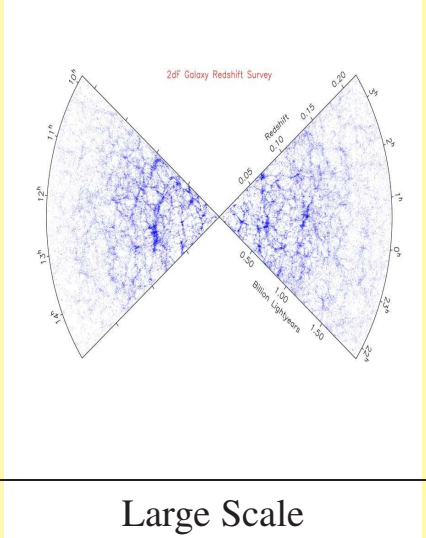


Lectures of C. Patrick

Light massive ν in Cosmology

Relic $\nu's$: Effects in several cosmological observations at several epochs

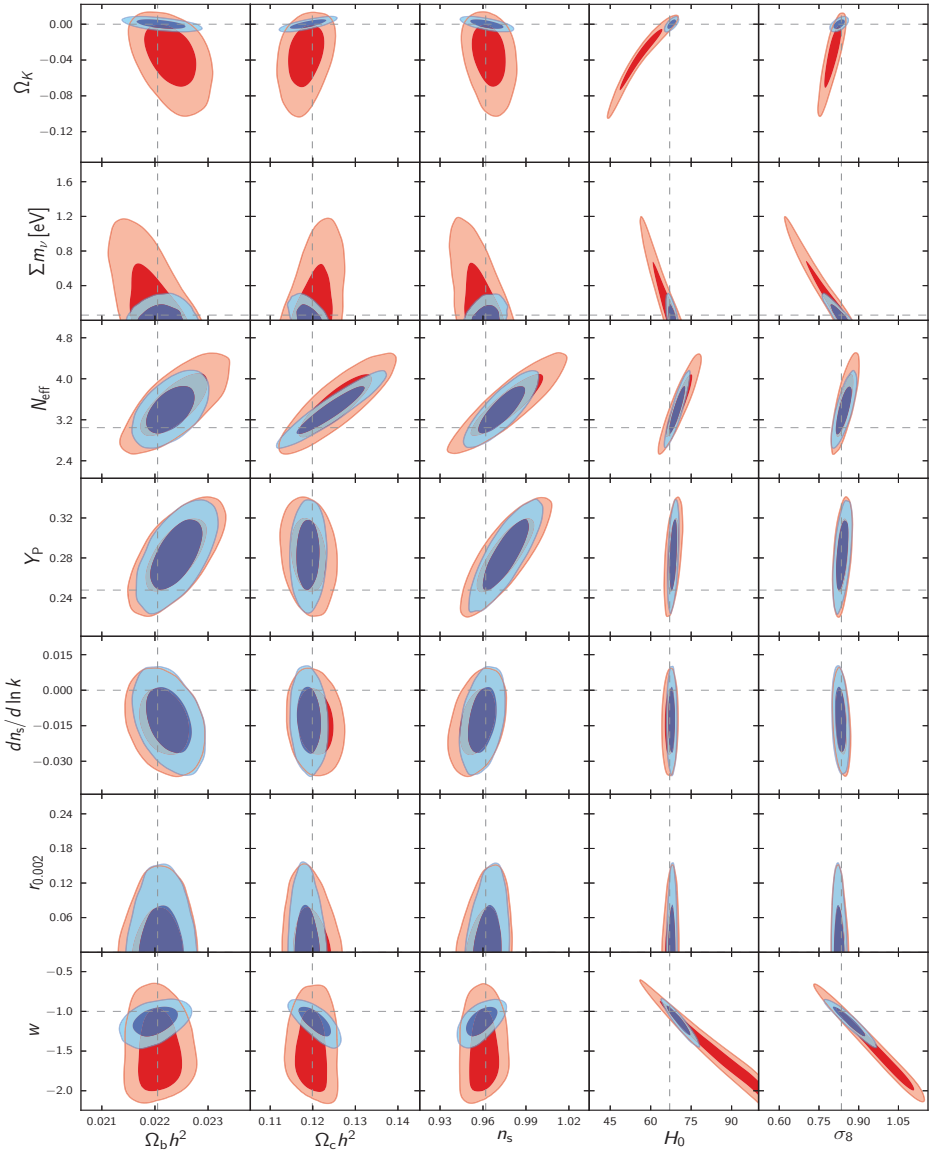
Mainly via two effects: $\rho_r = \left[1 + \frac{7}{8} \times \left(\frac{4}{11}\right)^{\frac{4}{3}} N_{\rm eff}\right] \rho_\gamma$ and $\sum_i m_{\nu_i}$

		
Primordial Nucleosynthesis BBN	Cosmic Microwave Background CMB	Large Scale Structure Formation LSS
$T \sim$ MeV	$T \lesssim$ eV	
Number of $\nu's$ ($N_{\rm eff}$)	$N_{\rm eff}$ and $\sum m_\nu$	

BUT: Observables also depend on all other cosmo parameters (and assumptions)

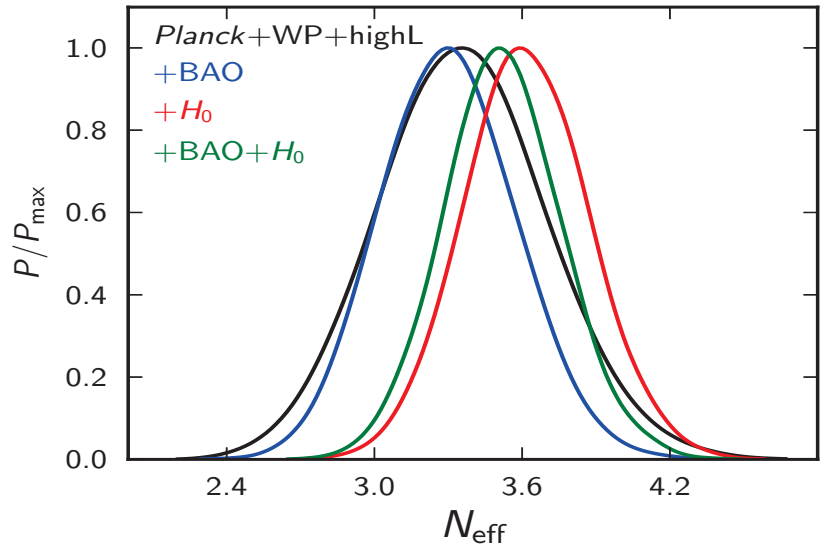
Example: Cosmological Analysis by Planck

arXiv:1502.01589



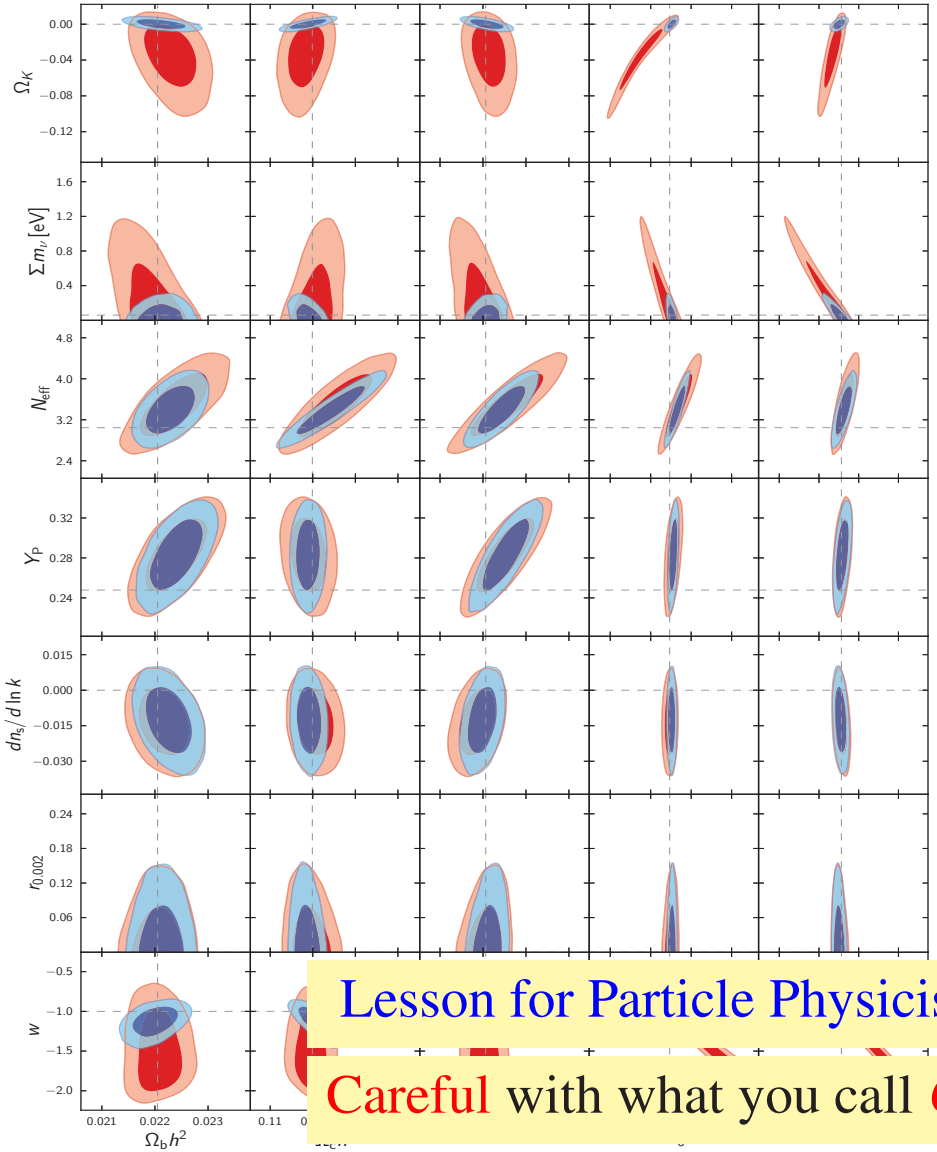
Range of Bounds in ΛCDM

Model	Observables	Σm_ν (eV) 95%
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT + lowP	≤ 0.72
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT + lowP + lensing	≤ 0.68
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT,TE,EE + lowP+lensing	≤ 0.59
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT,TE,EE + lowP	≤ 0.49
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT + lowP + lensing + BAO + SN + H_0	≤ 0.23
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT,TE,EE + lowP+ BAO	≤ 0.17



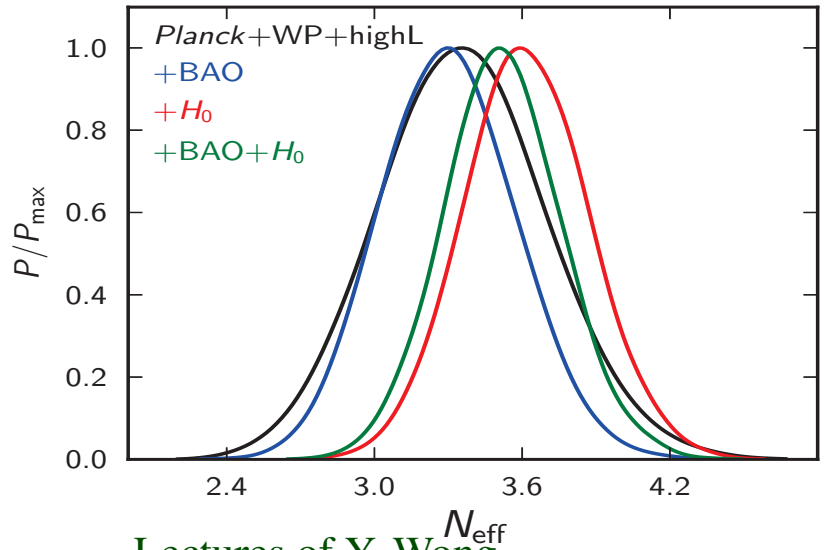
Example: Cosmological Analysis by Planck

arXiv:1502.01589



Range of Bounds in ΛCDM

Model	Observables	Σm_ν (eV) 95%
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT + lowP	≤ 0.72
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT + lowP + lensing	≤ 0.68
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT,TE,EE + lowP+lensing	≤ 0.59
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT,TE,EE + lowP	≤ 0.49
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT + lowP + lensing + BAO + SN + H_0	≤ 0.23
$\Lambda\mathrm{CDM} + m_\nu$	Planck TT,TE,EE + lowP+ BAO	≤ 0.17



Lectures of Y. Wong

Lesson for Particle Physicists:

Careful with what you call *Cosmological bound on m_ν*

Summary I

Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos

Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$

Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- From direct searches of ν-mass: $m_\nu \leq \mathcal{O}(eV)$

Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- From direct searches of ν-mass: $m_\nu \leq \mathcal{O}(eV)$

  Question: How to search for $m_\nu \ll \mathcal{O}(eV)$?

  Answer:

Summary I

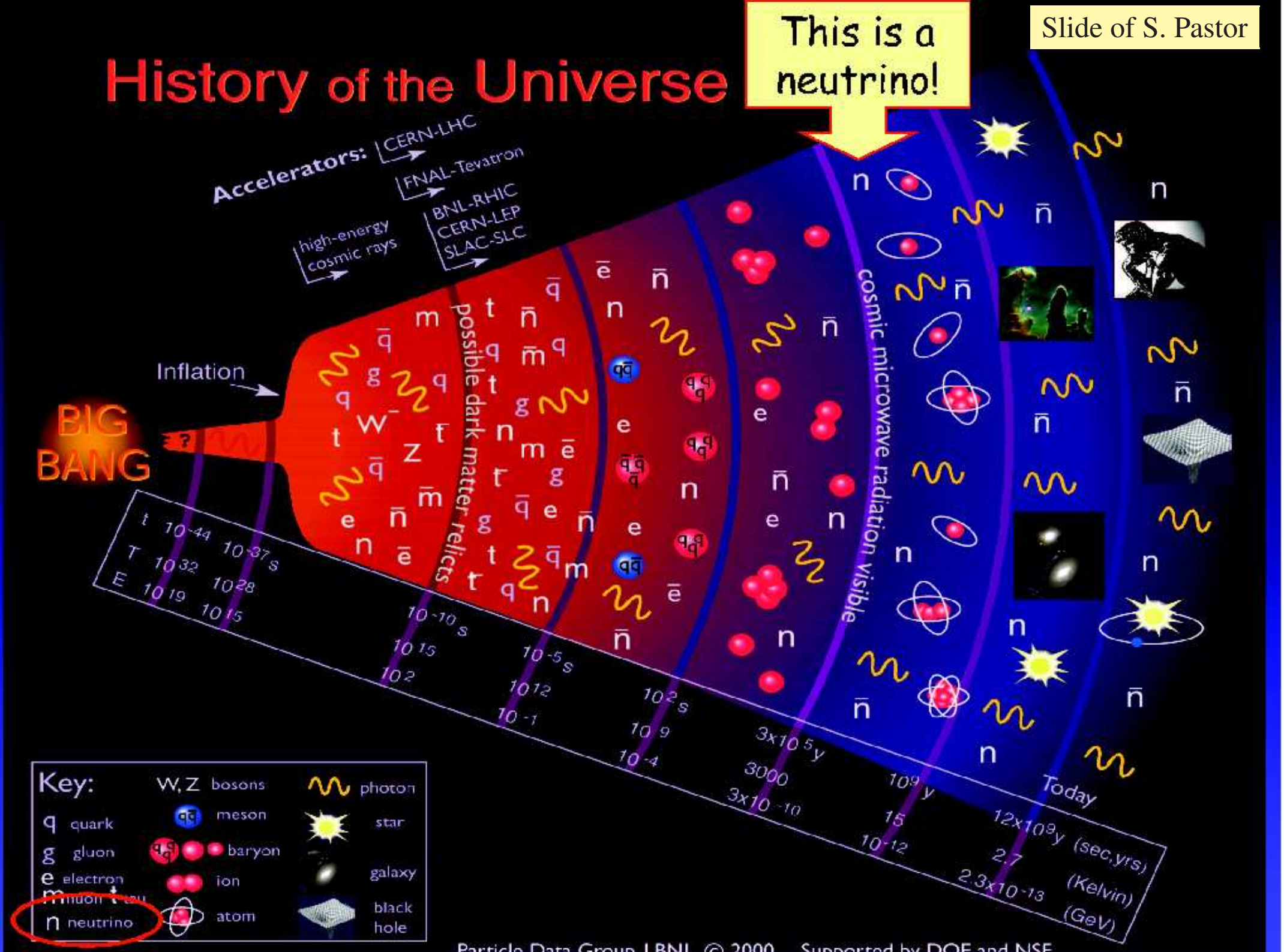
- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

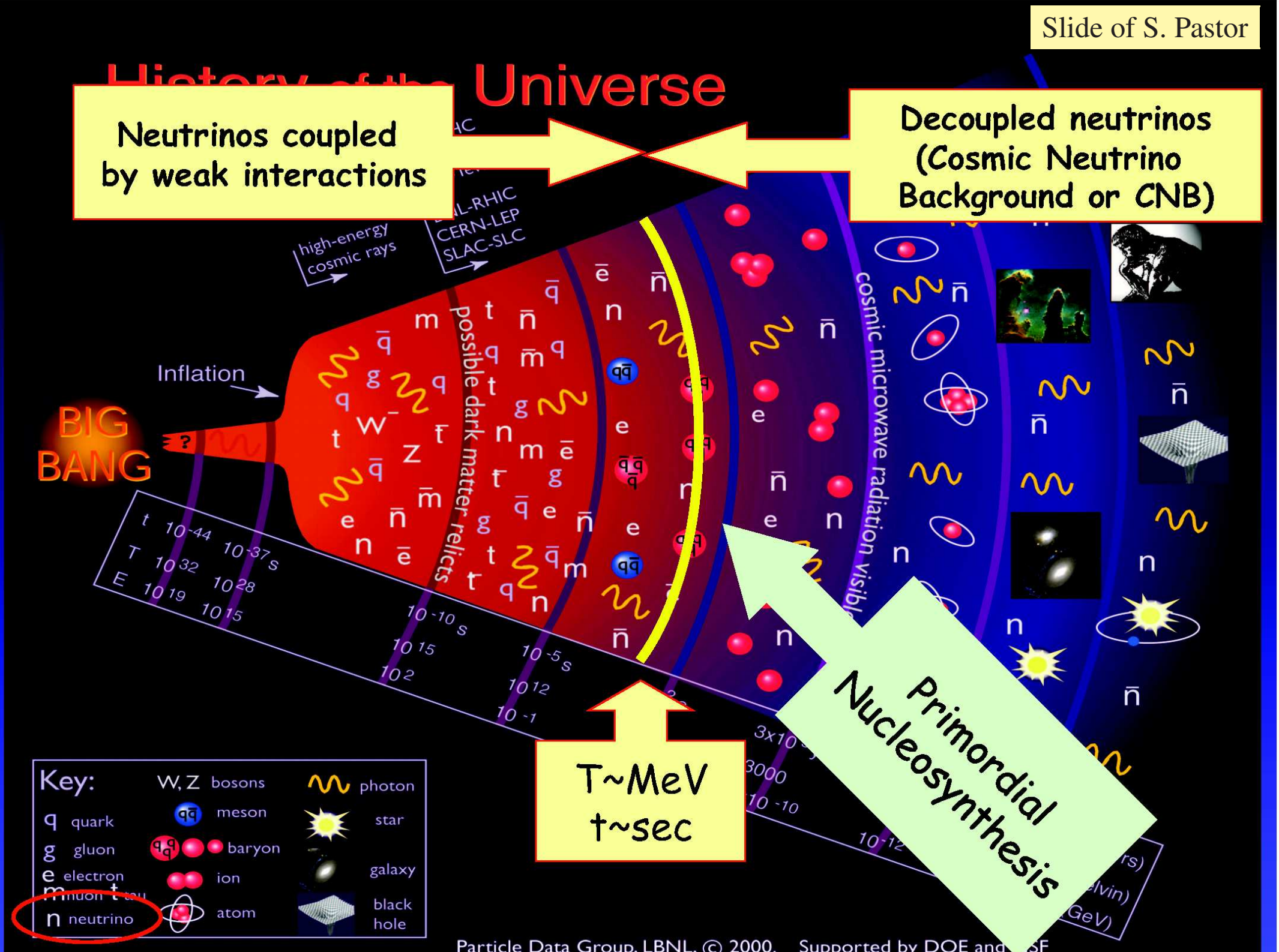
  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- From direct searches of ν-mass: $m_\nu \leq \mathcal{O}(eV)$

  Question: How to search for $m_\nu \ll \mathcal{O}(eV)$?

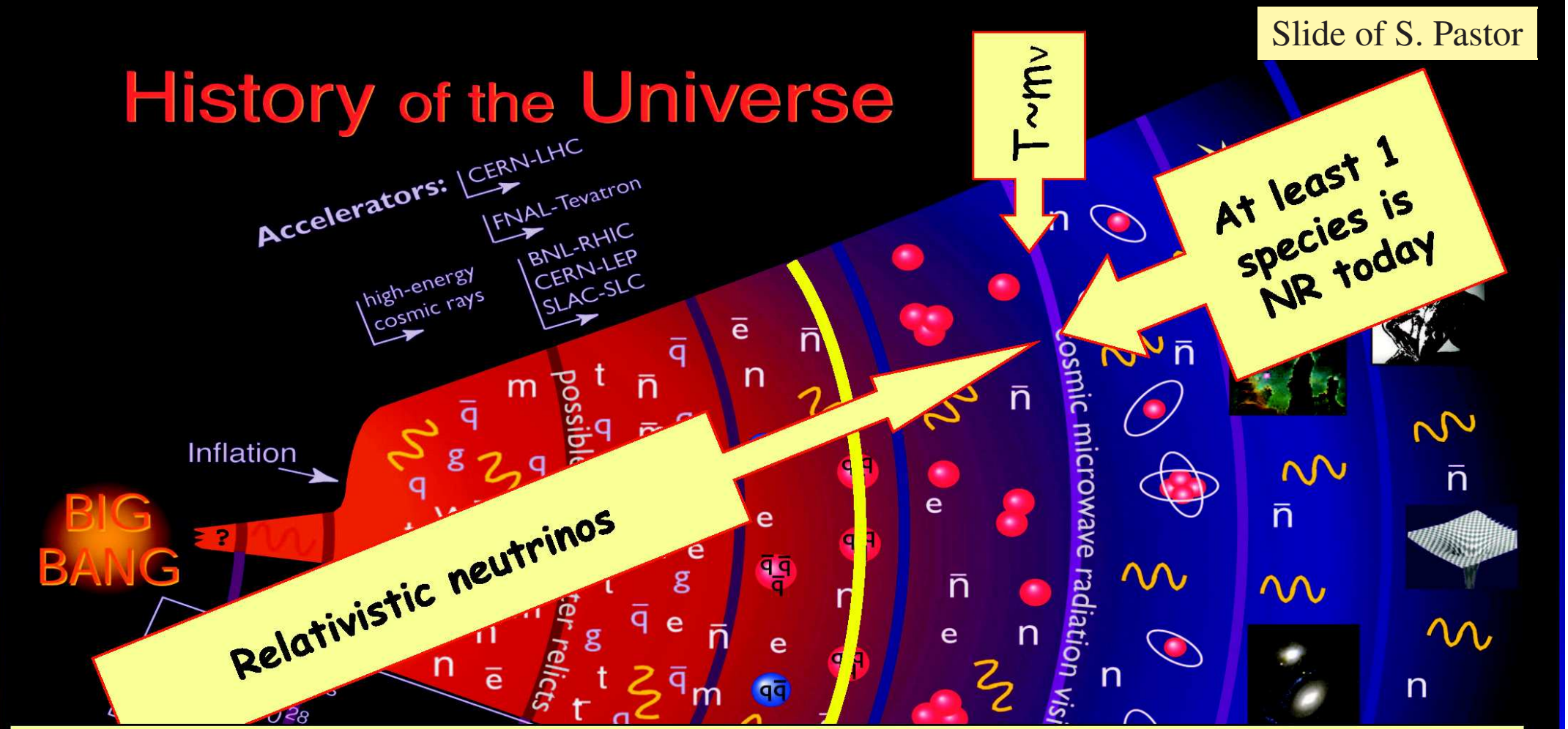
  Answer: Soon....


Slide of S. Pastor
This is a neutrino!
History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
Today
Key:
q quark
g gluon
e electron
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
(sec,yrs)
(Kelvin)
(GeV)
Particle Data Group LBNL © 2000 Supported by DOE and NSF

Slide of S. Pastor



Slide of S. Pastor



Neutrino cosmology is interesting because Relic neutrinos are very abundant:

- The CNB contributes to radiation at early times and to matter at late times (info on the number of neutrinos and their masses)
- Cosmological observables can be used to test standard or non-standard neutrino properties