Long Baseline Oscillation Experiments: Question #1

Offered by Patricia Vahle and Jeff Hartnell for the International Neutrino Summer School

Solution by (using Prob3++ software):

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Group *#7, August 15th, 2019*

Restatement of the Question

Long-baseline neutrino oscillation experiments seek to measure CP violation, the mass hierarchy, and the octant of θ_{23} using electron (anti)neutrino appearance and muon (anti)neutrino disappearance in a muon (anti)neutrino beam. The NOνA experiment has a baseline of $810\ km$ and a peak neutrino energy of $1.9\ GeV$. For the purposes of this problems, except as noted, consider the NOνA electron (anti)neutrino appearance and muon (anti)neutrino disappearance measurements as a "counting" experiment where you consider the beam to be monochromatic. Similarly, consider backgrounds and systematic uncertainties to be negligible compared to statistical uncertainties, again except as noted in individual problems.

For this question, you should ***consider three flavor neutrino oscillations***. The standard two flavor approximations will be insufficient. **Consider the duration of the NOνA experiment as an exposure of 36×10^{20} protons on target**, which may be divided between neutrino and antineutrino beams.

For an exposure of 6×10^{20} protons on target, the following $\{\nu_e, \overline{\nu_e}\}$ signal (S) and background (B) counts are expected for $\delta_{CP} = \left\{0, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$ in the NH(IH) and assuming $\sin^2(\theta_{23}) = 0.5$, $\sin^2(2\theta_{13}) = 0.085$, $\sin^2(2\theta_{12}) = 0.87$, $\Delta m^2_{12} = 7.5 \times 10^{-5} eV^2$ and $\Delta m^2_{23} = 2.5 \times 10^{-3} eV^2$…

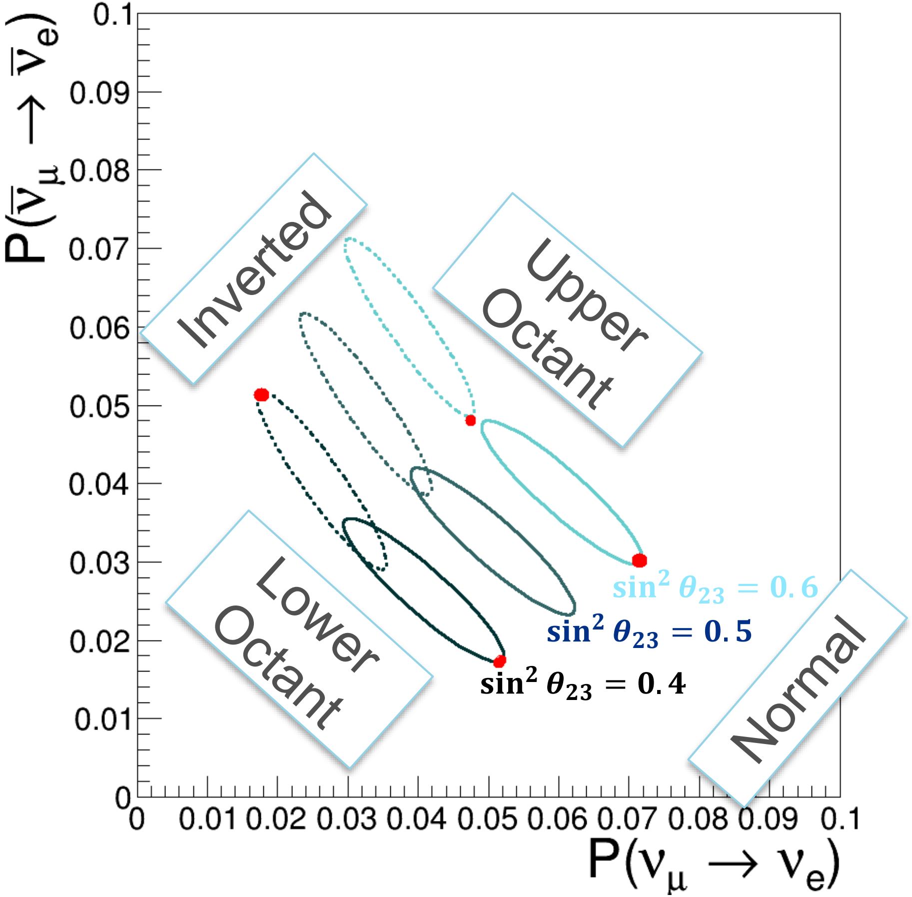
- **…in neutrino-mode**: $B = 7.75$, $S = 24.19(13.76), 17.93(11.07), 27.85(18.88)$
- **…in antineutrino-mode**: $B = 2.87$, $S = 7.58(8.91), 8.53(11.40), 5.58(7.68)$

Assume the backgrounds are independent of the oscillation parameters. For background information please have a look at arXiv:1210.1778.

1. NOνA must decide how to operate their beam. **The choice ranges from operating the beam in neutrino mode 100% of the time, through to 100% in antineutrino mode. What is the optimal run plan for NOνA to determine specifically the mass hierarchy?** Consider the following scenarios for true oscillation parameters:
   - NH, $\sin^2(\theta_{23}) = 0.6$ and $\delta_{CP} = \frac{3\pi}{2}$
   - NH, $\sin^2(\theta_{23}) = 0.4$ and $\delta_{CP} = \frac{3\pi}{2}$
   - IH, $\sin^2(\theta_{23}) = 0.6$ and $\delta_{CP} = \frac{3\pi}{2}$
   - IH, $\sin^2(\theta_{23}) = 0.4$ and $\delta_{CP} = \frac{\pi}{2}$

   How does your proposed run plan depend on the oscillation parameters that Nature has chosen? What would your run plan be in those specific scenarios? What should the run plan be when we don't know what Nature has chosen?! Do you need to run antineutrinos? Invent a physics scenario of your own choosing that might cause you to make the incorrect hierarchy selection. Do a quantitative analysis of this scenario to see if such a thing is really possible. If you have time, consider the sensitivity to the octant (and CP violation) and consider the optimal run plan to measure those parameters.

Points of interest



Method of Solution

- Using Prob3++, we consider all values of δ_{CP} across 2D slices of $\sin^2\theta_{23}$ for both hierarchies within $P(\nu_\mu \rightarrow \nu_e): P(\overline{\nu_\mu} \rightarrow \overline{\nu_e})$ parameter space
  - These appear as “bi-probability” ovals when matter effects are included, and change their characteristic size and angle depending on the value of θ_{23}
  - Can determine points of interest in this space using *given* oscillation parameters
- Can scale estimated event counts for each appearance type processes and their associated errors from count space to oscillation probability space…

$$N = S_G + B_G$$

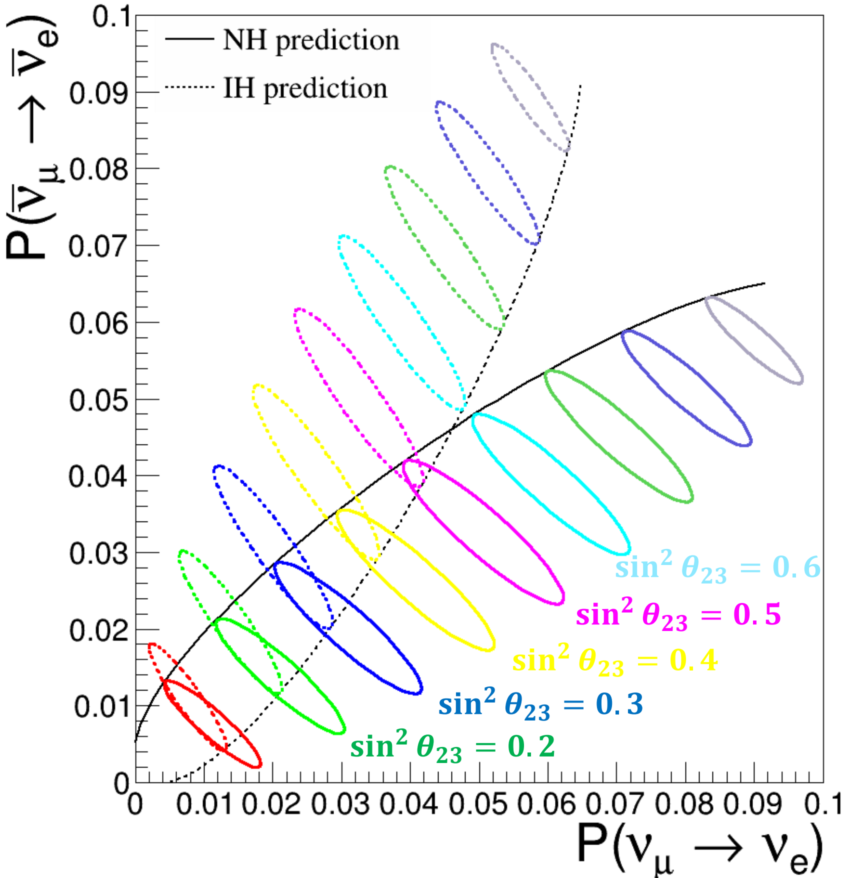
$$N_{\delta_{CP}^A \theta_{23}^A H^A} = \left(\frac{P_{\delta_{CP}^A \theta_{23}^A H^A}}{P_{\delta_{CP}^A \theta_{23}^G H^A}} S_{G;\delta_{CP}^A \theta_{23}^G H^A} \right) + B_{G;\delta_{CP}^A \theta_{23}^G H^A}$$

$$\sigma_A = \pm \frac{P_{\delta_{CP}^A \theta_{23}^A H^A}}{\sqrt{6 f N_{\delta_{CP}^A \theta_{23}^A H^A}}}$$

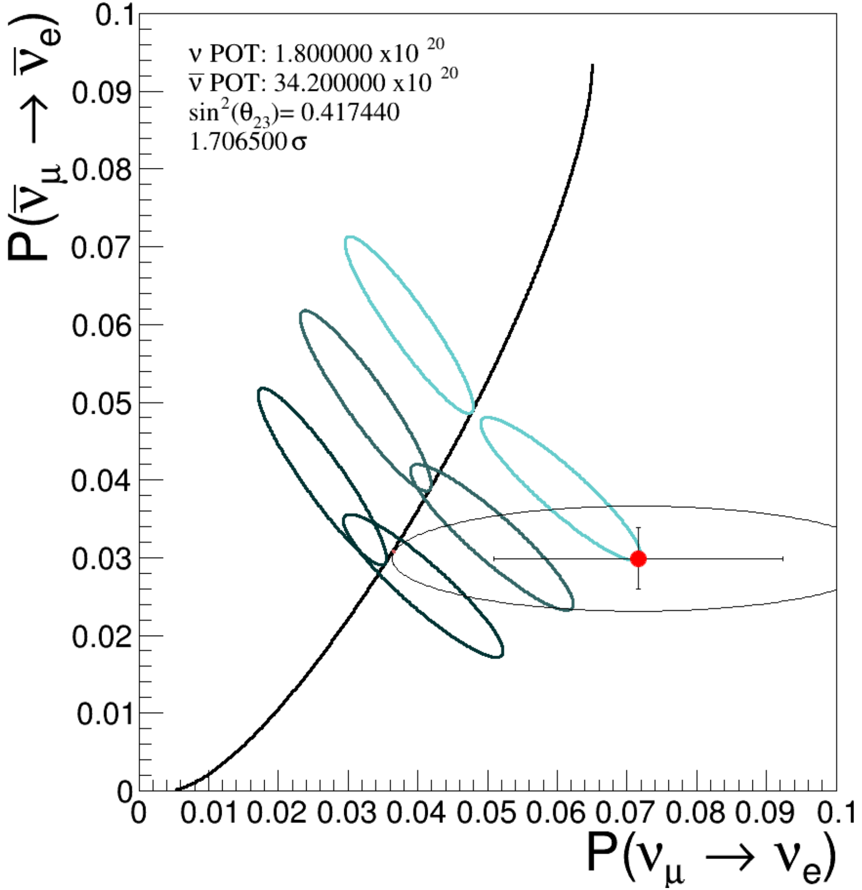
Bi-probability plot scan over δ_{CP} and θ_{23} for both hierarchies

Method:

- Extrapolate over all values of $\sin^2\theta_{23}$ to construct a boundary between NH and IH using the closest $\delta_{CP} = \frac{\pi}{2}$
- Iterate over possible beam fractions (FHC vs RHC)
- Find which beam fraction rejects the alternative hierarchy at ***any*** $\sin^2\theta_{23}$ at the highest σ

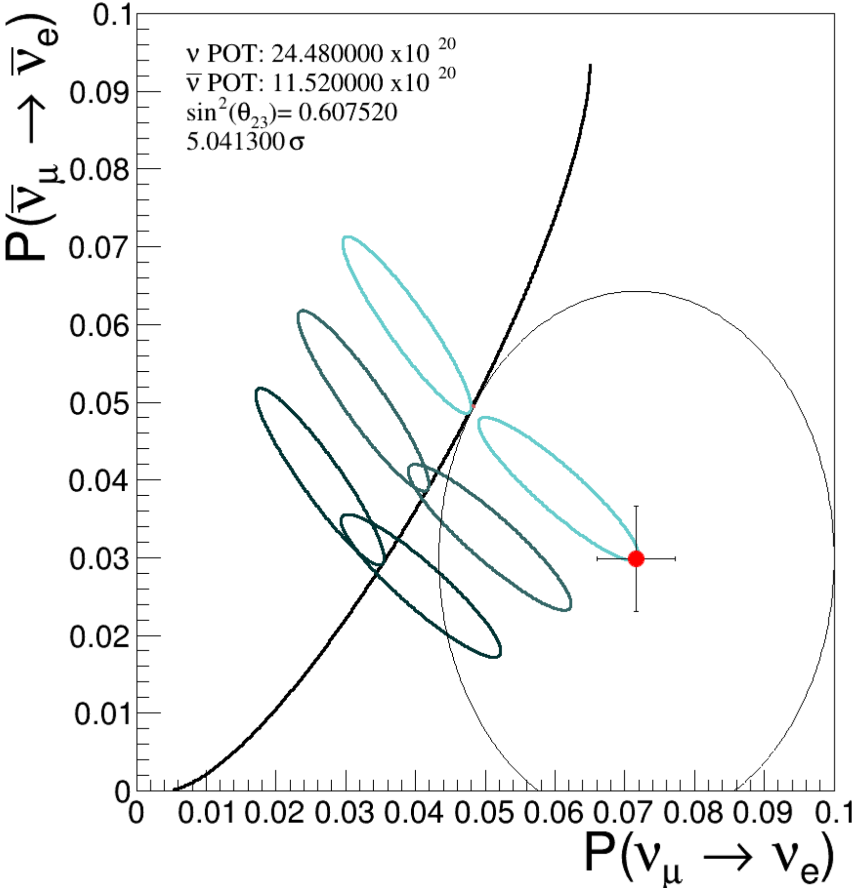


NH, $\sin^2(\theta_{23}) = 0.6$ and $\delta_{CP} = \frac{3\pi}{2}$



NH, $\sin^2(\theta_{23}) = 0.6$ and $\delta_{CP} = \frac{3\pi}{2}$

68% FHC excludes the IH with the highest certainty!



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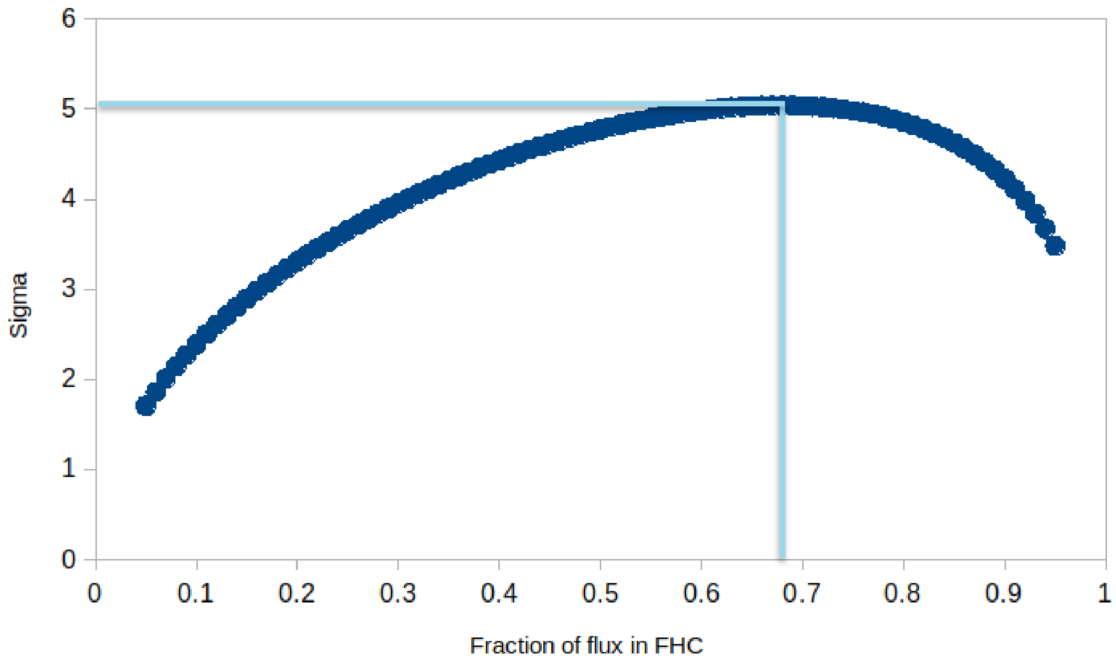

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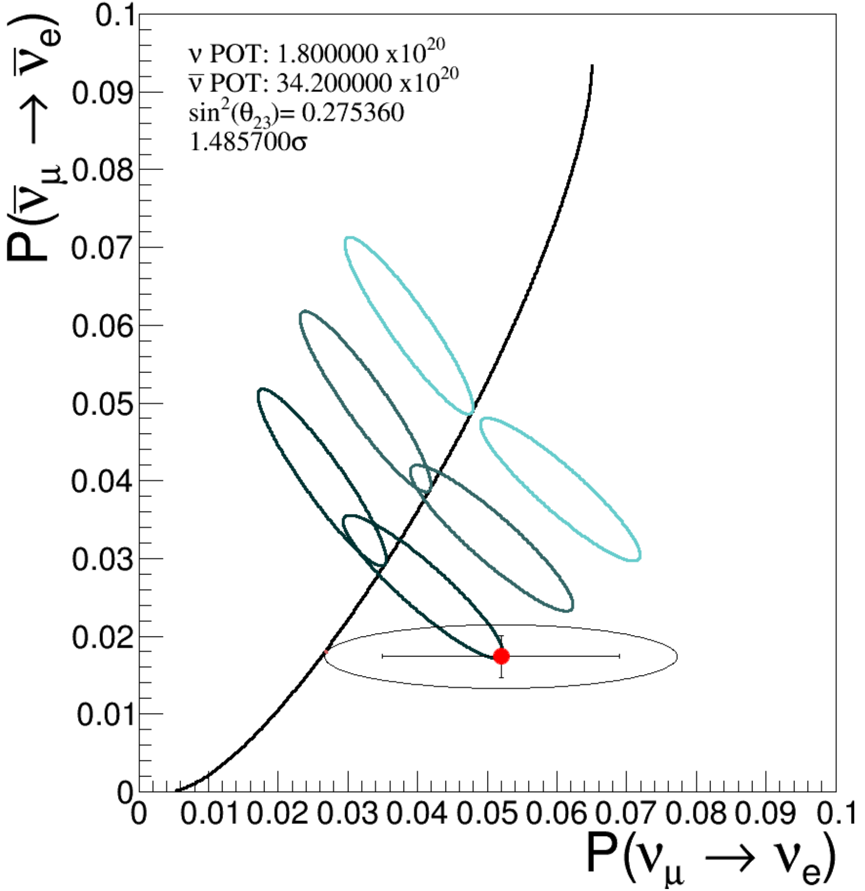
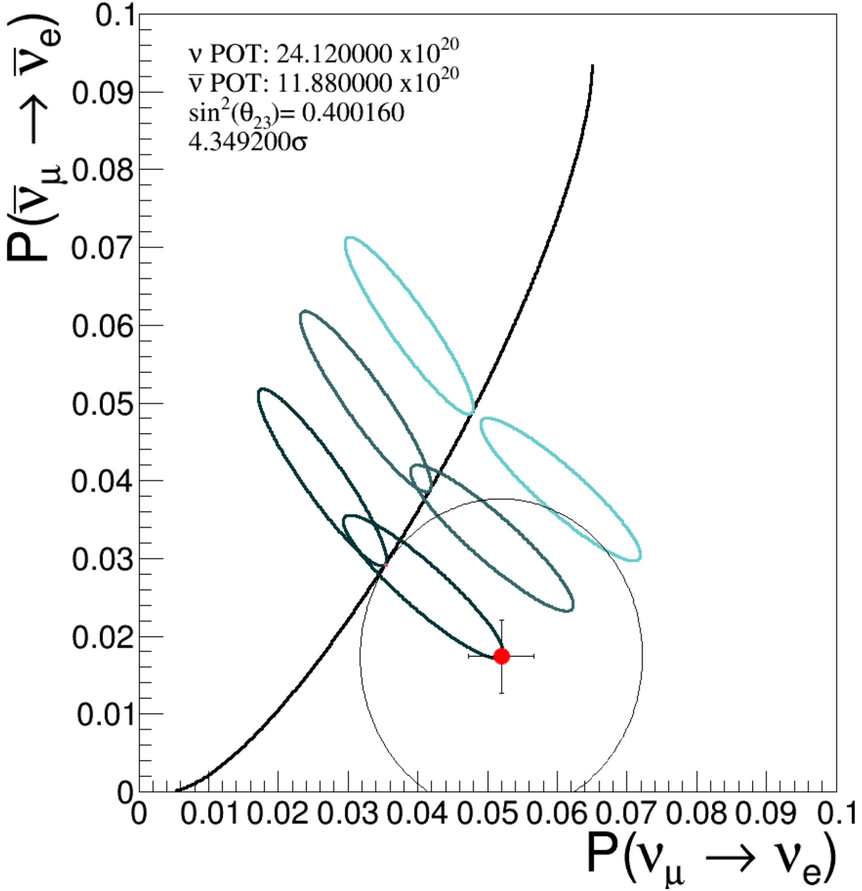


NH, $\sin^2(\theta_{23}) = 0.4$ and $\delta_{CP} = \frac{3\pi}{2}$

67% FHC



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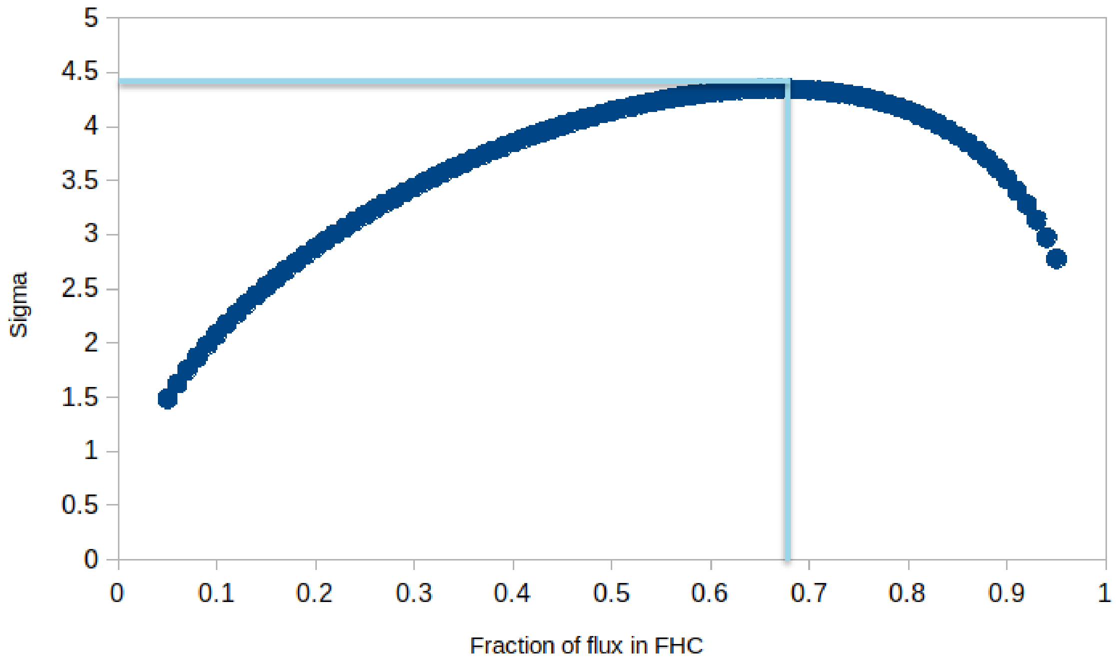
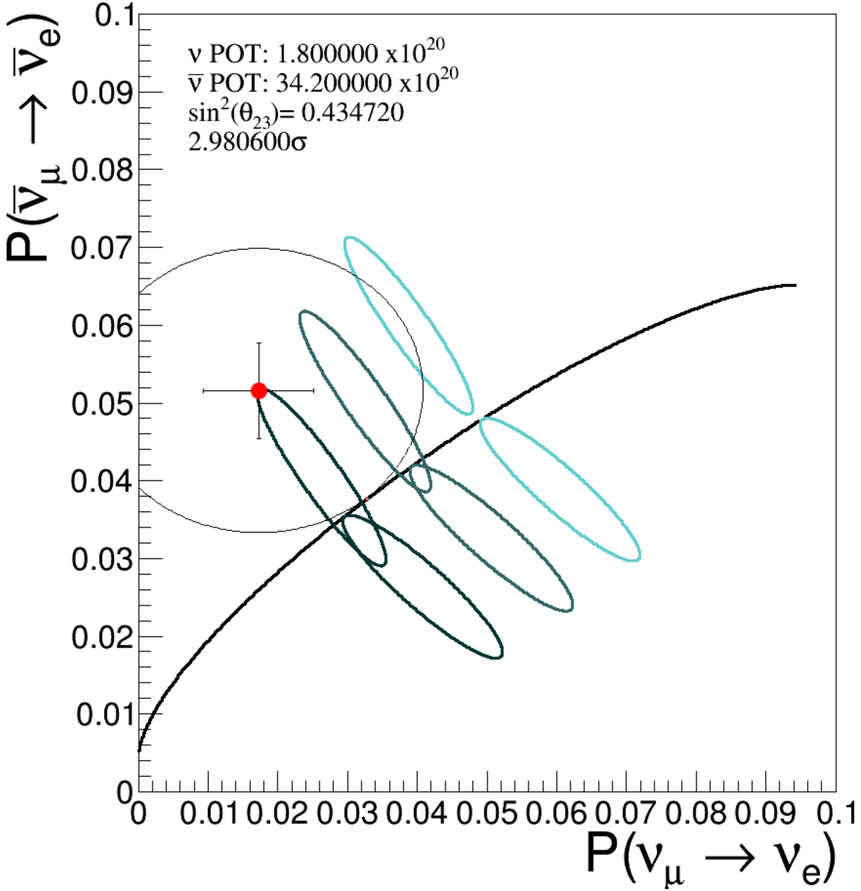
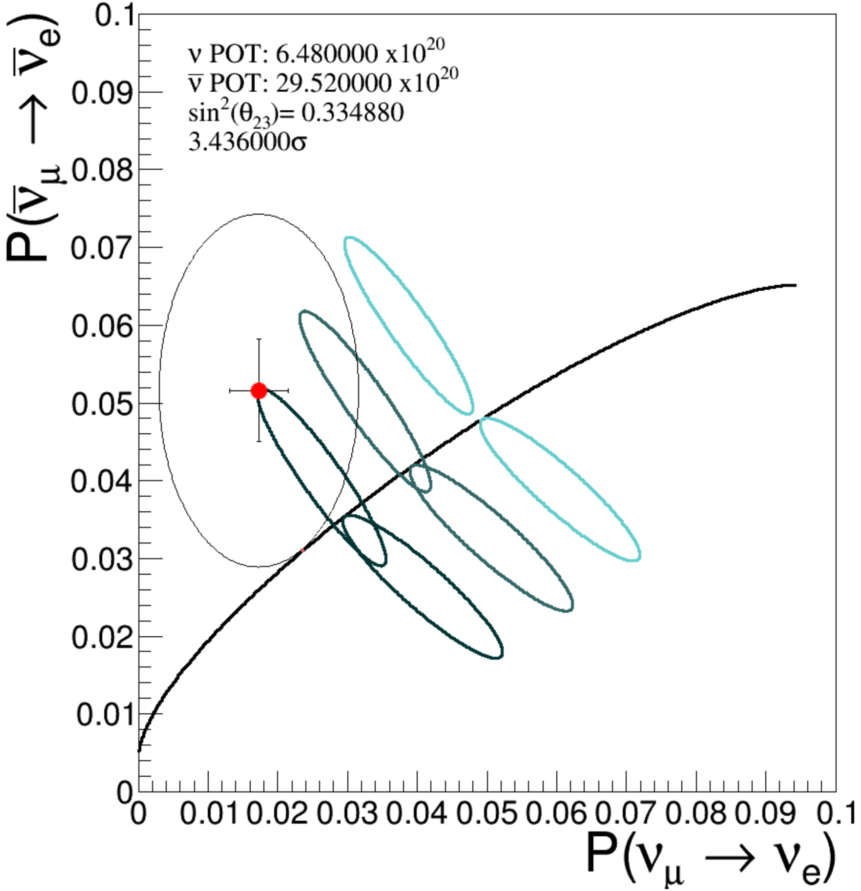


IH, $\sin^2(\theta_{23}) = 0.4$ and $\delta_{CP} = \frac{\pi}{2}$



IH, $\sin^2(\theta_{23}) = 0.4$ and $\delta_{CP} = \frac{\pi}{2}$

18% FHC



Fermilab

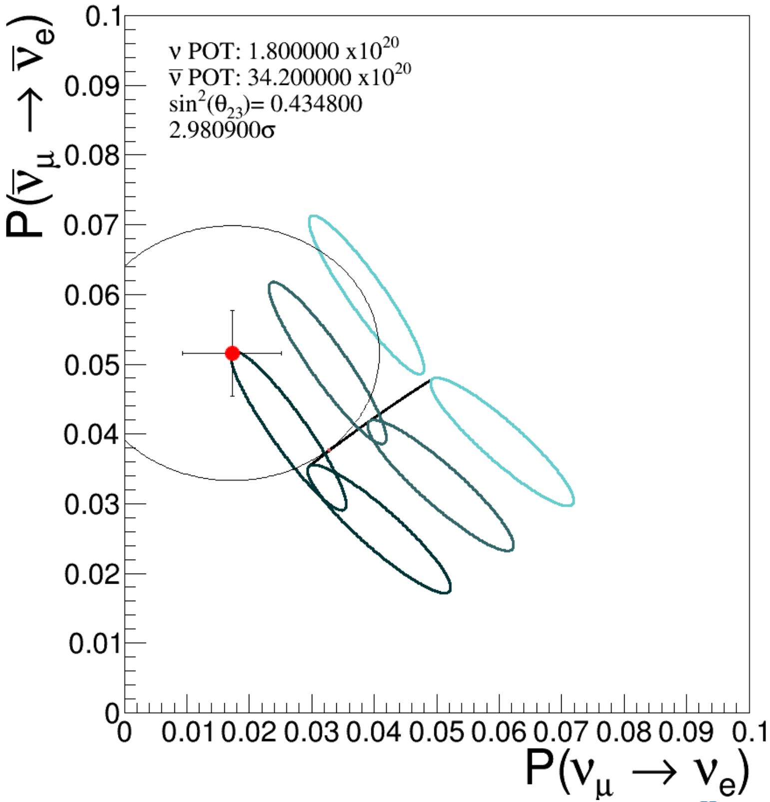
IH, $\sin^2(\theta_{23}) = 0.4$ and $\delta_{CP} = \frac{\pi}{2}$

But, using <u>only</u>…

$$0.4 \leq \sin^2(\theta_{23}) \leq 0.6$$

(a *possible* prior constraint)

$\Longrightarrow$ 100% FHC!



Fermilab

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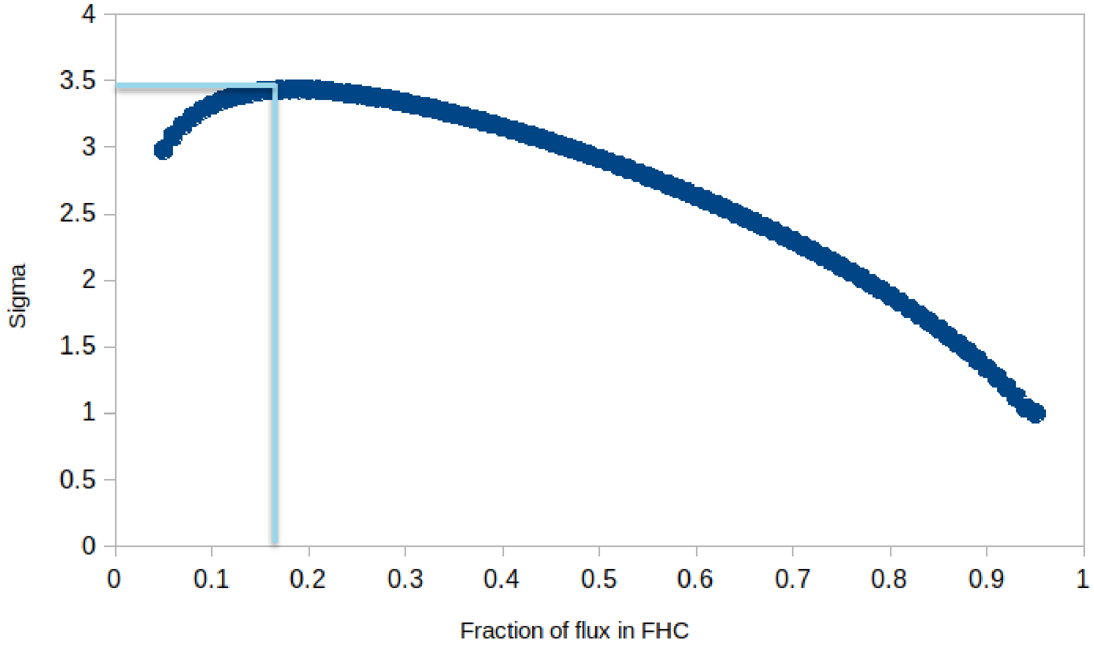
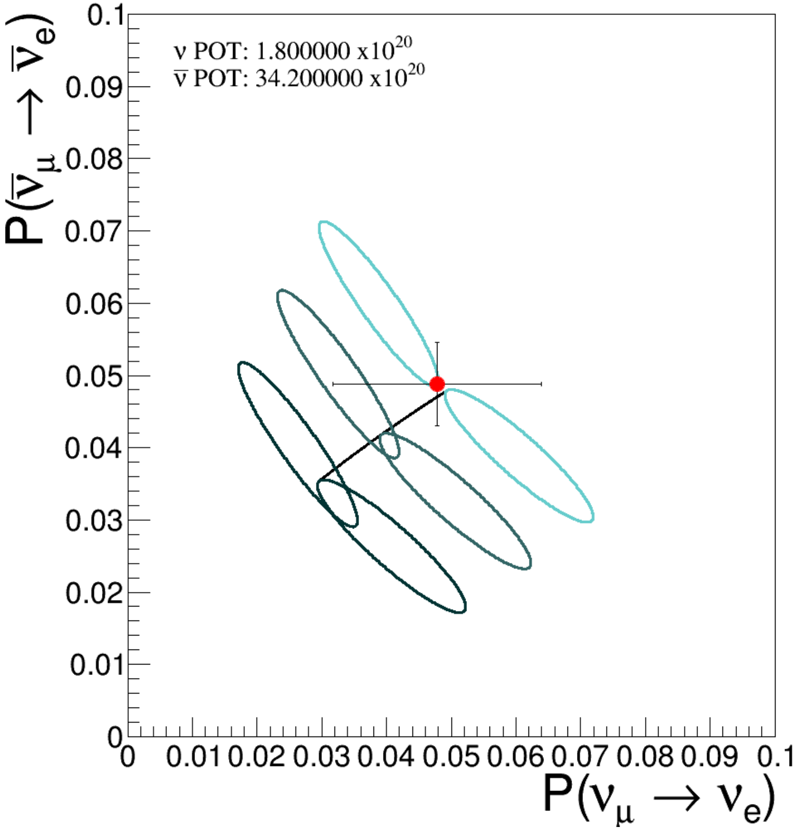
18% FHC



IH, $\sin^2(\theta_{23}) = 0.6$ and $\delta_{CP} = \frac{3\pi}{2}$

Cannot reject any hierarchical hypothesis significantly!

Must break this degeneracy through other measurements…



Fermilab

Summary

- For points at the extrema of the *normal hierarchy*, $67 - 68\%$ FHC is ideal
- For points at the extrema of the *inverted hierarchy*, 18% FHC is ideal
- If the true parameters lie in the area in which the normal and inverted hierarchy are degenerate (or nearly degenerate), *no beam configuration will allow you to determine the hierarchy*
- For a *totally* unknown parameter space, we would want to average the optimal beam fraction over all possible values of δ_{CP} and θ_{23}
  - Extrapolating from *only* these four points, a plan of $\sim 32\%$ FHC, 68% RHC would be ideal
  - Would need to include distributions of priors for a full analysis