PARAMETERIZATION AND APPLICATIONS OF THE LOW-Q^2 NUCLEON VECTOR FORM FACTORS

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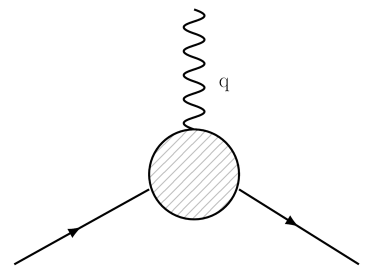
INTRODUCTION

We present the proton and neutron vector form factors in a convenient parametric form that is model independent and optimized for $Q^2 \lesssim$ few GeV^2. The form factors are determined from a global fit to electron scattering data and precise charge radii measurements. A primary goal of this work is to provide a consistent framework for applications such as neutrino event generators and to propagate form factor constraints and uncertainties into cross-section predictions.

REVIEW OF FORM FACTORS

The nucleon electromagnetic current is expressed in terms of Dirac (F_1) and Pauli (F_2) form factors,



$$\Gamma^\mu(q^2) = F_1(q^2)\gamma^\mu + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

In terms of Sachs form factors G_E and G_M,

$$F_1 = \frac{G_E + \tau G_M}{1+\tau}, \quad F_2 = \frac{G_E - G_M}{1+\tau}, \quad \tau = -\frac{q^2}{4M^2}$$

The Rosenbluth formula for electron-nucleon scattering cross section,

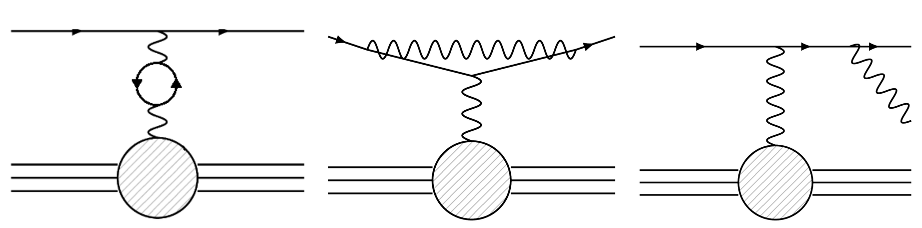
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1+\tau}\left\{G_E^2 + \frac{\tau}{\epsilon}G_M^2\right\}$$

where, $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{Z^2\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}\frac{E'}{E}$$

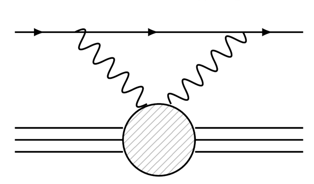
$$\frac{1}{\epsilon} = 1 + 2(2+\tau)\tan^2\frac{\theta}{2}$$

RADIATIVE CORRECTION

We have considered radiative corrections to compare the form factors extracted from experimental data,



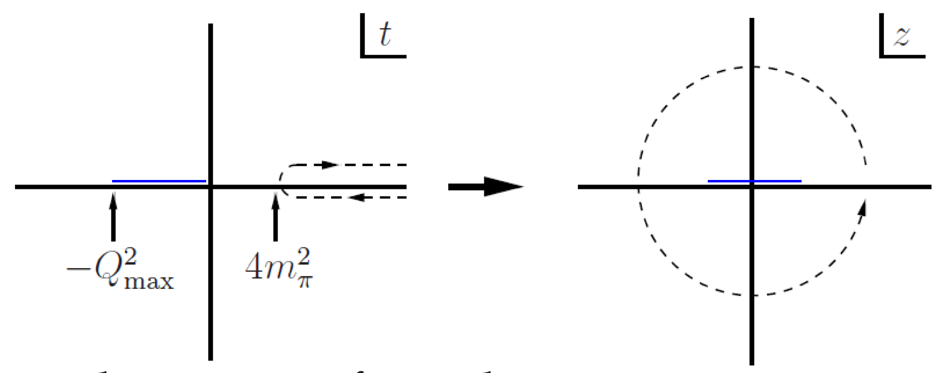
Standard QED radiative corrections are extended by modern calculations of two-photon exchange contributions allowing one floating parameter.



$$d\sigma_{\text{expt}} = d\sigma_{\text{Born}}(1+\delta_{\text{RC}})$$

THE BOUNDED z EXPANSION

According to QCD constraint, nucleon form factors must be analytic in $t \equiv q^2 \equiv -Q^2$ outside of a timelike cut starting at $t_{\text{cut}} = 4m_\pi^2$, the two-pion production threshold ($t_{\text{cut}} = 9m_\pi^2$ for isoscalar combinations).



Now, applying a conformal map we get a true small-expansion variable z for the physical kinematic region of scattering experiments that lies on the negative real axis. It is represented by the blue line for a set of data with maximum momentum transfer Q^2_{max}.

$$z(t:t_{\text{cut}},t_0) = \frac{\sqrt{t_{\text{cut}}-t} - \sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}-t} + \sqrt{t_{\text{cut}}-t_0}}$$

$$G_E = \sum_{k=0}^{k_{\text{max}}} a_k[z(q^2)]^k, \quad G_M = \sum_{k=0}^{k_{\text{max}}} b_k[z(q^2)]^k$$

pQCD requires that the form factors fall off faster than $1/Q^3$ in the large Q^2 limit $\longrightarrow$ four sum rules

$$\sum_{k=n}^{k_{\text{max}}} k(k-1)(k-n+1)a_k = 0, \quad n = 0,1,2,3.$$

We choose $k_{\text{max}} = 8$ and estimate fitting uncertainty as a difference to $k_{\text{max}} \to k_{\text{max}}+1$. Four parameters (e.g., a_1, a_2, a_3, a_4) are determined by fitting to data.

νN CCQE CROSS SECTION

Neutrino-nucleon charged-current quasielastic cross section is expressed in terms of form factors as

$$\frac{d\sigma}{dQ^2}(Q^2,E_\nu) = \frac{G_F^2|V_{ud}|^2}{8\pi}\frac{M^2}{E_\nu^2}$$

$$\left[A(Q^2)\frac{k_l^2+Q^2}{M^2} - B(Q^2)\frac{s-u}{M^2} + C(Q^2)\left(\frac{s-u}{M^2}\right)^2\right]$$

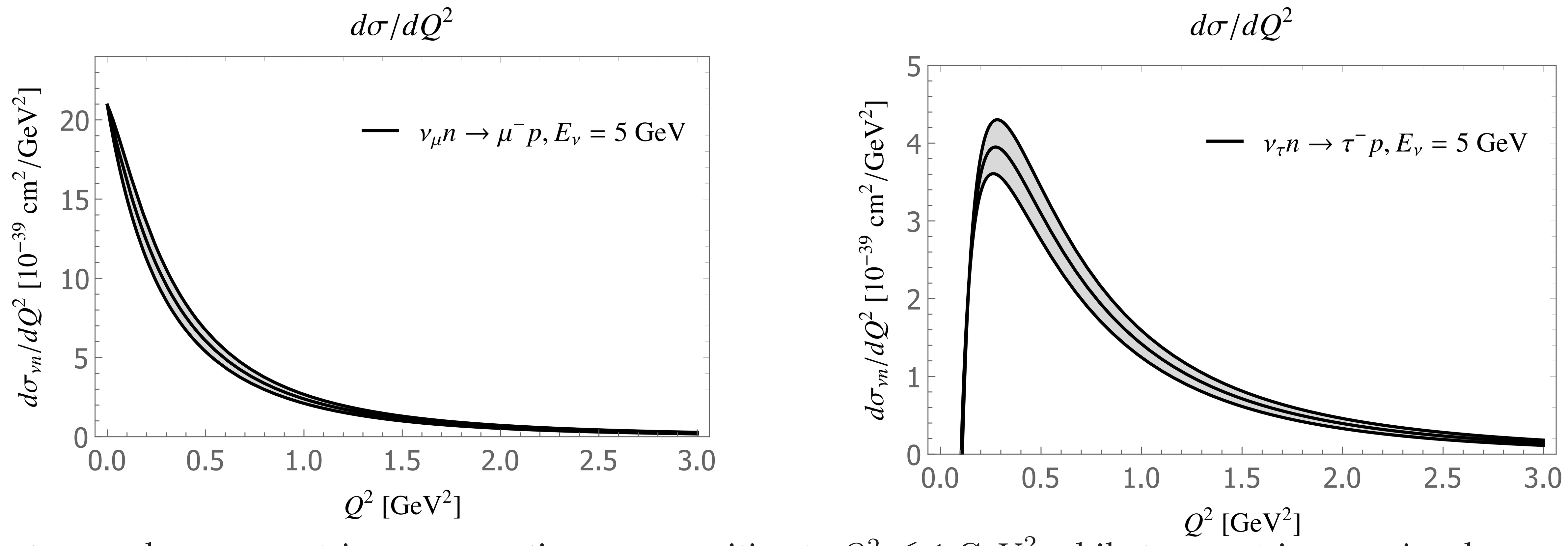
where parameters A, B and C depend on the nucleon isovector form factors $F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$, axial form factor F_A and pseudoscalar form factor F_P

$$A = A(F_1^V, F_2^V, F_A, F_P)$$

$$B = 4\tau F_A(F_1^V + F_2^V)$$

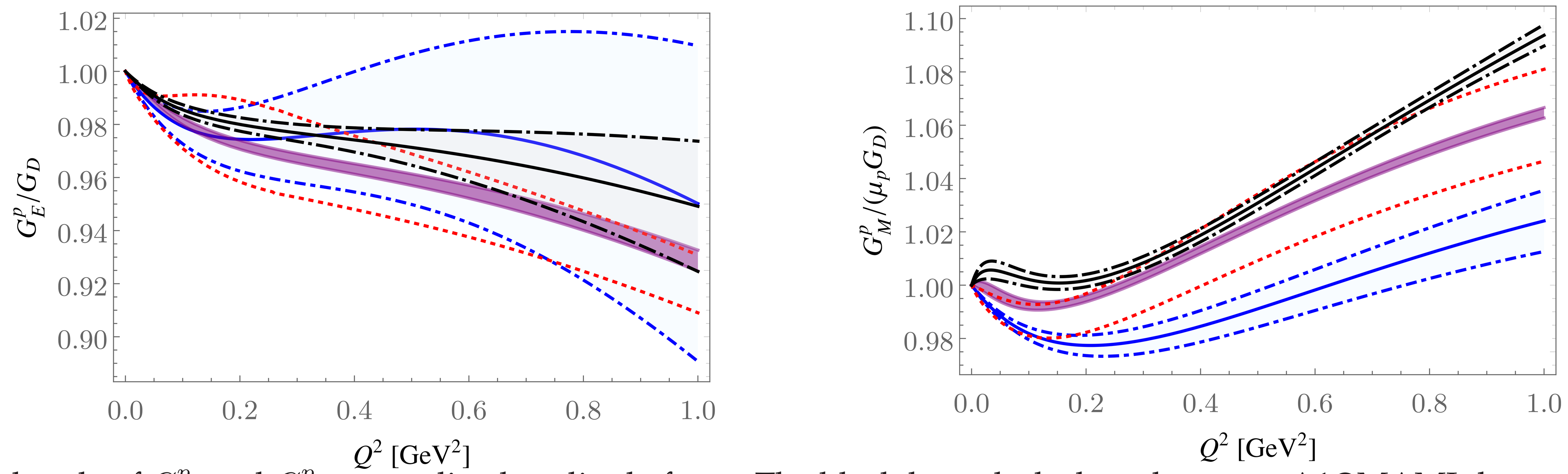
$$C = \frac{1}{4}\left\{(F_1^V)^2 + \tau(F_2^V)^2 + (F_A)^2\right\}$$

RELEVANT KINEMATICS FOR DUNE AND HYPER-K



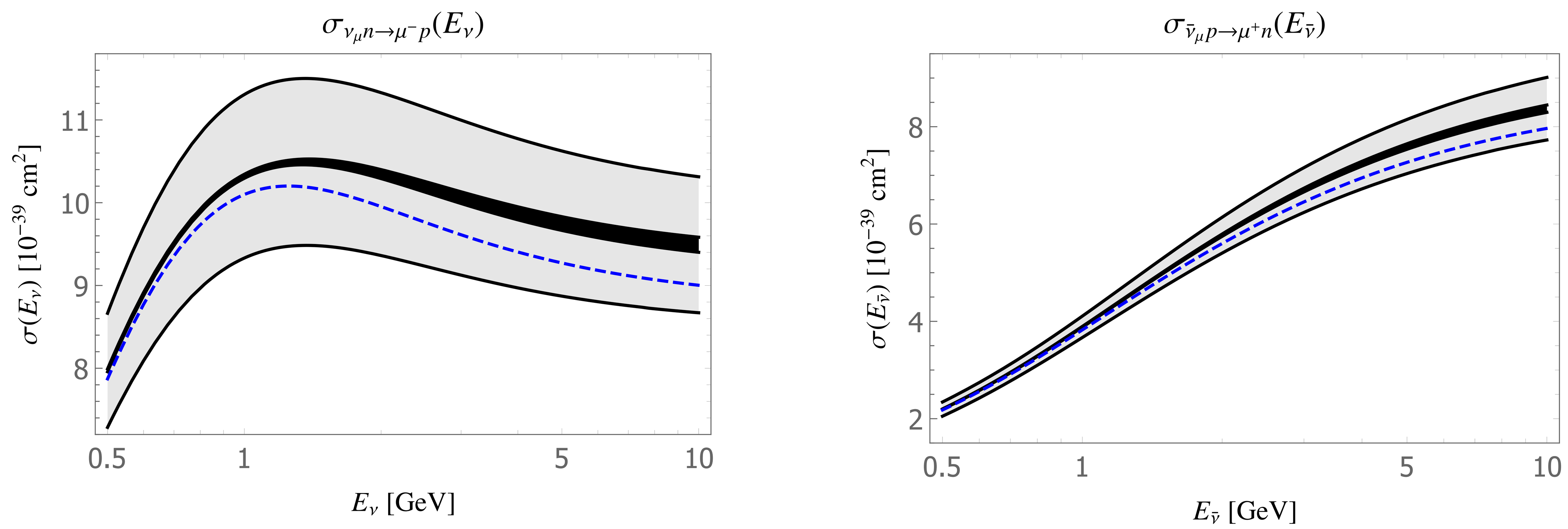
Electron and muon neutrino cross sections are sensitive to $Q^2 \lesssim 1$ GeV^2 while tau neutrino requires larger Q^2. Two isospin-decomposed fits with data below $Q^2 < 1$ GeV^2 and $Q^2 < 3$ GeV^2 are performed.

PROTON FORM FACTORS INCLUDING A1@MAMI DATA



1σ bands of G_E^p and G_M^p normalized to dipole form. The black long dash-dotted curves: A1@MAMI data with electric charge radius constraint; the purple bands: world data including A1@MAMI; the red dotted curves: Ye et al.; the blue dash-dotted curve: BBBA2005. G_M^p from A1@MAMI is significantly different to previous results.

INFLUENCE ON CROSS SECTIONS



CCQE cross section differs by 3–5% compared to commonly-used form factor models (dashed line) when the vector form factors are constrained by recent high-statistics electron–proton scattering data from A1@MAMI.