Singular values as neutrino mixing quantifiers

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Aim of singular values approach to mixing matrices

- Investigation of the region of physically admissible mixing matrices.
- Deeper understanding of the neutrino mixing phenomenon.
- Study of scenarios with different number of additional neutrinos.
- Establishing new restrictions on light-heavy neutrino mixings.

Neutrino masses and mixing

$$\mathcal{L} = -\bar{\nu}_L M \nu_R + H.c. \to M = U^\dagger m V \to \nu_{\alpha L}^{(f)} = (U_{\text{PMNS}})_{\alpha i} \nu_{iL}^{(m)}$$

Current status of masses and mixings [1,2]

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\theta_{12} \in [31.61^\circ, 36.27^\circ]$, $\theta_{23} \in [41.1^\circ, 51.3^\circ]$,
$\theta_{13} \in [8.22^\circ, 8.98^\circ]$, $\delta \in [144^\circ, 357^\circ]$,
$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5}\ [\text{eV}^2]$, $\Delta m_{32}^2 = (2.453 \pm 0.034) \times 10^{-3}\ [\text{eV}^2]$

(A) Deviation from unitarity in the neutrino sector [3]

$$U_{\text{PMNS}} = (1-\alpha)U = TU$$

Entry	(I): $m >$ EW	(II): $\Delta m^2 \gtrsim 100\ \text{eV}^2$	(III): $\Delta m^2 \sim 0.1 - 1\ \text{eV}^2$
$T_{11} = 1 - \alpha_{11}$	$0.99870 \div 1$	$0.976 \div 1$	$0.990 \div 1$
$T_{22} = 1 - \alpha_{22}$	$0.99978 \div 1$	$0.978 \div 1$	$0.986 \div 1$
$T_{33} = 1 - \alpha_{33}$	$0.99720 \div 1$	$0.900 \div 1$	$0.900 \div 1$
$T_{21} = \|\alpha_{21}\|$	$0.0 \div 0.00068$	$0.0 \div 0.025$	$0.0 \div 0.017$
$T_{31} = \|\alpha_{31}\|$	$0.0 \div 0.00270$	$0.0 \div 0.069$	$0,0 \div 0.045$
$T_{32} = \|\alpha_{32}\|$	$0.0 \div 0.00120$	$0.0 \div 0.012$	$0.0 \div 0.053$

(B) Knowledge from matrix analysis [4]

1) Matrix Norm

A matrix norm is a function $\|\cdot\|$ from the set of all complex matrices into $\mathbb{R}$ that satisfies the following properties

$$\|A\| \geq 0 \text{ and } \|A\| = 0 \Leftrightarrow A = 0,$$
$$\|\alpha A\| = |\alpha|\|A\|,$$
$$\|A + B\| \leq \|A\| + \|B\|,$$
$$\|AB\| \leq \|A\|\|B\|.$$

2) Weyl's Inequalities

Let A and B be $n \times n$ Hermitian matrices. Then

$$\lambda_j(A+B) \leq \lambda_i(A) + \lambda_{j-i+1}(B) \text{ for i} \leq \text{j},$$
$$\lambda_j(A+B) \geq \lambda_i(A) + \lambda_{j-i+n}(B) \text{ for i} \geq \text{j}.$$

3) Singular values

Singular values σ_i of a given matrix A are positive square roots of the eigenvalues λ_i of the matrix $AA^\dagger$

$$\sigma_i(A) = \sqrt{\lambda_i(AA^\dagger)}.$$

4) Contractions

$$\|A\| \leq 1$$

Operator norm (spectral norm)

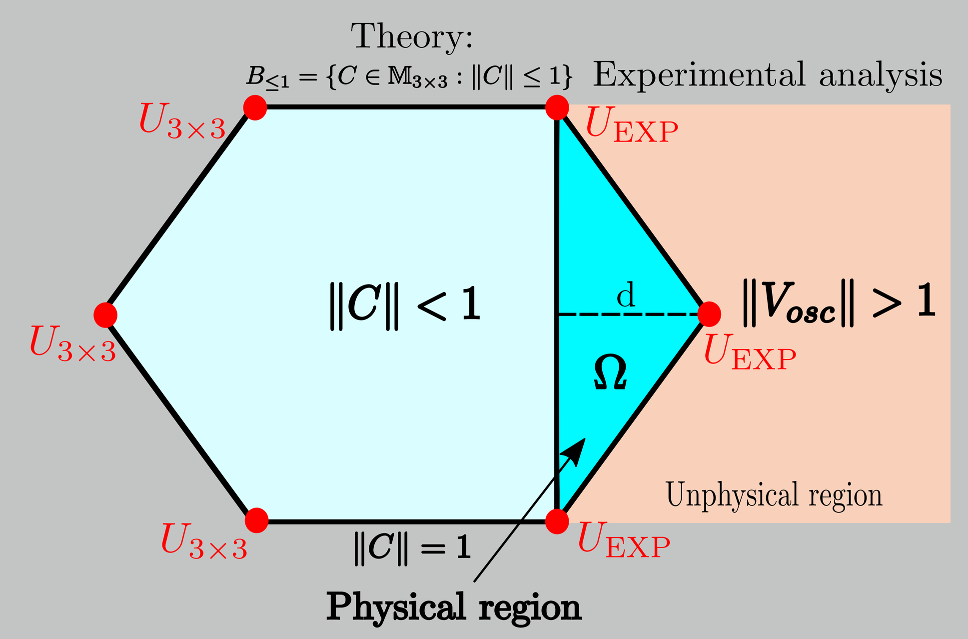
$$\|A\| := \sup_{\|x\|=1} \|Ax\| = \sigma_{\max}(A)$$

5) Unitary dilation

$$U_{\text{PMNS}} \xrightarrow{\text{dilation}} \begin{pmatrix} U_{\text{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} \equiv U \to UU^\dagger = I$$

(C) Region of physically admissible mixing matrices [5]

$$\Omega := \text{conv}(U_{\text{PMNS}}) = \{\sum_{i=1}^m \alpha_i U_i \mid U_i \in U(3), \alpha_1, ..., \alpha_m \geq 0, \sum_{i=1}^m \alpha_i = 1,$$
$$\theta_{12}, \theta_{13}, \theta_{23} \text{ and } \delta \text{ given by experimental values}\}$$



It is divided into four disjoint subsets:

Ω_1 : 3+1 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 = 1.0, \sigma_3 < 1.0\}$,
Ω_2 : 3+2 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 < 1.0, \sigma_3 < 1.0\}$,
Ω_3 : 3+3 scenario: $\Sigma = \{\sigma_1 < 1.0, \sigma_2 < 1.0, \sigma_3 < 1.0\}$,
Ω_4 : PMNS scenario: $\Sigma = \{\sigma_1 = 1, \sigma_2 = 1, \sigma_3 = 1\}$.

(D) Numerical studies with singular values [6]

- Analysis of the amount of the space for additional neutrinos based on deviations of singular values from unity.
  Results: e.g. $\Delta m^2 \sim 0.1 - 1\ \text{eV}^2 \to \sigma_3 = 0.889$
- A study of possible distinction between three scenarios with different number of additional neutrinos on the level of experimental data using singular values and corresponding division of the neutrino mixing space Ω.
  Results:
  - 3+2 and 3+3 scenarios cannot be distinguished.
  - **3+1 scenario differs.**

(E) Estimation of the light-heavy U_{lh} mixing [6]

Ω_1 : 3+1 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 = 1.0, \sigma_3 < 1.0\}$

$$\begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c & -s \\ 0 & 0 & s & c \end{pmatrix} \begin{pmatrix} Q_1^\dagger & 0 \\ 0 & Q_2^\dagger \end{pmatrix}.$$

We are interested in the estimation of the light-heavy mixing sector which is given by

$$U_{lh} = W_1 S_{12} Q_2^\dagger,$$

where $W_1 \in \mathbb{C}^{3\times3}$ is unitary, $S_{12} = (0, 0, -s)^T$ and $Q_2 = e^{i\theta}, \theta \in (0, 2\pi]$.
Taking into account exact values of the W_1 we can estimate the light-heavy mixing by the analytical formula

$$|U_{i4}| = |w_{i3}| \cdot |\sqrt{1 - \sigma_3^2}|, \quad i = e, \mu, \tau.$$

Estimation of the light-heavy mixing via CS decomposition

- (I) $m >$ EW :
  $|U_{e4}| \leq 0.021$, $|U_{\mu4}| \leq 0.021$, $|U_{\tau4}| \leq 0.075$.
  $|U_{e4}| \leq 0.055$ [7], $|U_{\mu4}| \leq 0.057$ [7], $|U_{\tau4}| \leq 0.079$ [7].
- (II): $\Delta m^2 \gtrsim 100\ \text{eV}^2$:
  $|U_{e4}| \leq 0.082$, $|U_{\mu4}| \leq 0.099$, $|U_{\tau4}| \leq 0.44$.
- (III) $\Delta m^2 \sim 0.1 - 1\ \text{eV}^2$:
  $|U_{e4}| \leq 0.130$, $|U_{\mu4}| \leq 0.167$, $|U_{\tau4}| \leq 0.436$.
  $|U_{e4}| \leq 0.167$ [8], $|U_{\mu4}| \leq 0.148$ [8], $|U_{\tau4}| \leq 0.361$ [9].

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