

1 Objective

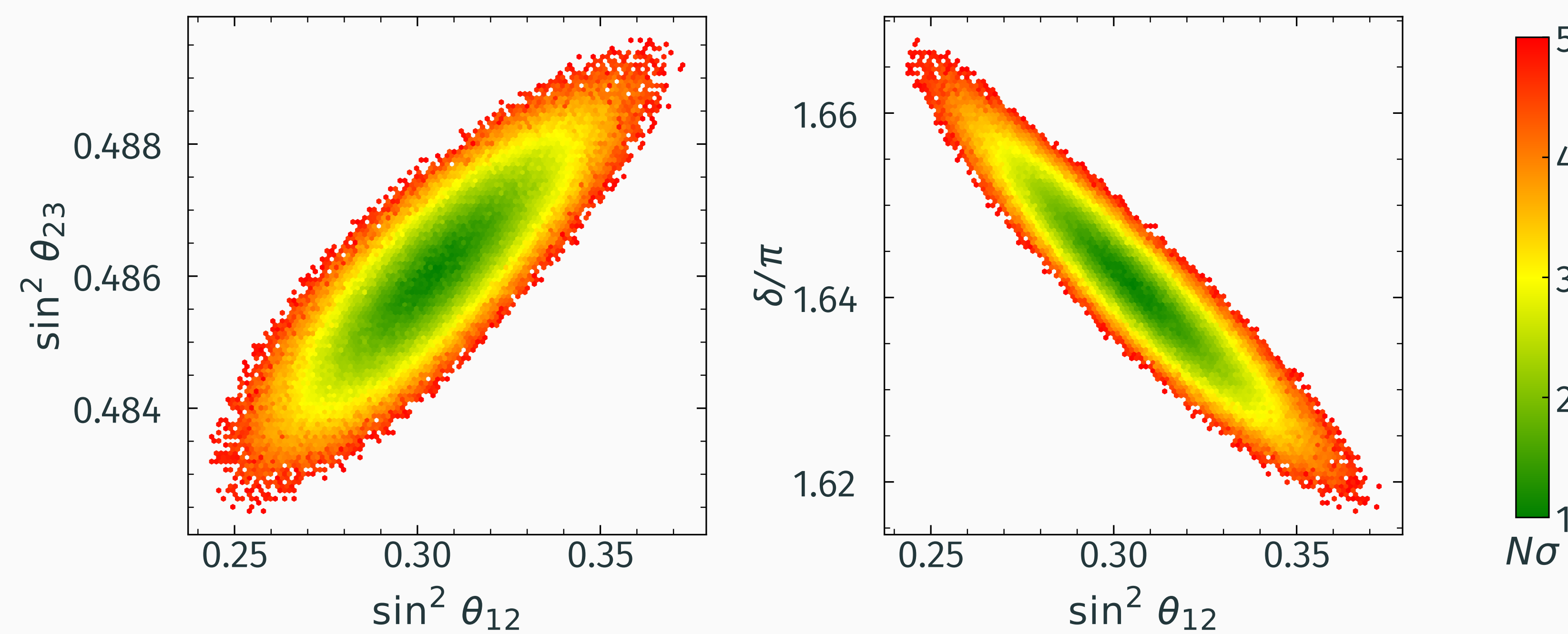
- Predict the absolute neutrino mass scale and the lepton CPV phases
- Predict testable correlations between the observables

2 Main result: predictive modular S_4 model^{1,2}

- Predictive model of lepton masses and mixing based on modular S_4 flavour symmetry
- 7 parameters to fit 8 observables ($N\sigma = 1.012$) and predict 4 observables
- Predictions at a few percent level:

$$\begin{aligned} m_1 &= 0.012 \text{ eV}, & \Sigma m &= 0.078 \text{ eV}, & \delta/\pi &= \pm 1.64, \\ m_2 &= 0.015 \text{ eV}, & |\langle m \rangle| &= 0.012 \text{ eV}, & \alpha_{21}/\pi &= \pm 0.35, \\ m_3 &= 0.051 \text{ eV}, & \sin^2 \theta_{23} &= 0.486, & \alpha_{31}/\pi &= \pm 1.25 \end{aligned}$$

- Correlations:



- Both CP and flavour symmetry are spontaneously broken by the modulus field τ

3 Model specification

- Seesaw type-1 with 3 heavy neutrinos N^c
- Modular group representations (ρ, k) (see Section 4):

$$L \sim (\mathbf{3}, 2), \quad E^c \sim (\mathbf{1}', 2) \oplus (\mathbf{1}, 4) \oplus (\mathbf{1}', 4), \quad N^c \sim (\mathbf{3}, 0)$$

- Superpotential:

$$W = \alpha (E_1^c L Y_{3'}^{(2)})_{\mathbf{1}} H_d + \beta (E_2^c L Y_{\mathbf{3}}^{(4)})_{\mathbf{1}} H_d + \gamma (E_3^c L Y_{3'}^{(4)})_{\mathbf{1}} H_d + g (N^c L Y_{\mathbf{2}}^{(2)})_{\mathbf{1}} H_u + g' (N^c L Y_{3'}^{(2)})_{\mathbf{1}} H_u + \Lambda (N^c N^c)_{\mathbf{1}}$$

- CP invariance condition: $\text{Im}(g'/g) = 0$ (see Section 5)

References

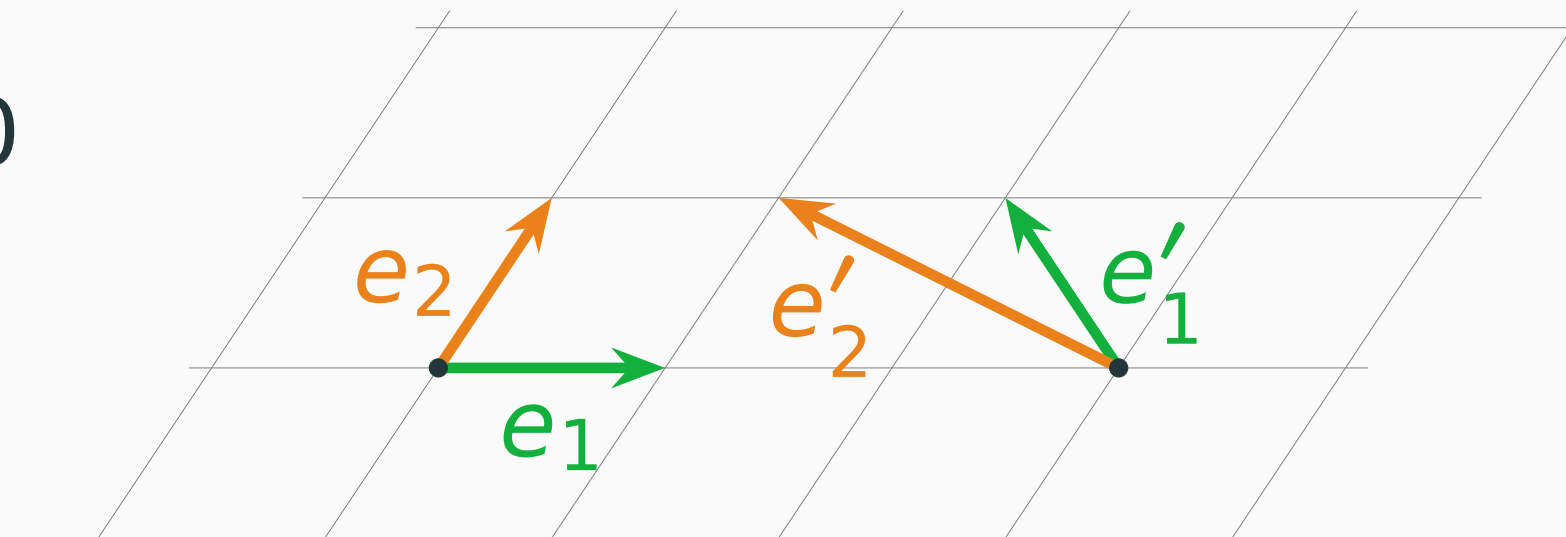
1. P. P. Novichkov, J. T. Penedo, S. T. Petcov, A. V. Titov, JHEP 07 (2019), 165
2. P. P. Novichkov, J. T. Penedo, S. T. Petcov, A. V. Titov, JHEP 04 (2019), 005
3. F. Feruglio, 1706.08749, in book "From My Vast Repertoire ...: Guido Altarelli's Legacy"
4. P. P. Novichkov, J. T. Penedo, S. T. Petcov, 2006.03058

4 Modular invariance framework³

- Naturally leads to a discrete flavour symmetry
- Discrete flavour symmetry can explain 2 large + 1 small lepton mixing angles
- More predictive than conventional discrete symmetry approach: avoids multiple new scalar fields, baroque potentials and shaping symmetries

$$\text{modulus } \tau = \frac{e_2}{e_1} \in \mathbb{C}, \text{ Im } \tau > 0$$

$$\tau' = \frac{e_2'}{e_1'}$$



$$\tau' = \frac{a\tau + b}{c\tau + d} \quad \text{modular group } \Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1 \right\}$$

N	Γ_N
2	S_3
3	A_4
4	S_4
5	A_5

$$\phi'_i = (c\tau + d)^{-k} \rho \left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right]_{ij} \phi_j$$

weight $k \in \mathbb{Z}$

unitary representation of Γ_N

$$Y(\tau) \phi_1 \dots \phi_n \quad k = k_1 + \dots + k_n$$

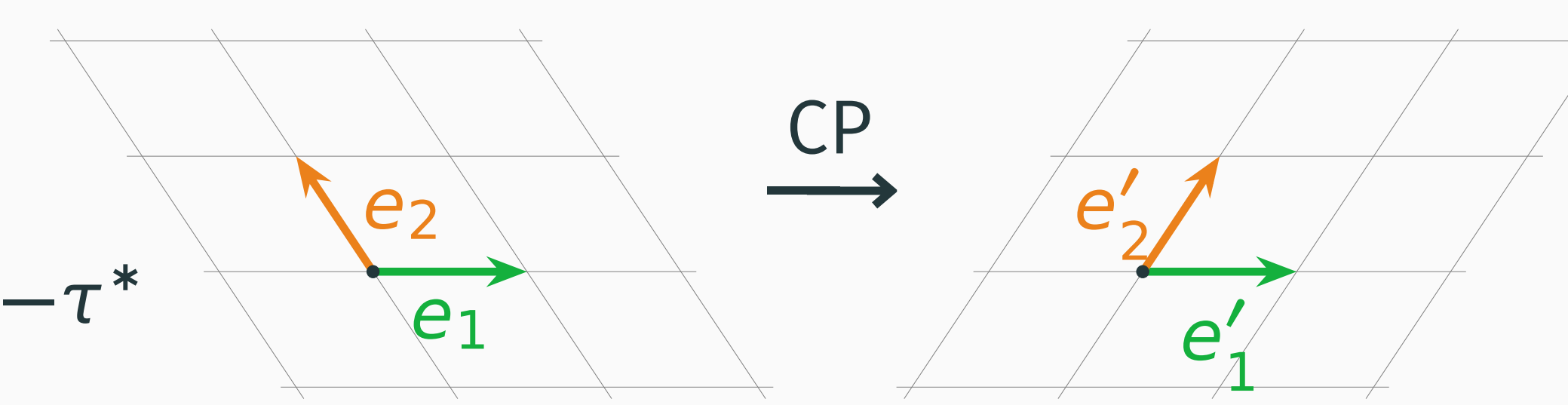
$$\rho \otimes \rho_1 \otimes \dots \otimes \rho_n \supset \mathbf{1}$$

coupling is a modular form (very constrained)

5 CP invariance in modular-symmetric theories^{1,4}

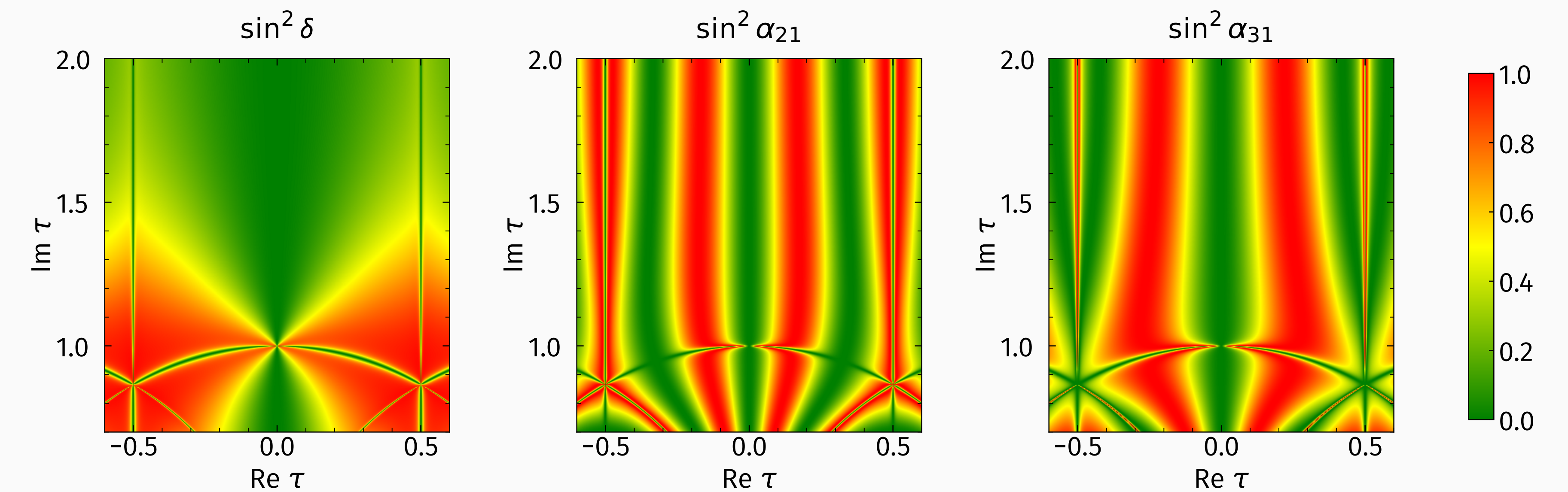
$$\tau = \frac{e_2}{e_1}$$

$$\tau' = \frac{e_2'}{e_1'} = -\tau^*$$



- in certain basis, $\phi'_i = \phi_i^\dagger$
- CP invariance condition: all couplings are real

CP is broken spontaneously by τ , unless $\text{Re } \tau = 0$ or $\text{Re } \tau = \pm 1/2$ or $|\tau| = 1$:



In our model, $\tau \approx 0.099 + i 1.016$, which allows for large CP violation.