Tri-Partite Entanglement In Neutrino Oscillations (ID 337)

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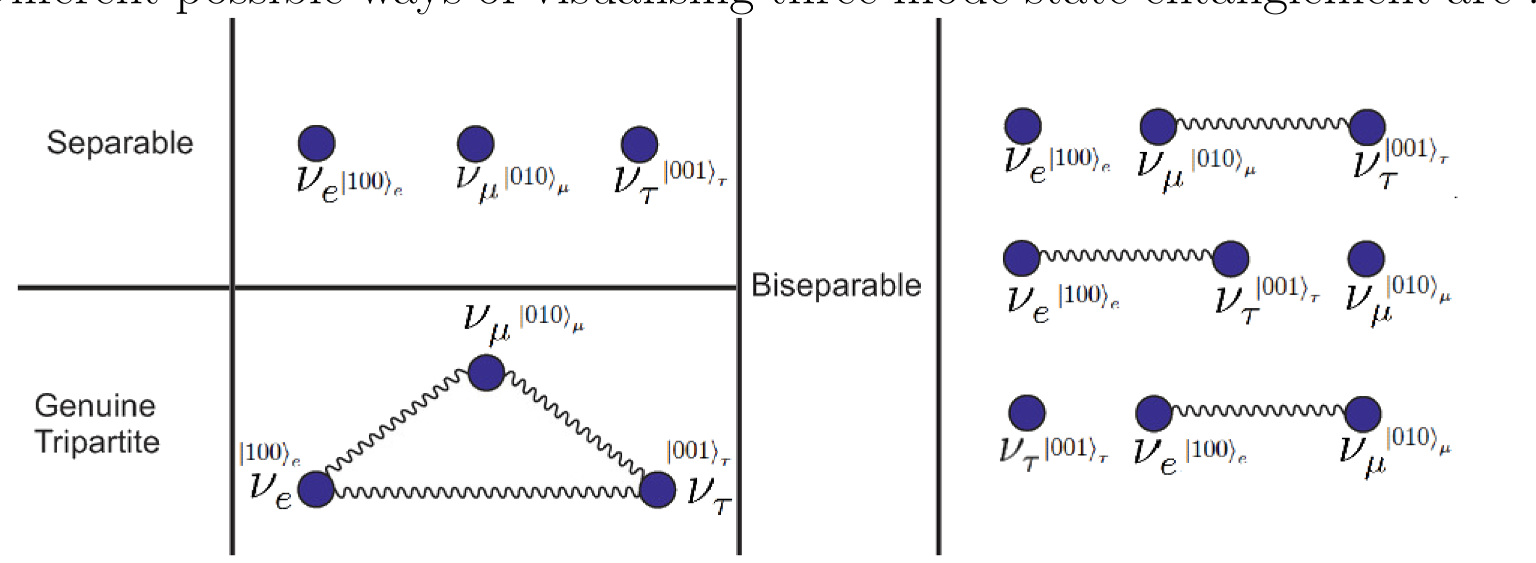
Abstract

We investigate and quantify various measures of entanglement in two and three flavor neutrino oscillations. We identify quantum information systems that bear resemblance to neutrino systems.

Introduction

- The neutrino flavor states $|\nu_\alpha\rangle$ are linear superposition of mass eigenstates $(|\nu_j\rangle)$, $|\nu_\alpha\rangle = \Sigma_j U_{\alpha j}|\nu_j\rangle$. The time evolution follows $|\nu_\alpha(t)\rangle = \Sigma_j e^{-iE_j t} U_{\alpha j}|\nu_j\rangle$, where $U_{\alpha j}$ are the elements of the PMNS (Pontecorvo-Maki-Nakagawa-Sakita) matrix and E_j is the energy associated with the mass eigenstates $|\nu_j\rangle$. This is a superposition state.
- In the three mode basis, we identify each flavor state $(\alpha = e, \mu, \tau)$ at t=0 as [1,2,3] : $|\nu_e\rangle = |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau \equiv |100\rangle_e$; $|\nu_\mu\rangle = |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau \equiv |010\rangle_\mu$; and $|\nu_\tau\rangle = |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau \equiv |001\rangle_\tau$
- Different possible ways of visualising three mode state entanglement are :



Bi-Partite Entanglement in Two-Flavor Neutrino Oscillations

- In two-flavor$(\nu_\alpha \to \nu_\beta)$ mixing, $|\nu_e(t)\rangle = \tilde{U}_{ee}(t)|10\rangle_e + \tilde{U}_{e\mu}(t)|01\rangle_\mu$, where $|10\rangle$ and $|01\rangle$ are two-qubit states, the density matrix is $\rho^{e\mu}(t) = |\nu_e(t)\rangle\langle\nu_e(t)| = (\tilde{U}_{ee}(t)|10\rangle + \tilde{U}_{e\mu}(t)|01\rangle)(\tilde{U}^*_{ee}(t)\langle 10| + \tilde{U}^*_{e\mu}(t)\langle 01|)$
- Positive Partial Transpose (PPT) criterion is a condition for determining entanglement in bi-partite system. It states that if the partial transposition $\rho^{T_e}_{pq,rs}(t) = \rho^{e\mu}_{rq,ps}(t)$ or $\rho^{T_\mu}_{pq,rs}(t) = \rho^{e\mu}_{ps,rq}(t)$ of a density matrix is a positive operator with all positive eigenvalues then the system is un-entangled. If the system has even one negative eigenvalues then it is entangled.
- Various measures of bi-partite entanglement are :

Entanglement Measures	Results obtained from $\rho^{T_\mu}(t)$
1.PPT Criterion for an entanglement	Eigenvalues of $\rho^{T_\mu}(t)$ are $\lambda_1 = P_d$, $\lambda_2 = P_a$, $\lambda_3 = \sqrt{P_d P_a}$, $\lambda_4 = -\sqrt{P_d P_a}$
2.Negativity $N(\rho^{e\mu}) = \lVert\rho^{T_\mu}\rVert - 1$	$N_{e\mu} = 2\sqrt{P_a P_d}$

- Using the "Spin-flipped" density matrix, $\tilde{\rho}^{e\mu}(t) = (\sigma_y \otimes \sigma_y)\rho^{*e\mu}(t)(\sigma_y \otimes \sigma_y)$ where σ_x and σ_y are Pauli matrices, we calculate concurrence: $C(\rho^{e\mu}(t)) \equiv [max(\mu_1 - \mu_2 - \mu_3 - \mu_4, 0)]$, in which $\mu_1, ..., \mu_4$ are the eigenvalues of the matrix $\rho^{e\mu}(t)\tilde{\rho}^{e\mu}(t)$, tangle: $\tau(\rho^{e\mu}) \equiv [max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)]^2$ and linear entropy: $S(\rho^{e\mu}) = 1 - Tr(\rho^{e\mu})^2$.

Entanglement Measures	Results obtained from $\rho^{e\mu}(t)$
1.Concurrence $C(\rho^{e\mu}(t))$	Only one eigenvalue(μ) is non zero: $2\sqrt{P_a P_d}$ thus $C_{e\mu} = 2\sqrt{P_a P_d}$
2.Tangle $\tau(\rho^{e\mu})$	$\tau_{e\mu} = 4P_a P_d$
5. Linear Entropy $S(\rho^{e\mu})$	$S_{e\mu} = 4P_a P_d = \tau_{e\mu}$

- We see that tangle between e and μ modes as $\tau_{e\mu} = C^2_{e\mu} = 4P_a P_d$.
- The figure below shows the time evolution of the various measures of entanglement compared to the oscillation probabilities in a typical reactor experiment.

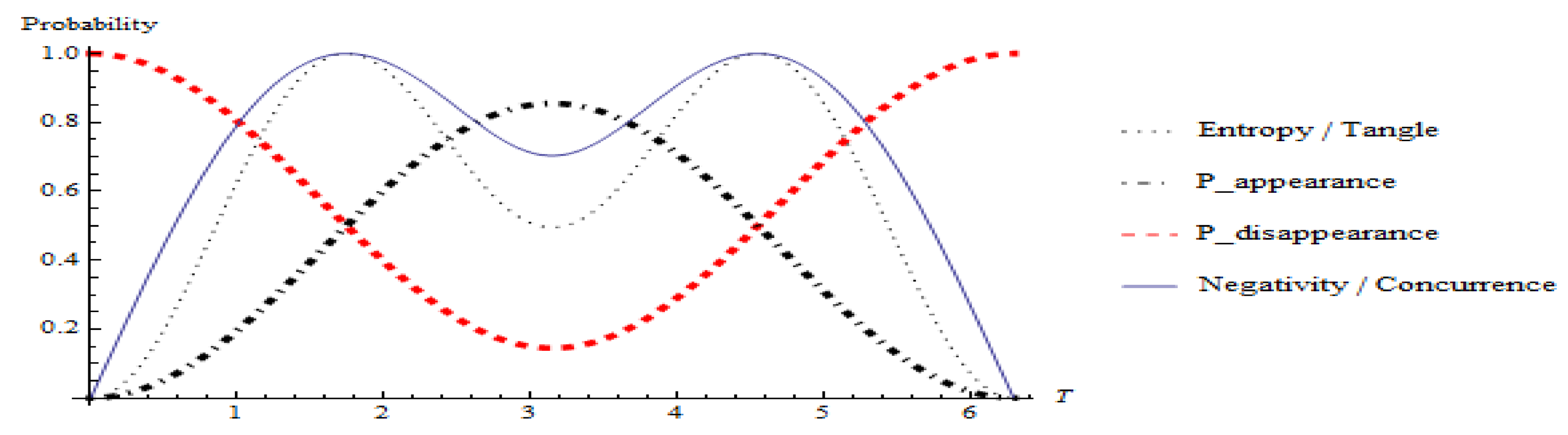


Figure 1:$\tau_{e\mu}$, $S_{e\mu}$ $N_{e\mu}$ and $C_{e\mu}$, P_d, P_a as functions of the scaled time $T \equiv \Delta m^2 t/2E$ for $|\nu_e(t)\rangle$, considering the experimental value $\sin^2\theta = 0.310$, where θ= mixing angle [4].

- We see that when $P_a = P_d = 0.5$, all measures of entanglement tend to 1 i.e, $N_{e\mu} = \tau_{e\mu} = C_{e\mu} = S_{e\mu} = 1$, which corresponds to maximally entangled state.

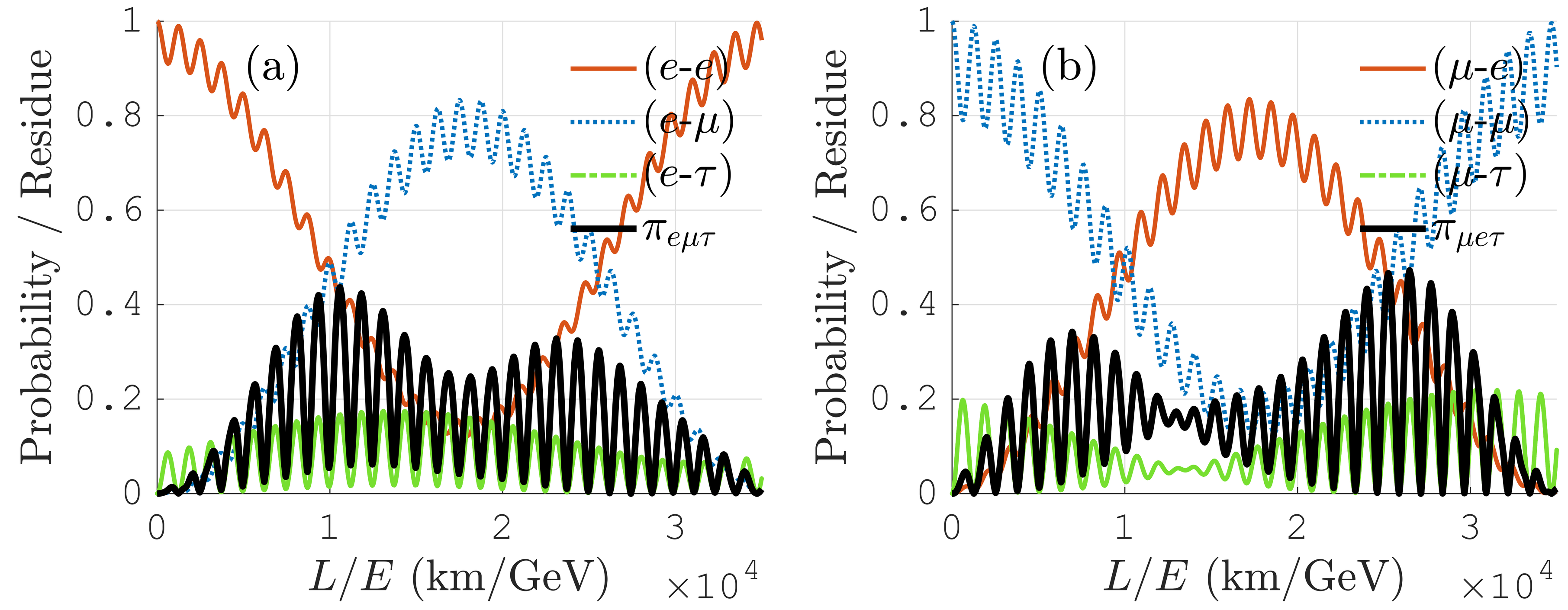
Tri-Partite Entanglement in Three-Flavor Neutrino Oscillations

- Biseparable states are formed in a three particle system, by considering two out of three modes state as a single state.
- The density matrix of the time evolved electron neutrino flavor state is $\rho^{e\mu\tau}(t) = |\tilde{U}_{ee}(t)|^2|100\rangle\langle 100| + |\tilde{U}_{e\mu}(t)|^2|010\rangle\langle 010| + |\tilde{U}_{e\tau}(t)|^2|001\rangle\langle 001| + \tilde{U}_{ee}(t)\tilde{U}^*_{e\mu}(t)|100\rangle\langle 010| + \tilde{U}_{ee}(t)\tilde{U}^*_{e\tau}(t)|100\rangle\langle 001| + \tilde{U}_{e\mu}(t)\tilde{U}^*_{e\tau}(t)|010\rangle\langle 001| + |\tilde{U}_{e\tau}(t)\tilde{U}^*_{ee}(t)|001\rangle\langle 100| + \tilde{U}_{e\tau}(t)\tilde{U}^*_{e\mu}(t)|001\rangle\langle 010|$.
- The pairwise measures of entanglement are negativity $(N^2_{e(\mu\tau)})$, Concurrence $(C^2_{e(\mu\tau)})$, tangle $(\tau_{e(\mu\tau)})$ and Linear entropy $(S_{e(\mu\tau)})$. Because we have effectively reduced the three ν system to bi-partite system, the measures remain the same. $N^2_{e(\mu\tau)} = C^2_{e(\mu\tau)} = \tau_{e(\mu\tau)} = S_{e(\mu\tau)} = 4P_a P_d$.
- For tri-partite entanglement, criterion is known as Coffman-Kundu-Wooters (CKW) inequality. It states that the sum of quantum correlations between e and μ, and between e and τ, is either less than or equal to the quantum correlations between e and $\mu\tau$ (treating it as a single object) [4,5,6]: $C^2_{e\mu} + C^2_{e\tau} \leq C^2_{e(\mu\tau)}$, $\tau_{e\mu} + \tau_{e\tau} \leq \tau_{e(\mu\tau)}$ and $N^2_{e\mu} + N^2_{e\tau} \leq N^2_{e(\mu\tau)}$.

Bi-separable entanglement measures	Results from $\rho^{e\mu\tau}(t)$
1. Concurrence equality	$C^2_{e\mu} + C^2_{e\tau} = C^2_{e(\mu\tau)}$
2. Tangle equality	$\tau_{e\mu} + \tau_{e\tau} = \tau_{e(\mu\tau)}$
3. Negativity inequality	$N^2_{e\mu} + N^2_{e\tau} < N^2_{e(\mu\tau)}$

- There is one more genuine measures of tri-partite entanglement quantified by three-tangle and three-π negativity known as residual entanglement.
- The residual entanglement three-π for electron neutrino flavor state $|\nu_e(t)\rangle$ is, (see Fig.2)

$$\pi_{e\mu\tau} = \frac{4}{3}[|\tilde{U}_{ee}(t)|^2\sqrt{|\tilde{U}_{ee}(t)|^4 + 4|\tilde{U}_{ee}(t)|^2|\tilde{U}_{e\tau}(t)|^2} + |\tilde{U}_{e\mu}(t)|^2\sqrt{|\tilde{U}_{e\mu}(t)|^4 + 4|\tilde{U}_{ee}(t)|^2|\tilde{U}_{e\tau}(t)|^2} + |\tilde{U}_{e\tau}(t)|^2\sqrt{|\tilde{U}_{e\tau}(t)|^4 + 4|\tilde{U}_{ee}(t)|^2|\tilde{U}_{e\mu}(t)|^2} - |\tilde{U}_{ee}(t)|^4 - |\tilde{U}_{e\mu}(t)|^4 - |\tilde{U}_{e\tau}(t)|^4] > 0.$$



Figure(2):(a) Time evolved electron neutrino flavor state $|\nu_e(t)\rangle$ (relevant to reactor experiment) and (b) a muon flavor state $|\nu_\mu(t)\rangle$ (relevant to accelerator experiment) vs scale of distance per energy unit $\frac{L}{E}$. At $\frac{L}{E} > 0$ entanglement among three-flavor modes occurs i.e, $\pi_{e\mu\tau} > 0$ and $\pi_{\mu e\tau} > 0$, and exhibits a typical oscillatory behavior [4,8].

- The residual entanglement tr-partite results are :

Residual Entanglement	Tri-Partite results
Three-tangle $\tau_{e\mu\tau} = C^2_{e(\mu\tau)} - C^2_{e\mu} + C^2_{e\tau}$	$\tau_{e\mu\tau} = 0$
Three-π $\pi_{e\mu\tau} = \frac{1}{3}(N^2_{e(\mu\tau)} + N^2_{\mu(e\tau)} + N^2_{\tau(e\mu)} - 2N^2_{e\mu} - 2N^2_{e\tau} - 2N^2_{\mu\tau})$	$\pi_{e\mu\tau} > 0$

Results and Conclusions

- Two-flavor neutrino oscillations shows bi-partite entanglement.
- We show that the entanglement measures can be expressed in terms of neutrino appearance (P_a) and disappearance (P_d) probabilities.
- We find that the larger mixing leads to more entanglement which is intuitive.
- For two ν system, we find a laboratory analog of entanglement swapping due to a beam splitter placed at an angle. We intend to exploit this analogy to quantum information systems based on neutrinos.
- The tri-partite entanglement is quantified in two ways: (a) in terms of measures of bi-partite entanglement. (b) genuine tri-partite entanglement. Both are related to neutrino appearance (P_a) and disappearance (P_d) probabilities.
- We find that $\pi_{e\mu\tau}$ reaches the maximum value 0.436629 (see Fig.2(a)) when transition probabilities are $P_{\nu_{e\to e}} = 0.39602$, $P_{\nu_{e\to\mu}} = 0.435899$, and $P_{\nu_{e\to\tau}} = 0.168081$. Similarly, for $|\nu_\mu(t)\rangle$, $\pi_{\mu e\tau}$ reaches the maximum value 0.472629 (see Fig.2(b)).
- The tri-partite result $\pi_{e\mu\tau} > \tau_{e\mu\tau} = 0$ or $\pi_{\mu e\tau} > \tau_{\mu e\tau} = 0$ imply that the three-neutrino state shows the remarkable property of having a genuine form of three way entanglement.
- **Future Work:** Correlations exhibited by neutrino oscillations in tri-partite system are like the W-states which are legitimate physical resources for quantum information tasks. We will seek for laboratory systems like beam splitter for simulating tri-partite entanglement of neutrinos.

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