Examination of reliability of nuclear matrix elements of neutrinoless double-β decay by QRPA

NEUTRINO 2020

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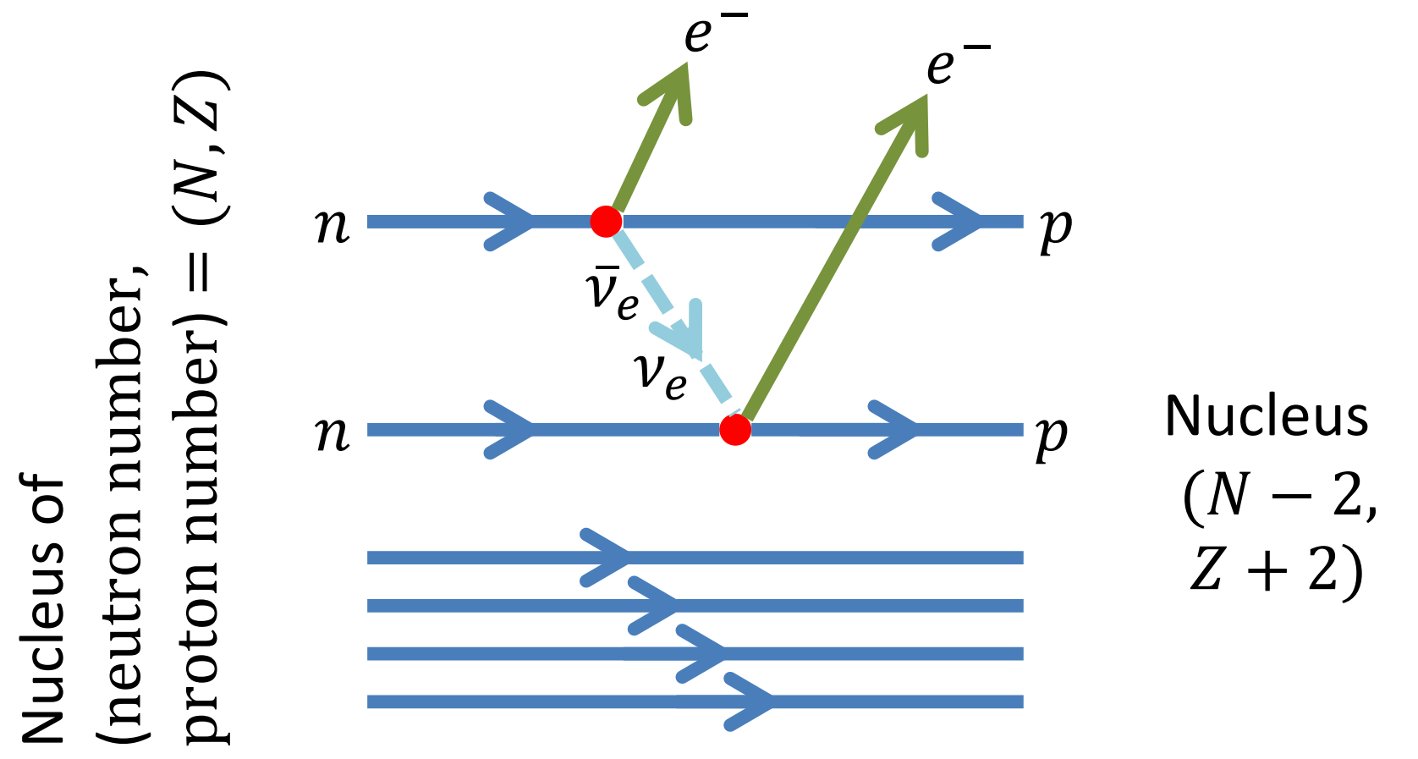
The goal is to determine the effective mass of the neutrino. The double-β decay of nucleus is used for this purpose. We aim at the theoretical contribution.



Possible change of two neutrons to two protons in a nucleus emitting two electrons with neutrino exchange (neutrinoless double-β decay). This decay occurs, if the neutrino (ν) is a Majorana particle ($\nu = \bar{\nu}$), and the effective neutrino mass can be determined; see the equations below. Determination of the effective neutrino mass is one of the most important subjects in modern physics.

Why nuclei?

Because E(final state) $< E$(initial state) is necessary.

Other conditions for the nuclei used in the experiments:

- Single beta decay is suppressed.
- The energy spectrum of two electrons in $2\nu\beta\beta$ decay can be distinguished well from that of $0\nu\beta\beta$.
- Large Q value [$\approx E$(initial state) – E(final state)].
- The parent nuclei can be produced massively with high purity.

List of decays searched in the experiments

^{76}Ge→^{76}Se
^{130}Te→^{130}Xe
^{136}Xe→^{136}Ba
^{150}Nd→^{150}Sm
^{48}Ca→^{48}Ti
^{82}Se→^{82}Kr
^{96}Zr→^{96}Mo
^{100}Mo→^{100}Ru
^{110}Pd→^{110}Cd
^{116}Cd→^{116}Sn
^{124}Sn→^{124}Te
and more

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

U: Pontecorvo–Maki–Nakagawa–Sakata matrix
m_i: eigen mass (i=1,2,3)

$$\underbrace{1/T_{0\nu}(\mathrm{g.s.} \rightarrow \mathrm{g.s.})}_{\text{Half-life}} = \underbrace{\left|M^{(0\nu)}\right|^2}_{\text{Nuclear matrix element}} \underbrace{G_{0\nu}}_{\text{Phase-space factor}} \underbrace{g_A^4}_{\text{Axial-vector current coupling}} \underbrace{\left(\frac{\langle m_\nu \rangle}{m_e}\right)^2}_{\text{Effective } \nu \text{ mass}}$$

Half-life ← Experimental measurement

Nuclear matrix element, Phase-space factor: Theoretical calculation

m_e: electron mass

Phase-space factor ← Wave functions of emitted electrons
Nuclear matrix element ← Nuclear wave functions

Nuclear matrix element ↓ Accurate calculation more difficult than the phase-space factor

Nuclear wave functions ↑ Approximation is indispensable.

Nuclear matrix element

$$M^{(0\nu)} = \sum_b \sum_{pp'} \sum_{nn'} \langle pp' | V(r_{12}, E_b) | nn' \rangle \langle 0_f^+ | c_{p'}^\dagger c_{n'} | b \rangle \langle b | c_p^\dagger c_n | 0_i^+ \rangle$$

neutron→proton

$\langle 0_f^+|$: Final state, ground state of nucleus(N-2,Z+2)

$|b\rangle$: intermediate state, nucleus (N-1,Z+1). Energy is E_b.

$|0_i^+\rangle$: Initial state, ground state of nucleus (N,Z)

The transition operator used in our calculation is

$$V(r_{12}, E_b) \cong h_+(r_{12}, E_b)\ \{\text{linear combination of Double-Gamow-Teller and Double-Fermi operators}\}$$

$h_+(r_{12}, E_b)$: neutrino potential; energy denominator included

Status and purpose of this study:
The calculated nuclear matrix elements by various approximation methods and groups are distributed in a range of factor of 2–3. The nuclear matrix element cannot be obtained by experiment. Thus, examination and improvement of the calculation are essential.

Approximation to the method to obtain the nuclear wave functions in our study:

Quasiparticle random-phase approximation (QRPA)

Nuclear excitation is described as the superposition of two-quasiparticle excitations.

How good?

- Transition-strength function can be well reproduced.
- Sum rule is satisfied.
- Widely used in nuclear and condensed-matter physics.

IMPROVEMENT 1

Under the approximation to the equation of the nuclear matrix element by

$$V(r_{12}, E_b) \cong V(r_{12}, \bar{E}_b)$$

$\bar{E}_b$: average energy of the intermediate states $|b\rangle$,

a virtual decay path is possible to use for calculation as two-neutron removal followed by two-proton addition.

The nuclear matrix element of the virtual path must be equal to that of the double-β path. This is a constraint to the effective interactions used for the approximation.

↓

The strength of the isoscalar proton-neutron pairing interaction is determined. J. Terasaki, Phys. Rev. C **91**, 034318 (2015); ibid **93**, 024317 (2016)

NEW

Why important?

This interaction is necessary for calculating the nuclear matrix element of **two-neutrino double-β decay**.

There are exp. data of half-life to this decay for those nuclei used for the neutrinoless double-β decay exp. The two-neutrino decay operator $V_{2\nu}(E_b)$ cannot be replaced by $V_{2\nu}(\bar{E}_b)$, thus, the double-β-path calculation is necessary. We use those data for determining the axial-vector current coupling g_A.

IMPROVEMENT 2

NEW

We included the many-body correlations rigorously in the contribution of the intermediate state $|b\rangle$ to the nuclear matrix element. This improvement is a mathematically complicated story. See
J. Terasaki, Phys. Rev. C **87**, 024316 (2013).

IMPROVEMENT OF CHECK METHOD

NEW

Comparison of Gamow-Teller strength function derived from charge-change reaction data is made with the QRPA calculation. This is a check of the charge-change transition density used for the calculation of the double-β decay. It has been clarified that this charge-change reaction is induced by
Gamow-Teller ***+ isovector spin monopole*** **(**$\propto r^2$**)** *operators*.
J. Terasaki Phys. Rev. C **97**, 034304 (2018)

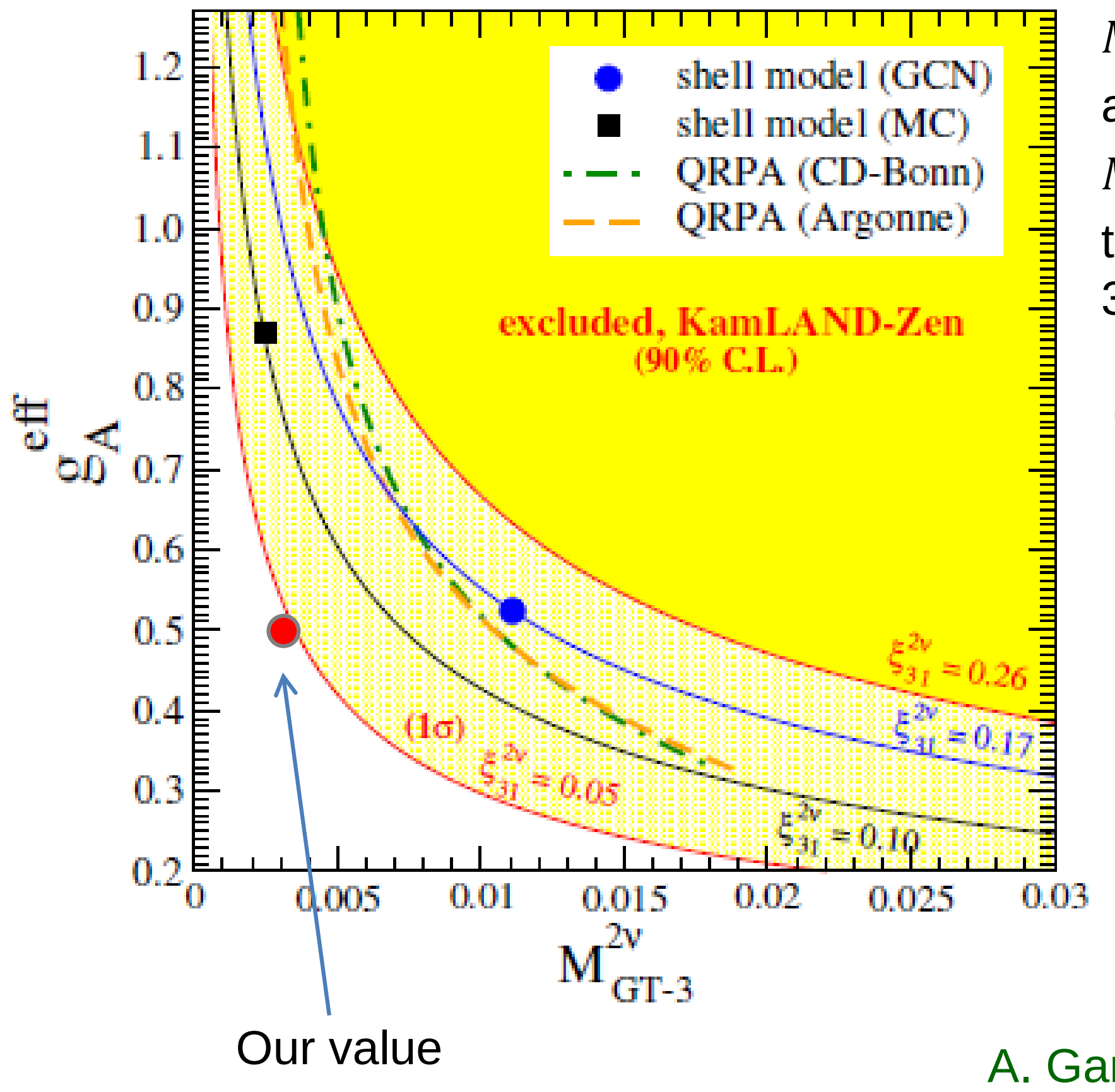
PROGRESS IN THE PAST YEAR

We have tested intensively the calculation of ^{136}Xe→^{136}Ba;
J. Terasaki and Y. Iwata, Phys. Rev. C, **100**, 034325 (2019).

1. The self-check of the dual intermediate states using the 2νββ decay; the two sets of intermediate-state energies obtained by the QRPA based on the initial state and those based on the final state give the same nuclear matrix element (NME).
2. The convergence of the NME with respect to the dimension of the single-particle space.
3. The comparison of the two spectra of the intermediate nucleus (*N*–1,*Z*+1) obtained by the QRPA based on the initial and final states and comparison of them with the experimental data.
4. The Gamow-Teller (GT) sum rule.
5. The comparison of the GT$^-$ strength with the data of the charge-change reaction.
6. The comparison of the β decay spectrum and the GT transition strengths with the experimental data of a neighboring nucleus.
7. Test using a new quantity expressing a higher-order effect in the 2νββ NME; F. Šimkovic et al., PRC **97**, 034315 (2018); A. Gando et al, PRL **122**, 192501 (2019).

In any of the seven tests, no problem was found.

Test No. 7



$M_{GT\text{-}3}^{(2\nu)}$: analogous to the 2νββ NME $M_{GT}^{(2\nu)}$ but the exponent of the energy denominator is 3.
F. Šimkovic et al., PRC **97**, 034315 (2018)

$g_A^{\rm eff}$: effective axial-vector current coupling

Statistical analysis with the exp. data of ββ spectrum of ^{136}Xe has been made. The confidence level of exclusion of $(g_A^{\rm eff}, M_{GT\text{-}3}^{(2\nu)})$ is shown by the intensity of color.

A. Gando et al., PRL **122**, 192501 (2019)

0 νββ decay

The final output of the calculation is the reduced half-life

$$R_{1/2}^{(0\nu)} = (G_{0\nu} g_A^4)^{-1} \left|M^{(0\nu)}\right|^{-2} m_e^2.$$

This is the quantity necessary for determining $\langle m_\nu \rangle$;

$$\langle m_\nu \rangle = \left(R_{1/2}^{(0\nu)} / T_{1/2}^{(0\nu)}\right)^{1/2}.$$

We obtained for ^{136}Xe→^{136}Ba

$$R_{1/2}^{(0\nu)} = 1.223 \times 10^{13} \mathrm{MeV}^2 \mathrm{yr},$$

using the effective g_A = 0.49 for the 2νββ NME.

Conclusion

The QRPA is a good approximation for ^{136}Xe and ^{136}Ba; the higher-order particle-hole correlations are weak. It is possible by neglecting these minor correlations to use the limited computational resource for obtaining the convergence of the NME with respect to the extension of the single-particle space. We did it. Our nuclear wave functions are appropriate for estimating the half-life of ^{136}Xe to the 0νββ decay.

Choosing the decay instances for which the approximation used is reliable is our solution to the discrepancy problem of the NME by the methods to calculate the nuclear wave functions.