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# Atomic Shannon entropy in astrophysical nonthermal plasmas

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# Abstract

The nonthermal effects on the Shannon entropy for the atomic states are investigated in astrophysical Lorentzian plasmas. The Shannon entropies for the ground and excited states in astrophysical Lorentzian plasmas are also obtained as functions of the spectral index, effective screening lengths, and plasma parameters including the radial and angular parts. It is shown that the nonthermal characteristics of the Lorentzian plasma suppresses the entropy changes in the ground state as well as in the excited states. In addition, it is found that the entropy change in excited states is more effective than that in the ground state in Lorentzian astrophysical plasmas.

# Atomic Shannon Entropy

$$S_\rho = -\int dr^3 \rho(\boldsymbol{r}) \ln \rho(\boldsymbol{r}) = -\int dr^3 \, |\psi(\boldsymbol{r})|^2 \ln \, |\psi(\boldsymbol{r})|^2,$$

where $\psi(\boldsymbol{r})$ is the atomic wave function : $\psi(\boldsymbol{r})[=R_{nl}(r)Y_l^m(\Omega)]$

$$S_\rho = S(R_{nl}) + S(Y_{lm}),$$

where $S(R_{nl})$ is the radial part of the Shannon entropy with the radial wave function $R_{nl}(r)$:

$$S(R_{nl}) = -\int drr^2 \, |R_{nl}(r)|^2 \ln \, |R_{nl}(r)|^2,$$

$S(Y_{lm})$ is the angular part of the Shannon entropy with the spherical harmonics $Y_l^m(\Omega)$:

$$S(Y_l^m) = -\int d\Omega \, |Y_l^m(\Omega)|^2 \ln \, |Y_l^m(\Omega)|^2,$$

and $d\Omega(=\sin\theta d\theta d\phi)$ is the differential solid angle in spherical coordinates.

# Lorentzian Nonthermal Distribution

$$f_{L-\kappa}(v) = N(\kappa)\left(1 + \frac{mv^2}{2\kappa E(\kappa)}\right)^{-(\kappa+1)},$$

where $N(\kappa)$ is the normalization factor with $\int d^3v f_{L-\kappa}(v)/n = 1$:

$$N(\kappa) = n\left(\frac{m}{2\pi\kappa E(\kappa)}\right)^{3/2} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)},$$

$\kappa(>3/2)$ is the spectral index of the Lorentzian plasma, $v$ is the electron velocity, $n$ is the plasma density, $m$ is the electron mass, $E(\kappa)[\equiv E_M(\kappa-3/2)/\kappa]$ is the characteristic energy in generalized Lorentzian plasmas, $E_M \equiv k_B T$, $k_B$ is the Boltzmann constant, $T$ is the plasma temperature, and $\Gamma$ represents the gamma function.

# Atomic States in Nonthermal Plasmas

Schrödinger equation for the $nl$-shell electron of the hydrogen atom in generalized Lorentzian plasmas would be represented by

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dR_{nl}(r)}{dr}\right]+\left\{-\frac{l(l+1)}{r^2}+\frac{2m}{\hbar^2}\left[E_{nl}+\frac{e^2}{r}\right.\right.$$

$$\left.\left.\times\exp\left(-\sqrt{\frac{\kappa-1/2}{\kappa}}\frac{r}{\lambda_D}\right)\right]\right\}R_{nl}(r)=0,$$

Hence, the analytic ansatzes $R_{10}(r)$ and $R_{21}(r)$ for the normalized screened variational $1s$ and $2p$ wave radial functions are then presumed to be, respectively, as follows:

$$R_{10}(r;\,\xi_{1s})=\frac{2}{\xi_{1s}^{3/2}(\lambda_D)}\exp(-r/\xi_{1s}(\lambda_D)),$$

$$R_{21}(r,\,\xi_{2p})=\frac{1}{2\sqrt{6}}\frac{r}{\xi_{2p}^{5/2}(\lambda_D)}\exp(-r/2\xi_{2p}(\lambda_D)),$$

# Energy Eigenvalues in Nonthermal Plasmas

$$E_{10}(\xi_{1s};\,\lambda_D) = \langle \psi_{1s}|H|\,\psi_{1s}\rangle$$
$$= -\frac{e^2}{\xi_{1s}(\lambda_D)[1+\sqrt{(\kappa-1/2)/\kappa}\,(\xi_{1s}/2\lambda_D)]^2}$$
$$+\frac{\hbar^2}{2m\xi_{1s}^2(\lambda_D)},$$

$$E_{21}(\xi_{2p};\,\lambda_D) = \langle \psi_{2p}|H|\,\psi_{2p}\rangle$$
$$= -\frac{e^2}{4\xi_{2p}(\lambda_D)[1+\sqrt{(\kappa-1/2)/\kappa}\,(\xi_{2p}/\lambda_D)]^4}$$
$$+\frac{\hbar^2}{8m\xi_{2p}^2(\lambda_D)},$$

## Screening parameters in Nonthermal Plasmas

Rayleigh–Ritz method

$$\partial E_{10}(\xi_{1s};\, \lambda_D)/\partial \xi_{1s} = 0 \;\; \text{and} \;\; \partial E_{21}(\xi_{2p};\, \lambda_D)/\partial \xi_{2p} = 0$$

$$\xi_{2p}(\lambda_D) \cong a_0\left[1 - 10\left(\frac{\kappa - 1/2}{\kappa}\right)\left(\frac{a_0}{\lambda_D}\right)^{2} + 40\left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\left(\frac{a_0}{\lambda_D}\right)^{3}\right]^{-1},$$

$$\xi_{2p}(\lambda_D) \cong a_0\left[1 - 10\left(\frac{\kappa - 1/2}{\kappa}\right)\left(\frac{a_0}{\lambda_D}\right)^{2} + 40\left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\left(\frac{a_0}{\lambda_D}\right)^{3}\right]^{-1},$$

where $a_0(=\hbar^2/me^2)$ is the Bohr radius of the hydrogen atom.

# Shannon Entropy for the 1$s$ state

$$S_{1s} = S(R_{10}) + S(Y_0^0).$$

$$\begin{aligned} S(R_{10}) &= -\int drr^2\,|R_{10}(r;\,\xi_{1s})|^2 \ln\,|R_{10}(r;\,\xi_{1s})|^2 \\ &= -\left(\frac{2}{\xi_{1s}^{3/2}}\right)^2 \int_0^\infty drr^2 \exp(-2r/\xi_{1s}) \\ &\quad \times \ln\left(\frac{2}{\xi_{1s}^{3/2}}\exp(-r/\xi_{1s})\right)^2 \\ &= 3 + \ln\left(\frac{\xi_{1s}^3(\lambda_D)}{4}\right). \end{aligned}$$

$$S(Y_{00}) = -\int d\Omega\,|Y_0^0(\Omega)|^2 \ln\,|Y_0^0(\Omega)|^2 = \ln(4\pi).$$

$$\begin{aligned} S_{1s} &= 3 + \ln(\pi\xi_{1s}^3(\lambda_D)) \\ &= 3 + \ln\left[\pi a_0^3\left(1 - \frac{3}{4}\left(\frac{\kappa - 1/2}{\kappa}\right)\bar{\lambda}_D^{-2}\right.\right. \\ &\quad \left.\left. + \left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\bar{\lambda}_D^{-3}\right)^{-3}\right], \end{aligned}$$

# Entropy Changes for the 1*s* state

In Maxwellian plasmas, i.e., in the limit of $\kappa \to \infty$, the entropy change $\triangle S_\infty(1s)$ in the $1s$ state becomes $\triangle S_\infty(1s) = 3\ln\left[\left(1 - \frac{3}{4}\bar{\lambda}_D^{-2} + \bar{\lambda}_D^{-3}\right)^{-1}\right]$. Hence, the entropy change $\triangle S_\kappa(1s)$ in the $1s$ state by the nonthermal character of the Lorentzian plasma with suprathermal populations is obtained by

$$\triangle S_\kappa(1s) = 3\ln\left[\left(1 - \frac{3}{4}\bar{\lambda}_D^{-2} + \bar{\lambda}_D^{-3}\right)\left(1 - \frac{3}{4}\left(\frac{\kappa - 1/2}{\kappa}\right)\bar{\lambda}_D^{-2} + \left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\bar{\lambda}_D^{-3}\right)^{-1}\right].$$

# Shannon Entropy for the 2*p* state

$$S_{2p} = S(R_{21}) + S(Y_1^m),$$

$$\begin{aligned} S(R_{21}) &= -\int drr^2\, |R_{21}(r,\, \xi_{2p})|^2 \ln\, |R_{21}(r,\, \xi_{2p})|^2 \\ &= -\left(\frac{1}{2\sqrt{6}}\frac{1}{\xi_{2p}^{5/2}}\right)^2 \int_0^\infty drr^4 \exp(-r/\xi_{2p}) \\ &\quad \times \ln\left(\frac{1}{2\sqrt{6}}\frac{r}{\xi_{2p}^{5/2}}\exp(-r/2\xi_{2p})\right)^2 \\ &= \frac{5}{6} + \ln(24\xi_{2p}^3(\lambda_D)) + 2\gamma, \end{aligned}$$

where $\gamma = -\Gamma'(1)(=0.5772157\ \ldots\ )$

$$S(Y_{1m}) = -\int d\Omega\, |Y_1^m(\Omega)|^2 \ln\, |Y_1^m(\Omega)|^2 (m = 0, \pm 1).$$

# Shannon Entropy for the 2*p* state

Then, the total Shannon entropies $S_{2p_0}$ and $S_{2p_{\pm 1}}$ for the $2p_0(m=0)$ and $2p_{\pm 1}(m=\pm 1)$ states are, respectively, found to be

$$
\begin{aligned}
S_{2p_0} &= \frac{9}{6} + \ln(32\pi\xi_{2p}^3) + 2\gamma \\
&= \frac{9}{6} + 2\gamma + \ln\left[32\pi a_0^3\left(1 - 10\left(\frac{\kappa - 1/2}{\kappa}\right)\bar{\lambda}_D^{-2}\right.\right. \\
&\quad \left.\left. + 40\left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\bar{\lambda}_D^{-3}\right)^{-3}\right],
\end{aligned}
$$

$$
\begin{aligned}
S_{2p_{\pm 1}} &= \frac{15}{6} + \ln(16\pi\xi_{2p}^3) + 2\gamma \\
&= \frac{15}{6} + 2\gamma + \ln\left[16\pi\left(1 - 10\left(\frac{\kappa - 1/2}{\kappa}\right)\bar{\lambda}_D^{-2}\right.\right. \\
&\quad \left.\left. + 40\left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\bar{\lambda}_D^{-3}\right)^{-3}\right].
\end{aligned}
$$

# Entropy Changes for the 2*s* state

In Maxwellian plasmas, the entropy change $\triangle S_{\infty}(2p)$ in the $2p$ state becomes $\triangle S_{\infty}(1s) = 3\ln[(1 - 10\bar{\lambda}_D^{-2} + 40\bar{\lambda}_D^{-3})^{-1}]$. Hence, the entropy change $\triangle S_{\kappa}(2p)$ in the $2p$ state by the nonthermal character of the Lorentzian plasma with suprathermal populations is given by

$$\triangle S_{\kappa}(2p) = 3\ln\left[(1 - 10\bar{\lambda}_D^{-2} + 40\bar{\lambda}_D^{-3})\left(1 - 10\left(\frac{\kappa - 1/2}{\kappa}\right)\bar{\lambda}_D^{-2} + 40\left(\frac{\kappa - 1/2}{\kappa}\right)^{3/2}\bar{\lambda}_D^{-3}\right)^{-1}\right].$$

# Results

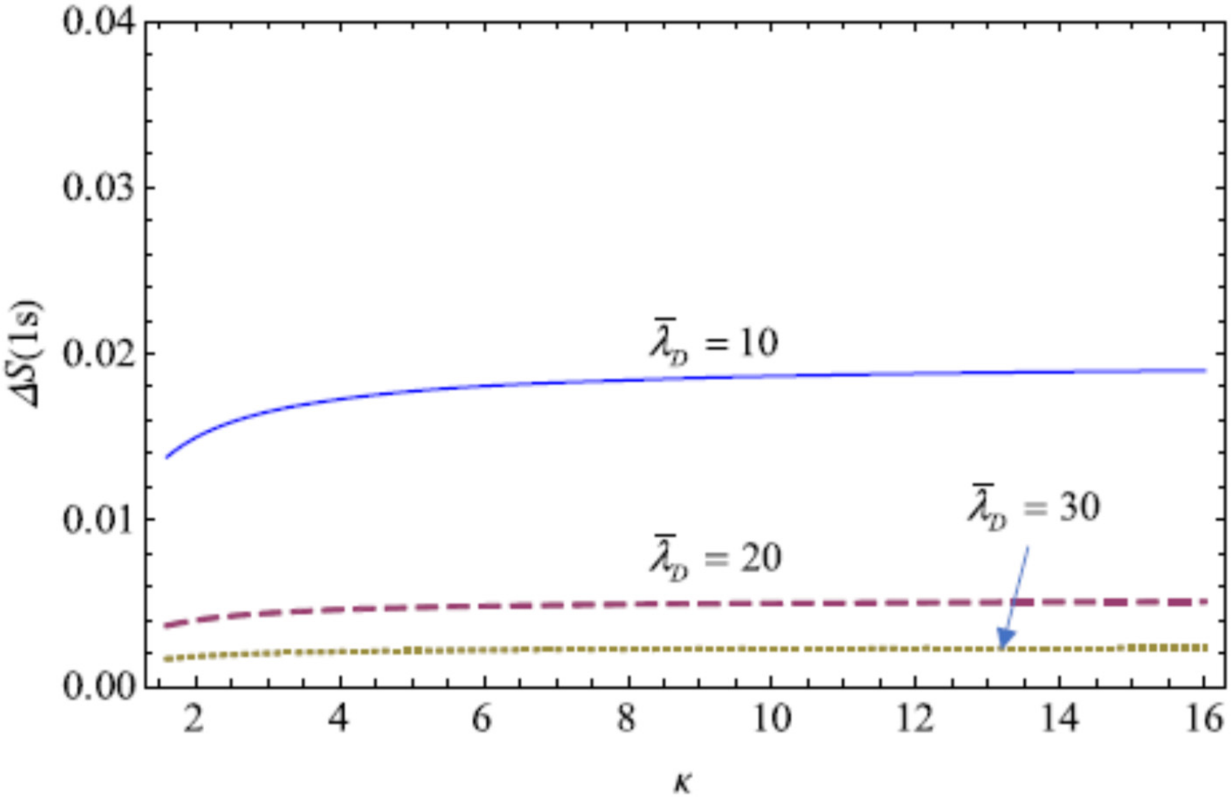


**Figure 1.** Entropy change $\Delta S(1s)$ in the $1s$ state as a function of the spectral index $\kappa$ of the Lorentzian plasma. The solid line represents the case of $\bar{\lambda}_D = 10$. The dashed line represents the case of $\bar{\lambda}_D = 20$. The dotted line represents the case of $\bar{\lambda}_D = 30$.

# Results

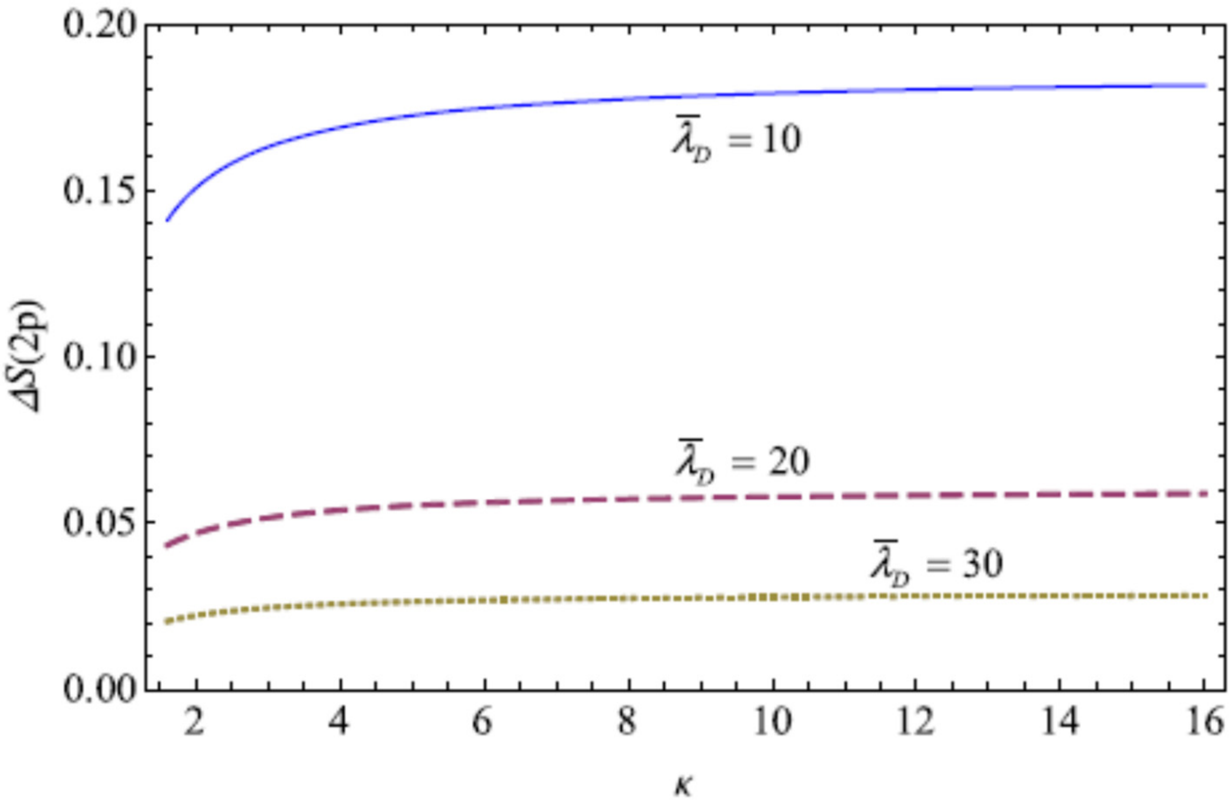


**Figure 2.** Entropy change $\Delta S(2p)$ in the $2p$ state as a function of the spectral index $\kappa$ of the Lorentzian plasma. The solid line represents the case of $\bar{\lambda}_D = 10$. The dashed line represents the case of $\bar{\lambda}_D = 20$. The dotted line represents the case of $\bar{\lambda}_D = 30$.

# Results

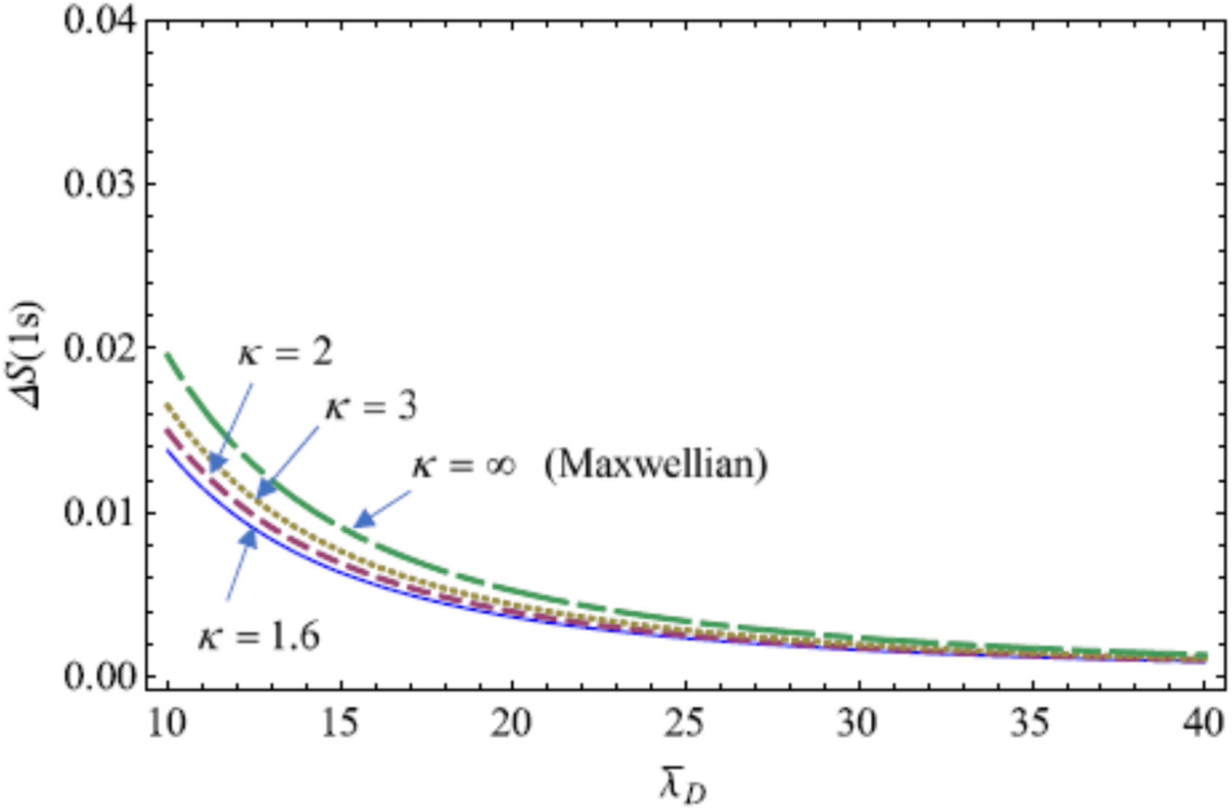


**Figure 3.** Entropy change $\Delta S(1s)$ in the $1s$ state as a function of the scaled Debye length $\bar{\lambda}_D$. The solid line represents the case of $\kappa = 1.6$. The dashed line represents the case of $\kappa = 2$. The dotted line represents the case of $\kappa = 3$. The dashed–dotted line represents the Maxwellian limit of the entropy change.

# Results

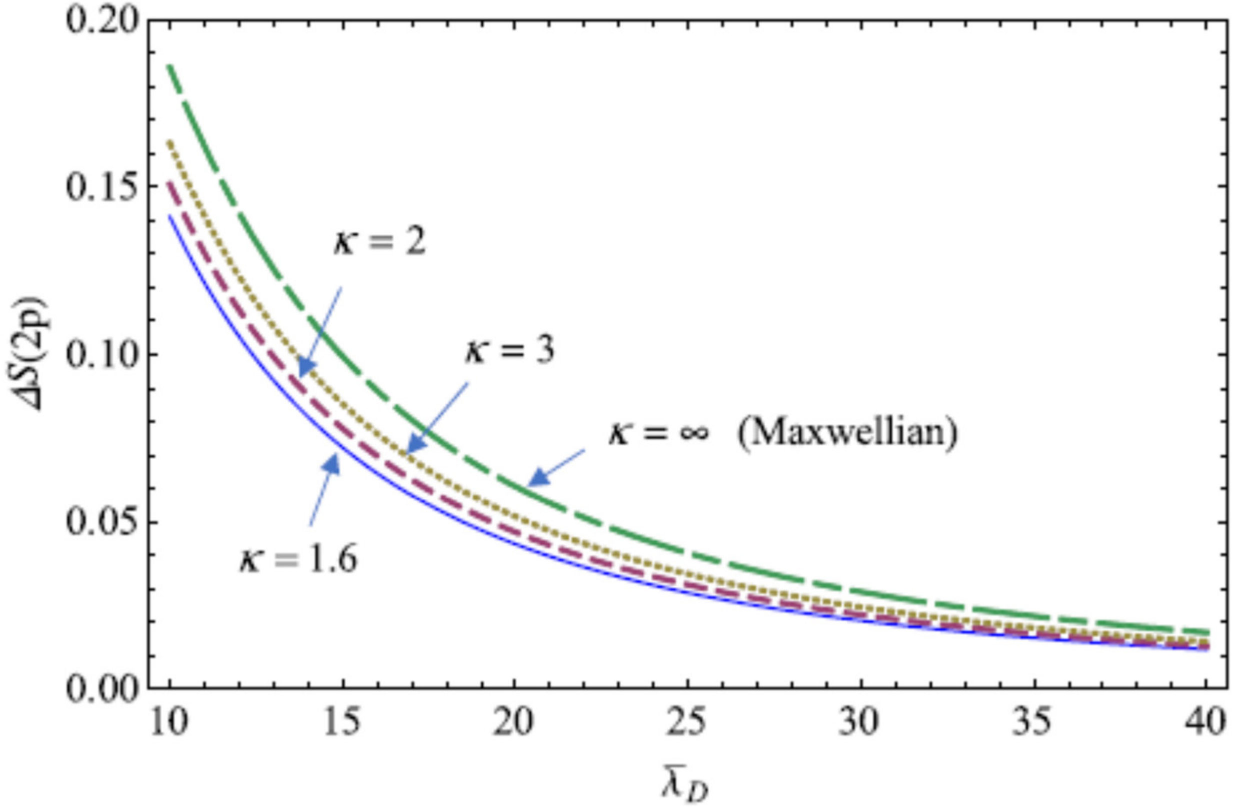


**Figure 4.** Entropy change $\Delta S(2p)$ in the $2p$ state as a function of the scaled Debye length $\bar{\lambda}_D$. The solid line represents the case of $\kappa = 1.6$. The dashed line represents the case of $\kappa = 2$. The dotted line represents the case of $\kappa = 3$. The dashed–dotted line represents the Maxwellian limit of the entropy change.

# Results

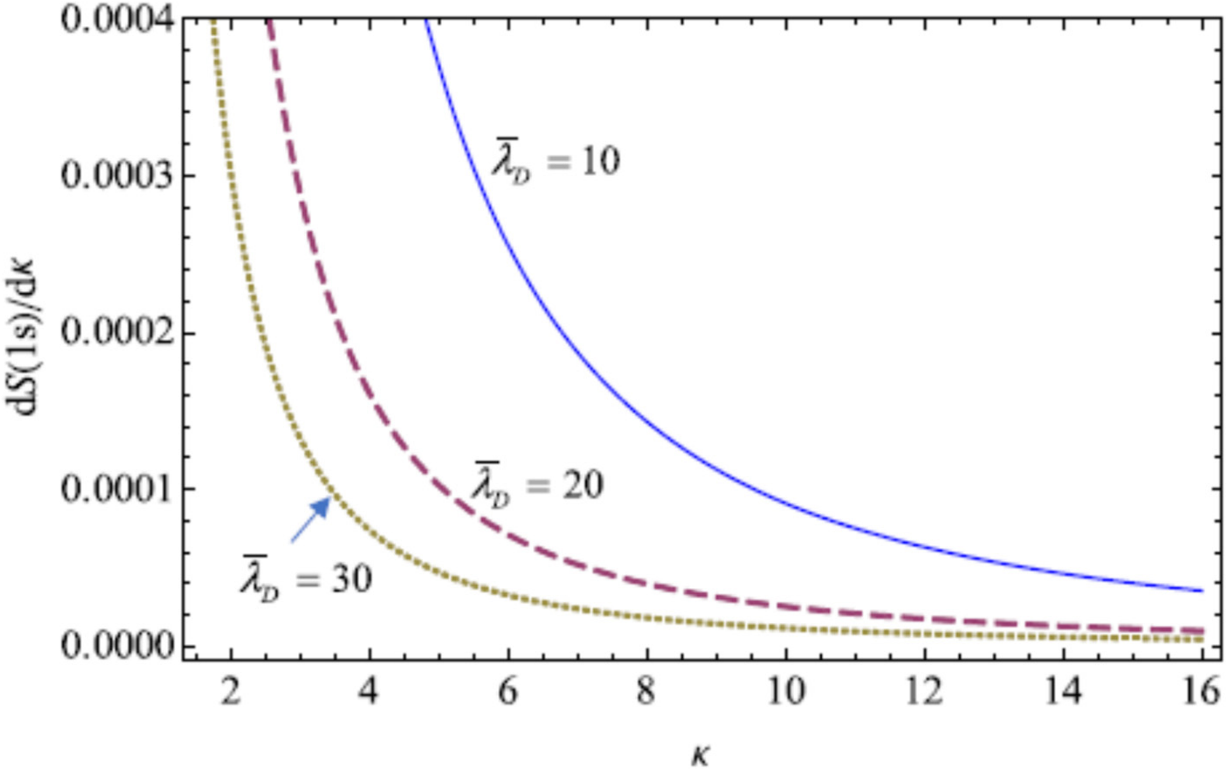


**Figure 5.** Kappa-gradient of the entropy change $dS(1s)/d\kappa$ in the 1$s$ state as a function of the spectral index $\kappa$ of the Lorentzian plasma. The solid line represents the case of $\bar{\lambda}_D = 10$. The dashed line represents the case of $\bar{\lambda}_D = 20$. The dotted line represents the case of $\bar{\lambda}_D = 30$.

# Results

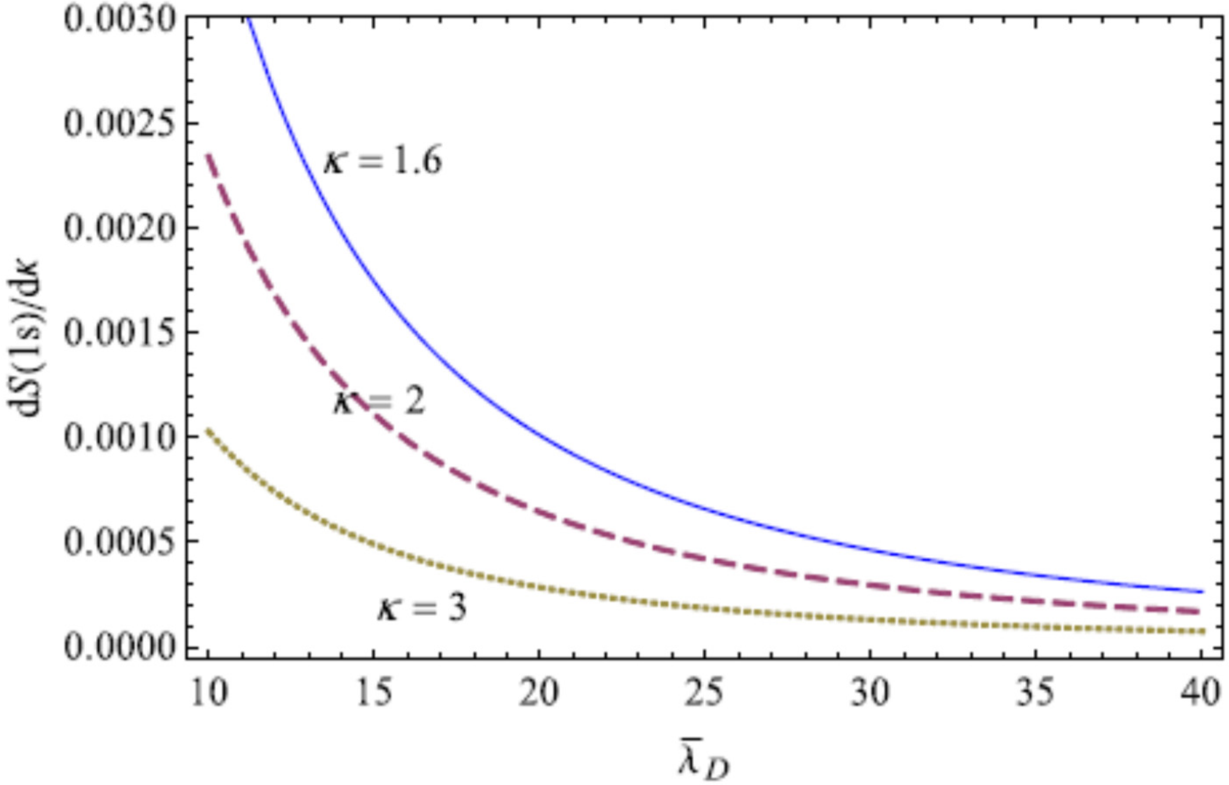


**Figure 6.** Kappa-gradient of the entropy change $dS(1s)/d\kappa$ in the $1s$ state as a function of the scaled Debye length $\bar{\lambda}_D$. The solid line represents the case of $\kappa = 1.6$. The dashed line represents the case of $\kappa = 2$. The dotted line represents the case of $\kappa = 3$.

# Results

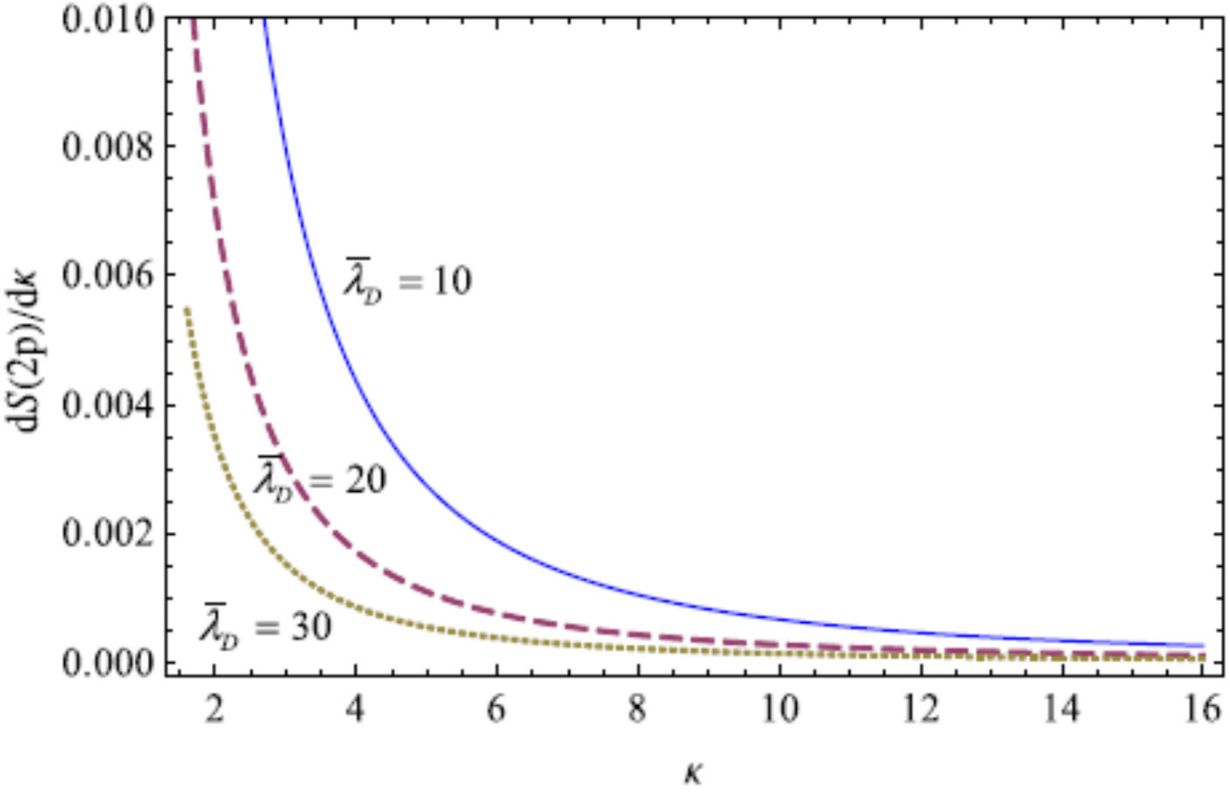


**Figure 7.** Kappa-gradient of the entropy change $dS(2p)/d\kappa$ in the $2p$ state as a function of the spectral index $\kappa$ of the Lorentzian plasma. The solid line represents the case of $\bar{\lambda}_D = 10$. The dashed line represents the case of $\bar{\lambda}_D = 20$. The dotted line represents the case of $\bar{\lambda}_D = 30$.

# Results

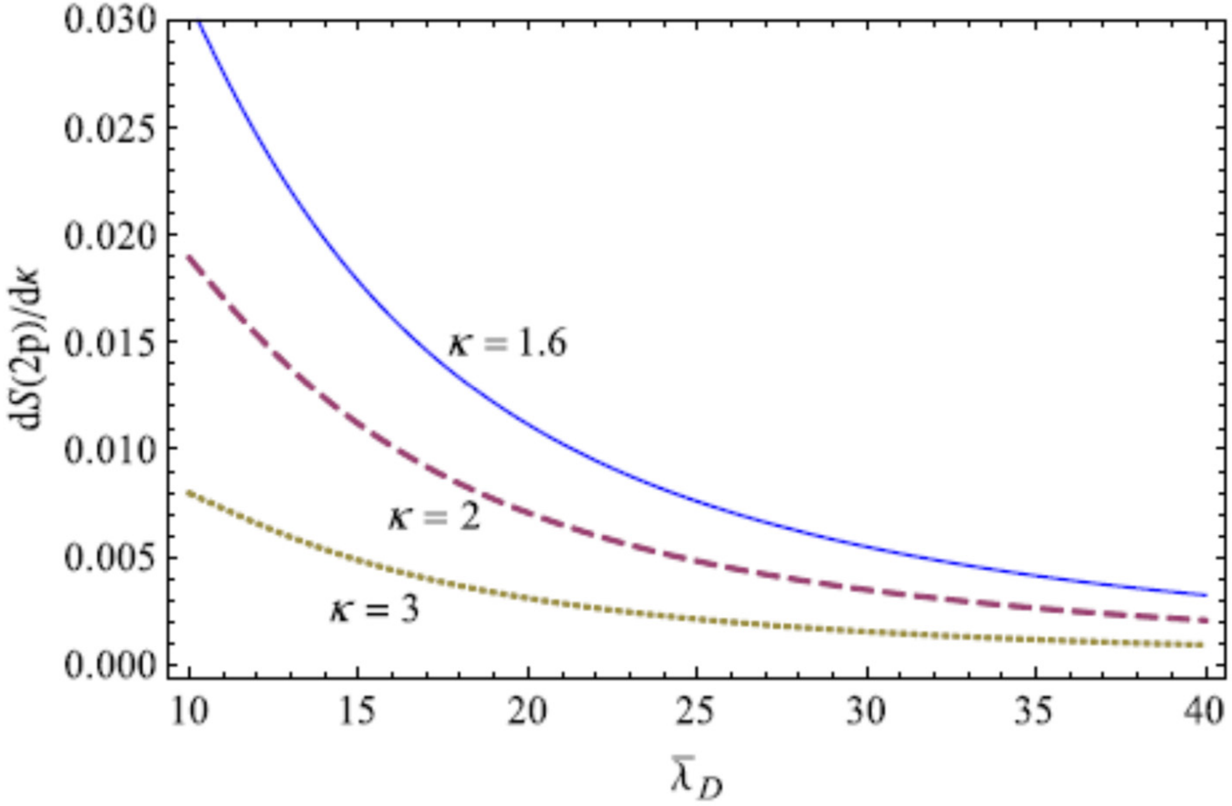


**Figure 8.** Kappa-gradient of the entropy change $dS(2p)/d\kappa$ in the $2p$ state as a function of the scaled Debye length $\bar{\lambda}_D$. The solid line represents the case of $\kappa = 1.6$. The dashed line represents the case of $\kappa = 2$. The dotted line represents the case of $\kappa = 3$.

# Conclusions

In this work, we investigated the nonthermal effects on the variation of the Shannon entropy for the atomic states in astrophysical Lorentzian plasmas. The analytic expression of the screened atomic wave functions, the energy eigenvalues, and the effective screening lengths for the hydrogen atom in Lorentzian plasmas are obtained by the Rayleigh–Ritz variational method.

The Shannon entropies for the 1*s* ground and 2*p* excited states in astrophysical Lorentzian plasmas are also obtained as functions of the spectral index, effective screening lengths, and plasma parameters including the radial and angular parts.

It is shown that the nonthermal character of the astrophysical Lorentzian suppresses the entropy changes in the 1s ground and 2p excited states.

In this work, we have found that the nonthermal electrons with a low-spectral index $\kappa$ suppresses $\Delta S(1s)$ since the electron with a low-spectral index contributes more effectively to the plasma shielding in Lorentzian plasmas. It is also found that the contribution of low-energy electron suppresses $\Delta S(1s)$ in Lorentzian plasmas.

In addition, we have found that the influence of the nonthermal electrons with a low-spectral index κ suppresses $\Delta S(2p)$ since the electrons with low-spectral indices contribute more efficiently to the shielding in Lorentzian plasmas. It is also shown that the influence of low-energy electrons suppresses $\Delta S(2p)$ in Lorentzian plasmas.

These results would provide useful information on the plasma transport, the information flow of the atomic data, and the physical characteristics of astrophysical nonthermal plasmas..

**Thank you very much for your attention.**