Probing the two-neutrino exchange force using atomic parity violation

Mitrajyoti Ghosh[1], Yuval Grossman[1], and Walter Tangarife[2]

[1]Cornell University, [2]Loyola University Chicago

Preparing people to lead extraordinary lives

Motivation

- It is well-known that pair of massless neutrinos mediate a long-range force via one-loop diagrams; however the force is very weak and behaves like $\sim G_F^2/r^5$.
- Because the neutrino only couples weakly, and the weak interaction violates parity, one expects that the force due to the two-neutrino exchange violates parity.
- However, what is different here from the exchange of weakly interacting gauge bosons such as W or Z bosons is that the neutrino force is necessarily longer in range since the neutrinos are almost massless.

The two-neutrino exchange force

A classical force is mediated by a boson. The two-neutrino exchange, as shown in Fig. 1, can also give rise to a long-range force since two fermions, and so to some extent, can be treated as a boson. This force is also called "a quantum force" as it arises at the loop level. The force has an interaction range which goes inversely to the mass of the mediating particle. Thus, the neutrino force leads to a long range effect as the neutrino is almost massless. In our work, we claim that it is the largest parity violating force in the Standard Model.

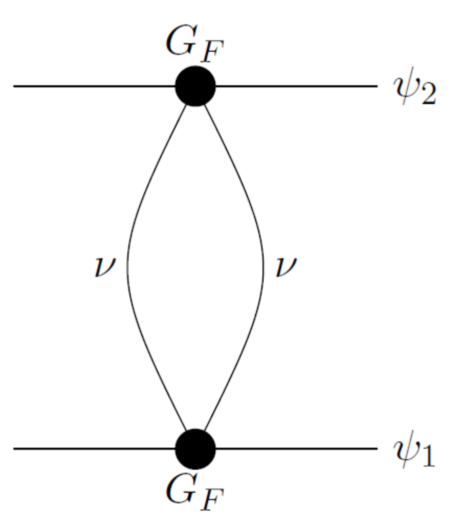

Figure: The four-Fermi effective diagram for two-neutrino exchange forces between two fermions.

For future reference, the parity-conserving form of the two-neutrino potential to leading order in v for the case of a single flavor of neutrinos with mass m_ν is given by

$$V_{\nu\nu}^{\text{Dirac}}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3}\frac{K_3(2m_\nu r)}{r^2}, \quad V_{\nu\nu}^{\text{Majorana}}(r) = \frac{G_F^2 m_\nu^2}{2\pi^3}\frac{K_2(2m_\nu r)}{r^3}, \tag{1}$$

where $K_n(x)$ is the nth order modified Bessel functions of the second kind.

Atomic parity violation (APV)

In the presence of a parity violating interaction, energy eigenstates in an atomic system are no longer states of definite parity. Atomic processes which were once completely forbidden by parity conservation are allowed by a parity-violating perturbation. Selection rules allow electric dipole ($E1$) transitions between states of opposite parity and magnetic dipole ($M1$) transitions between states with the same parity. With an additional parity violating interaction, we expect an interference of the electric and magnetic dipole effects in the electromagnetic transitions between any two states.The APV between two states that are predominantly of the same parity is quantified by:

$$R = \mathcal{I}m\left(\frac{E1_{PV}}{M1}\right), \tag{2}$$

where $E1_{PV}$ is the electric dipole transition amplitude between the opposite-parity parts of the two states. Measurement of R is done through optical rotation experiments. Because of parity violation, the transition amplitude is different for electromagnetic radiation of different polarization, leading to a difference of refractive index between left and right circular polarized radiation, causing optical rotation in the sample.

The parity violating potential

In a generic atom, with nucleon mass m_N, the most generic form of the parity-violating potential is given by

$$V_{PNC}(r) = H_1 F(r)\vec{\sigma}_e\cdot\vec{v}_e + H_2 F(r)\vec{\sigma}_N\cdot\vec{v}_e + C(\vec{\sigma}_e\times\vec{\sigma}_N)\cdot\vec{\nabla}\left[F(r)\right], \tag{3}$$

where $\vec{\sigma}_e/2$ is the spin of the electron, $\vec{\sigma}_N/2$ is the net nuclear spin, H_1, H_2, and C are real constants, and $F(r)$ is a radial real function. The values of the H_1, H_2, C, and $F(r)$ depend on the specific diagram.

Many effects contribute to Eq. (3), the most well known being the $Z-$boson exchange (tree-level). At loop level, we have the two-neutrino exchange and some other interactions with other fermions in the loop, or parity violating penguin diagrams with a photon exchange. However, only the neutrino exchange constitutes a long range force because of the small neutrino mass.

Applications in the hydrogen atom

In the hydrogen atom, the parity violating part of the neutrino force takes the following form:

$$V_{PNC}^{\text{loop}} \approx \frac{G_A}{m_e}\left(-\frac{1}{4}+s_W^2+\frac{1}{2}|U_{ei}|^2\right)\left[(2\vec{\sigma}_p\cdot\vec{p}_e)V_{\nu_i\nu_i}(r) + (\vec{\sigma}_e\times\vec{\sigma}_p)\cdot\vec{\nabla}V_{\nu_i\nu_i}(r)\right], \tag{4}$$

where G_A is the axial form factor of the proton, m_e is the mass of the electron and $V_{\nu_i\nu_i}(r)$ is the radial force between two fermions due to the exchange of two Dirac neutrinos in mass eigenstate i, defined in Eq. (1). We assume an implicit sum over the neutrino mass eigenstates.

For low-angular-momentum states, the short-range force mediated by the Z boson dominates over the neutrino-exchange force since the four-Fermi approximation breaks down for very short distances. To isolate the effects of the neutrino potential, one must therefore look at high-ℓ states, where the Z force is significantly weaker than the neutrino force. We performed calculations for the $4F_{5/2,F=3}$ and $4F_{7/2,F=3}$ in hydrogen, which both have $\ell=3$ and are the first few states for which the neutrino effect is much larger than the effect of the Z boson. Defining a small parameter ν_i by:

$$\nu_i \equiv \frac{1}{\alpha}\frac{m_{\nu_i}}{m_e}, \tag{5}$$

we obtain

$$R \approx \sum_i\left(-\frac{1}{4}+s_W^2+\frac{1}{2}|U_{ei}|^2\right)\left(-7.7\times10^{-33}+3.7\times10^{-32}\nu_i^2\right). \tag{6}$$

where U_{ei} are the matrix elements of the PMNS matrix to incorporate flavor mixing), and s_W is the sine of the weak mixing angle.

For reference, the value of R obtained for Z exchange in the lower ℓ states is of the order of 10^{-10}.

Conclusions

- The effect of the neutrino force in the states considered is too small to be measured by current experiment. However, the neutrino force is the longest parity violating force possible.
- The value of R is dependent on neutrino masses, so this may be an interesting way to get some idea about the neutrino masses through atomic physics experiments.
- More complicated systems have enhancements of R due to various factors, which would make our analysis more useful in those systems rather than in hydrogen.

[1] M. Ghosh, Y. Grossman and W. Tangarife, [arXiv:1912.09444 [hep-ph]]. Phys. Rev. D 101, 116006.