General Relativistic hydrodynamics and neutrino transport in supernova explosion and helical effects

Carlo Salvattore Cruz Sanchez, Fulgencio Villegas Silva, Teofilo Vargas Auccalla

Theorical Physics Group, Faculty of Physics Sciences

National University of San Marcos

Introduction

Neutrinos play a key role in supernova explosions. When a massive star explodes, most of the gravitational binding energy is carried away by neutrinos. In this work we derive the equations of radiative hydrodynamics for a coupled system of neutrino radiation and matter using neutrino transport by frequency integrated methods starting with the metric in the comoving frame and the energy-momentum tensors for matter and radiation which are often used in stellar collapse calculations. Thus, it is obtained the neutrino transport theory without taking into account the chirality of neutrinos. Recently, it has been proved that chirality of particles leads to dramatic modification to the conventional theory because of that we introduce the chirality of neutrino to understand which new fenomena we can find.

Aim

- We will obtain the general relativistic equations for neutrino and matter with an specific metric which will be understood as a core collapsed supernovae.
- We are gonna obtain the hydrodynamics equations for neutrino and matter assuming matter to be a perfect fluid.
- We will show that the energy momentum tensor and the equations of motion of neutrinos will be reduced to the energy momentum tensor and equations of motion for matter for specific conditions.
- We are gonna show that the total mass-energy conservation will be related to external work and the flux crossing the surface.
- Finally, we will show that the neutrino density can be transformed to the fluid helicity

References

1. E. Baron. General relativistic neutrino transport in stellar collapse.The AstrophysicalJournal, 2016.
2. A. Lichnerowicz.Theories Relativistes de La Gravitation et de LElectromagnetisme.Masson et Cie, 1st edition, 1955
3. M. A. Stephanov and Y. Yin, Chiral Kinetic Theory, Phys.Rev. Lett. 109, 162001 (2012)

Radiative Hydrodynamics

We choose the metric as Baron[1]:

$$ds^2 = e^{2\phi}dt^2 - e^{2\lambda}dm^2 - R^2 d\Omega^2 \tag{1}$$

where m is the total rest mass in a shell of radius R, t is a time coordinate and $d\Omega^2 = d\theta^2 + sin^2\theta d\theta^2$. We can define:

$$U = e^{-\phi}\frac{\partial R}{\partial t} = D_t R \tag{2}$$

$$\Gamma = e^{-\lambda}\frac{\partial R}{\partial m} = D_m R \tag{3}$$

The continuity equation from [2] is $(nv^\alpha)_{;\alpha} = 0$, where v^α is the fluid four-velocity and n is the number density of baryons, in the comoving frame,

$$v^\alpha = (v^0, 0, 0, 0), v^\alpha v_\alpha = 1, v^0 = e^{-\phi}$$

Since $\sqrt{-g} = 4\pi R^2 e^{\phi+\lambda}$ the continuity equation give us:

$$e^\lambda = \frac{1}{4\pi R^2 n} \tag{4}$$

The total enery-momentum tensor is

$$\tau^{\alpha\beta} = T^{\alpha\beta} + t^{\alpha\beta} \tag{5}$$

where

$$\begin{aligned} T^{\alpha\beta} &= nwv^\alpha v^\beta - p_m g^{\alpha\beta}, \\ t^{\alpha\beta} &= nE_\nu v^\alpha v^\beta + F^\alpha_\nu v^\beta + v^\alpha F^\beta_\nu + P^{\alpha\beta}_\nu \end{aligned} \tag{6}$$

$T^{\alpha\beta}$ is the energy momentum tensor of matter which is consider a perfect fluid, where $w = 1 + E_m + p_m/n$ and E_m is the energy per nucleon of matter and p_m is the matter pressure. $t^{\alpha\beta}$ is the energy momentum tensor for neutrinos, where E_ν is the energy of neutrinos per baryon, $P^{\alpha\beta}_\nu$ is the neutrino pressure tensor and F^α_ν is the flux. For extremely relativistic particles we know $TrP^{\alpha\beta}_\nu = nE_\nu$ and specifically in the comoving frame we have

$$F^\alpha_\nu = (0, F_\nu, 0, 0)$$

$$P^\alpha_{\nu\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & P_\nu & 0 & 0 \\ 0 & 0 & (nE_\nu - p_\nu)/2 & \\ 0 & 0 & 0 & (nE_\nu - p_\nu)/2 \end{pmatrix}$$

where p_ν is the radial component of the pressure and F_ν is the radial component of the flux.

Einsteins field equation is

$$\Re^\alpha_\beta - 1/2\delta^\alpha_\beta \Re = 8\pi\tau^\alpha_\beta \tag{7}$$

In terms of the Ricci tensor the equation $\Re^1_0$ gives

$$e^{-\phi}\frac{\partial}{\partial t}\left(\frac{1}{n}\right) = \frac{1}{\Gamma}\frac{\partial}{\partial m}(4\pi R^2 U) - \frac{4\pi R^2 F_\nu}{\Gamma n R} \tag{8}$$

Now let us define the gravitational mass from Baron[1] as

$$\widetilde{m} = \frac{1}{2}R(1 + U^2 - \Gamma^2). \tag{9}$$

Using equation (7) for R^0_0 we get

$$\frac{\partial \widetilde{m}}{\partial m} = 4\pi R^2[n(1 + E_m) + nE_\nu + \frac{UF_\nu}{\Gamma}]\frac{\partial R}{\partial m} \tag{10}$$

Radiative Hydrodynamics

From R^1_1 equation we get

$$\frac{\partial U}{\partial t} = e^{-\phi}[-4\pi R^2 n\Gamma\frac{\partial\phi}{\partial m} + \frac{\widetilde{m}}{R^2} + 4\pi R(p_m + p_\nu)] \tag{11}$$

The hydrodynamic equations are given by

$$T^\alpha_{\beta;\alpha} = -t^\alpha_{\beta;\alpha} = -Q_\beta \tag{12}$$

where the source term Q_β is a vector such that

$$v_\alpha Q^\alpha = Q \tag{13}$$

where Q is the energy transfered and $\triangle^\alpha_\beta Q^\beta = \Upsilon^\alpha$ and the quantity $\triangle = g^{\alpha\beta} - v^\alpha v^\beta$ is a projection operator orthogonal to the four-velocity. where Υ is the momentum transfer in the rest frame.

From (12) the $T^\alpha_{0;\alpha}$ and equation gives

$$\frac{\partial E_m}{\partial t} = -p_m\frac{\partial}{\partial t}\left(\frac{1}{n}\right) - \frac{Qe^\phi}{n} \tag{14}$$

while the equation $T^\alpha_{1;\alpha}$ gives an equation for the lapse function ϕ

$$nw\frac{\partial\phi}{\partial m} = -\frac{\partial p_m}{\partial m} - \frac{\Upsilon}{4\pi R^2 n} \tag{15}$$

The evolution of neutrinos is derived from equation (12) for $t^\alpha_{0;\alpha}$

$$\begin{aligned} &\frac{\partial}{\partial t}E_\nu + e^{-\phi}\frac{\partial}{\partial m}(e^{2\phi}4\pi R^2 F_\nu) + \\ &p_\nu\frac{\partial}{\partial t}\left(\frac{1}{n}\right) + \frac{U}{nR}(nE_\nu - 3p_\nu)e^\phi = e^\phi\frac{Q}{n} \end{aligned} \tag{16}$$

the equation $t^\alpha_{1;\alpha}$

$$\begin{aligned} &4\pi R^2 n\frac{\partial}{\partial m}p_\nu + \frac{\Gamma}{R}(3p_\nu - E_\nu n) + (nE_\nu + p_\nu)\frac{\partial\phi}{\partial m}\times \\ &(4\pi R^2 n) + e^{-\phi}\frac{\partial F_\nu}{\partial t} + 2nF_\nu e^{-\phi}\frac{\partial}{\partial t}\left(\frac{1}{n}\right) - \frac{2F_\nu U}{R} = \Upsilon \end{aligned} \tag{17}$$

From the equation of motion (15) and (10) we obtain the total mass-energy conservation:

$$e^{-\phi}\frac{\partial\widetilde{m}}{\partial t} = -[4\pi R^2 U(p_m + p_\nu) + 4\pi R^2\Gamma F_\nu] \tag{18}$$

When we solve the Boltzmann equation for this system we can find (16) and (17) if we integrate the monochromatic radiation energy and momentum transport equations.

Conclusions

- We have presented the general relativistic hydrodynamics in the context of flux-limited diffusion theory.
- In the completed trapped regime one sees that the neutrino energy and momentum transport equations reduce to the same as matter equations.
- We show that the chiral energy released from neutrinos transform into fluid energy by the hydrodynamic evolution due to the conservation law of helicity.

Chiral Hydrodynamics

The relativistic hydrodynamic equations for single charged chiral fermions are given by the energy and momentum conservations for the energy-momentum tensor $T^{\mu\nu}$ and the anomaly relation for the electric current j^μ

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\lambda}j_\lambda \\ \partial_\mu j^\mu &= -CE^\mu B_\mu \end{aligned} \tag{19}$$

Here $F^{\mu\nu}$ is the field strenght, the electric and magnetic fields are defined in the fuild rest frame. The right hand side of this equations represent the work done by electromagnetic fields and the quantum anomaly with $C = \pm 1/(4\pi^2)$ for right and left handed fermions, respectively.

We choose the Landau-Lifshitz frame, because the variables ϵ, T^{0i}, n concide with conserved quantities, where $T^{\mu\nu}$ and j^μ are given by

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \tau^{\mu\nu} \\ j^\mu &= nu^\mu + \xi_B B^\mu + \xi\omega^\mu + \nu^\mu \end{aligned} \tag{20}$$

Here n is the charge density and ω^ν is the vorticity, ϵ is the energy density, P is the pressure. ν^μ and $\tau^{\mu\nu}$ denote particle diffusion and viscous stress tensor. The terms ξ_B and ξ are the transport coefficients of the *Chiral Magnetic Effect* and the *Chiral Vortical Effect*. It can be prove that chiral effects don't affect the transport equation for ν and $\mathbf{B}$.

Performing the volumen integral of the anomaly relation for the electric current and assuming that $\mathbf{j}$ vanishes at infinity we obtain the **Conservation law of helicity**

$$\begin{aligned} &\frac{d}{dt}Q_{total} = 0, \\ &Q_{total} \equiv Q_{chi} + \frac{C}{2}Q_{mag} + \xi Q_{flu} + \xi_B Q_{mix}, \end{aligned} \tag{21}$$

Q_{mag}, Q_{flu}, Q_{mix} are called the magnetic helicity, fluid helicity, and cross helicity. When a massive star explodes we only have Q_{chi} and Q_{flu} by the conservation law this means that Q_{chi} can be converted into Q_{flu} by the hydrodynamic evolution.

Forthcoming Research

We have in mind that our calculations will be employed in the calculations of neutrino transport equations and then it will be employed in modelling stellar collapse. Also we want to show that the helicity of neutrinos induces effects in the context of Chiral Kinetic Theory in curved space-time and see what kind of different fenomena we find in the supernova explosion.