Monte Carlo Simulation of Reverse Emittance Exchange in MICE

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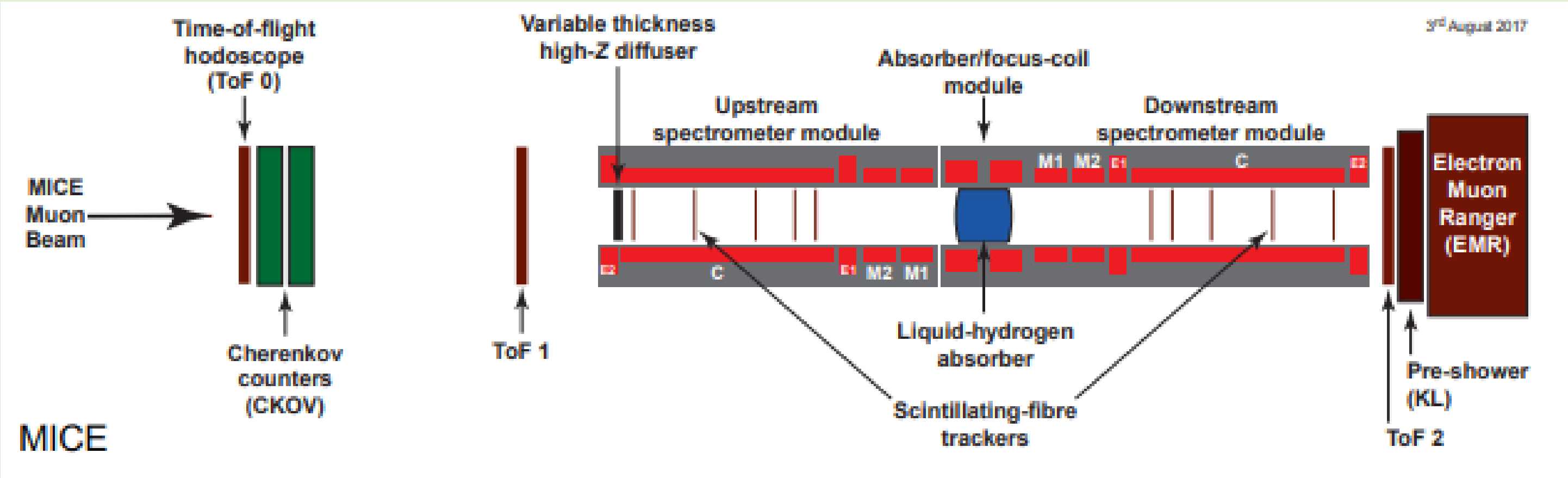
Muon Ionization Cooling Experiment



In MICE (formerly at the Rutherford Appleton Laboratory, Harwell, UK), a titanium target was dipped into a circulating proton beam creating pions, which were subsequently captured in a quadrupole magnet. These pions then decayed to muons in the transport line.

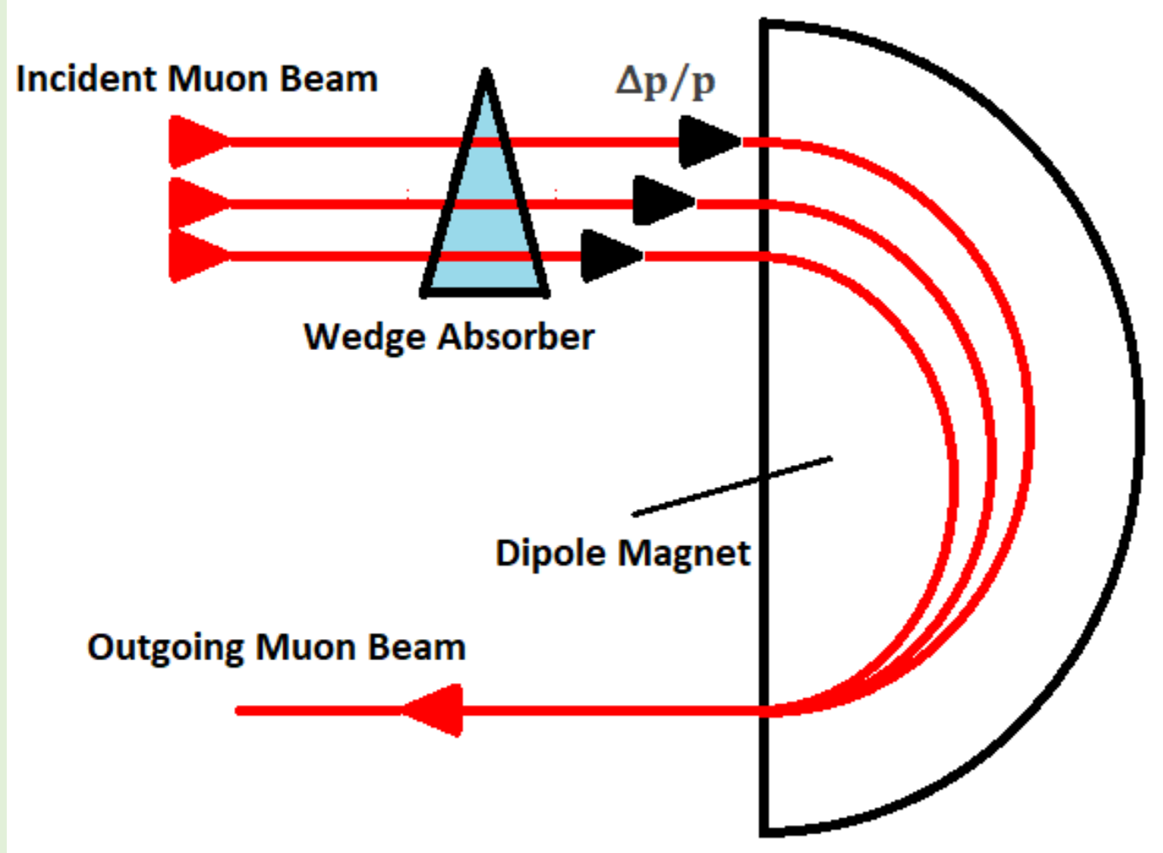
The MICE muon beam was passed through a series of detectors in the cooling channel. The Time-of-Flight stations, Kloe-Light and Electron-Muon Ranger were used to distinguish muons from electrons and pions, while the two tracking detectors either side of the absorber immersed in a uniform multi-Tesla magnetic field measured the position and momentum of each particle. Each tracker consisted of 5 stations of three planes of scintillating fibres. The three planes were at 60 degrees to each other.

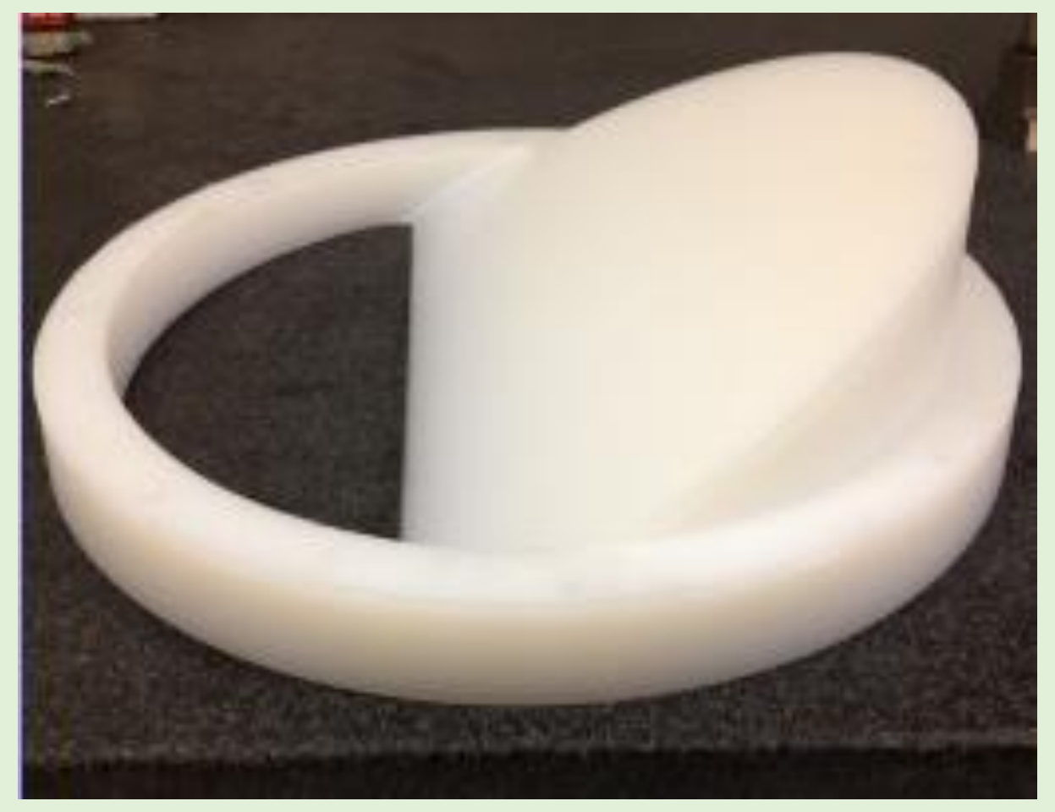
Position and momentum measurements made in each tracker allow the comparison of phase-space density and emittance (the spread of a beam in position-momentum space) at the reference planes before and after an absorber. Absorbers used in MICE include liquid Hydrogen and Lithium Hydride. In Ionization Cooling, when a beam passes through a low Atomic number absorber, electrons are ionized in the absorber and the beam loses momentum. The longitudinal momentum is recovered in an RF cavity. This results in the beam having a reduced transverse emittance.

Emittance Exchange and Reverse Emittance Exchange were investigated in MICE by placing a Polyethylene wedge between the reference planes, to observe the change in longitudinal and transverse phase space densities.

Reverse Emittance Exchange

When muons are produced by proton collision with a target, they occupy a large phase-space volume and emittance (their spread in position-momentum space). To meet the acceptance requirements of a storage ring, these muons must be cooled, with ionization cooling the only viable process to cool the muon beam to the required phase-space density on a shorter timescale than the muon lifetime.





Ionization cooling only reduces the transverse emittance. Emittance Exchange allows the transfer of emittance between longitudinal and transverse phase-space, and thus overall 6D cooling can be achieved.

In Emittance Exchange a muon beam is passed through a dipole magnet to create both a position spread and a position-energy correlation in the beam. The beam is then passed through a wedge of specific thickness to eliminate the momentum dispersion. The beam then has an increased longitudinal phase-space density and a reduced transverse phase-space density. The emittance has been exchanged. The transverse emittance can then be reduced by ionisation cooling .

In Reverse Emittance Exchange the beam is first passed through a wedge and then through a magnetic dipole. This allows one to increase the transverse phase-space density at the expense of decreased longitudinal phase-space density. Thus the emittance exchange has been reversed.

Acknowledgments

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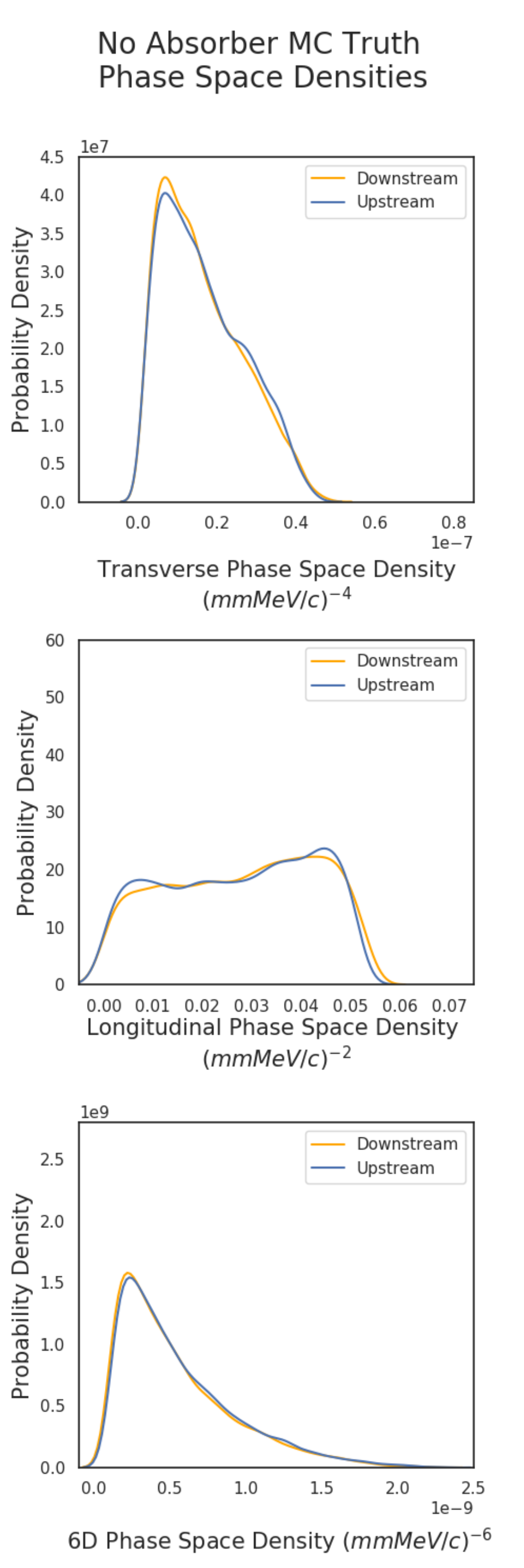
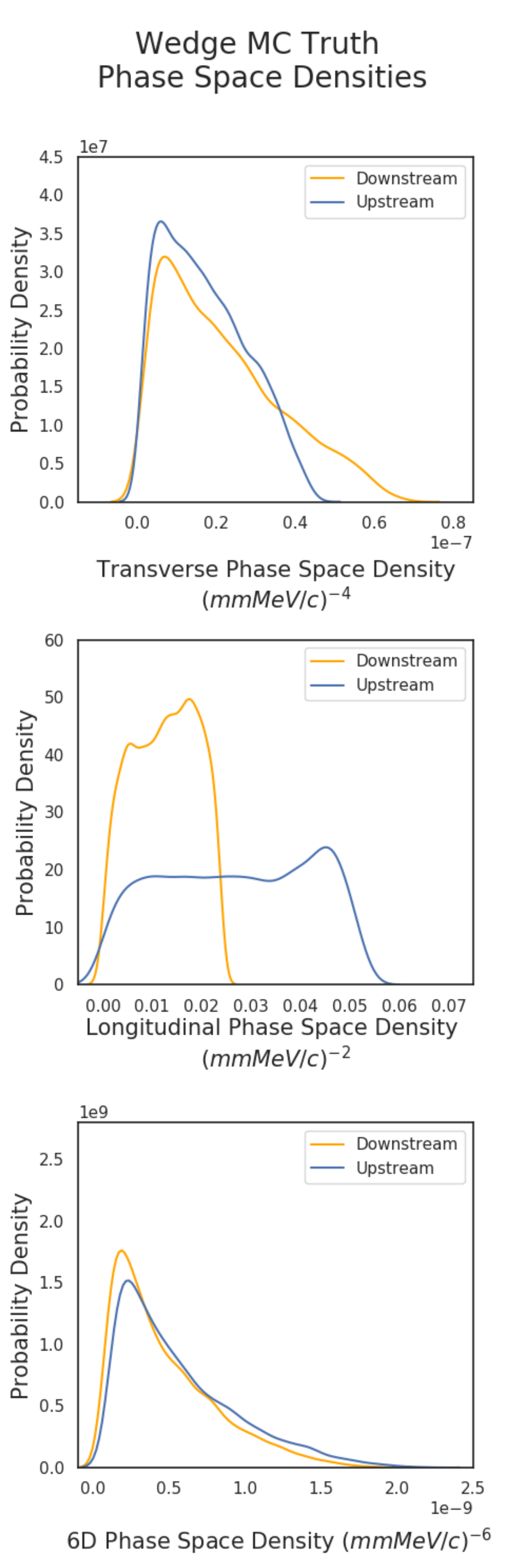
Particle Distribution Functions

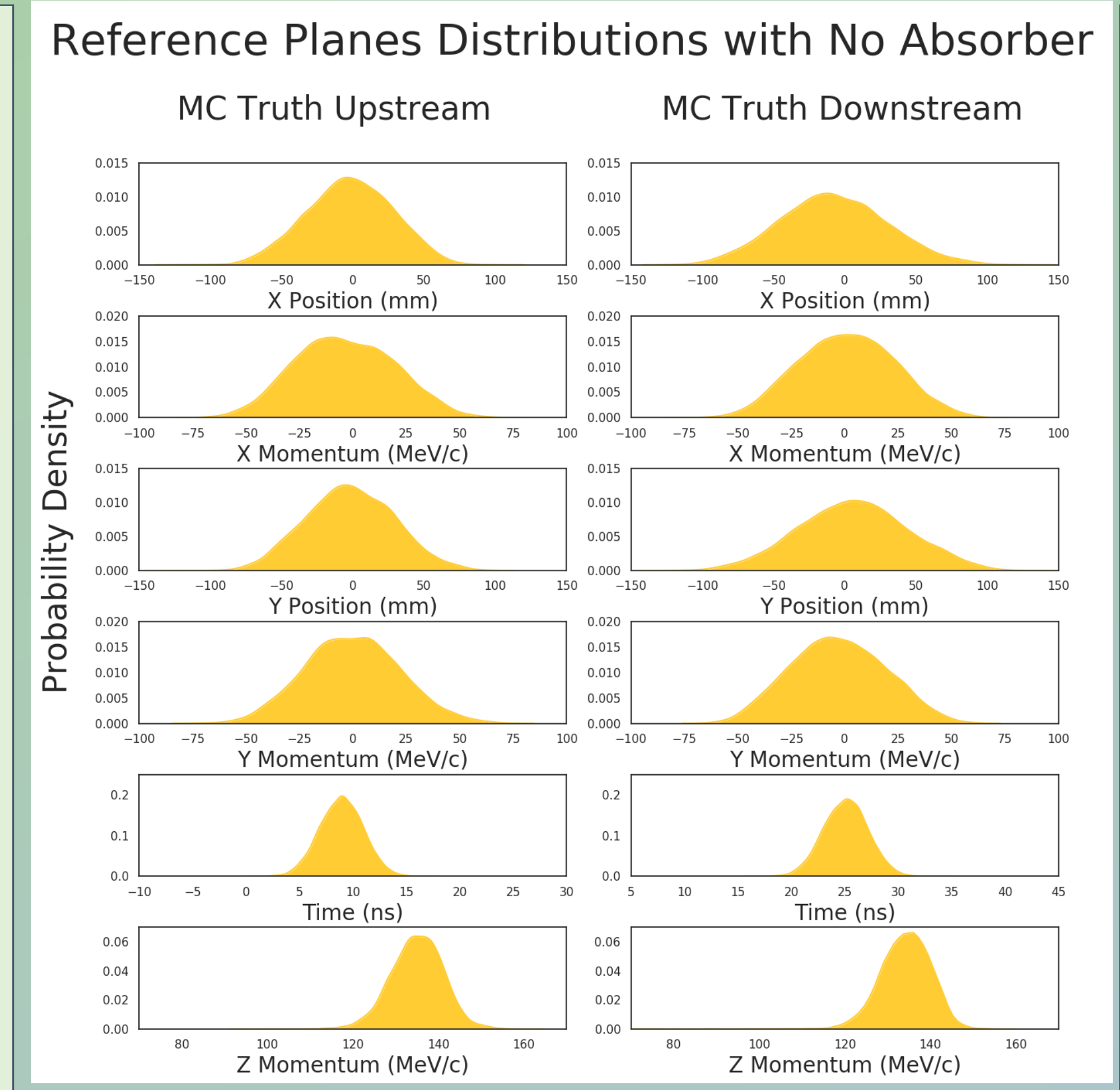
MICE measured the position and momentum particle by particle, with the beam being assembled from these individual measurements. On the right shows the Monte Carlo measurements at the upstream and downstream reference planes for when no absorber was present and when a polyethylene wedge was placed between the reference planes.

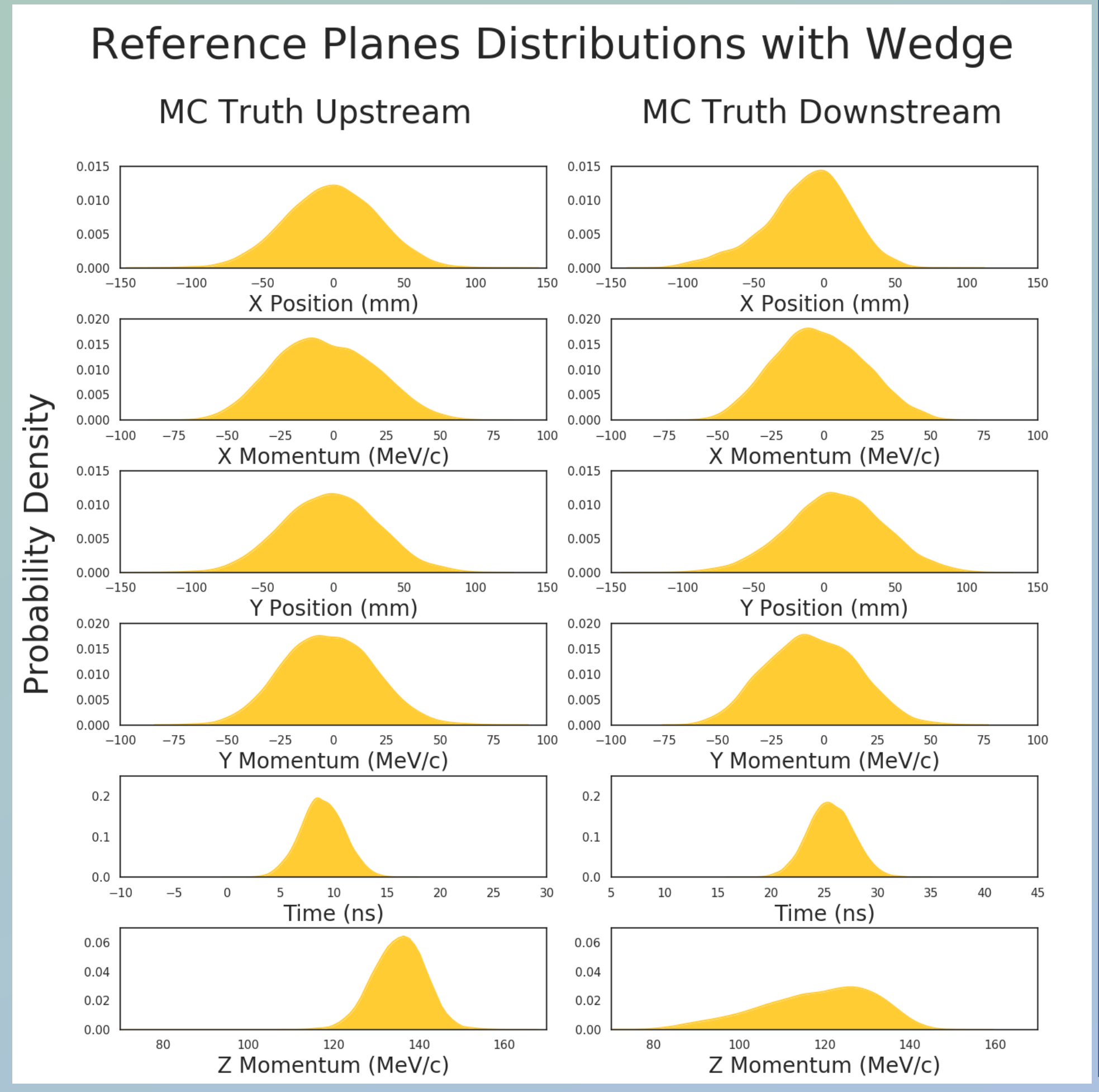
Both samples had the same input beam with a nominal transverse emittance of 6 mm and 140MeV/c momentum. However a cut on transmission has been made (insisting on the particle being transmitted to TOF2) to allow for the comparison of the upstream and downstream phase space densities. This results in slightly different input distributions for each case as the wedge may scatter out some particles beyond the aperture of the experiment.

The bias created by both the transmission losses as well as the resulting change in the particle distribution functions have not been accounted for in the phase space density calculations.


No Absorber MC Truth Phase Space Densities


Wedge MC Truth Phase Space Densities


Reference Planes Distributions with No Absorber


Reference Planes Distributions with Wedge

Estimating the Phase Space Density

A particle beam can be described by the distribution of the particles in the beam also known as the phase space density $\rho(x,y,z,p_x,p_y,p_z)$. Liouville's theorem states that the density of particles in phase space is a constant i.e. ${}^{d\rho}/_{dt} = 0$ (providing there are no dissipative forces), where the number of particles in that phase-space volume is given by:

$$N = \int \rho(x,y,z,p_x,p_y,p_z)\,dx\,dy\,dz\,dp_x\,dp_y\,dp_z = \int \rho dV$$

The x, y, p_x, p_y, p_z components used to calculate the density are the measurements at the reference planes, with the z component calculated from the difference in arrival time at the reference plane and the mean arrival time.

Kernel Density Estimation (KDE) has been used to determine the probability of a particle having a particular phase space density in a given distribution. KDE is a non-parametric density estimation technique which makes fewer assumptions about the underlying distribution. This is done by calculating the kernel, a multivariate Gaussian centred on each data point. All of the kernels are then summed to arrive at the KDE. The KDE is then given by:

$$\rho(\vec{x}) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left\{\frac{1}{h}(\vec{x} - \vec{X_i})\right\}$$

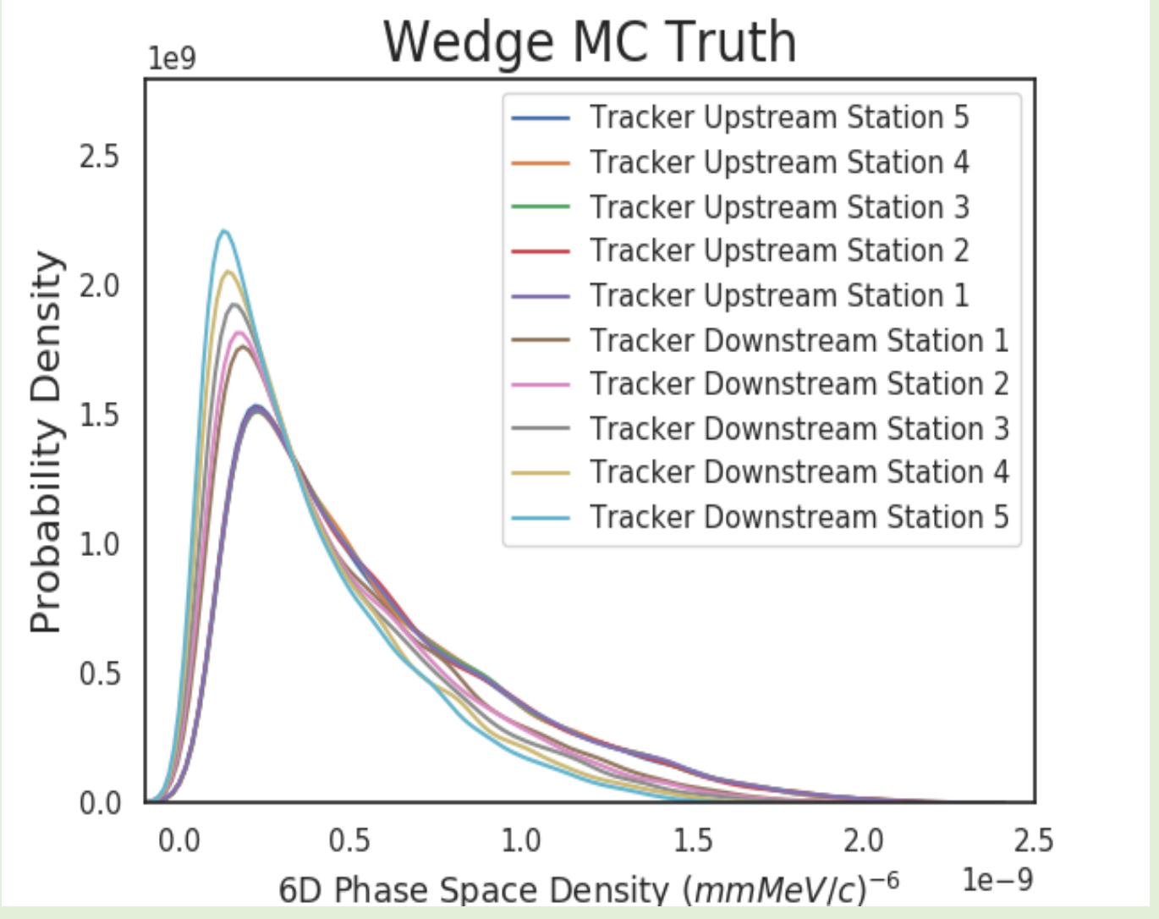
where K is the kernel written as a function of reference point $\vec{x}$, i^{th} data point X_i and kernel width h. The dimensionality is given by d, with sample size n.

Change in Phase Space Density

On the left are shown the change in the Monte Carlo Truth phase space densities (assuming the 6D density can be separated into transverse and longitudinal components) for the No Absorber and Wedge cases between the reference planes.

For the No Absorber case, the density remains conserved (6D in particular), while the Wedge shows a decrease in longitudinal density and a slight increase in transverse density. Note, the calculated densities are biased by not including all the heated particles outside of the aperture of the experiment.

The measurements are taken at the reference planes, longitudinal z-direction (space-like) planes and not at a moment in time (time-like plane). When those two planes no longer coincide due to the dispersion created at the wedge, an apparent decrease in the 6D density can be seen, as seen by the evolution of the 6D density through the 10 tracking stations (right) from Upstream Tracker Station 5 through to Downstream Tracker Station 5.


Wedge MC Truth

Conclusion

The conservation of the 6D phase space density in the No Absorber case shows the capability of this analysis. The Emittance Exchange effect, as to be used for a Muon Collider or Neutrino Factory, can then be quantified once corrections are made to achieve a time-like state as well as the corrections for the transmission losses.