Neutrino quantum decoherence in supernovae explosions

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1 Introduction

We study the influence of the neutrino quantum decoherence (see [1–4] and references therein) on collective neutrino oscillations. Collective neutrino oscillations is a phenomenon engendered by neutrino-neutrino interaction. It is significant in different astrophysical environments where the neutrino density is extremely high. Examples of such environments are the early universe, supernovae explosions, binary neutron stars, accretion discs of black holes. The effect of collective neutrino oscillations attracts the growing interest in sight of appearance of multi-messenger astronomy and constructing of new large-volume neutrino detectors that will be highly efficient for observing neutrino fluxes from supernovae explosions.

Previously, it was shown that neutrino quantum decoherence can significantly modify the neutrino fluxes from reactors and the sun. Here below we study the influence of the neutrino quantum decoherence on supernovae neutrino fluxes. The peculiarity of the supernovae fluxes is that one of the main modes of neutrino oscillations in supernovae is engendered by the collective effects. We note, that previous works dedicated to collective neutrino oscillations (see [5] and references therein) accounted only for the kinematical decoherence.

2 Equations of motion

Consider the two-flavor neutrino mixing scenarios, i.e. the mixing between ν_e and ν_x states where ν_x stands for ν_μ or ν_τ. Here below we focus on the derivation of the neutrino oscillation probability and highlight the interplay between collective neutrino oscillations and neutrino quantum decoherence. We use the simplified model of supernova neutrinos that was considered in [6, 7]. In such a model neutrinos are produced and emitted with a single energy and a single emission angle.

The neutrino evolution in supernovae environment that accounts for neutrino quantum decoherence is determined by the following equation

$$i\frac{d\rho_f}{dt} = [H, \rho_f] + D[\rho_f]\,, \qquad i\frac{d\bar{\rho}_f}{dt} = [\bar{H}, \bar{\rho}_f] + D[\bar{\rho}_f], \tag{1}$$

where ρ_f ($\bar{\rho}_f$) is the density matrix for neutrino (antineutrino) in the flavour basis and H ($\bar{H}$) are total neutrino (antineutrino) Hamiltonian. Neutrino quantum decoherence is described by the dissipation term $D[\rho]$ that we define in the next section.

Hamiltonian H contains the three terms

$$H = H_{vac} + H_M + H_{\nu\nu}, \tag{2}$$

where H_{vac} is the vacuum Hamiltonian, H_M and $H_{\nu\nu}$ are Hamiltonians that describe matter potential and neutrino-neutrino interaction correspondingly. In the flavour basis they are given by

$$H_{vac} = \frac{\delta m^2}{4E}\begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}, \tag{3}$$

$$H_M = \frac{\sqrt{2}}{2}G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{4}$$

$$H_{\nu\nu} = \sqrt{2}G_F n_\nu \left((1+\beta)\rho_f - \alpha(1+\bar{\beta})\bar{\rho}_f\right), \tag{5}$$

where β represents the initial asymmetry between the electron and x-type neutrinos, and $\bar{\beta}$ the asymmetry between electron and x-type antineutrinos, α is the ratio of electron antineutrinos relative to electron neutrinos, n_e and n_ν describe the electron and neutrino density profiles.

3 Neutrino quantum decoherence

In our previous studies [1–4] we developed a new theoretical framework that enabled one to consider a concrete process of particles interactions as a source of the decoherence. In particular, in [1] a new mechanism of neutrino quantum decoherence engendered by the neutrino radiative decay. In parallel, another framework was developed [8, 9] for the description of the neutrino quantum decoherence due to the non-forward neutrino scattering processes. Both mechanisms are described by the Lindblad master equation in form [10, 11].

In this paper we are are not interested in a specific mechanism of neutrino quantum decoherence. Therefore, we use the Lindblad master equation for the description of the neutrino quantum decoherence and do not fix an analytical expression for the decoherence and relaxation parameters. The dissipation term $D[\rho]$ is expressed within neutrino effective mass basis

$$D\left[\rho_{\tilde{m}}(t)\right] = \frac{1}{2}\sum_{k=1}^{3}\left[V_k, \rho_{\tilde{m}}V_k^\dagger\right] + \left[V_k\rho_{\tilde{m}}, V_k^\dagger\right], \tag{6}$$

where V_k are the dissipative operators that arise from interaction between the neutrino system and the external environment, $\rho_{\tilde{m}}$ is the neutrino density matrix in the effective mass basis. Here below, we omit index "$\tilde{m}$" in order not overload formulas.

The operators V_k, ρ_f and H can be expanded over the Pauli matrices $O = a_\mu\sigma_\mu$, where σ_μ are composed by an identity matrix and the Pauli matrices. In this case eq. (1) can be written in the following form

$$\frac{\partial P_k(t)}{\partial t}\sigma_k = 2\epsilon_{ijk}H_iP_j(t)\sigma_k + D_{kl}P_l(t)\sigma_k, \tag{7}$$

where the matrix $D_{ll} = -diag\{\Gamma_1, \Gamma_1, \Gamma_2\}$ and Γ_1, Γ_2 are the parameters that describe two dissipative effects: 1) the decoherence effect and 2) the relaxation effect, correspondingly. In the case of the energy conservation in the neutrino system there is an additional requirement on a dissipative operators [12]

$$[H_S, V_k] = 0. \tag{8}$$

In this case the relaxation parameter is equal to zero $\Gamma_2 = 0$. Here below, we consider only the case of the energy conservation, i.e. $\Gamma_2 = 0$. For further consideration we use the flavour basis. It can be shown that the dissipation matrix D_{ij} in the flavour basis is expressed as

$$\tilde{D}_{lk} = -\frac{\Gamma_1}{2}\begin{pmatrix} 1+\cos 4\tilde{\theta} & 0 & \sin 4\tilde{\theta} \\ 0 & 2 & 0 \\ \sin 4\tilde{\theta} & 0 & 1-\cos 4\tilde{\theta} \end{pmatrix}, \tag{9}$$

where $\tilde{\theta}$ is the in-medium (effective) mixing angle that is given by

$$\sin^2 2\tilde{\theta}_{ij} = \frac{\Delta m_{ij}^2 \sin^2 2\theta_{ij}}{(\Delta m_{ij}\cos 2\theta_{ij} - 2\sqrt{2}G_F n_e E)^2 + \Delta m_{ij}^2 \sin^2 2\theta_{ij}}. \tag{10}$$

In a particular environments of a supernova where the collective oscillations occur the electron density is extremely high and the effective mixing angle $\theta \approx 0$ and $\tilde{D}_{kl} \approx D_{kl}$. We consider only the case of high electron density, thus we use the latter equality and substitute w$\tilde{D}_{kl}$ by D_{kl}.

4 Linearized (in)stability analysis

In this section we consider analytical conditions for the occurrence of the neutrino collective effects. The onset of these collective effects has been related to the presence of an instability (see [6] and references therein). In order to study this instabilities we will apply to eq. (1) the linearization procedure described in [6, 13].

Consider a time dependent small amplitude variation δP_k around the initial configuration P_k^0 and a corresponding variation of the density dependent Hamiltonian δH_k around the initial Hamiltonian H_k^0

$$P_k = P_k^0 + \delta P_k, \quad \text{where} \quad \delta P_k = P_k' e^{-i\omega t} + H.c., \tag{11}$$

$$H_k = H_k^0 + \delta H_k, \quad \text{where} \quad \delta H_k = H_k' e^{-i\omega t} + H.c., \tag{12}$$

$$H_k' = \frac{\partial H_k}{\partial P_k}P_k' + \frac{\partial H_k}{\partial \bar{P}_k}\bar{P}_k'. \tag{13}$$

In the case of high electron density the in-medium eigenstates initially coincide with the flavor states. Therefore, the initial conditions are given by

$$H_k^0 = \begin{pmatrix} 0 \\ 0 \\ H^0 \end{pmatrix}, \quad P_k^0 = \begin{pmatrix} 0 \\ 0 \\ P^0 \end{pmatrix}. \tag{14}$$

Putting (11)-(14) into (1) and considering only the non-diagonal elements ($\rho_{12} = P_x + iP_y$) one obtains the following equation for eigenvalues (we neglect the higher-order corrections)

$$(\omega - i\Gamma_1)\begin{pmatrix} \rho_{12}' \\ \bar{\rho}_{21}' \end{pmatrix} = \begin{pmatrix} A_{12} & B_{12} \\ \bar{A}_{21} & \bar{B}_{21} \end{pmatrix}\begin{pmatrix} \rho_{12}' \\ \bar{\rho}_{21}' \end{pmatrix}, \tag{15}$$

where on the right-hand side of equation is the stability matrix that coincides with one from [6, 13]. In case of a single energy and single emission angle it is expressed as

$$\begin{aligned} A_{12} &= (H_{11}^0 - H_{22}^0) - \frac{\partial H_{12}}{\partial \rho_{12}}(\rho_{11}^0 - \rho_{22}^0), \\ B_{12} &= \frac{\partial H_{12}}{\partial \bar{\rho}_{21}^0}(\bar{\rho}_{22}^0 - \bar{\rho}_{11}^0), \\ \bar{A}_{21} &= (\bar{H}_{22}^0 - \bar{H}_{11}^0) - \frac{\partial \bar{H}_{21}}{\partial \bar{\rho}_{21}}(\bar{\rho}_{22}^0 - \bar{\rho}_{11}^0), \\ \bar{B}_{21} &= \frac{\partial \bar{H}_{21}}{\partial \rho_{12}^0}(\rho_{11}^0 - \rho_{22}^0). \end{aligned} \tag{16}$$

The eigenvalues are given by

$$\omega = i\Gamma_1 + \frac{1}{2}\left(A_{12} + \bar{A}_{21} \pm \sqrt{(A_{12} - \bar{A}_{21})^2 + 4B_{12}\bar{B}_{21}}\right). \tag{17}$$

From eq. (11) it follows that if the eigenvalues have an imaginary part, the non-diagonal elements of the neutrino density matrix can grow exponentially and thus the system become unstable, that is, if

$$\begin{cases} (A_{12} - \bar{A}_{21})^2 + 4B_{12}\bar{B}_{21} < 0, \\ \mathcal{I}m\left((A_{12} - \bar{A}_{21})^2 + 4B_{12}\bar{B}_{21}\right) > \Gamma_1. \end{cases} \tag{18}$$

The first condition is the same as was derived in [6, 13]. The second term is a new one that was not considered before. From eq. (18) one can see, that neutrino quantum decoherence prevents a system from an exponential growth of non-diagonal elements, i.e. neutrino quantum decoherence leads to the damping of the neutrino collective oscillations.

5 Numerical calculations

For numerical calculations we use the supernovae model that was considered in [7]. The initial neutrino flux is characterized by $\alpha = 0.8$, $\beta = 0.48$ and $\bar{\beta} = 0.6$ and neutrino energy $E = 20$ MeV. The electron density profile is given by

$$n_e(r) = n_0\left(\frac{R_\nu}{r}\right)^3\left[a + b\tan^{-1}\left(\frac{r - R_\nu}{R_s}\right)\right], \tag{19}$$

where $R_\nu = 10$ km is the radius of the neutrinosphere, $n_0 = 10^{-4}$ eV is the electron density at the neutrinosphere, $a = 0.308$, $b = 0.121$ and $R_s = 42$ km are the parameters that characterise the electron fraction in the supernovae. The neutrino density profile is given by

$$n_\nu(r) = n_\nu^0\left(\frac{R_\nu}{r}\right)^4, \tag{20}$$

where $n_\nu^0 = 10^{-4}$ eV is the neutrino density at the neutrinosphere.

With the use of numerical calculations we plot the survival probability $P_{\nu_e\nu_e}$ of the electron neutrino depending on the distance for two particular cases: (i) the survival probability for the case when the neutrino decoherence effect is not accounted for is shown in Fig. (1a), and (ii) the survival probability for the case when the neutrino decoherence effect is accounted for is shown in Fig. (1b) as a function of the neutrino quantum decoherence (decoherence parameter is set to be $\Gamma_1 = 10^{-21}$ GeV). We assumed that the decoherence parameters does not depend on neutrino energy.

6 Conclusion

In this paper we considered for the first time the effect of the neutrino quantum decoherence in supernovae fluxes. We derive new conditions of the collective neutrino oscillations accounting for neutrino quantum decoherence which can appear as a result of the physics beyond the standard model. Therefore, the importance of the neutrino quantum decoherence studies are highlighted by the new opportunity for searching of new physics.

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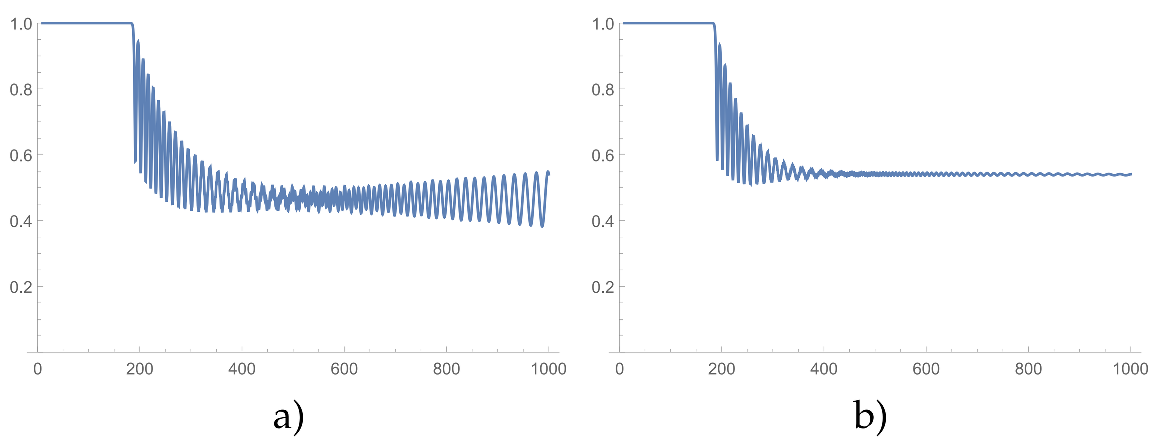

Figure 1: The survival probability of the electron neutrino in the absence of quantum decoherence (a) and for the case when the neutrino decoherence parameter is $\Gamma_1 = 10^{-21}$ GeV (b).

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