LEPTON NUMBER VIOLATION IN A UNIFIED FRAMEWORK

Y. KAWAMURA*, Y. MATSUO*, T. MOROZUMI†, **APRIADI SALIM ADAM**, Y. SHIMIZU†, Y. TOKUNAGA, AND N. TOYOTA*

*GRADUATE SCHOOL OF SCIENCE; †GRADUATE SCHOOL OF ADVANCED SCIENCE AND ENGINEERING, HIROSHIMA UNIVERSITY

BACKGROUND AND MOTIVATION

1 The existence of the Cosmic Neutrino Background (CNB) is predicted,

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \sim 1.95\ \mathrm{K} \sim 1.68 \times 10^{-4}\ \mathrm{eV}.$$

- It would be interesting to study their properties, such as lepton numbers stored in CNB.
- Since we have known a lots of information on neutrinos, such as their mass and mixing, then we are able to study how CNB exists.

2 If neutrino is Majorana particles, it seems difficult to assign any lepton number for them.

- The particle number can be defined for the complex field, $\phi^c = \phi^\dagger \neq \phi$. It is associated to the phase transformation, $\phi' = e^{i\alpha}\phi$.
- Since Majorana field satisfies $\psi^c = \psi$, how one can define their lepton numbers?
- How lepton number evolves by time?

THE SET UP

A neutrino carries the lepton number L_α ($\alpha = e, \mu, \tau$) associated with charged lepton.

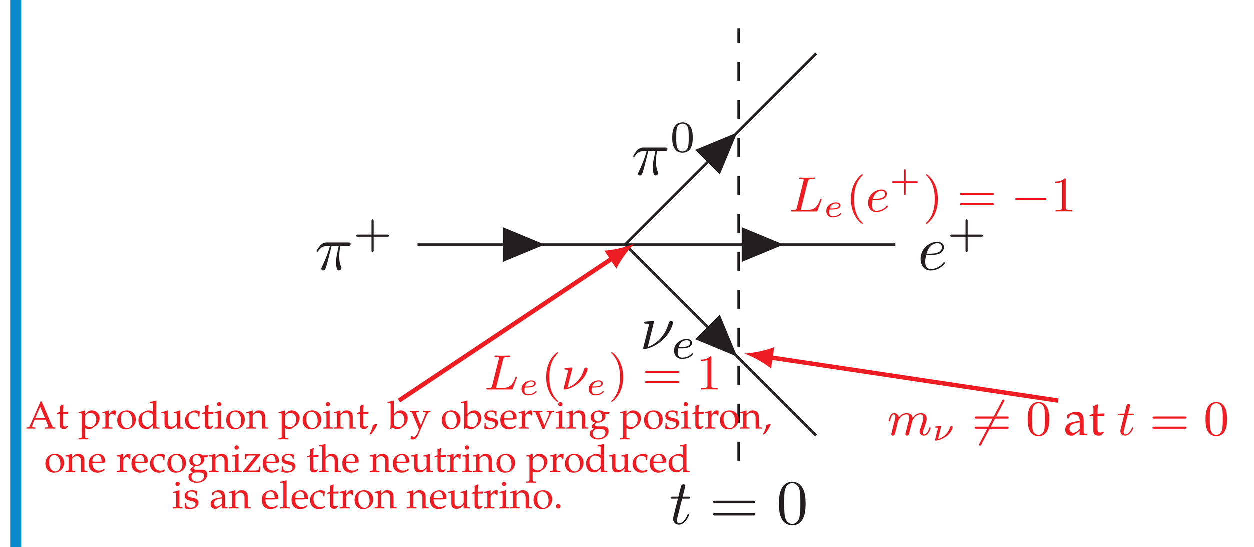


Figure 1: The production of the electron neutrino and its propagation. At $t < 0$, the mass term vanishes.

We turn on the Majorana mass $m_{\alpha\beta}$ at $t = 0$.

$$\mathcal{L}_m = -\theta(t)\{\frac{m_{\alpha\beta}}{2}\overline{(\nu_{\alpha L})^c}\nu_{\beta L} + h.c.\},$$

The neutrino field is expanded with massless spinor,

$$\nu_{\alpha L}(\mathbf{x}, t) = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}(a_\alpha(\mathbf{p})e^{-i|\mathbf{p}|t + i\mathbf{p}\cdot\mathbf{x}}u_L(\mathbf{p}) + b_\alpha^\dagger(\mathbf{p})e^{i|\mathbf{p}|t - i\mathbf{p}\cdot\mathbf{x}}v_L(\mathbf{p})).$$

One can define lepton number operator for $t < 0$,

$$L_\alpha(t) = \int d^3\mathbf{x} : \overline{\nu_{\alpha L}}\gamma^0\nu_{\alpha L} : = \int' \frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3}\left(a_\alpha^\dagger(\mathbf{p})a_\alpha(\mathbf{p}) - b_\alpha^\dagger(\mathbf{p})b_\alpha(\mathbf{p})\right).$$

Continuity condition for operators at $t = 0$,

$$\nu_{L\alpha}(t = 0_-) = V_{\alpha i}L\psi_i(t = 0_+), \quad V\text{: PMNS matrix}$$

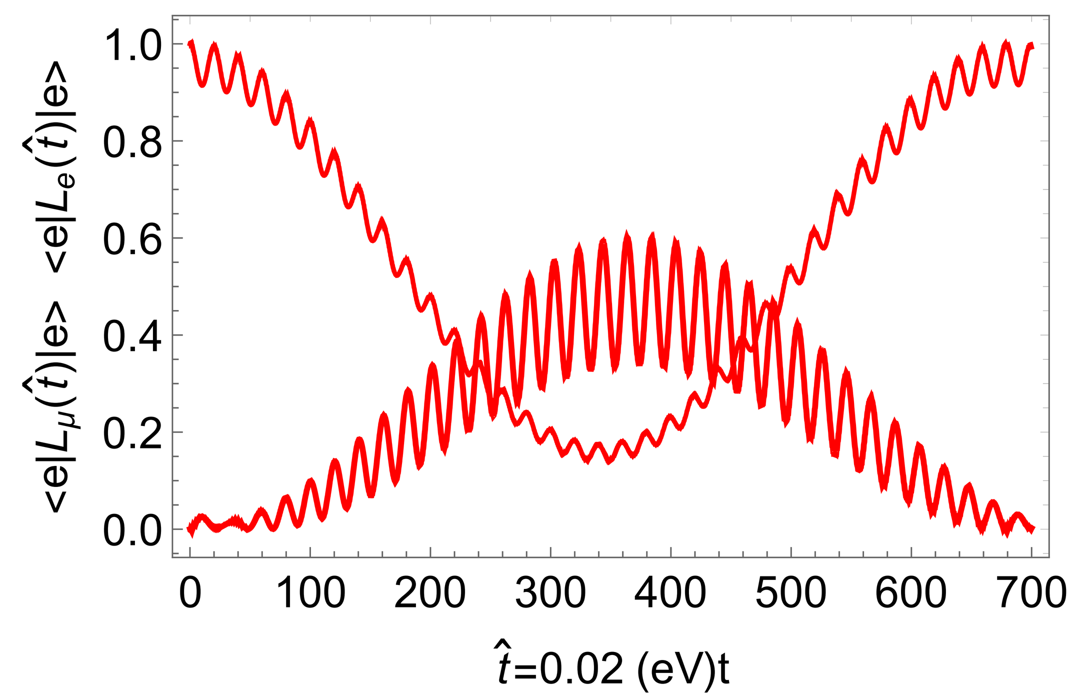
$$\psi_i(\mathbf{x}, 0_+) = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i(\mathbf{p})}\sum_{\lambda=\pm 1} \times (a_{Mi}(\mathbf{p},\lambda)u_i(\mathbf{p},\lambda)e^{i\mathbf{p}\cdot\mathbf{x}} + a_{Mi}^\dagger(\mathbf{p},\lambda)v_i(\mathbf{p},\lambda)e^{-i\mathbf{p}\cdot\mathbf{x}})$$

Time evolution of lepton family number operator for $t \geq 0$,

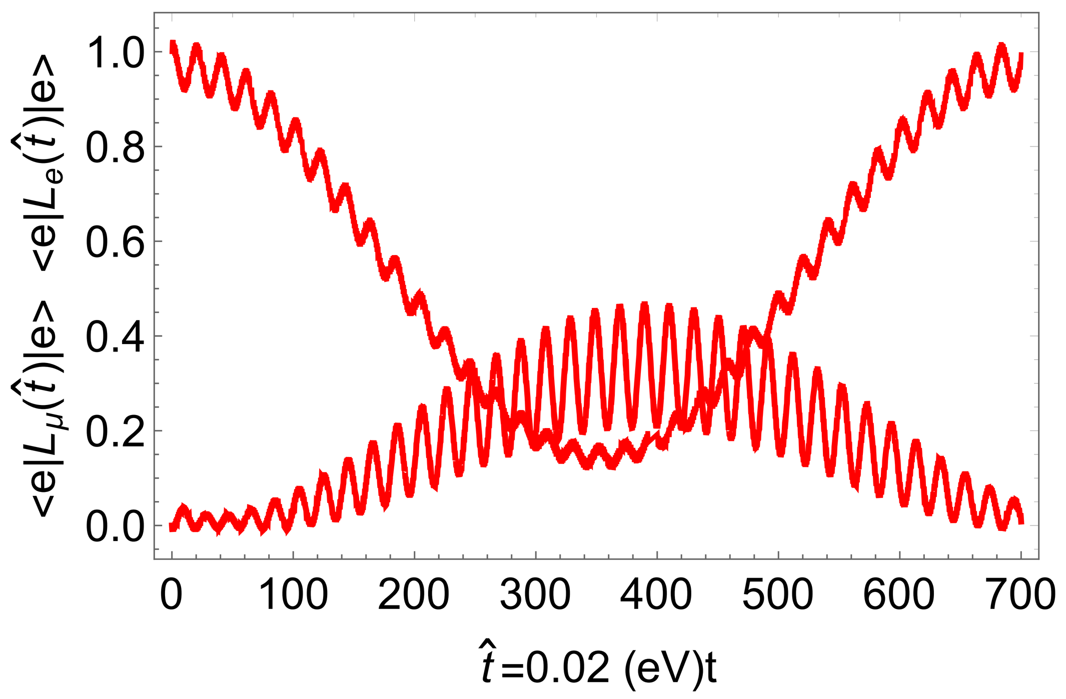
$$E_i(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_i^2}$$

$$\begin{aligned}
L_\alpha(t) &= \int' \frac{d^3\mathbf{p}}{(2\pi)^3|2\mathbf{p}|}\left\{a_\alpha^\dagger(\mathbf{p},t)a_\alpha(\mathbf{p},t) - b_\alpha^\dagger(\mathbf{p},t)b_\alpha(\mathbf{p},t)\right\} \\
&= \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}\sum_{\beta,\gamma,i,j}\Big[\{V_{\alpha i}^*V_{\beta i}^*V_{\alpha j}V_{\gamma j}a_\beta(-\mathbf{p})a_\gamma^\dagger(-\mathbf{p}) - V_{\alpha i}V_{\beta i}V_{\alpha j}^*V_{\gamma j}^*b_\beta(-\mathbf{p})b_\gamma^\dagger(-\mathbf{p})\}\frac{m_i m_j \sin(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t)}{|\mathbf{p}|^2} \\
&+ \{V_{\alpha i}^*V_{\beta i}V_{\alpha j}V_{\gamma j}^*a_\beta^\dagger(\mathbf{p})a_\gamma(\mathbf{p}) - V_{\alpha i}V_{\beta i}^*V_{\alpha j}^*V_{\gamma j}b_\beta^\dagger(\mathbf{p})b_\gamma(\mathbf{p})\}\{\cos E_i(\mathbf{p})t\cos E_j(\mathbf{p})t \\
&+ \sin E_i(\mathbf{p})t\sin E_j(\mathbf{p})t + i\frac{E_i(\mathbf{p})\sin E_i(\mathbf{p})t\cos E_j(\mathbf{p})t - E_j(\mathbf{p})\sin E_j(\mathbf{p})t\cos E_i(\mathbf{p})t}{|\mathbf{p}|}\Big\} \\
&+ i\{V_{\alpha i}^*V_{\beta i}V_{\alpha j}V_{\gamma j}a_\beta^\dagger(\mathbf{p})a_\gamma^\dagger(-\mathbf{p}) + V_{\alpha i}V_{\beta i}^*V_{\alpha j}^*V_{\gamma j}^*b_\beta^\dagger(\mathbf{p})b_\gamma^\dagger(-\mathbf{p})\}\frac{m_j\sin E_j(\mathbf{p})t\{\cos E_i(\mathbf{p})t + i\frac{E_i(\mathbf{p})}{|\mathbf{p}|}\sin E_i(\mathbf{p})t\}}{|\mathbf{p}|} \\
&- i\{V_{\alpha i}^*V_{\beta i}^*V_{\alpha j}V_{\gamma j}^*a_\beta(-\mathbf{p})a_\gamma(\mathbf{p}) + V_{\alpha i}V_{\beta i}V_{\alpha j}^*V_{\gamma j}b_\beta(-\mathbf{p})b_\gamma(\mathbf{p})\}\frac{m_i\sin E_i(\mathbf{p})t\{\cos E_j(\mathbf{p})t - i\frac{E_j(\mathbf{p})}{|\mathbf{p}|}\sin E_j(\mathbf{p})t\}}{|\mathbf{p}|}\Big]
\end{aligned}$$

NUMERICAL RESULTS

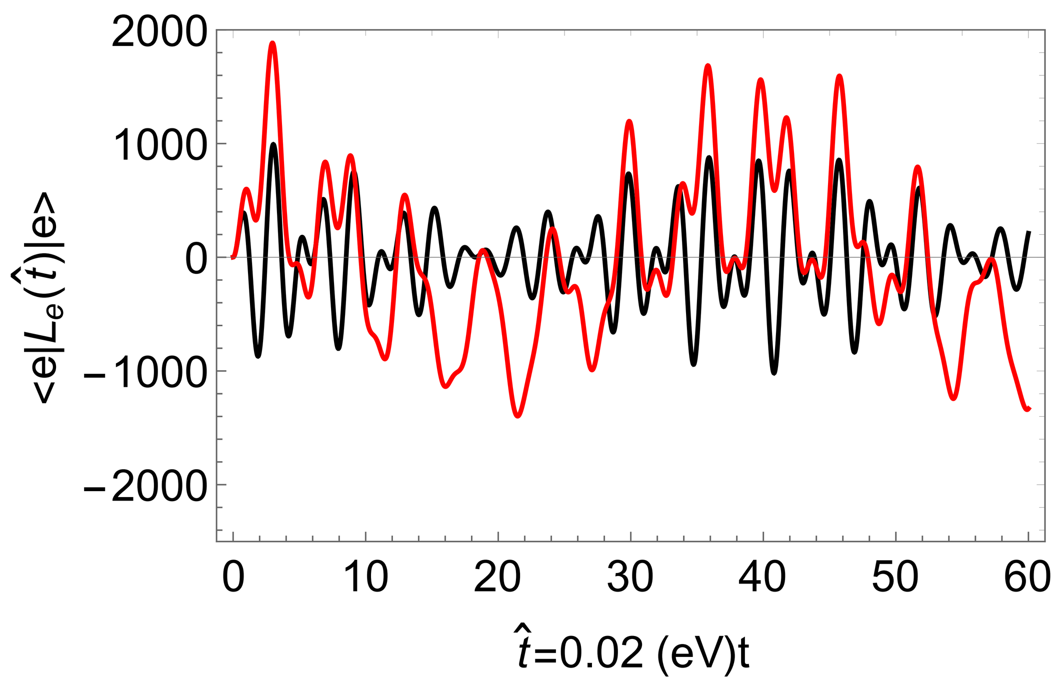


NH case

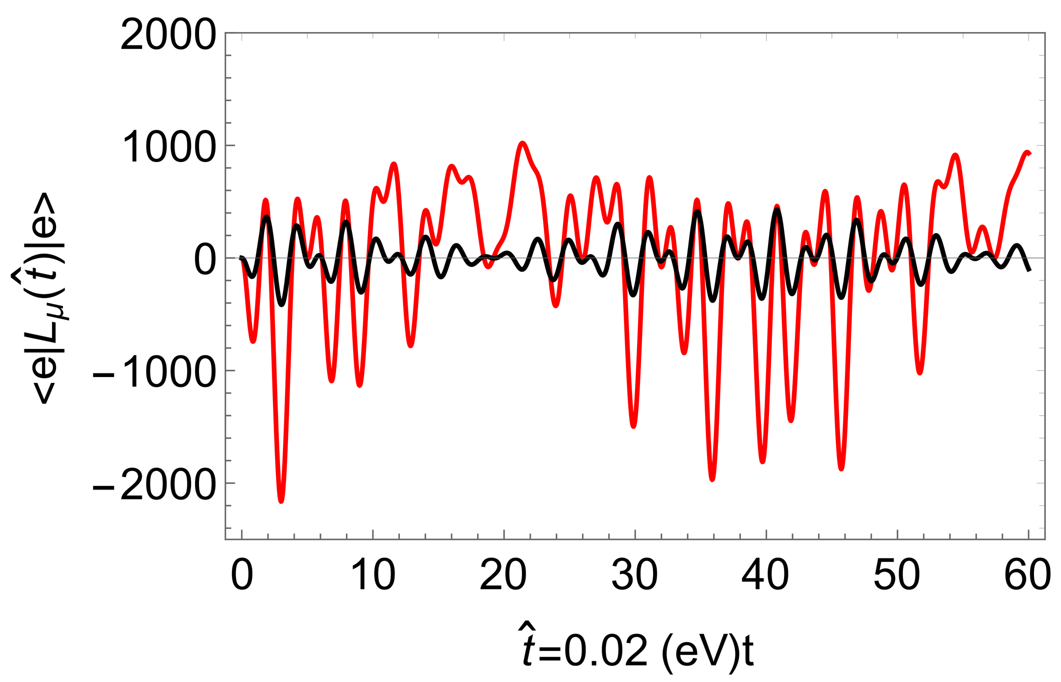


IH case

Time evolution of the expectation value for lepton family number, both $\langle e|L_e(\hat{t})|e\rangle$ and $\langle e|L_\mu(\hat{t})|e\rangle$, with a specific initial condition. $|e\rangle$ stands for the electron neutrino with momentum $|\mathbf{q}| = 0.2$ (eV) and Majorana phases, $(\alpha_{21}, \alpha_{31}) = (\frac{\pi}{2}, \frac{\pi}{3})$. Dimensionless time $\hat{t} = 600 = 20 \times 10^{-12}$ sec.



The electron number



The muon number

Time dependence of electron family number $\langle e|L_e(\hat{t})|e\rangle$ (left figure) and muon family number $\langle e|L_\mu(\hat{t})|e\rangle$ (right figure) for NH case. The neutrino momentum $|\mathbf{q}| = 0.0002$ (eV) and the lightest neutrino mass is 0.01 (eV). Majorana phases $(\alpha_{21}, \alpha_{31})$: black $(0, 0)$ and red $(\frac{\pi}{2}, \frac{\pi}{3})$ lines.

CONCLUSION

- We have formulated the time evolution of lepton family number operators under the presence of Majorana mass terms.
- The lepton family number is sensitive to Majorana phases and neutrino mass hierarchies, inverted and normal cases.
- Starting with an electron neutrino state, electron number can change its sign which implies neutrino to anti-neutrino transition occurs during their time evolution.

REFERENCES

[1] Y. Kawamura, Y. Matsuo, T. Morozumi, A. S. Adam, Y. Shimizu, Y. Tokunaga and N. Toyota, arXiv:2004.07664 [hep-ph].

CONTACT INFORMATION

Presenter Apriadi Salim Adam
Email apriadi.adam@gmail.com
Phone (+62) 822 3505 3354