QCD Calculation for hadronic B decays

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Heavy flavor physics is a very important hot topic in particle physics recently

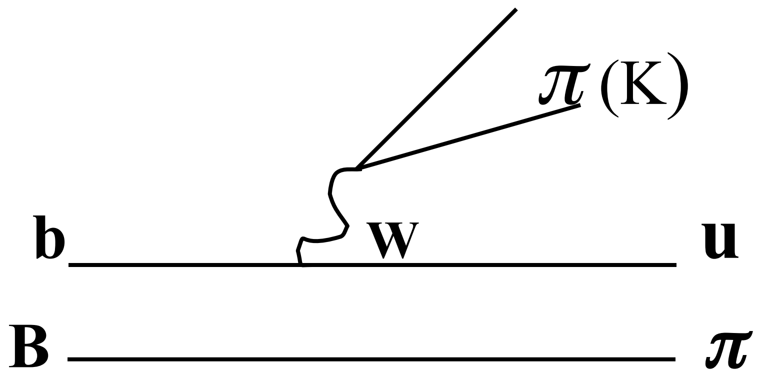
- People expect the new physics signal from the heaviest top quark, since it is very close to the electroweak breaking scale
- But there are too few data of the heavy top quark production
- Therefore beauty quark is our best chance for new physics signals, since it also belongs to the third family

Current Flavor Anomalies

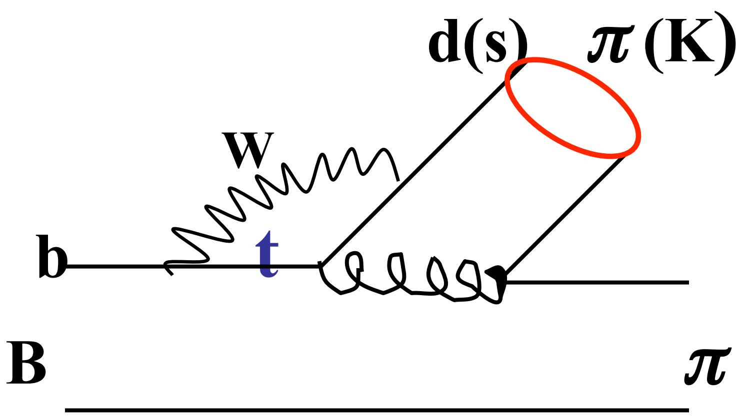
$\sim 3.5\sigma$	$(g-2)_\mu$ anomaly	
$\sim 3.5\sigma$	non-standard like-sign dimuon charge asymmetry	
$\sim 3.5\sigma$	enhanced $B \to D^{(*)}\tau\nu$ rates	$R_{D(*)}$
$\sim 3.5\sigma$	suppressed branching ratio of $B_s \to \phi\mu^+\mu^-$	
$\sim 3\sigma$	tension between inclusive and exclusive determination of $\lvert V_{ub}\rvert$	
$\sim 3\sigma$	tension between inclusive and exclusive determination of $\lvert V_{cb}\rvert$	
$2-3\sigma$	anomaly in $B \to K^*\mu^+\mu^-$ angular distributions	P_5'
$2-3\sigma$	SM prediction for ϵ'/ϵ below experimental result	
$\sim 2.5\sigma$	lepton flavor non-universality in $B \to K\mu^+\mu^-$ vs. $B \to Ke^+e^-$	R_K
$\sim 2.5\sigma$	non-zero $h \to \tau\mu$	

Rich physics in hadronic B decays

CP violation, FCNC, sensitive to new physics contribution…



The standard model describes interactions amongst quarks and leptons



In experiments, we can only observe hadrons

How can we test the standard model without solving QCD?

Perturbative calculations

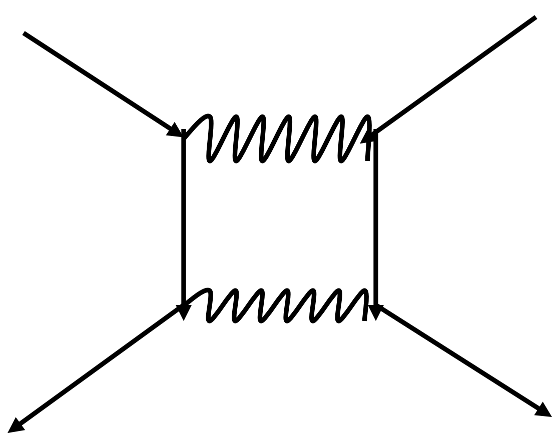
- In principle, all hadronic physics should be calculated by QCD
- In fact, you can always use QCD to calculate any process,

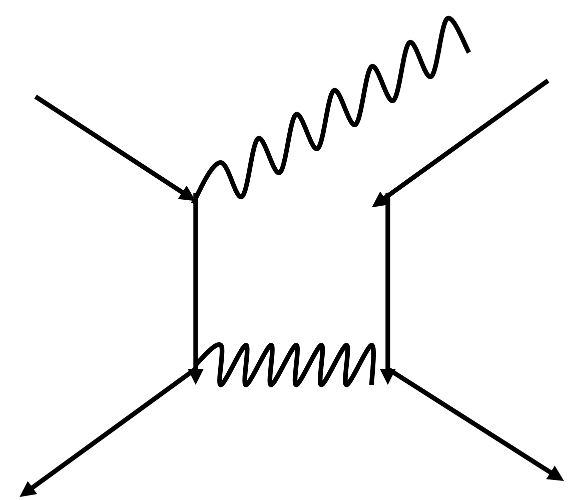
provided you can renormalize the infinities and do all order calculations.

- Perturbation calculation means order by order
- Involving loop diagrams
- Therefore divergences unavoidable

Divergences

- Ultraviolet divergences → renormalization
- Infrared divergences ? **Infrared divergence in virtual corrections should be canceled by real emission**
- In exclusive QCD processes → factorization





Factorization can only be proved in power expansion by operator product expansion. To achieve that, we need a hard scale Q

- In the certain order of 1/Q expansion, the hard dynamics characterized by Q factorize from the soft dynamics
- Hard dynamics is process-dependent, but calculable
- Soft dynamics are universal (process-independent) ⇨ predictive power of factorization theorem
- Factorization theorem holds up to all orders in α_s, but to certain power in 1/Q
- In B decays the hard scale Q is just the b quark mass

QCD-methods based on factorization work well for the leading power of $1/m_b$ expansion

collinear QCD Factorization approach
[Beneke, Buchalla, Neubert, Sachrajda, 99']

Perturbative QCD approach based on k_T factorization
[Keum, Li, Sanda, 00'; Lu, Ukai, Yang, 00']

Soft-Collinear Effective Theory
[Bauer, Pirjol, Stewart, 01']

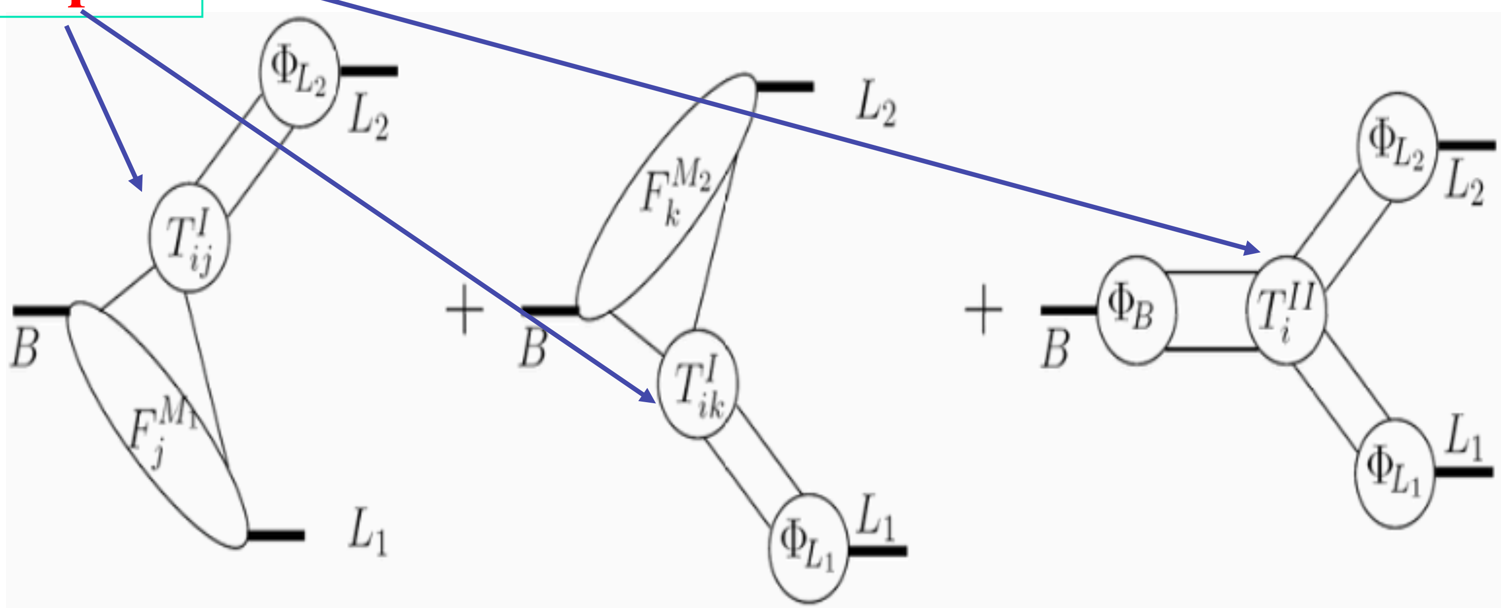
Unavailable for $1/m_b$ power corrections

- Work well for most of charmless B decays, except for ππ, πK puzzle etc.

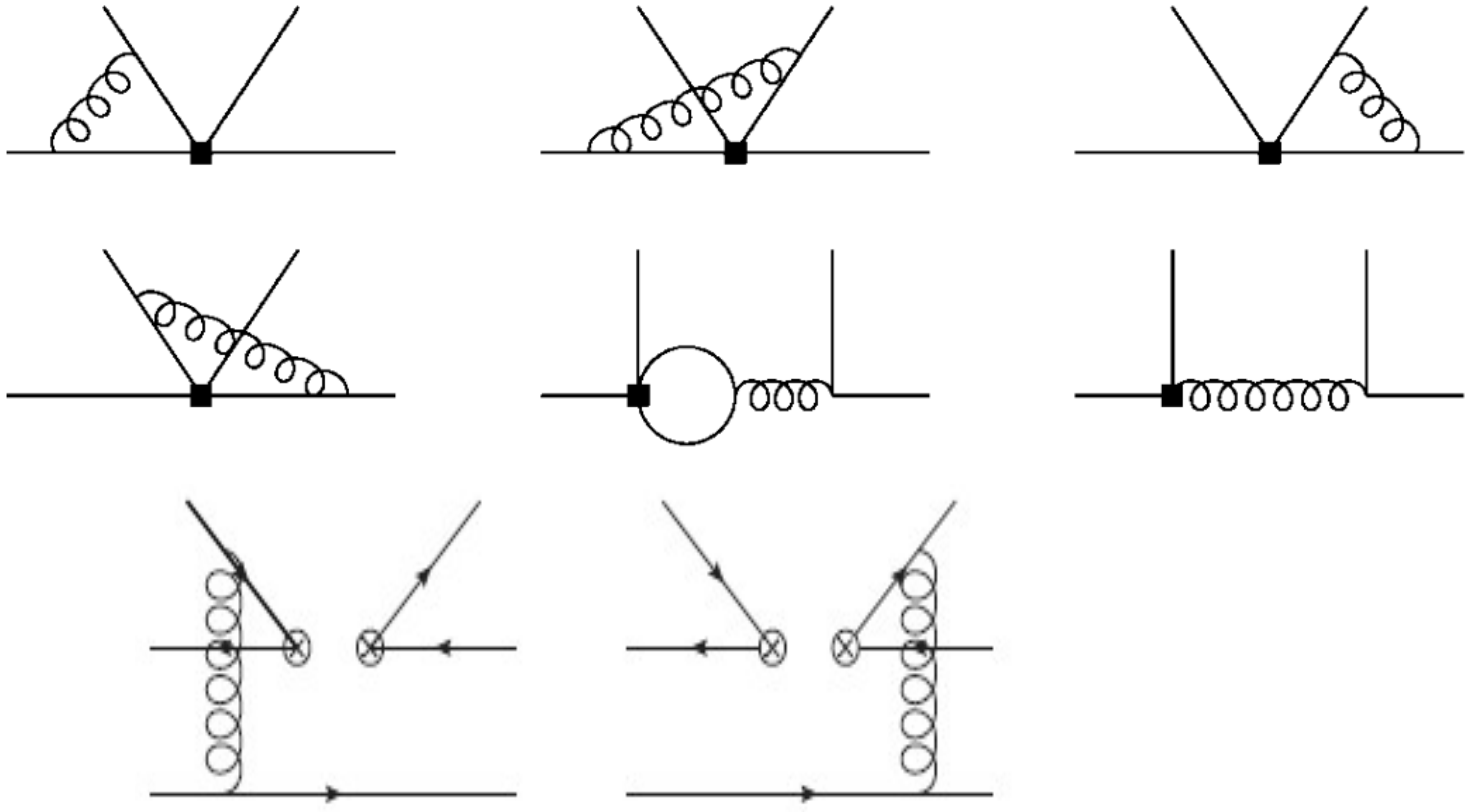
QCD factorization by BBNS: **PRL 83 (1999) 1914; NPB591 (2000) 313**

$$
\begin{aligned}
-\langle L_1 L_2 | Q_i | \bar{B} \rangle \;=\;& \sum_j F_j^{B\to L_1}(m_2^2) \int_0^1 du \, T_{ij}^I(u)\, \Phi_{L_2}(u) \\
&+ \sum_k F_k^{B\to L_2}(m_1^2) \int_0^1 dv \, T_{ik}^I(v)\, \Phi_{L_1}(v), \\
&+ \int_0^1 d\xi du dv \, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{L_1}(v)\, \Phi_{L_2}(u)
\end{aligned}
$$

hard part

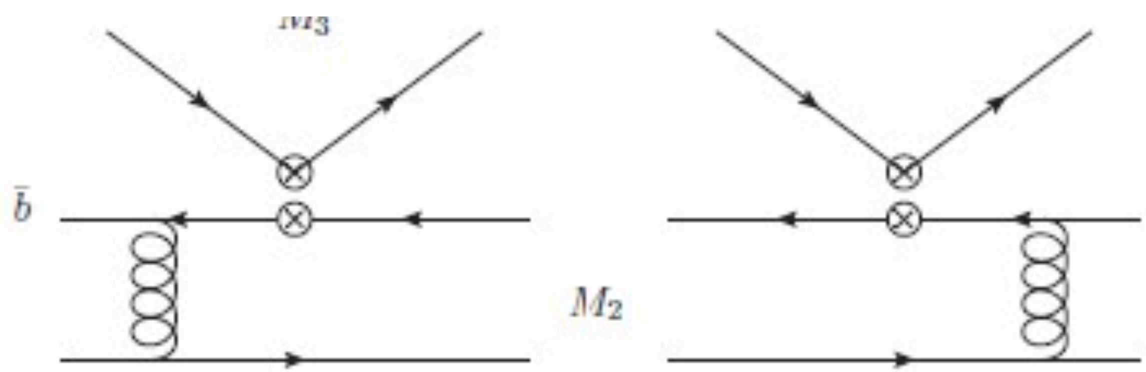


α_s corrections to the hard part T



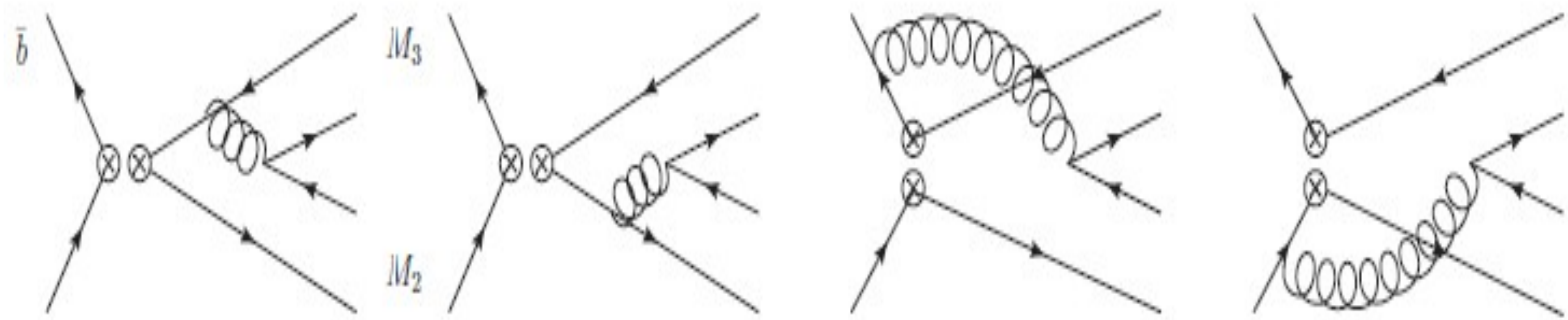
The missing diagrams, which contribute to the renormalization of decay constant or form factors

(e) (f)



Endpoint divergence appears in these calculations

The annihilation type diagrams are important to the source of strong phases



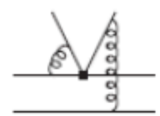
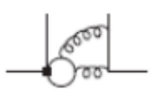
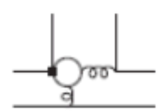
- **However, these diagrams are similar to the form factor diagrams, which have endpoint singularity, not perturbatively calculable.**
- **These divergences are not physical, can only be treated in QCDF as free parameters, which makes CP asymmetry not predictable:**

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

Current status of NNLO QCD factorization calculations

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} = \sum_{\text{terms}} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\mathrm{I}}(\mu_h,\mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2}\Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\mathrm{II}}(\mu_h,\mu_I)}_{1+\ldots} \star \underbrace{J^{\mathrm{II}}(\mu_I,\mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1}\Phi_{M_1}(\mu_s) \star f_{M_2}\Phi_{M_2}(\mu_s) \Big\}$$

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 in progress	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

Analyses of complete sets of final states

- **PP, PV**
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237
- **VV**
  MB, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237
- **AP, AV, AA**
  Cheng, Yang, 0709.0137, 0805.0329
- **SP, SV**
  Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403
- **TP, TV**
  Cheng, Yang, 1010.3309

Based on NLO hard-scattering functions.

Most phenomenological analysis based on NLO hard scattering functions

- **PP, PV**
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237
- **VV**
  MB, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237
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- **TP, TV**
  Cheng, Yang, 1010.3309

Amplitudes and phenomenology with NNLO results are under way

With more and more precise data, power corrections are urgently needed

□ Form factors from LCSRs with light hadron DAs

- $B \to \pi$: gradual improvements of OPE

[V.Belyaev, A.K., R.Rückl (1993)]; [V.Belyaev, V.M.Braun, A.K., R.Rückl (1995)]
[A.K., R.Rückl, S.Weinzierl, O.I.Yakovlev (1997)]; [E.Bagan,P.Ball, V.M. Braun (1997)]
[P.Ball, R.Zwicky (2004)]; [G.Duplancic, A.K., T.Mannel, B.Melic, N.Offen (2008)]
[A.K., T.Mannel, N.Offen, Y.M. Wang (2011)]
[A. Bharucha (2012)], [A.Rusov (2016)]

- $D \to \pi, K$: byproduct of $B \to \pi$ LCSR [A.K., C.Klein, T.Mannel, N.Offen, (2009)]

- $B \to K$, $B_s \to K$: $SU(3)$ breaking: $m_s \neq 0$, in kaon DAs, $f_{B_s} \neq f_B$

the latest update in [A.K., A.Rusov (2017)]

- $B_{(s)} \to \rho, \omega, K^*, \phi$: with (zero-width) ρ, K^* DAs

[P.Ball, R. Zwicky (2004)], [A.Bharucha, D.Straub, R.Zwicky (2015)]

- $\Lambda_b \to p$: with nucleon DAs, no NLO corrections yet

[AK, Th.Mannel, Ch. Klein, Y.-M. Wang (2011)]

B-meson DAs

- definition of two-particle DA in HQET:

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega)-\frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

$\oplus$ higher twists

- key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

  - possible to extract λ_B from $B \to \gamma\ell\nu_\ell$ using QCDF$\oplus$LCSR

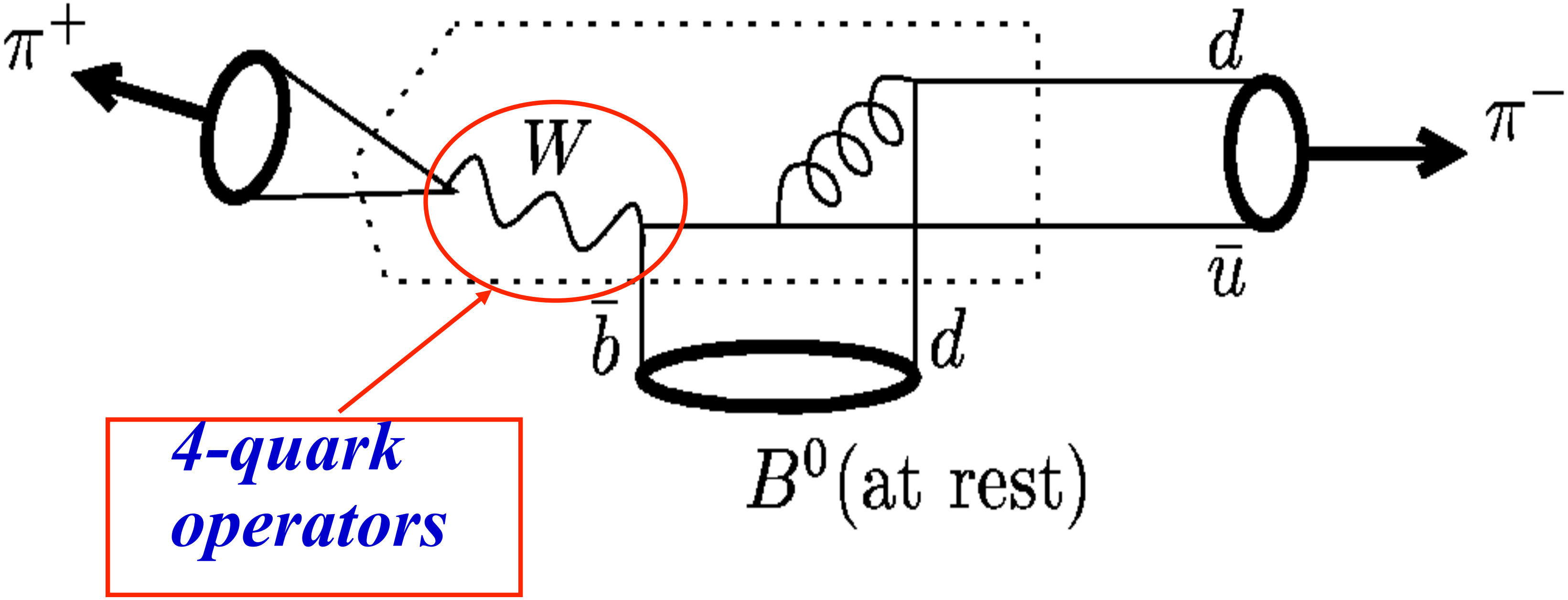
    [M.Beneke, V.M. Braun, Y.Ji, Y.B. Wei (2018)]

  - current limit from Belle measurement (2018): $\lambda_B > 240$ MeV
  - QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV

    [V.Braun, D.Ivanov, G.Korchemsky (2004)]

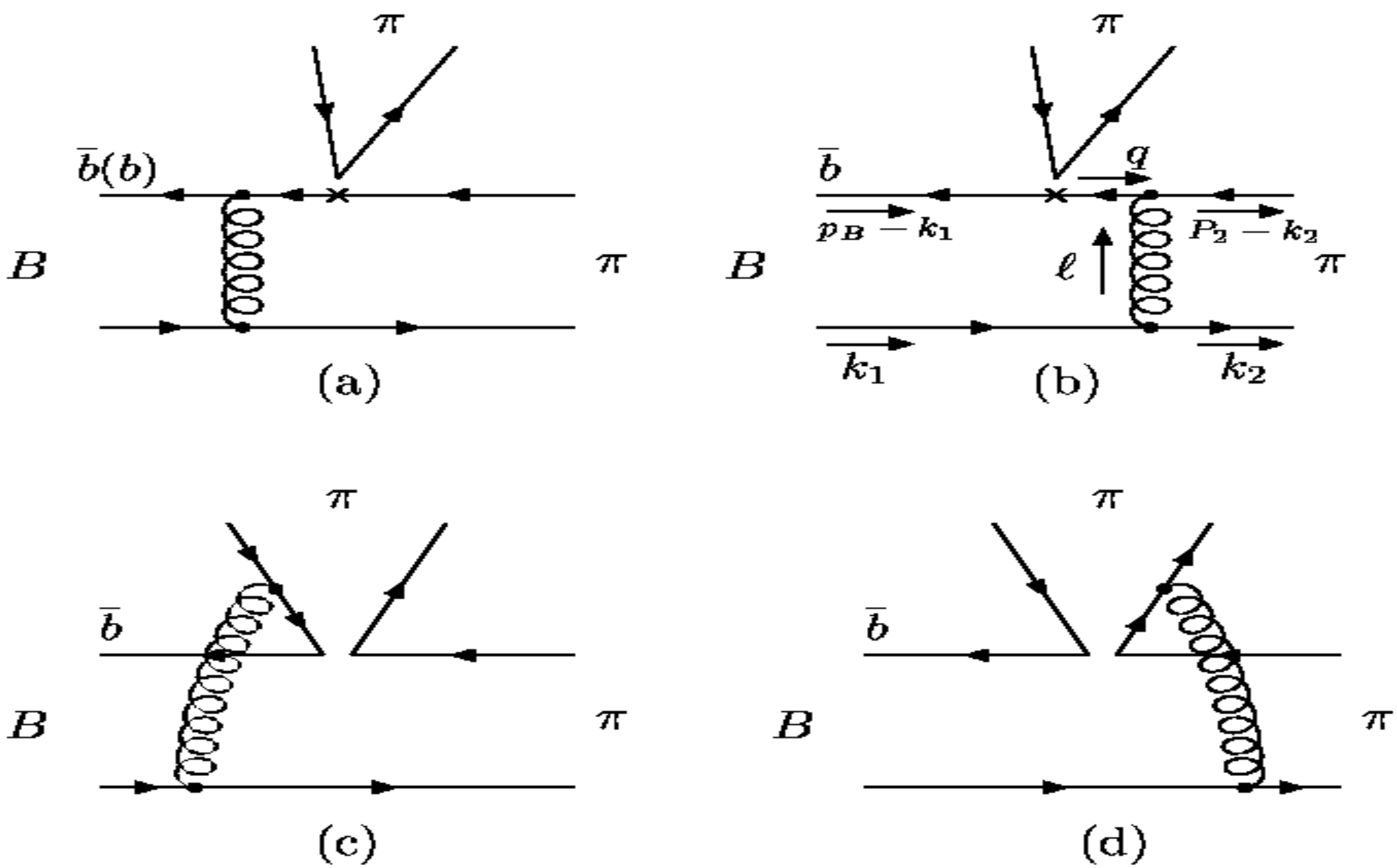
- higher twists DAs recently worked out [V. Braun, Y. Ji and A. Manashov (2017)]

Picture of PQCD Approach



Inside the dotted square, is the 6-quark interaction, which is perturbative calculable

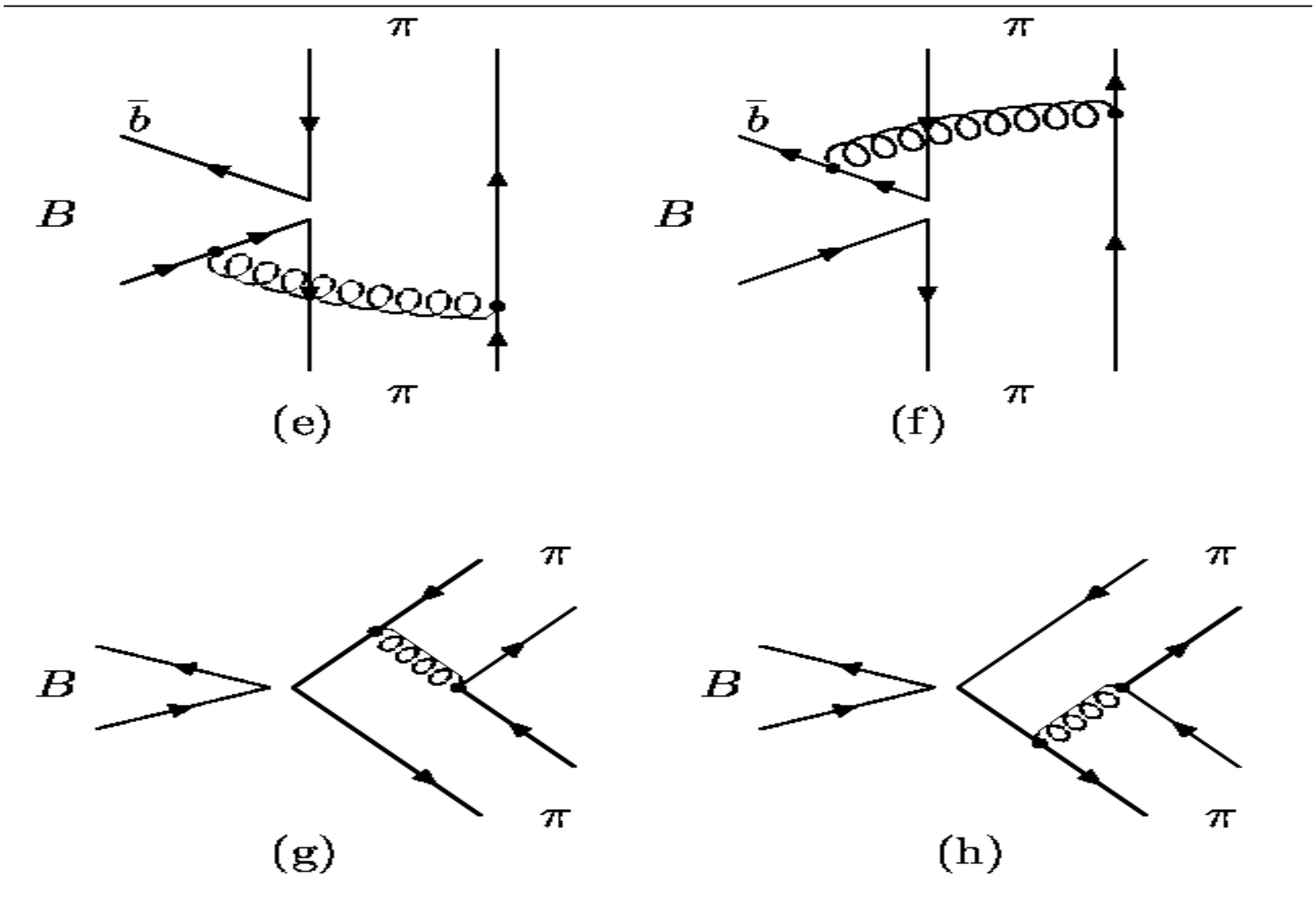
The leading order emission Feynman diagram in PQCD approach



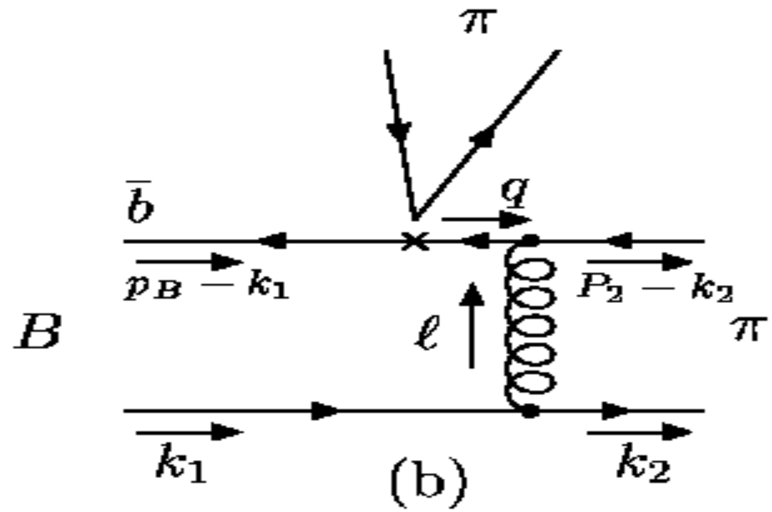
Form factor diagram

Hard scattering diagram

The leading order Annihilation type Feynman diagram in PQCD approach



Endpoint singularity



- **Gluon propagator**

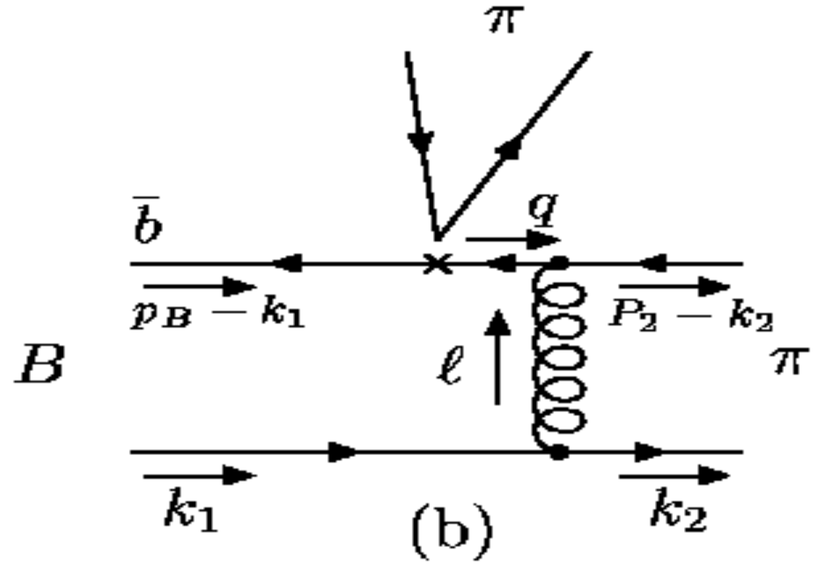
$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2}$$

- **x,y Integrate from 0→1, that is endpoint singularity**
- **The reason is that, one neglects the transverse momentum of quarks, which is not applicable at endpoint.**
- **If we pick back the transverse momentum, the divergence disappears**

$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2-(k_1^T-k_2^T)^2}$$

Endpoint singularity

- **It is similar for the quark propagator**

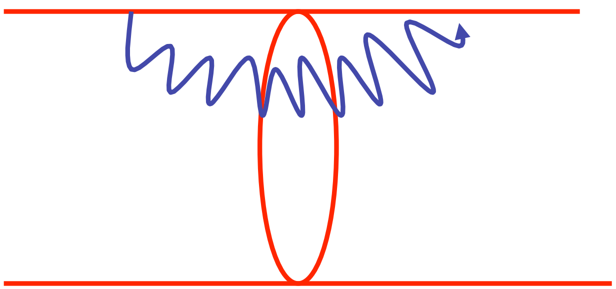


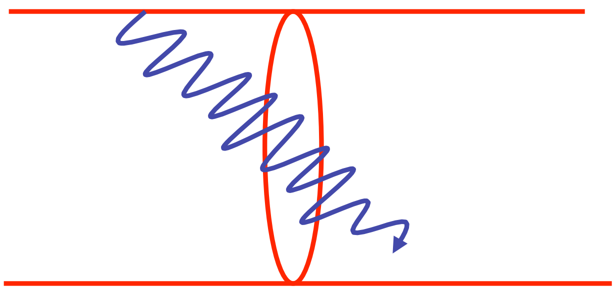
$$\int_0^1 \frac{1}{x} dx = \ln \frac{1}{\varepsilon}$$

$$\int_0^1 \frac{1}{x+k} dx dk = \int dk \left[\ln(x+k)\right]_0^1 = \int dk \left[\ln(1+k) - \ln k\right]$$

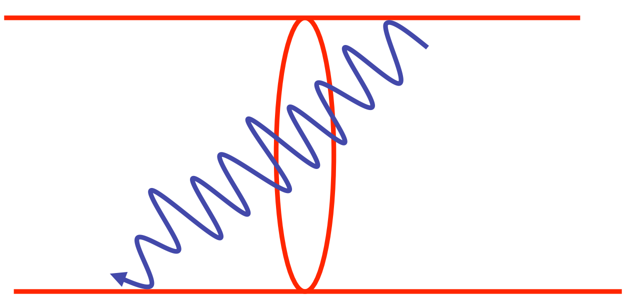
The logarithm divergence disappear if one has an extra dimension

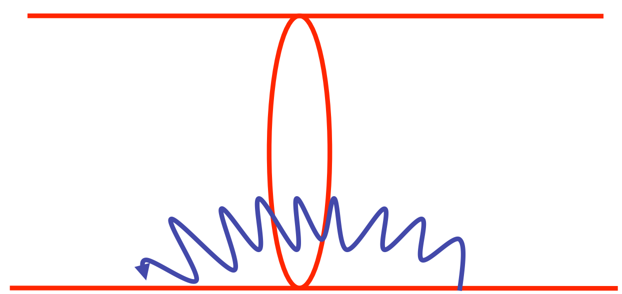
However, with transverse momentum, means one extra energy scale





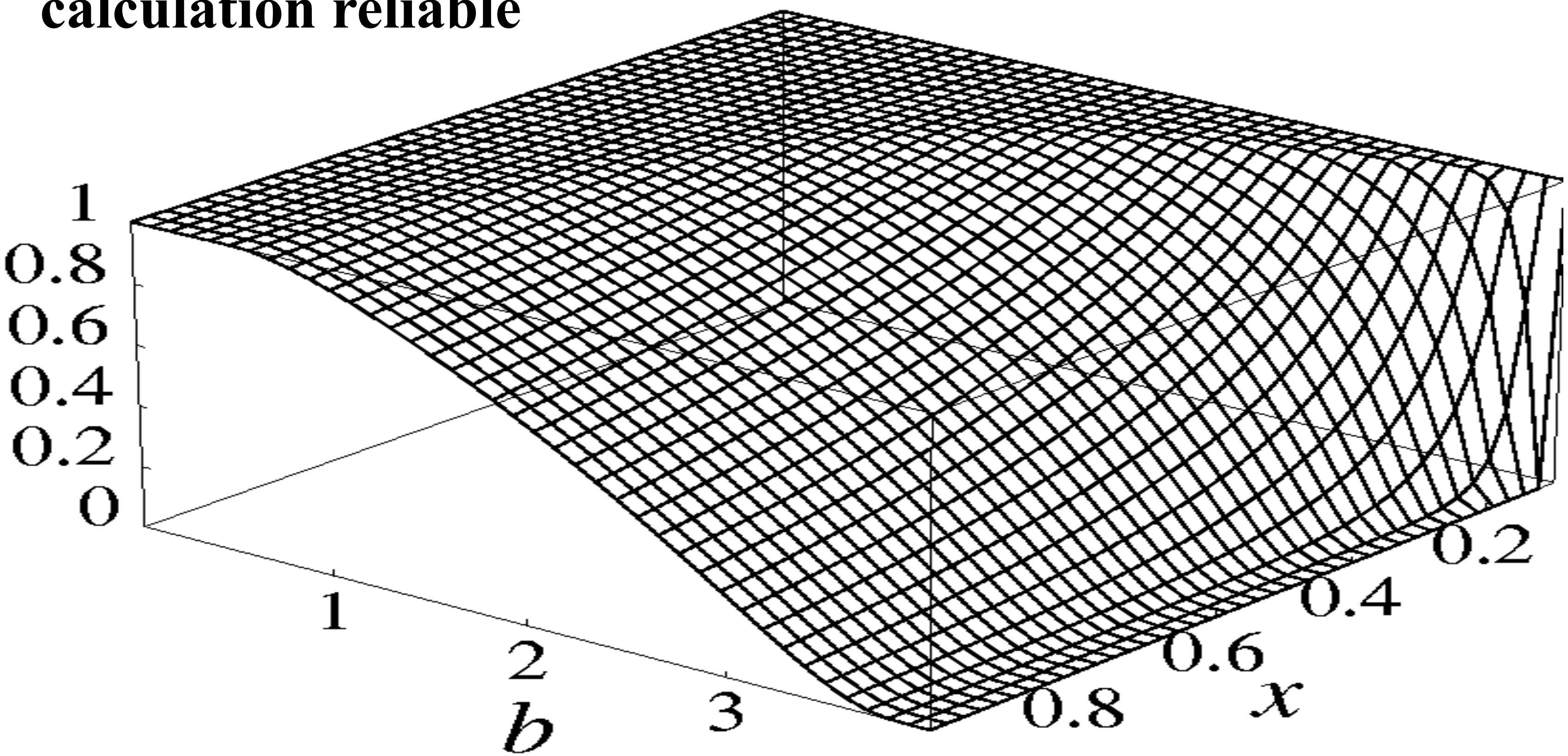
The overlap of Soft and collinear divergence will give double logarithm ln^2Pb, which is too big to spoil the perturbative expansion. We have to use renormalization group equation to resum all of the logs to give the so called Sudakov Form factor





Sudakov Form factor exp{–S(x,b)}

This factor exponentially suppresses the contribution at the endpoint (small k_T), makes our perturbative calculation reliable



CP Violation in $B \to \pi\pi(K)$ *(real prediction before exp.)*

CP(%)	FA	BBNS	PQCD *(2001)*	Exp *(2004)*
π^+K^-	+9±3	+5±9	–17±5	–11.5±1.8
π^0K^+	+8 ± 2	7 ±9	–13 ±4	+4 ± 4
π^+K^0	1. 7± 0. 1	1 ±1	–1. 0±0.5	–2 ±4
$\pi^+\pi^-$	–5±3	–6±12	+30±10	+37±10

CP Violation in $B \to \pi\pi(K)$

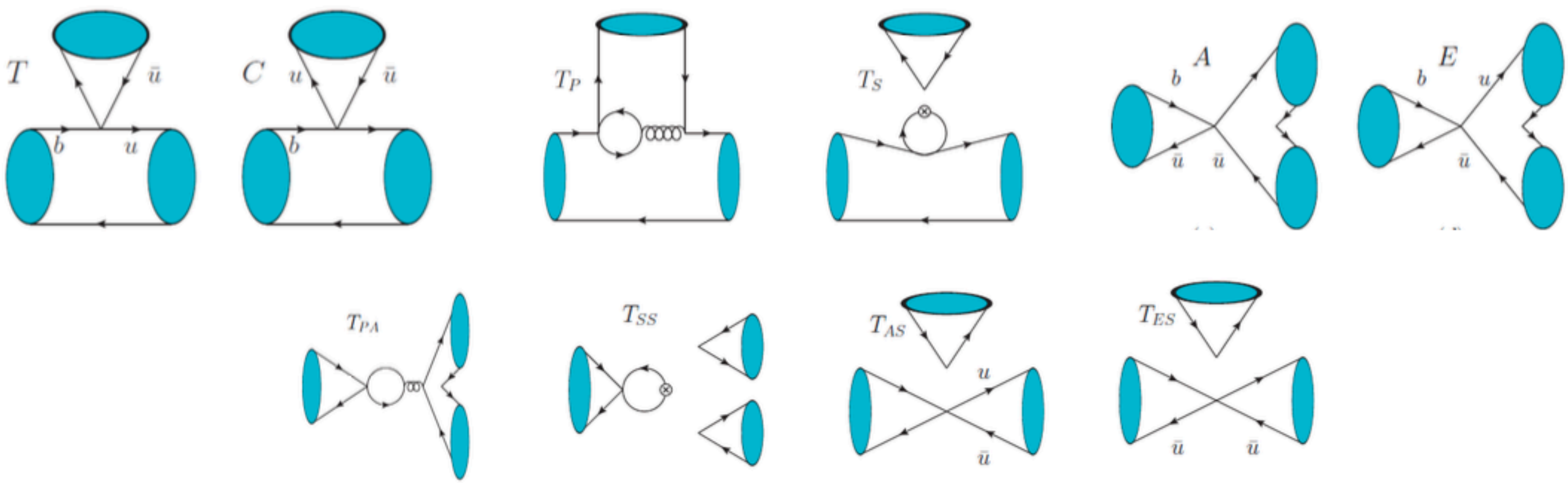
Including large annihilation fixed from exp.

CP(%)	FA	Cheng,HY	PQCD (2001)	Exp
π^+K^-	+9±3	-7.4 ± 5.0	–17±5	–9.7±1.2
π^0K^+	+8 ± 2	0.28±0.10	–13 ±4	4. 7 ± 2.6
π^+K^0	1. 7± 0. 1	4.9 ± 5.9	–1. 0±0.5	0.9 ±2.5
$\pi^+\pi^-$	–5±3	17 ± 1.3	+30±10	+38±7

The prove of factorization of QCD from electroweak is not needed

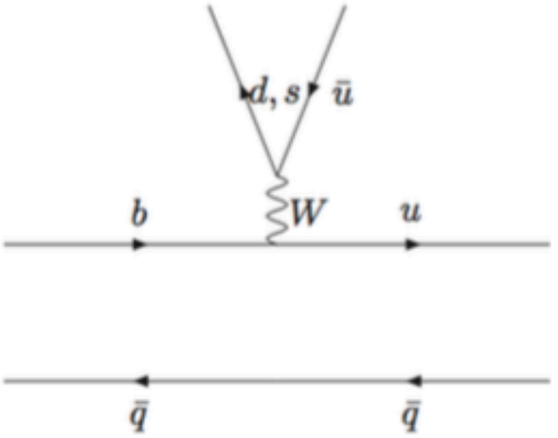
- Flavour SU(3) irreducible matrix elements
- Topological amplitudes (often with flavour SU(3) or SU(2))

$$T, C, P, P_{\mathrm{EW}}, S, E, A, \ldots$$



Tree topology diagram contributing to Charmless B decays

For the color favored diagram (T), it is proved factorization to all order of α_s expansion in soft-collinear effective theory,



(a) T

The decay amplitudes is just the decay constants and form factors times Wilson coeficients of four quark operators. The SU(3) breaking effect is automatically kept

$$T^{P_1P_2} = i\frac{G_F}{\sqrt{2}}V_{ub}V_{uq'}a_1(\mu)f_{p_2}(m_B^2 - m_{p_1}^2)F_0^{BP_1}(m_{p_2}^2),$$

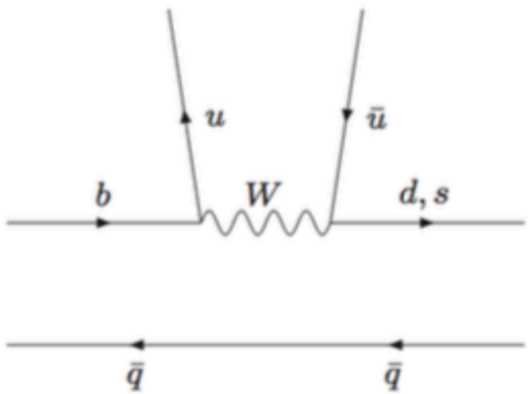
$$T^{PV} = \sqrt{2}G_F V_{ub}V_{uq'}a_1(\mu)f_V m_V F_1^{B-P}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

$$T^{VP} = \sqrt{2}G_F V_{ub}V_{uq'}a_1(\mu)f_P m_V A_0^{B-V}(m_P^2)(\varepsilon_V^* \cdot p_B),$$

No free parameter

For other diagrams, we extract the amplitude and strong phase from experimental data by χ^2 fit

We factorize out the decay constants and form factor to keep the SU(3) breaking effect



(b) C

For the color suppressed tree diagram (C), we have two kinds of contributions

$$C^{P_1P_2} = i\frac{G_F}{\sqrt{2}} V_{ub} V_{uq'} \chi^{C} e^{i\phi^{C}} f_{P_2} (m_B^2 - m_{p_1}^2) F_0^{BP_1}(m_{p_2}^2),$$

$$C^{PV} = \sqrt{2} G_F V_{ub} V_{uq'} \chi^{C'} e^{i\phi^{C'}} f_V m_V F_1^{B-P}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

$$C^{VP} = \sqrt{2} G_F V_{ub} V_{uq'} \chi^{C} e^{i\phi^{C}} f_P m_V A_0^{B-V}(m_P^2)(\varepsilon_V^* \cdot p_B),$$

Global Fit for all B→PP, VP and PV decays

with $\chi^2/\text{d.o.f} = 45.2/34 = 1.3.$

35 branching Ratios and 11 CP violation observations data are used for the fit

$$
\begin{aligned}
\chi^{C} &= 0.48 \pm 0.06, & \phi^{C} &= -1.58 \pm 0.08,\\
\chi^{C'} &= 0.42 \pm 0.16, & \phi^{C'} &= 1.59 \pm 0.17,\\
\chi^{E} &= 0.057 \pm 0.005, & \phi^{E} &= 2.71 \pm 0.13,\\
\chi^{P} &= 0.10 \pm 0.02, & \phi^{P} &= -0.61 \pm 0.02.\\
\chi^{P_C} &= 0.048 \pm 0.003, & \phi^{P_C} &= 1.56 \pm 0.08,\\
\chi^{P'_C} &= 0.039 \pm 0.003, & \phi^{P'_C} &= 0.68 \pm 0.08,\\
\chi^{P_A} &= 0.0059 \pm 0.0008, & \phi^{P_A} &= 1.51 \pm 0.09,
\end{aligned}
$$

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i^{\text{th}} - x_i}{\Delta x_i} \right)^2.$$

Large strong phase

Zhou, Zhang, Lyu and Lü, EPJC (2017) 77: 125

FAT global fit results of B→ VV decays

18 branching fractions, 20 polarization fractions, 6 relative phases, and 2 direct CP asymmetries as input

10 free parameters to be fitted

$$\chi_C^0 = 0.23 \pm 0.05, \quad \phi_C^0 = 0.48 \pm 0.29; \quad \chi_E^0 = 0.082 \pm 0.026, \quad \phi_E^0 = 1.69 \pm 0.16;$$

$$\chi_S^0 = 0.018 \pm 0.003, \quad \phi_S^0 = 1.29 \pm 0.22; \quad \chi_{P_A}^0 = 0.012 \pm 0.002, \quad \phi_{P_A}^0 = -0.07 \pm 0.18;$$

$$\chi_{P_A}^{\parallel,\perp} = 0.0098 \pm 0.0003, \quad \phi_{P_A}^{\parallel,\perp} = -0.21 \pm 0.09;$$

The $\chi^2/\text{d.o.f} = 82.0/(46-10)$ is 2.28.

Wang, Zhang, Li and Lü, EPJC (2017) 77: 333

SCET is an effective theory classifying operators and amplitudes

- Similar results are obtained with SU(3) symmetry

$$\zeta = (33.3 \pm 1.6) \times 10^{-2}, \qquad \zeta_J = (1.6 \pm 1.0) \times 10^{-2},$$
$$|A_{ccL}| = (38.1 \pm 1.1) \times 10^{-4}, \qquad \arg[A_{ccL}] = -0.29 \pm 0.11,$$
$$|A_{cc\parallel}| = (18.8 \pm 0.8) \times 10^{-4}, \qquad \arg[A_{cc\parallel}] = 1.98 \pm 0.18,$$
$$|A_{cc\perp}| = (17.1 \pm 0.7) \times 10^{-4}, \qquad \arg[A_{cc\perp}] = 2.11 \pm 0.18,$$

with $\chi^2/\text{d.o.f.} = 67.1/(35-8) = 2.5.$

Wang, Zhou, Li and Lü，PRD 96, 073004 (2017)

Summary/Challenges

- Hadronic B Decays are important in the test of standard model and search for signals of new physics.
- Power corrections in QCDF are very important that need to be calculated precisely
- Such as The annihilation type diagrams are the key point in explaining the K pi puzzle and large direct CP asymmetry found in B decays
- Next-to-leading order perturbative calculations in PQCD is needed to explain the more and more precise experimental data

Thanks

Comparison of different contributions from FAT and QCDF

Table 1 The amplitudes and strong phases of topological diagrams in the FAT corresponding to contributions in the QCDF. The topology A and P_E are neglected in the FAT. The electroweak penguin contributions of α_4^{EW}, β_3^{EW} and β_4^{EW} in the QCDF are also neglected in the FAT

Diagram	T	C	P_C	P(PP)	P_{EW}	E	A	P_A(PV)	P_E
FAT	a_1	$\chi^{C^{(\prime)}} e^{i\phi^{C^{(\prime)}}}$	$\chi^{P_C^{(\prime)}} e^{i\phi^{P_C^{(\prime)}}}$	$a_4(\mu) + \chi^P e^{i\phi^P} r_\chi$	$a_9(\mu)$	$\chi^E e^{i\phi^E}$	–	$-i\chi^{P_A} e^{i\phi^{P_A}}$	–
	–	$0.48e^{-1.58i}$	$0.048e^{1.56i}$	$-0.12e^{-0.24i}$	-0.009	$0.057e^{2.71i}$		$0.0059e^{-0.006i}$	
QCDF	α_1	α_2	α_3	α_4	α_3^{EW}	β_1	β_2	β_3	β_4
	–	$0.22e^{-0.53i}$	$0.011e^{2.23i}$	$-0.089e^{0.11i}$	$-0.009e^{0.04i}$	0.025	-0.011	-0.008	-0.003