QCD Calculation for hadronic B decays

Cai-Dian Lü

lucd@ihep.ac.cn
CFHEP, IHEP, Beijing
Heavy flavor physics is a very important hot topic in particle physics recently

- People expect the new physics signal from the heaviest top quark, since it is very close to the electroweak breaking scale
- But there are too few data of the heavy top quark production
- Therefore beauty quark is our best chance for new physics signals, since it also belongs to the third family
Current Flavor Anomalies

$\sim 3.5\sigma$ $(g - 2)_\mu$ anomaly

$\sim 3.5\sigma$ non-standard like-sign dimuon charge asymmetry

$\sim 3.5\sigma$ enhanced $B \to D^{(*)}\tau\nu$ rates

$\sim 3.5\sigma$ suppressed branching ratio of $B_s \to \phi\mu^+\mu^-$

$\sim 3\sigma$ tension between inclusive and exclusive determination of $|V_{ub}|$

$\sim 3\sigma$ tension between inclusive and exclusive determination of $|V_{cb}|$

$2 - 3\sigma$ anomaly in $B \to K^*\mu^+\mu^-$ angular distributions

$2 - 3\sigma$ SM prediction for $\epsilon'/\epsilon$ below experimental result

$\sim 2.5\sigma$ lepton flavor non-universality in $B \to K\mu^+\mu^-$ vs. $B \to Ke^+e^-$

$\sim 2.5\sigma$ non-zero $h \to \tau\mu$
Rich physics in hadronic $B$ decays

CP violation, FCNC, sensitive to new physics contribution…

The standard model describes interactions amongst quarks and leptons

In experiments, we can only observe hadrons

How can we test the standard model without solving QCD?
Perturbative calculations

- In principle, all hadronic physics should be calculated by QCD
- In fact, you can always use QCD to calculate any process, provided you can renormalize the infinities and do all order calculations.
- Perturbation calculation means order by order
- Involving loop diagrams
- Therefore divergences unavoidable
Divergences

- Ultraviolet divergences $\rightarrow$ renormalization
- Infrared divergences? **Infrared divergence in virtual corrections should be canceled by real emission**
- In exclusive QCD processes $\rightarrow$ factorization
Factorization can only be proved in power expansion by operator product expansion. To achieve that, we need a hard scale $Q$

- In the certain order of $1/Q$ expansion, the hard dynamics characterized by $Q$ factorize from the soft dynamics
- Hard dynamics is process-dependent, but calculable
- Soft dynamics are universal (process-independent)
- Predictive power of factorization theorem
- Factorization theorem holds up to all orders in $\alpha_s$, but to certain power in $1/Q$
- In $B$ decays the hard scale $Q$ is just the $b$ quark mass
QCD-methods based on factorization work well for the leading power of $1/m_b$ expansion

collinear QCD Factorization approach

Perturbative QCD approach based on $k_T$ factorization
[Keum, Li, Sanda, 00’; Lu, Ukai, Yang, 00’]

Soft-Collinear Effective Theory
[Bauer, Pirjol, Stewart, 01’]

Unavailable for $1/m_b$ power corrections

Daniels’ Work well for most of charmless B decays, except for $\pi\pi$, $\pi K$ puzzle etc.

\[-\langle L_1 L_2 | Q_i | \bar{B} \rangle = \sum_j F_j^{B \rightarrow L_1}(m_2^2) \int_0^1 du \, T_{ij}^I(u) \Phi_{L_2}(u) + \sum_k F_k^{B \rightarrow L_2}(m_1^2) \int_0^1 dv \, T_{ik}^I(v) \Phi_{L_1}(v), \]

\[+ \int_0^1 d\xi du dv \, T_{i}^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{L_1}(v) \Phi_{L_2}(u)\]
$\alpha_s$ corrections to the hard part T
The missing diagrams, which contribute to the renormalization of decay constant or form factors

Endpoint divergence appears in these calculations
The annihilation type diagrams are important to the source of strong phases

- However, these diagrams are similar to the form factor diagrams, which have endpoint singularity, not perturbatively calculable.
- These divergences are not physical, can only be treated in QCDF as free parameters, which makes CP asymmetry not predictable:

\[
\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \quad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2} (X_A^{M_1})^2
\]
Current status of NNLO QCD factorization calculations

\[
\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\text{eff}} = \sum_{\text{terms}} C(\mu_h) \times \left\{ F_{B \rightarrow M_1} \times T^I(\mu_h, \mu_s) \ast f_{M_2} \Phi_{M_2}(\mu_s) \right. \\
+ f_B \Phi_B(\mu_s) \ast \left. \left[ T^{\Pi}(\mu_h, \mu_I) \ast J^{\Pi}(\mu_I, \mu_s) \right] \ast f_{M_1} \Phi_{M_1}(\mu_s) \ast f_{M_2} \Phi_{M_2}(\mu_s) \right\}
\]

<table>
<thead>
<tr>
<th>Status</th>
<th>2-loop vertex corrections ($T_{i}^I$)</th>
<th>1-loop spectator scattering ($T_{i}^{II}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trees</td>
<td>![Tree Diagram] [GB 07, 09] [Beneke, Huber, Li 09]</td>
<td>![Tree Diagram] [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]</td>
</tr>
<tr>
<td>Penguins</td>
<td>![Penguin Diagram] in progress</td>
<td>![Penguin Diagram] [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]</td>
</tr>
</tbody>
</table>
Analyses of complete sets of final states

- **PP, PV**
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237

- **VV**
  MB, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237

- **AP, AV, AA**
  Cheng, Yang, 0709.0137, 0805.0329

- **SP, SV**
  Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403

- **TP, TV**
  Cheng, Yang, 1010.3309

Based on NLO hard-scattering functions.
Most phenomenological analysis based on NLO hard scattering functions

- **PP, PV**
  
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237

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Amplitudes and phenomenology with NNLO results are under way

With more and more precise data, power corrections are urgently needed
Form factors from LCSRs with light hadron DAs

- $B \to \pi$: gradual improvements of OPE
  
  
  
  
  
  [A. Bharucha (2012)], [A.Rusov (2016)]

- $D \to \pi, K$: byproduct of $B \to \pi$ LCSR
  
  [A.K., C.Klein, T.Mannel, N.Offen, (2009)]

- $B \to K, B_s \to K$: $SU(3)$ breaking: $m_s \neq 0$, in kaon DAs, $f_{B_s} \neq f_B$
  
  the latest update in [ A.K., A.Rusov (2017)]

- $B(s) \to \rho, \omega, K^*, \phi$: with (zero-width) $\rho, K^*$ DAs
  

- $\Lambda_b \to \rho$: with nucleon DAs, no NLO corrections yet
  
  [AK, Th.Mannel, Ch. Klein, Y.-M. Wang (2011)]
**$B$-meson DAs**

- **definition of two-particle DA in HQET:**

  \[
  \langle 0| \bar{q}_{2\alpha}(x)[x,0]h_{\nu\beta}(0)|\bar{B}_v \rangle = - \frac{if_{Bm_B}}{4} \int_0^\infty d\omega e^{-i\omega v \cdot x} (1 + y) \left\{ \phi_+^B(\omega) - \frac{\phi_+^B(\omega) - \phi_-^B(\omega)}{2v \cdot x} \right\} \gamma_5 \right\}_{\beta\alpha}
  \]

  \(\oplus\) higher twists

- **key input parameter: the inverse moment**

  \[
  \frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}
  \]

  - possible to extract $\lambda_B$ from $B \rightarrow \gamma\ell\nu_\ell$ using QCDF $\oplus$ LCSR
  - current limit from Belle measurement (2018): $\lambda_B > 240$ MeV
  - QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV
  - higher twists DAs recently worked out
  - V. Braun, Y. Ji and A. Manashov (2017)
Picture of PQCD Approach

Inside the dotted square, is the 6-quark interaction, which is perturbative calculable.
The leading order emission Feynman diagram in PQCD approach.

Form factor diagram

Hard scattering diagram
The leading order Annihilation type Feynman diagram in PQCD approach

\[ B \rightarrow \bar{b} \pi \]

\[ \bar{b} \rightarrow B \pi \]

\[ B \rightarrow \bar{b} \pi \]

\[ \bar{b} \rightarrow B \pi \]
Endpoint singularity

- Gluon propagator
  \[ \frac{i}{(k_1 - k_2)^2} = \frac{i}{-2xym_B^2} \]

- x, y Integrate from 0 \rightarrow 1, that is endpoint singularity

- The reason is that, one neglects the transverse momentum of quarks, which is not applicable at endpoint.

- If we pick back the transverse momentum, the divergence disappears
  \[ \frac{i}{(k_1 - k_2)^2} = \frac{i}{-2xym_B^2 - (k_1^T - k_2^T)^2} \]
Endpoint singularity

- It is similar for the quark propagator

\[ \int_0^1 \frac{1}{x} \, dx = \ln \frac{1}{\varepsilon} \]

\[ \int_0^1 \frac{1}{x + k} \, dx \, dk = \int dk \left[ \ln(x + k) \right]_0^1 = \int dk \left[ \ln(1 + k) - \ln k \right] \]

The logarithm divergence disappear if one has an extra dimension
However, with transverse momentum, means one extra energy scale

The overlap of Soft and collinear divergence will give double logarithm \( \ln^2 P_b \), which is too big to spoil the perturbative expansion. We have to use renormalization group equation to resum all of the logs to give the so called **Sudakov Form factor**
Sudakov Form factor \( \exp\{-S(x,b)\} \)

This factor exponentially suppresses the contribution at the endpoint (small \( k_T \)), makes our perturbative calculation reliable.
CP Violation in $B \rightarrow \pi \pi (K)$
(real prediction before exp.)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\pi^+K^-$</td>
<td>+9±3</td>
<td>+5±9</td>
<td>−17±5</td>
<td>−11.5±1.8</td>
</tr>
<tr>
<td>$\pi^0K^+$</td>
<td>+8 ± 2</td>
<td>7 ±9</td>
<td>−13 ±4</td>
<td>+4 ± 4</td>
</tr>
<tr>
<td>$\pi^+K^0$</td>
<td>1.7± 0.1</td>
<td>1 ±1</td>
<td>−1.0±0.5</td>
<td>−2 ±4</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>−5±3</td>
<td>−6±12</td>
<td>+30±10</td>
<td>+37±10</td>
</tr>
</tbody>
</table>
including large annihilation fixed from exp.

<table>
<thead>
<tr>
<th>CP(%)</th>
<th>FA</th>
<th>Cheng, HY</th>
<th>PQCD (2001)</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+K^-$</td>
<td>$+9\pm3$</td>
<td>$-7.4 \pm 5.0$</td>
<td>$-17\pm5$</td>
<td>$-9.7\pm1.2$</td>
</tr>
<tr>
<td>$\pi^0K^+$</td>
<td>$+8 \pm 2$</td>
<td>$0.28\pm0.10$</td>
<td>$-13 \pm 4$</td>
<td>$4.7 \pm 2.6$</td>
</tr>
<tr>
<td>$\pi^+K^0$</td>
<td>$1.7\pm0.1$</td>
<td>$4.9 \pm 5.9$</td>
<td>$-1.0\pm0.5$</td>
<td>$0.9 \pm 2.5$</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$-5\pm3$</td>
<td>$17 \pm 1.3$</td>
<td>$+30\pm10$</td>
<td>$+38\pm7$</td>
</tr>
</tbody>
</table>
The prove of factorization of QCD from electroweak is not needed

- Flavour SU(3) irreducible matrix elements
- Topological amplitudes (often with flavour SU(3) or SU(2))

\[ T, C, P, \overline{P}_{EW}, S, E, A, \ldots \]
The decay amplitudes is just the decay constants and form factors times Wilson coefficients of four quark operators. The SU(3) breaking effect is automatically kept

\[ T^{P_1P_2} = i \frac{G_F}{\sqrt{2}} V_{ub} V_{uq'} a_1(\mu) f_{p_2} (m_B^2 - m_{p_1}^2) F_0^{BP_1}(m_{p_2}^2), \]

\[ T^{PV} = \sqrt{2} G_F V_{ub} V_{uq'} a_1(\mu) f_V m_V F_1^{B-P}(m_V^2) (\varepsilon^*_V \cdot p_B), \]

\[ T^{VP} = \sqrt{2} G_F V_{ub} V_{uq'} a_1(\mu) f_P m_V A_0^{B-V}(m_P^2) (\varepsilon^*_V \cdot p_B), \]
For other diagrams, we extract the amplitude and strong phase from experimental data by $\chi^2$ fit.

We factorize out the decay constants and form factor to keep the SU(3) breaking effect.

For the color suppressed tree diagram (C), we have two kinds of contributions:

\[
C^{P_1P_2} = i \frac{G_F}{\sqrt{2}} V_{ub} V_{q'd}^* \chi^{C} e^{i\phi^C} f_{P_2}(m_B^2 - m_{P_1}^2) F_0^{BP_1}(m_{P_2}^2),
\]

\[
C^{PV} = \sqrt{2} G_F V_{ub} V_{q'd}^* \chi^{C'} e^{i\phi^{C'}} f_v m_V F_1^{B-P}(m_V^2)(\epsilon_V^* \cdot p_B),
\]

\[
C^{VP} = \sqrt{2} G_F V_{ub} V_{q'd}^* \chi^{C} e^{i\phi^C} f_p m_V A_0^{B-V}(m_P^2)(\epsilon_V^* \cdot p_B),
\]
Global Fit for all $B \rightarrow PP$, VP and PV decays with $\chi^2$/d.o.f = 45.2/34 = 1.3.

35 branching Ratios and 11 CP violation observations data are used for the fit

$$\chi^C = 0.48 \pm 0.06, \quad \phi^C = -1.58 \pm 0.08,$$
$$\chi^{C'} = 0.42 \pm 0.16, \quad \phi^{C'} = 1.59 \pm 0.17, \quad \chi^2 = \sum_{i=1}^{n} \left( \frac{x_i^{\text{th}} - x_i}{\Delta x_i} \right)^2,$$
$$\chi^E = 0.057 \pm 0.005, \quad \phi^E = 2.71 \pm 0.13,$$
$$\chi^P = 0.10 \pm 0.02, \quad \phi^P = -0.61 \pm 0.02.$$
$$\chi^{P_C} = 0.048 \pm 0.003, \quad \phi^{P_C} = 1.56 \pm 0.08,$$
$$\chi^{P_C'} = 0.039 \pm 0.003, \quad \phi^{P_C'} = 0.68 \pm 0.08,$$
$$\chi^{P_A} = 0.0059 \pm 0.0008, \quad \phi^{P_A} = 1.51 \pm 0.09.$$

Large strong phase

Zhou, Zhang, Lyu and Lü, EPJC (2017) 77: 125
FAT global fit results of $B \rightarrow VV$ decays

18 branching fractions, 20 polarization fractions, 6 relative phases, and 2 direct CP asymmetries as input

10 free parameters to be fitted

\[ \chi_C^0 = 0.23 \pm 0.05, \quad \phi_C^0 = 0.48 \pm 0.29; \quad \chi_E^0 = 0.082 \pm 0.026, \quad \phi_E^0 = 1.69 \pm 0.16; \]
\[ \chi_S^0 = 0.018 \pm 0.003, \quad \phi_S^0 = 1.29 \pm 0.22; \quad \chi_{PA}^0 = 0.012 \pm 0.002, \quad \phi_{PA}^0 = -0.07 \pm 0.18; \]
\[ \chi_{PA}^{\parallel,\perp} = 0.0098 \pm 0.0003, \quad \phi_{PA}^{\parallel,\perp} = -0.21 \pm 0.09; \]

The $\chi^2$/d.o.f = 82.0/(46 − 10) is 2.28.

Wang, Zhang, Li and Lü, EPJC (2017) 77: 333
SCET is an effective theory classifying operators and amplitudes

- Similar results are obtained with SU(3) symmetry

\[ \zeta = (33.3 \pm 1.6) \times 10^{-2}, \quad \zeta_J = (1.6 \pm 1.0) \times 10^{-2}, \]
\[ |A_{ccL}| = (38.1 \pm 1.1) \times 10^{-4}, \quad \arg[A_{ccL}] = -0.29 \pm 0.11, \]
\[ |A_{cc\parallel}| = (18.8 \pm 0.8) \times 10^{-4}, \quad \arg[A_{cc\parallel}] = 1.98 \pm 0.18, \]
\[ |A_{cc\perp}| = (17.1 \pm 0.7) \times 10^{-4}, \quad \arg[A_{cc\perp}] = 2.11 \pm 0.18, \]

with \( \chi^2 / \text{d.o.f.} = 67.1 / (35 - 8) = 2.5. \)

Wang, Zhou, Li and Lü, PRD 96, 073004 (2017)
Hadronic B Decays are important in the test of standard model and search for signals of new physics.

Power corrections in QCDF are very important that need to be calculated precisely.

Such as The annihilation type diagrams are the key point in explaining the K pi puzzle and large direct CP asymmetry found in B decays.

Next-to-leading order perturbative calculations in PQCD is needed to explain the more and more precise experimental data.
Thanks
Comparison of different contributions from FAT and QCDF

<table>
<thead>
<tr>
<th>Diagram</th>
<th>T</th>
<th>C</th>
<th>$P_C$</th>
<th>P(PP)</th>
<th>$P_{EW}$</th>
<th>E</th>
<th>A</th>
<th>$P_A(PV)$</th>
<th>$P_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAT</td>
<td>$a_1$</td>
<td>$\chi^{C(t)} e^{i\phi C(t)}$</td>
<td>$\chi^{P_c(t)} e^{i\phi P_c(t)}$</td>
<td>$a_4(\mu) + \chi^P e^{i\phi P} r_\chi$</td>
<td>$a_9(\mu)$</td>
<td>$\chi^E e^{i\phi^E}$</td>
<td>$-i \chi^P A e^{i\phi^PA}$</td>
<td>$-$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-$</td>
<td>$0.48 e^{-1.58i}$</td>
<td>$0.048 e^{1.56i}$</td>
<td>$-0.12 e^{-0.24i}$</td>
<td>$-0.009$</td>
<td>$0.057 e^{2.71i}$</td>
<td>$0.0059 e^{-0.006i}$</td>
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<tr>
<td>QCDF</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$\alpha_4$</td>
<td>$\alpha_3^{EW}$</td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
<td>$\beta_3$</td>
<td>$\beta_4$</td>
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<tr>
<td></td>
<td>$-$</td>
<td>$0.22 e^{-0.53i}$</td>
<td>$0.011 e^{2.23i}$</td>
<td>$-0.089 e^{0.11i}$</td>
<td>$-0.009 e^{0.04i}$</td>
<td>$0.025$</td>
<td>$-0.011$</td>
<td>$-0.008$</td>
<td>$-0.003$</td>
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