Leading and subleading jet functions

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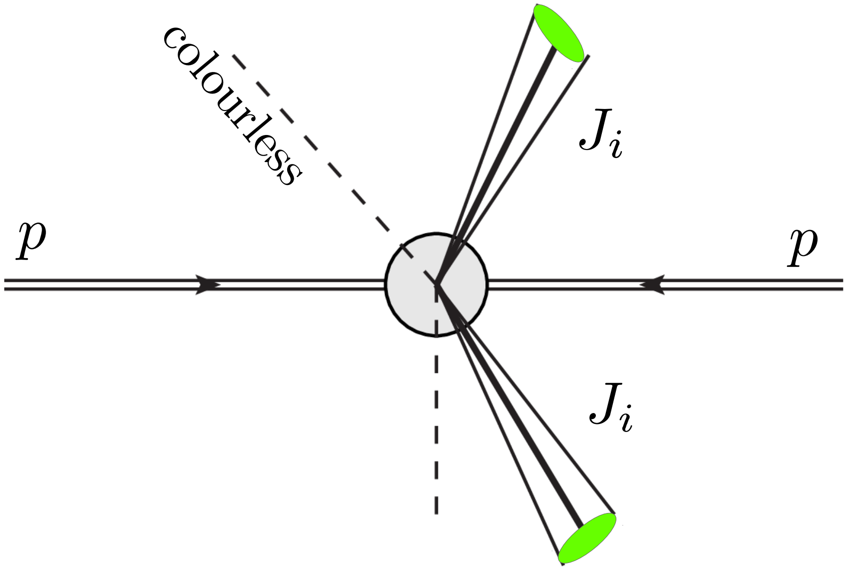
Jets & Jet functions

Jet functions J_i:

- Enter in factorization theorems with final state partons
- Describe collinear radiation of final state partons
- Various types already studied: Exclusive, semi-inclusive, ….

[Ellis, Vermilion, Walsh, Hornig, Lee: JHEP 1011 (2010) 101]

[Kang, Ringer, Vitev: JHEP 1610 (2016) 125]



Jets & Jet functions

Here we consider:

$$\frac{d\sigma}{dp_{T,J}} = \sum_{i\in\text{partons}} \int dz\, dp_{T,i}\, \frac{d\hat{\sigma}_{pp\to X+i}}{dp_{T,i}}\, J_i(z)\delta(p_{T,J} - z p_{T,i})$$

Energy fraction $z = p_{T,J}/p_{T,i}$

Specifically:

- Leading jet functions: energy fraction of leading jet
- Subleading jet functions: energy fraction of leading and subleading jets

Will use the jet functions to resum $(\alpha_s \ln R)^n$.

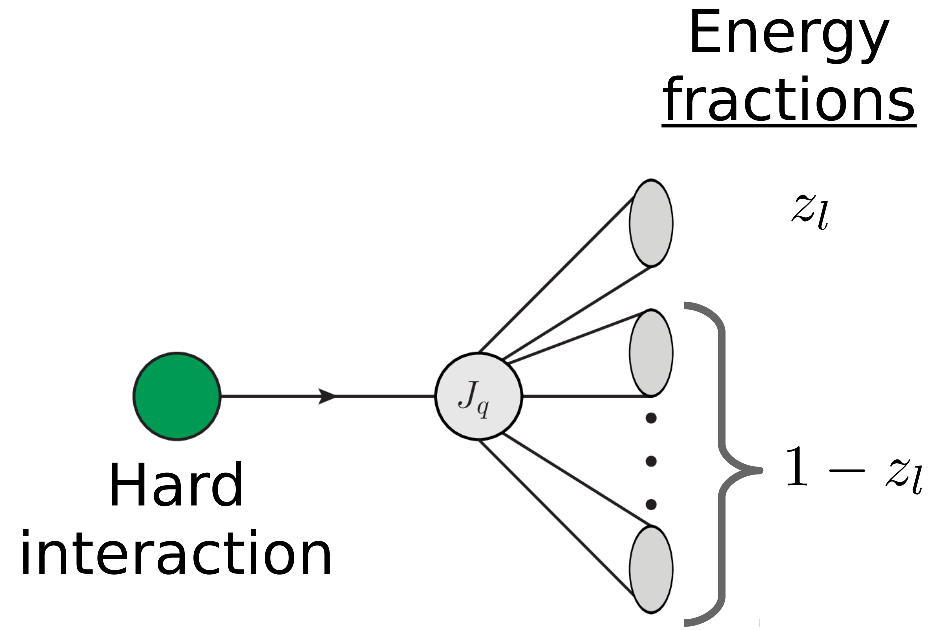
Leading Jet function

Final state partons fragment into jets.

Leading-jet function: probability that the ***hardest*** jet has energy fraction z_l.

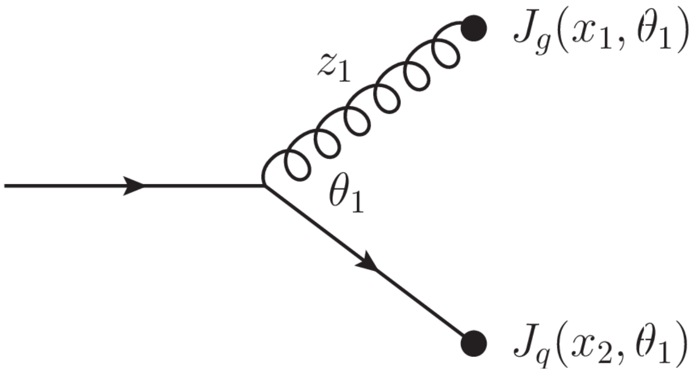
$$J_i(z_l,\mu) = \sum_{n=0} \left(\frac{\alpha_s}{\pi}\right)^n J_i^{(n)}(z_l,\mu)$$

$$J_q^{(0)}(z_l,\mu) = J_g^{(0)}(z_l,\mu) = \delta(1-z_l)$$



Leading Jet function: $\mathcal{O}(\alpha_s)$ calculation

$$
\begin{aligned}
J_q(z_l, \theta^{\max}) &= \delta(1-z_l) \\
&\quad + \frac{\alpha_s}{\pi} \int_0^1 dz_1\, P_{gq}(z_1) \int_R^{\theta^{\max}} \frac{d\theta_1}{\theta_1} \\
&\quad \times \int_0^1 dx_1\, J_g(x_1,\theta_1) \int_0^1 dx_2\, J_q(x_2,\theta_1) \\
&\quad \times \delta\Big(z_l - \max[z_1 x_1, (1-z_1)x_2]\Big)
\end{aligned}
$$

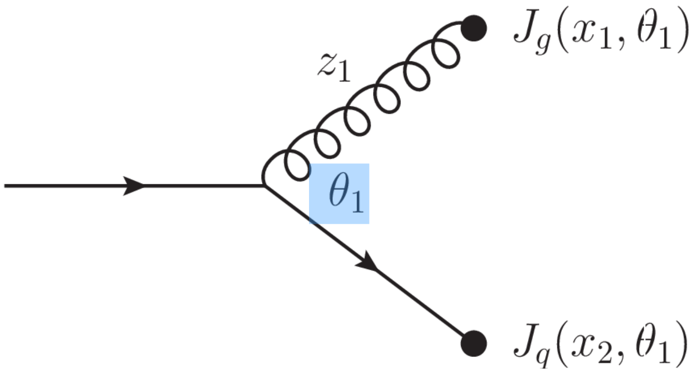


Using a parton shower picture, one can write down a recursive definition at LL.

[Elder, Procura, Thaler, Waalewijn, Zhou: JHEP 1706 (2017) 085]
[Waalewijn: Phys.Rev. D86 (2012) 094030]

Leading Jet function: $\mathcal{O}(\alpha_s)$ calculation

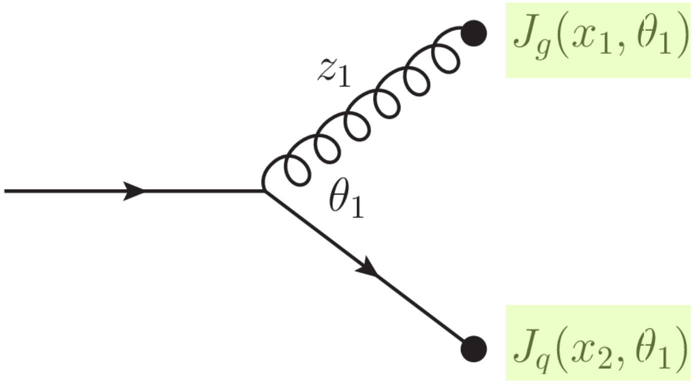
$$J_q(z_l, \theta^{\max}) = \delta(1 - z_l) + \frac{\alpha_s}{\pi} \int_0^1 dz_1\, P_{gq}(z_1) \int_R^{\theta^{\max}} \frac{d\theta_1}{\theta_1} \times \int_0^1 dx_1\, J_g(x_1, \theta_1) \int_0^1 dx_2\, J_q(x_2, \theta_1) \times \delta\Big(z_l - \max[z_1 x_1, (1 - z_1) x_2]\Big)$$



Usual angular integration. Emissions smaller than R finish in the same jet, so they do not affect z_l.

Leading Jet function: $\mathcal{O}(\alpha_s)$ calculation

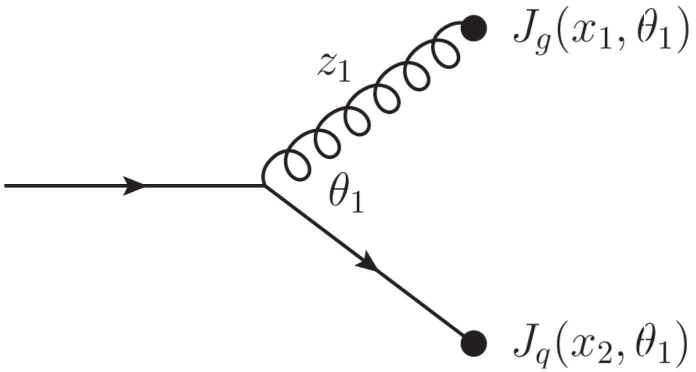
$$
\begin{aligned}
J_q(z_l, \theta^{\max}) &= \delta(1 - z_l) \\
&+ \frac{\alpha_s}{\pi} \int_0^1 dz_1 \, P_{gq}(z_1) \int_R^{\theta^{\max}} \frac{d\theta_1}{\theta_1} \\
&\times \int_0^1 dx_1 \, J_g(x_1, \theta_1) \int_0^1 dx_2 \, J_q(x_2, \theta_1) \\
&\times \delta\Big(z_l - \max[z_1 x_1, (1 - z_1) x_2]\Big)
\end{aligned}
$$



- Integrate over possible energy fractions of new ‘leading’ jets.
- At LL emissions are strongly angular ordered: θ_1 is new $\theta^{\max}$ for further emissions.

Leading Jet function: $\mathcal{O}(\alpha_s)$ calculation

$$
\begin{aligned}
J_q(z_l, \theta^{\max}) = \delta(1 - z_l) \\
&+ \frac{\alpha_s}{\pi} \int_0^1 dz_1 \, P_{gq}(z_1) \int_R^{\theta^{\max}} \frac{d\theta_1}{\theta_1} \\
&\times \int_0^1 dx_1 \, J_g(x_1, \theta_1) \int_0^1 dx_2 \, J_q(x_2, \theta_1) \\
&\times \delta\Big(z_l - \max[z_1 x_1, (1 - z_1) x_2]\Big)
\end{aligned}
$$



Ensures z_l corresponds to the jet with the highest energy fraction.

Leading Jet function: LL RGE

- Recursive definition allows us to obtain the LL RGE
- Jet function scale is set by $p_T R$, evolve to hard scale p_T
- Convenient to re-write angular integral in terms of p_T
- Dependence on the scale μ explicit (lower bound on θ integral)

Derivative w.r.t. μ gives

$$\mu \frac{d}{d\mu} J_q(z_l, \mu) = \frac{\alpha_s}{\pi} \int_0^1 dz_1\, P_{gq}(z_1) \int_0^1 dx_1\, J_g(x_1, \mu) \int_0^1 dx_2\, J_q(x_2, \mu) \\ \times \delta\Big(z_l - \max[z_1 x_1, (1 - z_1) x_2]\Big)$$

Difficult to solve analytically.
Can generate LL solutions to higher order for the jet function.

Leading Jet function: LL RGE Solutions

Using lower order solutions as input to the RGE, we obtain:

$$J_q^{(0)}(z_l,\mu) = \delta(1-z_l)$$

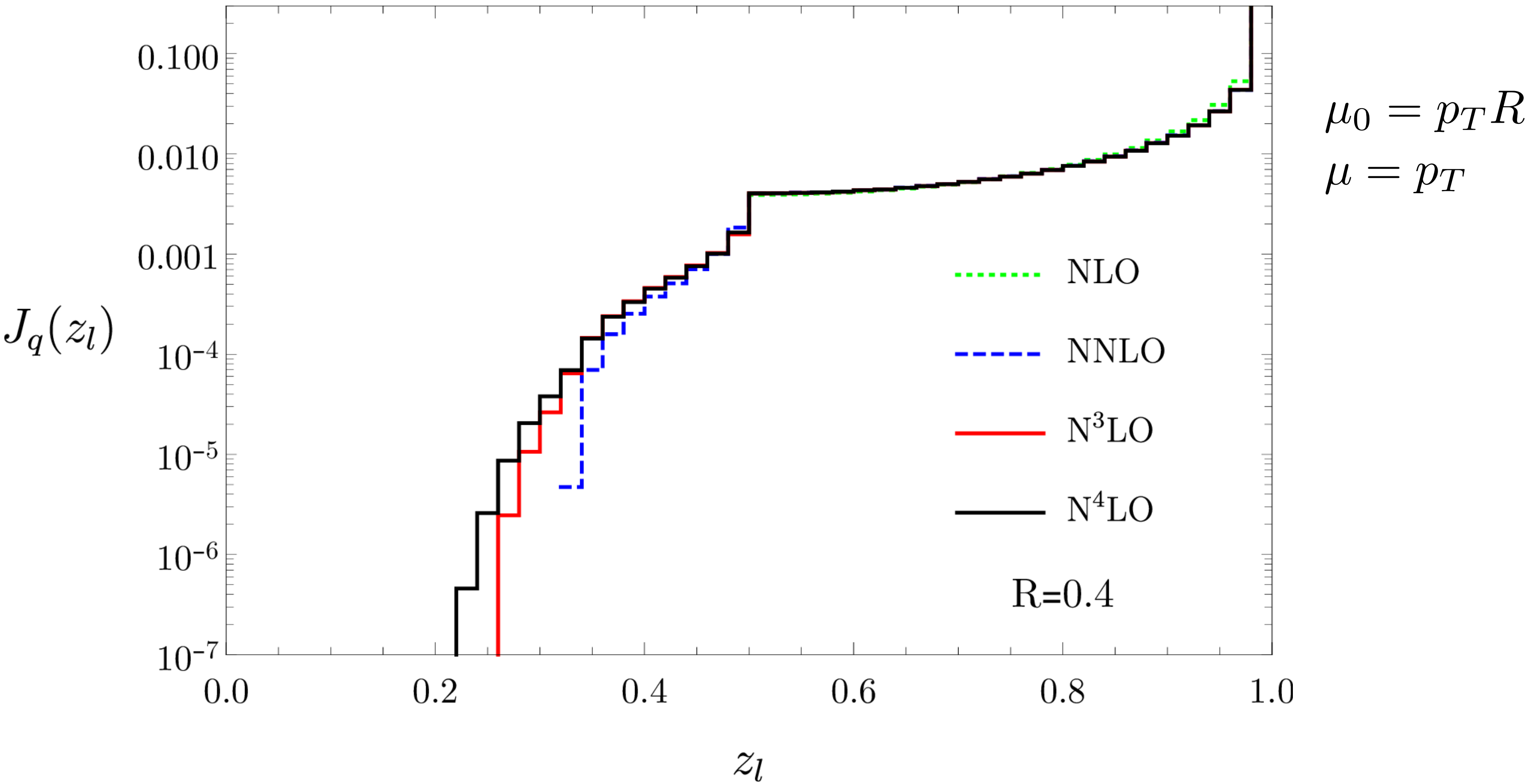
$$J_q^{(1)}(z_l,\mu) = \ln\frac{\mu}{\mu_0}\Big[P_{qq}(z_l)+P_{gq}(z_l)\Big]\Theta\left(z_l-\frac{1}{2}\right)$$

$$J_q^{(2)}(z_l,\mu) = \frac{1}{2}\ln^2\frac{\mu}{\mu_0}\Bigg\{\Theta\left(z_l-\frac{1}{2}\right)\left[\frac{\beta_0}{2}\left(P_{qq}(z_l)+P_{gq}(z_l)\right)+\int_z^1\frac{dx}{x}F_q\left(x,\frac{z_l}{x}\right)\right]$$
$$+\Theta\left(\frac{1}{2}-z_l\right)\Theta\left(z_l-\frac{1}{3}\right)\int_{\frac{1}{2}}^{\frac{z_l}{1-z_l}}dx\left[\frac{1}{x}F_q\left(x,\frac{z_l}{x}\right)+F_q(x,1-z_l)\right]\Bigg\}$$

$F_q(x,y)$ - products of splitting functions.

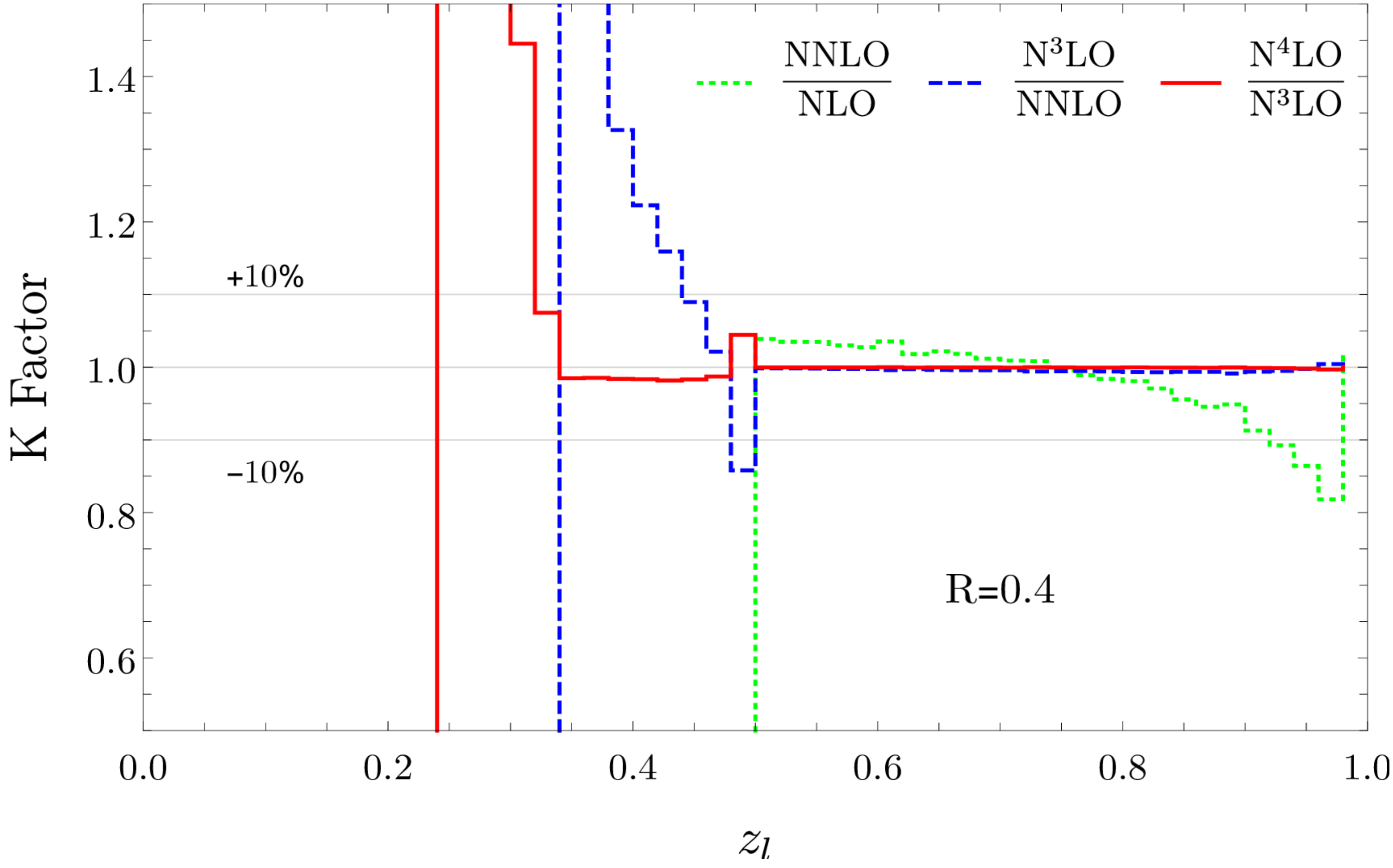
At $\mathcal{O}(\alpha_s)$ only one splitting – leading parton must have $z_l > 0.5$.
At $\mathcal{O}(\alpha_s^2)$ we have 3 partons. Implies $z_l > 1/3$.

Leading Jet function: LL RGE Solutions



Good convergence for $z_l > 0.5$. Slight shift to lower z_l.

Leading Jet function: LL RGE Solutions



$\mu_0 = p_T R$

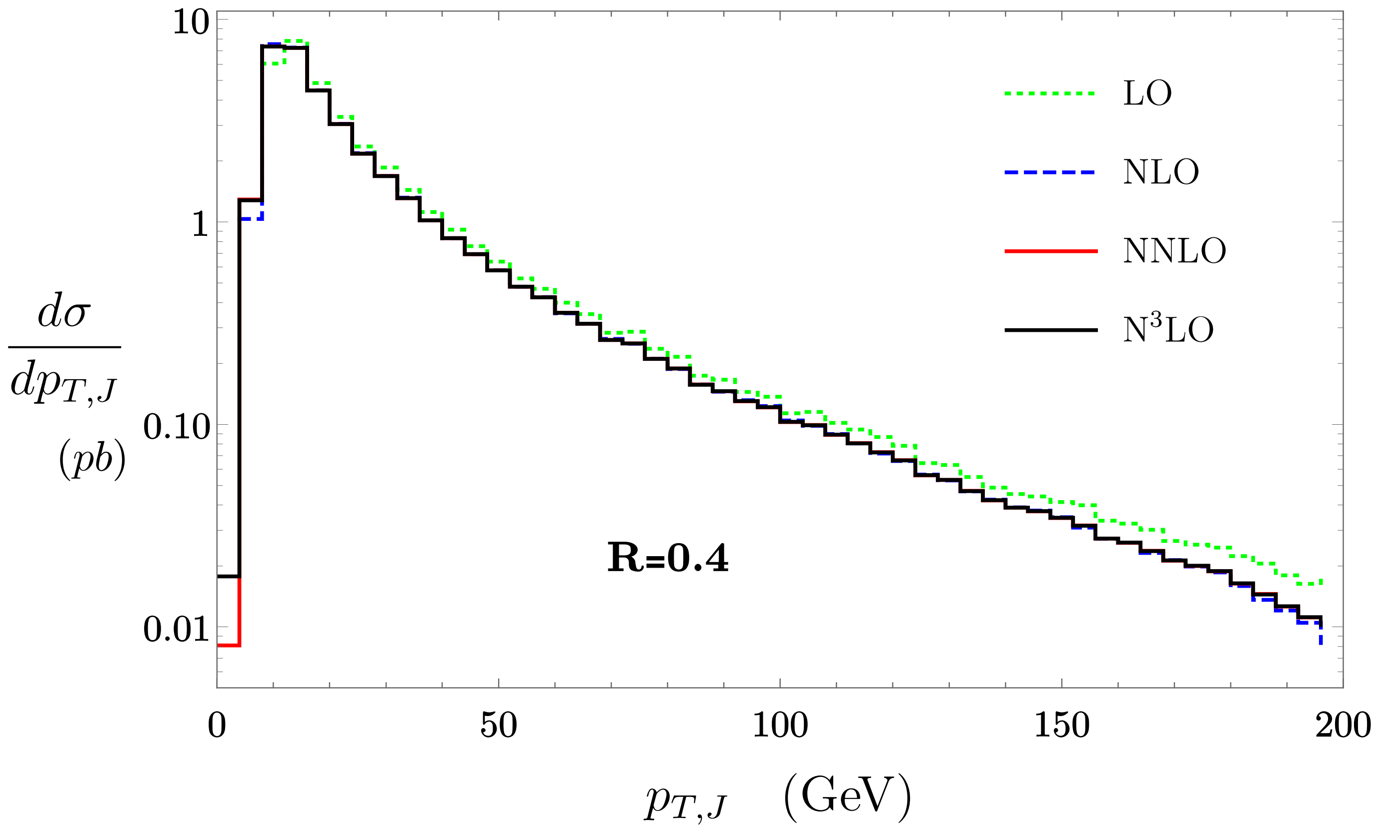
$\mu = p_T$

Good convergence for $z_l > 0.5$. Slight shift to lower z_l.

Leading jet function in H+1j production

Consider $pp \to H + j$ at LO. Inclusion of higher order $\ln R$ terms.

$$\frac{d\sigma_{pp\to H+1j}}{dp_{T,J}} = \sum_{i\in \text{partons}} \int_{p_{T,J}} \frac{dp_{T,i}}{p_{T,i}} \frac{d\hat{\sigma}_{pp\to H+i}}{dp_{T,i}} J_i\left(\frac{p_{T,J}}{p_{T,i}}\right)$$



Slight softening of the high $p_{T,J}$ tail.

Leading jet function in H+1j production

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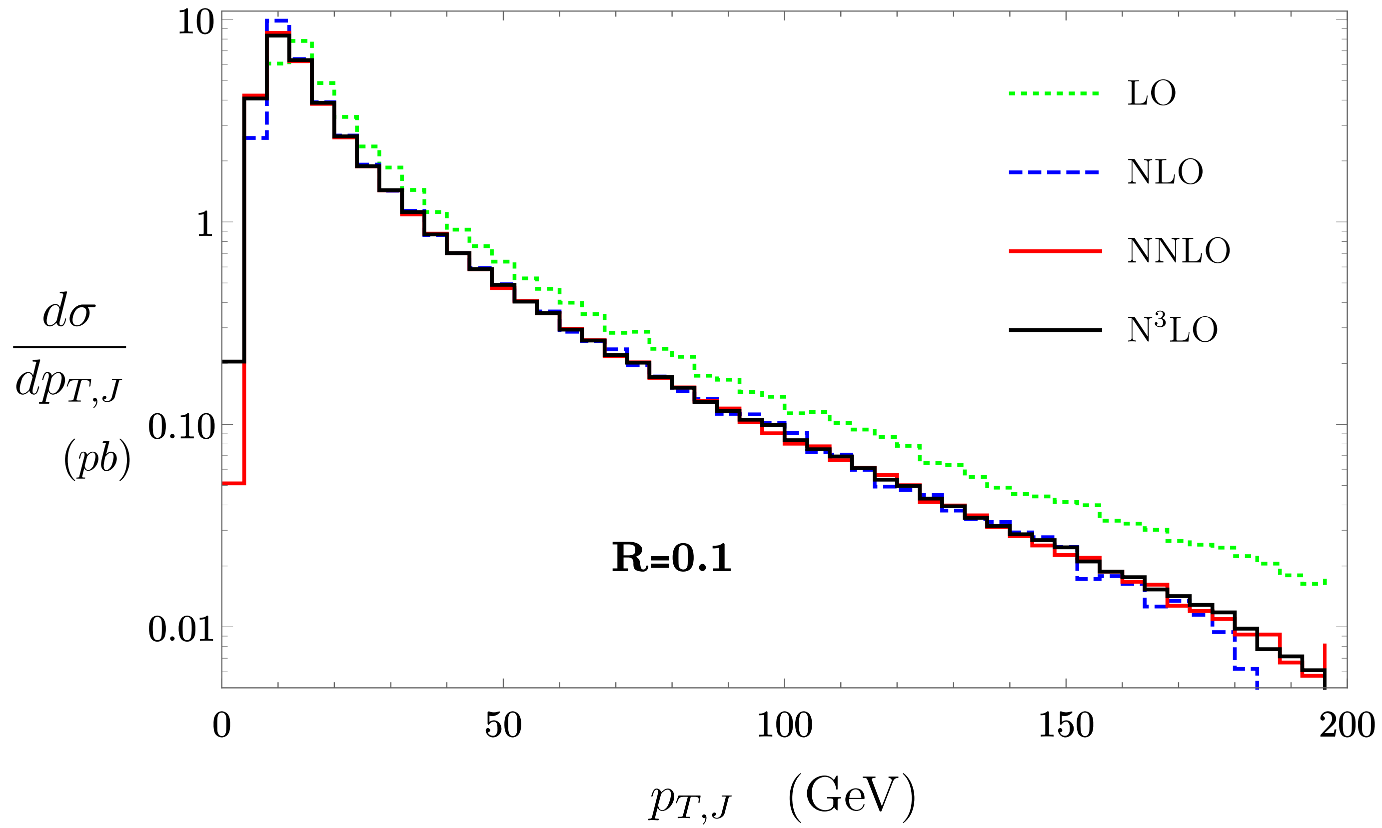
$$\frac{d\sigma_{pp\to H+1j}}{dp_{T,J}} = \sum_{i\in\text{partons}} \int_{p_{T,J}} \frac{dp_{T,i}}{p_{T,i}} \frac{d\hat{\sigma}_{pp\to H+i}}{dp_{T,i}} J_i\left(\frac{p_{T,J}}{p_{T,i}}\right)$$



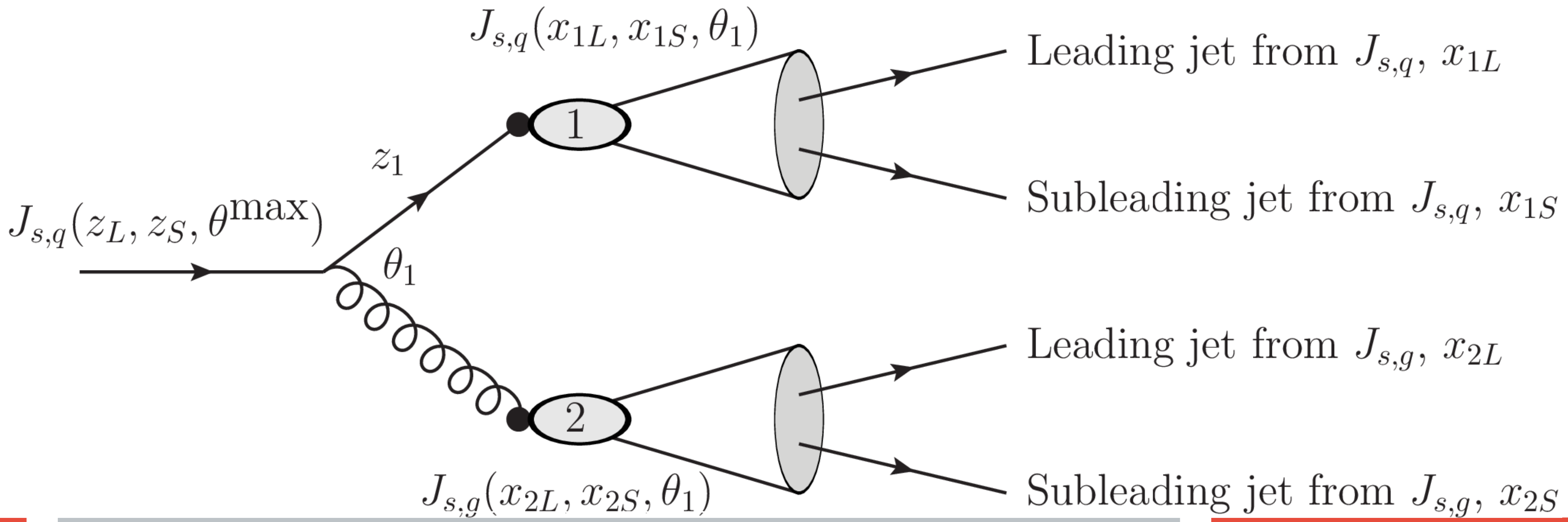
Slight softening of the high $p_{T,J}$ tail.

Subeading Jet function: LL Definition

A subleading-jet function can be used to track the energy fraction of the 2nd hardest jet.

Can be used to place jet vetos in exclusive production.

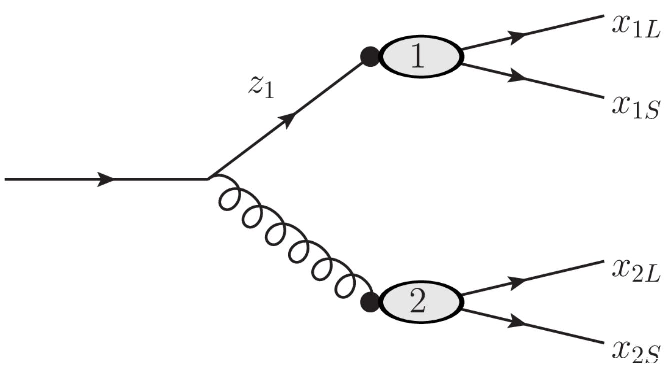
Use a parton shower picture to determine the evolution at LL.



Subeading Jet function: LL Definition

As before we derive an RG equation for our subleading jet function:

$$\mu\frac{d}{d\mu}J_i(z_L,z_S,\mu)=\frac{\alpha_s}{2\pi}\int_0^1 dz_1\,dx_{1L}\,dx_{2L}\int_0^{x_{1L}}dx_{1S}\int_0^{x_{2L}}dx_{2S}$$
$$\times F_i(x_{1L},x_{2L},x_{1S},x_{2S},z_1)$$
$$\times\Big\{\Theta\big(z_1x_{1L}-(1-z_1)x_{2L}\big)\delta\big(z_L-z_1x_{1L}\big)\delta\big(z_S-\max\left[z_1x_{1S},(1-z_1)x_{2L}\right]\big)$$
$$+\Theta\big((1-z_1)x_{2L}-z_1x_{1L}\big)\delta\big(z_L-(1-z_1)x_{2L}\big)\delta\big(z_S-\max\left[z_1x_{1L},(1-z_1)x_{2S}\right]\big)\Big\}$$



Subeading Jet function: LL Definition

As before we derive an RG equation for our subleading jet function:

$$\mu \frac{d}{d\mu} J_i(z_L, z_S, \mu) = \frac{\alpha_s}{2\pi} \int_0^1 dz_1\, dx_{1L}\, dx_{2L} \int_0^{x_{1L}} dx_{1S} \int_0^{x_{2L}} dx_{2S}$$

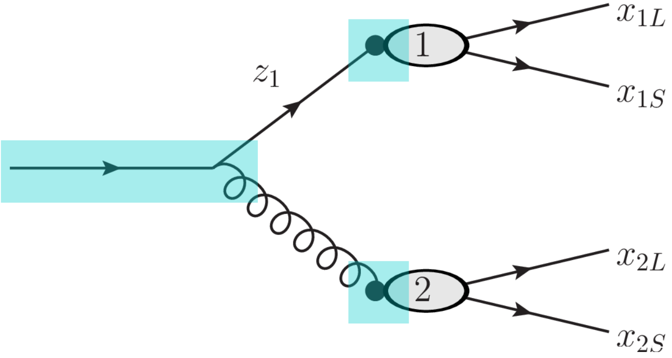
$$\times\, F_i(x_{1L}, x_{2L}, x_{1S}, x_{2S}, z_1)$$

$$\times \Big\{ \Theta\big(z_1 x_{1L} - (1-z_1)x_{2L}\big)\delta\big(z_L - z_1 x_{1L}\big)\delta\big(z_S - \max\left[z_1 x_{1S}, (1-z_1)x_{2L}\right]\big)$$

$$+\, \Theta\big((1-z_1)x_{2L} - z_1 x_{1L}\big)\delta\big(z_L - (1-z_1)x_{2L}\big)\delta\big(z_S - \max\left[z_1 x_{1L}, (1-z_1)x_{2S}\right]\big) \Big\}$$

Splitting & jet functions. E.g.

$$F_q = 2P_{qq}(z_1) J_{s,q}(x_{1L}, x_{1S}) J_{s,g}(x_{2L}, x_{2S})$$



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As before we derive an RG equation for our subleading jet function:

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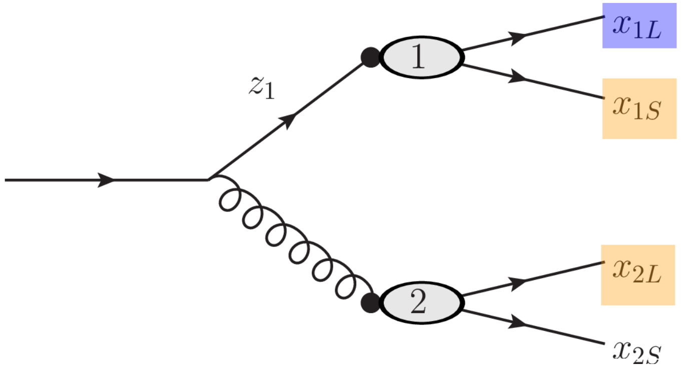
$$\times F_i(x_{1L}, x_{2L}, x_{1S}, x_{2S}, z_1)$$
$$\times \Big\{ \Theta\big(z_1 x_{1L} - (1-z_1)x_{2L}\big)\delta\big(z_L - z_1 x_{1L}\big)\delta\big(z_S - \max\left[z_1 x_{1S}, (1-z_1)x_{2L}\right]\big)$$
$$+ \Theta\big((1-z_1)x_{2L} - z_1 x_{1L}\big)\delta\big(z_L - (1-z_1)x_{2L}\big)\delta\big(z_S - \max\left[z_1 x_{1L}, (1-z_1)x_{2S}\right]\big) \Big\}$$

Accounting for the subleading jet.
Depends on where the leading jet is.



Subeading Jet function: RGE solutions

We can again derive LL results to higher orders in α_s :

$$J_{s,i}^{(0)} = \delta(1-z_L)\delta(z_S)$$

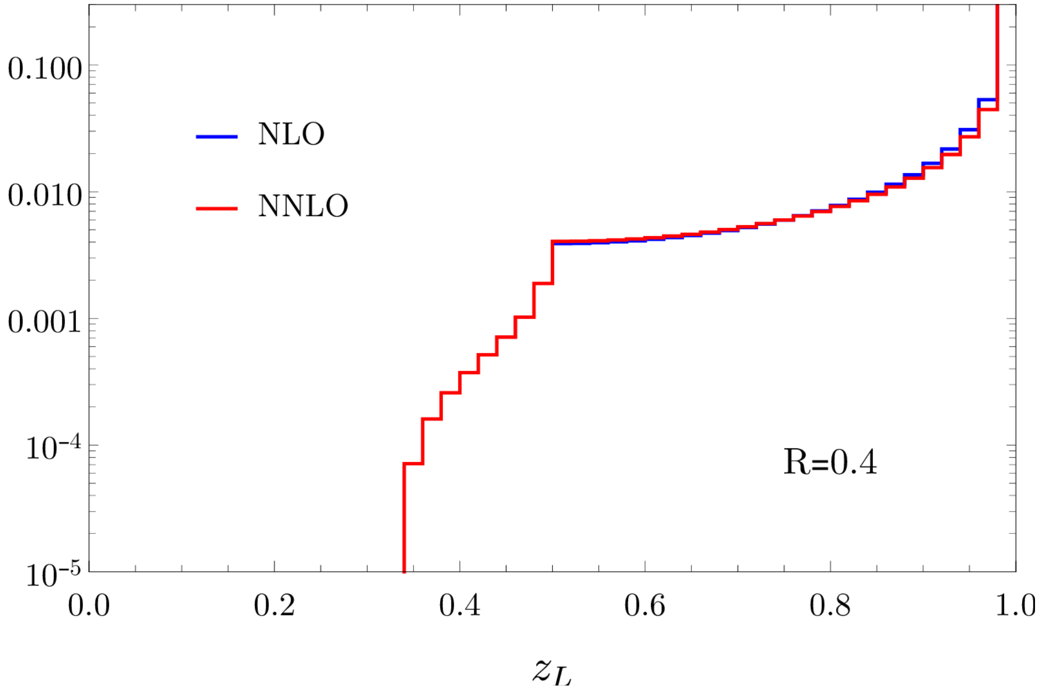
$$J_{s,q}^{(1)} = \left[P_{qq}(z_L) + P_{gq}(z_L)\right] \ln\frac{\mu}{\mu_0}\,\delta(z_S - (1-z_L))\Theta\left(z_L - \frac{1}{2}\right)$$

$$\begin{aligned} J_{s,q}^{(2)}(z_L, z_S) &= \frac{\beta_0}{4}\left[P_{qq}(z_L) + P_{gq}(z_L)\right]\delta(z_S - (1-z_L))\Theta\left(z_L - \frac{1}{2}\right)\ln^2\frac{\mu}{\mu_0} \\ &+ \frac{1}{2}\Theta\left(z_L - z_s\right)\Theta\left(z_L + 2z_S - 1\right)\Theta\left(1 - z_L - z_S\right)\Bigg\{ \\ &\frac{1}{1-z_L}F_{1,q}\left(z_L, \frac{z_S}{1-z_L}\right) + \frac{1}{1-z_S}F_{1,q}\left(z_S, \frac{z_L}{1-z_S}\right) \\ &+ \frac{1}{z_L+z_S}F_{2,q}\left(z_L+z_S, \frac{z_L}{z_L+z_S}\right)\Bigg\}\ln^2\frac{\mu}{\mu_0} \end{aligned}$$

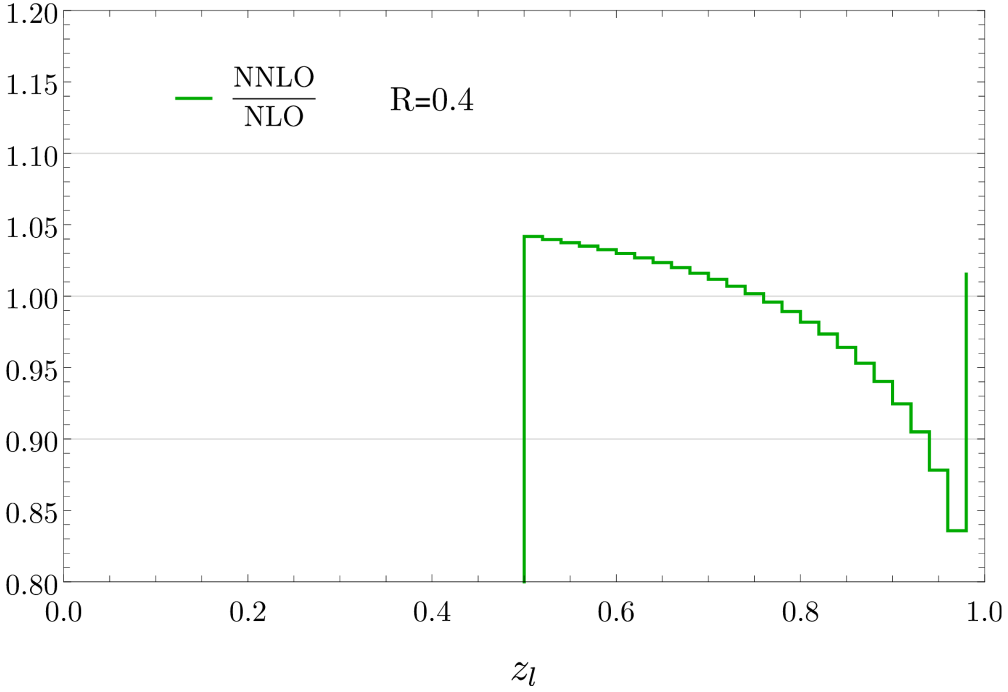
Subeading Jet function: LL RGE solutions

$$z_S \leq z_{\mathrm{v}} = 0.5$$

$J_{s,q}(z_L)$



K factor

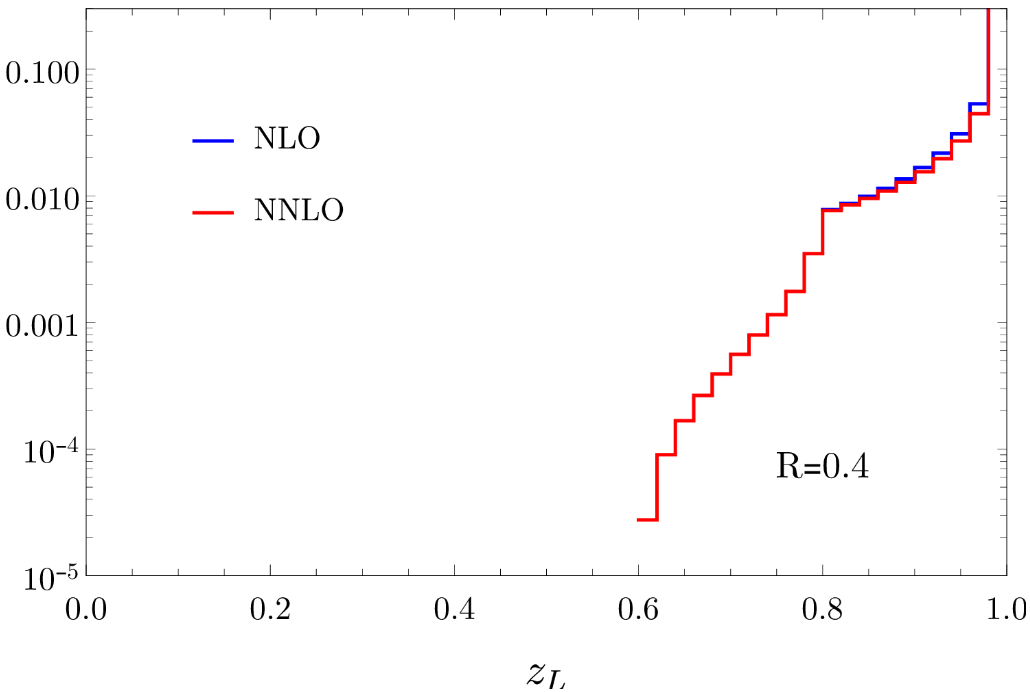


$$J_{s,q}(z_L) = \int_0^{z_{\mathrm{v}}} dz_s \, J_{s,q}(z_L, z_S)$$

Subeading Jet function: LL RGE solutions

$$z_S \leq z_{\rm v} = 0.2$$

$J_{s,q}(z_L)$



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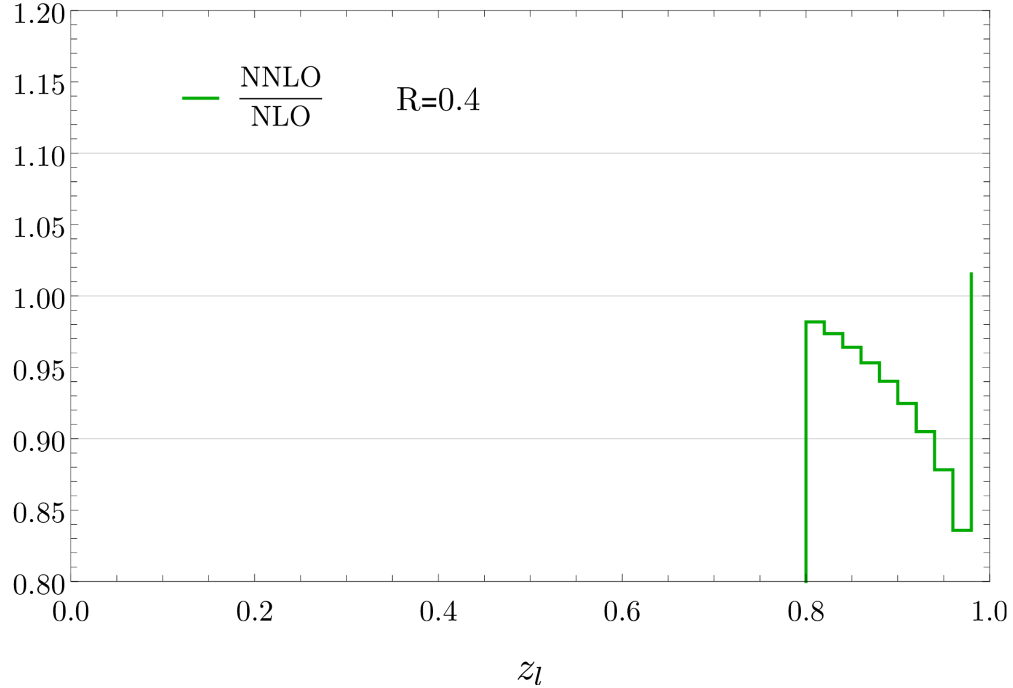


$$J_{s,q}(z_L) = \int_0^{z_{\rm v}} dz_s \, J_{s,q}(z_L, z_S)$$

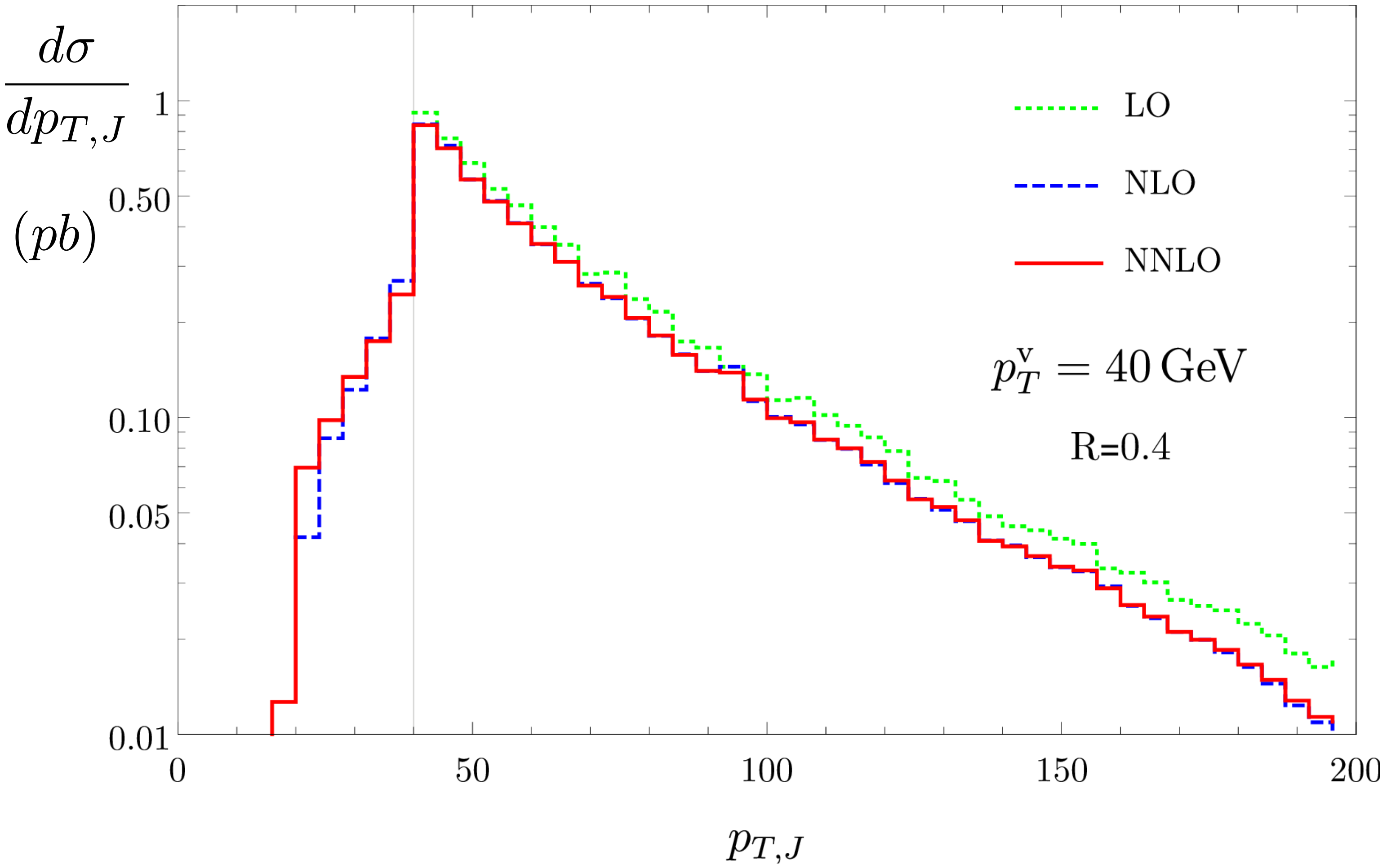
Subleading jet function in H+1j production

Provided $p_T^{\mathrm{v}} \lesssim p_{T,J},\ R \ll 1$, the subleading jet function can be used to obtain exclusive cross sections.

$$\frac{d\sigma_{pp\to H+1j}}{dp_{T,J}} = \sum_{i\in\mathrm{partons}} \int_{p_{T,J}} \frac{dp_{T,i}}{p_{T,i}} \int dz_S \, \frac{d\hat{\sigma}^{\mathrm{LO}}_{pp\to H+i}}{dp_{T,i}} \, J_{s,i}\left(\frac{p_{T,J}}{p_{T,i}}, z_S\right) \Theta\left(p_T^{\mathrm{v}} - z_S p_{T,i}\right)$$

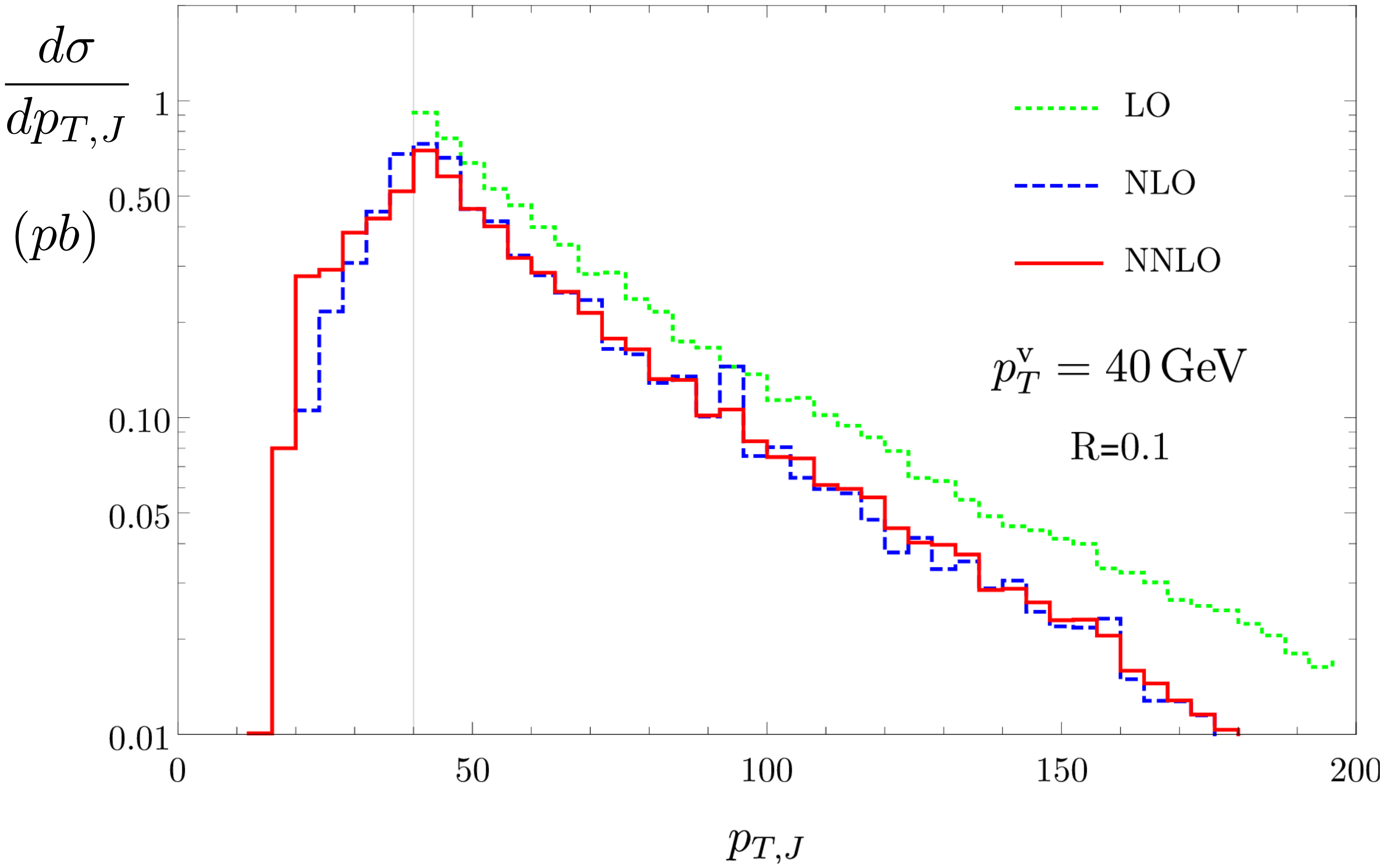
Subleading jet below veto

Subleading jet function in H+1j production



Part of cross section pushed below veto – softer spectrum

Subleading jet function in H+1j production



Part of cross section pushed below veto – softer spectrum

Conclusions

- Examined jet functions describing leading & subleading jet
- Parton shower approach to construct non-linear RGE structure
- Obtained analytic perturbative solutions for (sub)leading jet functions at LL accuracy
- Reasonably fast convergence for moderate values of R
- Examined the impact of these jet functions on $H+j$ production