Leading and subleading jet functions

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Jet functions $J_i$:

- Enter in factorization theorems with final state partons
- Describe collinear radiation of final state partons
- Various types already studied: Exclusive, semi-inclusive, ....

[Kang, Ringer, Vitev: JHEP 1610 (2016) 125]
Jets & Jet functions

Here we consider:

\[
\frac{d\sigma}{dp_{T,J}} = \sum_{i \in \text{partons}} \int dz\, dp_{T,i} \frac{d\hat{\sigma}_{pp \to X+i}}{dp_{T,i}} J_i(z) \delta(p_{T,J} - zp_{T,i})
\]

Energy fraction \( z = \frac{p_{T,J}}{p_{T,i}} \)

Specifically:
- Leading jet functions: energy fraction of leading jet
- Subleading jet functions: energy fraction of leading and subleading jets

Will use the jet functions to resum \((\alpha_s \ln R)^n\)
Leading Jet function

Final state partons fragment into jets.

**Leading-jet function**: probability that the **hardest** jet has energy fraction \( z_l \).

\[
J_i(z_l, \mu) = \sum_{n=0} \left( \frac{\alpha_s}{\pi} \right)^n J_i^{(n)}(z_l, \mu)
\]

\[
J_q^{(0)}(z_l, \mu) = J_g^{(0)}(z_l, \mu) = \delta(1 - z_l)
\]
Leading Jet function: $\mathcal{O}(\alpha_s)$ calculation

\[ J_q(z_l, \theta_{\text{max}}) = \delta(1 - z_l) \]

\[ + \frac{\alpha_s}{\pi} \int_0^1 dz_1 P_{gq}(z_1) \int_R^{\theta_{\text{max}}} d\theta_1 \frac{d\theta_1}{\theta_1} \]

\[ \times \int_0^1 dx_1 J_g(x_1, \theta_1) \int_0^1 dx_2 J_q(x_2, \theta_1) \]

\[ \times \delta(z_l - \max[z_1 x_1, (1 - z_1) x_2]) \]

Using a parton shower picture, one can write down a recursive definition at LL.

Leading Jet function: $O(\alpha_s)$ calculation

$$J_q(z_l, \theta_{\text{max}}) = \delta(1 - z_l)$$

$$+ \frac{\alpha_s}{\pi} \int_0^1 dz_1 \ P_{gq}(z_1) \int_{\theta_1}^{\theta_{\text{max}}} \frac{d\theta_1}{\theta_1}$$

$$\times \int_0^1 dx_1 \ J_g(x_1, \theta_1) \int_0^1 dx_2 \ J_q(x_2, \theta_1)$$

$$\times \delta(z_l - \max[z_1 x_1, (1-z_1)x_2])$$

Usual angular integration. Emissions smaller than $R$ finish in the same jet, so they do not affect $z_l$. 
Leading Jet function: $\mathcal{O}(\alpha_s)$ calculation

$$J_q(z_l, \theta_{\text{max}}) = \delta(1 - z_l)$$

$$+ \frac{\alpha_s}{\pi} \int_0^1 dz_1 \, P_{qg}(z_1) \int_{\theta_{\text{max}}}^{\theta_1} d\theta_1 \frac{d\theta_1}{\theta_1}$$

$$\times \int_0^1 dx_1 \, J_g(x_1, \theta_1) \int_0^1 dx_2 \, J_q(x_2, \theta_1)$$

$$\times \delta\left(z_l - \max[z_1 x_1, (1 - z_1)x_2]\right)$$

- Integrate over possible energy fractions of new ‘leading’ jets.
- At LL emissions are strongly angular ordered: $\theta_1$ is new $\theta_{\text{max}}$ for further emissions.
Leading Jet function: $O(\alpha_s)$ calculation

\[
J_q(z_l, \theta^\text{max}) = \delta(1 - z_l) \\
+ \frac{\alpha_s}{\pi} \int_0^1 dz_1 \, P_{gq}(z_1) \int_R^{\theta^\text{max}} \frac{d\theta_1}{\theta_1} \\
\times \int_0^1 dx_1 \, J_g(x_1, \theta_1) \int_0^1 dx_2 \, J_q(x_2, \theta_1) \\
\times \delta \left( z_l - \max[z_1 x_1, (1 - z_1)x_2] \right)
\]

Ensures $z_l$ corresponds to the jet with the highest energy fraction.
Leading Jet function: LL RGE

- Recursive definition allows us to obtain the LL RGE
- Jet function scale is set by $p_T R$, evolve to hard scale $p_T$
- Convenient to re-write angular integral in terms of $p_T$
- Dependence on the scale $\mu$ explicit (lower bound on $\theta$ integral)

Derivative w.r.t. $\mu$ gives

$$
\mu \frac{d}{d\mu} J_q(z_l, \mu) = \frac{\alpha_s}{\pi} \int_0^1 dz_1 \, P_{gq}(z_1) \int_0^1 dx_1 \, J_g(x_1, \mu) \int_0^1 dx_2 \, J_q(x_2, \mu) \\
\times \delta \left( z_l - \max[z_1 x_1, (1-z_1)x_2] \right)
$$

Difficult to solve analytically.
Can generate LL solutions to higher order for the jet function.
Leading Jet function: LL RGE Solutions

Using lower order solutions as input to the RGE, we obtain:

\[ J_q^{(0)}(z_l, \mu) = \delta(1 - z_l) \]

\[ J_q^{(1)}(z_l, \mu) = \ln \frac{\mu}{\mu_0} \left[ P_{qq}(z_l) + P_{gq}(z_l) \right] \Theta \left( z_l - \frac{1}{2} \right) \]

\[ J_q^{(2)}(z_l, \mu) = \frac{1}{2} \ln^2 \frac{\mu}{\mu_0} \left\{ \Theta \left( z_l - \frac{1}{2} \right) \left[ \frac{\beta_0}{2} (P_{qq}(z_l) + P_{gq}(z_l)) + \int_z^1 \frac{dx}{x} F_q \left( x, \frac{z_l}{x} \right) \right] \\
+ \Theta \left( \frac{1}{2} - z_l \right) \Theta \left( z_l - \frac{1}{3} \right) \int_{\frac{1}{2}}^{\frac{z_l}{1-z_l}} dx \left[ \frac{1}{x} F_q \left( x, \frac{z_l}{x} \right) + F_q(x, 1 - z_l) \right] \right\} \]

\[ F_q(x, y) - \text{products of splitting functions.} \]

At \( \mathcal{O}(\alpha_s) \) only one splitting – leading parton must have \( z_l > 0.5 \).
At \( \mathcal{O}(\alpha_s^2) \) we have 3 partons. Implies \( z_l > 1/3 \).
Leading Jet function: LL RGE Solutions

Good convergence for $z_l > 0.5$. Slight shift to lower $z_l$. 

$\mu_0 = p_T R$  
$\mu = p_T$
Leading Jet function: LL RGE Solutions

Good convergence for $z_l > 0.5$. Slight shift to lower $z_l$. 

$\mu_0 = p_T R$
$\mu = p_T$
Leading jet function in $H+1j$ production

Consider $pp \rightarrow H + j$ at LO. Inclusion of higher order $\ln R$ terms.

$$\frac{d\sigma_{pp \rightarrow H+1j}}{dp_{T,J}} = \sum_{i \in \text{partons}} \int_{p_{T,J}} dp_{T,i} \frac{d\hat{\sigma}_{pp \rightarrow H+i}}{dp_{T,i}} J_i \left( \frac{p_{T,J}}{p_{T,i}} \right)$$

Slight softening of the high $p_{T,J}$ tail.
Leading jet function in $H+1j$ production

Consider $pp \rightarrow H + j$ at LO. Inclusion of higher order $\ln R$ terms.

$$\frac{d\sigma_{pp\rightarrow H+1j}}{dp_{T,J}} = \sum_{i\in \text{partons}} \int_{p_{T,J}} \frac{dp_{T,i}}{p_{T,i}} \frac{d\hat{\sigma}_{pp\rightarrow H+i}}{dp_{T,i}} J_i \left( \frac{p_{T,J}}{p_{T,i}} \right)$$

Slight softening of the high $p_{T,J}$ tail.
A subleading-jet function can be used to track the energy fraction of the $2^{\text{nd}}$ hardest jet.

Can be used to place jet vetos in exclusive production.

Use a parton shower picture to determine the evolution at LL.
As before we derive an RG equation for our subleading jet function:

\[
\mu \frac{d}{d\mu} J_i(z_L, z_S, \mu) = \frac{\alpha_s}{2\pi} \int_0^1 dz_1 \int_0^{x_{1L}} dx_{1L} \int_0^{x_{2L}} dx_{2L} \int_0^{x_{1S}} dx_{1S} \int_0^{x_{2S}} dx_{2S} 
\times F_i(x_{1L}, x_{2L}, x_{1S}, x_{2S}, z_1) 
\times \left\{ \Theta(z_1 x_{1L} - (1 - z_1) x_{2L}) \delta(z_L - z_1 x_{1L}) \delta(z_S - \max \left[ z_1 x_{1S}, (1 - z_1) x_{2L} \right]) 
+ \Theta((1 - z_1) x_{2L} - z_1 x_{1L}) \delta(z_L - (1 - z_1) x_{2L}) \delta(z_S - \max \left[ z_1 x_{1L}, (1 - z_1) x_{2S} \right]) \right\}
\]
Subleading Jet function: LL Definition

As before we derive an RG equation for our subleading jet function:

\[ \mu \frac{d}{d\mu} J_i(z_L, z_S, \mu) = \frac{\alpha_s}{2\pi} \int_0^1 dz_1 \, dx_{1L} \, dx_{2L} \int_0^{x_{1L}} dx_{1S} \int_0^{x_{2L}} dx_{2S} \]

\[ \times F_i(x_{1L}, x_{2L}, x_{1S}, x_{2S}, z_1) \]

\[ \times \left\{ \Theta(z_1 x_{1L} - (1 - z_1) x_{2L}) \delta(z_L - z_1 x_{1L}) \delta(z_S - \max [z_1 x_{1S}, (1 - z_1) x_{2L}]) \right. \]

\[ + \Theta((1 - z_1) x_{2L} - z_1 x_{1L}) \delta(z_L - (1 - z_1) x_{2L}) \delta(z_S - \max [z_1 x_{1L}, (1 - z_1) x_{2S}]) \right\} \]

Splitting & jet functions. E.g.

\[ F_q = 2P_{qq}(z_1) J_{s,q}(x_{1L}, x_{1S}) J_{s,g}(x_{2L}, x_{2S}) \]
As before we derive an RG equation for our subleading jet function:

\[
\mu \frac{d}{d\mu} J_i(z_L, z_S, \mu) = \frac{\alpha_s}{2\pi} \int_0^1 dz_1 dx_{1L} dx_{2L} \int_0^{x_{1L}} dx_{1S} \int_0^{x_{2L}} dx_{2S} \\
\times F_i(x_{1L}, x_{2L}, x_{1S}, x_{2S}, z_1) \\
\times \left\{ \Theta(z_1 x_{1L} - (1 - z_1) x_{2L}) \delta(z_L - z_1 x_{1L}) \delta(z_S - \max [z_1 x_{1S}, (1 - z_1) x_{2L}]) \\
+ \Theta((1 - z_1) x_{2L} - z_1 x_{1L}) \delta(z_L - (1 - z_1) x_{2L}) \delta(z_S - \max [z_1 x_{1L}, (1 - z_1) x_{2S}]) \right\}
\]
Subleading Jet function: RGE solutions

We can again derive LL results to higher orders in $\alpha_s$:

\[
J_{s,i}^{(0)} = \delta(1 - z_L) \delta(z_S)
\]

\[
J_{s,q}^{(1)} = \left[ P_{qq}(z_L) + P_{gq}(z_L) \right] \ln \frac{\mu}{\mu_0} \delta(z_S - (1 - z_L)) \Theta \left( z_L - \frac{1}{2} \right)
\]

\[
J_{s,q}^{(2)}(z_L, z_S) = \frac{\beta_0}{4} \left[ P_{qq}(z_L) + P_{gq}(z_L) \right] \delta(z_S - (1 - z_L)) \Theta \left( z_L - \frac{1}{2} \right) \ln^2 \frac{\mu}{\mu_0}
\]

\[
+ \frac{1}{2} \Theta(z_L - z_s) \Theta(z_L + 2z_S - 1) \Theta(1 - z_L - z_S) \left\{ \frac{1}{1 - z_L} F_{1,q} \left( z_L, \frac{z_S}{1 - z_L} \right) + \frac{1}{1 - z_S} F_{1,q} \left( z_S, \frac{z_L}{1 - z_S} \right) \right. \\
\left. + \frac{1}{z_L + z_S} F_{2,q} \left( z_L + z_S, \frac{z_L}{z_L + z_S} \right) \right\} \ln^2 \frac{\mu}{\mu_0}
\]
Subleading Jet function: LL RGE solutions

\[ z_S \leq z_v = 0.5 \]

\[ J_{s,q}(z_L) = \int_0^{z_v} dz_S J_{s,q}(z_L, z_S) \]
Subleading Jet function: LL RGE solutions

\[ z_S \leq z_V = 0.2 \]

\[ J_{s,q}(z_L) = \int_0^{z_V} dz_S \, J_{s,q}(z_L, z_S) \]

K factor
Subleading jet function in $H+1j$ production

Provided $p_T^\gamma \ll p_T,J$, $R \ll 1$, the subleading jet function can be used to obtain exclusive cross sections.

\[
\frac{d\sigma_{pp\rightarrow H+1j}}{dp_{T,J}} = \sum_{i\in \text{partons}} \int_{p_T,J} dp_{T,i} \int dZ_S \frac{d\hat{\sigma}_{pp\rightarrow H+i}^{LO}}{dp_{T,i}} J_{s,i} \left( \frac{p_{T,J}}{p_{T,i}}, Z_S \right) \Theta (p_T^\gamma - Z S p_{T,i})
\]
Subleading jet function in \( H+1j \) production

\[
\frac{d\sigma}{dp_{T,J}} \quad (pb)
\]

\( p_T^y = 40 \text{ GeV} \)

\( R=0.4 \)

Part of cross section pushed below veto – softer spectrum
Subleading jet function in H+1j production

Part of cross section pushed below veto – softer spectrum
Conclusions

- Examined jet functions describing leading & subleading jet
- Parton shower approach to construct non-linear RGE structure
- Obtained analytic perturbative solutions for (sub)leading jet functions at LL accuracy
- Reasonably fast convergence for moderate values of $R$
- Examined the impact of these jet functions on $H + j$ production