Higgs Pair Production at High-Energy

Stephen Jones

Borowka, Davies, Greiner, Heinrich, Jahn, Kerner,
Luisoni, Mishima, Schlenk, Schubert, Steinhauser,
Wellmann, Zirke,

[1907.06408]
JHEP 10 (2016) 107 [1608.04798]
PRL 117 (2016) 012001, Erratum 079901 [1604.06447]

Motivation - Higgs self-coupling

Standard Model Higgs Lagrangian:

$$\mathcal{L} \supset -V(\phi), \quad V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

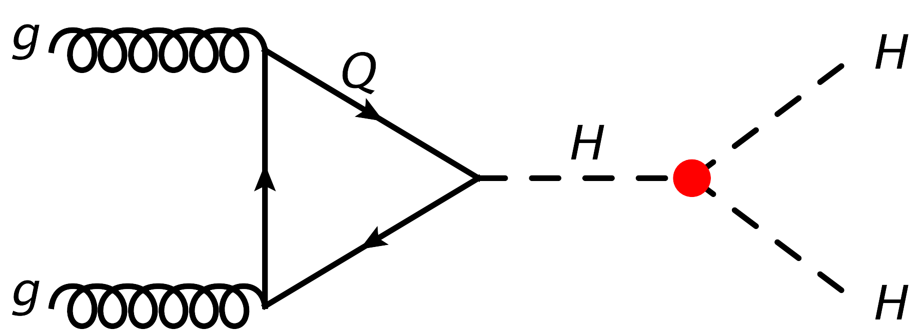
EW symmetry breaking

$$V(H) = \frac{1}{2}m_H^2 H^2 + \boxed{\lambda v H^3} + \frac{\lambda}{4}H^4,$$

$$\mu^2 = \lambda v^2$$
$$m_H^2 = 2\lambda v^2$$

SM: self-couplings determined by m_H, v

Higgs pair production probes triple-Higgs coupling



Higgs self-coupling not yet measured!
Extremely challenging to measure at LHC due to $\mathcal{O}(\mathrm{fb})$ cross section and difficult backgrounds

Experimental Prospects

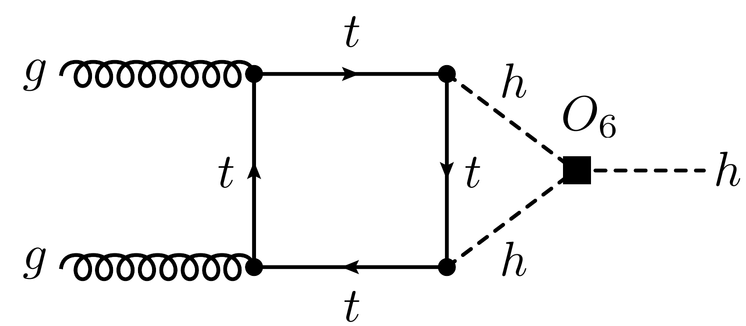
HH extremely challenging to measure, combining $b\bar{b}b\bar{b}, b\bar{b}\tau^+\tau^-, b\bar{b}\gamma\gamma$:

$$\sigma_{\text{tot}} \leq 6.7\sigma_{\text{SM}} \text{ and } -5.0 < \kappa_\lambda < 12.0$$ CERN-EP-2019-099

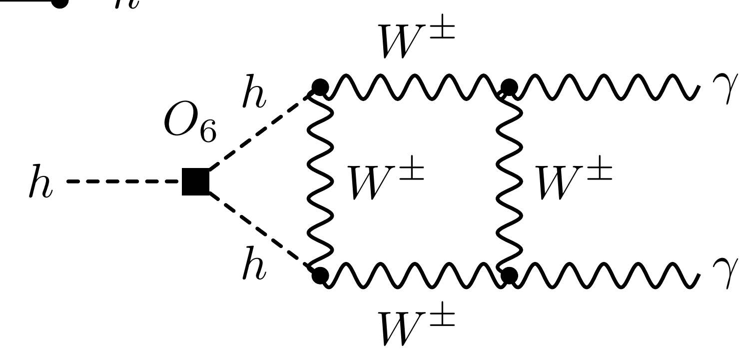
Can obtain complementary limits on λ_3 from single Higgs:

$$-3.2 < \kappa_\lambda < 11.9$$ ATL-PHYS-PUB-2019-009

EW corr. to single H production (also VBF, VH)

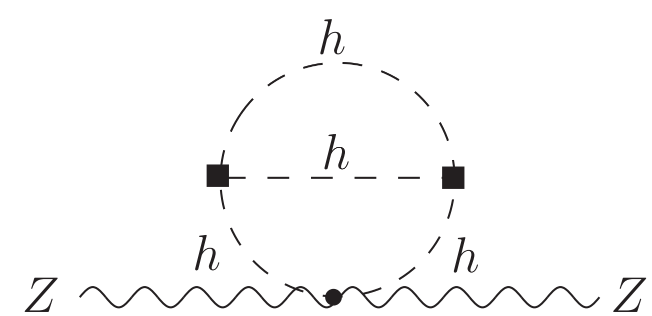


Gorbahn, Haisch 16; Bizoń, Gorbahn, Haisch, Zanderighi 16; Degrassi, Giardino, Maltoni, Pagani 16; Maltoni, Pagani, Shivaji, Zhao 17; Di Vita, Grojean, Panico, Riembau, Vantalon 17



Modification of precision EW observables (EW oblique corrections) S, T

Degrassi, Fedele, Giardino 17; Kribs, Maier, Rzehak, Spannowsky, Waite 17;

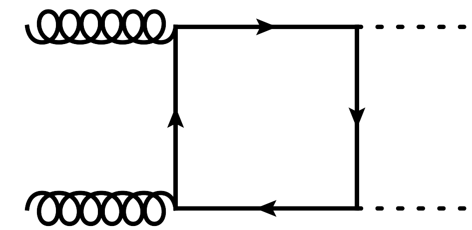


Limits on λ_4 : from (partial) EW corrections to HH

Bizon, Haisch, Rottoli 18; Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 18

Production History

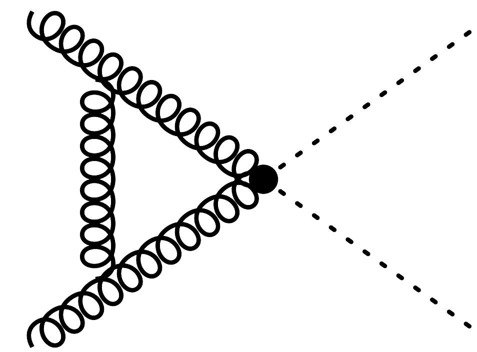
1. LO (full m_t dependence)
   Glover, van der Bij 88



2. NLO **K≈1.9**

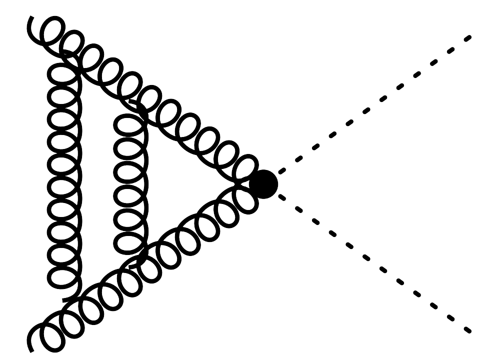
   Heavy Top Quark Limit
   Dawson, Dittmaier, Spira 98

   Including m_t in Real Radiation or $1/m_t$ Expansion
   Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14;
   Grigo, Hoff, Steinhauser 15; Maltoni, Vryonidou, Zaro 14;

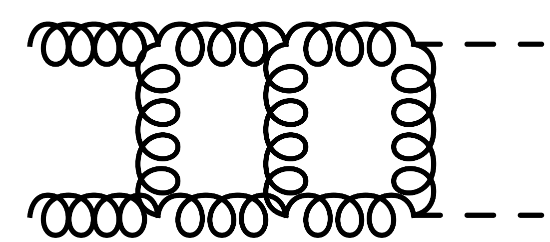


3. NNLO Heavy Top Quark Limit **K≈1.2**
   de Florian, Mazzitelli 13; Grigo, Melnikov, Steinhauser 14; Shao, Li, Li, Wang 13; de Florian, Mazzitelli 15; de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16

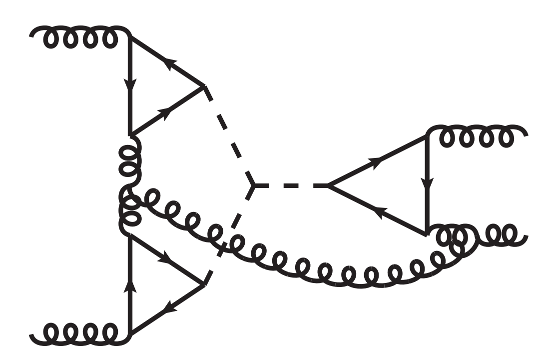
   Expansion in $1/m_t$
   Grigo, Hoff, Steinhauser 15;



4. (Partial) N3LO Heavy Top Quark Limit
   Banerjee, Borowka, Dhani, Gehrmann, Ravindran 18

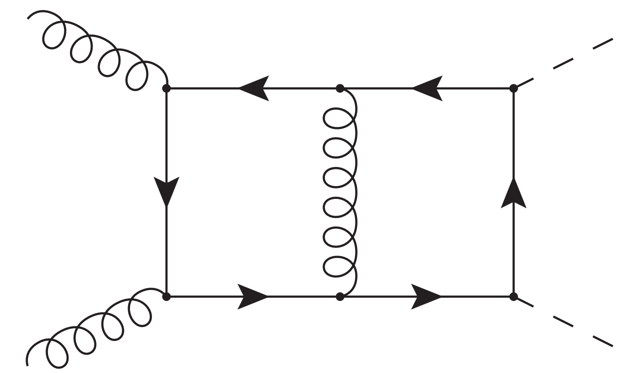


5. (Partial) NNLO Real-Virtual in $1/m_t$ Expansion
   Davies, Herren, Mishima, Steinhauser 19



Production History (II)

1. NLO QCD (2-loop) with Full Top Mass
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16;
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16;
   Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18;
   (Transverse momentum) NLO + NLL
   Ferrera, Pires 16
   Including Parton Shower
   Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; SPJ, Kuttimalai 17



2. NLO with Full Top Mass + NNLO HTL
   Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18
   (Soft gluon) NNLO + NNLL
   de Florian, Mazzitelli 18
3. Padé ($1/m_t$ + threshold)
   Gröber, Maier, Rauh 17
4. Expansion in $p_t^2 + m_h^2$
   Bonciani, Degrassi, Giardino, Gröber 18
5. High-Energy Expansion
   Davies, Mishima, Steinhauser, Wellmann 18, 18; Mishima 18

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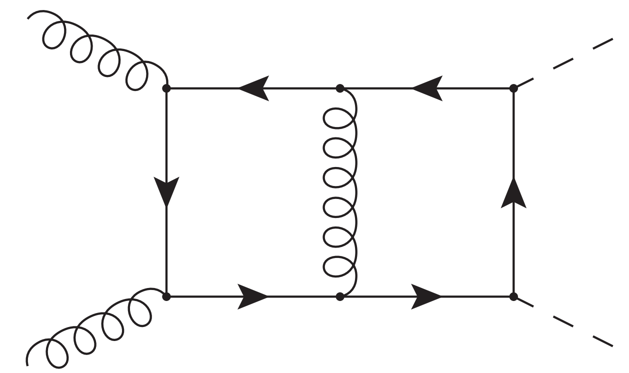
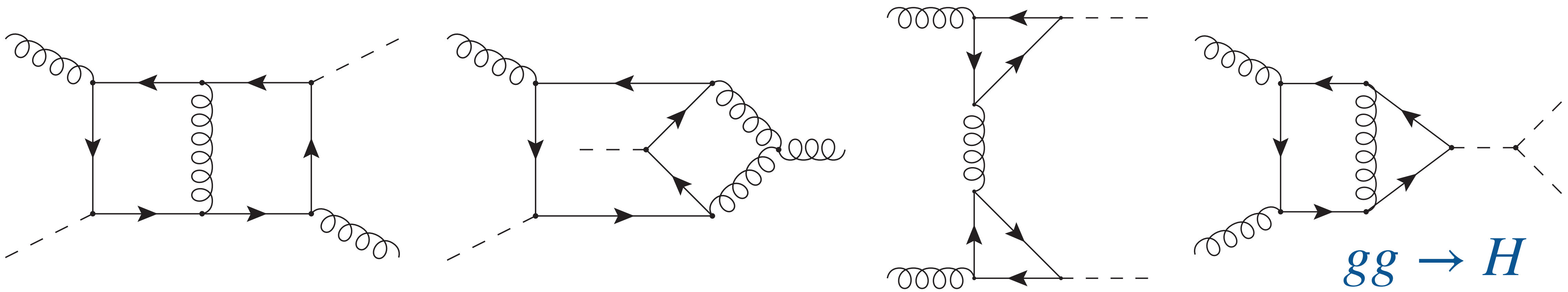


This work combines these two approaches

Full Numerical Results

NLO QCD corrections to HH production with full top quark mass:
≤7 propagator, 4-point, 2-loop diagrams, 4 mass scales (s, t, m_t, m_h)



Planar **Non-planar** **Reducible**

Degrassi, Giardino, Gröber 16

Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

Reduced to master integrals (planar part) and evaluated numerically using sector decomposition & quasi Monte Carlo integration on GPUs

Pro:

Amplitude is valid in all phase-space regions

Con:

Amplitude is slow to evaluate (1.5-48 GPU hours/PS point)

Goal of this work is to supplement this result with an analytic expansion

High-Energy Expansion

Amplitude for $gg \to HH$ can be decomposed into 2 Lorentz structures:

$$\mathcal{M}^{ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\mathcal{M}^{\mu\nu,ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\delta^{ab}X_0(F_1A_1^{\mu\nu} + F_2A_2^{\mu\nu})$$

Defined such that:
$$\mathcal{M}^{++} = \mathcal{M}^{--} = -F_1$$
$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -F_2$$

Form factors may be expanded in α_s as: $F = F^{(0)} + \dfrac{\alpha_s(\mu_R)}{\pi}F^{(1)} + \ldots$

We may cast the one- and two-loop corrections in the form:

$$F_1^{(0)} = \frac{3m_h^2}{s - m_h^2}F_{\text{tri}}^{(0)} + F_{\text{box1}}^{(0)}, \qquad F_2^{(0)} = F_{\text{box2}}^{(0)},$$

$$F_1^{(1)} = \frac{3m_h^2}{s - m_h^2}\underline{F_{\text{tri}}^{(1)} + F_{\text{box1}}^{(1)}} + F_{\text{dt1}}^{(1)}, \qquad F_2^{(1)} = \underline{F_{\text{box2}}^{(1)}} + F_{\text{dt2}}^{(1)},$$

Only underlined terms are expanded, other terms included exactly

High-Energy Expansion (II)

Each $gg \to HH$ form factor can be expanded in m_h^2, m_t^2:

Davies, Mishima, Steinhauser, Wellmann 18; Mishima 18

1) Express amplitude in terms of Feynman integrals:
   $I(m_h^2, m_t^2)$

2) Taylor expand around $m_h^2 = 0$:
   $I(0,m_t^2) + m_h^2 I'(0,m_t^2) + \dots$

Note: possible since no $\log(m_h^2)$ in the limit $m_h^2 \to 0$ since Higgs couples only to top

3) IBP reduce Feynman integrals to master integrals:
   $J(0,m_t^2)$

4) Compute master integrals in expansion around $m_t^2 = 0$:
   $$J(0,m_t^2) = \sum_{m,n} C_{m,n} \, (m_t^2)^m \, \log(m_t^2)^n$$

Results expanded up to m_h^2, m_t^{32}

Expansion expected to be valid for: $m_h^2 < m_t^2 \ll |t|, s$

Padé Improvement

After UV renormalisation and IR subtraction the finite virtual piece is interfered with the Born result to obtain $\mathscr{V}_{\text{fin}}$

For expanded results we have:

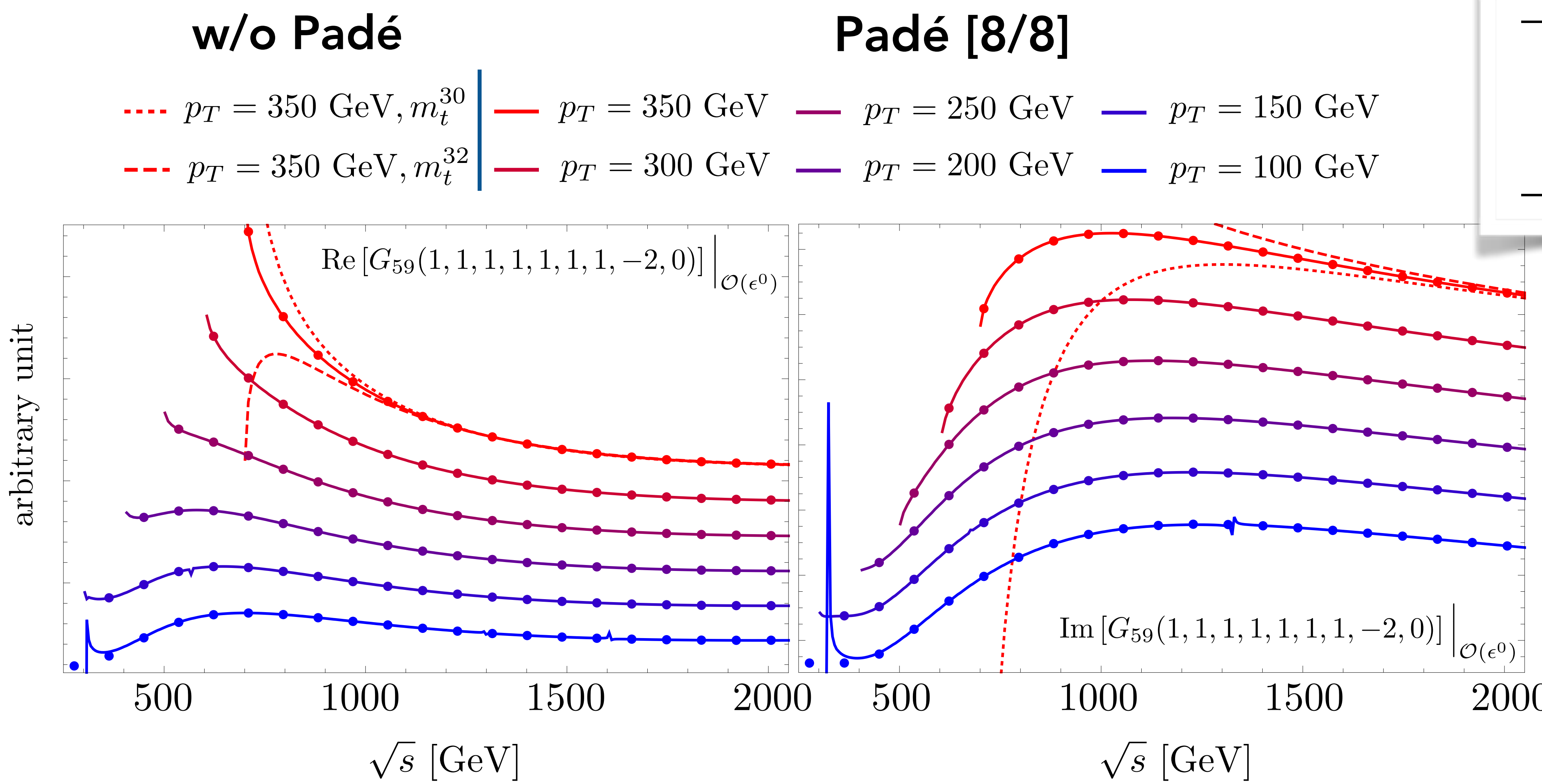
$$\mathscr{V}^N_{\text{fin}} = V_0 + \sum_{i=2}^{N} V_i m_t^i, \quad \text{where } V_0 \text{ contains terms exact in } m_t, m_h$$

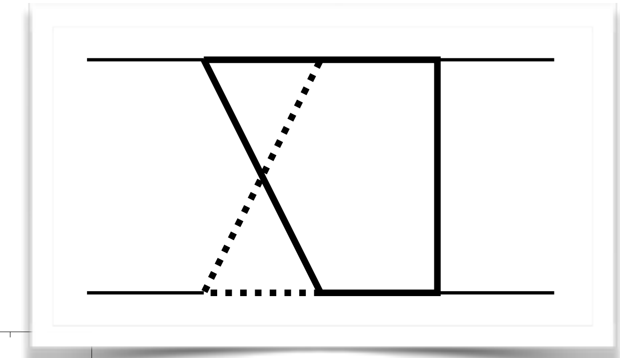
For each phase-space point expanded results can be used to fit a Padé ansatz: See: Gröber, Maier, Rauh 17

Note: no replacement in $\log(m_t)$

1) Fix s, t, m_h
2) Replace $m_t^{2k} \to m_t^{2k} x^k$ and $m_t^{2k-1} \to m_t^{2k-1} x^k$ and fix m_t
3) Match to $\mathscr{V}^N_{\text{fin}} = \dfrac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m} = [n/m](x)$
4) Consider nearly `diagonal' Padé approximants $|n - m| \leq 2$ which include at least terms up to m_t^{30}: $\{[7/8], [8/7], [7/9], [8/8], [9/7]\}$

Padé Improved Integrals





Dots: Numerical results obtained with pySecDec

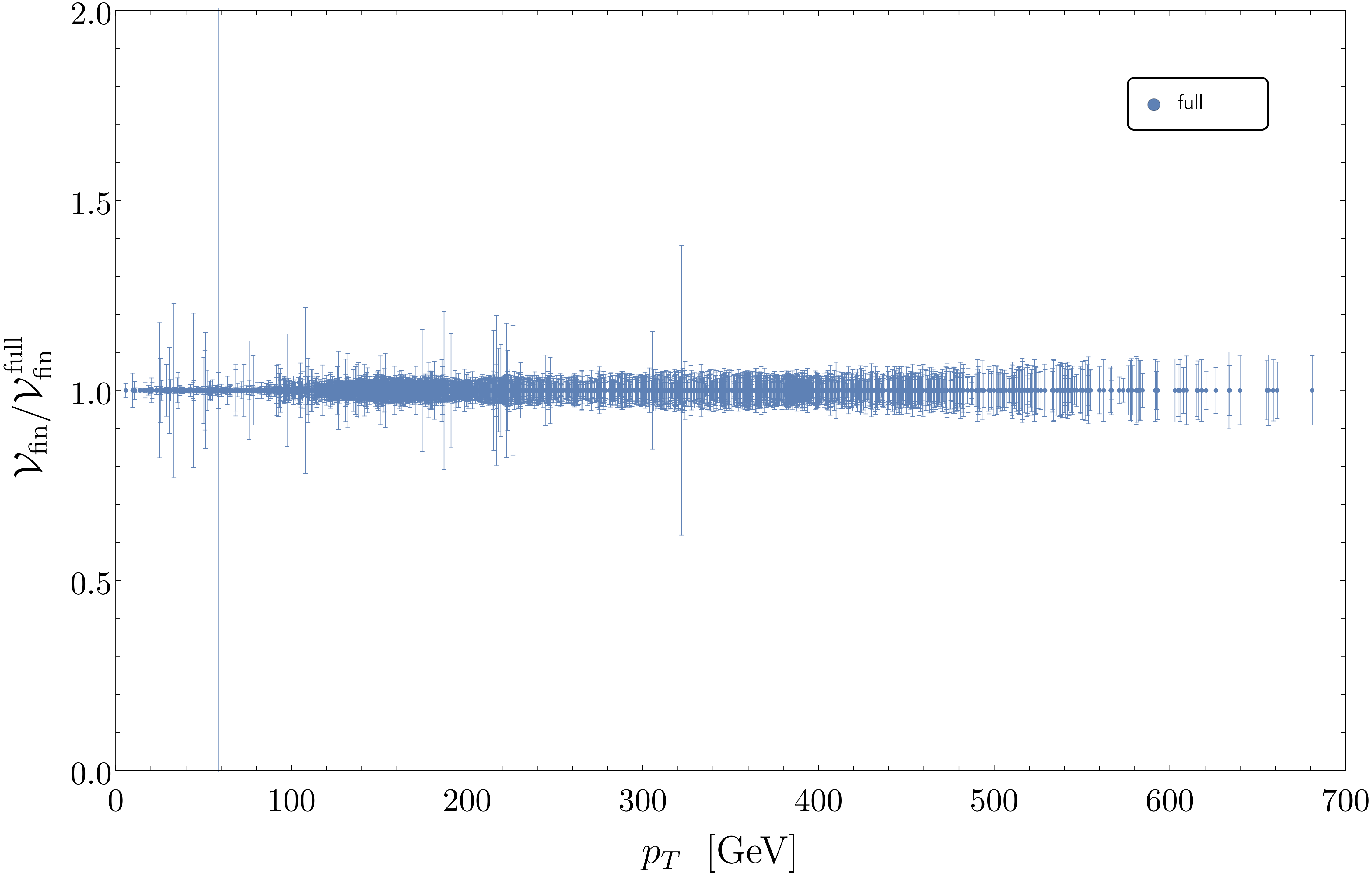
Padé improves agreement between expansion and numerical results for smaller $\sqrt{s}$

Small fluctuations for $\sqrt{s} > 500$ GeV caused by poles close to $x = 1$ in $\mathbb{C}$, can be damped by averaging over Padé approx. α_i with proximity of nearest pole β_i as weight:

$$\text{value}: \quad \alpha = \sum_i w_i \alpha_i, \quad \text{weight}: \quad w_i = \frac{\beta_i^2}{\sum_j \beta_j^2}, \quad \text{error}: \quad \delta_\alpha = \sqrt{\frac{\sum_i w_i (\alpha_i - \alpha)^2}{1 - \sum_i w_i^2}}$$

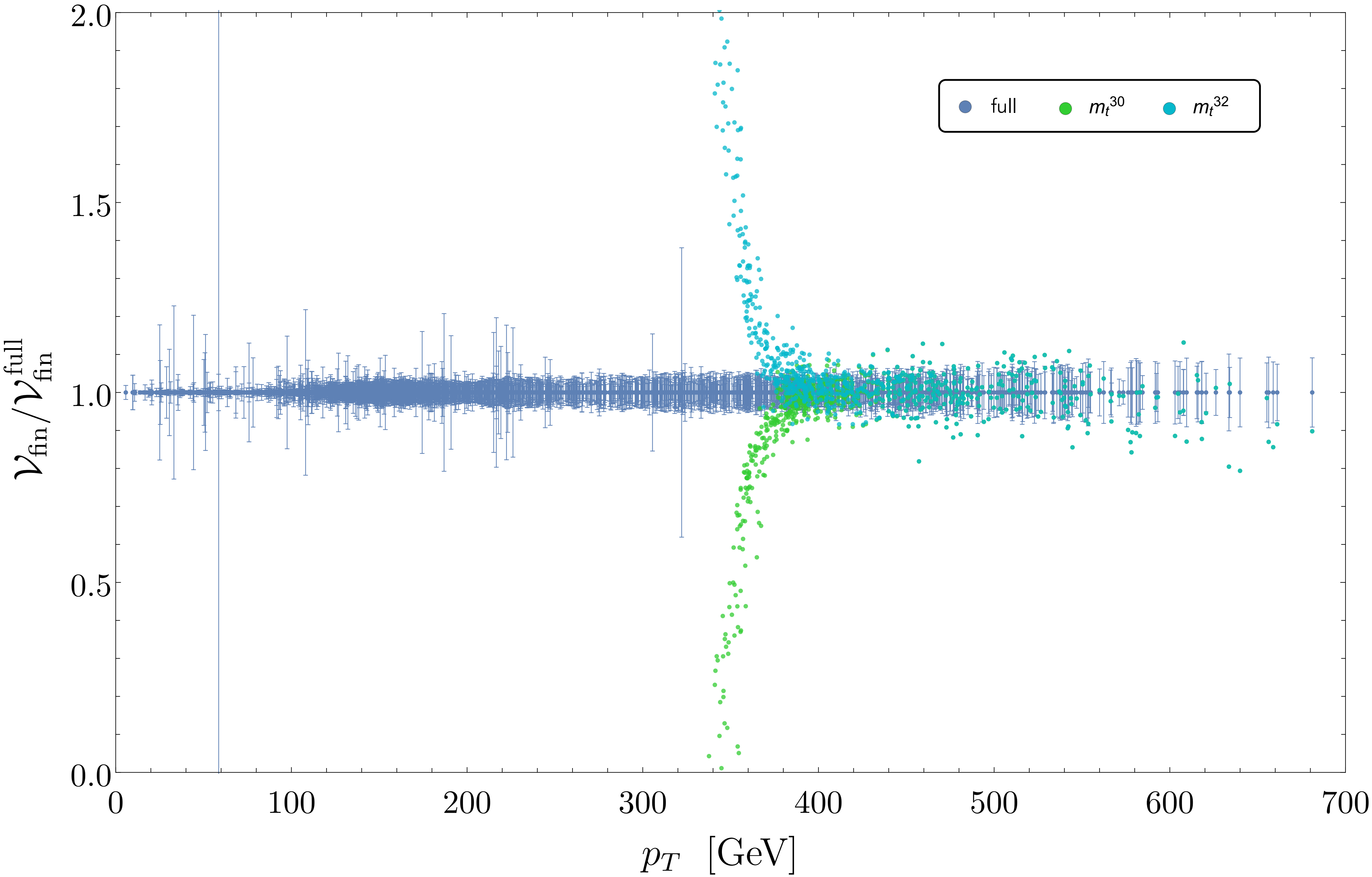
Padé Improved Amplitude

Can compare approximated results to full result



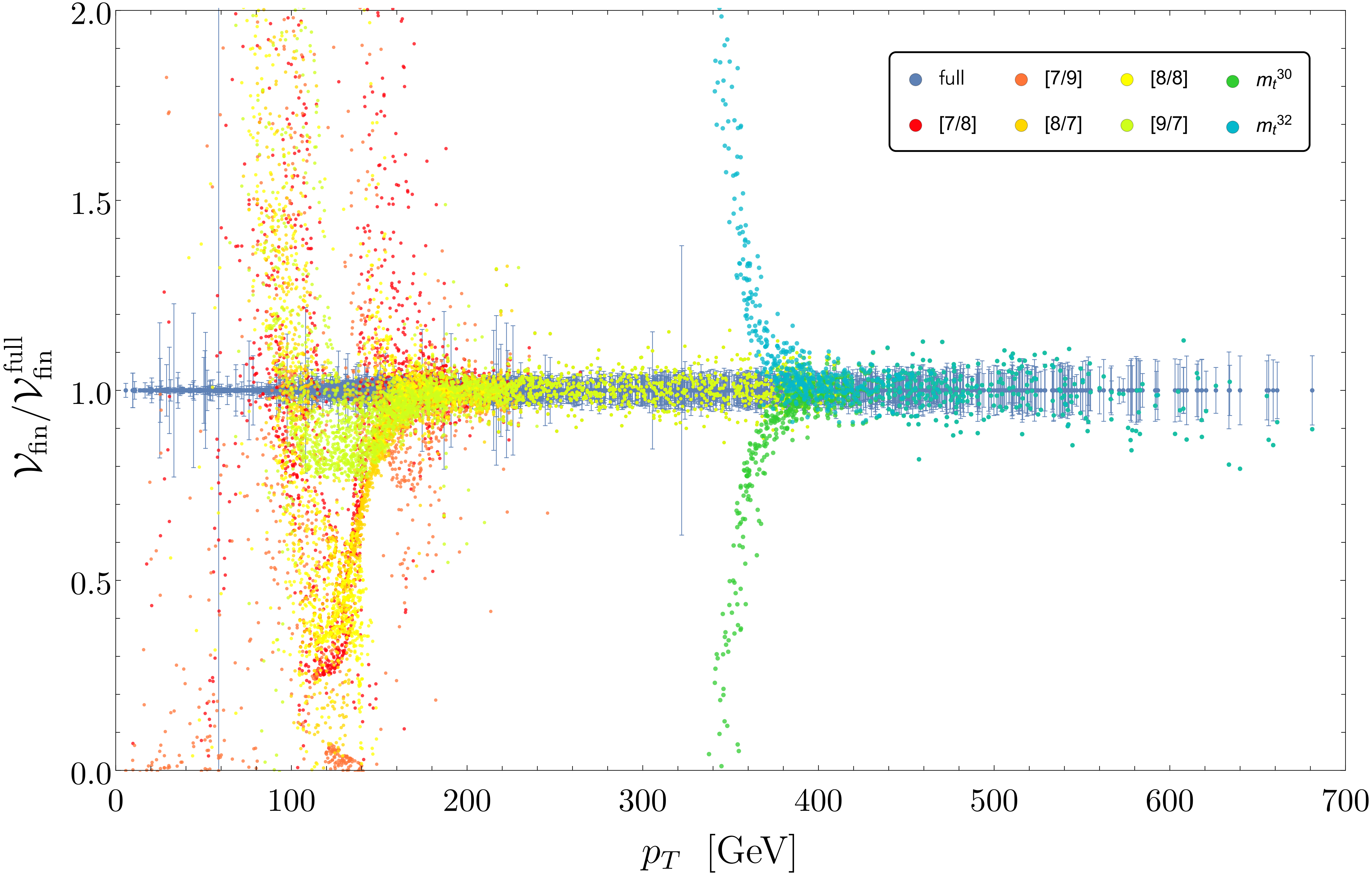
Padé Improved Amplitude

Expansion (no Padé) agrees well with full result for $p_T \geq 400$ GeV



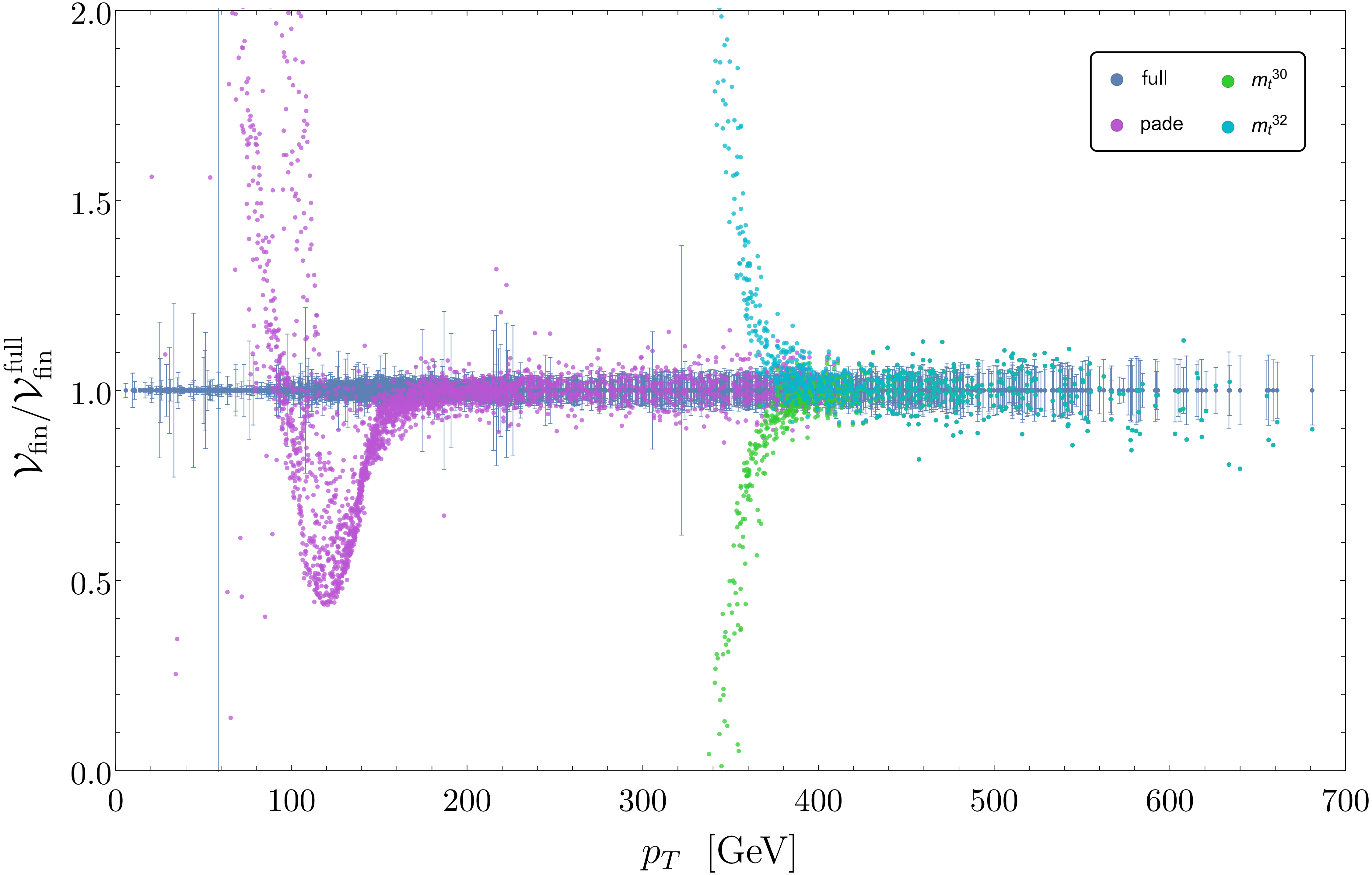
Padé Improved Amplitude

Padé improved expansion agrees well for $p_T \geq 200$ GeV



Padé Improved Amplitude

Weighted average Padé approximation slightly fewer fluctuations



Grid: Parametrisation

Fixing m_t, m_h the virtual matrix element depends on only 2 parameters: (s, t)

Can build 2D grid of our phase-space points and interpolate between 6320 pre-calculated points (added 2922 points for this work)

Parametrisation:

$$x = f(\beta(s)), \quad c_\theta = |\cos\theta| = \left| \frac{s + 2t - 2m_h^2}{s\beta(s)} \right|, \quad \beta = \left(1 - \frac{4m_h^2}{s}\right)^{\frac{1}{2}}$$

Choose $f(\beta(s))$ according to cumulative distribution function of phase space points used to populate the grid

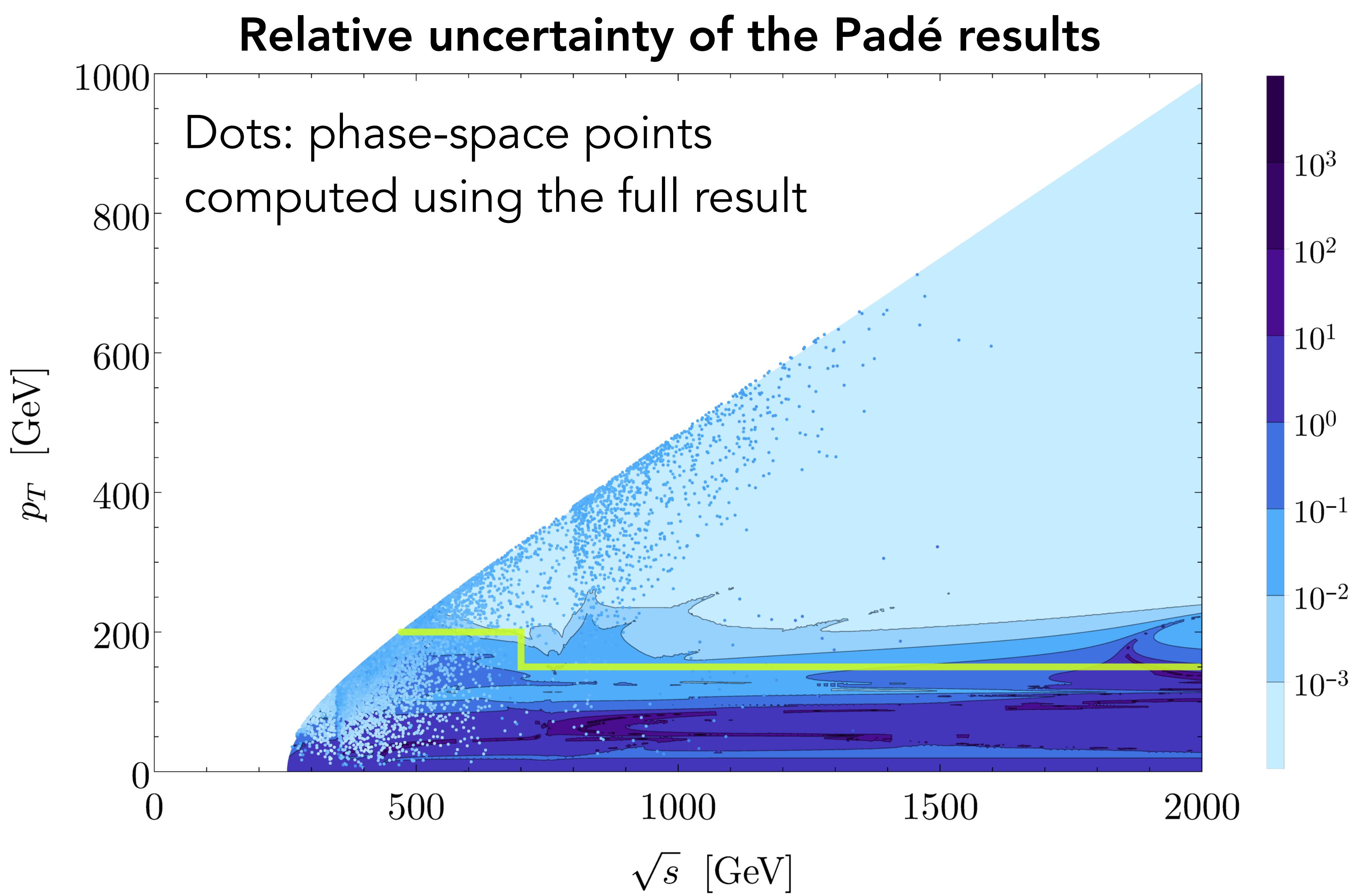
Obtain nearly uniform distribution in (x, c_θ) unit square

Two-step interpolation procedure:

1. Choose equidistant grid points, estimate result at each grid point with linear interpolation of amplitude results in vicinity
2. Clough-Tocher interpolation (as implemented in scipy) to estimate amplitude at arbitrary sampling points

Procedure reduces fluctuations due to uncertainties of input data points

Grid: Point Input

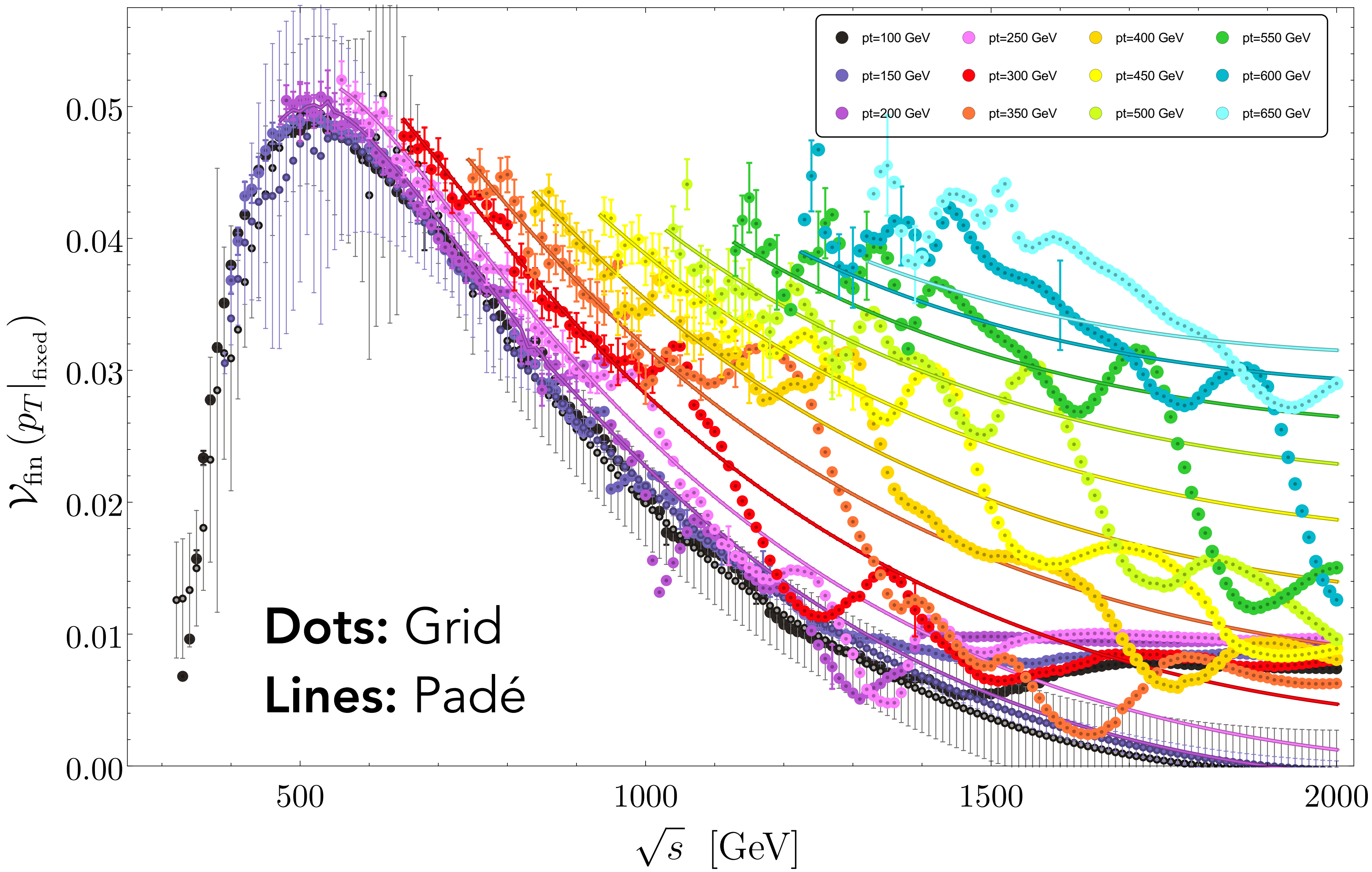


Quality of expanded results degrades for small $p_T^2 = \dfrac{tu - m_H^4}{s}$ due to the break down of the assumption $m_h^2, m_t^2 \ll |t|$

Construct grid based on:

- 6320 points computed using the full NLO result
- Supplemented with Padé approximated results for $\sqrt{s} < 700$ GeV, $p_T \geq 200$ GeV and $\sqrt{s} \geq 700$ GeV, $p_T \geq 150$ GeV

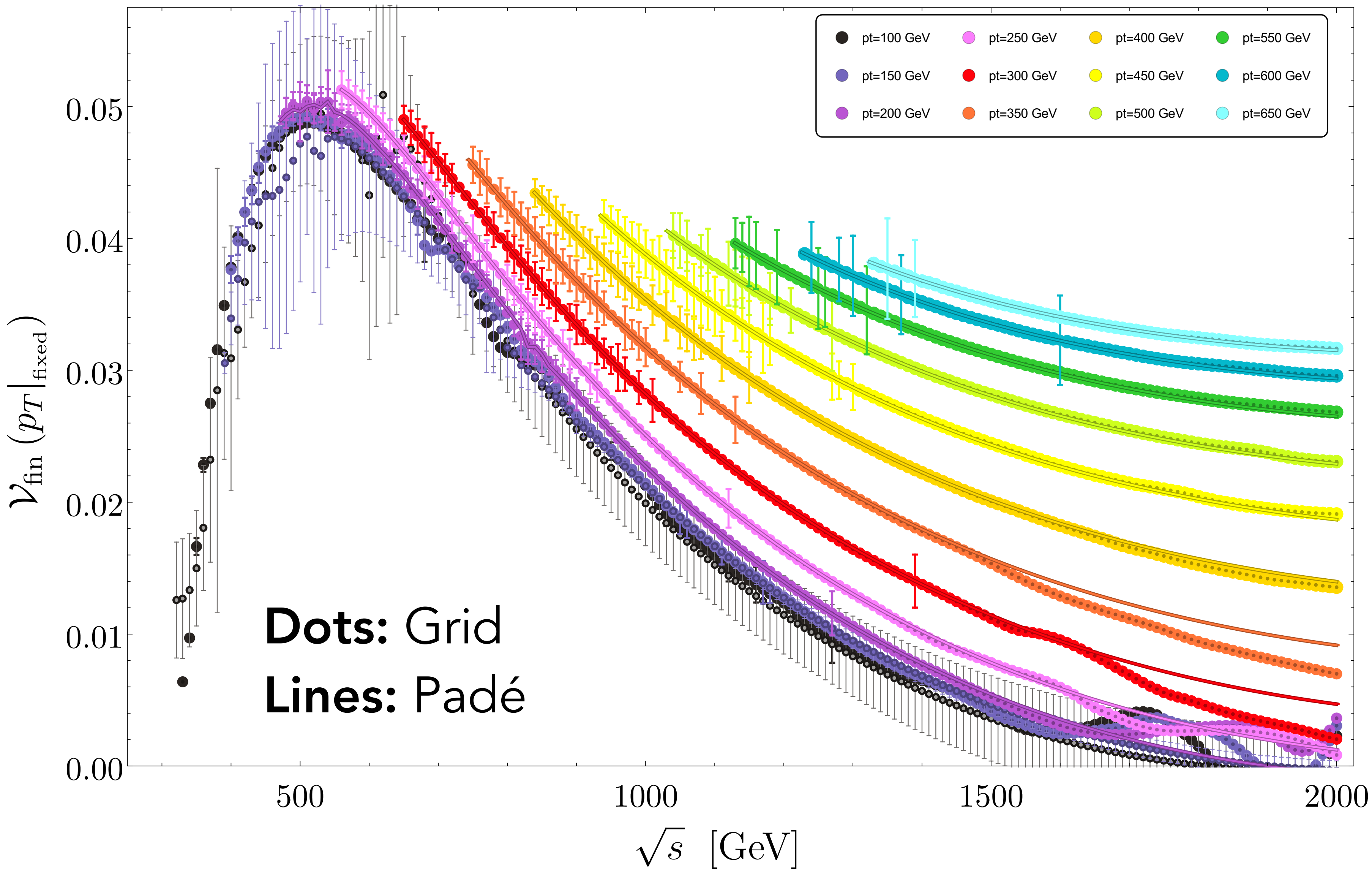
Full Result Grid vs Padé Improved Expansion



Low m_{hh}, $p_{T,h}$:
Grid is densely populated and result is reliable, agrees well with Padé

Large m_{hh}:
Grid has no support from input points, results fluctuate wildly
Padé gives stable results (especially for sufficiently large $p_{T,h}$)

Full Result Grid + Padé Improved Expansion



Low m_{hh}, $p_{T,h}$:

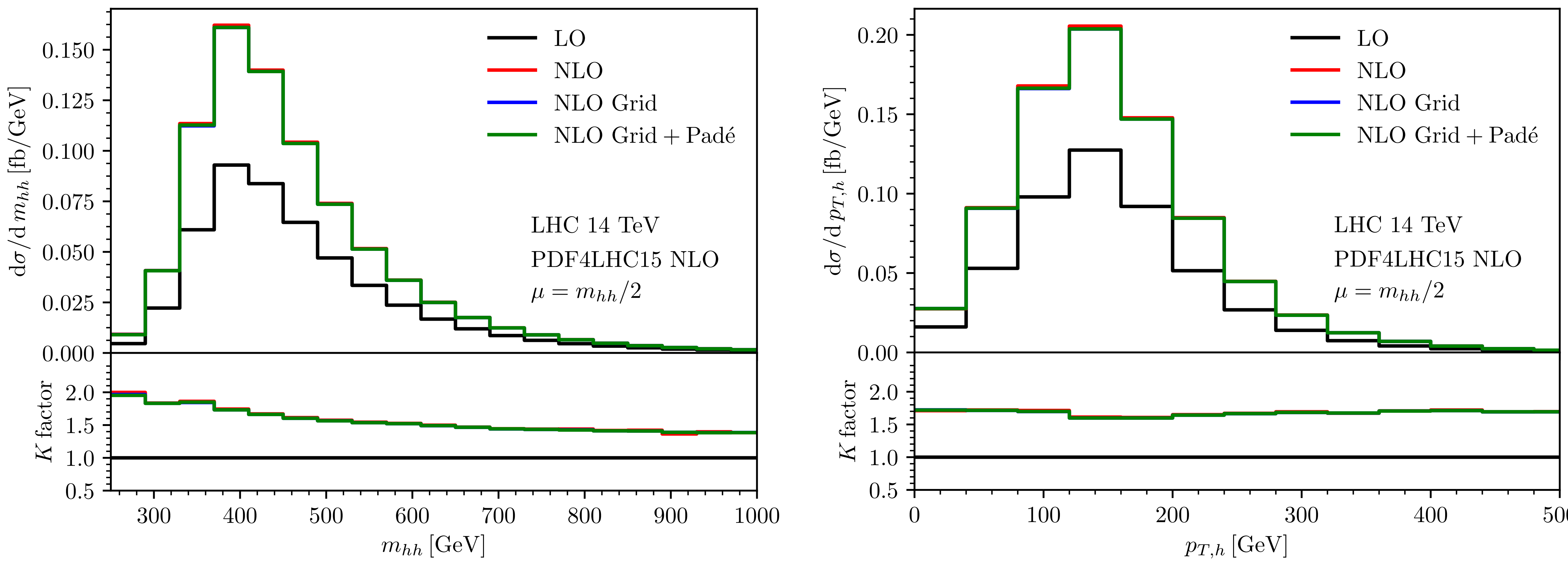
Including Padé points (above cut) has limited impact on grid (full result maintained)

Large m_{hh}:

Grid faithfully reproduces Padé results, much reduced fluctuations

At very large m_{hh} & small $p_{T,h}$ some fluctuations remain, can in principle be pushed to higher m_{hh} by increasing grid density in this region

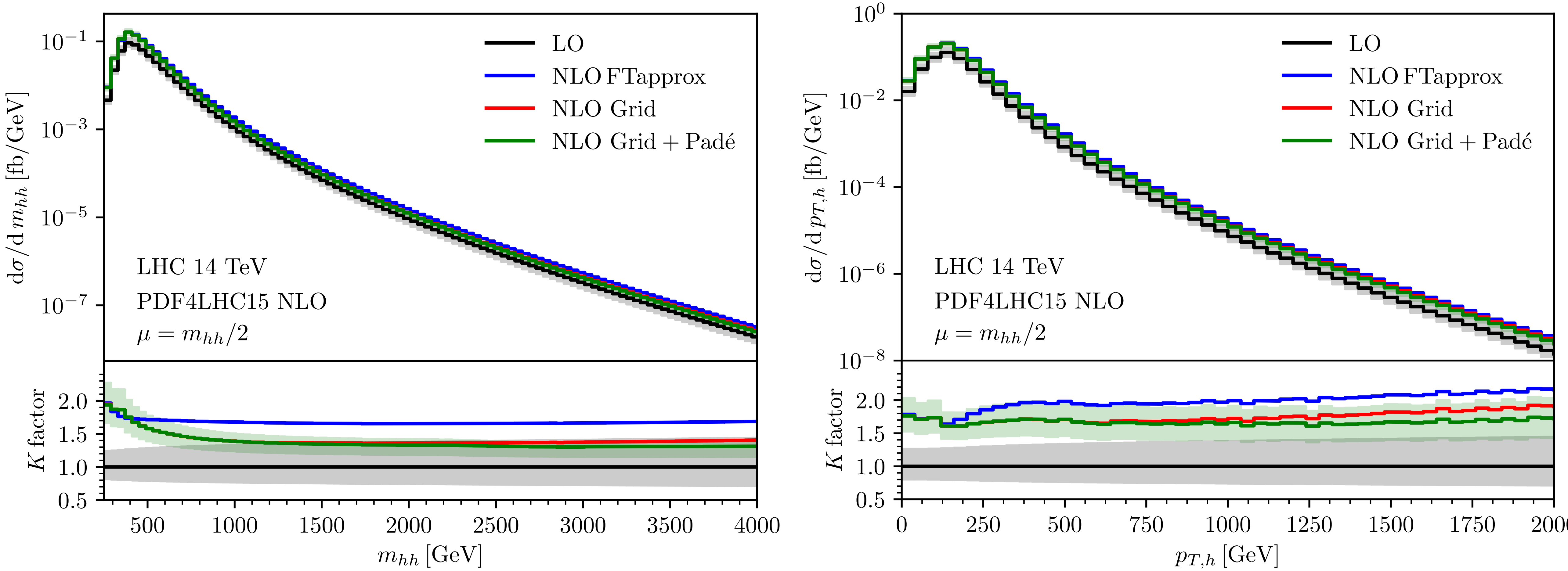
Grid Validation



Original result sampled according to LO unweighted events for σ_{tot}, expect result valid for $m_{hh} \lesssim 1$ TeV, $p_{T,h} \lesssim 500$ GeV

Observe excellent agreement between original result, grid result based on 6320 samples and grid result enhanced with Padé approx.

Results 14 TeV

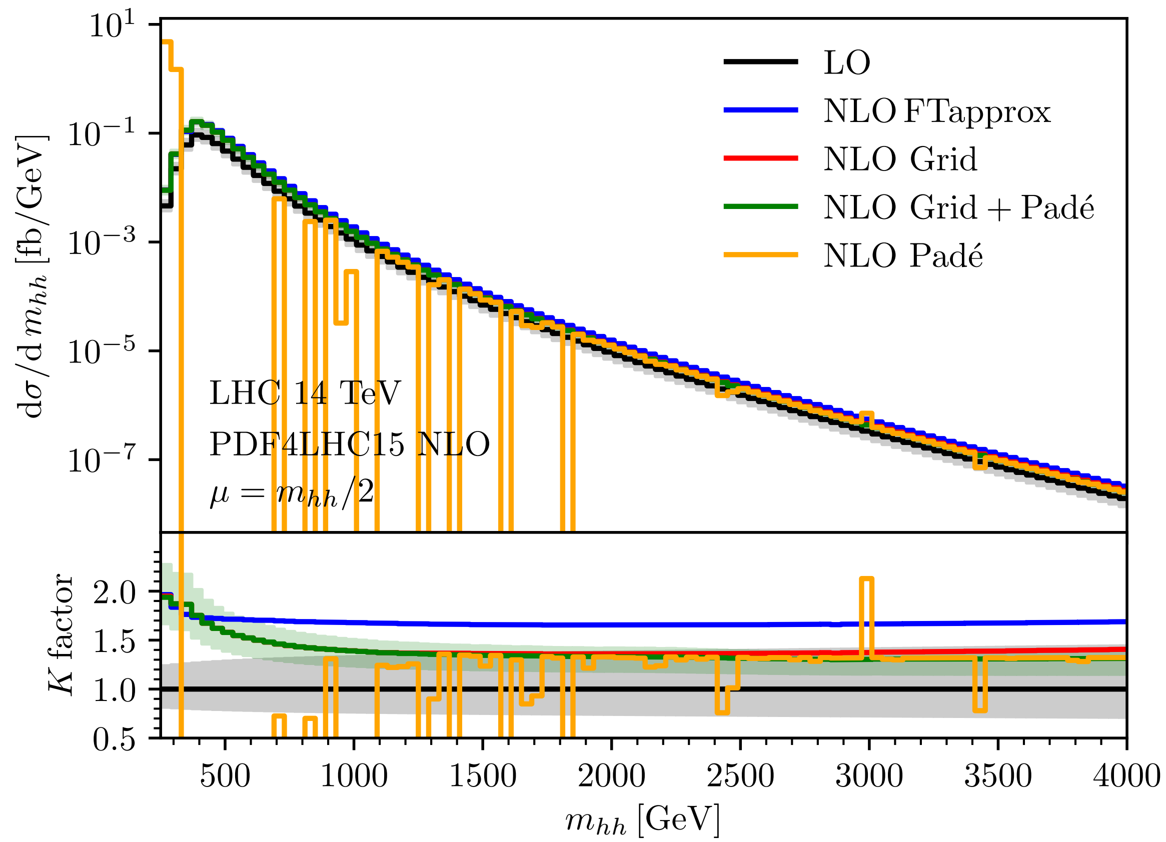


For $m_{hh} \gtrsim 1.5$ TeV or $p_{T,h} \gtrsim 800$ GeV grid result has no support and relies purely on extrapolation which leads to unphysical behaviour

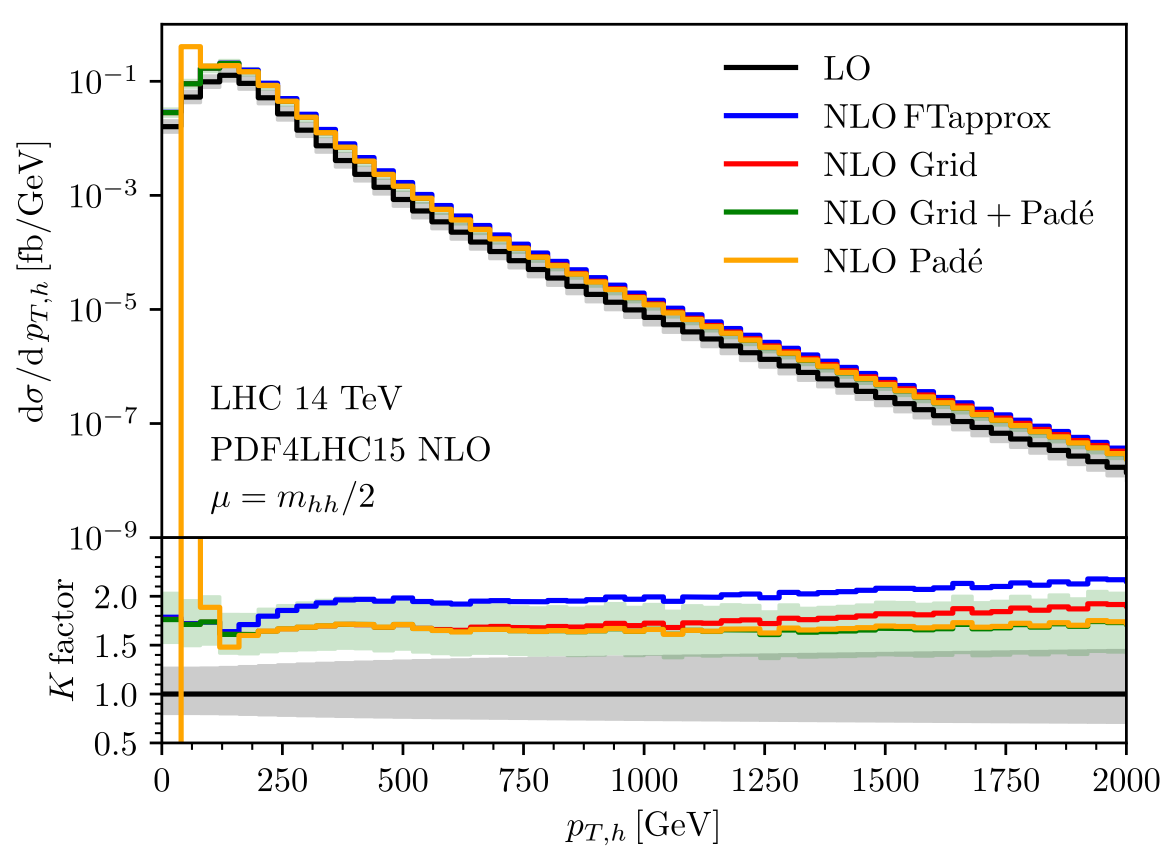
Observe difference between grid and Padé improved grid which grows for large m_{hh} or $p_{T,h}$

Including high-energy expansion tames the growth of the amplitude

Validation 14 TeV: Invariant Mass & pT

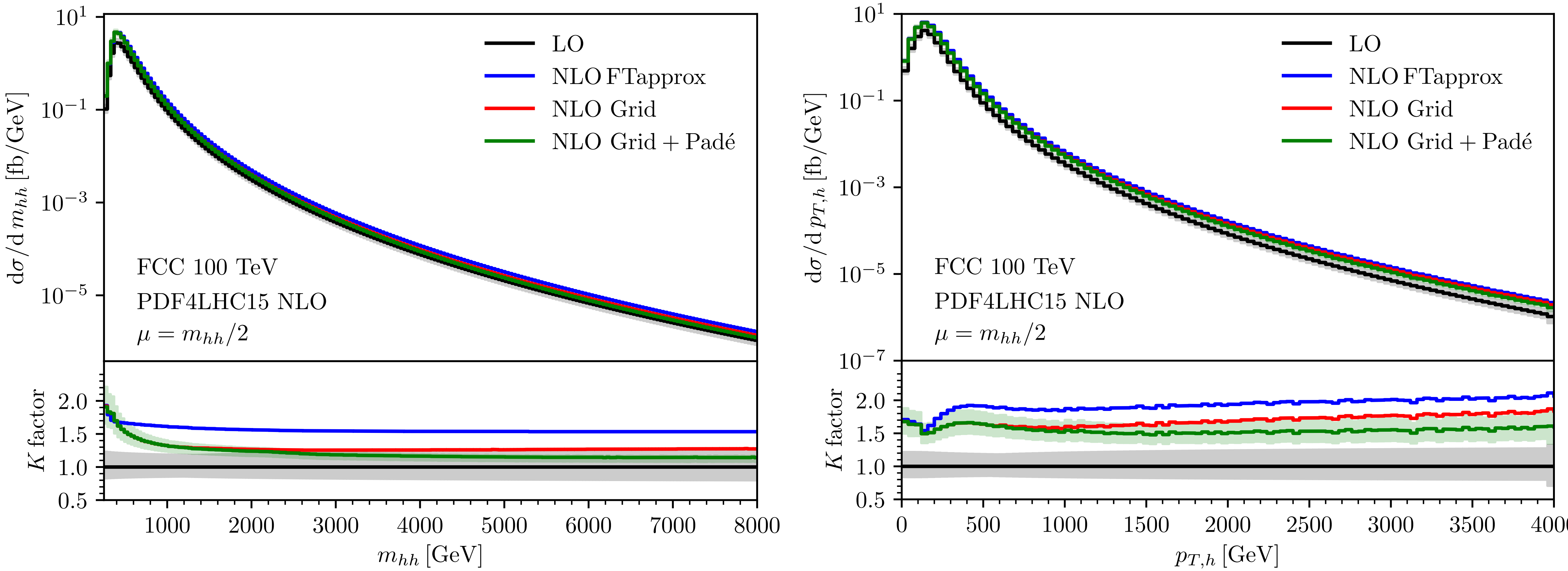


Re-evaluating all virtual samples using only the Padé approximation: find broad agreement but with extremely large fluctuations over the entire range of m_{hh} (expected since we do not always have $m_h^2, m_t^2 \ll |t|$)



Looking instead at $p_{T,h}$ we see that the fluctuations happen only for low $p_{T,h} \lesssim 180$ GeV, for large $p_{T,h}$ we always satisfy $m_h^2, m_t^2 \ll |t|, s$ and find excellent agreement between Padé and grid + Padé

Results 100 TeV



Impact of high-energy expanded results more significant for $\sqrt{s_H} = 100$ TeV, at large m_{hh} and $p_{T,h}$ the Padé improved result goes outside the NLO scale bands

For $p_{T,h} = 2000$ GeV: K-factor reduced from $K \approx 1.7$ to $K \approx 1.5$

Note: same PS points used for all curve (all differences due to virtuals)

Conclusion

Padé Improved Results

- Matching high-energy expanded results to Padé approximations can extend their region of validity
- Obtain good agreement with full result at integral and amplitude level even at fairly low energy

Combined Result

- Grid: https://github.com/mppmu/hhgrid
- Interface to improved grid identical to that used previously for HH e.g. in Parton Shower programs

Outlook

- Expect the combination of numerical results with suitably chosen expansions can yield fast and precise results even for extreme kinematic configurations

Thank you for listening!

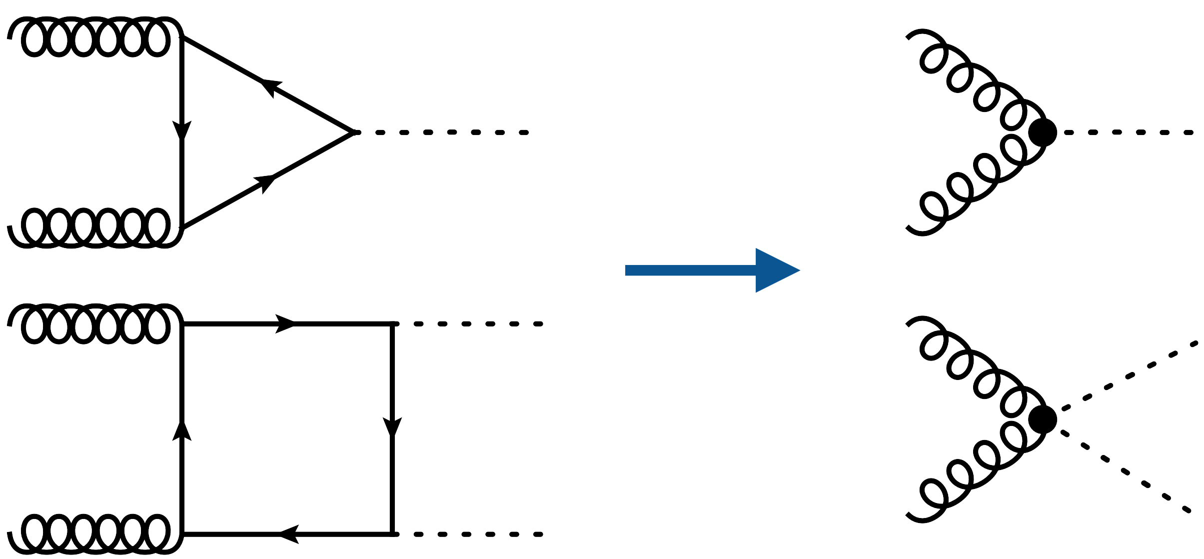
Backup

Heavy Top Limit

Heavy Top Limit (HTL): $m_T \to \infty$

Effective tree-level couplings between gluons and Higgs

Lowers number of loops by 1



HTL valid for

$$\sqrt{\hat{s}} \ll 2m_T$$

HH production for

$$2m_H < \sqrt{\hat{s}}$$

Small energy range in which HTL is technically justified

Born improved NLO HTL:

$$\mathrm{d}\sigma_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}_{\mathrm{NLO}}(m_T) \equiv \underbrace{\frac{\mathrm{d}\sigma_{\mathrm{LO}}(m_T)}{\mathrm{d}\sigma_{\mathrm{LO}}(m_T \to \infty)}}_{\mathrm{N}} \mathrm{d}\sigma_{\mathrm{NLO}}(m_T \to \infty)$$

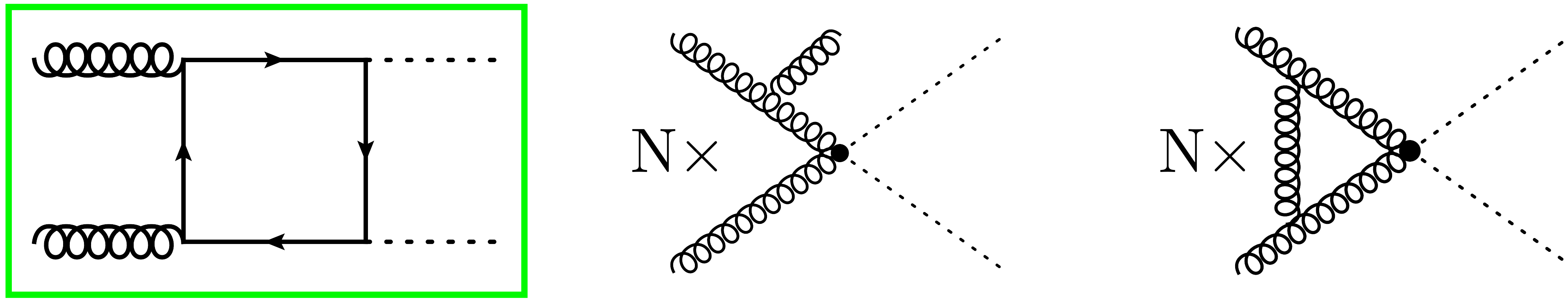
Spira et al. (HPAIR)

HH Approximations @ NLO (Schematically)

B R V

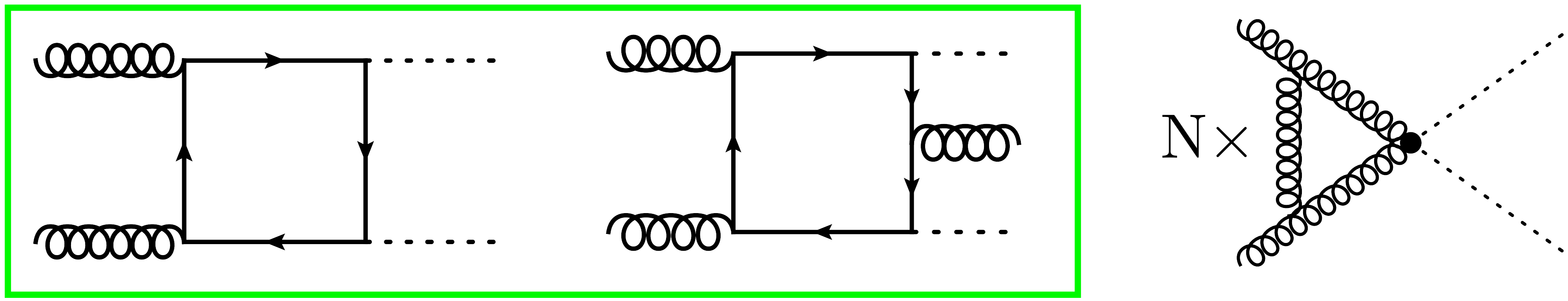
B-i HTL:

Dawson,
Dittmaier, Spira 98

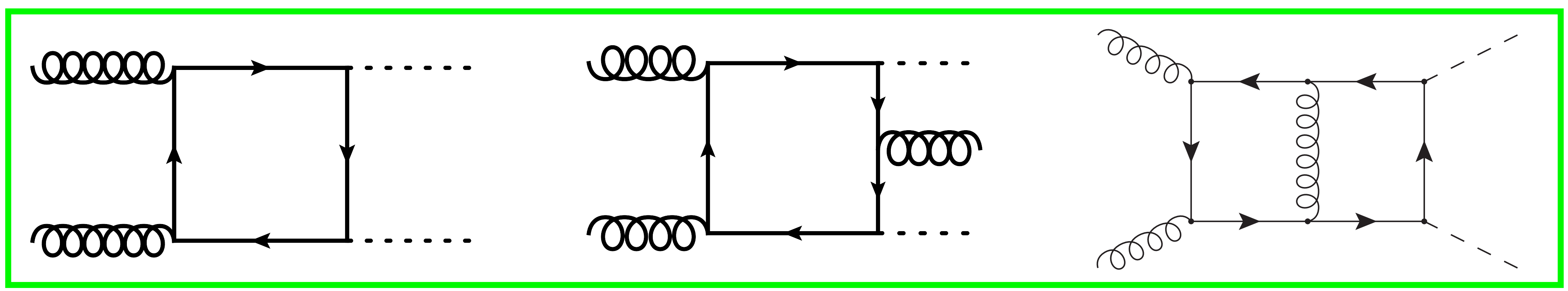


FTapprox:

Maltoni, Vryonidou,
Zaro 14



Full Theory:

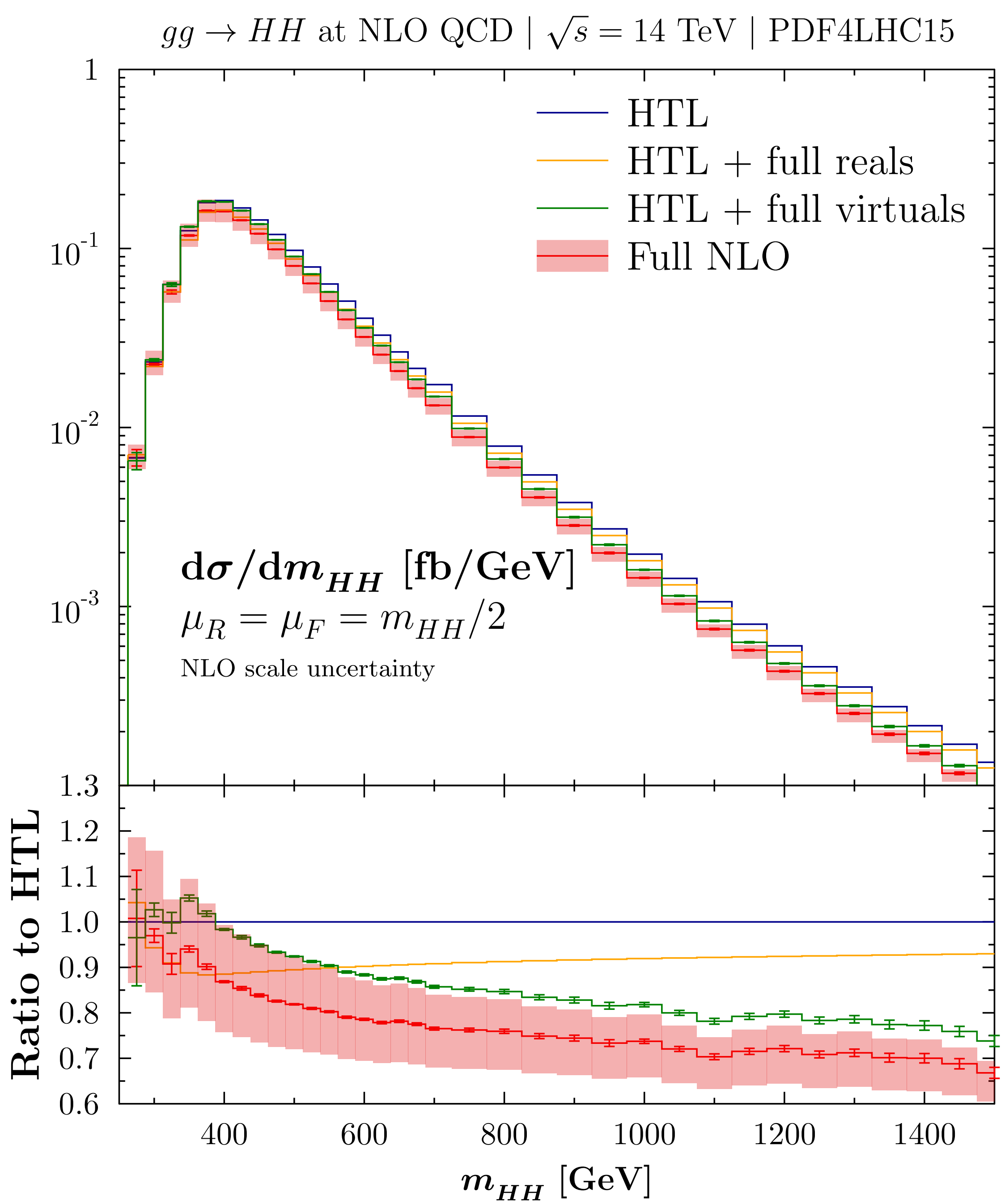


Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16;
Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16;
Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18;

HH Top Quark Mass Scheme Uncertainties

HH recently recomputed by another group (also using numerical methods)

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18



Mutual agreement with our result

Studied top quark mass scheme/scale uncertainties:

$$\frac{d\sigma(gg\to HH)}{dQ}\Big|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\,\text{fb/GeV},$$

$$\frac{d\sigma(gg\to HH)}{dQ}\Big|_{Q=400\text{ GeV}} = 0.1609(4)^{+7\%}_{-7\%}\,\text{fb/GeV},$$

$$\frac{d\sigma(gg\to HH)}{dQ}\Big|_{Q=600\text{ GeV}} = 0.03204(9)^{+0\%}_{-26\%}\,\text{fb/GeV},$$

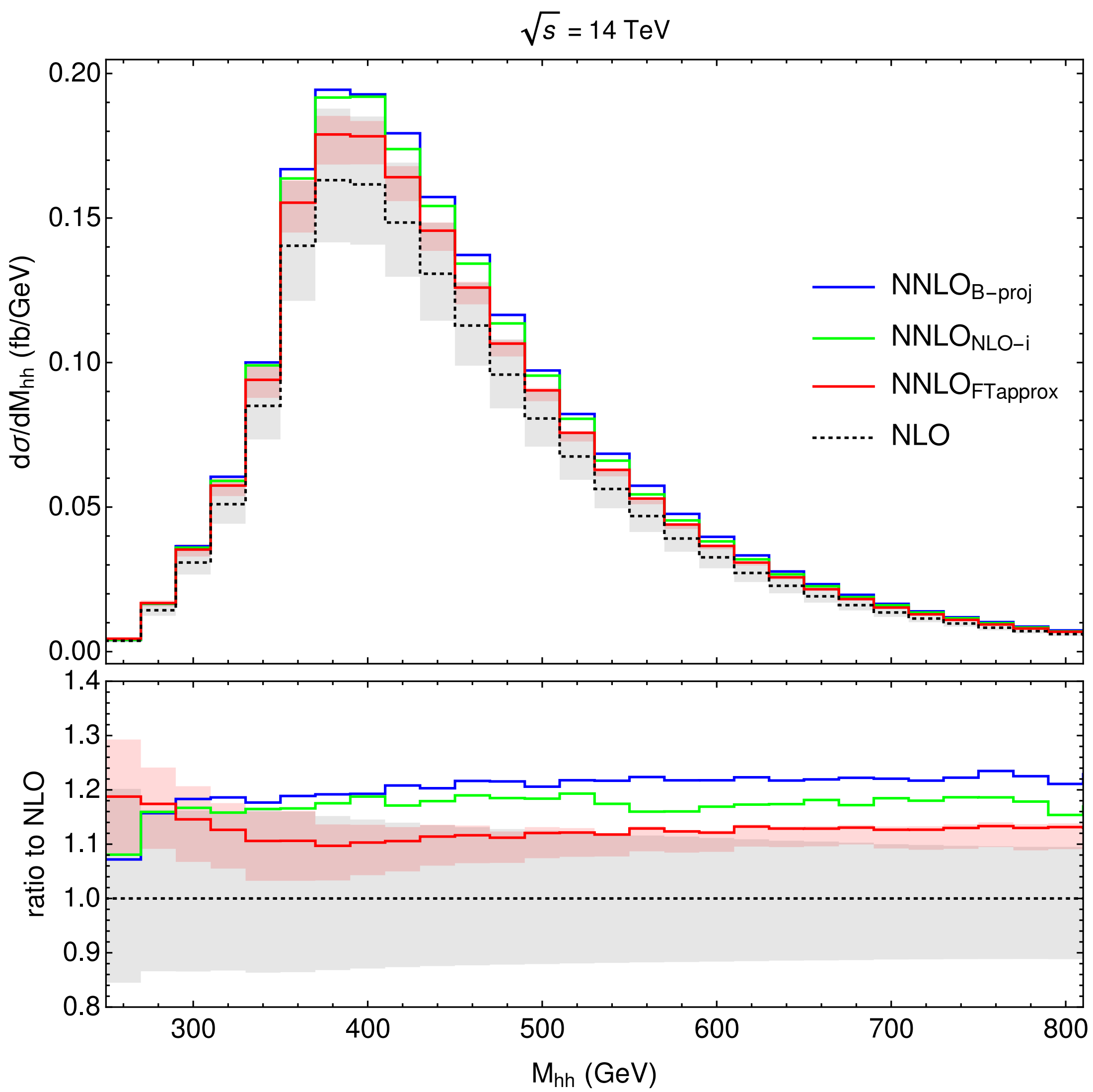
$$\frac{d\sigma(gg\to HH)}{dQ}\Big|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\,\text{fb/GeV},$$

Large uncertainty obtained varying scale of top quark mass (in $\overline{\text{MS}}$) by factor 2 up/down

HH: NNLO EFT Combined with NLO SM

Differential NNLO HTL + NLO SM

Top quark mass effects studied using 3 different approximations



$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\mathrm{NLO_{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\mathrm{NNLO_{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\mathrm{NNLO_{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\mathrm{NNLO_{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\mathrm{NNLO_{FTapprox}}$	±2.6%	±2.7%	±3.4%	±4.6%
$\mathrm{NNLO_{FTapprox}/NLO}$	1.118	1.116	1.096	1.067

1) **$\mathrm{NNLO_{NLO\text{-}i}}$**

Rescale NLO by $K_{NNLO} = NNLO_{HTL}/NLO_{HTL}$

2) **$\mathrm{NNLO_{B\text{-}proj}}$**

Project real radiation contributions to Born configurations, rescale by LO/LO_{HEFT}

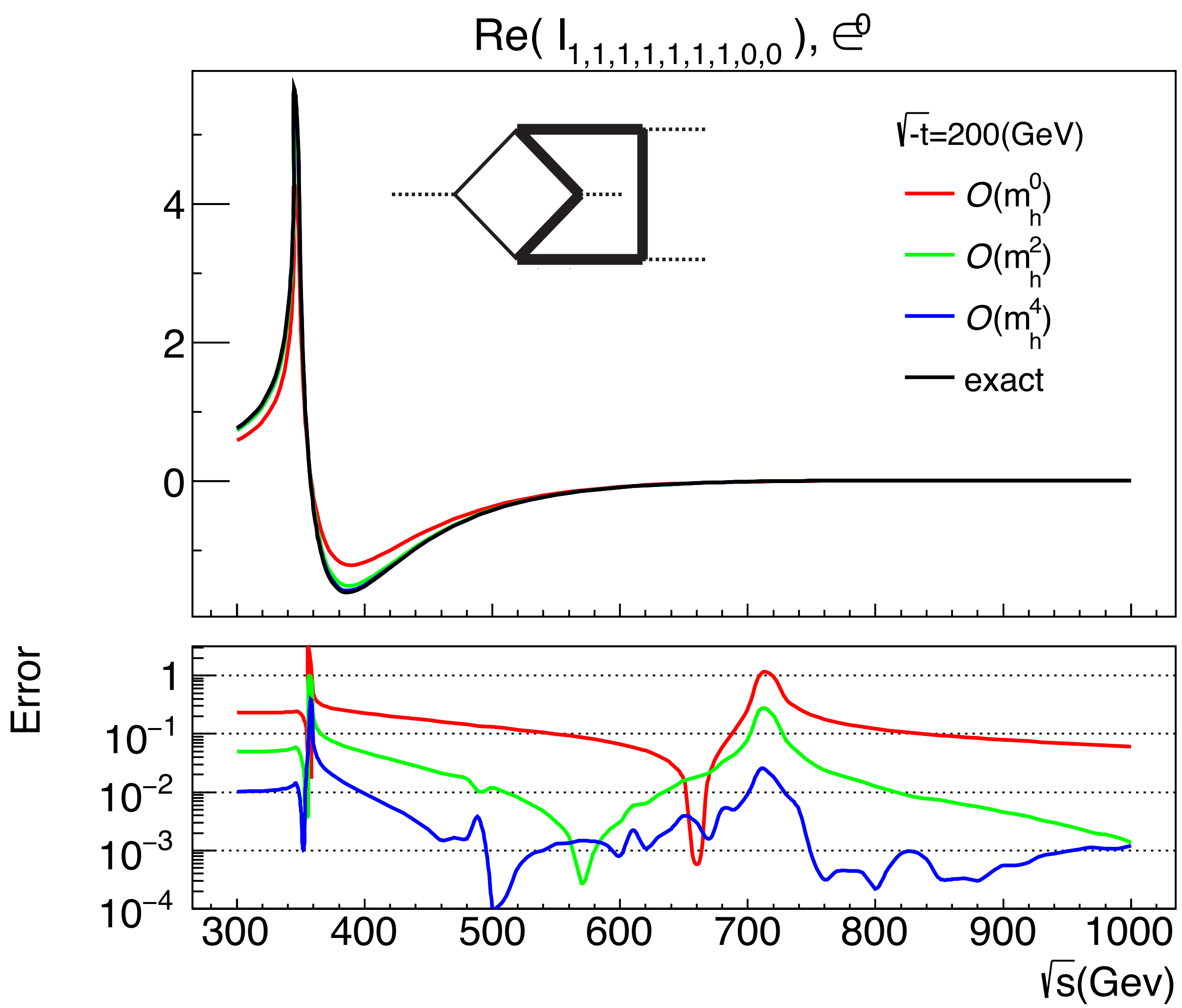
3) **$\mathrm{NNLO_{FTapprox}}$**

NNLO EFT correction rescaled for each multiplicity by:

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\mathrm{Born}}_{\mathrm{Full}}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\mathrm{HEFT}}(ij \to HH + X)}$$

Grazzini, Heinrich, SJ, Kallweit, Kerner, Lindert, Mazzitelli 18; (+NNLL) de Florian, Mazzitelli 18;

HH Production via Expansions



Alternatively:

Expand only about small m_H^2

Larger range of validity, some integrals significantly more involved (Elliptic) Xu, Yang 18

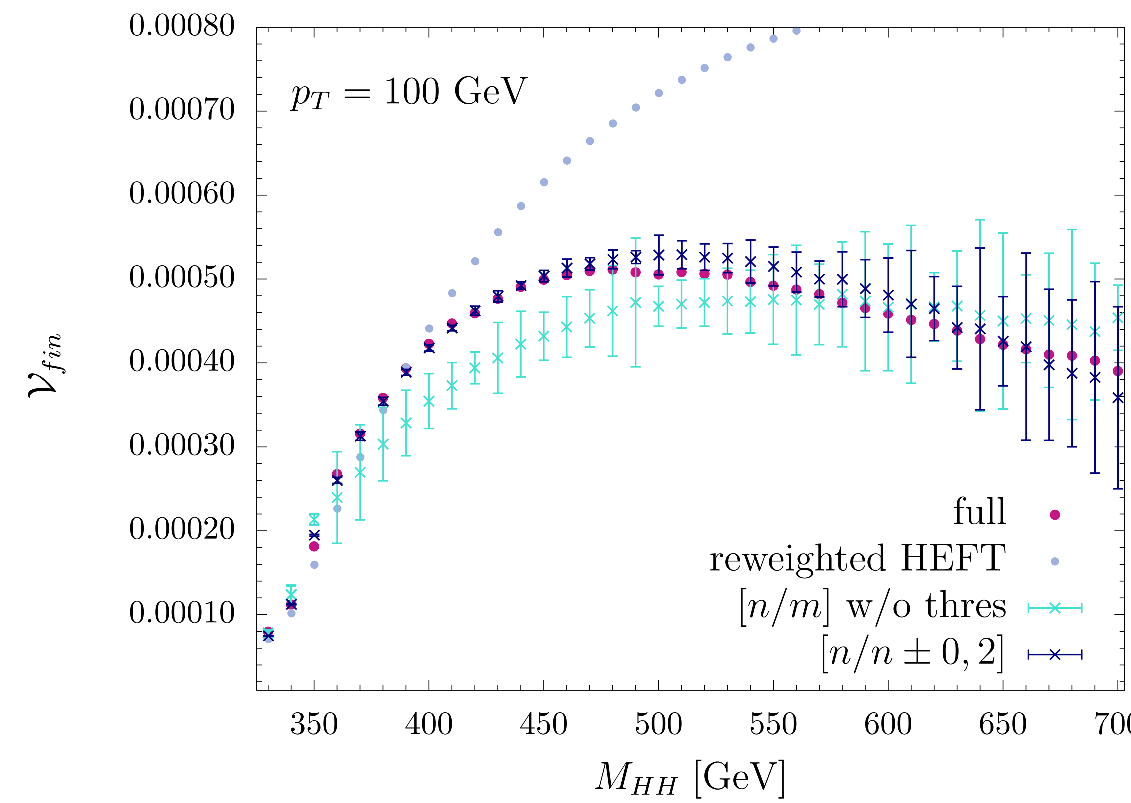
HH Production via Expansions (II)

Produce a Padé approximation using:
Large- m_T expansion + threshold expansion

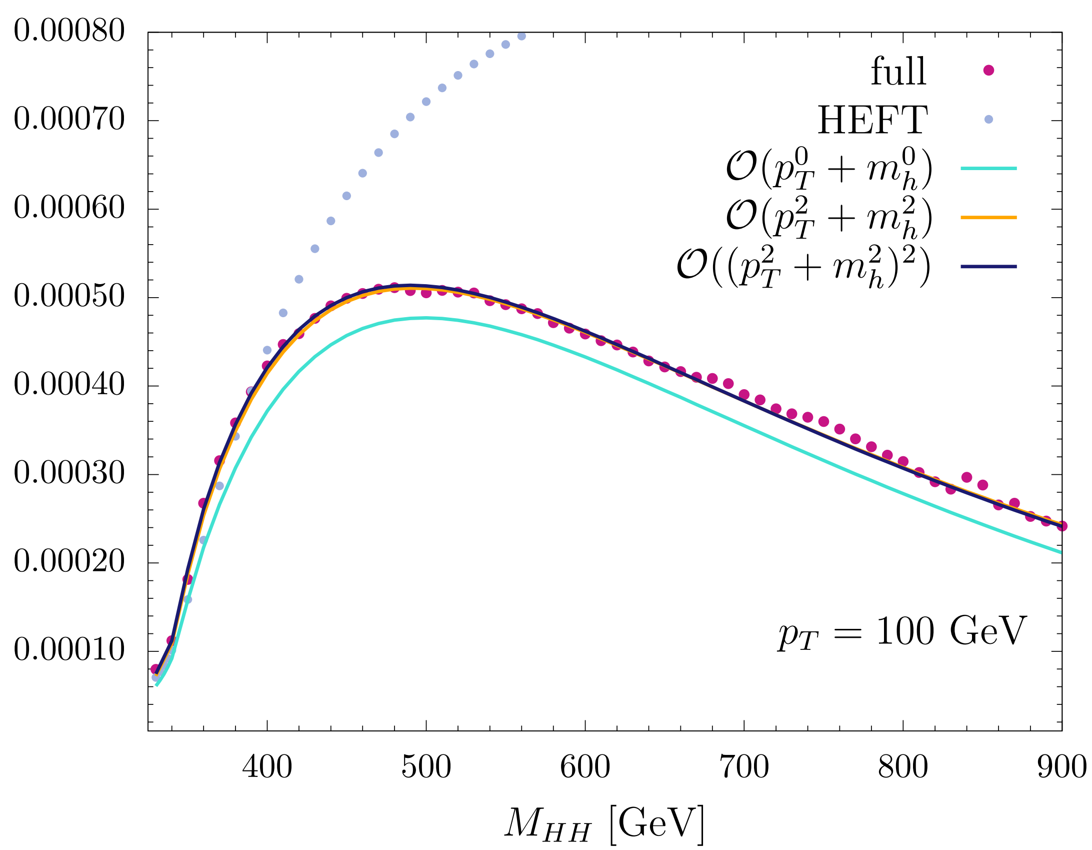
Incorporate non-analytic threshold corrections into approximation

Method applicable to more processes:

$$gg \to H^{(*)}, HZ, ZZ$$



Gröber, Maier, Rauh 17



Have: $p_T^2 + m_H^2 \leq \hat{s}/4$

Expand in: $p_T^2 + m_H^2$

Solve remaining dependence on $\hat{s}, m_T$

Invariant mass distribution agrees well with full (numerical) result up to ~900 GeV

Bonciani, Degrassi, Giardino, Gröber 18

Compute masters: differential equations

Differentiate master integrals wrt $X \in \{s,t,m_t^2\}$. IBP reduce result:

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{J} = M(s,t,m_t^2,\epsilon)\cdot\vec{J}.$$

Slide: J. Davies
ACAT 2019

m_t^2 **equation:** substitute high-energy ansatz for each master integral,

$$J = \sum_i \sum_j \sum_k C_{ijk}(s,t)\,\epsilon^i\,(m_t^2)^j\,\log(m_t^2)^k.$$

Obtain a system of linear equations for coefficients $C_{ijk}(s,t)$. **Solve!**

... we require **Boundary Conditions**

- determine leading powers in $m_t^2 \to$ fixes some $C_{ijk}(s,t)$

Here we determine the amplitude to m_t^{32} with `Mathematica` ... difficult!