Higher-order calculations and precision phenomenology

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Outline

- Introduction
- NLO calculations
- NNLO
  - jets
  - heavy-quarks
- Beyond NNLO
  - NNLO QCD + NLO EW for dibosons
  - N3LO
- Summary & Outlook

Disclaimer: a (personal) selection of recent fixed order QCD results!
QCD at hadron colliders

\[
\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij}(p_1, p_2, \alpha_s(\mu_R), Q^2; \mu_F^2, \mu_R^2) + \mathcal{O} \left( \frac{\Lambda_{QCD}}{Q} \right)
\]

Parton distributions: universal but not perturbatively computable

Hard partonic cross section: process dependent but computable in perturbation theory

Power-suppressed contributions

The factorisation picture is systematically improvable (until the power-suppressed contributions become quantitative relevant…)

Talks by Kassabov, Alekhin, Nadolsky…

\[\begin{align*}
h_1 & \quad x_1 \\
h_2 & \quad x_2
\end{align*}\]
The NLO revolution has left us with flexible tools that make possible to carry out NLO QCD+EW computations

- Realistic final states with off-shell effects and interferences
- Merging to Parton Shower and full deployment into Monte Carlo tools used in experimental analyses

Treatment of QCD IR singularities based on well established CS and FKS methods

Focus is now on NNLO (and beyond) but....

....NLO for loop-induced processes require two-loop amplitudes!
NLO: Higgs at high $p_T$  

Higgs production at high-$p_T$ can be useful to test new physics scenarios

New Physics could change the high-$p_T$ spectrum mildly affecting the inclusive rates

For example: current constraints on the charm Yukawa $y_c$ are rather weak but if $y_c$ is very different from its SM value  

$\rightarrow$ effect on Higgs $p_T$ distribution  

see e.g. Bishara, Haisch, Monni, Re (2016)

Up to very recently the theoretical predictions beyond LO only available in the large-$m_t$ limit

De Florian, Kunszt, MG (1999)  
Glosser, Schmidt (2002)  
Ravindran, Smith, van Neerven (2002)

Exact NLO calculation requires 2-loop amplitudes with different mass scales: this is at the forefront of current technologies!
NLO: Higgs at high $p_T$

First exact NLO calculation recently completed numerically

Jones, Kerner, Luisoni (2018)

Trick used: $m_H^2/m_{top}^2 = 12/23$

eliminates one scale

K-factor similar to the one obtained in the large-$m_{top}$ limit

Consistent with approximate result valid at large $p_T$

Lindert et al (2018)

Combined with NNLO in EFT leads to accurate reference predictions for boosted analyses

<table>
<thead>
<tr>
<th>$p^\text{cut}_T$</th>
<th>NNLO$^{\text{approximate}}_{\text{quad.unc.}}$ [fb]</th>
<th>NNLO$^{\text{approximate}}_{\text{lin.unc.}}$ [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 GeV</td>
<td>$32.0^{+9.1}_{-11.6}$%</td>
<td>$32.0^{+9.4}_{-11.9}$%</td>
</tr>
<tr>
<td>430 GeV</td>
<td>$22.1^{+9.0}_{-11.4}$%</td>
<td>$22.1^{+9.3}_{-11.8}$%</td>
</tr>
<tr>
<td>450 GeV</td>
<td>$17.4^{+8.9}_{-11.5}$%</td>
<td>$17.4^{+9.3}_{-11.9}$%</td>
</tr>
</tbody>
</table>

HXSWG ggF subgroup, preliminary
NLO: $gg \rightarrow ZH$

Despite highly accurate NNLO QCD+NLO-EW predictions still ZH not fully under control

$gg$ induced loop contribution (first appears at NNLO and leads to large uncertainties!)

S. Dittmaier et al. HXSWG YR4 (2016)

<table>
<thead>
<tr>
<th>$\sigma$ (fb)</th>
<th>NLO</th>
<th>NNLO (DY-like)</th>
<th>NNLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC8</td>
<td>$0.2820^{+2%}_{-2%}$</td>
<td>$0.2574^{+3%}_{-4%}$</td>
<td>$0.3112^{+3%}_{-2%}$</td>
</tr>
<tr>
<td>LHC14</td>
<td>$0.2130^{+10%}_{-12%}$</td>
<td>$0.1770^{+7%}_{-6%}$</td>
<td>$0.2496^{+5%}_{-2%}$</td>
</tr>
</tbody>
</table>

Impact of $gg \rightarrow ZH$

Very important in the boosted region

NLO corrections known only in large $m_t$ limit ($\sim 100\%$)

Altenkamp et al. (2012)
NNLO: building blocks

- Tree-level amplitudes with two additional partons

- One-loop amplitudes with one additional parton (to be evaluated in unresolved regions where instabilities may arise)

- Two-loop amplitudes currently the major bottleneck (new class of functions, charting new territory…)

Crucial to keep the calculation fully differential: corrections for fiducial and inclusive rates may be significantly different (H in VBF, WW…)

All the three contributions separately divergent!
NNLO methods

Broadly speaking there are two approaches that we can follow:

- **Organise the calculation from scratch so as to cancel all the singularities**
  - Sector Decomposition (SD)
  - antenna subtraction
  - colourful subtraction
  - subtraction+sector decomposition
    (stripper, nested subtractions…)

  Anastasiou, Melnikov, Petriello (2004)
  Gehrmann, Glover (2005)
  Somogyi, Trocsanyi, Del Duca (2005, 2007)
  Czakon (2010,2011)
  Boughezal, Melnikov, Petriello (2011)
  Caola, Melnikov, Rontsch (2017)]

- **Start from an inclusive NNLO calculation (sometimes obtained through resummation) and combine it with an NLO calculation for n+1 parton process**
  - $q_T$ subtraction
  - N-jettiness method
  - born projection (P2B) method

  [References: Catani, MG (2007)
  Boughezal, Focke, Liu, Petriello (2015)
  Tackmann et al. (2015)
  Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)]

Search for an “ideal” subtraction method that can be applied as easily as CS or FKS at NLO is still subject of intense work
NNLO results lead to much better description of the data.
Triple differential di-jet cross section as a function of the average $p_T$ of the leading jets $y^*=|y_1- y_2|/2$ and $y_b=|y_1+ y_2|/2$

NNLO, NPxEW of the same order

EW correction included assuming factorisation
Heavy quarks

Extension of $q_T$ subtraction to heavy-quark production now completed

\[ d\sigma_{tt}^{NLO} = H_{tt}^{NLO} \otimes d\sigma_{LO}^{tt} + \left[ d\sigma_{t\bar{t} + \text{jets}}^{NLO} - d\sigma_{\text{CT}}^{NLO} \right] \]
Extension of $q_T$ subtraction to heavy-quark production now completed

$$d\sigma_{(N)NLO}^{t\bar{t}} = \mathcal{H}_{(N)NLO}^{t\bar{t}} \otimes d\sigma_{LO}^{t\bar{t}} + \left[d\sigma_{(N)LO}^{t\bar{t}+\text{jets}} - d\sigma_{CT(N)LO}^{C}\right]$$

✅ **Modified subtraction counterterm fully known**

Additional perturbative ingredient: soft anomalous dimension $\Gamma_t$ known at NNLO

Catani, Devoto, Kallweit, Mazzitelli, Sargsyan, MG (2019)
Mitov, Sterman, Sung (2009)
Extension of $q_T$ subtraction to heavy-quark production now completed

$$d\sigma_{\overline{t}t}^{NNLO} = \mathcal{H}_{\overline{t}t}^{NNLO} \otimes d\sigma_{\overline{t}t}^{LO} + \left[ d\sigma_{\overline{t}t+\text{jets}}^{NNLO} - d\sigma_{\text{CT}}^{NNLO} \right]$$

✅ Modified subtraction counterterm fully known

Additional perturbative ingredient: soft anomalous dimension $\Gamma_t$ known at NNLO

✅ Additional soft contributions needed to evaluate $\mathcal{H}_{\overline{t}t}^{NNLO}$

Catani, Devoto, Kallweit, Mazzitelli, Sargsyan, MG (2019)

Mitov, Sterman, Sung (2009)

Catani, Devoto, Mazzitelli, MG, to appear
Heavy quarks

Catani, Devoto, Kallweit, Mazzitelli, Sargsyan, MG (2019)

Extension of $q_T$ subtraction to heavy-quark production now completed

\[ d\sigma_{t\bar{t}}^{(N)NLO} = \mathcal{H}_{t\bar{t}}^{(N)NLO} \otimes d\sigma_{t\bar{t}}^{LO} + \left[ d\sigma_{t\bar{t}+\text{jets}}^{(N)LO} - d\sigma_{CT}^{(N)LO} \right] \]

- Modified subtraction counterterm fully known
- Additional perturbative ingredient: soft anomalous dimension $\Gamma_t$ known at NNLO
- Additional soft contributions needed to evaluate $\mathcal{H}_{t\bar{t}}^{(N)NLO}$

Inclusive cross section

<table>
<thead>
<tr>
<th>$\sigma_{NNLO}$ [pb]</th>
<th>Matrix</th>
<th>Top++</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 TeV</td>
<td>238.5(2)$^{+3.9%}_{-6.3%}$</td>
<td>238.6$^{+4.0%}_{-6.3%}$</td>
</tr>
<tr>
<td>13 TeV</td>
<td>794.0(8)$^{+3.5%}_{-5.7%}$</td>
<td>794.0$^{+3.5%}_{-5.7%}$</td>
</tr>
<tr>
<td>100 TeV</td>
<td>35215(74)$^{+2.8%}_{-4.7%}$</td>
<td>35216$^{+2.9%}_{-4.8%}$</td>
</tr>
</tbody>
</table>

Tree and loop amplitudes from Openloops 2 (cross check with Recola)

Two-loop amplitudes from Czakon et al. (0.1% effect at 13 TeV)
LO, NLO and NNLO predictions obtained using NNPDF3.1 PDFs with $\alpha_S(m_Z)=0.118$ at the corresponding order

CMS data of CMS-TOP-17-002 in the lepton+jets channel

Extrapolation to parton level in the inclusive phase space

Our calculation is carried out without cuts

To compare with data we multiply our absolute predictions by 0.438 (semileptonic BR of the $t\bar{t}$ pair) times $2/3$ (only electrons and muons)
Heavy quarks

As noted in various previous analyses the measured $p_T$ distribution is slightly softer than the NNLO prediction.

Perturbative prediction relatively stable when going from NLO to NNLO.

Data and theory are consistent within uncertainties.
Heavy quarks

Good description of the data except in the first bin

Issues in extrapolation? Smaller $m_t$?

A smaller $m_t$ (just by about 2 GeV) leads to a higher theoretical prediction in this bin and to small changes at higher $m_{tt}$

CMS-TOP-18-004: leptonic channel: a fit with the same PDFs leads to $m_t = 170.81 \pm 0.68$ GeV
Heavy quarks

As for the single-differential distribution the $p_T$ distribution is softer than the NNLO prediction in all the rapidity intervals.
Heavy quarks

At LO there is a kinematical boundary \( m_{tt} > 2m_{T_{\text{min}}} \) starting from NLO, smeared by the relatively large bin size.

NNLO result nicely describes the data except in the first \( m_{tt} \) (first two panels).
Comparison with Czakon et al.

Excellent agreement even in extreme kinematical regions
More NNLO progress

We have now even NNLO computations for production + decay

- t-channel single top with $t \rightarrow Wb$ (N-jettiness + P2B)
- VH with $H \rightarrow bb$
  - $q_T$ + colourful
  - nested subtractions
  - antenna
- $t \bar{t}$ with $t \rightarrow Wb$ (stripper)

Interesting issues with extrapolation

Berger, Gao, Yuan, Zhu (2016)
Ferrera, Somogyi, Tramontano (2017)
Caola, Luisoni, Melnikov, Röntsch (2017)
Gauld, Gehrmann De Ridder, Glover, Huss, Mayer (2019)
Czakon et al (2019)
NNLO: deployment of results

NNLO computations are generally rather expensive (may need up to $O(10^6)$ CPU hours for a production run): most results obtained through private codes

- Fast tool for total cross sections and repository for differential distributions
  - Top-quark pairs

- NTUPLES
  - Viable at NNLO? (LH17 estimate: 2jets should require $O(100 \text{ TB})$)

- Applegrid (fast interpolation grids)
  - Makes use in PDF fits possible

- Public codes for limited sets of processes
  - Process specific: FEWZ, DYNLO, HNNLO, 2γNNLO, proVBFH...
  - General purpose: MCFM, MATRIX

Talk by Rabbertz
MCFM has marked the Tevatron era as “the tool” for NLO computations

An increasing number of processes now implemented at NNLO accuracy by using N-jettiness

<table>
<thead>
<tr>
<th>Process</th>
<th>nproc</th>
<th>(\sigma_{\text{NLO}} \pm \delta\sigma_{\text{MC}}^{\text{NLO}})</th>
<th>(\sigma_{\text{NNLO}} \pm \delta\sigma_{\text{MC}}^{\text{NNLO}} \pm \delta\sigma_{\text{NNLO}}^{\text{be}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W^+)</td>
<td>1</td>
<td>(4.220 \pm 0.002) nb</td>
<td>(4.19 \pm 0.02 \pm 0.043) nb</td>
</tr>
<tr>
<td>(W^-)</td>
<td>6</td>
<td>(3.315 \pm 0.001) nb</td>
<td>(3.23 \pm 0.01 \pm 0.033) nb</td>
</tr>
<tr>
<td>(Z)</td>
<td>31</td>
<td>(885.2 \pm 0.3) pb</td>
<td>(878 \pm 3 \pm 9) pb</td>
</tr>
<tr>
<td>(H)</td>
<td>112</td>
<td>(1.395 \pm 0.001) pb</td>
<td>(1.865 \pm 0.004 \pm 0.019) pb</td>
</tr>
<tr>
<td>(\gamma\gamma)</td>
<td>285</td>
<td>(27.94 \pm 0.01) pb</td>
<td>(43.60 \pm 0.06 \pm 0.44) pb</td>
</tr>
<tr>
<td>(W^+H)</td>
<td>91</td>
<td>(2.208 \pm 0.002) fb</td>
<td>(2.268 \pm 0.007 \pm 0.023) fb</td>
</tr>
<tr>
<td>(W^-H)</td>
<td>96</td>
<td>(1.494 \pm 0.001) fb</td>
<td>(1.519 \pm 0.004 \pm 0.015) fb</td>
</tr>
<tr>
<td>(ZH)</td>
<td>110</td>
<td>(0.7535 \pm 0.0004) fb</td>
<td>(0.846 \pm 0.001 \pm 0.0085) fb</td>
</tr>
<tr>
<td>(Z\gamma)</td>
<td>300</td>
<td>(959 \pm 8) fb</td>
<td>(1268 \pm 22) fb</td>
</tr>
</tbody>
</table>

Talk by T. Neumann

New implementation of H+jet helped to solve long standing discrepancies with other calculations

Campbell, Ellis, Seth (2019)
MATRIX

Kallweit, Wiesemann, MG (2017) + Devoto, Mazzitelli, Yook....

MUNICH
S. Kallweit

NNLO (+NNLL)

q_T Subtraction
S. Catani and M. Grazzini (2007)

OpenLoops
F. Cascioli, P. Maierhöfer and S. Pozzorini (2011)
F. Cascioli, J. Lindert, P. Maierhöfer and S. Pozzorini (2014)
F. Bucconi, S. Pozzorini, M. Zoller (2018)

COLLIER

VVAMP

GiNaC

TDHPL
T. Gehrmann and E. Remiddi

Munich -- the Multi-channel Integrator at swiss (CH) precision -- Automates qT-subtraction and Resummation to Integrate X-sections

\[ \text{\textsc{Matrix}} \]

Version: 1.0.0
Reference: arXiv:1711.06631

Nov 2017
MATRIX

- pp → Z/γ° (→ l+l−) ✔
- pp → W(→ lν) ✔
- pp → H ✔
- pp → γγ ✔
- pp → Wγ → lνγ ✔
- pp → Zγ → l+l−γ ✔
- pp → ZZ/WW → llνν ✔
- pp → WZ → ll ll ✔
- pp → ZZ(→ 4l) ✔
- pp → WW → (lνl′ν′) ✔
- pp → HH ✔
- pp → t¯t ✔

First public release out in November 2017

Runtime estimate for per mille accurate fiducial cross sections:

From O(10) CPU days for the simplest processes to O(1000) CPU days for t¯t

Talk by Yook

Plus NLO for gluon fusion (not yet in public release)

not in public release
Beyond 2 $\rightarrow$ 2

Current NNLO results limited to 2$\rightarrow$1 and 2$\rightarrow$2

A number of important processes would benefit from NNLO extension: ttH, V+2j, 3j....

- Analytical approach
  - five point amplitudes at leading colour
  - all master integrals for five point
  - master integrals for $t\bar{t}$
    ..................

- Numerical
  - $t\bar{t}$
  - PySecDec (HH, H+jet...)
  - HH

E.g. ttH: statistical accuracy could go down from $O(15\%)$ to $O(2\%)$ at the end of HL-LHC

Talks by Badger, Tancredi....

Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov (2019)

Gehrmann et al. (2019)

Bonciani et al. (2019)

Czakon et al (2013)

Borowka, Heinrich, Jones, Kerner....

Spira et al. (2018)
Beyond NNLO QCD
NNLO QCD+NLO EW for dibosons

Kallweit, Lindert, Pozzorini, Wiesemann, MG (to appear)

Different combination prescriptions
Different combination prescriptions
NNLO QCD+NLO EW for dibosons

Kallweit, Lindert, Pozzorini, Wiesemann, MG (to appear)
Kallweit, Lindert, Pozzorini, Wiesemann, MG (to appear)

**Giant NLO QCD K-factor driven by V+jet subprocess**
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Photon induced processes should not be included in multiplicative combination!
**NNLO QCD+NLO EW for dibosons**

Giant NLO QCD K-factor driven by V+jet subprocess

High-$p_T$ Sudakov suppression spoiled for WZ and WW

Photon induced processes should not be included in multiplicative combination!
NNLO QCD+NLO EW for dibosons

Kallweit, Lindert, Pozzorini, Wiesemann, MG (to appear)
NNLO QCD+NLO EW for dibosons

Jet veto $H_{T,jet} < 0.2 H_{T,lep}$ restores expected behaviour
For some benchmark processes $N^3LO$ leads to a reduction of theoretical uncertainties and increases our confidence on the perturbative convergence.

- Rapidity distribution in Higgs production  
  Dulat, Mistlberger, Pelloni (2019)

- Fully differential Higgs production (qT subtraction)  
  Cieri et al (2018)

- $H\rightarrow bb$ (N-jettiness+P2B)  
  Mondini, Schiavi, Williams (2019)

- Inclusive $bb\rightarrow H$  
  Duhr, Dulat, Mistlberger (2019)

- Inclusive $H$ and $HH$ in VBF  
  Karlberg, Dreyer (2018, 2019)

I expect the major impact of $N^3LO$ in the near future could be in the description of the Drell-Yan process where the data are already extremely precise and $N^3LO$ could help constraining the $p_T$ distribution at low $p_T$. 
Summary & Outlook

- LHC precision phenomenology is becoming a tool for BSM searches with new opportunities

- NNLO results now available for essentially all the relevant 2->1 and 2->2 processes and lead to an improved description of the data

- Cross validation of different computations essential in consolidating the results but improvements in subtraction/slicing techniques expected/needed

- Extension to 2->3 requires facing new challenges in the computations of two-loop amplitudes

- NNLO computations challenging also from the point of view of computing resources

  Only a limited subset of the results are publicly available

- N³LO era started with new exciting results