Resummation of Jet Rates in e^+e^- collisions

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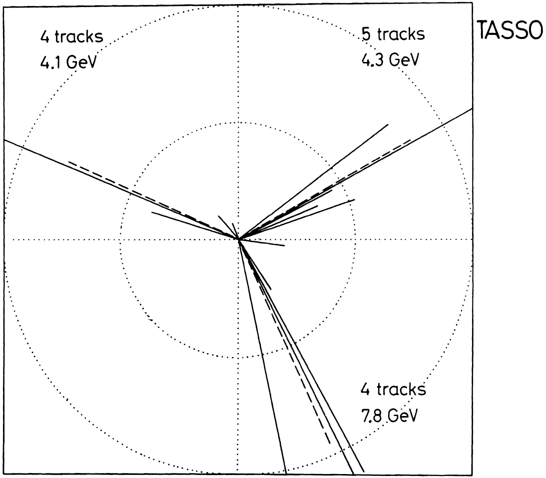


Fig.20g Another 3-jet event projected into the event plane.

[TASSO Collaboration 1997]

- How many jets in this picture?
- By eye: 3 jets?
- Better: It depends, on the jet algorithm (for this talk: Durham algorithm) and its resolution
- $\Rightarrow$ Important to understand this relation as good as we can

- Typically studied: $2 \to 3$ resolution scale y_3
- high accuracy possible, e.g. NNLL+NNLO [Banfi, McAslan, Monni, Zanderighi 2016]
- Aim of this talk: at least NLL accuracy for higher multiplicities
  - Fits of α_s in e^+e^- collisions [Verbytskyi et. al. 2019]
  - LHC: k_T splitting scales in $Z + \mathrm{jets}$ measured [ATLAS Collaboration 2017], prediction could be obtained by extending this study to colored initial states
  - Easy to define higher multiplicity equivalent $\to$ convenient to study effects like color correlations that become more important with higher multiplicities.

Outline

- Durham clustering:
  - Define

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{Q^2}(1 - \cos\theta_{ij})$$

    between each two objects i and j in the event.
- For n objects, find i, j that minimize $y_{ij} := y_n$.
- Recombine i and j into one object (here: by adding their four-momenta).
- Continue until left with only 2 objects (or until $y_n < y_{\mathrm{cut}}$)

- In the soft limit:

$$y_{\mathrm{n}} \approx k_T/Q$$

  with k_T transverse momentum to direction of closest hard leg.
- Specific to this study:
  - We want to resum soft gluons around some hard ($n-1$ parton) born event with well behaved fixed order description $\Rightarrow$ require $y_{\mathrm{n}-1} > \{0.008, 0.02, 0.08\}$
  - $\rightarrow$ different from the usual (experimental) definition
  - Results shown for LEP1 energy $Q = 91.2$ GeV.

Outline

- Based on the CAESAR formalism [Banfi, Salam, Zanerighi 2005]
- Independent implementation within the Sherpa framework [Gerwick, Höche, Marzani, Schumann 2015]
- Write cumulative cross section as

$$\Sigma(v) = \sum_{\delta} \int d\mathcal{B}_{\delta} \frac{d\sigma_{\delta}}{d\mathcal{B}_{\delta}} \exp\left[-\sum_{l} R_{l}^{\mathcal{B}_{\delta}}(v)\right] \mathcal{S}^{\mathcal{B}_{\delta}}(v) \mathcal{F}^{\mathcal{B}_{\delta}}(v).$$

- exponent and $\mathcal{F}$ already present in 2-jet case
- numerical evaluation of $\mathcal{F}$ and computation of color correlations in $\mathcal{S}$ main challenge in going to higher multiplicities.

- CAESAR method (with two hard legs):
  - Parametrize observable in the presence of single emission
    $V(k_i) = d_l g_l(\Phi) \left(\frac{k_T}{Q}\right)^a e^{-b_l \eta_l} \to k_T/Q$
  - For suitable observables $\Rightarrow \Sigma(v) = e^{-R_{NLL}(v)} \mathcal{F}(v)$
  - Single emission integral with α_s in CMW scheme

$$\mathrm{R}_{NLL}(v) = 2\int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz\, \frac{\alpha_s\left(\xi(1-z)^{\frac{2b}{a+b}}\right)}{2\pi} \frac{2\, C_F}{1-z} \Theta\left(\ln \frac{(1-z)^{\frac{2a}{a+b}}}{\xi/Q^2}\right) - \frac{\alpha_s(\xi)}{\pi} C_F B_q\right]$$

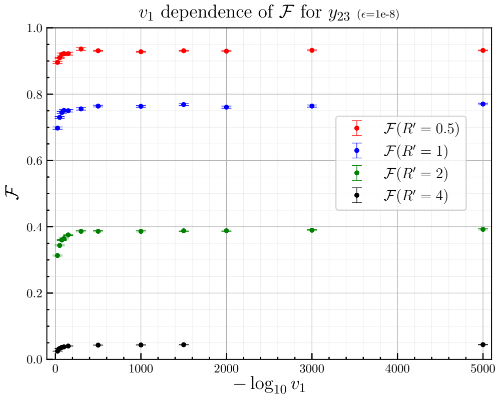
  - $\mathcal{F}(v) = \lim_{\epsilon\to 0} \lim_{\bar{v}\to 0} \mathcal{F}_{\epsilon,\bar{v}}(v)$,

$$\mathcal{F}_{\epsilon,\bar{v}}(v) = e^{R'_{\mathrm{NLL}}(v)\ln\epsilon} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m} \int \frac{d\zeta_i}{\zeta_i} d\xi_i \frac{d\Phi}{2\pi} P(\zeta_i, \xi_i, \Phi)\right) \Theta\left(1 - \frac{V(k_i(\bar{v}))}{\bar{v}}\right)$$

- Validation: Convergence of numerical $v \to 0$ limit in calculation of $\mathcal{F}$.
- Calculation of fixed order coefficien $\mathcal{F}_2$:

$$\mathcal{F}_2 = -\frac{\pi^2}{8} \frac{\sum_l C_l^2}{2\left(\sum_l C_l\right)^2}$$

$\to$ confirmed in numerical calculation within uncertainties.
$(C_l = C_A/C_F)$



- The S-function and color correlations:

- Takes the general form:

$$\mathcal{S}(t) = \frac{\mathrm{Tr}\left[He^{-\frac{t}{2}\Gamma^\dagger}ce^{-\frac{t}{2}\Gamma}\right]}{\mathrm{Tr}\left[cH\right]}, \quad |\mathcal{M}|^2 = \mathrm{Tr}[cH]$$

- Soft anomalous dimension Γ (up to sum over hard legs that can be absorbed into R_l's) give by [Bonciani, Catani, Mangano, Nason 2003]

$$\Gamma = \sum_i \sum_{j>i} T_i T_j \log(Q_{ij}/\mu)$$

- Calculation automated for arbitrary number of legs [Gerwick, Höche, Marzani, Schumann 2015].

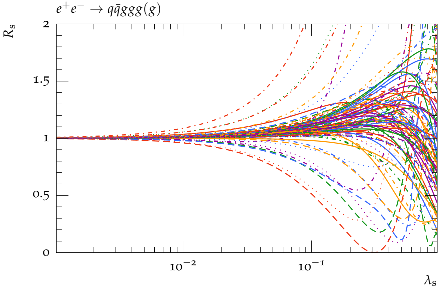
- Automation of color calculations:

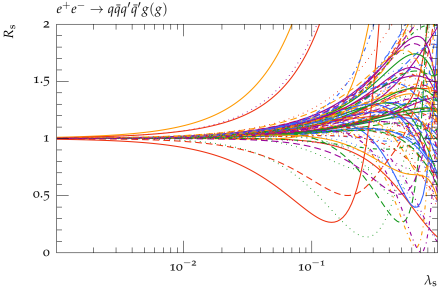
1. Pick a specific set of basis vectors t_α.
   - ★ Trace-basis sufficient.
2. Calculate $c_{\alpha\beta} = t_\alpha t_\beta$ and its inverse.
   - ★ If "Basis" over-complete $\rightarrow$ generalised inverse with the methods of [Gerwick, Höche, Marzani, Schumann 2015] (not necessary for multiplicities discussed here)
3. Calculate $T_i T_j$ in this basis
   - ★ Only once necessary for given number of quarks and gluons.
4. Hard matrix H from COMIX in Sherpa framework.

- Validation: Compare soft approximation to full matrix element

$$R = \frac{\mathrm{Tr}\left[H_n c_n \Gamma\right]}{\mathrm{Tr}\left[c_{n+1} H_{n+1}\right]}$$

- Take some hard configuration (non-collinear) and scale one of the momenta down $k \to \lambda_s k$ with $\lambda_s \to 0$.





Fixed order and Matching:

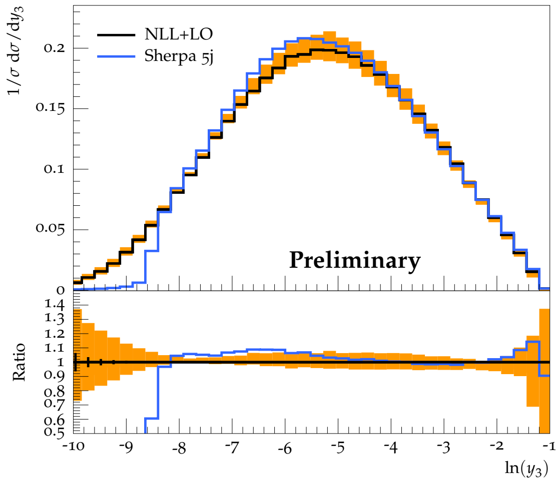
- Of course at least NLO available in principle for any multiplicity.
- Focus here: resummation with non-trivial color $\rightarrow$ so far only (additive) matching to LO.
- Modify logs so resummation goes to 0 at physical endpoints

$$\ln 1/y_n \rightarrow \ln\left(1/y_n - 1/y_n^{\max} + 1\right)$$

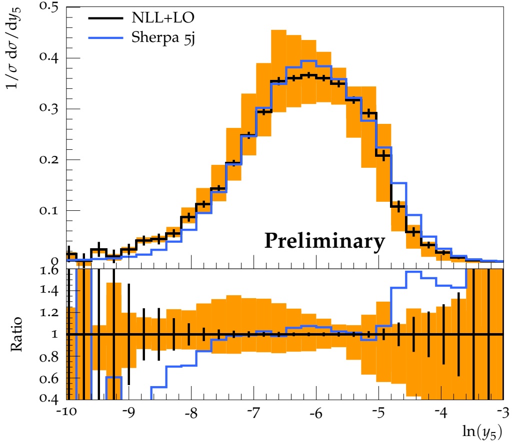
with endpoint $y_n^{\max} = 1/3$ for y_3 and $y_n^{\max} = y_{n-1}$ for $n > 3$.

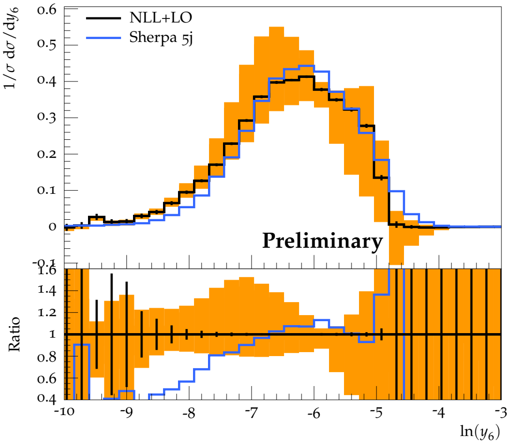
Outline

- Reproduce the known result for y_3
- Comparison to Sherpa results:
    - MEPS merging with up to 5 jets at LO
    - Cutoff effects dominate at low y_3.
    - Good agreement in bulk of distribution.

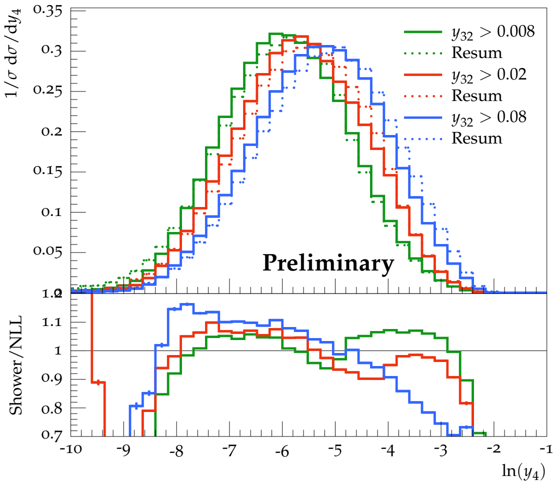


- Results for y_5 and y_6 with a cut on the born event $y_{n-1} > 0.008$
- Note: all results normalized to inclusive $(n-1)$-parton cross-section with corresponding cut





- Dependence on cut on born events for y_4.
- Take cuts to higher values $\rightarrow$ better behaved (hopefully) but not realistic for higher multiplicities.
- Varying cut in resummation mimics behaviour of shower.

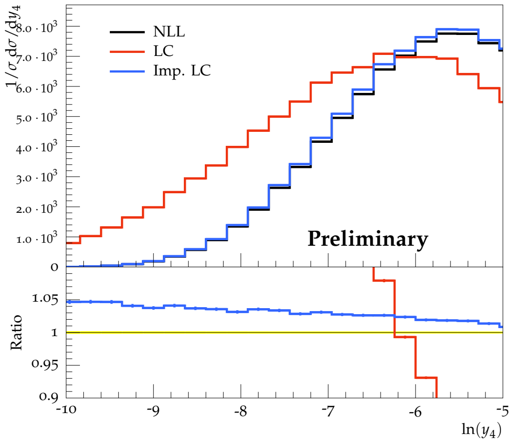


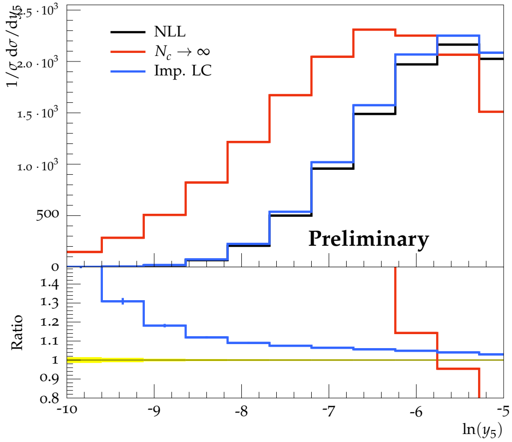
- Subleading color contributions:
  - Could repeat calculation with strictly $N_c \to \infty$ while $\alpha_\mathrm{s}/N_c = \mathrm{const.}$
  - We usually do "better":

$$\Sigma(v) = \sum_\delta \int d\mathcal{B}_\delta \frac{d\sigma_\delta}{d\mathcal{B}_\delta} \exp\left[-\sum_l R_l^{\mathcal{B}_\delta}(v)\right] \mathcal{S}^{\mathcal{B}_\delta}(v)\mathcal{F}^{\mathcal{B}_\delta}(v).$$

  - Everything apart from $\mathcal{S}$ has one of the hard legs associated with it.
  - Correct Casimir/anomalous dimensions simple to implement e.g. in showers (though maybe hard to analyse if correct).
  - Use this as an in between step to quantify "non-trivial" subleading contributions $\to$ "improved LC".

- Not really clear what to match to → no matching and restrict range to $\ln(1/y_n) > 5$.
- Improved LC reduces difference to full color, but growing with higher n.
- Results for y_4 und y_5.





Outline

- Conclusion
  - Presented preliminary results for jet resolution scales with high multiplicities in e^+e^- annihilations, at NLL+LO accuracy including non-trivial color correlations.
  - Calculation automated as plugin to Sherpa.
  - Observed good qualitative agreement between Sherpa parton shower predictions and analytic result in peak region.
  - Subleading color contributions can be large, but difference significantly reduced by simple adjustments.
- Outlook and To-Do's:
  - NLO calculation can be included.

Backup

- Color for y_3 trivial.
- $\rightarrow$ improved LC already full color structure.