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beam-driven **surface wave modes**  
in  
hollow **crystals**  
“fiber accelerator”

Aakash Sahai

Prof. Toshiki Tajima

[aakash.sahai@gmail.com](mailto:aakash.sahai@gmail.com)

## Bulk plasma acceleration

- plasma blowout regime - severe limitations **although** been very successfully demonstrated
- e<sup>-</sup>-beam acc. → **coupled** longitudinal & transverse wakes & **plasma-ion** problems
- e<sup>+</sup>-beam acc. → **overcome** ion-defocusing / phase-mix e<sup>-</sup>
- Hollow-channel → channel-wall current / fringe-fields

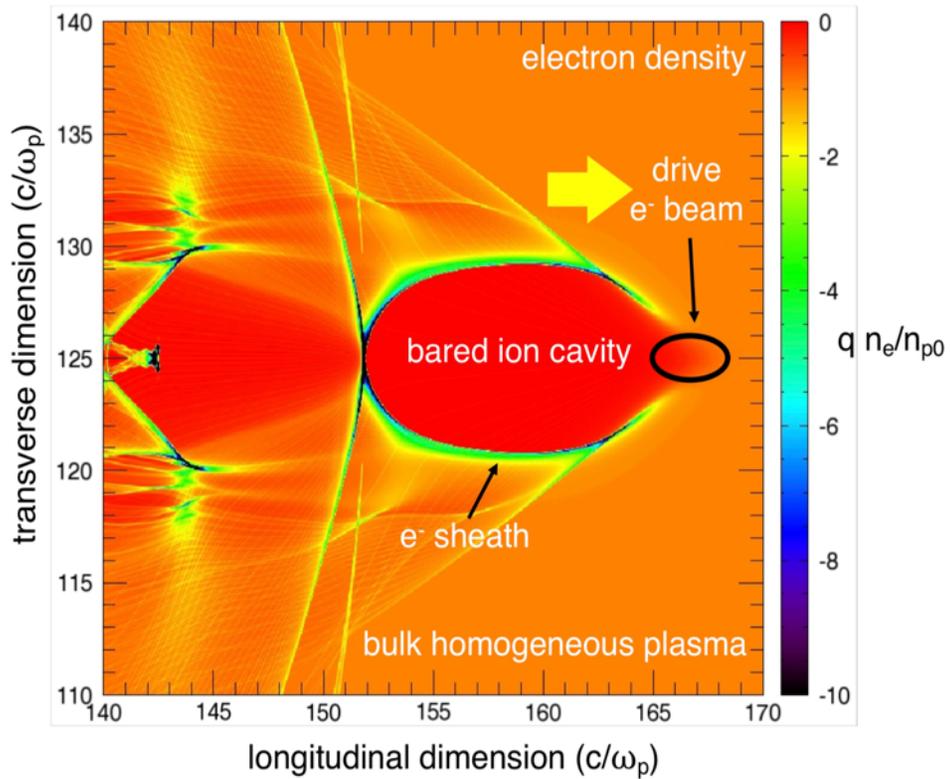
## Crystal hollow nanotube modes

- Surface plasmon polariton (short time-scales), surface phonon polariton mode (longer time-scales)
- SW-CNT, MW-CNT modes
- Beam-driven modes in the hollow region – plasmon-polariton vs phonon-polariton modes
- **3D PIC** results of **driver independence** → occurs for e<sup>+</sup> / e<sup>-</sup> drive beams

## Ongoing Work

- Acceleration / Focusing field → **decoupled – no ions**
- Driver affect → **head-erosion / etching – diff from bulk Plasma**
- Beam-loading → positron / electron beam in “crunch-in” wakefields

# Bulk plasma Blowout regime – fundamental limitations



- Degradation of accelerated beam qualities its emittance, energy spread etc.
- strong coupling between longitudinal and transverse wakefields – **radiation loss**
- particles scattering off background ions (heavier ion plasma) – **sec. ioniz. & dark current**
- field degradation due to ion motion & ion modes (lighter ion plasma)
- positron and bared ions in the blowout region both being positively charged

# Hollow “fiber” accelerator - 1983



Tajima, T., *Laser accelerators for ultra-high energies*, *Proceedings, 12th International Conference on High-Energy Accelerators*, pp. 470-472, C830811, HEACC 1983: Fermilab, Batavia, August 11-16, 1983

## LASER ACCELERATOR FOR ULTRA-HIGH ENERGIES

T. Tajima

Department of Physics and Institute for Fusion Studies  
 University of Texas, Austin, Texas 78712

### Abstract

It is shown that under appropriate conditions the laser electromagnetic wave is trapped in a plasma fiber and is capable of exciting a beat plasma wave with a flat phase front propagating with the speed of light  $c$  (or any desired speed). In an important special case where the fiber contains vacuum inside it is still possible to have a wave whose phase velocity is up to  $c$  in the direction of the particle propagation with a plasma slow wave structure or in curved geometry. While the parallel component of the laser field is used for acceleration, the perpendicular component is either cancelled by the imposed static magnetic field force or used directly to contain particles. In the latter way it does not seem impossible to reach multi-peta electronvolts ( $> 10^{15}$  eV) for protons. As examples of applications, I propose (i) direct method of collective excitations of quark-gluon systems; (ii) microscope for quark-gluon systems; (iii) method of direct interaction with spin or chiral structure of matter; and (iv) possible lasing by quark excitation.

### I. Introduction

The laser beat-wave accelerator scheme<sup>1</sup> is capable of creating a coherent large longitudinal electric field  $E_L = mc\omega_p/e$  of the order of 1 GeV/cm. Applying this scheme to an ultra-high energy accelerator has been considered.<sup>2,3,4</sup> Two of the difficulties associated with the original scheme identified are: (i) the longitudinal phase mismatch between the plasma wave and the particles; (ii) the tendency of laser light to spread in the transverse direction. It is crucial to overcome these

One important case of the plasma fiber accelerator is a circular or race-track accelerator for protons utilizing the laser electric field to bend their orbit in order to contain particles within a manageable size. If such a scheme proves feasible, proton energies of multi-petaelectronvolts are not out of scope. We touch upon this in Sec. III.

In the last Section applications of such an accelerator, if it is ever successfully built, are suggested. A novel aspect in these is to directly couple the macroscopic beam structure with collective modes of the microscopic subnuclear structure.

### II. Plasma Fiber Accelerator<sup>6</sup>

A way to match the phase of the accelerating field with high energy particles is to inject particles oblique to the electric field direction [see Fig. 1(a)]. In order to phase-lock, the angle  $\theta$  between the particle momentum and the electric field is given by

$$\cos \theta = \left(1 - \frac{\omega_p^2}{\omega_0^2}\right)^{1/2}. \quad (2)$$

Although we match the parallel phase, we have now introduced an extraneous perpendicular acceleration, which bends the particle orbit (perhaps undesirably). To correct this situation, it was proposed<sup>7</sup> that a static vertical magnetic field is imposed. The magnetic field is such that

$$\frac{B}{E} = \sin \theta = \frac{\omega_p}{\omega_0}, \quad (3)$$

for relativistic particles where  $B$  is out of the board in Fig. 1(c).

- hollow tube in plasma
- guide & self-focus high-intensity laser pulse
- phase velocity close to speed of light (low on-axis density)
- low ion concentration – on-axis minimize acc. beam plasma-ion interaction

# Hollow “fiber” acc. – further work



## laser-pulse driven

### Laser wakefield acceleration and optical guiding in a hollow plasma channel

T. C. Chiou, T. Katsouleas, C. Decker, W. B. Mori, J. S. Wurtele, G. Shvets, and J. J. Su

Citation: *Physics of Plasmas* (1994-present) **2**, 310 (1995); doi: 10.1063/1.871107

channel. The electromagnetic fringe fields of the wake extend to the center of the channel, enabling focusing and acceleration of particles down the axis. This scheme differs from conventional hollow fiber accelerators, so that the wake fields arise from surface currents at the channel edge rather than density bunching at the channel axis.

### Control of focusing forces and emittances in plasma-based accelerators using near-hollow plasma channels

*PHYSICS OF PLASMAS* **20**, 080701 (2013)

C. B. Schroeder, E. Esarey, C. Benedetti, and W. P. Leemans  
*Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

The wake excited in such a channel will consist of an electromagnetic wake owing to surface currents driven in the channel walls and a wake owing to the background ions in the channel. In the limit  $k_c^2 \ll k_w^2$ , the accelerating field in



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# Hollow “fiber” acc. – beam-driven

electron-beam driven

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PHYSICAL REVIEW LETTERS

19 OCTOBER 1998

**High Beam Quality and Efficiency in Plasma-Based Accelerators**

T. C. Chiou and T. Katsouleas

The focusing force is zero inside the channel for a very relativistic particle. The spikes at the channel walls are

positron-beam driven

LETTER

doi:10.1038/nature14890

**Multi-gigaelectronvolt acceleration of positrons  
in a self-loaded plasma wakefield**

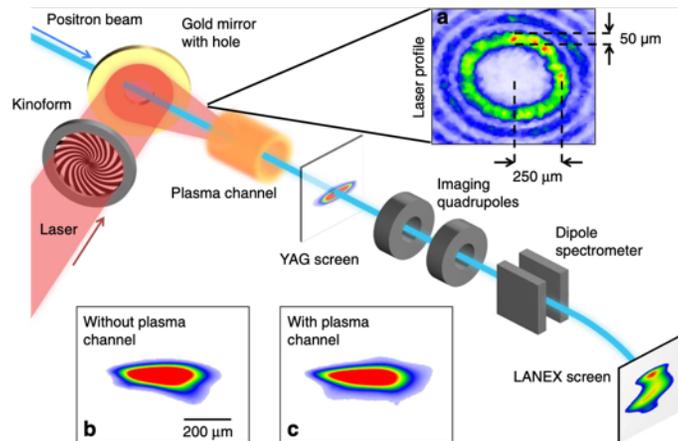
S. Corde<sup>1,2</sup>, E. Adli<sup>1,3</sup>, J. M. Allen<sup>1</sup>, W. An<sup>4,5</sup>, C. I. Clarke<sup>1</sup>, C. E. Clayton<sup>4</sup>, J. P. Delahaye<sup>1</sup>, J. Frederico<sup>1</sup>, S. Gessner<sup>1</sup>, S. Z. Green<sup>1</sup>, M. J. Hogan<sup>1</sup>, C. Joshi<sup>4</sup>, N. Lipkowitz<sup>1</sup>, M. Litos<sup>1</sup>, W. Lu<sup>6</sup>, K. A. Marsh<sup>4</sup>, W. B. Mori<sup>4,5</sup>, M. Schmeltz<sup>1</sup>, N. Vafaei-Najafabadi<sup>4</sup>, D. Walz<sup>1</sup>, V. Yakimenko<sup>1</sup> & G. Yocky<sup>1</sup>

vents positron acceleration. To avoid this problem, the use of a hollow plasma channel to produce wakes without a focusing force<sup>13–15</sup>, or the



## ARTICLE

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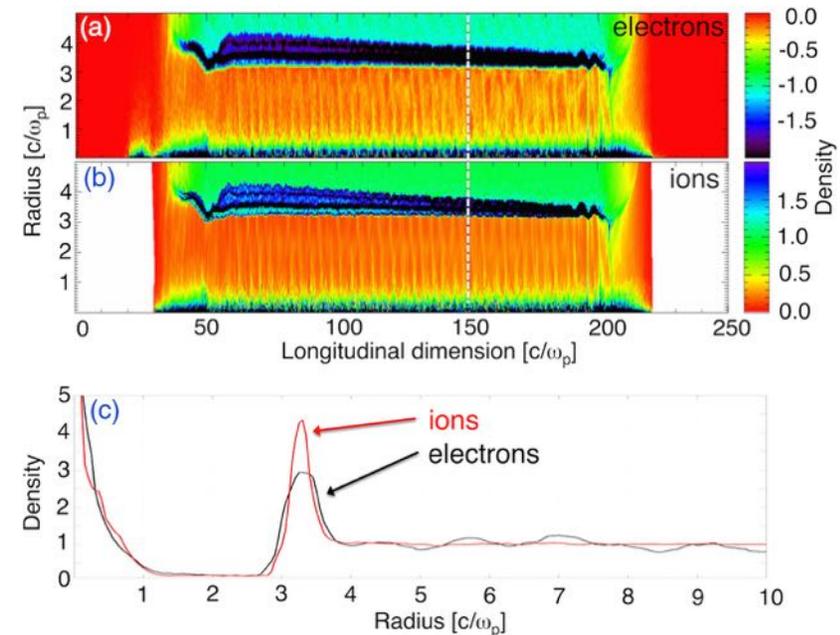
un-ionized gas within the channel

PHYSICAL REVIEW ACCELERATORS AND BEAMS **20**, 081004 (2017)

## Excitation of a nonlinear plasma ion wake by intense energy sources with applications to the crunch-in regime

Aakash A. Sahai\*

*Department of Physics, Blackett Laboratory and John Adams Institute for Accelerator Sciences, Imperial College London, London, SW7 2AZ, United Kingdom, and Department of Electrical Engineering, Duke University, Durham, North Carolina 27708, USA*  
(Received 5 February 2017; published 23 August 2017)





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# Hollow-channel – first expt.s



## e<sup>+</sup>-beam -driven

$$r_{ch} = 16 c/\omega_{pe}$$

$$n_b = 1.3 n_0$$

$$\sigma_z = 1.5 c/\omega_{pe}$$

$$\sigma_r = 0.3 c/\omega_{pe}$$

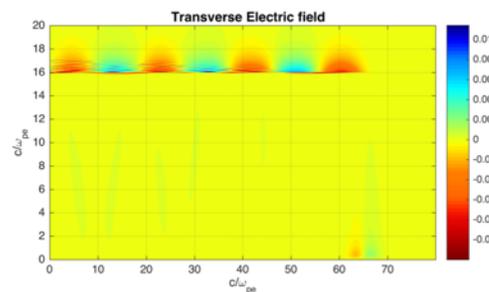
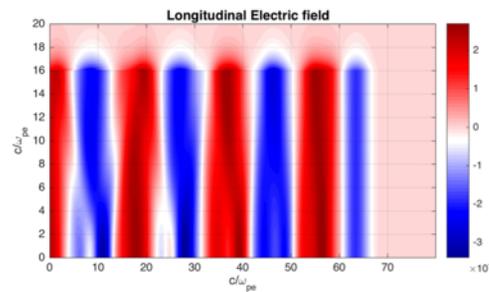
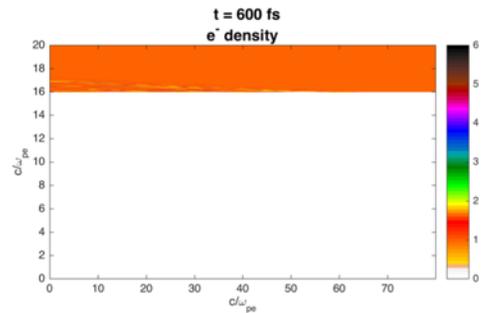
$$\gamma_b = 1000$$

## NO focusing forces

currents in the wall

EM acc. fields leak  
into the channel

$$10^{-2} E_{WB}$$



**CYL**  
geometry



ARTICLE

Received 17 Nov 2015 | Accepted 27 Apr 2016 | Published 2 Jun 2016

$$n_0 = 8 \times 10^{16} \text{ cm}^{-3}$$

$$\text{obs. field} = 220 \text{ MV/m}$$

$$\text{WB field} \sim 20 \text{ GV/m}$$

# e<sup>-</sup>-beam lin. regime - Hollow-channel



## e<sup>-</sup>-beam -driven

$$r_{ch} = 16 c/\omega_{pe}$$

$$n_b = 1.5 n_0$$

$$\sigma_z = 1.5 c/\omega_{pe}$$

$$\sigma_r = 0.5 c/\omega_{pe}$$

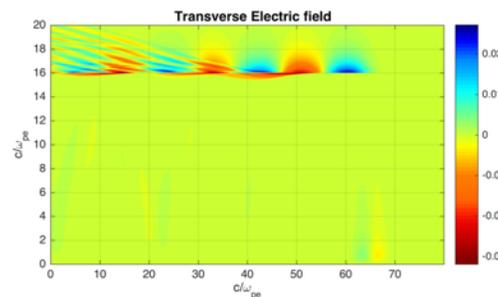
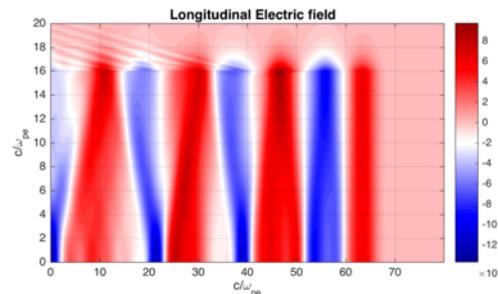
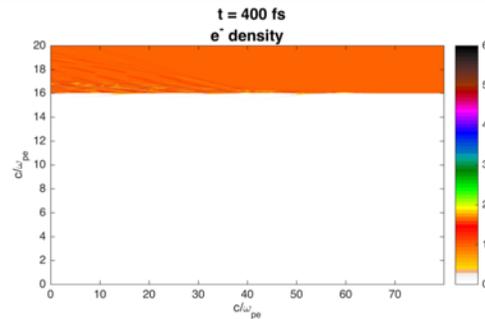
$$\Upsilon_b = 1000$$

**NO focusing forces**

currents in the wall

EM acc. fields leak into the channel

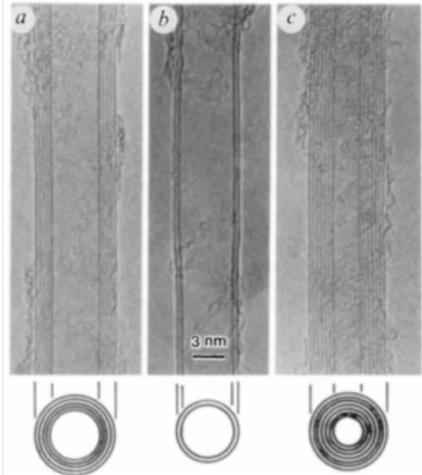
$$10^{-2} E_{WB}$$



**CYL**  
geometry

- pure EM mode (within channel) - **TM** mode
- not utilizing plasma space-charge electro-static mode
- just like metallic wall waveguide
- weak acceleration fields

# Nano-tubes – hollow crystal channels



LETTERS TO NATURE

## Helical microtubules of graphitic carbon

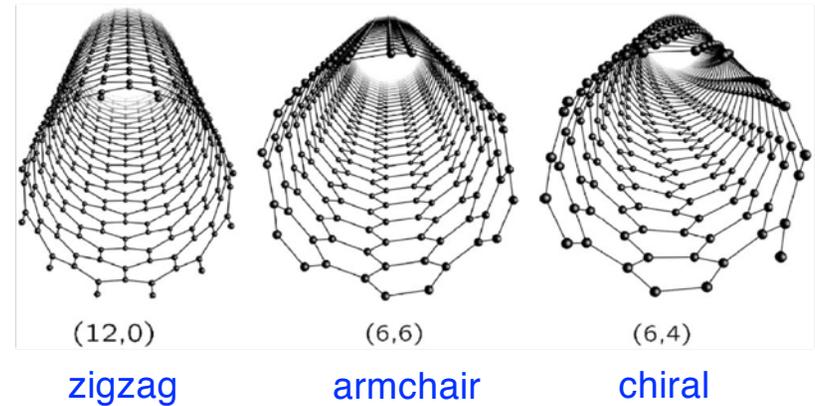
Sumio Iijima

NEC Corporation, Fundamental Research Laboratories,  
34 Miyukigaoka, Tsukuba, Ibaraki 305, Japan

NATURE · VOL 354 · 7 NOVEMBER 1991

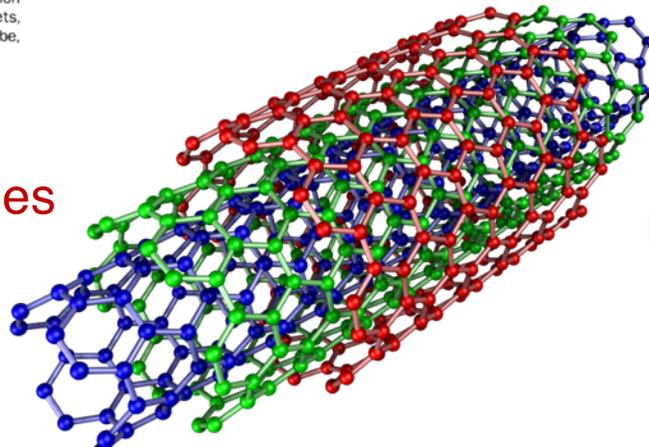
FIG. 1 Electron micrographs of microtubules of graphitic carbon. Parallel dark lines correspond to the (002) lattice images of graphite. A cross-section of each tubule is illustrated. *a*, Tube consisting of five graphitic sheets, diameter 6.7 nm. *b*, Two-sheet tube, diameter 5.5 nm. *c*, Seven-sheet tube, diameter 6.5 nm, which has the smallest hollow diameter (2.2 nm).

## single-walled nanotubes (SWNT)



SWNT diameter – 0.3nm to 4nm

## multi-walled nanotubes (MWNT)



MWNT diameter – 1.5nm to 100nm

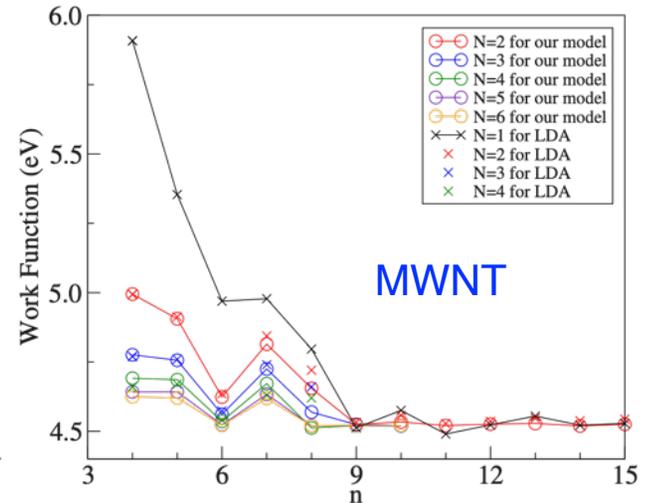
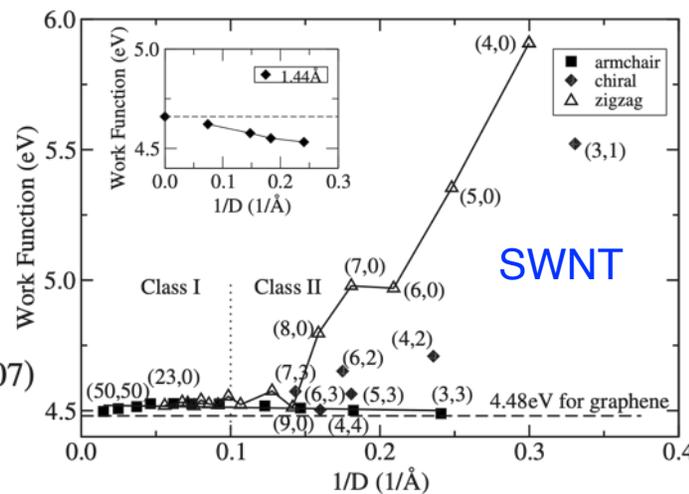
# Nano-tubes – properties



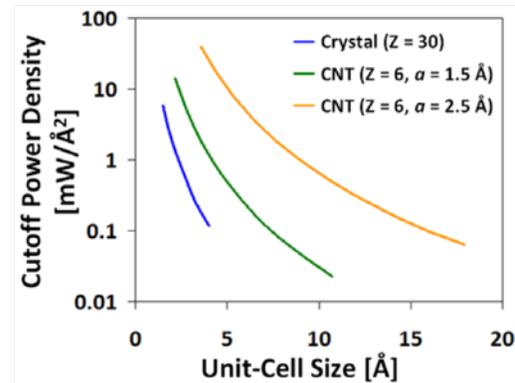
density  $\sim 10^{21} - 10^{22} \text{ cm}^{-3}$      $\lambda_{\text{mfp}} \simeq \mu \text{ m}$     collision-less plasma over few ten fs

work function  $\sim 5\text{eV}$

PHYSICAL REVIEW B **76**, 235413 (2007)



damage threshold



# solid-state modes - plasmonics



simplified wave equation in a dielectric

$$\mathbf{K}(\mathbf{K} \cdot \mathbf{E}) - K^2 \mathbf{E} = -\varepsilon(\mathbf{K}, \omega) \frac{\omega^2}{c^2} \mathbf{E}$$

$$K^2 = \varepsilon(\mathbf{K}, \omega) \frac{\omega^2}{c^2} \quad \text{purely transverse} \quad \mathbf{K} \cdot \mathbf{E} = 0$$

$$\varepsilon(\mathbf{K}, \omega) = 0 \quad \text{purely longitudinal}$$

low-frequency limit – metallic behavior

$$\omega < \omega_p$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

high-frequency limit – plasmonic behavior

$$\omega\tau \gg 1 \quad \tau \text{ tens of fs}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

bulk plasmon – dispersion  $\omega^2 = \omega_p^2 + \frac{6E_F K^2}{5m}$  electron time-scale dominated

bulk phonon – dispersion  $\varepsilon(k, \omega) = 1 - \frac{\omega_{pi}^2}{\omega^2 - \omega_{TO}^2} - \frac{\omega_{pe}^2}{\omega^2 - k_x^2 v_e^2}$  ion time-scale dominated

# surface plasmon polariton



propagating SPP mode

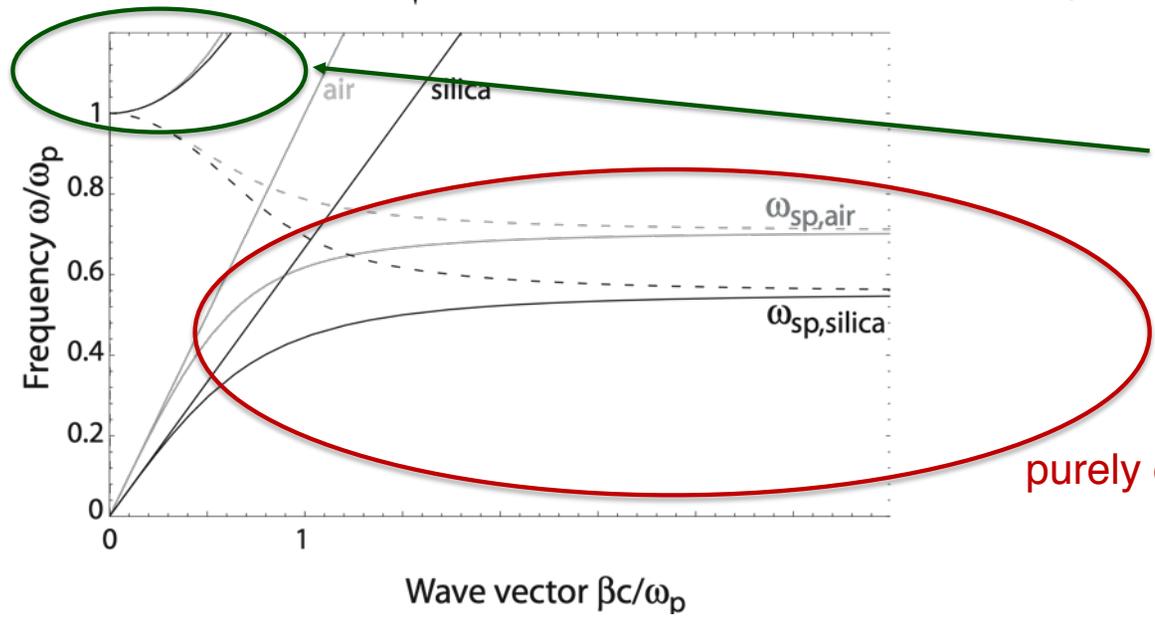
TM-mode existence condition – dispersion relation

surface plasmon frequency

$$\beta = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\omega_{sp} = \frac{\omega_p}{\sqrt{1 + \epsilon_2}}$$

also holds for cylindrical geometry



surface plasmon polariton

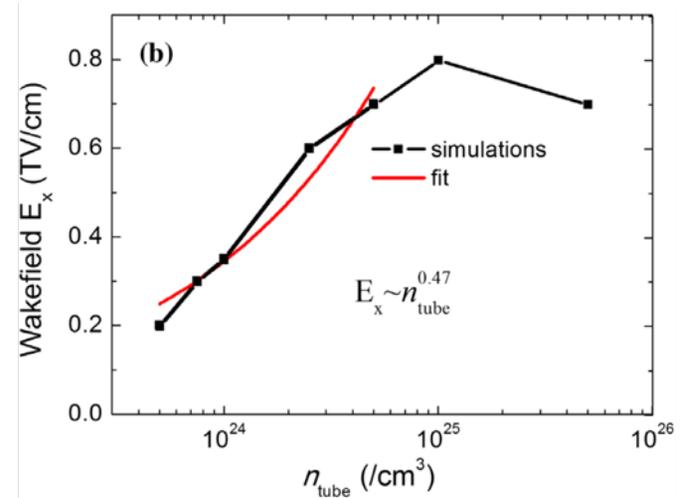
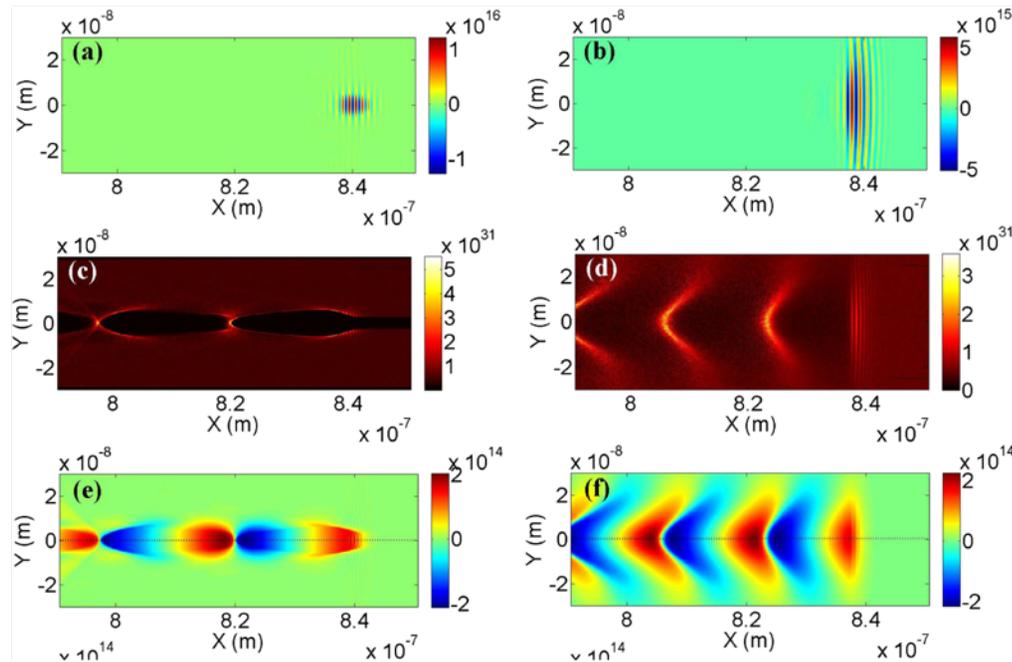
purely electrostatic modes: surface plasmon

## Particle-in-cell simulation of x-ray wakefield acceleration and betatron radiation in nanotubes

Xiaomei Zhang,<sup>1,2</sup> Toshiki Tajima,<sup>2</sup> Deano Farinella,<sup>2</sup> Youngmin Shin,<sup>3</sup> Gerard Mourou,<sup>4</sup> Jonathan Wheeler,<sup>4</sup> Peter Taborek,<sup>2</sup> Pisin Chen,<sup>5</sup> Franklin Dollar,<sup>2</sup> and Baifei Shen<sup>1</sup>

TABLE I. Summary of the laser and plasma parameters for our base case.

Laser wavelength $\lambda_L$	Peak amplitude $a_0$	Width radius $\sigma_L$	Length radius $\sigma_x$	Plasma density $n_{\text{tube}}$	Tube radius $\sigma_{\text{tube}}$
1 nm	4	5 nm	3 nm	$5 \times 10^{24}$ W/cm <sup>3</sup>	2.5 nm/0 nm
1 $\mu\text{m}$	4	5 $\mu\text{m}$	3 $\mu\text{m}$	$5 \times 10^{18}$ W/cm <sup>3</sup>	2.5 $\mu\text{m}$ /0 $\mu\text{m}$



## beam-driven Crunch-in regime model

single e<sup>-</sup> model  
**kinetic approach**

$$m_e \frac{d^2 r}{d\xi^2} + \frac{1}{c^2 \beta_b^2} \frac{n_p e^2}{\epsilon_0} \cdot \frac{1}{2r} (r^2 - r_{ch}^2) = 0$$

Weakly driven  
**(Linear regime)**

$$r^2 - r_{ch}^2 = \cancel{(r - r_{ch})^2} + 2r_{ch}(r - r_{ch})$$

$$\approx 2r_{ch}(r - r_{ch}),$$

$$\frac{d^2 r}{d\xi^2} = -\frac{(\omega_p/c)^2}{\beta_b^2} (r - r_{ch})$$

$$r(\xi) = r_{ch} + A \sin\left(\frac{\omega_p}{c\beta_b} \xi\right)$$

**SINUSOIDAL in the weak limit !**

## Crunch-in regime model - 2

single e<sup>-</sup> model  
 kinetic approach

$$r \sim r_{ch}$$

$$\frac{d^2 \rho}{d\xi^2} + \frac{1}{2\beta_b^2} \left(\frac{\omega_p}{c}\right)^2 \cdot \frac{1}{\rho} (\rho^2 - 1) = 0$$

$$\rho = r/r_{ch}$$

autonomous ODE

$$\frac{d}{d\xi} \left[ \frac{1}{2} \left( \frac{d\rho}{d\xi} \right)^2 \right] = \frac{d\rho}{d\xi} \frac{d^2 \rho}{d\xi^2}$$

$$\frac{1}{2} \rho'^2 + \frac{1}{2\beta_b^2} \left(\frac{\omega_p}{c}\right)^2 \cdot \left( \frac{1}{2} \rho^2 - \ln \rho \right) = C_1 \quad \rho' = d\rho/d\xi$$

initial condition

$$\rho'_0 \approx \frac{\omega_{pb}}{c} \cdot \frac{1}{\beta_b r_{ch}^{3/2}} \cdot \sqrt{\frac{\sigma_r^2 \sigma_z}{\sqrt{2\pi}}}$$

$$C_1 = \rho_0'^2 + \frac{1}{2\beta_b^2} \left(\frac{\omega_p}{c}\right)^2$$

## Crunch-in regime model - 3



**initial condition**

$$\rho'_0 \approx \frac{\omega_{pb}}{c} \cdot \frac{1}{\beta_b r_{ch}^{3/2}} \cdot \sqrt{\frac{\sigma_r^2 \sigma_z}{\sqrt{2\pi}}}$$

**Linear regime  
condition**

$$\frac{e}{m_e c \omega_{pe}} \frac{en_b}{r_{ch}^2} \left(\frac{\pi}{2}\right)^{3/2} \sigma_r^2 \sigma_z \ll 1$$

**Crunch-in regime  
condition**

$$R_N \simeq \frac{n_b}{n_e} \frac{\sigma_r}{c/\omega_{pe}} \sqrt{\frac{\sigma_z}{c/\omega_{pe}}} \frac{1}{2\sqrt{2}} \left(\frac{\pi}{2}\right)^{1/4}$$

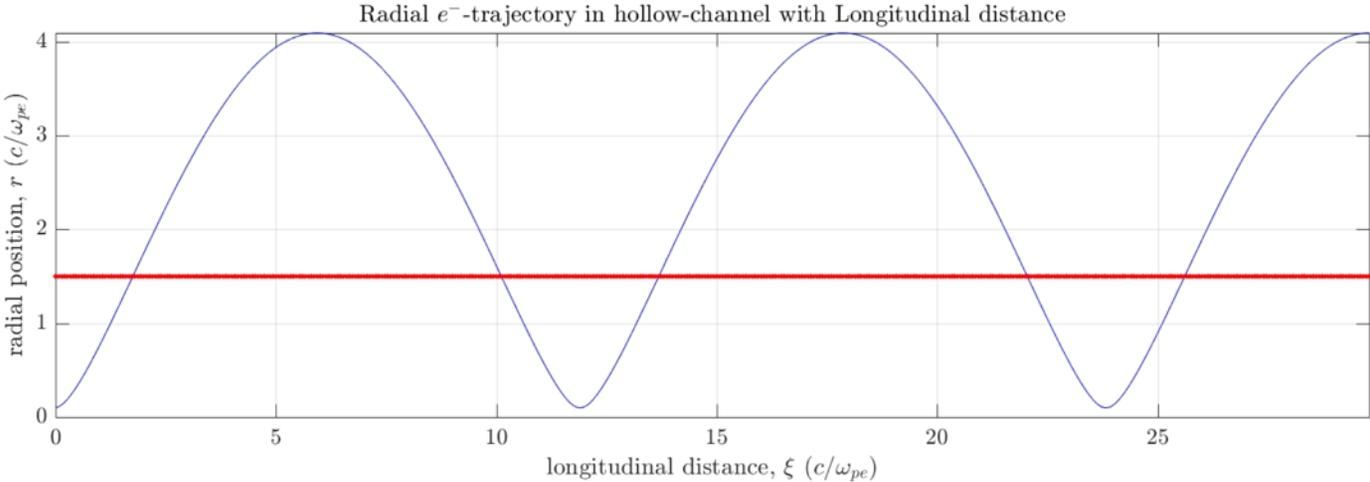
$$R_N = r_{ch} (c/\omega_{pe})^{-1}$$

# Crunch-in regime model - 4

single  $e^-$  model  
 kinetic approach

$$\frac{d^2 \rho}{d\xi^2} + \frac{1}{2\beta_b^2} \left(\frac{\omega_p}{c}\right)^2 \cdot \frac{1}{\rho} (\rho^2 - 1) = 0$$

numerical solution  $\rightarrow$  qualitative **Crunch-in** behavior



# e<sup>+</sup>-beam driven Hollow-channel

## Coherent on-axis compression of e<sup>-</sup> rings – Crunch-in



$$r_{ch} = 0.6 c/\omega_{pe}$$

$$n_b = 1.3 n_0$$

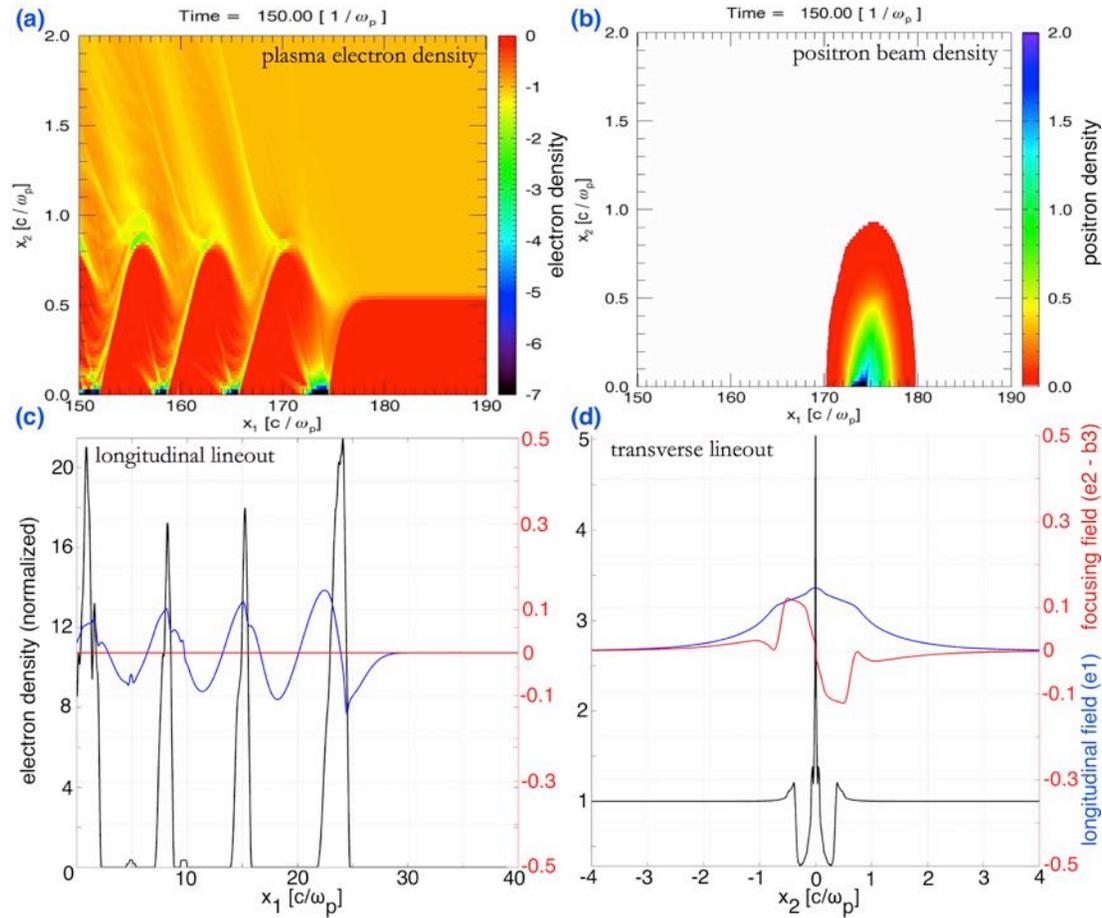
$$\sigma_z = 1.5 c/\omega_{pe}$$

$$\sigma_r = 0.3 c/\omega_{pe}$$

$$\gamma_b = 10000$$

on-axis dynamics

electron density  
 transverse field  
 longitudinal field



CYL geometry

# electron-beam driven “Crunch-in”



**matched**

$$r_{ch} = 1 \text{ c}/\omega_{pe}$$

$$n_b = 1.5 n_0$$

$$\sigma_z = 1.5 \text{ c}/\omega_{pe}$$

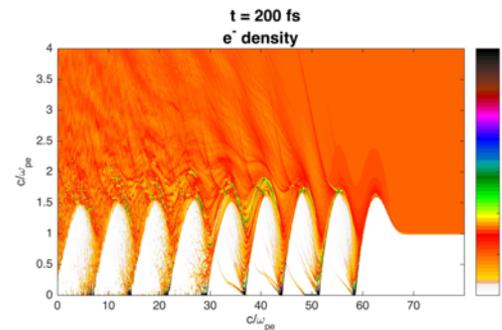
$$\sigma_r = 0.5 \text{ c}/\omega_{pe}$$

$$\gamma_b = 1000$$

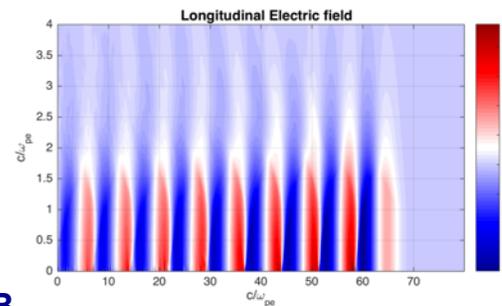
coherent radial oscillations  
 $e^-$  collapse to the channel axis

Coherent acc. fields  $\sim 0.4 E_{WB}$

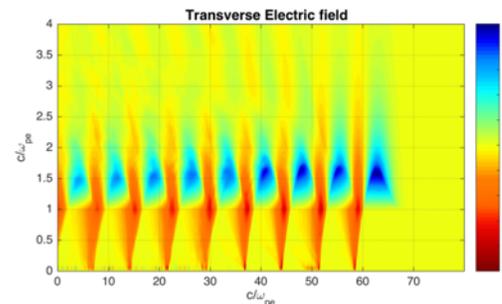
Focusing fields for positrons  
 Collocated with acc. fields  
 $\sim 0.4 E_{WB}$



**CYL**  
 geometry



**WB fields**  
 accln.  
 fields



**focusing**  
 fields for  
 positrons

# matching collapse-time - channel radius



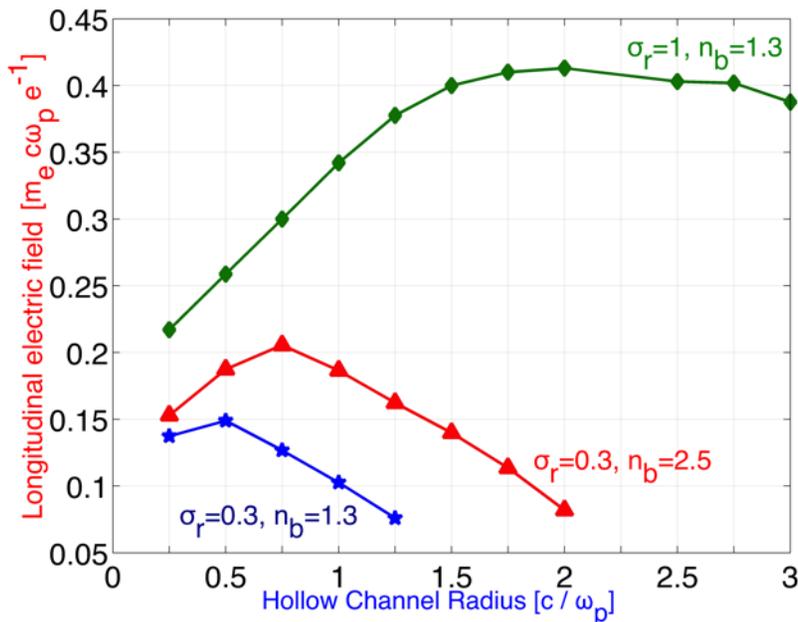
matching collapse time to  
“crunch-in” wavelength

$$\tau_c \approx \mathcal{D} \lambda_{Np} / c$$

$$\text{duty-cycle } \mathcal{D} = \frac{\tau_{back}}{\tau_{back} + \tau_{cav}}$$

$$r_{ch}^{opt} \approx 2 \sqrt{\pi} \mathcal{D} \frac{\lambda_{Np}}{\lambda_{pe}} \frac{\omega_{pb}}{\omega_{pe}} r_{pb}$$

**HIGHEST  
longitudinal  
fields**



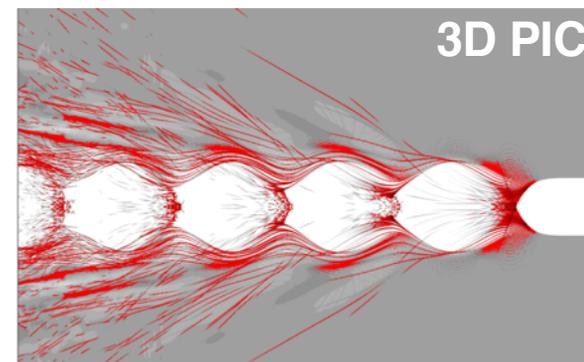
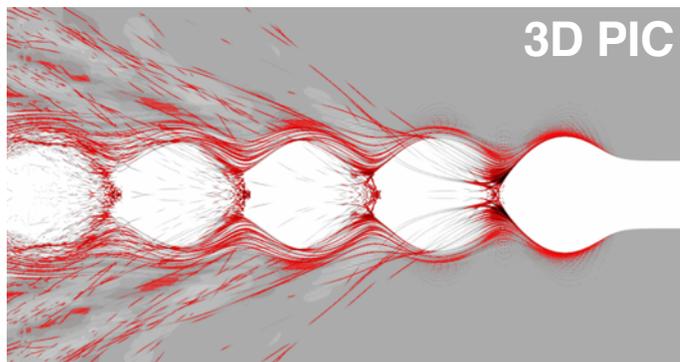
$E_{WB} (n_0 \sim 10^{22} \text{ cm}^{-3}) \sim [\text{few}] \text{ TVm}^{-1}$

$c/\omega_p \sim 100\text{s of nm}$

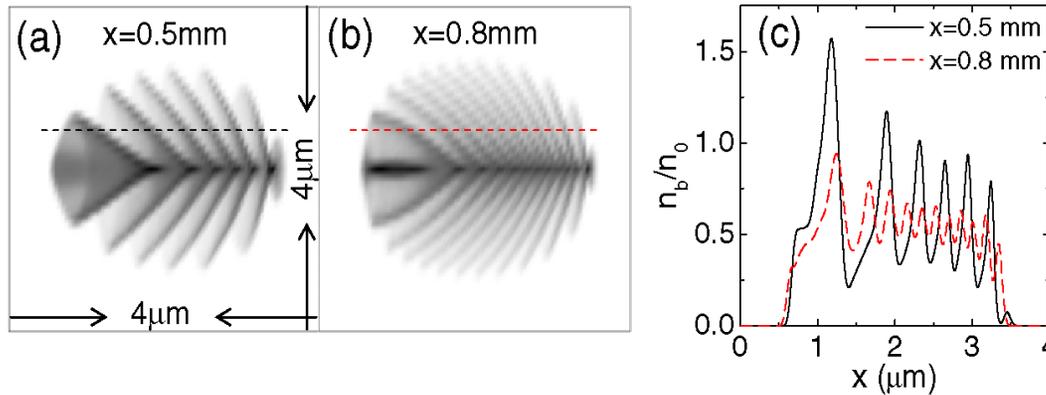
## Summary & Ongoing work



1. Proof provided – *existence of the crunch-in regime* – Analysis & PIC-based
2. **decoupled** long. (WB) & tran. fields  $\rightarrow$   $e^-$  acceleration
3. **ZERO** defocusing but **focusing** tran. fields  $\rightarrow$   $e^+$  acc.
4. Further investigating – positron / electron **beam-loading** in “crunch-in” regime
5. Further investigating – effect of loss of energy to the wakefields upon the driving energy-sources (**head erosion / etching** etc.)
6. Further investigating – evolution of beam guiding in hollow-channel plasma



beam micro-bunching – beam-plasma interactions



500 MeV

$$n_e = 2n_b \approx 4.4 \times 10^{19} \text{ cm}^{-3}$$

beam betatron  
frequency

$$\Omega_b = \omega_{pe} / \sqrt{2\gamma}$$

beam spot-size  
oscillation solution

$$\sigma_T(x, \xi) = \sigma_T(0, \xi) | \cos[\Omega_{b0}(1 + \xi/\sigma'_L)x/c] |.$$

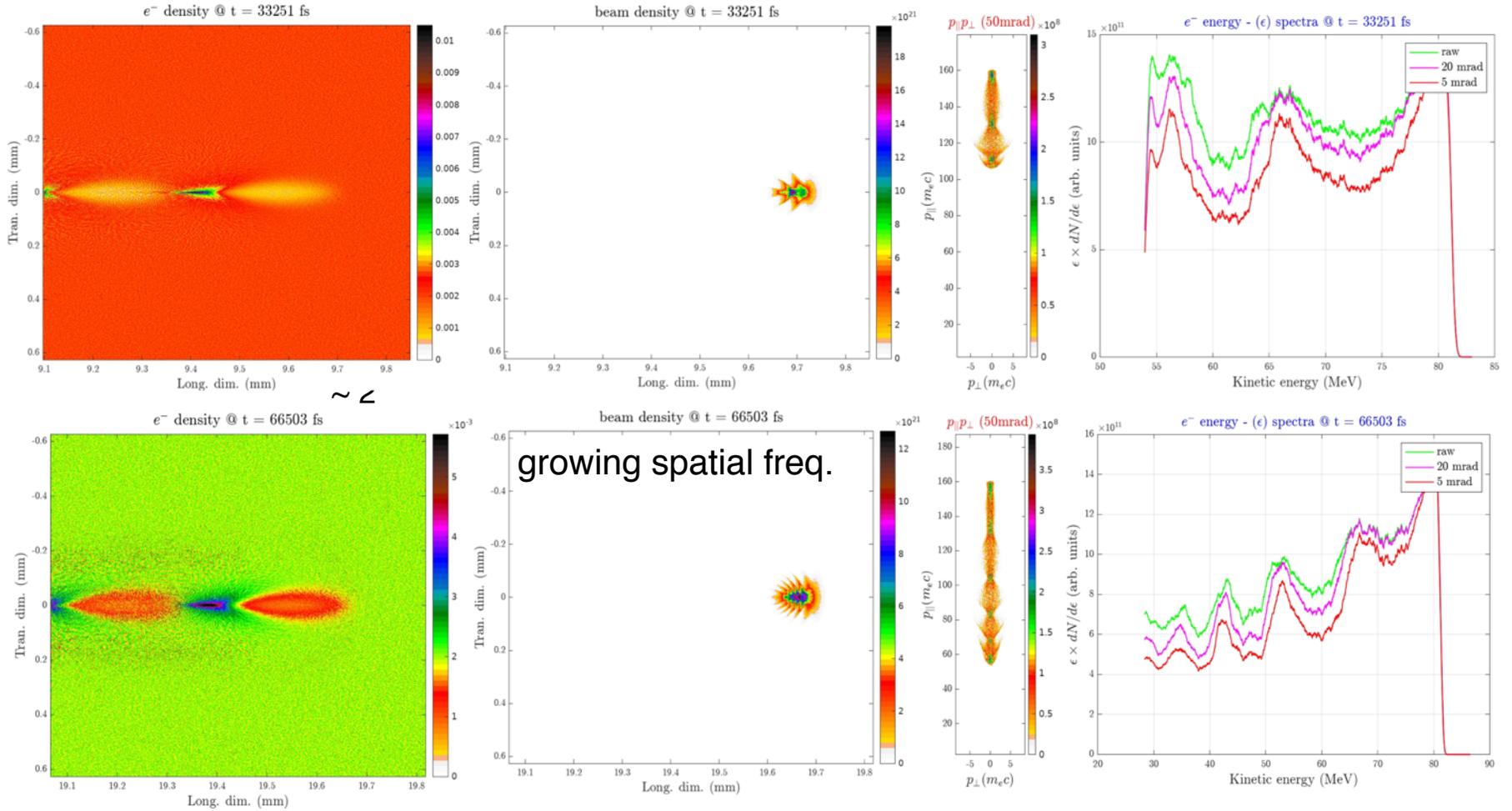
beam spot-size  
modulation period

$$\eta = \frac{\pi c \sigma'_L}{\Omega_{b0} x} = \sqrt{\frac{\gamma}{2}} \frac{\sigma'_L}{x} \lambda_{pe}$$



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Denver

**bunch-length** –  $\sigma_r = 50 \text{ } \mu\text{m}$ ,  $\sigma_z/c = 100 \text{ fs}$ ,  $Q = 100 \text{ pC}$ ,  $n_0 = 1 \text{e}16 \text{ cm}^{-3}$

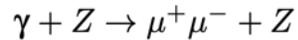


## Quasimonoenergetic laser plasma positron accelerator using particle-shower plasma-wave interactions

Aakash A. Sahai\*

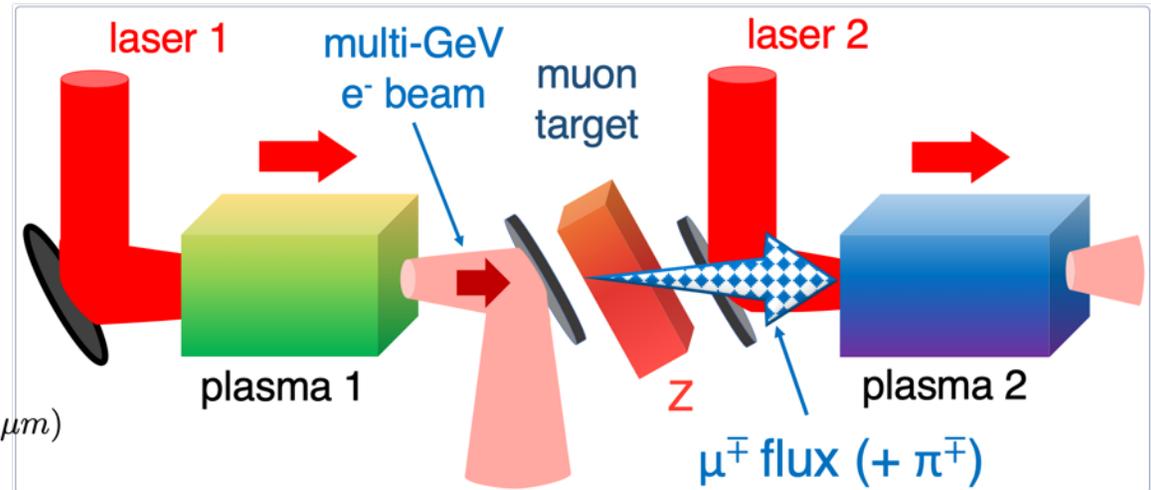
Department of Physics and John Adams Institute for Accelerator Science, Blackett Laboratory,  
Imperial College London, SW7 2AZ, United Kingdom

(R.2) Bethe-Heitler muon pair-production reaction:



$$\sigma_{\gamma Z_1 \rightarrow \mu^+ \mu^- Z_2} \simeq \frac{28}{9} Z^2 \alpha r_0^\mu{}^2 \left( \ln \frac{2E_\gamma}{m_\mu c^2} - \frac{109}{42} \right)$$

$$\mathcal{R}_{\gamma Z_1 \rightarrow \mu^+ \mu^- Z_2} (\text{in } 50\text{fs}) \simeq 10^5 \text{ pairs (1nC, 50fs, } \sigma_r \sim 20\mu\text{m)}$$



aakash.sahai@gmail.com