Higgs Boson Theory and Phenomenology: The Standard Model and Beyond

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Third CERN-Fermilab Hadron Collider Physics Summer School

Lectures

Lecture I: Electroweak symmetry breaking and the Standard Model Higgs boson

Lecture II: Weakly coupled Higgs bosons beyond the Standard Model

A brief bibliography

1. J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Perseus Publishing, Cambridge, MA, 1990).

2. H.E. Haber, "Lectures on Electroweak Symmetry Breaking," in *Testing the Standard Model*, Proceedings of the Theoretical Advanced Study Institute (TASI-90), Boulder, Colorado, 3–27 June, 1990, edited by M. Cvetic and P. Langacker (World Scientific, Singapore, 1991), pp. 340–475.

3. M. Carena and H.E. Haber, "Higgs boson theory and phenomenology," Prog. Part. Nucl. Phys. **50**, 63 (2003) [arXiv:hep-ph/0208209].

4. A. Djouadi, "The anatomy of electro-weak symmetry breaking. I: The Higgs boson in the Standard Model," Phys. Rept. **457**, 1 (2008) [arXiv:hep-ph/0503172].

5. A. Djouadi, "The anatomy of electro-weak symmetry breaking. II: The Higgs bosons in the minimal supersymmetric model," Phys. Rept. **459**, 1 (2008) [arXiv:hep-ph/0503173].

Lecture I: Electroweak symmetry breaking and the Standard Model Higgs boson

Outline

- The Standard Model—what's missing?
- mass generation and the Goldstone boson
- The significance of the TeV scale—Part 1
- theory of the Standard Model (SM) Higgs boson
- SM Higgs phenomenology—present and future

Particle content of the Standard Model

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, …

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0–0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009–0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04–0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

Something is missing…

BOSONS
force carriers
spin = 0, 1, 2, …

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

What's missing?

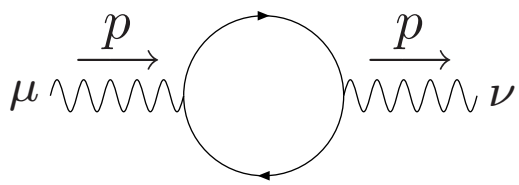
The theory of $W^{\pm}$ and Z gauge bosons must be *gauge invariant*; otherwise the theory is mathematically inconsistent. You may have heard that "gauge invariance implies that the gauge boson mass must be zero," since a mass term of the form $m^2 A^a_\mu A^{\mu a}$ is not gauge invariant.

So, what is the origin of the $W^{\pm}$ and Z boson masses? Gauge bosons are massless at tree-level, but perhaps a mass may be generated when quantum corrections are included. The tree-level gauge boson propagator $G^0_{\mu\nu}$ (in the Landau gauge) is:

$$G^0_{\mu\nu}(p) = \frac{-i}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) .$$

The pole at $p^2 = 0$ indicates that the tree-level gauge boson mass is zero. Let's now include the radiative corrections.

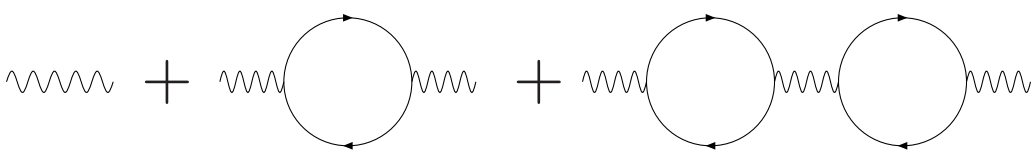
The polarization tensor $\Pi_{\mu\nu}(p)$ is defined as:



$$i\,\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2)$$

where the form of $\Pi_{\mu\nu}(p)$ is governed by gauge invariance, i.e. it satisfies $p^\mu \Pi_{\mu\nu}(p) = p^\nu \Pi_{\mu\nu}(p) = 0$.

The renormalized propagator is the sum of a geometric series



$$+ \ldots = \frac{-i(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{p^2[1+\Pi(p^2)]}$$

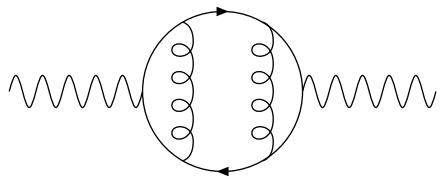
The pole at $p^2 = 0$ is shifted to a non-zero value if:

$$\Pi(p^2) \underset{p^2 \to 0}{\simeq} \frac{-g^2 v^2}{p^2}\,.$$

Then $p^2[1 + \Pi(p^2)] = p^2 - g^2 v^2$, yielding a gauge boson mass of gv.

Interpretation of the $p^2 = 0$ pole of $\Pi(p^2)$

The pole at $p^2 = 0$ corresponds to a propagating massless scalar. For example, the sum over intermediate states includes a quark-antiquark pair with many gluon exchanges, e.g.,

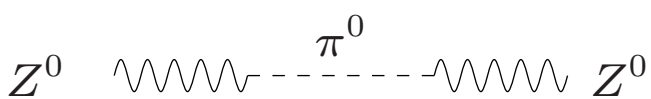


This is a strongly-interacting system—it is possible that one of the contributing intermediate states is a massless spin-0 state (due to the strong binding of the quark/antiquark pair).

We know that the Z and $W^{\pm}$ couple to neutral and charged weak currents

$$\mathcal{L}_{\rm int} = g_Z j^Z_\mu Z^\mu + g_W (j^W_\mu W^{+\mu} + {\rm h.c.}) \,,$$

which are known to create neutral and charged pions from the vacuum. In the absence of quark masses, the pions are massless bound states of $q\bar{q}$ [they are Goldstone bosons of chiral symmetry which is spontaneously broken by the strong interactions]. Thus, the diagram:



yields $\Pi(p^2) = -g_Z^2 f_\pi^2/p^2$, where $f_\pi = 93$ MeV is the amplitude for creating a pion from the vacuum. Thus, $m_Z = g_Z f_\pi$. Similarly $m_W = g_W f_\pi$.

Vector boson mass generation and the Goldstone boson

We have demonstrated a gauge-invariant mass generation mechanism for gauge bosons! The $p^2 = 0$ pole of $\Pi(p^2)$ corresponds to a propagating massless scalar state called the Goldstone boson. We showed that the W and Z are massive in the Standard Model (without Higgs bosons!!). Moreover, the ratio

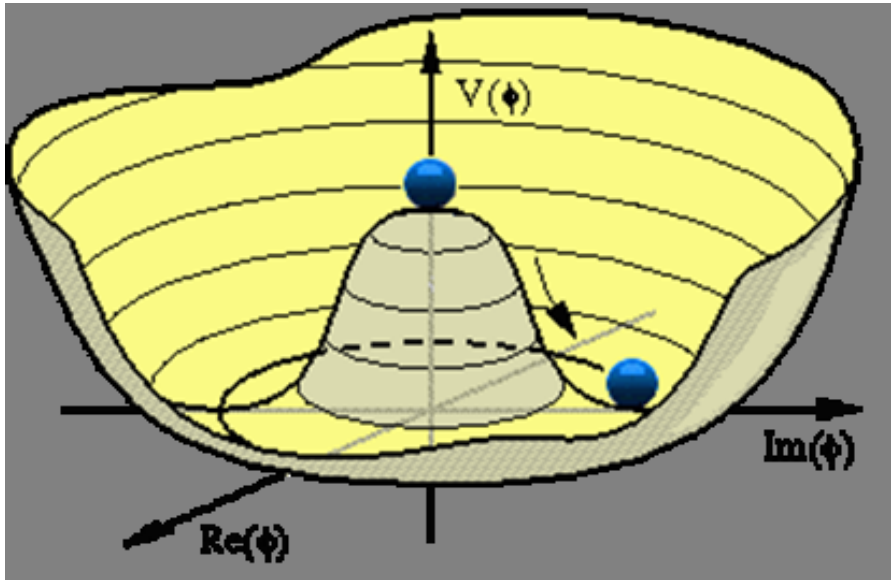
$$\frac{m_W}{m_Z} = \frac{g_W}{g_Z} \equiv \cos\theta_W \simeq 0.88$$

is remarkably close to the measured ratio. Unfortunately, since $g_Z \simeq 0.37$ we find $m_Z = g_Z f_\pi = 35$ MeV, which is too small by a factor of 2600. There must be another source for the vector boson masses, i.e. another source for the Goldstone boson.

The quest for electroweak symmetry breaking is the search for the dynamics that generates the Goldstone bosons that are the sources of mass for the W and Z.

Possible choices for electroweak-symmetry-breaking (EWSB) dynamics

- weakly-interacting self-coupled elementary (Higgs) scalar dynamics



- strong-interaction dynamics among new fermions (mediated perhaps by gauge forces) [see lectures by Sekhar Chivukula]

Both mechanisms generate new phenomena with significant experimental consequences.

Significance of the TeV Scale—Part 1

Let $\Lambda_{\rm EW}$ be energy scale of EWSB dynamics. For example:

- Elementary Higgs scalar ($\Lambda_{\rm EW} = m_h$).

- Strong EWSB dynamics ($e.g.$, $\Lambda_{\rm EW}^{-1}$ is the characteristic scale of bound states arising from new strong dynamics).

Consider $W_L^+ W_L^- \to W_L^+ W_L^-$ (L = longitudinal or equivalently, zero helicity) for $m_W^2 \ll s \ll \Lambda_{\rm EW}^2$. The corresponding amplitude, to leading order in g^2, but to all orders in the couplings that control the EWSB dynamics, is equal to the amplitude for $G^+ G^- \to G^+ G^-$ (where $G^\pm$ are the charged Goldstone bosons). The latter is universal, independent of the EWSB dynamics. This is a rigorous low-energy theorem.

Applying unitarity constraints to this amplitude yields a critical energy $\sqrt{s_c}$, above which unitarity is violated. This unitarity violation must be repaired by EWSB dynamics and implies that $\Lambda_{\rm EW} \lesssim \mathcal{O}\left(\sqrt{s_c}\right)$.

Unitarity of scattering amplitudes

Unitarity is equivalent to the conservation of probability in quantum mechanics. A violation of unitarity is tantamount to a violation of the principles of quantum mechanics—this is too sacred a principle to give up!

Consider the helicity amplitude $\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2)$ for a $2 \to 2$ scattering process with initial [final] helicities λ_1, λ_2 [λ_3, λ_4]. The Jacob-Wick partial wave expansion is:

$$\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2) = \frac{8\pi\sqrt{s}}{(p_i p_f)^1/2} e^{i(\lambda_i-\lambda_f)\phi} \sum_{J=J_0}^{\infty} (2J+1)\mathcal{M}^J_\lambda(s) d^J_{\lambda_i\lambda_f}(\theta)\,,$$

where p_i [p_f] is the incoming [outgoing] center-of-mass momentum, $\sqrt{s}$ is the center-of-mass energy, $\lambda \equiv \{\lambda_3\lambda_4\,;\,\lambda_1\lambda_2\}$ and

$$J_0 \equiv \max\{\lambda_i\,,\,\lambda_f\}\,, \quad \text{where} \quad \lambda_i \equiv \lambda_1 - \lambda_2\,, \quad \text{and} \quad \lambda_f \equiv \lambda_3 - \lambda_4\,.$$

Orthogonality of the d-functions allows one to project out a given partial wave amplitude. For example, for $W_L^+W_L^- \to W_L^+W_L^-$ (L corresponds to $\lambda = 0$),

$$\mathcal{M}^{J=0} = \frac{1}{16\pi s}\int_{-s}^{0} dt\,\mathcal{M}(L,L\,;\,L,L)\,,$$

where $t = -\frac{1}{2}s(1-\cos\theta)$ in the limit where $m_W^2 \ll s$.

For example, the $J = 0$ partial wave for $W_L^+ W_L^- \to W_L^+ W_L^-$ in the limit of $m_W^2 \ll s \ll \Lambda_{\rm EW}^2$ is equal to the corresponding amplitude for $G^+ G^- \to G^+ G^-$:

$$\mathcal{M}^{J=0} = \frac{G_F s}{16\pi\sqrt{2}}\,.$$

Partial wave unitarity implies that:

$$|\mathcal{M}^J|^2 \leq |{\rm Im}\ \mathcal{M}^J| \leq 1\,,$$

which gives

$$({\rm Re}\ \mathcal{M}^J)^2 \leq |{\rm Im}\ \mathcal{M}^J| \Big(1 - |{\rm Im}\ \mathcal{M}^J|\Big) \leq \tfrac{1}{4}\,.$$

Setting $|{\rm Re}\ \mathcal{M}^{J=0}| \leq \frac{1}{2}$ yields $\sqrt{s_c}$. The most restrictive bound arises from the isospin zero channel $\sqrt{\frac{1}{6}(2W_L^+ W_L^- + Z_L Z_L)}$:

$$s_c = \frac{4\pi\sqrt{2}}{G_F} = (1.2\ {\rm TeV})^2\,.$$

Since unitarity cannot be violated, we conclude that $\Lambda_{\rm EW} \lesssim \sqrt{s_c}$. That is,

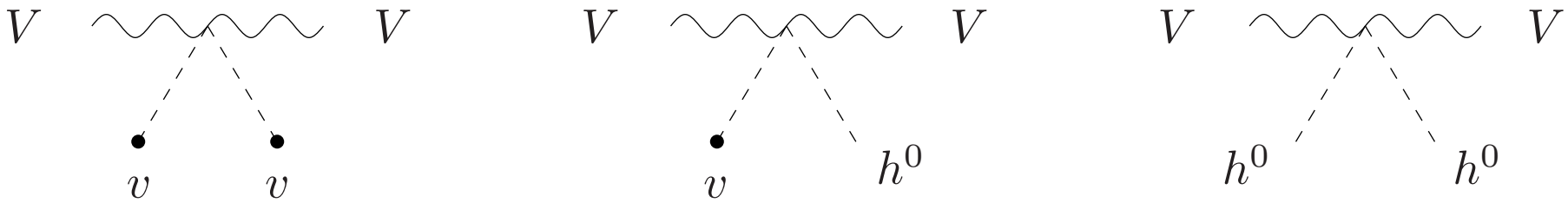
The dynamics of electroweak symmetry breaking must be exposed at or below the 1 TeV energy scale.

EWSB Dynamics of the Standard Model

- Add a new sector of "matter" consisting of a complex SU(2) doublet, hypercharge-one self-interacting scalar fields, $\Phi \equiv (\Phi^+ \;\; \Phi^0)$ with four real degrees of freedom. The scalar potential is:
$$V(\Phi) = \frac{\lambda}{4}(\Phi^\dagger\Phi - \tfrac{1}{2}v^2)^2 \,,$$
so that in the ground state, the neutral scalar field takes on a constant non-zero value $\langle\Phi^0\rangle = v/\sqrt{2}$, where $v = 246$ GeV.
- The non-zero scalar vacuum expectation value breaks the electroweak symmetry, thereby generating three Goldstone bosons (exactly massless), which become the longitudinal components of the $W^\pm$ and Z. Here, v plays the role of f_π, so we get $m_Z = g_Z v \simeq 91$ GeV.
- One scalar degree of freedom is left over—the Higgs boson, $h^0 \equiv \sqrt{2}\,\mathrm{Re}(\Phi^0 - \frac{v}{\sqrt{2}})$. It is a neutral CP-even scalar boson, whose interactions are precisely predicted, but whose mass $m_h = \frac{1}{2}\lambda v^2$ depends on the unknown strength of the scalar self-coupling—the only unknown parameter of the model.

Mass generation and Higgs couplings in the SM

Gauge bosons ($V = W^{\pm}$ or Z) acquire mass via interaction with the Higgs vacuum condensate.



Thus,

$$g_{hVV} = 2m_V^2/v\,, \qquad \text{and} \qquad g_{hhVV} = 2m_V^2/v^2\,,$$

i.e., the Higgs couplings to vector bosons are proportional to the corresponding boson squared-mass.

Likewise, by replacing V with the Higgs field h^0 in the above diagrams, the Higgs self-couplings are also proportional to the square of the Higgs mass:

$$g_{hhh} = \tfrac{3}{2}\lambda v = \frac{3m_h^2}{v}\,, \qquad \text{and} \qquad g_{hhhh} = \tfrac{3}{2}\lambda = \frac{3m_h^2}{v^2}\,.$$

Fermions in the Standard Model

Given a four-component fermion f, we can project out the right and left-handed parts:

$$f_R \equiv P_R f \,, \qquad f_L \equiv P_L f \,, \qquad \text{where} \quad P_{R,L} = \tfrac{1}{2}(1 \pm \gamma_5) \,.$$

Under the electroweak gauge group, the right and left-handed components of each fermion has different SU(2)×U(1)$_{\rm Y}$ quantum numbers:

fermions	SU(2)	U(1)$_{\rm Y}$
$(\nu \,,\, e^-)_L$	2	-1
e^-_R	1	-2
$(u \,,\, d)_L$	2	$1/3$
u_R	1	$4/3$
d_R	1	$-2/3$

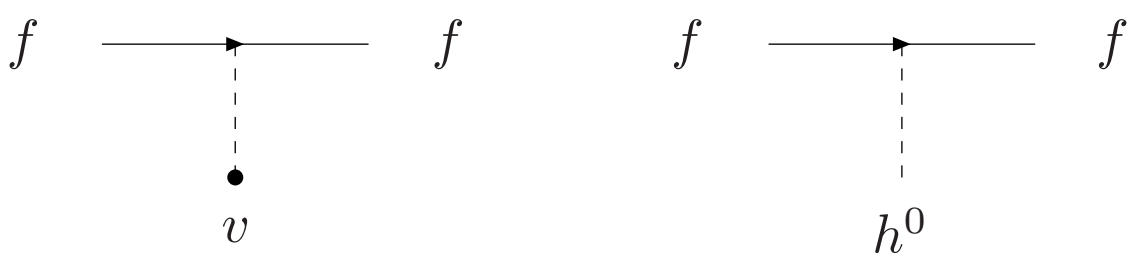
where the electric charge is related to the U(1)$_{\rm Y}$ hypercharge by $Q = T_3 + \frac{1}{2}Y$.

Before electroweak symmetry breaking, Standard Model fermions are massless, since the fermion mass term $\mathcal{L}_m = -m(\bar{f}_R f_L + \bar{f}_L f_R)$ is not gauge invariant.

The generation of masses for quarks and leptons is especially elegant in the SM (in other approaches to EWSB, fermion mass generation is often a challenge). The fermions couple to the Higgs field through the gauge invariant Yukawa couplings, e.g.,

$$\mathcal{L}_{\rm Yukawa} = -h_u(\bar{u}_R u_L \Phi^0 - \bar{u}_R d_L \Phi^+) - h_d(\bar{d}_R d_L \Phi^{0\,*} + \bar{d}_R u_L \Phi^-) + \text{h.c.}$$

The quarks and charged leptons acquire mass when Φ^0 acquires a vacuum expectation value:



Thus,

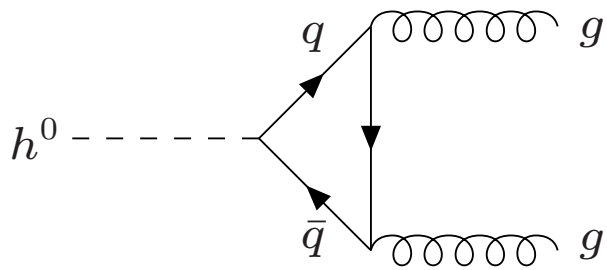
$$g_{hf\bar{f}} = m_f/v\,,$$

i.e., Higgs couplings to fermions are proportional to the corresponding fermion mass.

Loop induced Higgs boson couplings

Higgs boson coupling to gluons

At one-loop, the Higgs boson couples to gluons via a loop of quarks:



This diagram leads to an effective Lagrangian

$$\mathcal{L}^{\rm eff}_{hgg} = \frac{g\alpha_s N_g}{24\pi m_W} h^0 G^a_{\mu\nu} G^{\mu\nu a} \,,$$

where N_g is roughly the number of quarks heavier than h^0. More precisely,

$$N_g = \sum_i F_{1/2}(x_i) \,, \qquad\qquad x_i \equiv \frac{m^2_{q_i}}{m^2_h} \,,$$

where the loop function $F_{1/2}(x) \to 1$ for $x \gg 1$.

Note that heavy quark loops do *not* decouple. Light quark loops are negligible, as $F_{1/2}(x) \to \frac{3}{2}x^2 \ln x$ for $x \ll 1$.

The dominant mechanism for Higgs production at the LHC is gluon-gluon fusion. At leading order,

$$\frac{d\sigma}{dy}(pp \to h^0 + X) = \frac{\pi^2 \Gamma(h^0 \to gg)}{8m_h^3} g(x_+, m_h^2) g(x_-, m_h^2)\,,$$

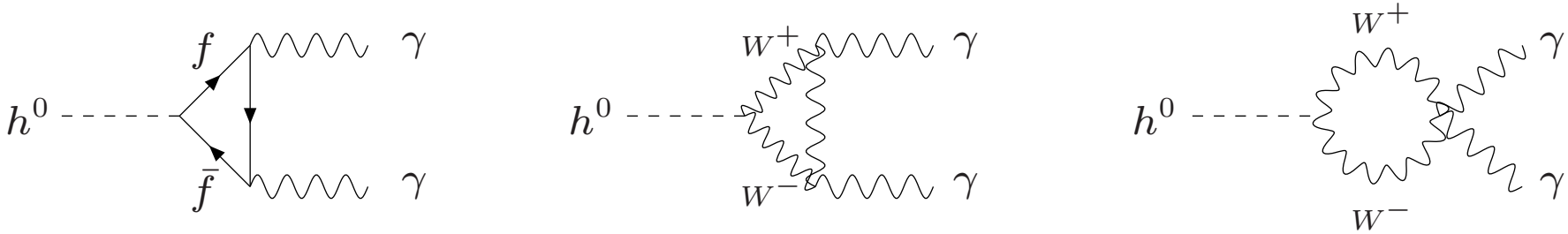
where $g(x, Q^2)$ is the gluon distribution function at the scale Q^2 and

$$x_\pm \equiv \frac{m_h e^{\pm y}}{\sqrt{s}}\,, \qquad y = \tfrac{1}{2} \ln\left(\frac{E + p_{||}}{E - p_{||}}\right)\,.$$

The rapidity y is defined in terms of the Higgs boson energy and longitudinal momentum in the pp center-of-mass frame.

Higgs boson coupling to photons

At one-loop, the Higgs boson couples to photons via a loop of charged particles:



If charged scalars exist, they would contribute as well. These diagrams lead to an effective Lagrangian

$$\mathcal{L}^{\rm eff}_{h\gamma\gamma} = \frac{g\alpha N_\gamma}{12\pi m_W} h^0 F_{\mu\nu}F^{\mu\nu} \,,$$

where

$$N_\gamma = \sum_i N_{ci} e_i^2 F_j(x_i) \,, \qquad x_i \equiv \frac{m_i^2}{m_h^2} \,.$$

In the sum over loop particles i of mass m_i, $N_{ci} = 3$ for quarks and 1 for color singlets, e_i is the electric charge in units of e and $F_j(x_i)$ is the loop function corresponding to ith particle (with spin j). In the limit of $x \gg 1$,

$$F_j(x) \longrightarrow \begin{cases} 1/4 \,, & j = 0 \,, \\ 1 \,, & j = 1/2 \,, \\ -21/4 \,, & j = 1 \,. \end{cases}$$

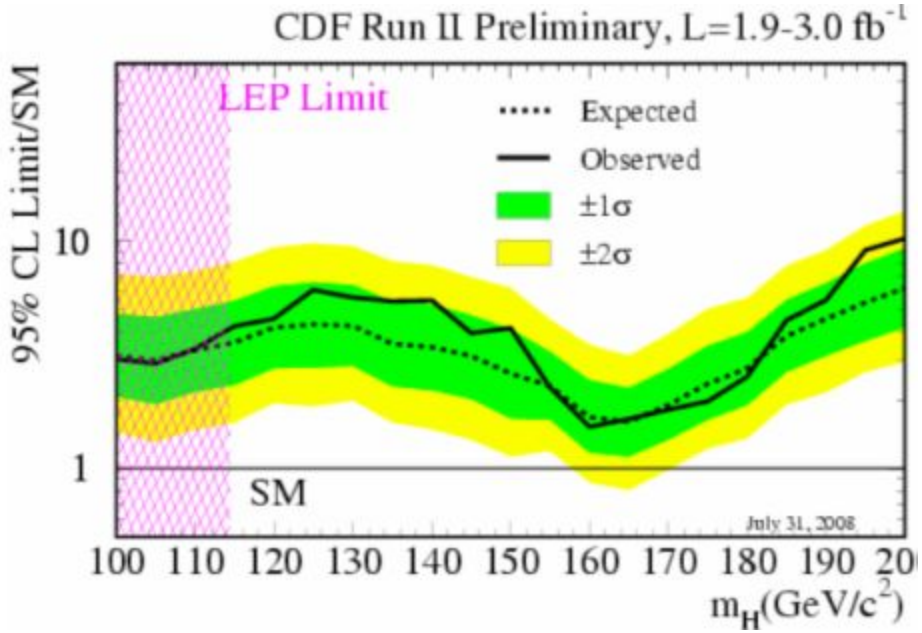
Expectations for the SM Higgs mass

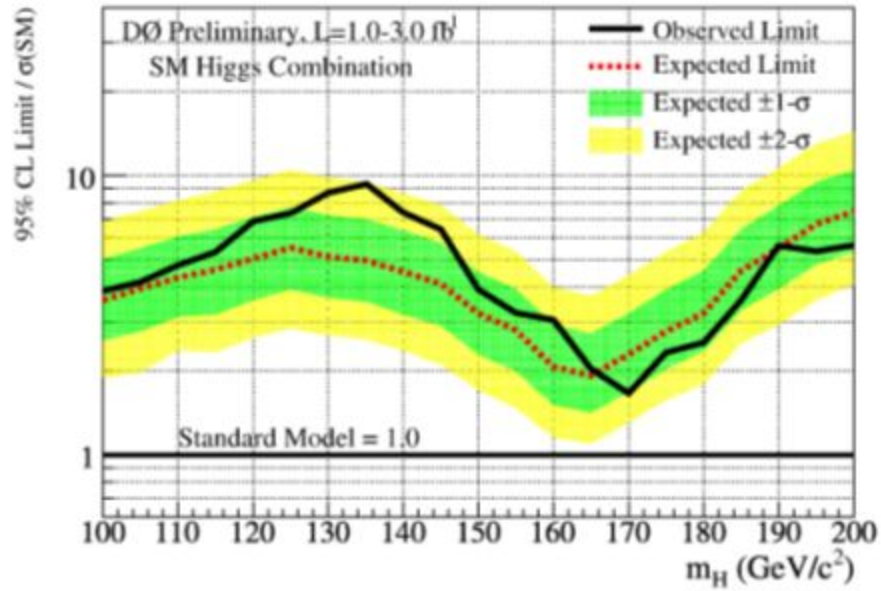
1. Lower experimental bound

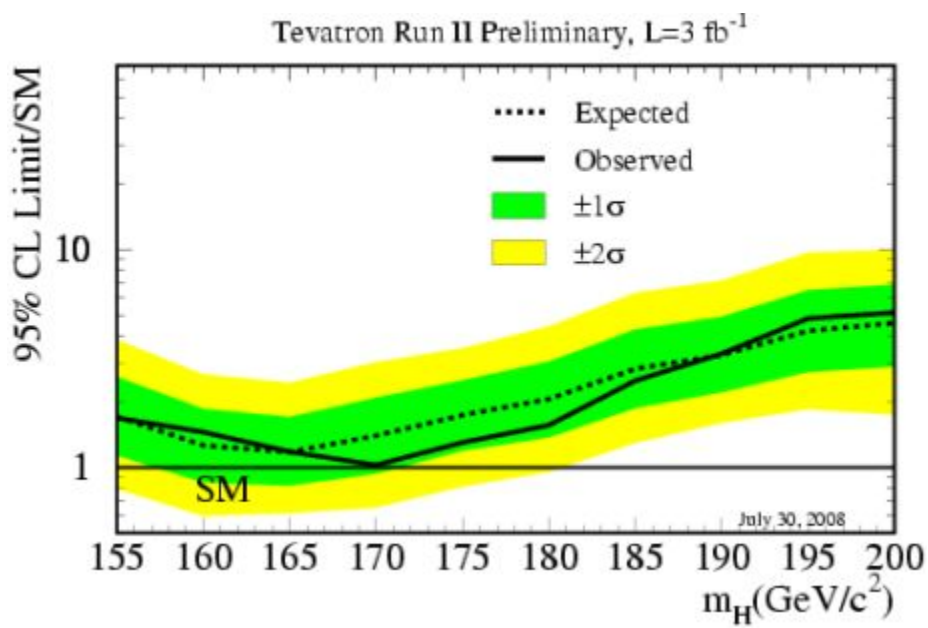
From 1989–2000, experiments at LEP searched for $e^+e^- \to Z \to h^0 Z$ (where one of the Z-bosons is on-shell and one is off-shell). No significant evidence was found leading to a lower bound on the Higgs mass

$$\boxed{m_h > 114.4 \text{ GeV} \quad \text{at} \quad 95\% \text{ CL.}}$$

In 2000, the report of the Higgs/Supersymmetry Tevatron Run-2 Workshop suggested that the Tevatron could extend the LEP Higgs reach with sufficient data. A few weeks ago, CDF and D0 announced that a small region of the SM Higgs mass range centered around 170 GeV is now excluded at 95% CL. With at least one more year of running, the Tevatron will further extend the Higgs mass reach.

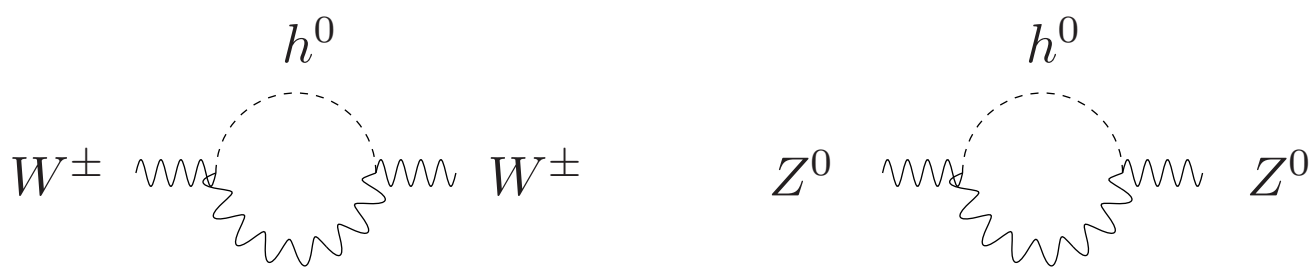

CDF Run II Preliminary, L=1.9-3.0 fb^{-1}
LEP Limit
Expected
Observed
±1σ
±2σ
95% CL Limit/SM
10
1
SM
July 31, 2008
100 110 120 130 140 150 160 170 180 190 200
m_H(GeV/c^2)


DØ Preliminary, L=1.0-3.0 fb^{-1}
SM Higgs Combination
Observed Limit
Expected Limit
Expected ±1-σ
Expected ±2-σ
95% CL Limit / σ(SM)
10
1
Standard Model = 1.0
100 110 120 130 140 150 160 170 180 190 200
m_H (GeV/c^2)


Tevatron Run II Preliminary, L=3 fb^{-1}
Expected
Observed
±1σ
±2σ
95% CL Limit/SM
10
1
SM
July 30, 2008
155 160 165 170 175 180 185 190 195 200
m_H(GeV/c^2)

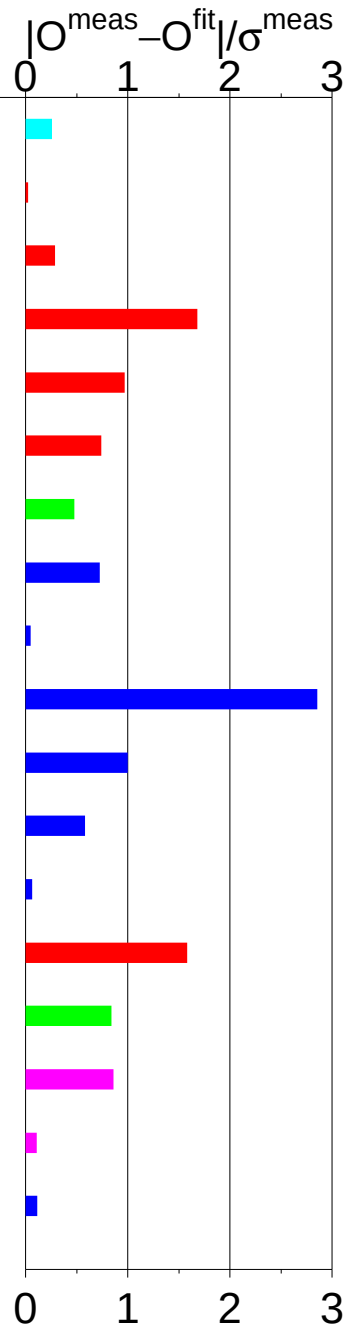
2. Upper bound from precision tests of the Standard Model

Very precise tests of the Standard Model are possible given the large sample of electroweak data from LEP, SLC and the Tevatron. Although the Higgs boson mass (m_h) is unknown, electroweak observables are sensitive to m_h through quantum corrections. For example, the W and Z masses are shifted slightly due to:



The m_h dependence of the above radiative corrections is logarithmic. Nevertheless, a global fit of many electroweak observables can determine the preferred value of m_h (assuming that the Standard Model is the correct description of the data).

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21581
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.377
Γ_W [GeV]	2.097 ± 0.048	2.092
m_t [GeV]	172.6 ± 1.4	172.8

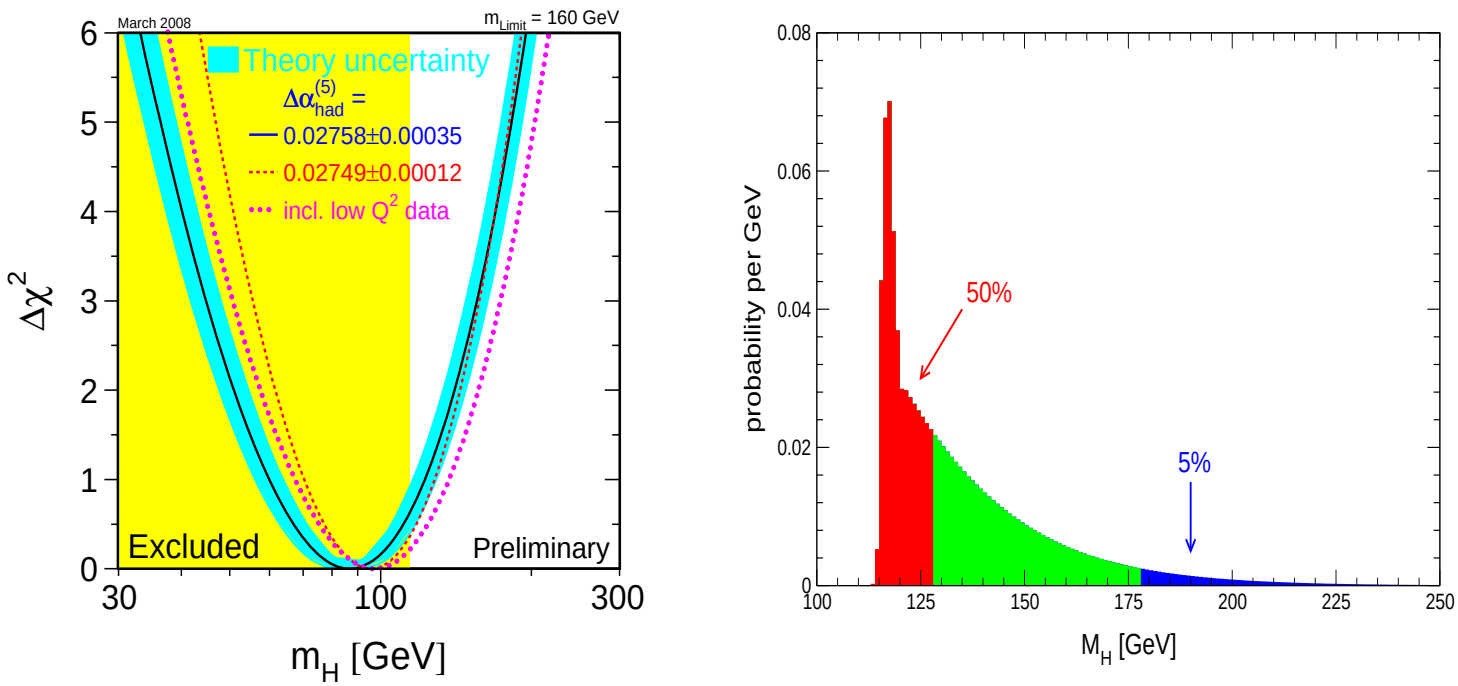


March 2008

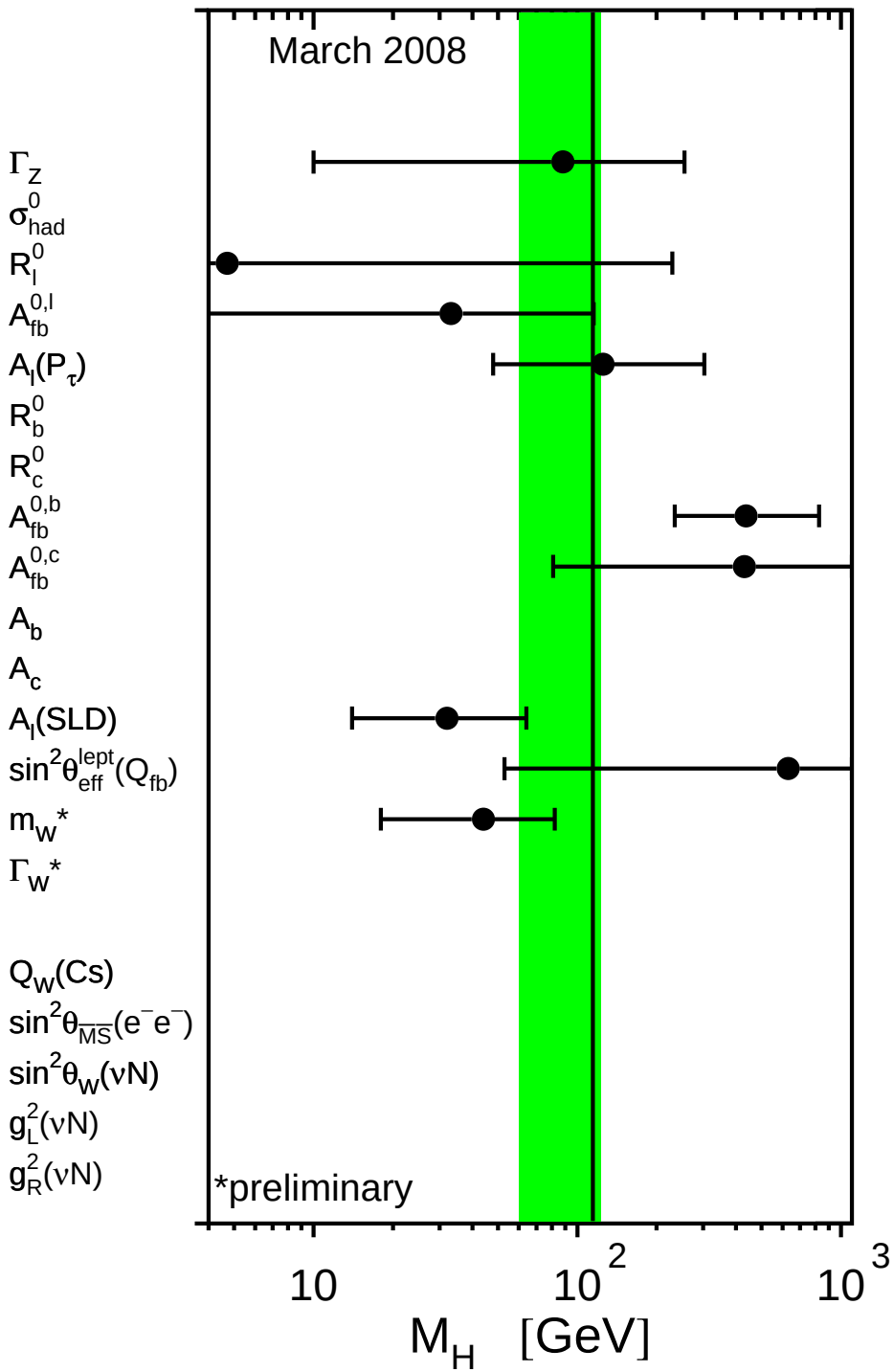
LEP/Tevatron Electroweak Working Groups: the SM global fit

$$m_h = 87^{+36}_{-27}\ \text{GeV} \qquad [m_h < 160\ \text{GeV}\ \ \text{one-sided 95\% CL}]\,.$$

Including the direct LEP search data yields $m_h < 190$ GeV at 95% CL.



(a) The *blueband* plot shows $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\rm min}$ as a function of m_h. (b) Probability distribution function for m_h [J. Erler, hep-ph/0701261].


March 2008
Γ_Z
σ^0_{had}
R^0_l
$A^{0,l}_{fb}$
$A_l(P_\tau)$
R^0_b
R^0_c
$A^{0,b}_{fb}$
$A^{0,c}_{fb}$
A_b
A_c
A_l(SLD)
$\sin^2\theta^{lept}_{eff}(Q_{fb})$
m_W*
Γ_W*
Q_W(Cs)
$\sin^2\theta_{\overline{MS}}(e^-e^-)$
$\sin^2\theta_W(\nu N)$
$g^2_L(\nu N)$
$g^2_R(\nu N)$
*preliminary
10
10^2
10^3
M_H [GeV]

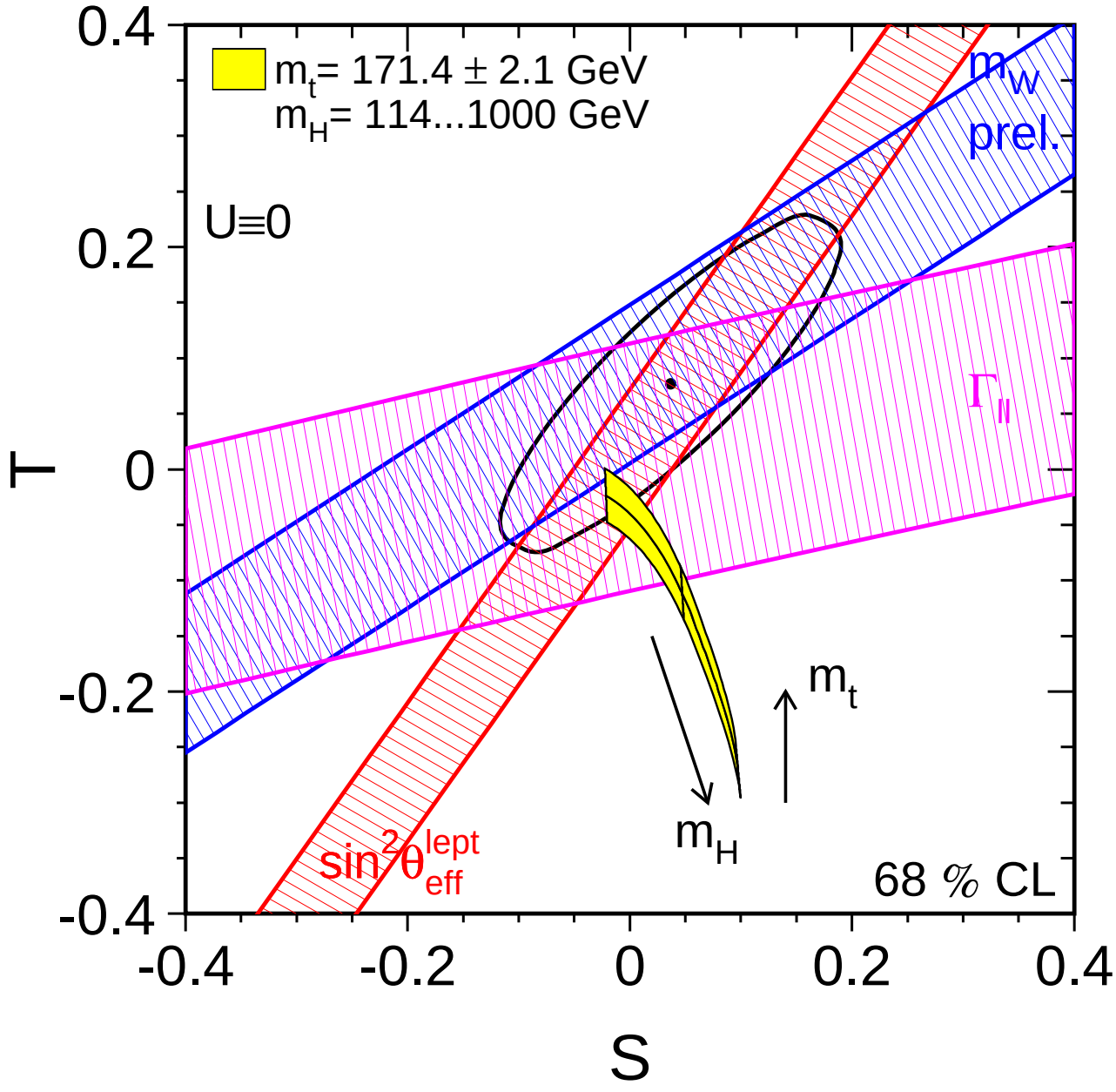
Can a Light Higgs Boson be avoided?

If new physics beyond the Standard Model (SM) exists, it almost certainly couples to W and Z bosons. Then, there will be additional shifts in the W and Z mass due to the appearance of new particles in loops. In many cases, these effects can be parameterized in terms of two quantities, S and T [Peskin and Takeuchi]:

$$\overline{\alpha}\, T \equiv \frac{\Pi^{\rm new}_{WW}(0)}{m_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{m_Z^2}\,,$$

$$\frac{\overline{\alpha}}{4\overline{s}_Z^2\overline{c}_Z^2}\, S \equiv \frac{\Pi^{\rm new}_{ZZ}(m_Z^2) - \Pi^{\rm new}_{ZZ}(0)}{m_Z^2} - \left(\frac{\overline{c}_Z^2 - \overline{s}_Z^2}{\overline{c}_Z\overline{s}_Z}\right)\frac{\Pi^{\rm new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{\rm new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\,,$$

where $s \equiv \sin\theta_W$, $c \equiv \cos\theta_W$, and barred quantities are defined in the $\overline{\rm MS}$ scheme evaluated at m_Z. The $\Pi^{\rm new}_{V_aV_b}$ are the new physics contributions to the one-loop V_a—V_b vacuum polarization functions.


m_t= 171.4 ± 2.1 GeV
m_H= 114...1000 GeV
U≡0
m_W
prel.
Γ_ll
m_t
m_H
sin²θ_eff^lept
68 % CL
T
S
0.4
0.2
0
-0.2
-0.4
-0.4
-0.2
0
0.2
0.4

In order to avoid the conclusion of a light Higgs boson, new physics beyond the SM must be accompanied by a variety of new phenomena at an energy scale between 100 GeV and 1 TeV. This new physics will be detected at future colliders

- either through direct observation of new physics beyond the Standard Model
- or by improved precision measurements that can detect small deviations from SM predictions.

Although the precision electroweak data is suggestive of a weakly-coupled Higgs sector, one cannot definitively rule out another source of EWSB dynamics (although the measured S and T impose strong constraints on alternative approaches).

Can the Higgs Boson mass be large?

A Higgs boson with a mass greater than 200 GeV almost certainly requires additional new physics beyond the Standard Model. But, how heavy can this Higgs boson be?

Let us return to the unitarity argument. Consider the scattering process $W_L^+(p_1)W_L^-(p_2) \to W_L^+(p_3)W_L^-(p_4)$ at center-of-mass energies $\sqrt{s} \gg m_W$. Each contribution to the tree-level amplitude is proportional to

$$[\varepsilon_L(p_1) \cdot \varepsilon_L(p_2)]\,[\varepsilon_L(p_3) \cdot \varepsilon_L(p_4)] \sim \frac{s^2}{m_W^4}\,,$$

after using the fact that the helicity-zero polarization vector at high energies behaves as $\varepsilon_L^\mu(p) \sim p^\mu/m_W$. Due to the magic of gauge invariance and the presence of Higgs-exchange contributions, the bad high-energy behavior is removed, and one finds for s, $m_h^2 \gg m_W^2$:

$$\mathcal{M} = -\sqrt{2}G_F m_H^2 \left(\frac{s}{s - m_h^2} + \frac{t}{t - m_h^2} \right) \,.$$

Projecting out the $J = 0$ partial wave and taking $s \gg m_h^2$,

$$\mathcal{M}^{J=0} = -\frac{G_F m_h^2}{4\pi\sqrt{2}}\,.$$

Imposing $|\mathrm{Re}\ \mathcal{M}^J| \leq \frac{1}{2}$ yields an upper bound on m_h. The most stringent bound is obtained by all considering other possible final states such as $Z_L Z_L$, $Z_L h^0$ and $h^0 h^0$. The end result is:

$$\boxed{m_h^2 \leq \frac{4\pi\sqrt{2}}{3G_F} \simeq (700\ \mathrm{GeV})^2\,.}$$

However, in contrast to our previous analysis of the unitarity bound, the above computation relies on the validity of a tree-level computation. That is, we are implicitly assuming that perturbation theory is valid. If $m_h \gtrsim 700$ GeV, then the Higgs-self coupling parameter, $\lambda = 2m_h^2/v^2$ is becoming large and our perturbative analysis is no longer valid.

Nevertheless, lattice studies suggest that an upper Higgs mass bound below 1 TeV remains valid even in the strong Higgs self-coupling regime.

Beyond the SM—The decoupling scenario

One message of the precision electroweak data is that the Standard Model is a good approximation to the theory of fundamental particles and their interactions at an energy scale of order 100 GeV. Thus, if new physics beyond the Standard Model exists, it is likely to consist of new degrees of freedom whose masses are somewhat larger than the scale of electroweak physics ($M_{\rm heavy} \gg m_W$).

Using effective field theory techniques, we can integrate out this "new heavy physics." What remains is the Standard Model Lagrangian, accompanied by higher-dimensional operators ($d \geq 5$) with coefficients suppressed by powers of $m_W/M_{\rm heavy}$.

If the Higgs sector is non-minimal, but the additional Higgs degrees of freedom are associated with the mass scale $M_{\rm heavy}$, then the effective low-energy theory will contain a single CP-even neutral Higgs boson, whose properties approximate those of the SM Higgs boson (up to corrections of order $m_W^2/M_{\rm heavy}^2$). This is the so-called *decoupling limit*.

These arguments provide additional motivation for studying in detail the phenomenology of the SM Higgs boson!

Higgs phenomenology at colliders

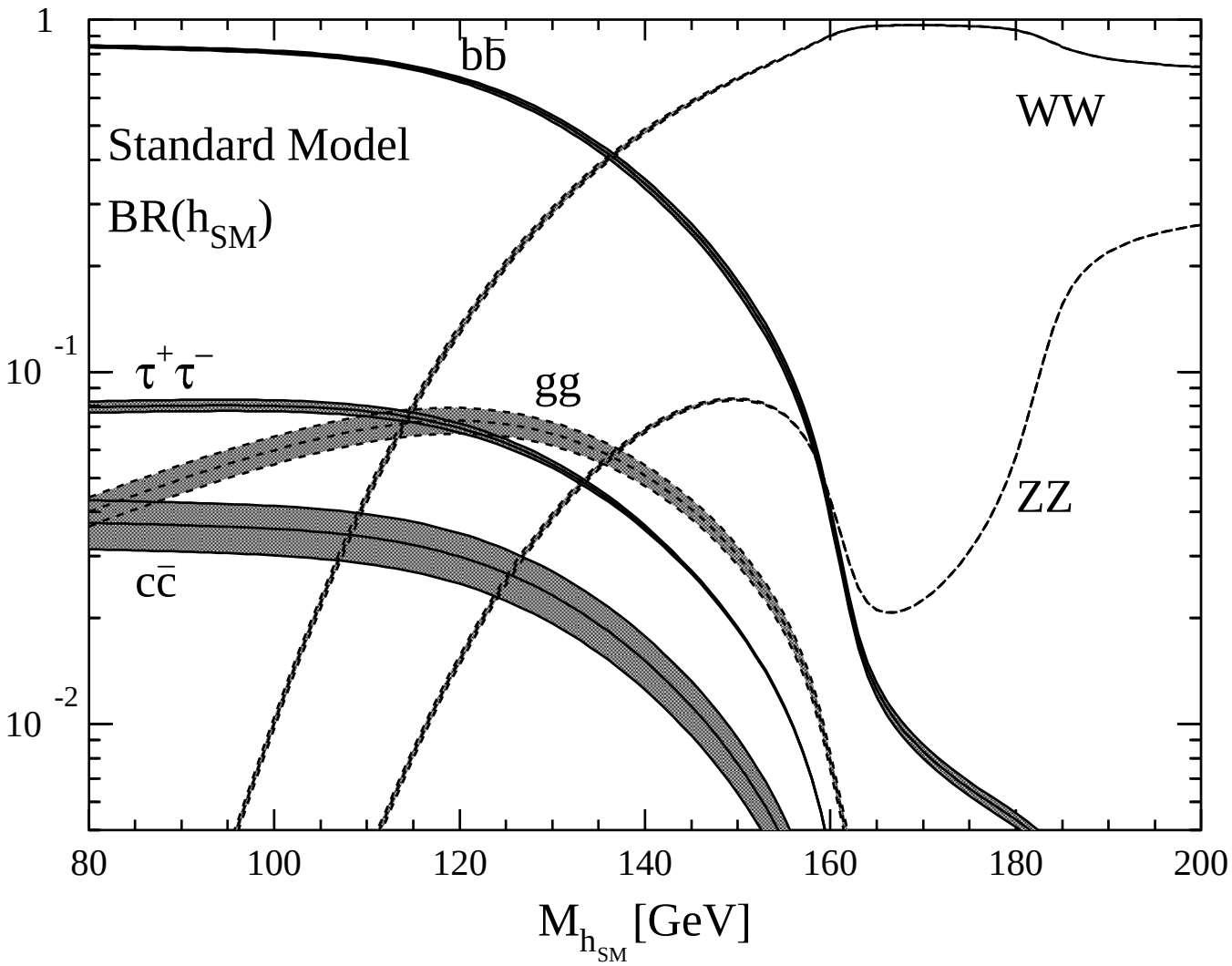
A program of Higgs physics at colliders must address:

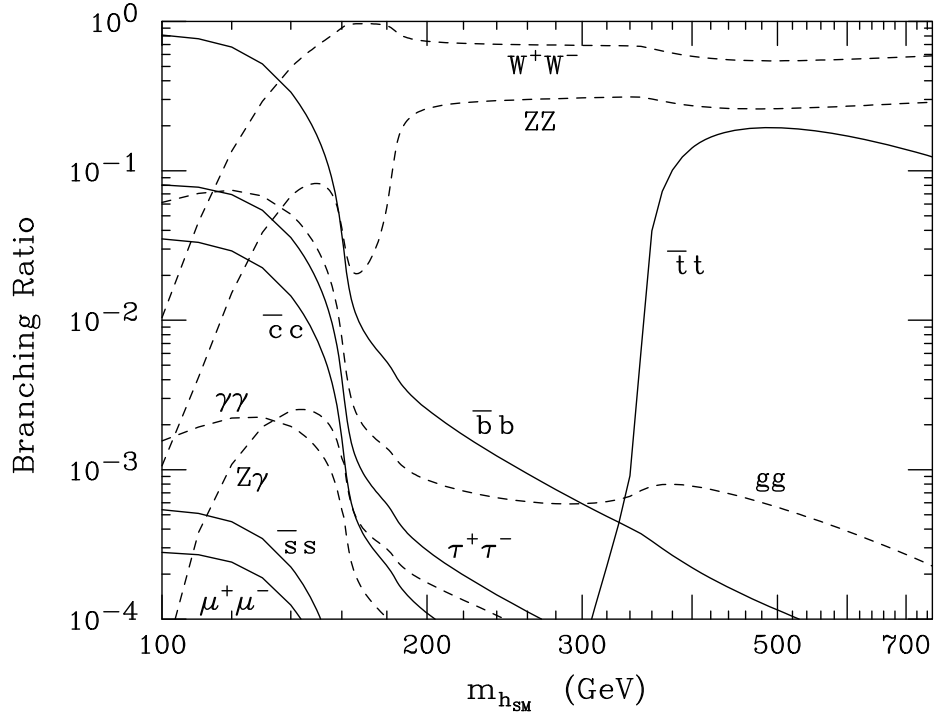
- Discovery reach for the SM Higgs boson
- How many Higgs states are there?
- Assuming one Higgs-like state is discovered
  - Is it a Higgs boson?
  - Is it the SM Higgs boson?

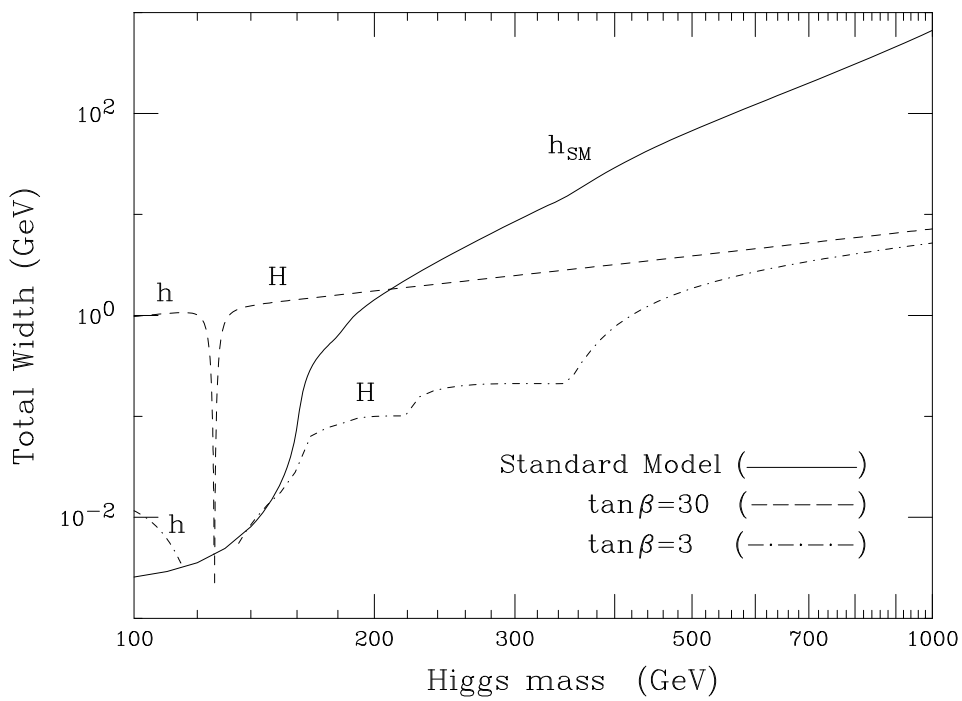
The measurement of Higgs boson properties will be critical in order to answer the last two questions:

- mass, width, CP-quantum numbers (CP-violation?)
- branching ratios and Higgs couplings
- reconstructing the Higgs potential

SM Higgs Branching Ratios and Width




Branching Ratio
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
100
200
300
500
700
$m_{h_{SM}}$ (GeV)
W^+W^-
ZZ
$\bar{t}t$
$\bar{c}c$
$\gamma\gamma$
$\bar{b}b$
$Z\gamma$
gg
$\bar{s}s$
$\tau^+\tau^-$
$\mu^+\mu^-$


Total Width (GeV)
10^2
10^0
10^{-2}
100
200
300
500
700
1000
Higgs mass (GeV)
h_{SM}
h
H
H
h
Standard Model (———)
$\tan\beta$=30 (– – – – –)
$\tan\beta$=3 (–·–·–·–)

Higgs production at hadron colliders

At hadron colliders, the relevant processes are

$$gg \to h^0 \to \gamma\gamma\,,$$

$$gg \to h^0 \to VV^{(*)}\,, \qquad [V = W \text{ or } Z]$$

$$qq \to qqV^{(*)}V^{(*)} \to qqh^0, \quad h^0 \to \gamma\gamma,\, \tau^+\tau^-,\, VV^{(*)}\,,$$

$$q\bar{q}^{(\prime)} \to V^{(*)} \to Vh^0\,, \quad h^0 \to b\bar{b}\,, WW^{(*)}\,,$$

$$gg, q\bar{q} \to t\bar{t}h^0, \quad h^0 \to b\bar{b},\, \gamma\gamma,\, WW^{(*)}\,.$$

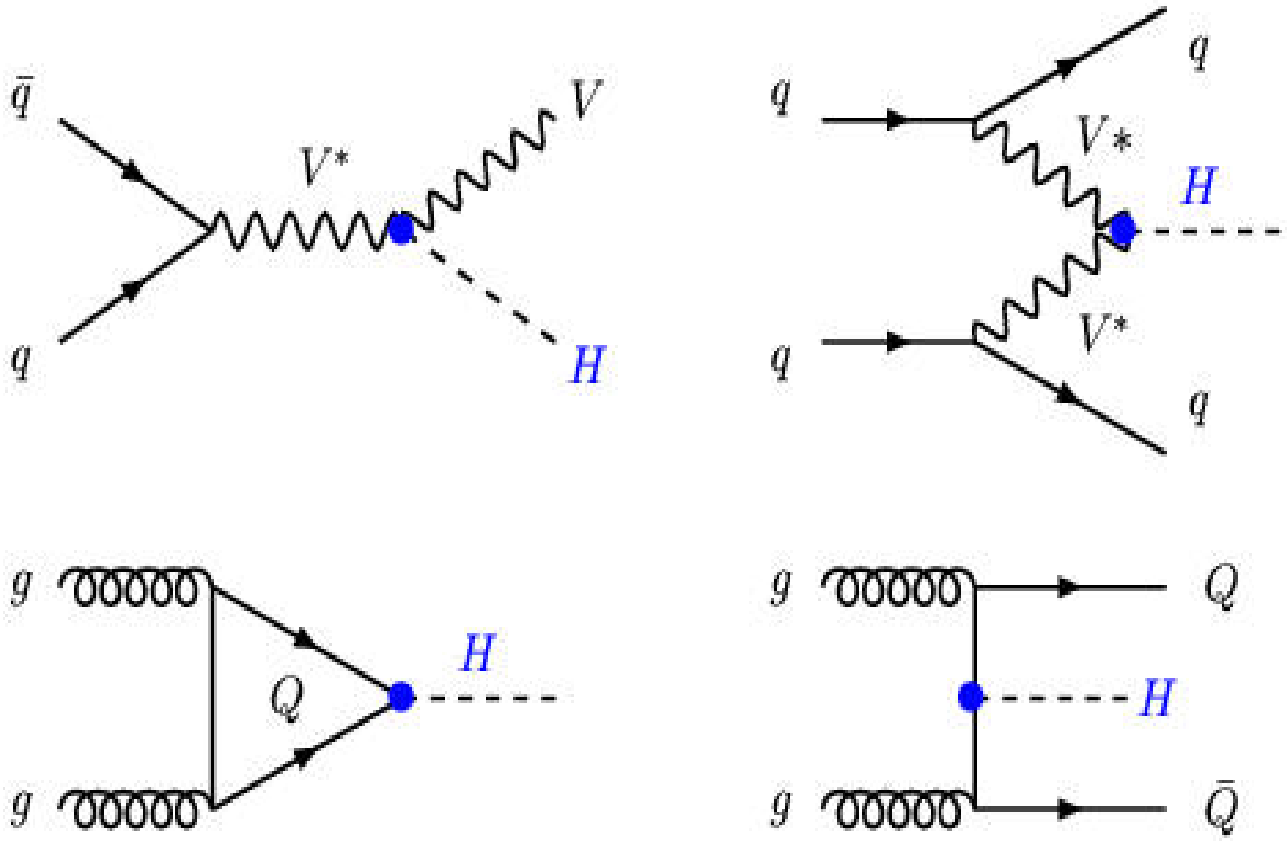
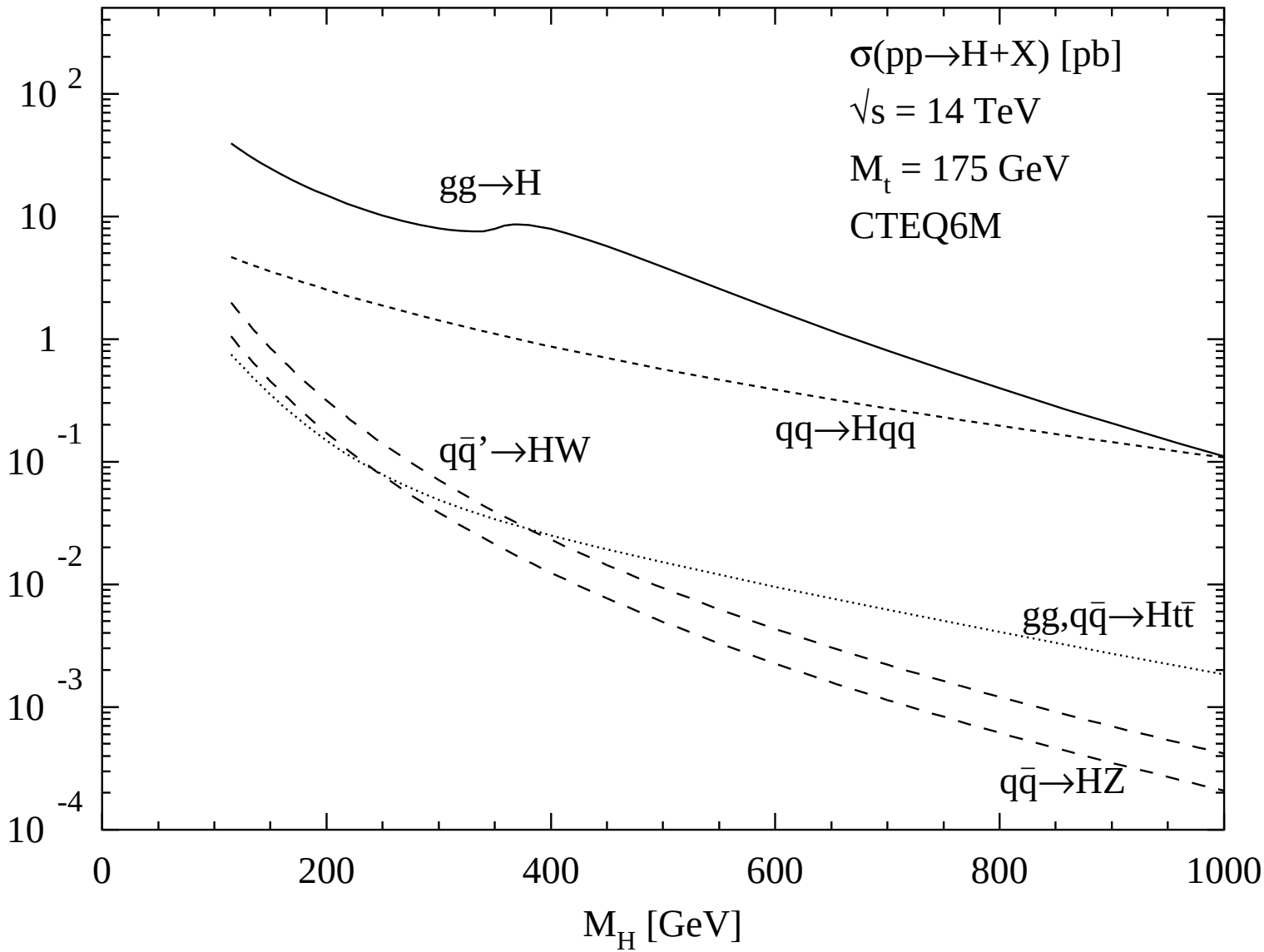
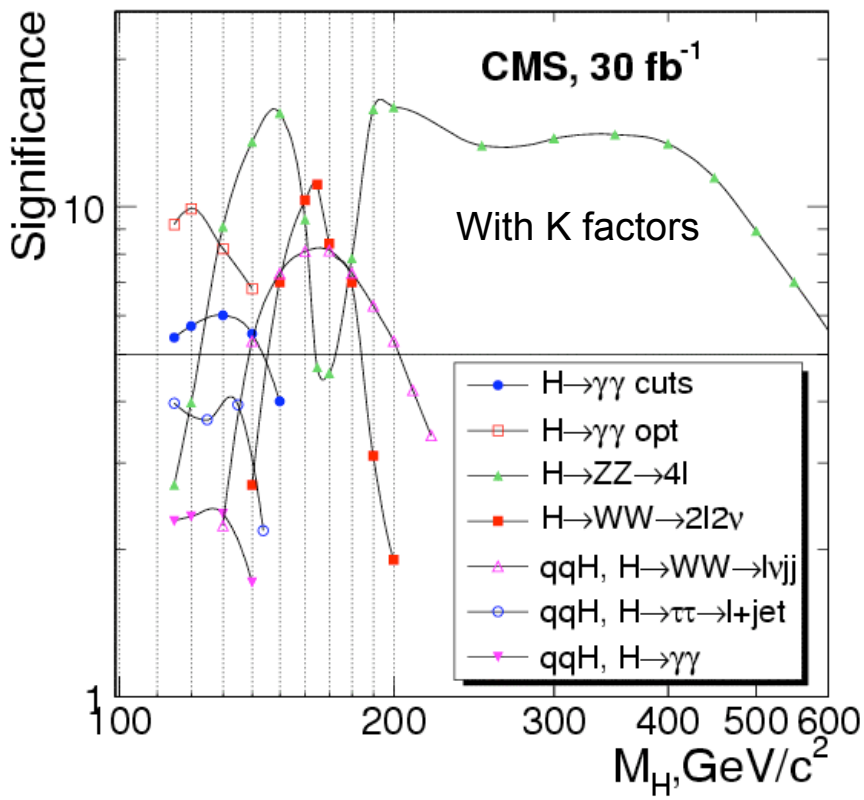


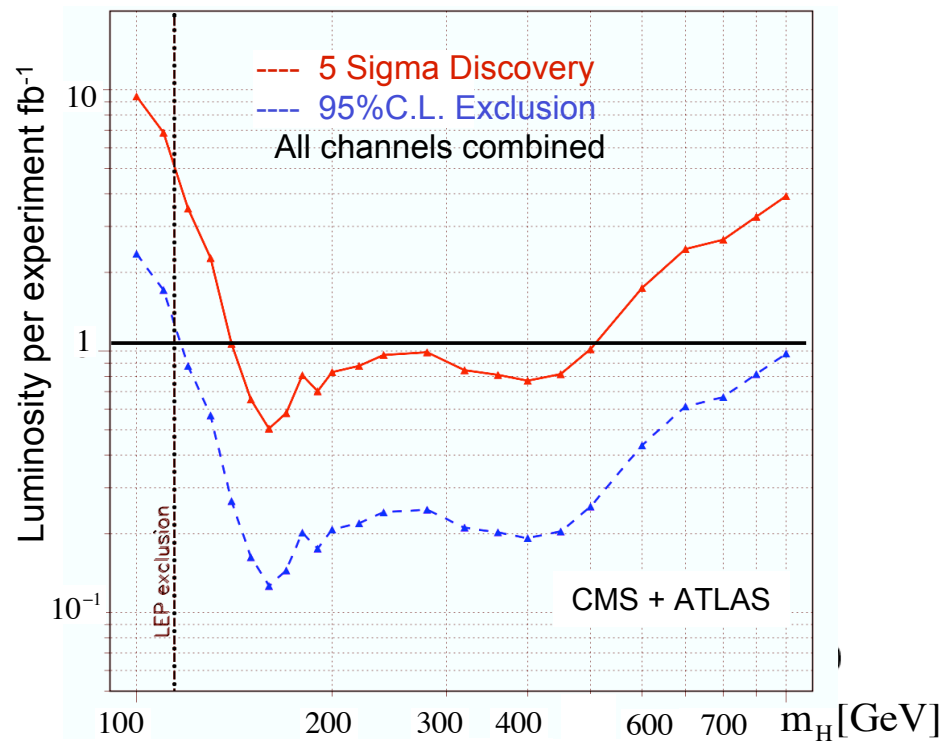
Figure 3.1: The dominant SM Higgs boson production mechanisms in hadronic collisions.

SM Higgs production cross-sections at the LHC



LHC Discovery Potential of a SM Higgs





- **Low mass range** $m_{H_{SM}} < 200$ GeV

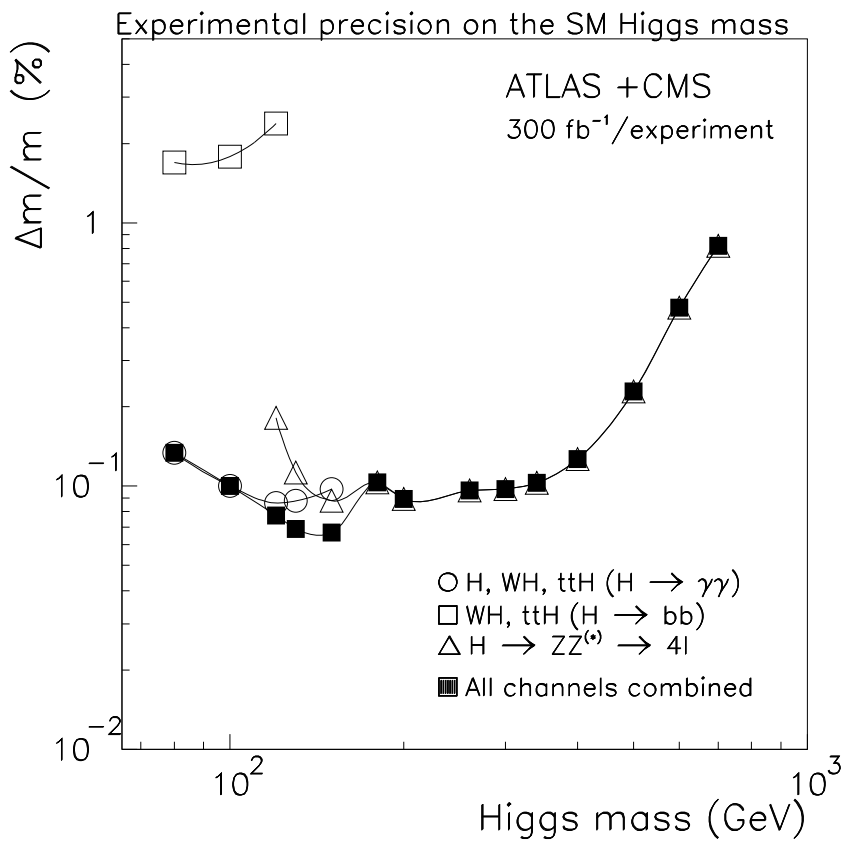
$$H \to \gamma\gamma, \tau\tau, bb, WW, ZZ$$

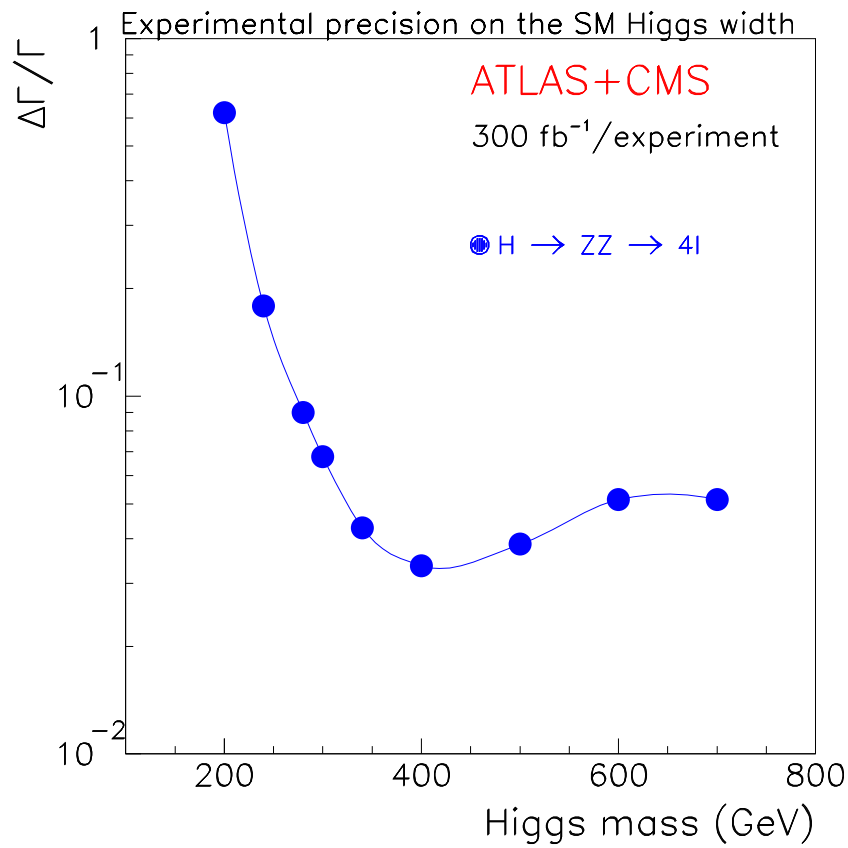
- **High mass range** $m_{H_{SM}} > 200$ GeV

$$H \to WW,\ ZZ$$

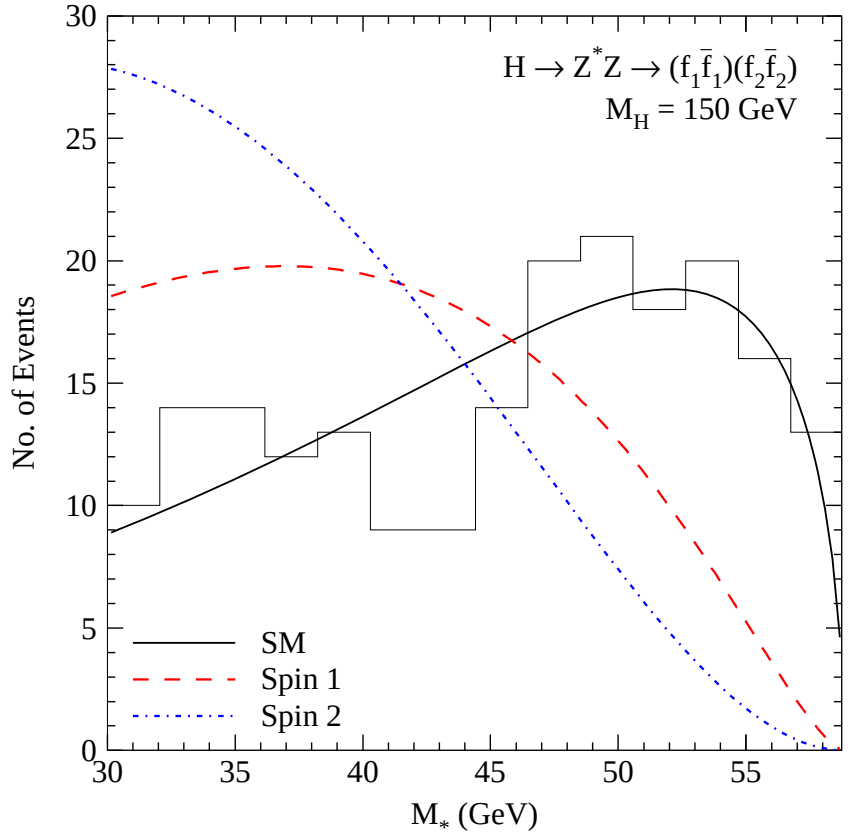
A SM Higgs cannot escape detection at the LHC

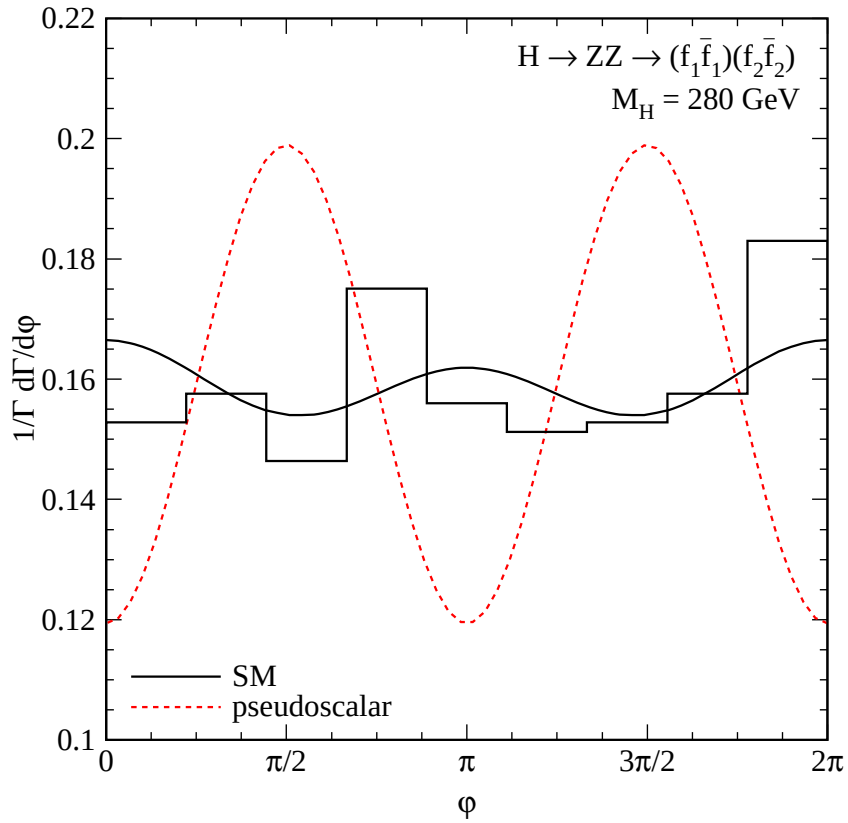
Higgs mass and width measurements at the LHC



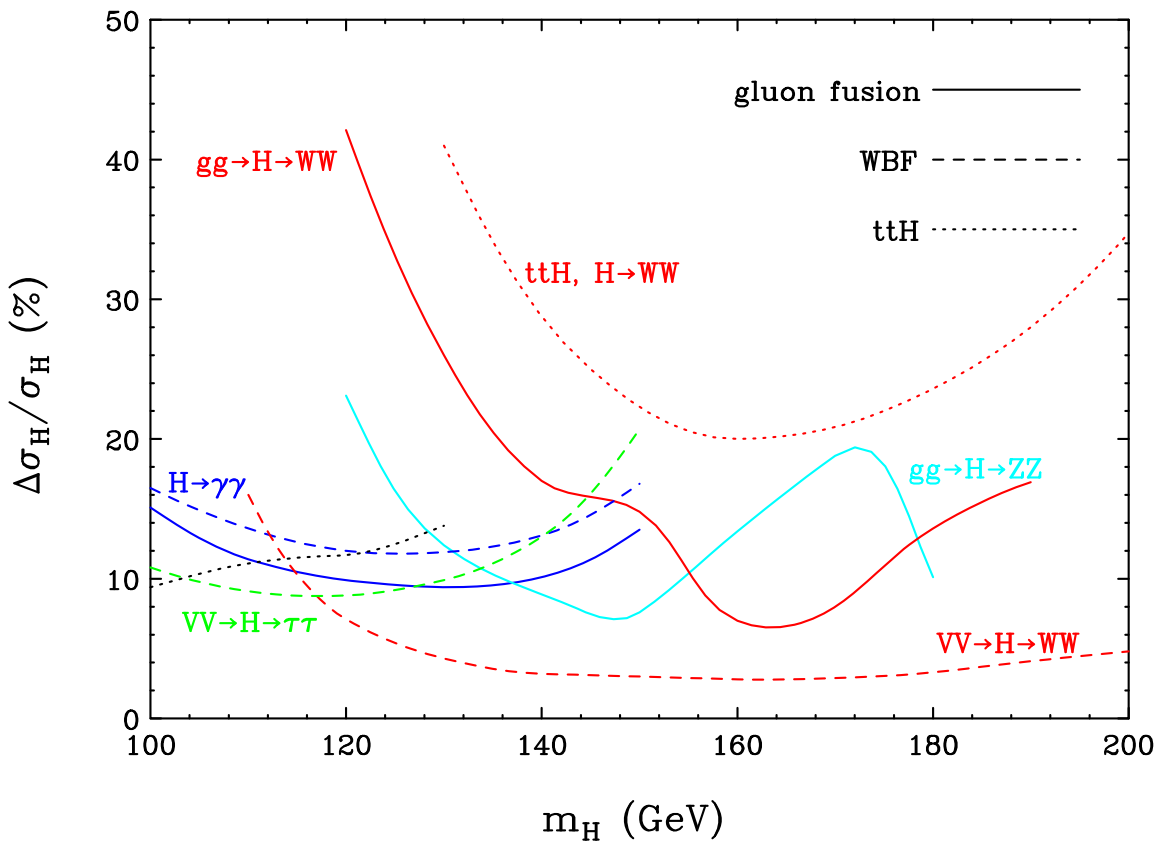


Determination of the Higgs quantum numbers

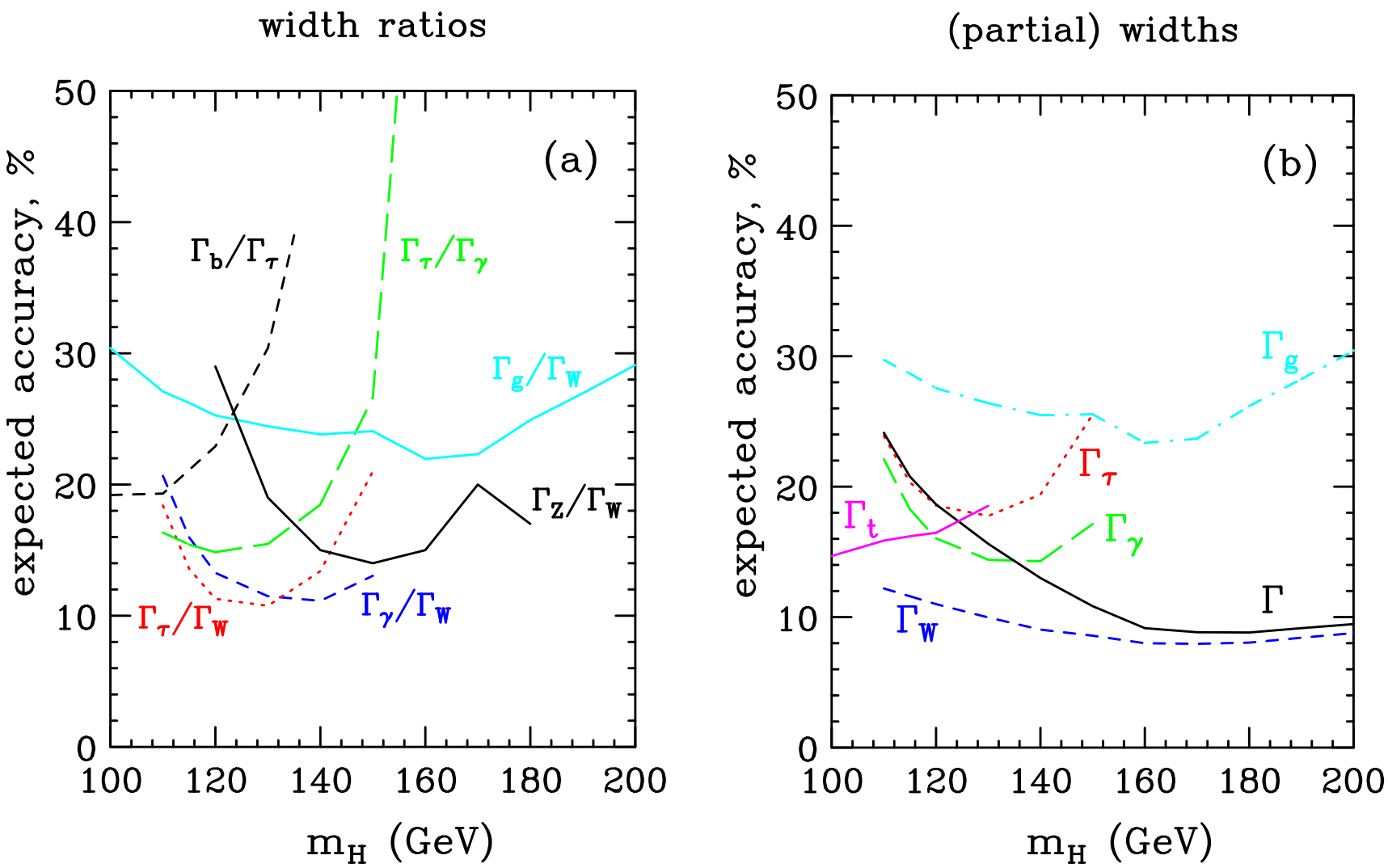




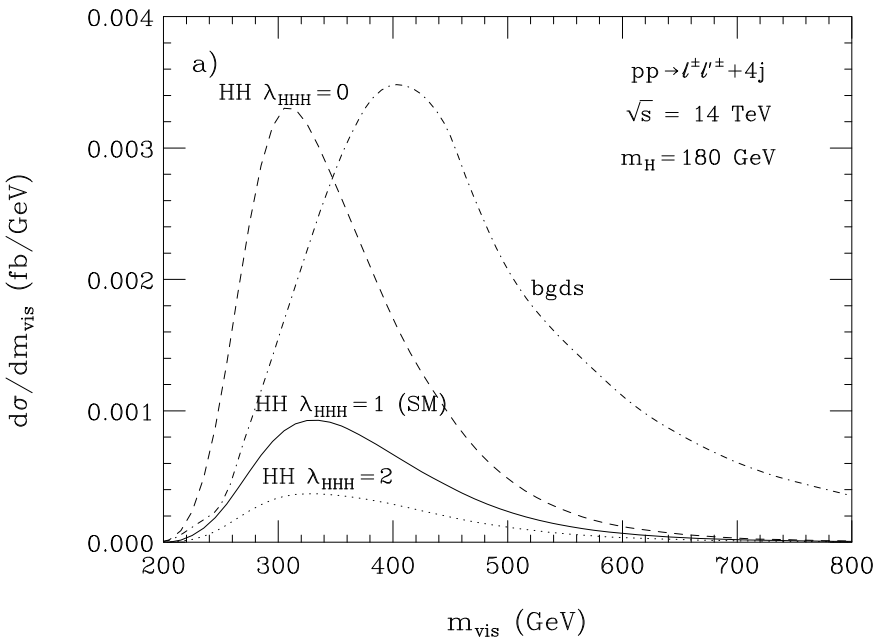
Accuracy of Higgs cross-section measurements

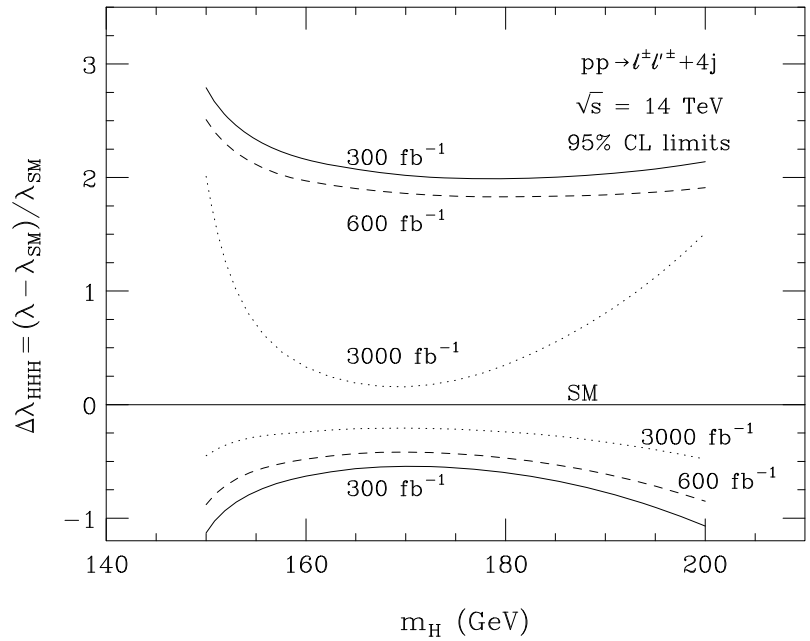


Precision Higgs coupling measurements at the LHC



The Higgs self-coupling at the LHC





Lecture II: Weakly coupled Higgs bosons beyond the Standard Model

Outline

- expanding the Higgs sector
- the significance of the TeV-scale—Part 2
- the MSSM Higgs sector at tree-level
- saving the MSSM Higgs sector—the impact of radiative corrections
- constraints from present data
- the MSSM Higgs sector—phenomenology
- Higgs physics beyond the MSSM

Constraints on the non-minimal Higgs sector

Three generations of fermions appear in nature, with each generation possessing the same quantum numbers under the SU(3)×SU(2)×U(1)$_{\rm Y}$ gauge group. So, why should the scalar sector be of minimal form?

For an arbitrary Higgs sector, the tree-level ρ-parameter is given by

$$\rho_0 \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_{T,Y}[4T(T+1)-Y^2]|V_{T,Y}|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2|V_{T,Y}|^2},$$

where $V_{T,Y} \equiv \langle\phi(T,Y)\rangle$ defines the vacuum expectation values (vevs) of each neutral Higgs field, and T and Y specify the total SU(2) isospin and the hypercharge of the Higgs representation to which it belongs. Y is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$, and

$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation}, \\ \frac{1}{2}, & (T,Y=0) \in \text{real representation}. \end{cases}$$

For the complex ($c = 1$) Higgs doublet of the Standard Model with $T = 1/2$ and $Y = 1$, it follows that $\rho_0 = 1$ as strongly suggested by the electroweak data. The same result follows from a Higgs sector consisting of multiple complex Higgs doublets (independent of the neutral Higgs vevs). One can also add Higgs singlets ($T = Y = 0$) without changing the value of ρ_0.

But, one cannot add arbitrary Higgs multiplets in general* unless their corresponding vevs are very small (typically $|V_{T,Y}| \lesssim 0.05v \sim 10\ \mathrm{GeV}$).

Thus, we shall consider non-minimal Higgs sectors consisting of multiple Higgs doublets (and perhaps Higgs singlets), but no higher Higgs representations, in order to avoid the fine-tuning of Higgs vevs.

*To automatically have $\rho_0 = 1$ independently of the Higgs vevs, one must satisfy

$$(2T+1)^2 - 3Y^2 = 1$$

for integer values of $(2T, Y)$. The smallest nontrivial solution beyond the complex $Y = 1$ Higgs doublet is a Higgs multiplet with $T = 3$ and $Y = 4$.

Danger: neutral Higgs-mediated flavor changing neutral currents

It is remarkable that in the SM, the neutral Higgs boson coupling to fermions is flavor-diagonal. This is a consequence of the Higgs-fermion Yukawa couplings:

$$\mathcal{L}_{\rm Yukawa} = -h_u^{ij}(\bar{u}_R^i u_L^j \Phi^0 - \bar{u}_R^i d_L^j \Phi^+) - h_d^{ij}(\bar{d}_R^i d_L^j \Phi^{0\,*} + \bar{d}_R^i u_L^j \Phi^-) + \text{h.c.}\,,$$

where i, j are generation labels and h_u and h_d are arbitrary complex 3×3 matrices. Writing $\Phi^0 = (v + h^0)/\sqrt{2}$, we identify the quark mass matrices as:

$$M_u^{ij} \equiv h_u^{ij} \frac{v}{\sqrt{2}}\,, \qquad M_d^{ij} \equiv h_d^{ij} \frac{v}{\sqrt{2}}\,.$$

One is free to redefine the quark fields:

$$u_L \to V_L^U u_L\,, \qquad u_R \to V_R^U u_R\,, \qquad d_L \to V_L^D d_L\,, \qquad d_R \to V_R^D d_R\,,$$

where V_L^U, V_R^U, V_L^D, and V_R^D are unitary matrices chosen such that

$$V_R^{U\,\dagger} M_u V_L^U = \text{diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad V_R^{D\,\dagger} M_d V_L^D = \text{diag}(m_d\,,\, m_s\,,\, m_b)\,,$$

such that the m_i are the positive quark masses (this is the *singular value decomposition* of linear algebra).

Having diagonalized the quark mass matrices, the neutral Higgs Yukawa couplings are automatically flavor-diagonal.[†] Hence the SM possesses no flavor-changing neutral currents (FCNCs) mediated by neutral Higgs boson (or gauge boson) exchange at tree-level.

In models with multiple Higgs doublets, this is no longer the case in general, since more than one Yukawa coupling matrix (one for each Higgs doublet) contributes to each of the up and down-type fermion mass matrices. Diagonalizing the quark mass matrix diagonalizes only one linear combination of the Yukawa coupling matrices.

However, one can recover flavor-diagonal Yukawa couplings by restricting the form of the Higgs-fermion Lagrangian. Glashow and Weinberg showed that a sufficient condition is to require that at most one neutral Higgs field couple to fermions of a given electric charge.

[†]Independently of the Higgs sector, the quark couplings to Z and γ are automatically flavor diagonal. Flavor dependence only enters the quark couplings to the $W^{\pm}$ via the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $K \equiv V_L^{U\,\dagger} V_L^D$.

Higgs boson phenomena beyond the SM

Consider the two-Higgs-doublet model (2HDM), consisting of two-complex hypercharge-one scalar doublets. Of the eight initial degrees of freedom, five are physical (after three Goldstone bosons provide masses for the $W^{\pm}$ and Z). The five physical scalars are: a charged Higgs pair, $H^{\pm}$, and three neutral scalars. In contrast to the SM, where the Higgs-sector is CP-conserving, the 2HDM allows for Higgs-mediated CP-violation. If CP is conserved, the three scalars can be classified as two CP-even scalars, h^0 and H^0 (where $m_h < m_H$ as the notation suggests) and a CP-odd scalar A^0.

Thus, new features of the extended Higgs sector include:

- Charged Higgs bosons
- A CP-odd Higgs boson (if CP is conserved in the Higgs sector)
- Higgs-mediated CP-violation (and neutral Higgs states of indefinite CP)

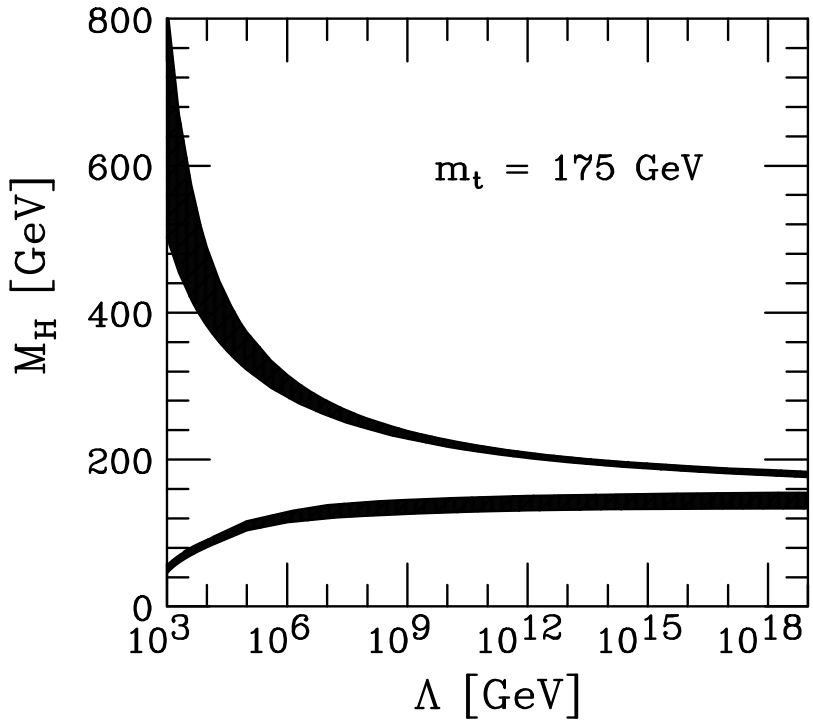
More exotic Higgs sectors allow for doubly-charged Higgs bosons, etc.

The significance of the TeV scale—Part 2

If a SM-like Higgs boson is discovered, should we expect any additional new physics phenomena at the TeV scale?

The Standard Model (SM) describes quite accurately physics near the EWSB scale [$v = 246$ GeV]. But, the SM is only a "low-energy" approximation to a more fundamental theory, whose degrees of freedom are revealed at some high energy scale Λ.

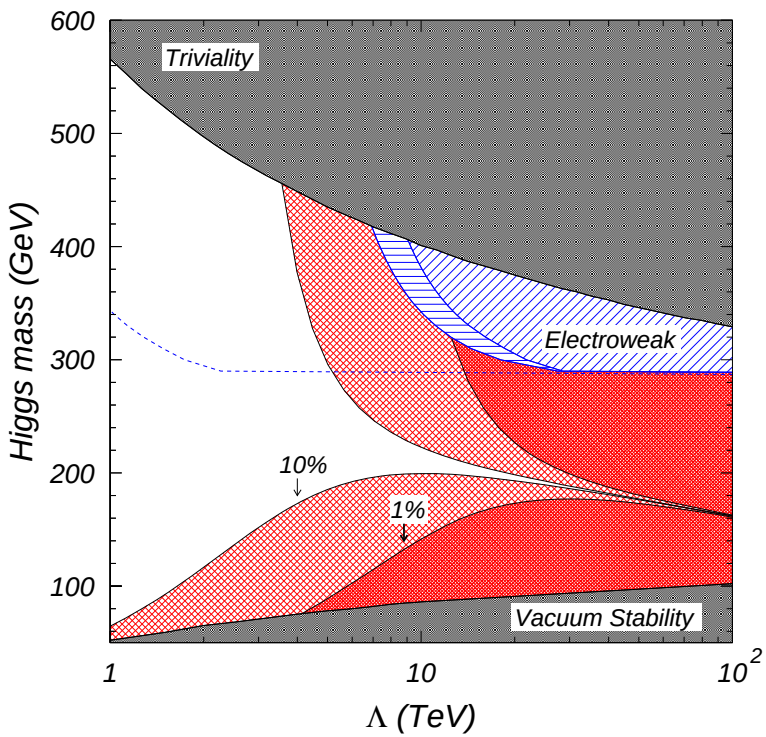
- The SM cannot be valid at energies above the Planck scale, $M_{\rm PL} \equiv (c\hbar/G_N)^{1/2} \simeq 10^{19}$ GeV, where gravity can no longer be ignored.
- Neutrinos are exactly massless in the Standard Model. But, the neutrino mixing data imply that neutrinos have very small masses ($m_\nu/m_e \lesssim 10^{-7}$). Neutrino masses can be incorporated in a theory whose fundamental scale is $M \gg v$. Neutrino masses of order v^2/M are generated, which suggest that $M \sim 10^{15}$ GeV.
- The radiatively-corrected Higgs potential is unstable at large values of the Higgs field ($|\Phi| > \Lambda$) if the Higgs mass is too small.
- The value of the Higgs self-coupling runs off to infinity at an energy scale above Λ if the Higgs mass is too large.



The present-day theoretical uncertainties on the lower [Altarelli and Isidori; Casas, Espinosa and Quirós] and upper [Hambye and Riesselmann] Higgs mass bounds as a function of energy scale Λ at which the Standard Model breaks down, assuming $m_t = 175$ GeV and $\alpha_s(m_Z) = 0.118$. The shaded areas above reflect the theoretical uncertainties in the calculations of the Higgs mass bounds.

Depending on the observed Higgs mass, we may be able to conclude that the SM breaks down at an energy Λ that is considerably below 10^{15} GeV.

The significance of the TeV-scale as the energy scale where new physics beyond the SM must emerge follows from the field-theoretic observation that m_h^2 (more precisely, the square of the Higgs vev) is sensitive to Λ^2. Demanding that the value of m_h is natural, $i.e.$, without substantial fine-tuning, then Λ cannot be significantly larger than 1 TeV.



Following Kolda and Murayama [*JHEP* **0007** (2000) 035], a reconsideration of the Λ $vs.$ Higgs mass plot with a focus on $\Lambda < 100$ TeV. Precision electroweak measurements restrict the parameter space to lie below the dashed line, based on a 95% CL fit that allows for nonzero values of S and T and the existence of higher dimensional operators suppressed by v^2/Λ^2. The unshaded area has less than one part in ten fine-tuning.

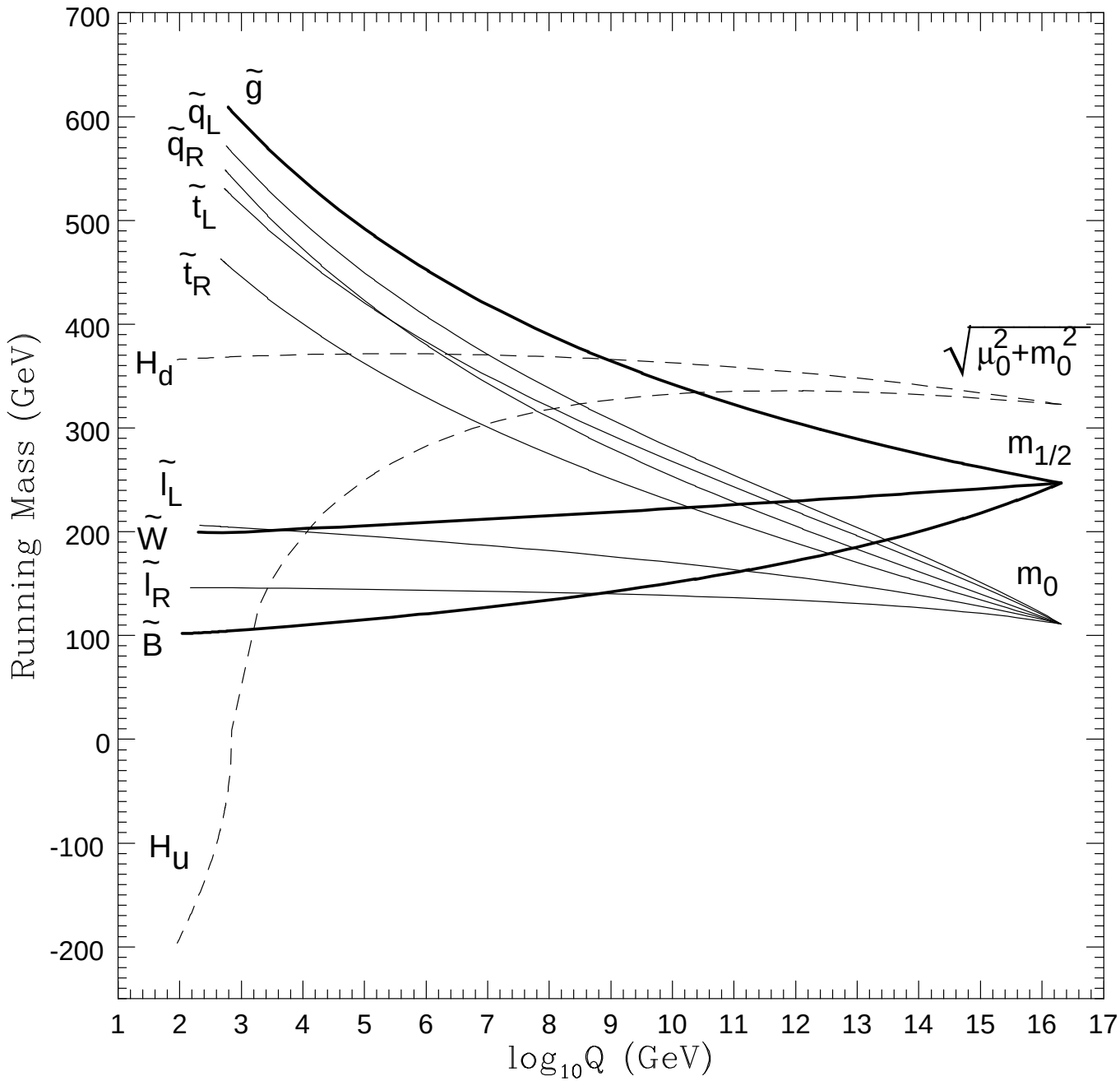
Low-Energy Supersymmetry

Supersymmetry (SUSY) provides a mechanism in which the quadratic sensitivity of scalar squared-masses to very high-energy scales is exactly canceled (see Steve Martin's lectures). Since SUSY is not an exact symmetry of nature, the supersymmetry must be broken. To maintain the naturalness of the theory, the SUSY-breaking scale cannot be significantly larger than 1 TeV.

> The scale of supersymmetry-breaking must be of order 1 TeV or less, if supersymmetry is associated with the scale of electroweak symmetry breaking.

We shall initially focus on the minimal supersymmetric extension of the Standard Model (MSSM), which is constructed by starting with the 2HDM and adding the associated superpartners.[‡] One bonus of this construction is the elegant way in which EWSB is radiatively generated (providing a nice connection between SUSY-breaking and the mechanism of EWSB).

[‡]Two Higgs doublets are required for anomaly cancellation by higgsino pairs of opposite hypercharge.


Running Mass (GeV)
$\log_{10}Q$ (GeV)
$\tilde{g}$
$\tilde{q}_L$
$\tilde{q}_R$
$\tilde{t}_L$
$\tilde{t}_R$
H_d
H_u
$\tilde{l}_L$
$\tilde{W}$
$\tilde{l}_R$
$\tilde{B}$
$\sqrt{\mu_0^2+m_0^2}$
$m_{1/2}$
m_0

The Higgs sector of the MSSM

The Higgs sector of the MSSM is a 2HDM, whose Yukawa couplings and Higgs potential are constrained by SUSY. Instead of employing to hypercharge-one scalar doublets $\Phi_{1,2}$, it is more convenient to introduce a $Y = -1$ doublet $H_d \equiv i\sigma_2 \Phi_1^*$ and a $Y = +1$ doublet $H_u \equiv \Phi_2$:

$$H_d = \begin{pmatrix} H_d^1 \\ H_d^2 \end{pmatrix} = \begin{pmatrix} \Phi_1^{0\,*} \\ -\Phi_1^- \end{pmatrix}, \qquad H_u = \begin{pmatrix} H_u^1 \\ H_u^2 \end{pmatrix} = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix}.$$

The origin of the notation originates from the Higgs Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = -h_u^{ij}(\bar{u}_R^i u_L^j H_u^2 - \bar{u}_R^i d_L^j H_u^1) - h_d^{ij}(\bar{d}_R^i d_L^j H_d^1 - \bar{d}_R^i u_L^j H_d^2) + \text{h.c.}\,.$$

Note that the neutral Higgs field H_u^2 couples exclusively to up-type quarks and the neutral Higgs field H_d^1 couples exclusively to down-type quarks.[§]

[§] This is an example of the so-called Type-II 2HDM, which satisfies the Glashow-Weinberg condition and has no tree-level Higgs-mediated FCNCs.

The Higgs potential of the MSSM is:

$$V = \left(m_d^2 + |\mu|^2\right) H_d^{i*} H_d^i + \left(m_u^2 + |\mu|^2\right) H_u^{i*} H_u^i - m_{ud}^2 \left(\epsilon^{ij} H_d^i H_u^j + \text{h.c.}\right)$$
$$+\tfrac{1}{8}\left(g^2 + g'^2\right)\left[H_d^{i*} H_d^i - H_u^{j*} H_u^j\right]^2 + \tfrac{1}{2} g^2 |H_d^{i*} H_u^i|^2 \,,$$

where $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon^{11} = \epsilon^{22} = 0$, and the sum over repeated indices is implicit. Above, μ is a supersymmetric Higgsino mass parameter and m_d^2, m_u^2, m_{ud}^2 are soft-supersymmetry-breaking masses. The quartic Higgs couplings are related to the SU(2) and $\mathrm{U(1)_Y}$ gauge couplings as a consequence of SUSY.

Minimizing the Higgs potential, the neutral components of the Higgs fields acquire vevs:[¶]

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} , \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} ,$$

where $v^2 \equiv v_d^2 + v_u^2 = 4m_W^2/g^2 = (246\ \mathrm{GeV})^2$. The ratio of the two vevs is an important parameter of the model:

$$\tan\beta \equiv \frac{v_u}{v_d} \,.$$

[¶]The phases of the Higgs fields can be chosen such that the vacuum expectation values are real and positive. That is, the tree-level MSSM Higgs sector conserves CP, which implies that the neutral Higgs mass eigenstates possess definite CP quantum numbers.

The five physical Higgs particles consist of a charged Higgs pair

$$H^{\pm} = H_d^{\pm} \sin\beta + H_u^{\pm} \cos\beta \,,$$

one CP-odd scalar

$$A^0 = \sqrt{2}\left(\mathrm{Im}\, H_d^0 \sin\beta + \mathrm{Im}\, H_u^0 \cos\beta\right) \,,$$

and two CP-even scalars

$$h^0 = -(\sqrt{2}\,\mathrm{Re}\, H_d^0 - v_d)\sin\alpha + (\sqrt{2}\,\mathrm{Re}\, H_u^0 - v_u)\cos\alpha \,,$$
$$H^0 = (\sqrt{2}\,\mathrm{Re}\, H_d^0 - v_d)\cos\alpha + (\sqrt{2}\,\mathrm{Re}\, H_u^0 - v_u)\sin\alpha \,,$$

where we have now labeled the Higgs fields according to their electric charge. The angle α arises when the CP-even Higgs squared-mass matrix (in the H_d^0—H_u^0 basis) is diagonalized to obtain the physical CP-even Higgs states.

All Higgs masses and couplings can be expressed in terms of two parameters usually chosen to be m_A and $\tan\beta$.

Tree-level MSSM Higgs masses

The charged Higgs mass is given by

$$m_{H^\pm}^2 = m_A^2 + m_W^2 \,,$$

and the CP-even Higgs bosons h^0 and H^0 are eigenstates of the squared-mass matrix

$$\mathcal{M}_0^2 = \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta & -(m_A^2 + m_Z^2)\sin\beta\cos\beta \\ -(m_A^2 + m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{pmatrix} \,.$$

The eigenvalues of $\mathcal{M}_0^2$ are the squared-masses of the two CP-even Higgs scalars

$$m_{H,h}^2 = \tfrac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta} \right) \,,$$

and α is the angle that diagonalizes the CP-even Higgs squared-mass matrix. It follows that

$$\boxed{m_h \leq m_Z |\cos 2\beta| \leq m_Z \,.}$$

Note the contrast with the SM where the Higgs mass is a free parameter, $m_h^2 = \frac{1}{2}\lambda v^2$. In the MSSM, all Higgs self-coupling parameters of the MSSM are related to the squares of the electroweak gauge couplings.

Aside: the decoupling limit of the MSSM

In the limit of $m_A \gg m_Z$, the expressions for the Higgs masses and mixing angle simplify and one finds

$$\begin{aligned} m_h^2 &\simeq m_Z^2 \cos^2 2\beta \,, \\ m_H^2 &\simeq m_A^2 + m_Z^2 \sin^2 2\beta \,, \\ m_{H^\pm}^2 &= m_A^2 + m_W^2 \,, \\ \cos^2(\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4 m_A^4} \,. \end{aligned}$$

Two consequences are immediately apparent. First, $m_A \simeq m_H \simeq m_{H^\pm}$, up to corrections of $\mathcal{O}(m_Z^2/m_A)$. Second, $\cos(\beta - \alpha) = 0$ up to corrections of $\mathcal{O}(m_Z^2/m_A^2)$. This is the decoupling limit, since at energy scales below approximately common mass of the heavy Higgs bosons $H^\pm$ H^0, A^0, the effective Higgs theory is precisely that of the SM.

In particular, we will see that in the limit of $\cos(\beta - \alpha) \to 0$, all the h^0 couplings to SM particles approach their SM limits.

Tree-level MSSM Higgs couplings

1. Higgs couplings to gauge boson pairs ($V = W$ or Z)

$$g_{h^0VV} = g_V m_V \sin(\beta - \alpha)\,, \qquad\qquad g_{H^0VV} = g_V m_V \cos(\beta - \alpha)\,,$$

where $g_V \equiv 2m_V/v$. There are no tree-level couplings of A^0 or $H^\pm$ to VV.

2. Higgs couplings to a single gauge boson

The couplings of V to two neutral Higgs bosons (which must have opposite CP-quantum numbers) is denoted by $g_{\phi A^0 Z}(p_\phi - p_A^0)$, where $\phi = h^0$ or H^0 and the momenta p_ϕ and p_A^0 point into the vertex, and

$$g_{h^0A^0Z} = \frac{g\cos(\beta - \alpha)}{2\cos\theta_W}\,, \qquad\qquad g_{H^0A^0Z} = \frac{-g\sin(\beta - \alpha)}{2\cos\theta_W}\,.$$

3. Summary of Higgs boson–vector boson couplings

The properties of the three-point and four-point Higgs boson-vector boson couplings are conveniently summarized by listing the couplings that are proportional to either $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$, and the couplings that are independent of α and β

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$	angle-independent
$H^0W^+W^-$	$h^0W^+W^-$	—
H^0ZZ	h^0ZZ	—
ZA^0h^0	ZA^0H^0	ZH^+H^- , γH^+H^-
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$	$W^\pm H^\mp A^0$
$ZW^\pm H^\mp h^0$	$ZW^\pm H^\mp H^0$	$ZW^\pm H^\mp A^0$
$\gamma W^\pm H^\mp h^0$	$\gamma W^\pm H^\mp H^0$	$\gamma W^\pm H^\mp A^0$
—	—	$VV\phi\phi$, VVA^0A^0 , VVH^+H^-

where $\phi = h^0$ or H^0 and $VV = W^+W^-$, ZZ, $Z\gamma$ or $\gamma\gamma$.

4. Higgs-fermion couplings

Since the neutral Higgs couplings to fermions are flavor-diagonal, we list only the Higgs coupling to 3rd generation fermions. The couplings of the neutral Higgs bosons to $f\bar{f}$ relative to the Standard Model value, $gm_f/2m_W$, are given by (the γ_5 indicates a pseudoscalar coupling):

$$
\begin{aligned}
h^0 b\bar{b} \quad (\text{or } h^0\tau^+\tau^-): &\qquad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,, \\
h^0 t\bar{t}: &\qquad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,, \\
H^0 b\bar{b} \quad (\text{or } H^0\tau^+\tau^-): &\qquad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,, \\
H^0 t\bar{t}: &\qquad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,, \\
A^0 b\bar{b} \quad (\text{or } A^0\tau^+\tau^-): &\qquad \gamma_5\tan\beta\,, \\
A^0 t\bar{t}: &\qquad \gamma_5\cot\beta\,.
\end{aligned}
$$

Similarly, the charged Higgs boson couplings to fermion pairs, with all particles pointing into the vertex, are given by[‖]

$$g_{H^- t\bar{b}} = \frac{g}{\sqrt{2} m_W} \left[m_t \cot\beta \, P_R + m_b \tan\beta \, P_L \right],$$
$$g_{H^- \tau^+ \nu} = \frac{g}{\sqrt{2} m_W} \left[m_\tau \tan\beta \, P_L \right].$$

Especially noteworthy is the possible $\tan\beta$-enhancement of certain Higgs-fermion couplings. The general expectation in MSSM models is that $\tan\beta$ lies in a range:

$$1 \lesssim \tan\beta \lesssim \frac{m_t}{m_b}.$$

Near the upper limit of $\tan\beta$, we have roughly identical values for the top and bottom Yukawa couplings, $h_t \sim h_b$, since

$$h_b = \frac{\sqrt{2}\, m_b}{v_d} = \frac{\sqrt{2}\, m_b}{v \cos\beta}, \qquad h_t = \frac{\sqrt{2}\, m_t}{v_u} = \frac{\sqrt{2}\, m_t}{v \sin\beta}.$$

[‖] Including the full flavor structure, the CKM matrix appears in the charged Higgs couplings in the standard way for a charged-current interaction.

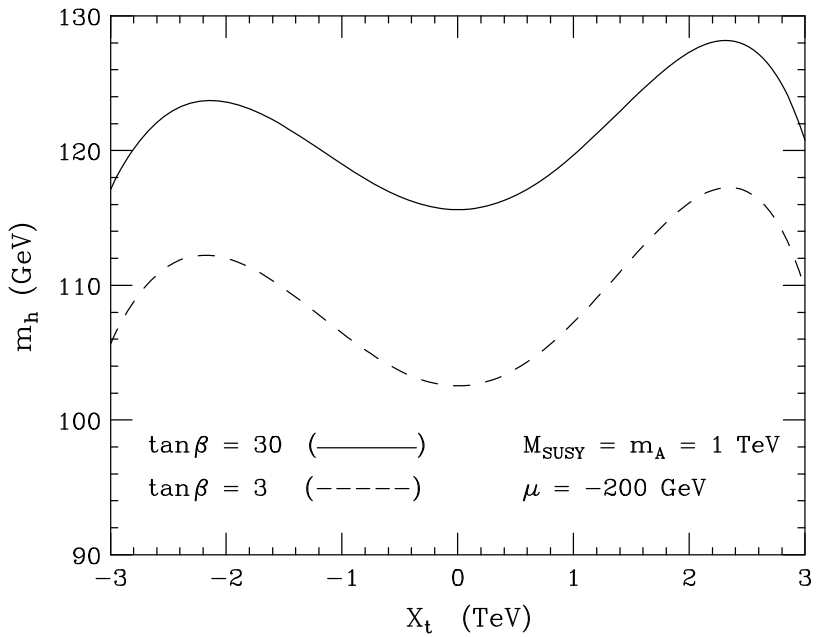
Saving the MSSM Higgs sector—the impact of radiative corrections

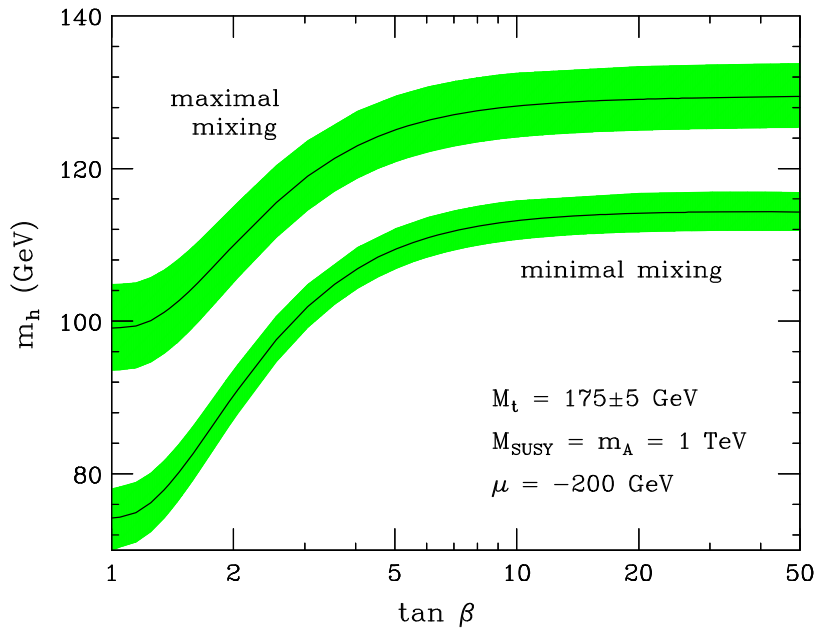
We have already noted the tree-level relation $m_h \leq m_Z$, which is already ruled out by LEP data. But, this inequality receives quantum corrections. The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancellation, which would have been exact if supersymmetry were unbroken):

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12 M_S^2}\right)\right],$$

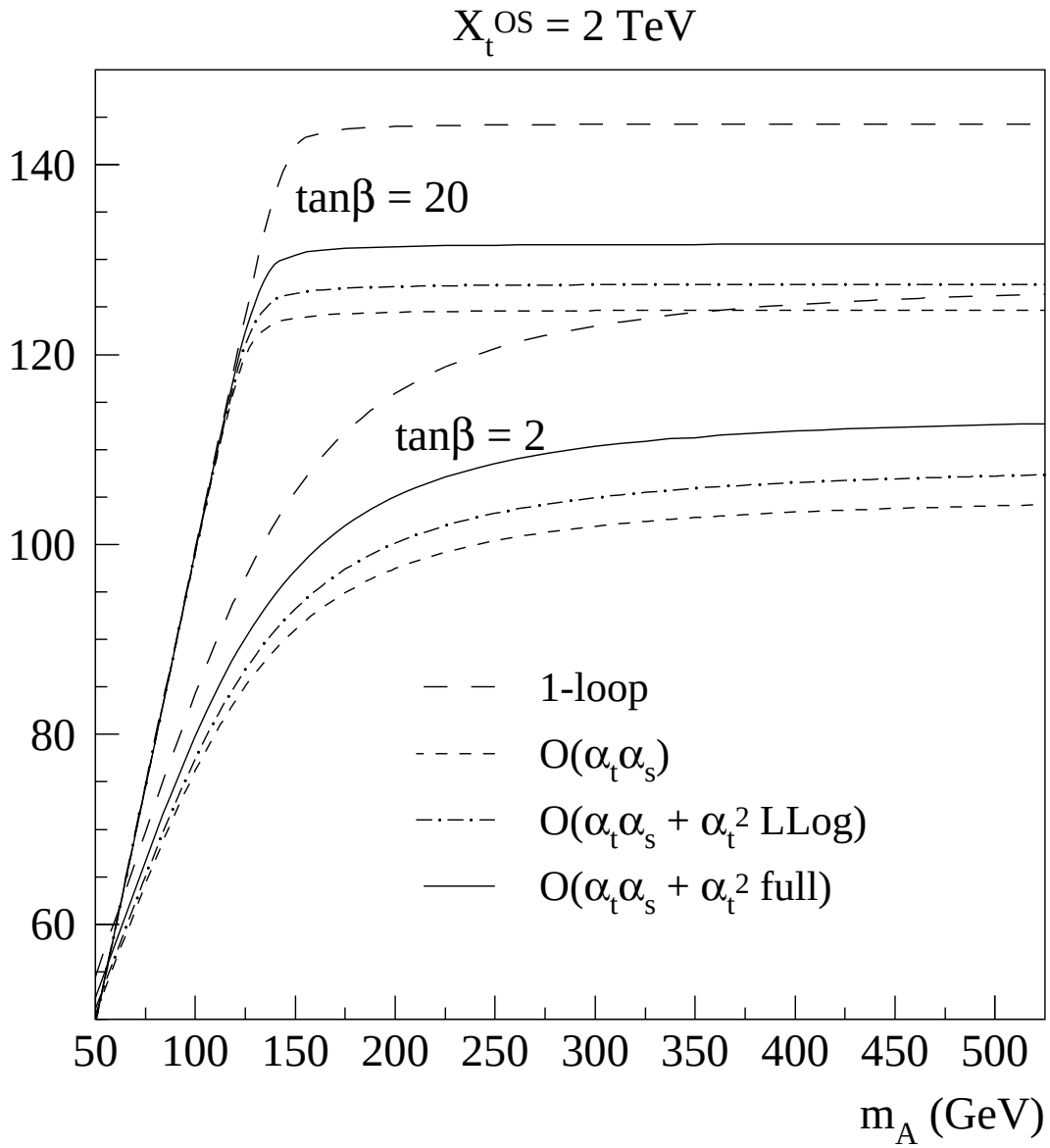
where $X_t \equiv A_t - \mu \cot\beta$ governs stop mixing and M_S^2 is the average top-squark squared-mass.

The state-of-the-art computation includes the full one-loop result, all the significant two-loop contributions, and renormalization-group improvements. The final conclusion is that $m_h \lesssim 130$ GeV [assuming that the top-squark mass is no heavier than about 2 TeV].

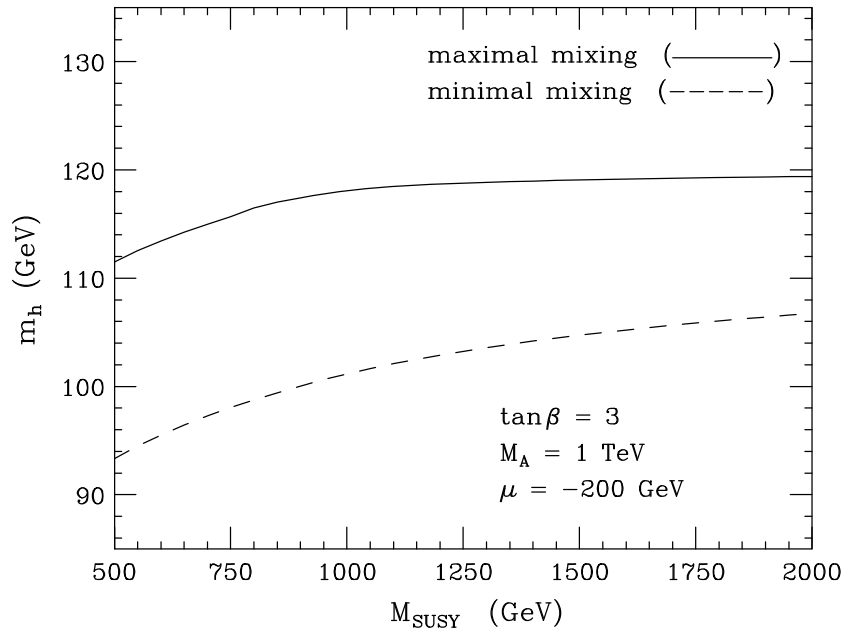


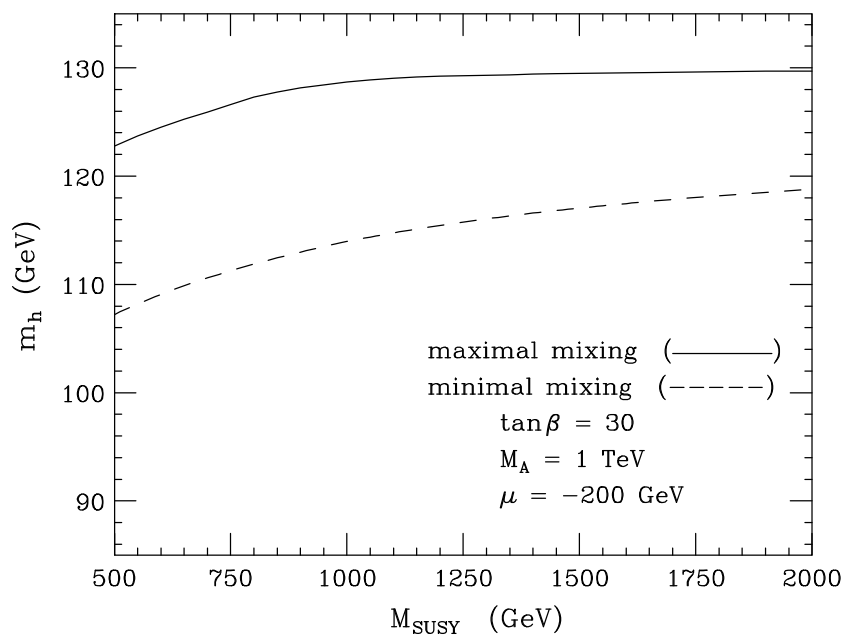


Maximal mixing corresponds to choosing the MSSM Higgs parameters in such a way that m_h is maximized (for a fixed $\tan\beta$). This occurs for $X_t/M_S \sim 2$. As $\tan\beta$ varies, m_h reaches is maximal value, $(m_h)_{\max} \simeq 130$ GeV, for $\tan\beta \gg 1$ and $m_A \gg m_Z$.



taken from Brignole et al., Nucl. Phys. **B631**, 195 (2002).


maximal mixing (———)
minimal mixing (- - - - -)
tanβ = 3
M_A = 1 TeV
μ = −200 GeV
m_h (GeV)
M_{SUSY} (GeV)
130
120
110
100
90
500
750
1000
1250
1500
1750
2000


maximal mixing (———)
minimal mixing (- - - - -)
tanβ = 30
M_A = 1 TeV
μ = −200 GeV
m_h (GeV)
M_{SUSY} (GeV)
130
120
110
100
90
500
750
1000
1250
1500
1750
2000

Radiatively-corrected Higgs couplings

Although radiatively-corrections to couplings tend to be at the few-percent levels, there is some potential for significant effects:

- large radiative corrections due to a $\tan\beta$-enhancement (assuming $\tan\beta \gg 1$)
- CP-violating effects induced by complex SUSY-breaking parameters that enter in loops

In the SUSY limit, bottom quarks only couple to H_d^0 and top quarks only couple to H_u^0. However, SUSY is broken and a small coupling of the bottom quark [top quark] to H_u^0 [H_d^0] will be generated from the one-loop Yukawa vertex corrections. These results can be summarized by an effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\rm eff} = \epsilon^{ij}\left[(h_b+\delta h_b)\bar{b}_R H_d^i Q_L^j + (h_t+\delta h_t)\bar{t}_R Q_L^i H_u^j\right] \\ +\Delta h_t \bar{t}_R Q_L^k H_d^{k*} + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \text{h.c.}$$

As a result, the tree-level relations between the Yukawa couplings and quark masses are modified. For simplicity, we neglect below possible CP-violating effects due to complex

couplings. Then, e.g.,

$$m_b = \frac{h_b v}{\sqrt{2}} \cos\beta \left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}} \cos\beta (1 + \Delta_b) \,,$$

The dominant contributions to Δ_b are $\tan\beta$-enhanced, with $\Delta_b \simeq (\Delta h_b / h_b) \tan\beta$. Explicitly, one finds that for large SUSY masses and $\tan\beta \gg 1$,

$$\Delta_b \simeq \left[\frac{2\alpha_s}{3\pi} \mu M_{\tilde{g}}\, I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}}) + \frac{h_t^2}{16\pi^2} \mu A_t\, I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, \mu^2)\right] \tan\beta \,,$$

where $M_{\tilde{g}}$ is the gluino mass, $M_{\tilde{b}_{1,2}}$ are the bottom squark masses, and smaller electroweak corrections have been ignored. The loop integral $I(a^2, b^2, c^2)$ is given by

$$I(a, b, c) = \frac{a^2 b^2 \ln(a^2/b^2) + b^2 c^2 \ln(b^2/c^2) + c^2 a^2 \ln(c^2/a^2)}{(a^2 - b^2)(b^2 - c^2)(a^2 - c^2)} \,,$$

and is of order $1/\max(a^2, b^2, c^2)$ when at least one of its arguments is large. That is, if all SUSY-mass parameters are simultaneously large, their effects do *not* decouple in Δ_b. Below the SUSY-scale, the effective theory is a general 2HDM, in which all possible Higgs-fermion Yukawa couplings are allowed!

From the effective Yukawa Lagrangian, we can obtain the couplings of the physical Higgs bosons to third generation fermions. Neglecting possible CP-violating effects,

$$\mathcal{L}_{\rm int} = -\sum_{q=t,b,\tau} \left[g_{h^0 q\bar{q}} h^0 q\bar{q} + g_{H^0 q\bar{q}} H^0 q\bar{q} - i g_{A^0 q\bar{q}} A^0 \bar{q}\gamma_5 q \right] + \left[\bar{b} g_{H^- t\bar{b}} t H^- + \text{h.c.} \right] \ .$$

For example,

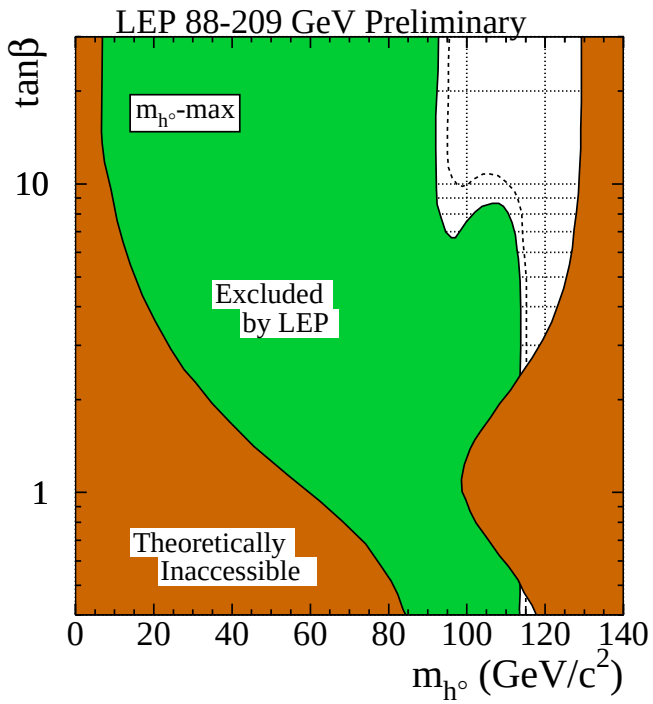
$$g_{h^0 b\bar{b}} = -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1 + \frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)(1+\cot\alpha\cot\beta)\right] \ .$$

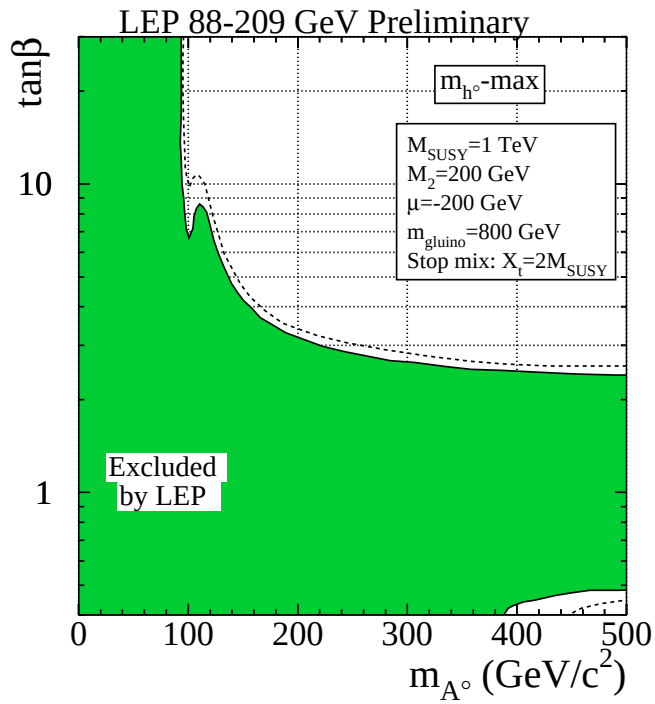
Two limits are noteworthy. First, in the decoupling limit where $\cos(\beta-\alpha) \ll 1$, we can put $\alpha \simeq \beta - \pi/2$ to obtain $g_{h^0 b\bar{b}} = m_b/v$ which is the expected SM result. Second, away from the decoupling limit, if $\tan\beta \gg 1$, then

$$g_{h^0 b\bar{b}} \simeq \frac{g^{(0)}_{h^0 b\bar{b}}}{1+\Delta_b} \ ,$$

where the superscript zero indicates the tree-level coupling. In some regions of SUSY parameter space, the $\tan\beta$-enhanced Δ_b can be as large as 25% and of either sign, leading to significant enhancements or suppressions of the $h^0 b\bar{b}$ coupling with respect to, e.g., the $h^0\tau^+\tau^-$ coupling.

Summary of the LEP MSSM Higgs Search [95% CL limits]





- Charged Higgs boson: $m_{H^\pm} > 79.3$ GeV
- MSSM Higgs: $m_h > 92.9$ GeV; $m_A > 93.4$ GeV [max-mix scenario]

WARNING: Allowing for possible CP-violating effects that can enter via radiative corrections, large holes open up in the Higgs mass exclusion plots.

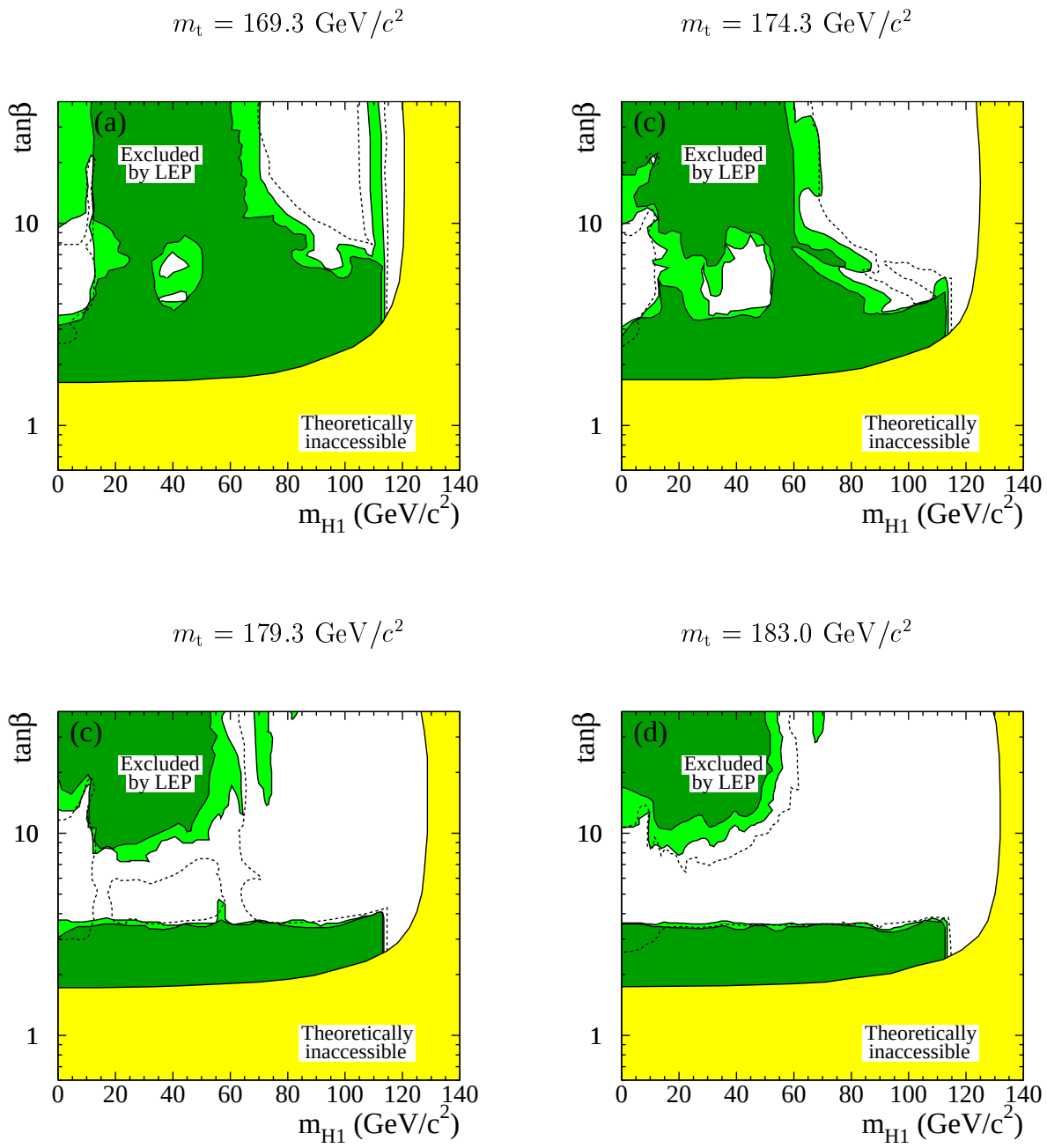


Figure 19: *Exclusions, at 95% c.l. (light-green) and at 99.7% c.l. (dark-green), for the CP-violating CPX scenario, for four top masses. From upper left, right to lower left, right:* m_t = 169.3 GeV/c^2, 174.3 GeV/c^2, 179.3 GeV/c^2 *and* 183.0 GeV/c^2. *In all cases and in each scan point the more conservative of the two theoretical calculations, FeynHiggs 2.0 or CPH, was used.*

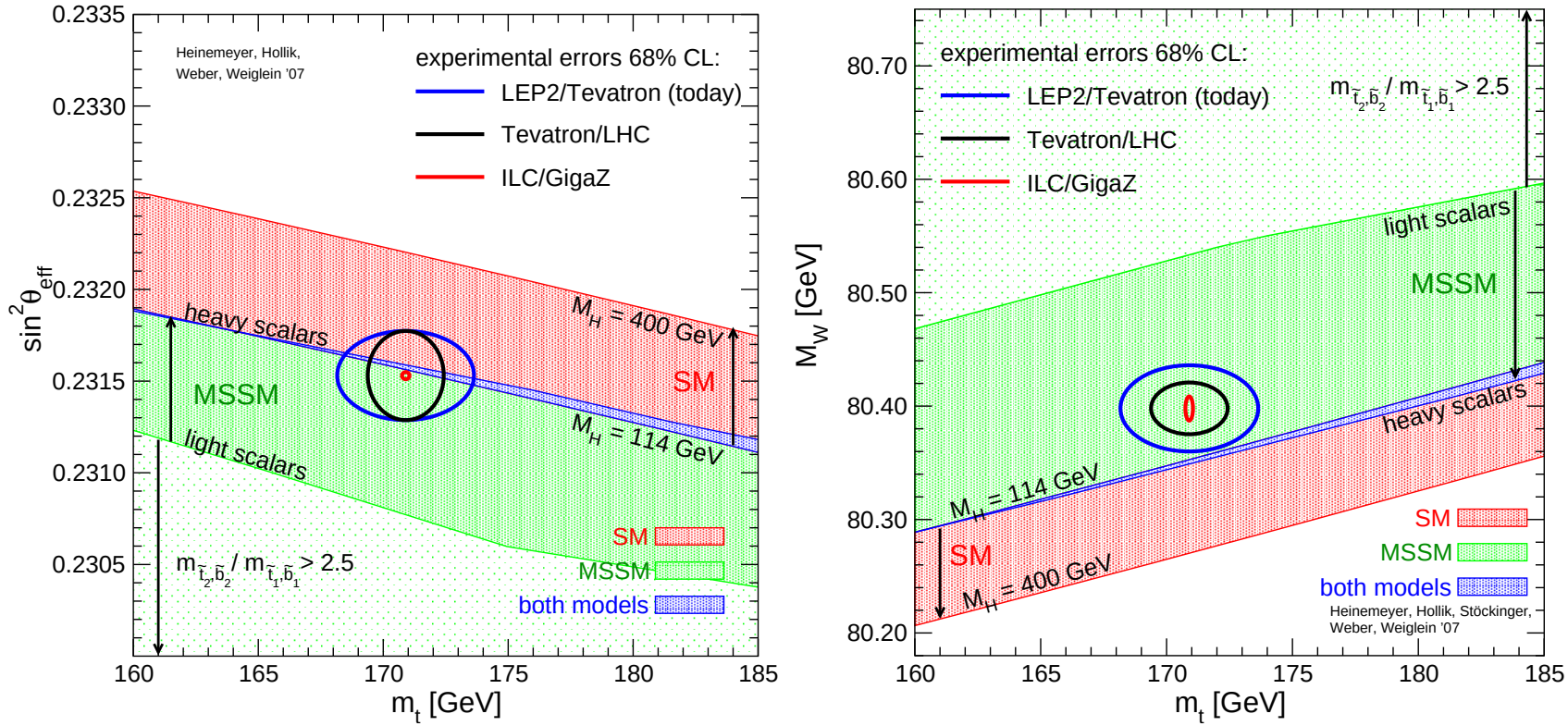
Implications of the precision electroweak data

- In the decoupling limit (assuming that the SUSY particles are somewhat heavy), the effects of the heavy Higgs states and the SUSY particles decouple and the global SM fit applies.

- In the latter case, h^0 is a SM-like Higgs boson whose mass lies below about 130 GeV in the *preferred* Higgs mass range!

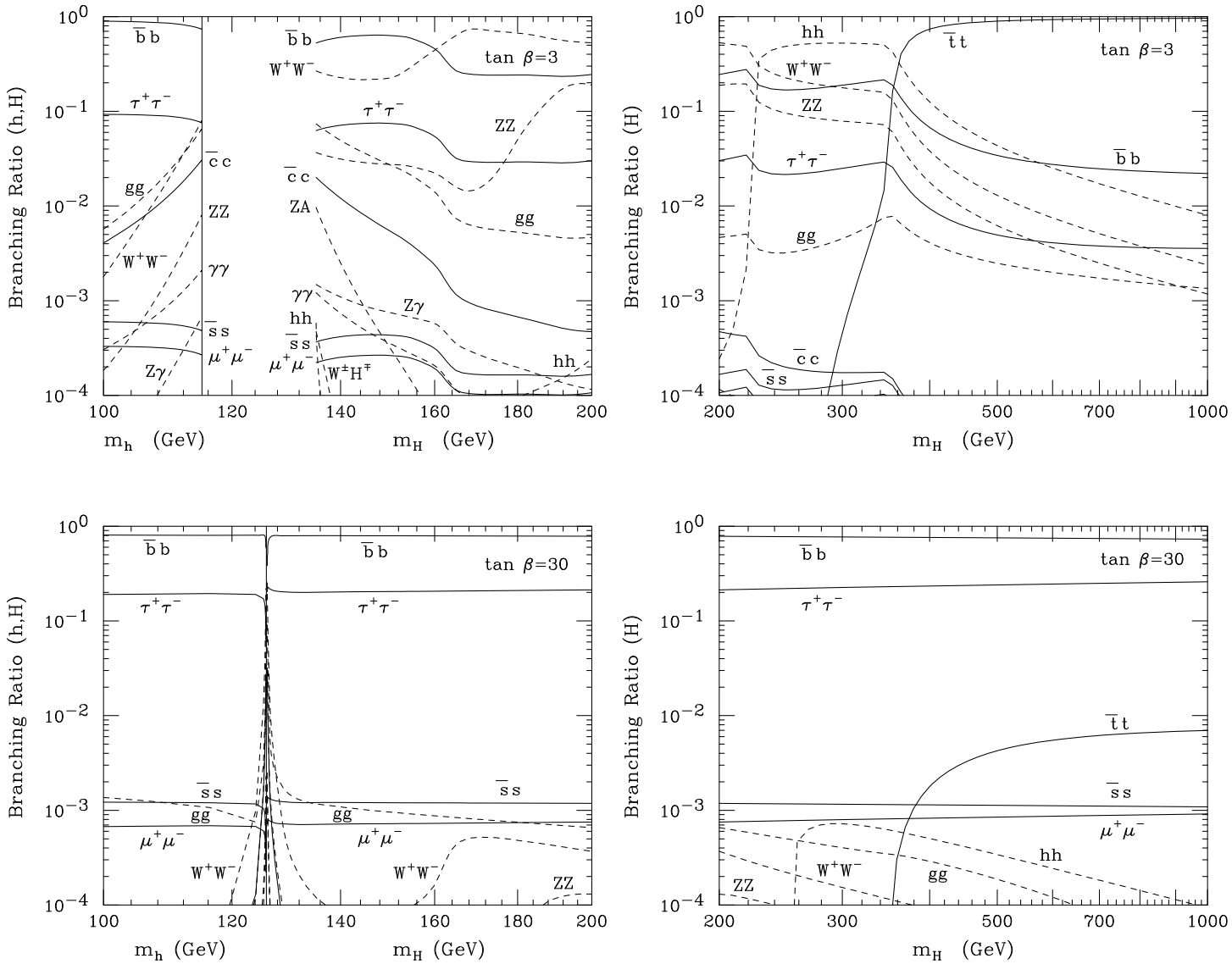
- If SUSY particle masses are not too heavy, they can have small effects on the fit to precision electroweak data. With additional degrees of freedom, the goodness of fit can be slightly improved (and possibly argue for SUSY masses close to their present experimental limits).

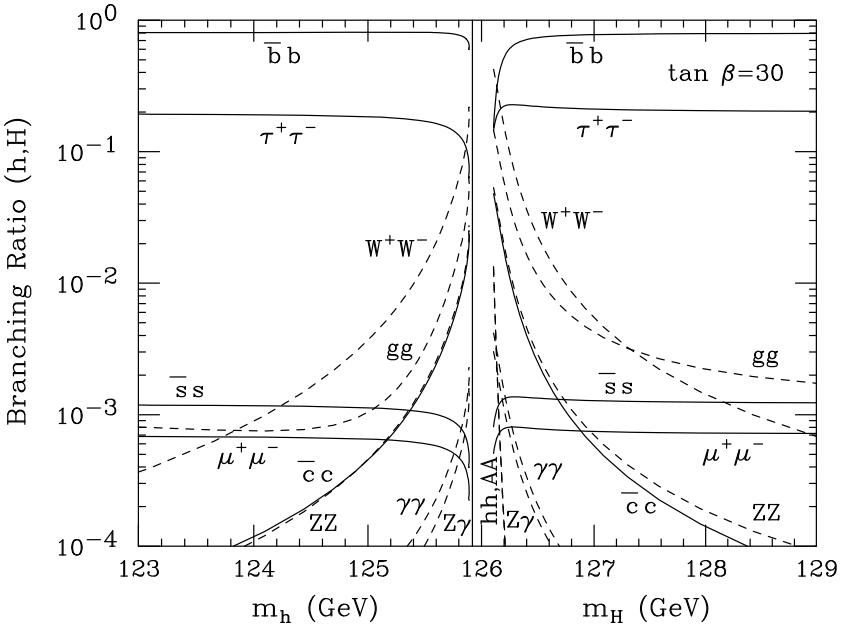
- The MSSM fit is further improved if one wishes to ascribe deviations of $(g-2)_\mu$ and $b \to s\gamma$ from their SM expectations to the effects of superpartners.

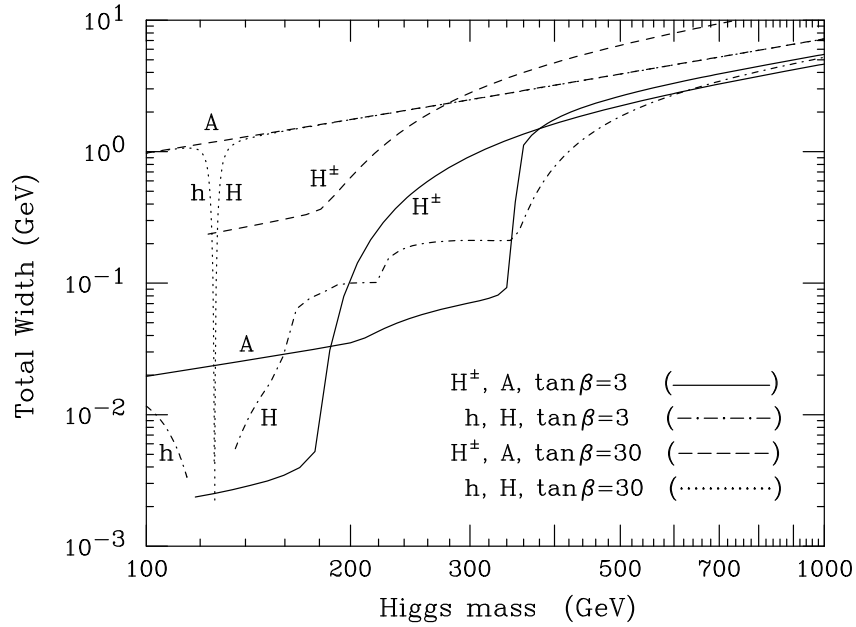


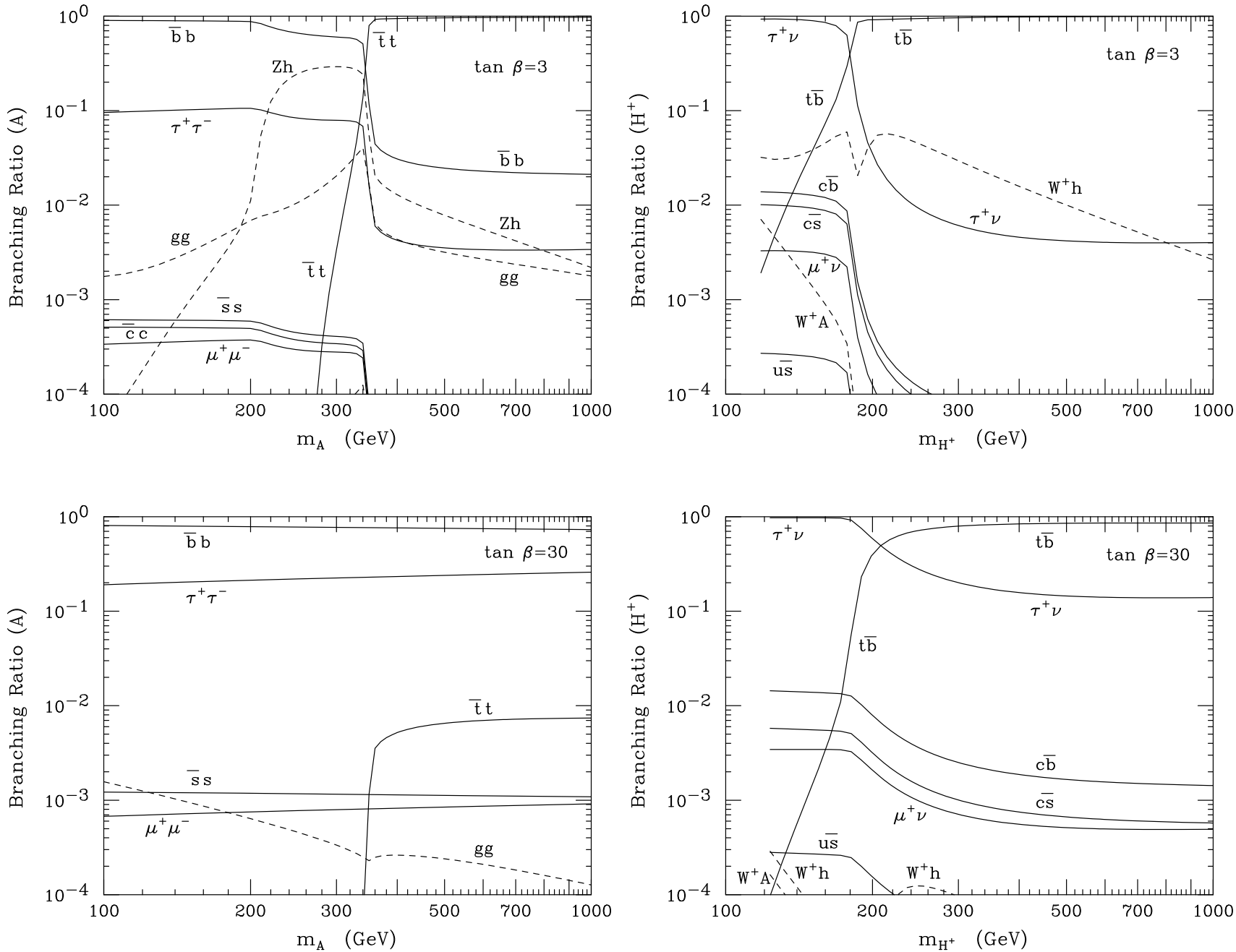
Taken from S. Heinemeyer, W. Hollik, A.M. Weber and G. Weiglein, JHEP **0804**, 039 (2008).

MSSM Higgs boson decay branching ratios

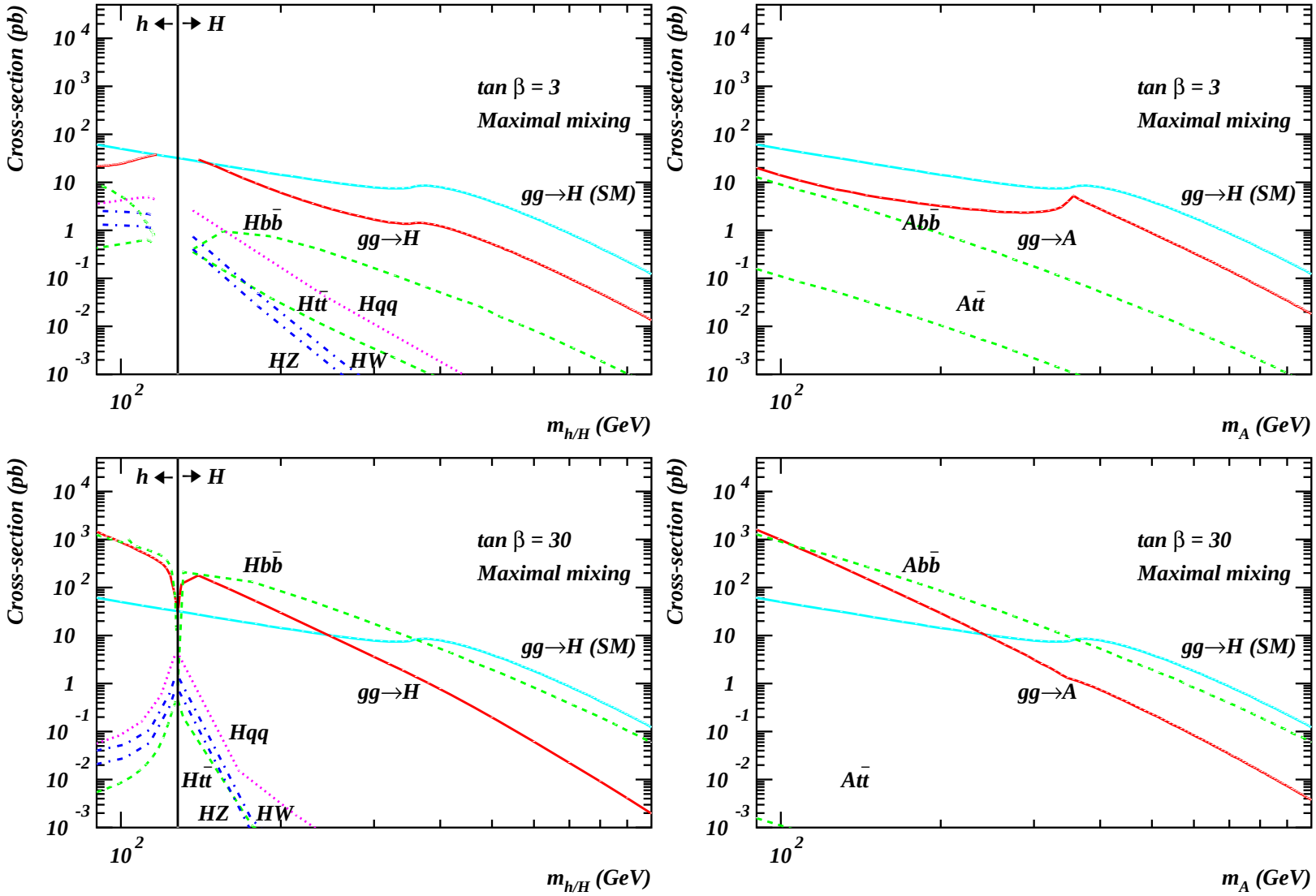


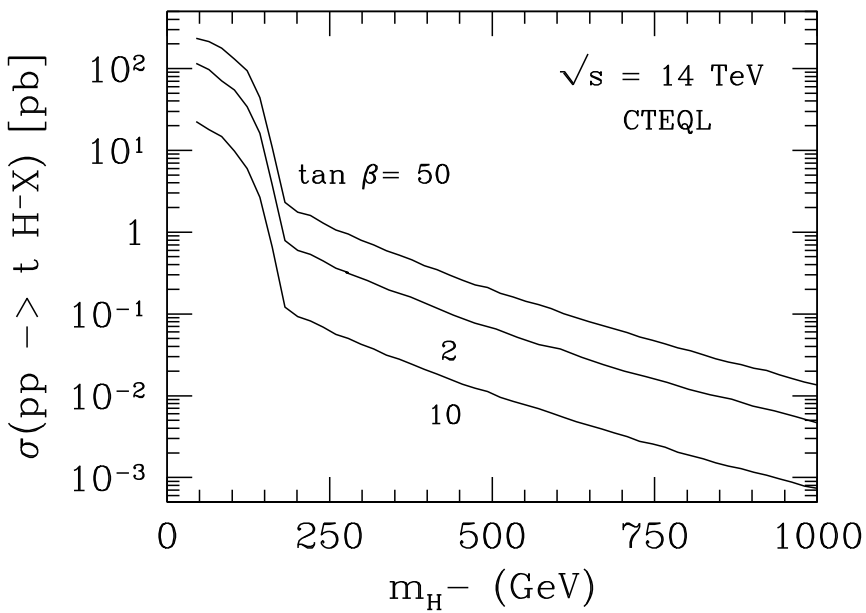

Branching Ratio (h,H)
m_h (GeV)
m_H (GeV)
tan β=30
$\bar{b}b$
$\tau^+\tau^-$
W^+W^-
gg
$\bar{s}s$
$\mu^+\mu^-$
$\bar{c}c$
ZZ
$\gamma\gamma$
$Z\gamma$
hh,AA

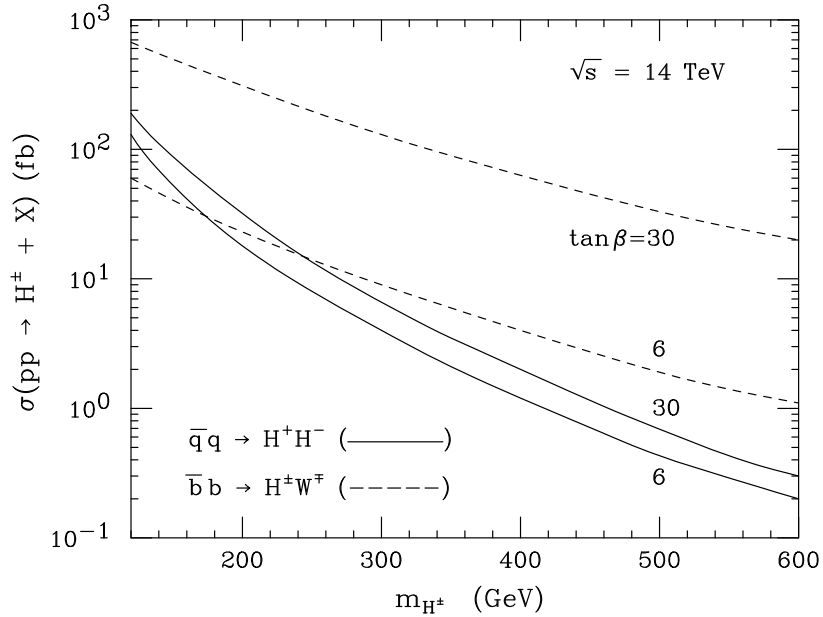

Total Width (GeV)
Higgs mass (GeV)
A
h
H
$H^\pm$
$H^\pm$, A, tanβ=3 (———)
h, H, tanβ=3 (–·–·–)
$H^\pm$, A, tanβ=30 (– – – – –)
h, H, tanβ=30 (·····)


Branching Ratio (A)
m_A (GeV)
$\bar{b}b$
$\bar{t}t$
Zh
tan β=3
$\tau^+\tau^-$
gg
$\bar{s}s$
$\bar{c}c$
$\mu^+\mu^-$
Branching Ratio (H^+)
m_{H^+} (GeV)
$\tau^+\nu$
$t\bar{b}$
$c\bar{b}$
$c\bar{s}$
$\mu^+\nu$
W^+A
$u\bar{s}$
W^+h
tan β=30

MSSM Higgs production cross-sections at the LHC




√s = 14 TeV
CTEQL
tan β= 50
2
10
σ(pp -> t H-X) [pb]
m_H− (GeV)

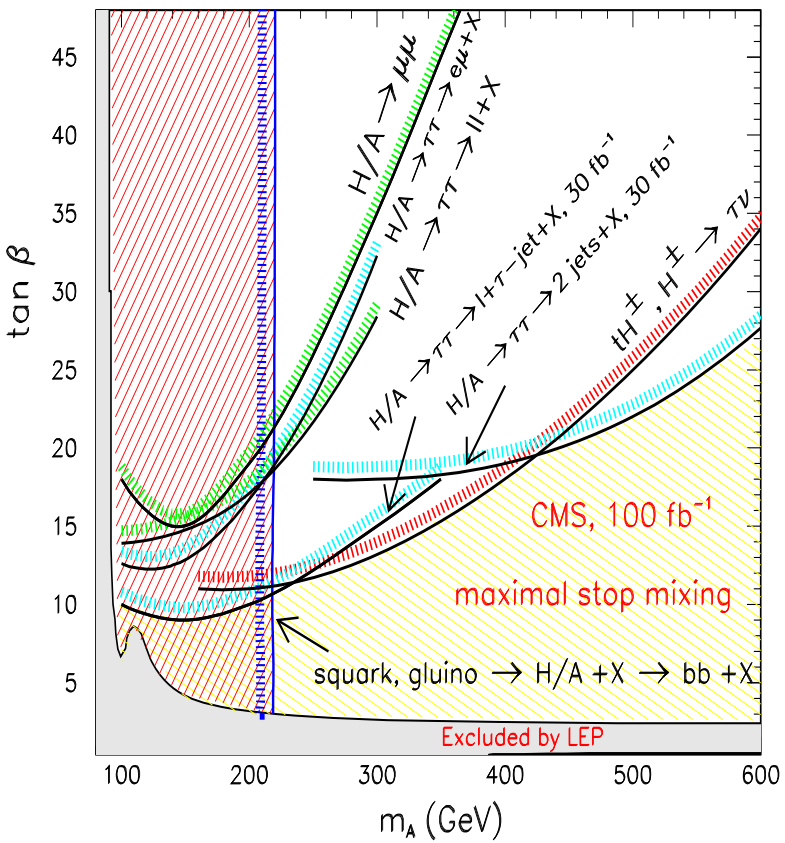

√s = 14 TeV
tan β=30
6
30
6
q̄q → H⁺H⁻ (———)
b̄b → H±W∓ (-----)
σ(pp → H± + X) (fb)
m_H± (GeV)

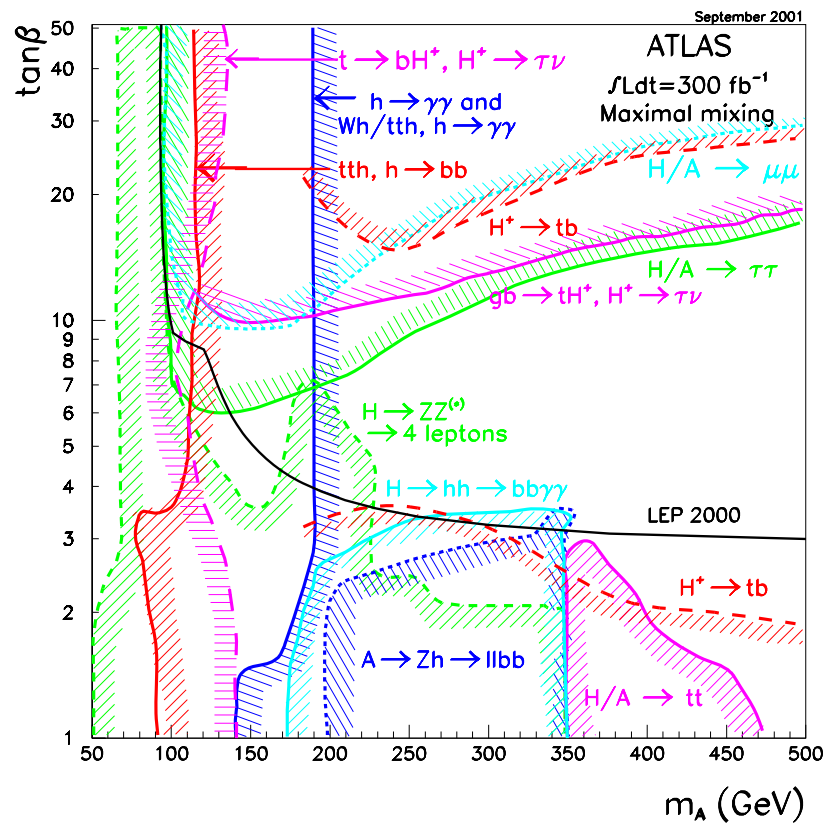
MSSM Higgs Searches at the LHC

In addition to the standard SM-Higgs searches, new possibilities arise:

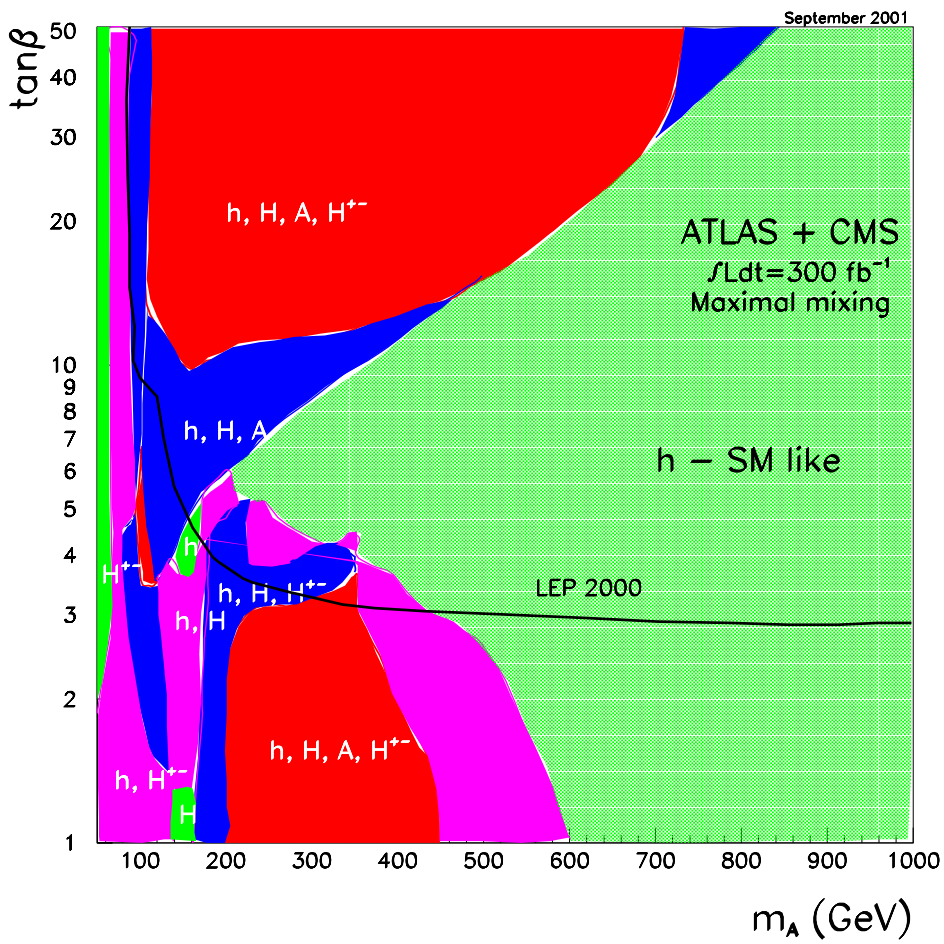
- gluon-gluon fusion can produce both CP-even and CP-odd Higgs bosons.
- VV fusion ($V = W$ or Z) can produce only CP-even Higgs bosons (at tree-level). Moreover, in the decoupling limit, the heavy CP-even Higgs boson is nearly decoupled from the VV channel.
- Neutral Higgs bosons can be produced in association with $b\bar{b}$ and with $t\bar{t}$ in gluon-gluon scattering.
- Charged Higgs bosons can be produced in association with $t\bar{b}$ in gluon-gluon scattering.
- If $m_{H^\pm} < m_t - m_b$, then $t \to bH^-$ is an allowed decay, and the dominant $H^\pm$ production mechanism is via $t\bar{t}$ production.

- Higgs bosons can be produced in pairs (e.g., H^+H^-, $H^\pm h^0$, h^0A^0).
- Higgs bosons can be produced in cascade decays of SUSY particles.
- Higgs search strategies depend on the region of m_A–$\tan\beta$ plane





Discovery potential for one, two, three, . . . many Higgs states at the LHC.



. . . although there is a large region of MSSM parameter space (the "infamous LHC wedge") where only a SM-like Higgs boson can be discovered.

Beyond the MSSM Higgs sector

The Higgs sector of the MSSM is minimal (as both doublets are needed). The most common extension is one where a complex Higgs singlet is added. The resulting model is called the NMSSM. Another possible extension is the expansion of the electroweak gauge group, e.g. adding an additional U(1). The corresponding Higgs sector is expanded as well, often with the addition of extra singlets.

For a motivation for such extensions, ask Steve Martin. Here, I will simply note a few interesting consequences:

- The upper bound on the MSSM Higgs mass is somewhat relaxed (depending on the model parameters). In particular, the NMSSM possesses a new Higgs self-coupling parameter λ that is not related to gauge couplings.

- The lightest Higgs boson of the model can be dominantly singlet and hence very weakly coupled. Mass limits on such a Higgs boson are not very stringent.

- The LEP lower limits on the neutral MSSM Higgs bosons can be evaded to some extent. In particular, the SM-like Higgs boson can dominantly decay into a pair of the light singlet-like Higgs scalars.

The last observation has spawned a minor industry—construct bizarre Higgs models to avoid the LEP Higgs mass bounds (SUSY may or may not be involved). To play the game, invent new physics that couples to a SM-like Higgs boson, Arrange the model so that the dominant decay mode of the Higgs boson is into new particles. Eventually, these will decay to SM particles—so the end result of one Higgs decay could be, e.g., six hadronic jets. Observe that LEP has no limit for such a strange possibility, and claim victory (and/or submit to the ArXiv).

Conclusions

• The Standard Model is not yet complete. The nature of the dynamics responsible for EWSB (and generating the Goldstone bosons that provide the longitudinal components of the massive $W^{\pm}$ and Z bosons) remains unresolved.

• There are strong hints that a weakly-coupled elementary Higgs boson exists in nature (although loopholes still exist). If a weakly-coupled SM-like Higgs boson is not discovered at the LHC, then other new phenomena (that are responsible for "fixing up" the precision electroweak data) will be detected.

• Strong theoretical arguments based on naturalness suggest that the Standard Model must be superseded by a more fundamental theory at an energy scale of order 1 TeV. This new physics is intimately connected with the dynamics of electroweak symmetry breaking.

• Low-Energy Supersymmetry provides a consistent framework for the weakly-coupled Higgs boson.

• Nature may still have some surprises up her sleeve. Perhaps extra dimensions will emerge at the TeV scale, with interesting implications for EWSB dynamics.

Conclusions (continued)

• Once (or if?) the Higgs boson is discovered, one must verify that its properties match expectations (a scalar state with couplings proportional to mass). Next, one must check whether its properties are consistent with SM Higgs predictions. Any departures from SM behavior will reveal crucial information about the nature of the EWSB dynamics.

• Ultimately, one must discover the TeV-scale dynamics associated with EWSB *e.g.*, low-energy supersymmetry and/or new particles and phenomena responsible for creating the Goldstone bosons. We expect the LHC to yield a very rich menu of new phenomena.

• But what if there is only a SM Higgs boson and no evidence for new physics beyond the SM? ...