Tracking 2

David Stuart
University of California
Santa Barbara

August 19, 2008

Review and Outline

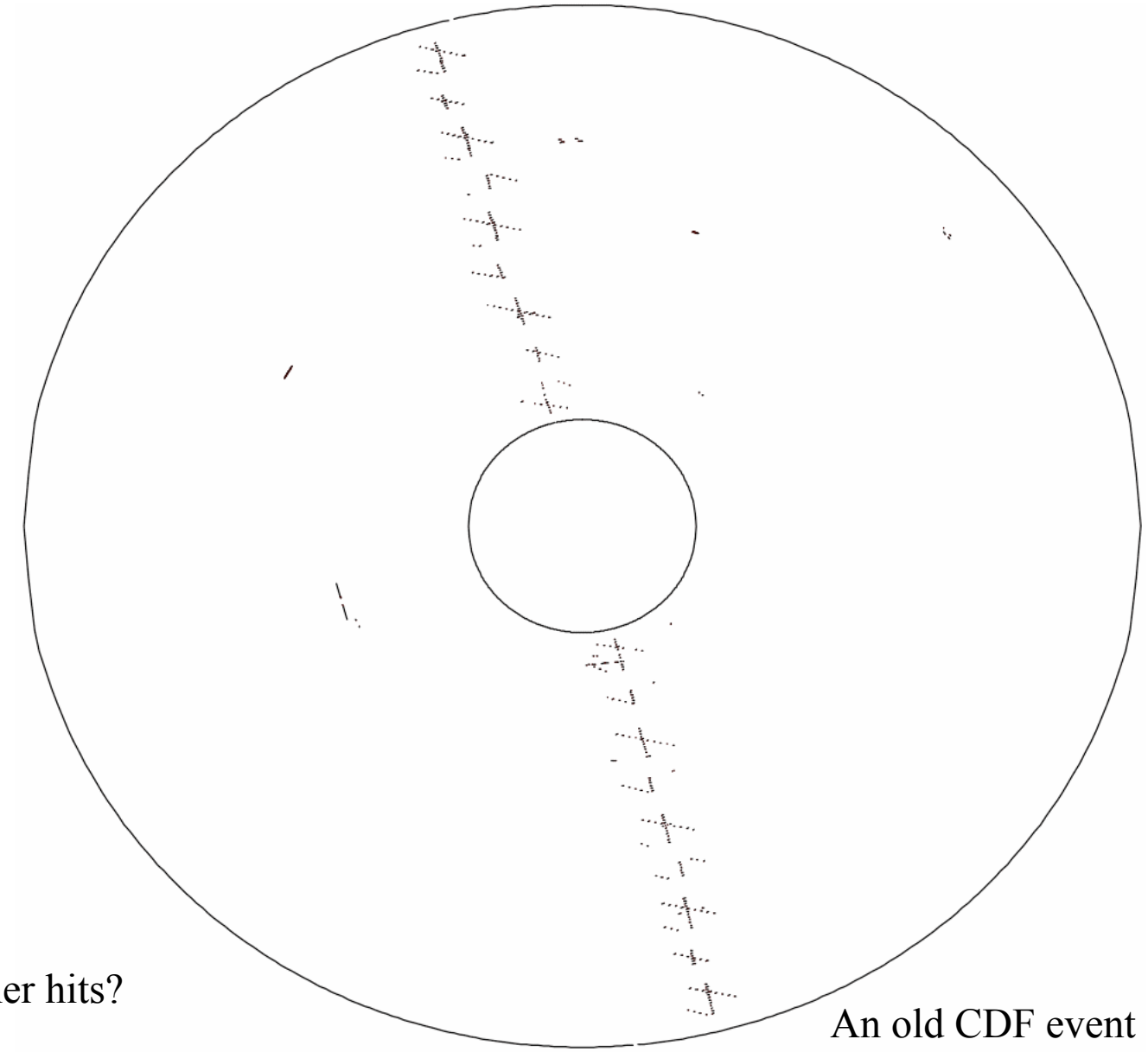
- Goals of tracking
  - Measure 4-vector and origin of particles.
  - Confirm and improve from other components
- Hardware
  - Measure position (“hits”) at points along path.
  - dE/dx ⇒ ionization ⇒ position
- Software
  - Collect measured hits and fit a helix.

Track Finding (AKA Tracking)

- Pattern recognition
- Fitting
- Performance assessment

Pattern recognition

Find the track(s).



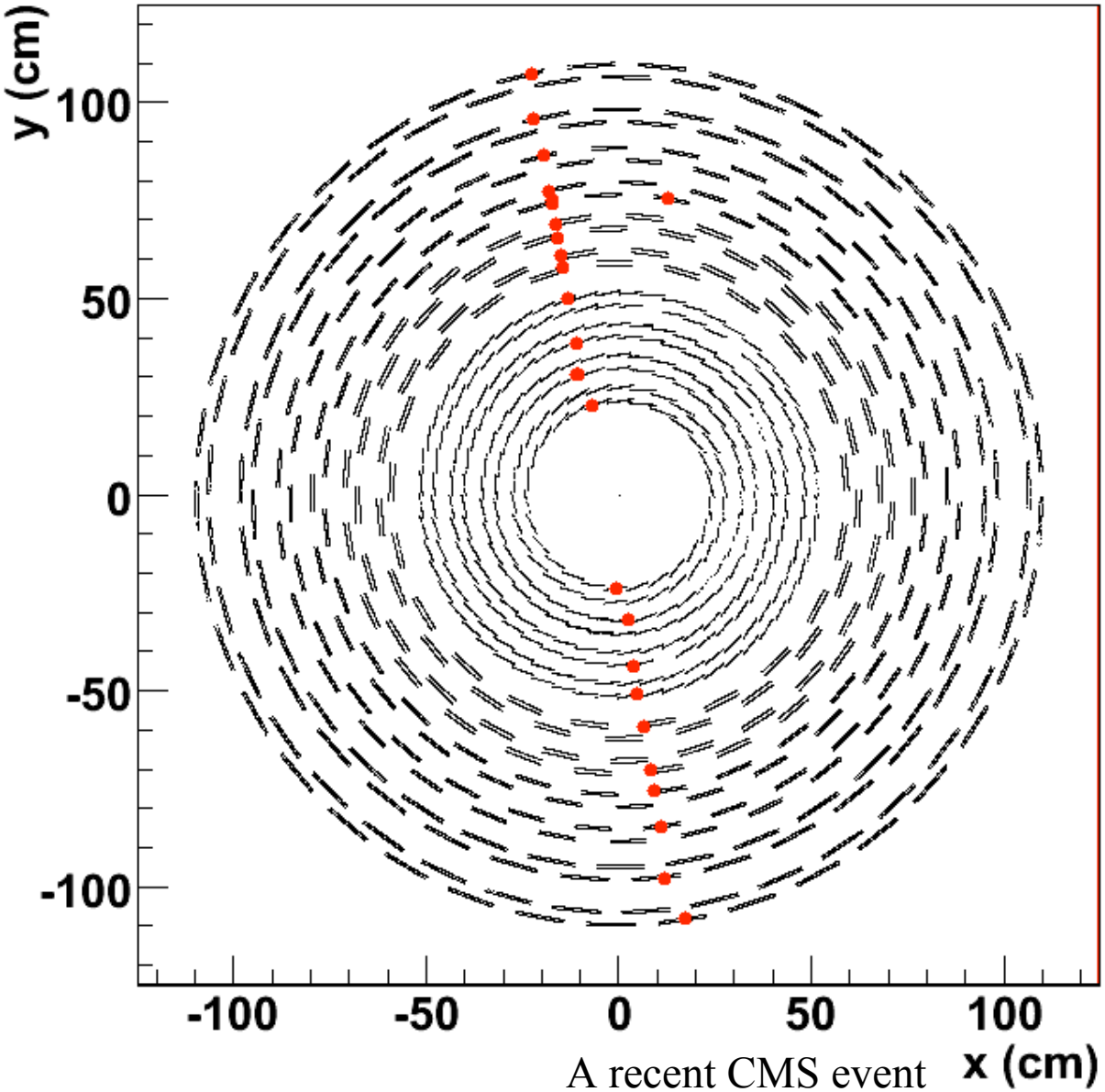
An old CDF event

Check: What causes the other hits?

Pattern recognition

Find the track(s).

Pretty easy. How to code it?



A recent CMS event

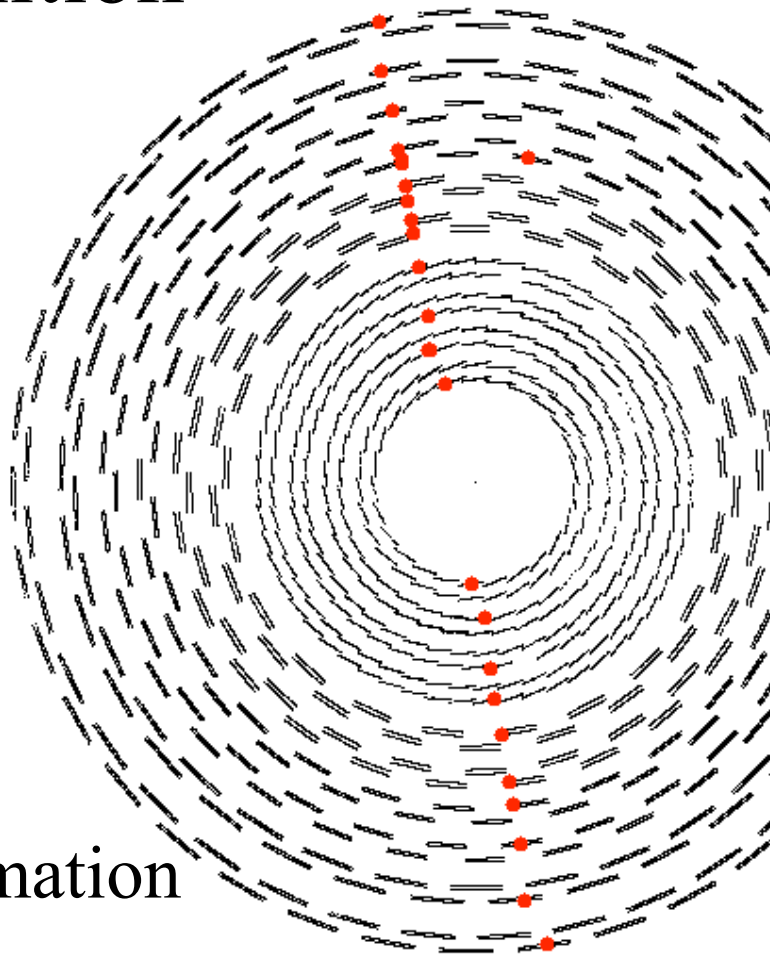
Pattern recognition

Three approaches:

1. Brute force combinatorics

2. Pattern matching

3. Seeding based on other information



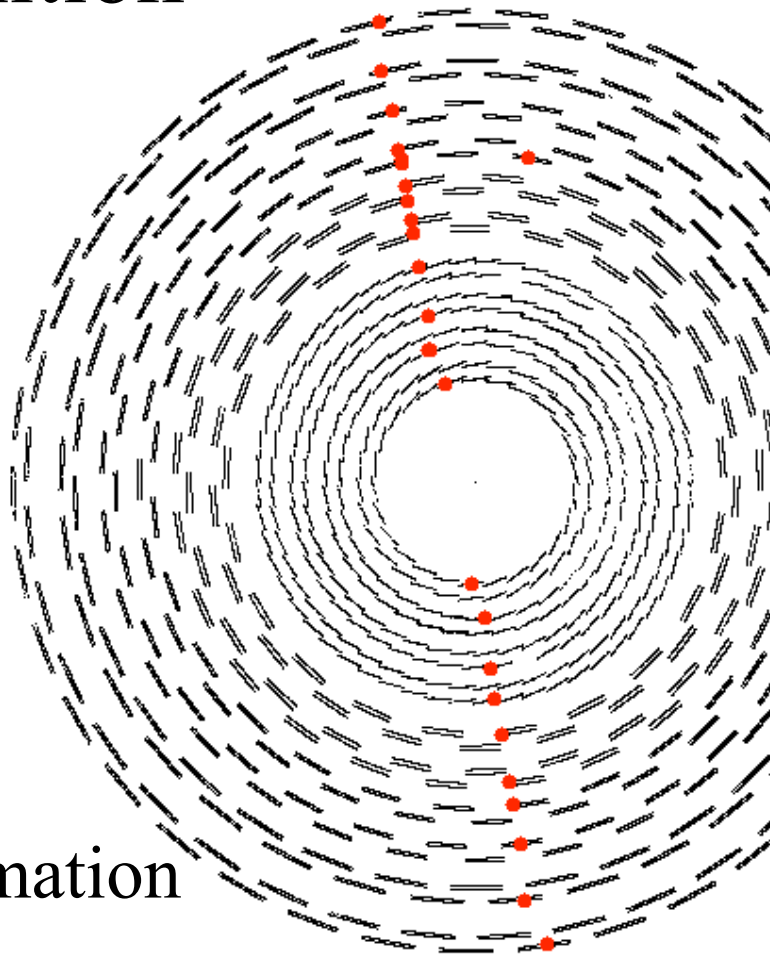
Pattern recognition

Three approaches:

1. Brute force combinatorics
   - Too slow

2. Pattern matching

3. Seeding based on other information



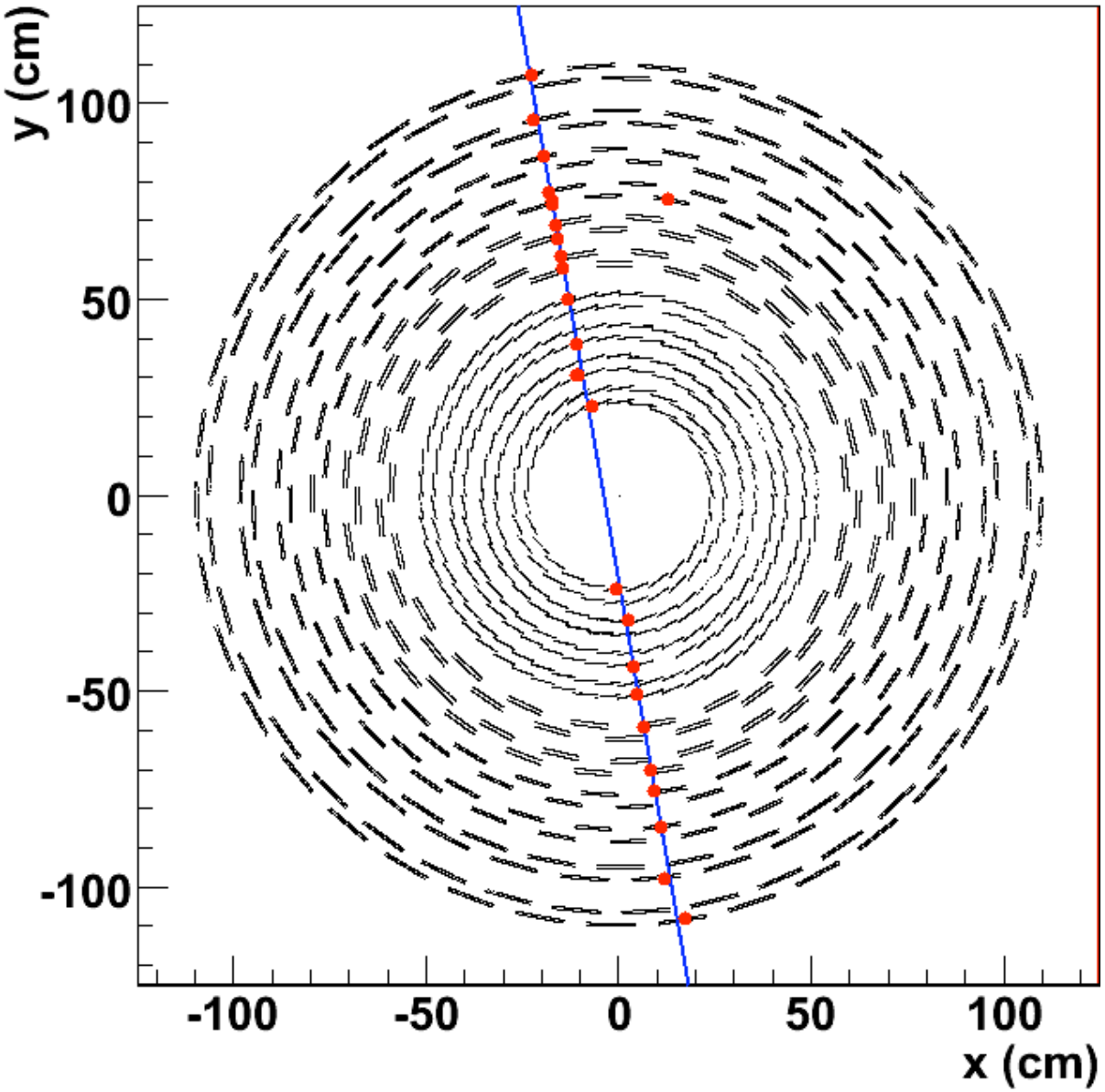
Pattern matching

Find the track(s).

Here is a hint…

Obtained ~ by fitting all hits to a straight line within root.

Cheating?



Pattern matching

Try several hints…

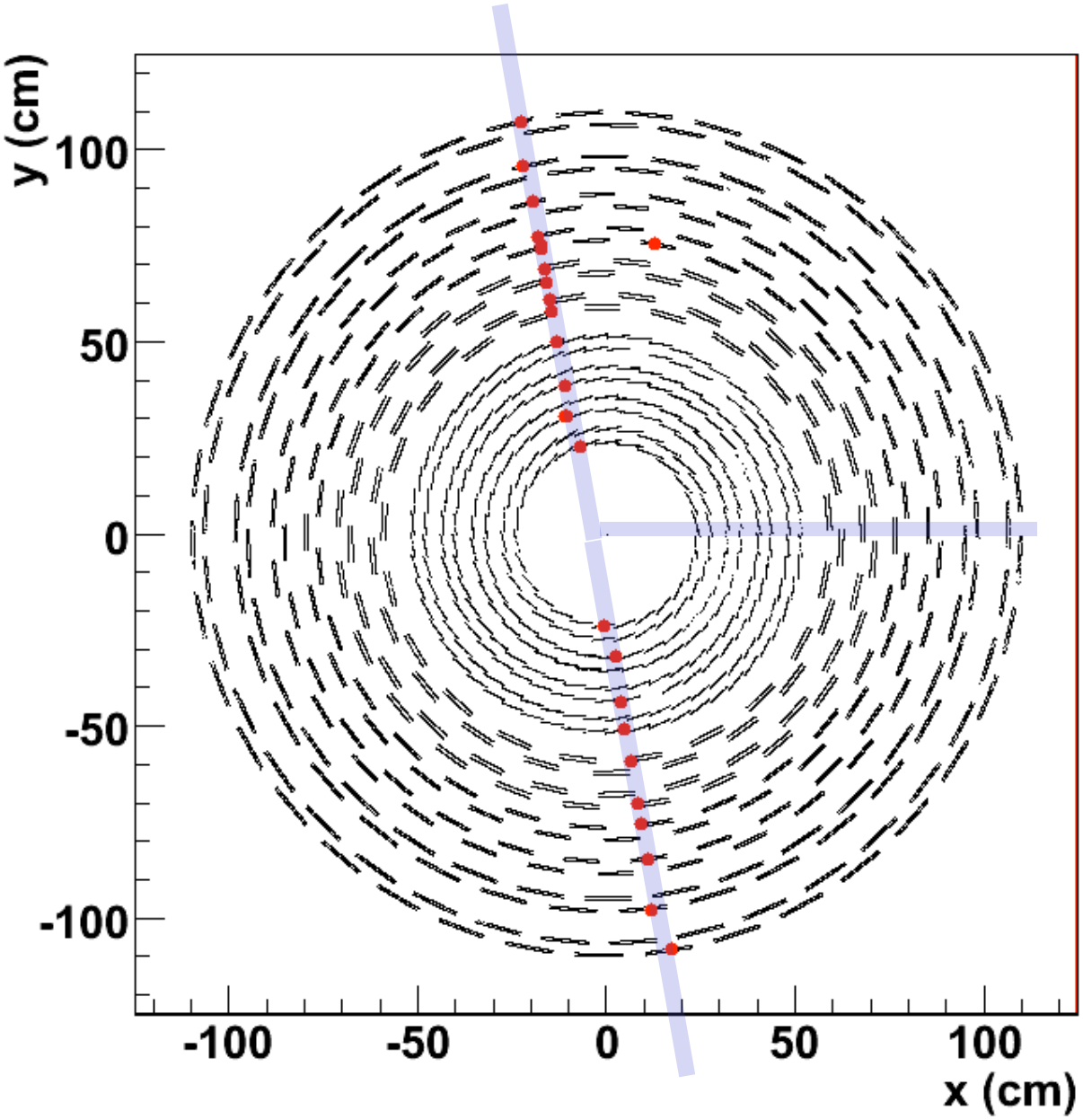
look for hits in a window.

```
if (NhitsInWindow>nMin)
    track = true
else
    track = false
```

Likewise, look for hits in all windows.

Well suited to a trigger using

parallel AND gates.



Pattern matching

What about curvature?

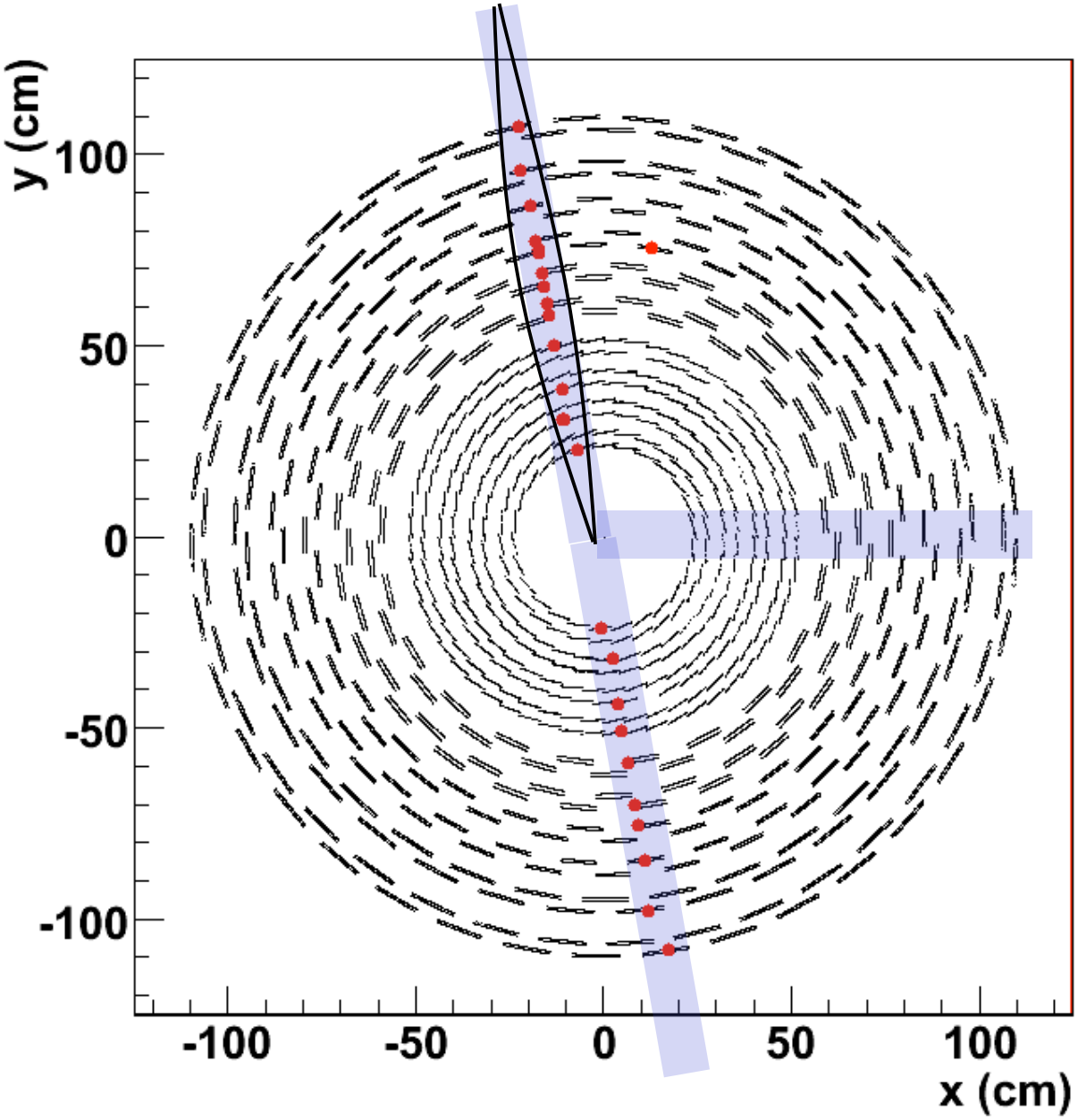
Widen window for

lower p_T.

If sufficient hits,

then track with $|p_T|>\min p_T$

Still well-suited to trigger.



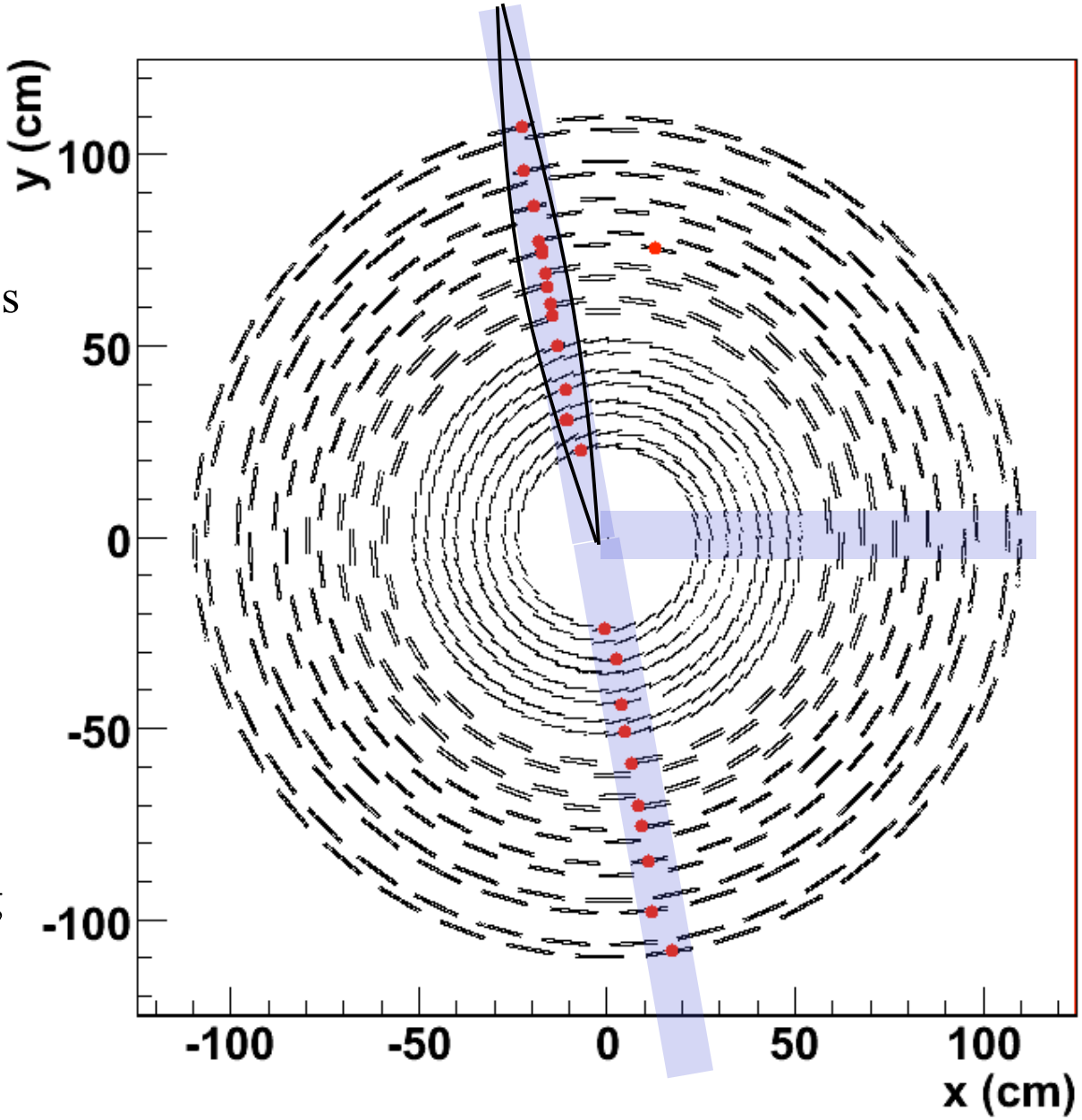
Pattern matching

Limitations:

- Confusable if several particles in the window.

- Assumes that the particle came from the beamspot.

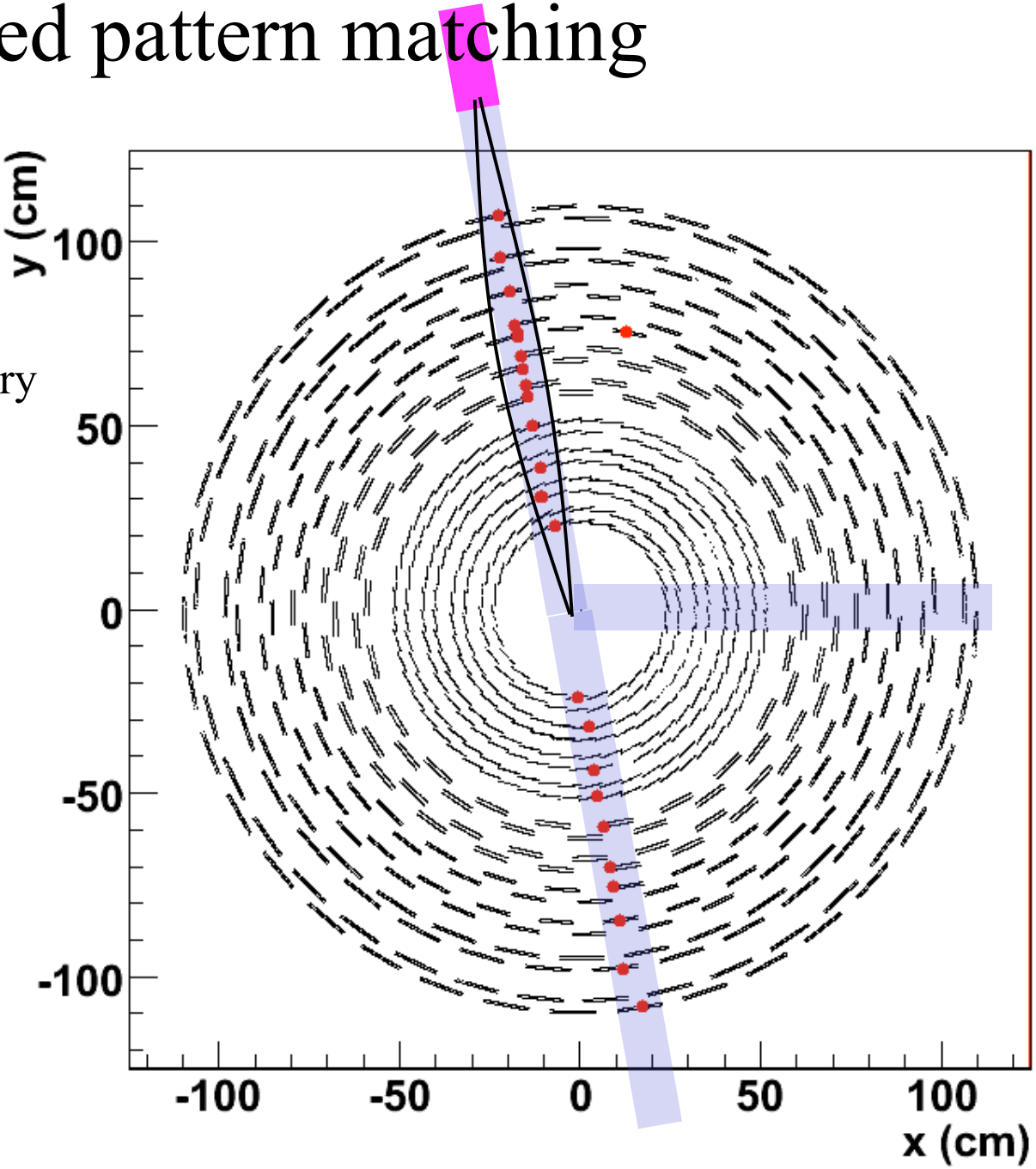
Neither is serious for matching tracks to primary leptons.



Matched pattern matching

For matching tracks to isolated lepton (e.g. electron) we already ~ know the trajectory from the calorimeter.

Only need to search in one p_T constrained window.



Pattern recognition

Three approaches:

1. Brute force combinatorics

2. Pattern matching:
   - Fast and parallelizable
   - Finds tracks "above some p_T cut"
   - Can be limited by other information

3. Seeding based on other information
   - Can be **constrained** by other information

Seeded hit search

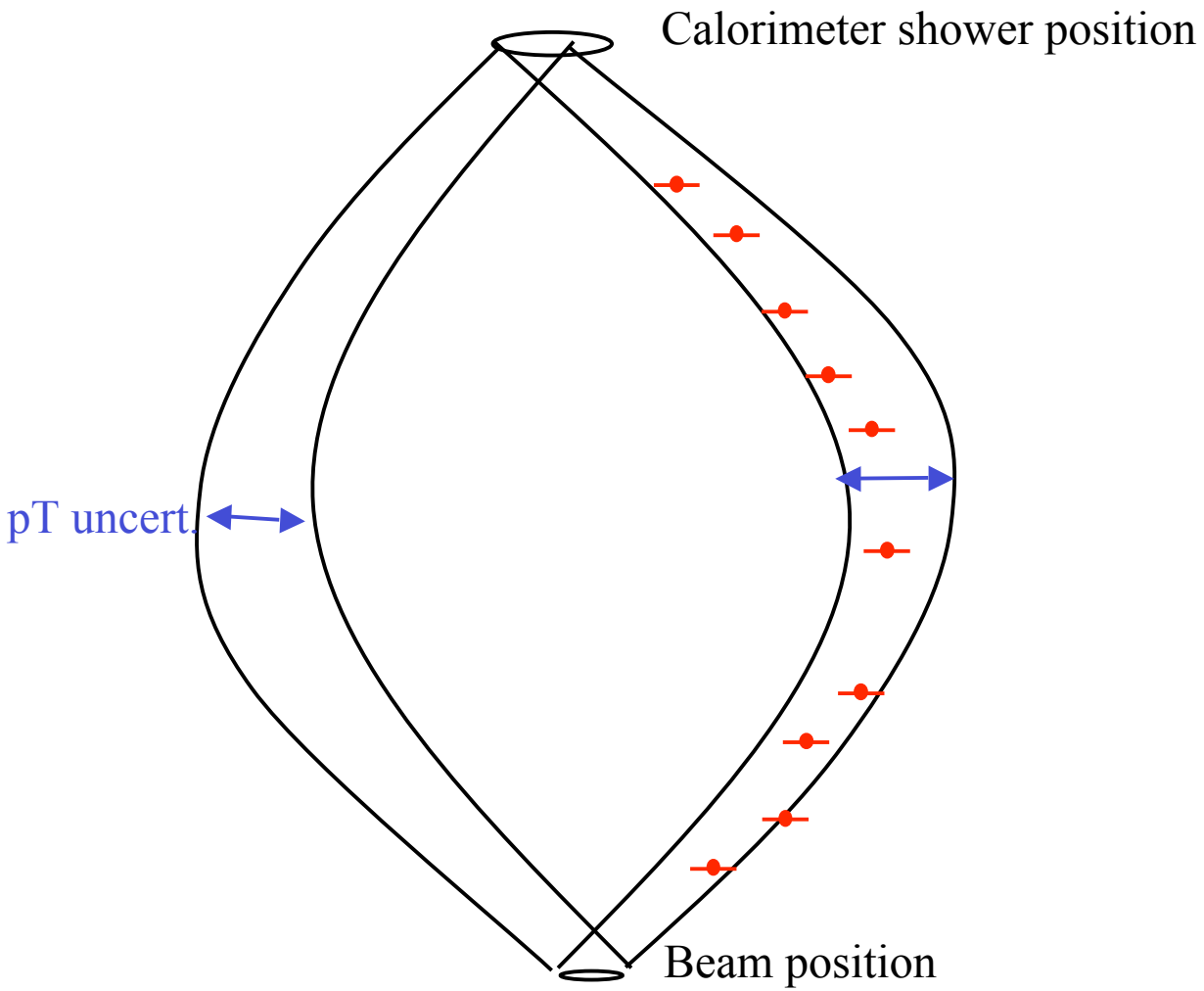
If we had a hypothesis, e.g., electron, use that "seed" fully.

Know position and p_T and their uncertainties.

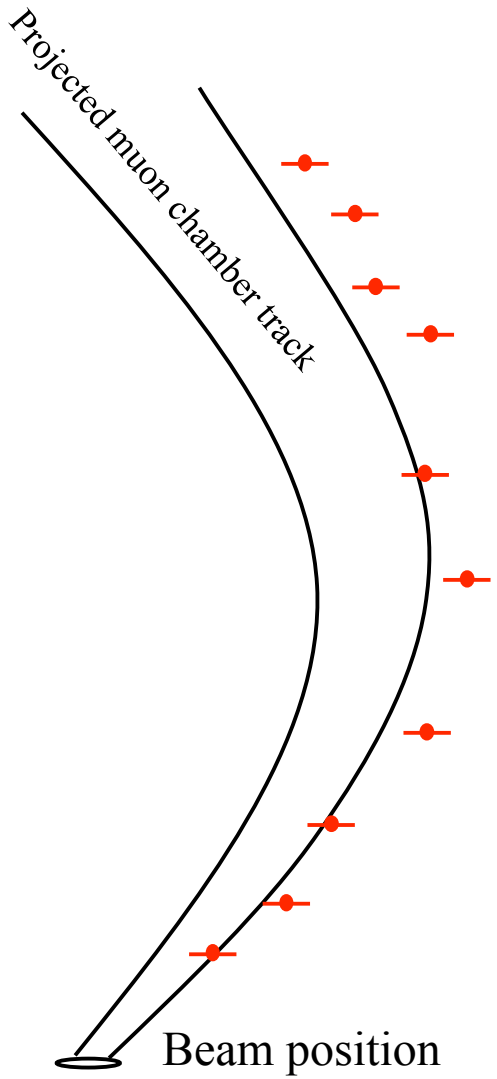
Don't know charge.

So…

Electron seeded hit search



Muon seeded hit search



Pattern recognition

Three approaches:

1. Brute force combinatorics

2. Pattern matching:
   - Fast and parallelizable
   - Finds tracks "above some p_T cut"
   - Can be limited by other information

3. Seeding based on other information
   - Can be **constrained** by other information
   - Useful e.g., for leptons.

4. A mixture: combinatoric pattern seeding?

Pattern recognition

Combinatoric segment finding in one supercell:

Pick end points ⇒line (with uncert.)

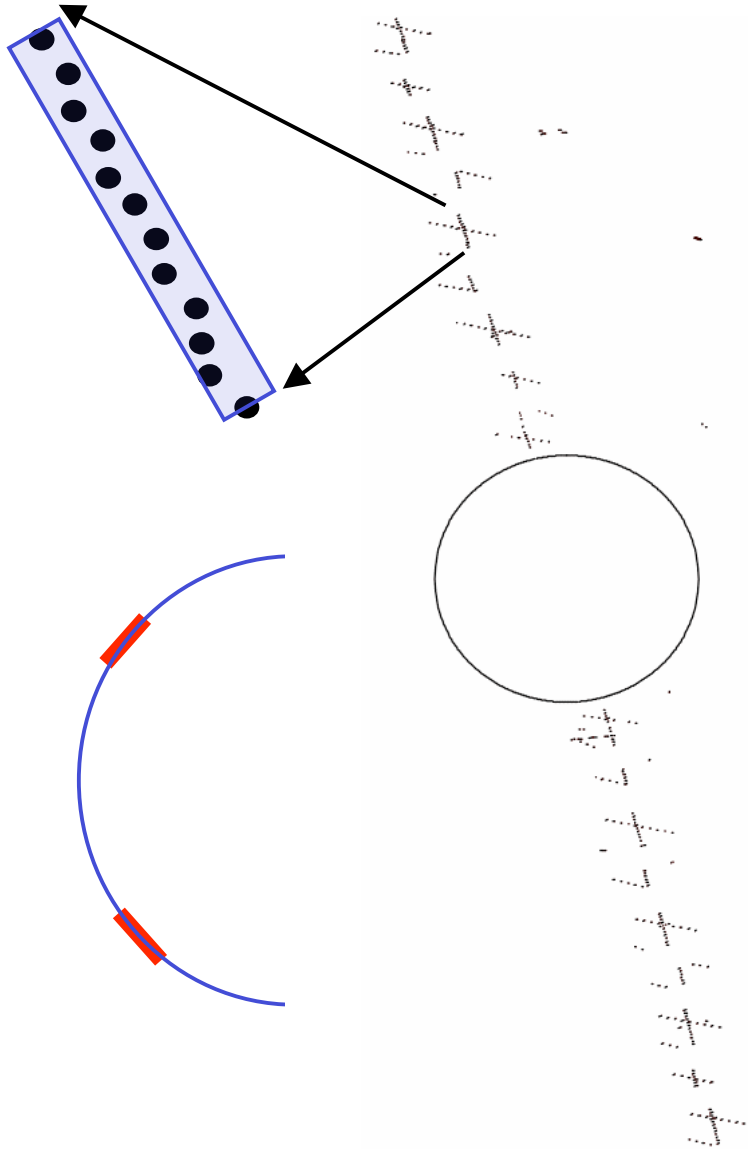
Pattern match in the window. Track?

Fit straight line (at this small lever arm, ~straight)

Remove outliers and refit.

Use this segment as a hypothesis (seed) to find

- hits in other super-layers
- matching segments in other super-layers



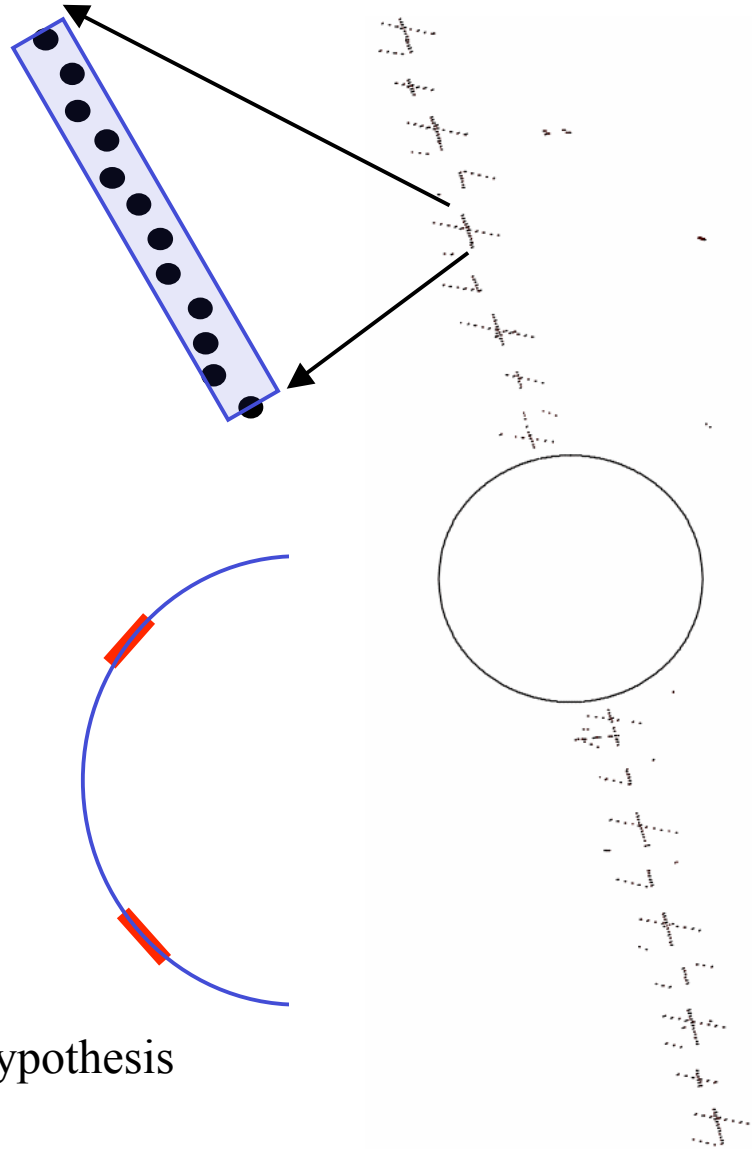
Pattern recognition

Combinatoric segment finding in one supercell:

Pick end points ⇒line (with uncert.)
Pattern match in the window. Track?
Fit straight line (at this small lever arm, ~straight)
Remove outliers and refit.

Use this segment as a hypothesis (seed) to find

- hits in other super-layers
- matching segments in other super-layers



In short: Use the scientific method.
Measurement → Hypothesis → Measurement → Better hypothesis

Fitting

The pattern recognition found a set of hits associated to one particle.

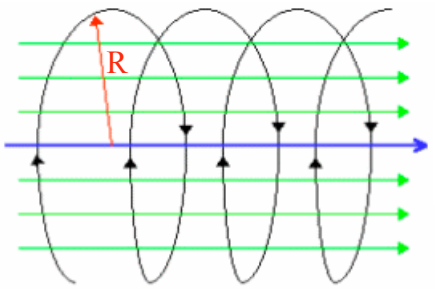
Now fit them to find the path.

For a straight line, that is easy:

```
myGraphXvsY->Fit("pol1");
myGraphRvsZ->Fit("pol1");
```

With a magnetic field, though, need to find:

- Momentum, actually p_T, actually R.
- Direction (phi and theta)
- Point of origin



$p_T = 0.3\ B\ R$

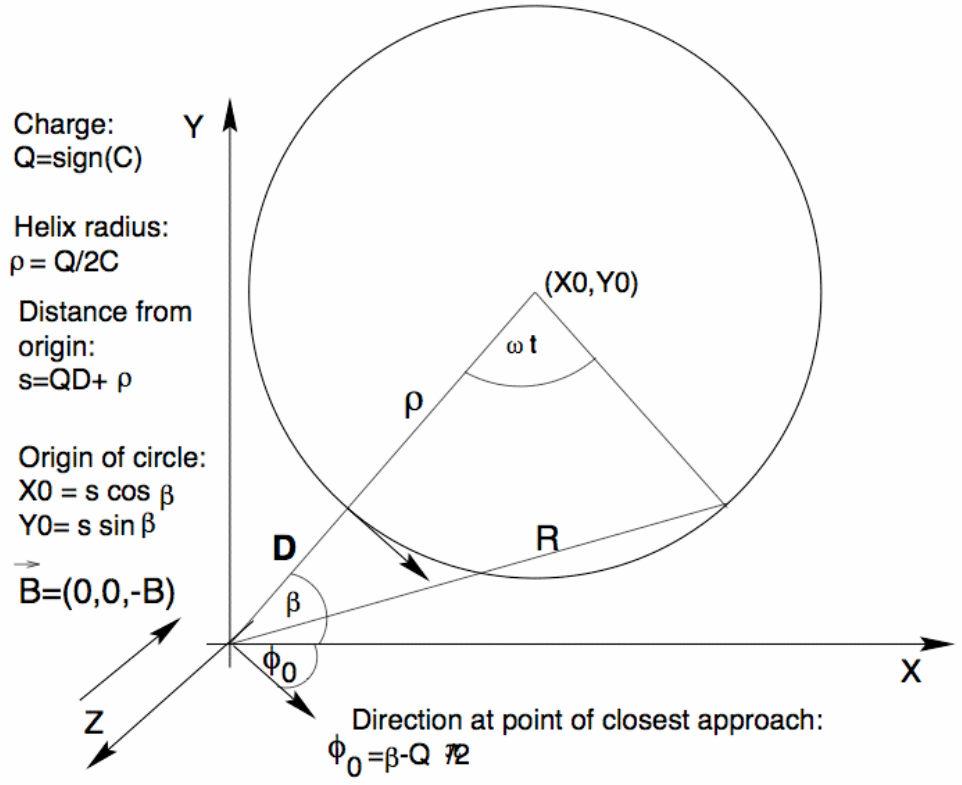
Describing a helix

$$
\begin{aligned}
x(t) &= x_0 + \rho\cos(\omega t - \alpha_0) \\
y(t) &= y_0 + \rho\sin(\omega t - \alpha_0) \\
z(t) &= z_0 + c\beta_z t = z_0 + \frac{\rho\omega t}{c}\cot\theta
\end{aligned}
$$

where:

$\omega = \frac{c\beta_\perp}{\rho}$: angular velocity

$\frac{\beta_z}{\beta_\perp} = \frac{P_z}{P_t} = \cot\theta$



This works, but direction and origin are vague

Hanz Wenzel, CDF Note 1790

Describing a helix

$$\left.\begin{aligned} x(t) &= x_0 + \rho\cos(\omega t - \alpha_0) \\ y(t) &= y_0 + \rho\sin(\omega t - \alpha_0) \\ z(t) &= z_0 + c\beta_z t = z_0 + \frac{\rho\omega t}{c}\cot\theta \end{aligned}\right\}$$

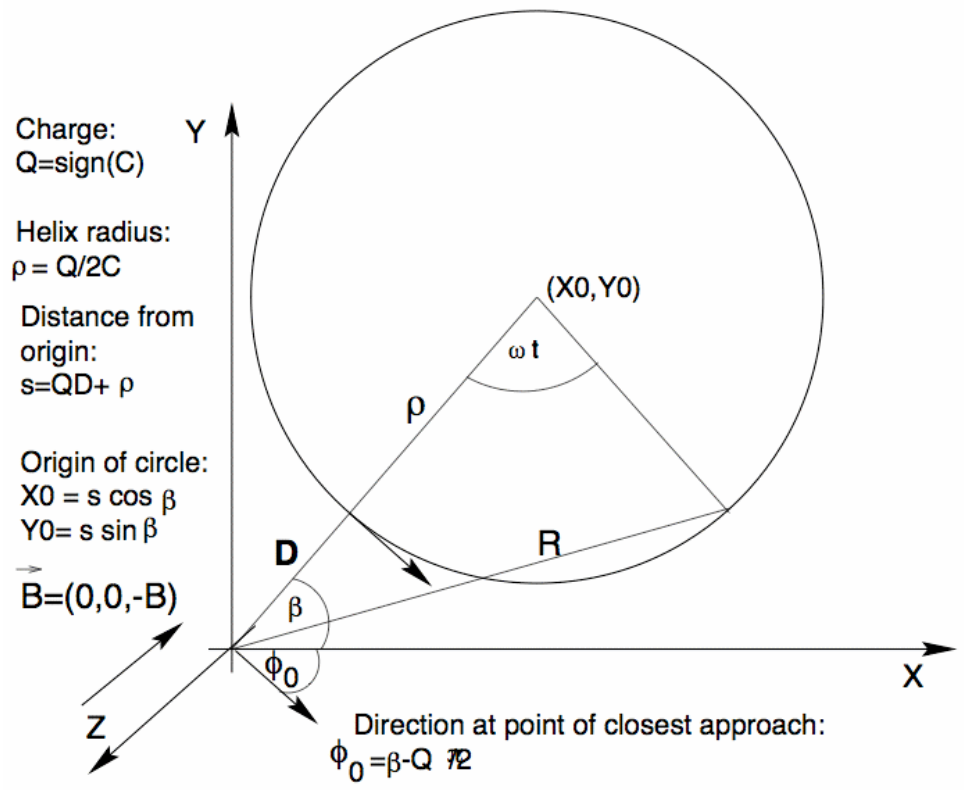
$$\begin{aligned} x &= x_0 + Q\rho\sin(Q\lambda + \varphi_0) \\ y &= y_0 - Q\rho\cos(Q\lambda + \varphi_0) \\ z &= z_0 + \rho\lambda\cot\theta \end{aligned}$$

Where

$\lambda = \omega t$

and

$$\rho = \frac{1}{2QC}$$



Hanz Wenzel, CDF Note 1790

Describing a helix

Five helix parameters:

C: [half] curvature, signed

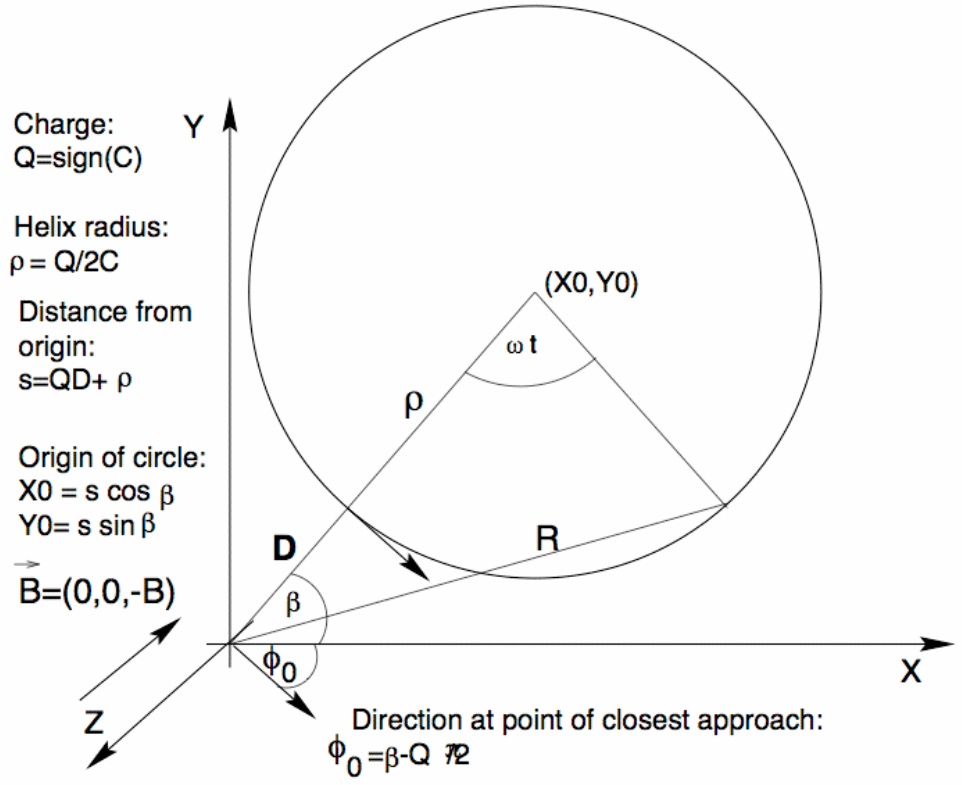
cot(θ): polar angle

D: Distance of closest approach to orign. (Also called impact parameter, d_0).

ϕ_0: Phi at closest approach

z_0: z at closest approach

$$
\begin{aligned}
x &= x_0 + Q\rho\sin(Q\lambda + \varphi_0) \\
y &= y_0 - Q\rho\cos(Q\lambda + \varphi_0) \\
z &= z_0 + \rho\lambda\cot\theta
\end{aligned}
$$



Hanz Wenzel, CDF Note 1790

Describing a helix

Five helix parameters:

C: [half] curvature, signed

cot(θ): polar angle

D: Distance of closest approach to orign. (Also called impact parameter, d_0).

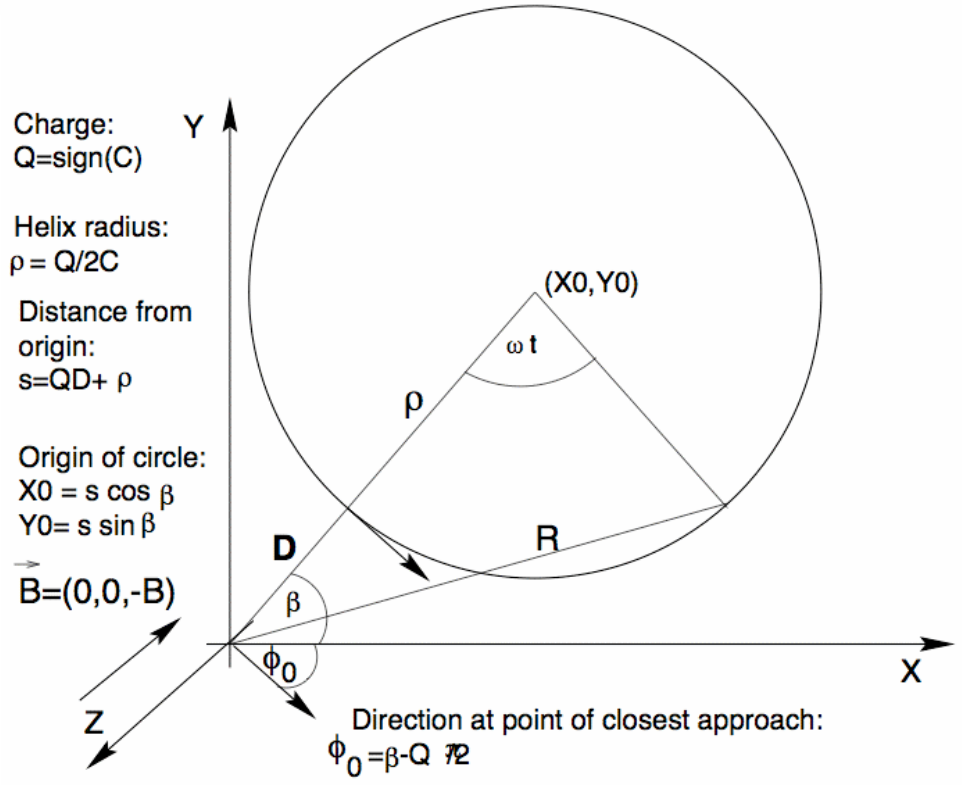
ϕ_0: Phi at closest approach

z_0: z at closest approach

$$x = -\frac{\sin\varphi_0(1+2CD)}{2C} + \frac{1}{2C}\sin(Q\lambda+\varphi_0)$$

$$y = \frac{\cos\varphi_0(1+2CD)}{2C} - \frac{1}{2C}\cos(Q\lambda+\varphi_0)$$

$$z = z_0 + \frac{Q}{2C}\lambda\cot\theta,\ Q = sign(C)$$



Hanz Wenzel, CDF Note 1790

Describing a helix

Five helix parameters:

C: [half] curvature, signed

cot(θ): polar angle

D: Distance of closest approach to orign. (Also called impact parameter, d_0).

ϕ_0: Phi at closest approach

z_0: z at closest approach

Simple functions for phi or z at any R:

$$\varphi(R,\vec{\alpha}) = \varphi_0 + \sin^{-1}\left(\frac{CR + (1+CD)D/R}{1+2CD}\right)$$

$$Z(R,\vec{\alpha}) = z_0 + \frac{\cot\theta}{C}\sin^{-1}\left(C\sqrt{\frac{R^2-D^2}{1+2CD}}\right)$$

…so, you can make your own simple event display in root.
(Beware that your software may not use half curv.)

Hanz Wenzel, CDF Note 1790

Fitting

So, fit the helix to the collected hits and obtain the five helix parameters and their uncertainties.

Then find momentum from:

$$C = Q \cdot \frac{1.49898 \cdot 10^{-4} \cdot B_{magnet}}{P_t}$$

and

$$cot\theta = \frac{P_z}{P_t}$$

and propagate the uncertainties.

Hanz Wenzel, CDF Note 1790

Fitting

That is pretty good, but it neglects two effects:

- dE/dx
- Multiple Coloumb Scattering

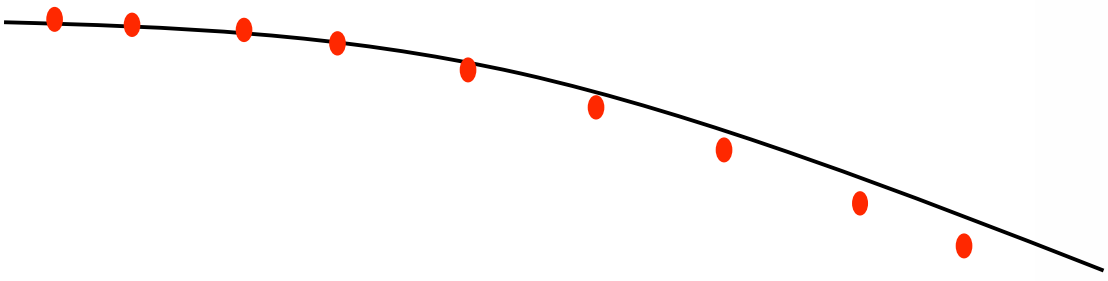
Ionization energy loss, dE/dx

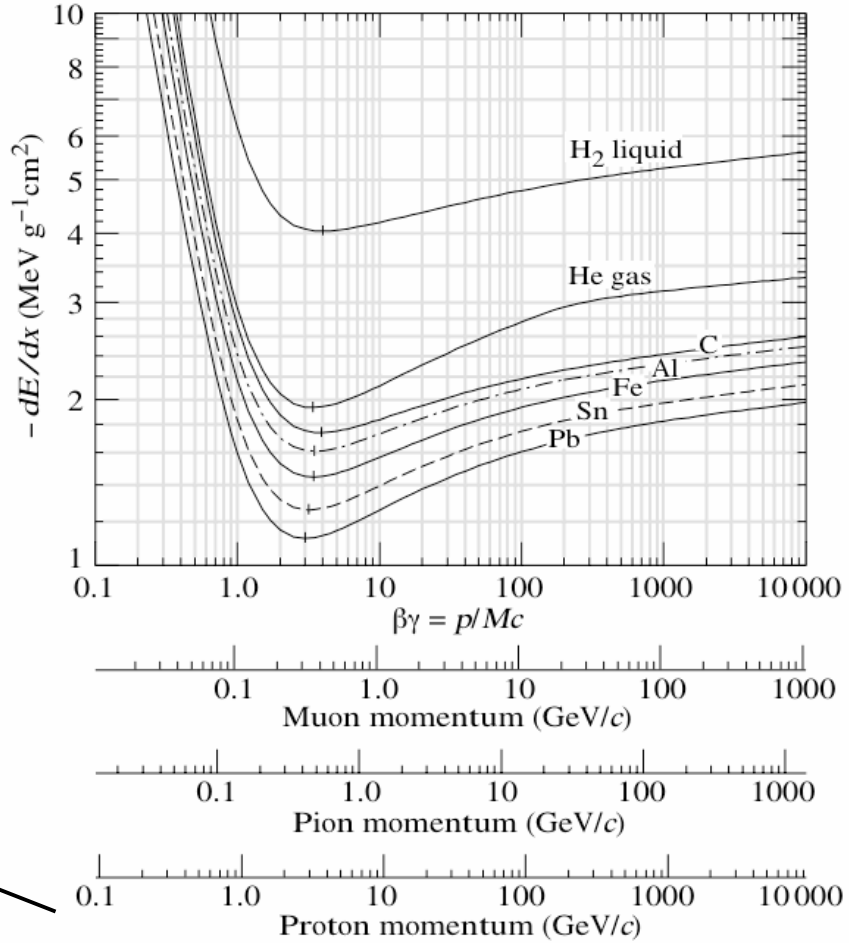
$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Air: ~2 keV/cm

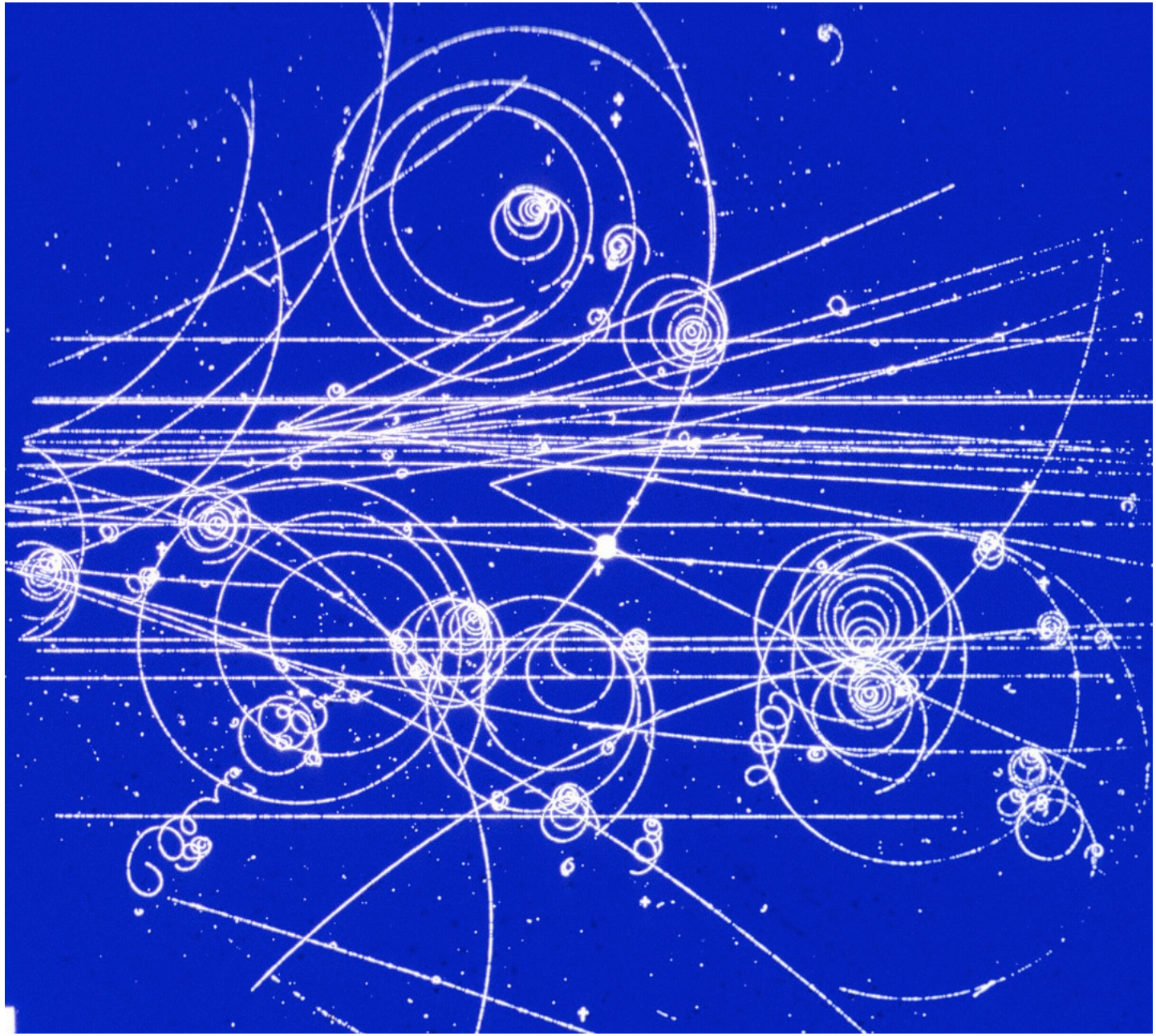
Silicon: ~120 keV/ 300 μm

Fit should include this
(but now we need to know M)...





An example of dE/dx affecting tracks. An old CERN bubble chamber event.



An example of dE/dx affecting tracks. An old CERN bubble chamber event.

Multiple Coulomb Scattering

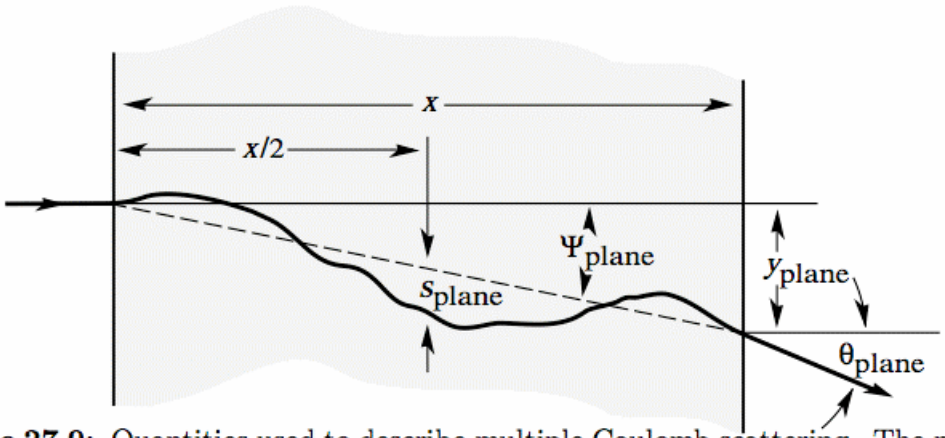


Figure 27.9: Quantities used to describe multiple Coulomb scattering. The particle is incident in the plane of the figure.

Often just called Coulomb scattering or simply scattering.

$$\theta_{rms} = \frac{13.6\ \text{MeV}}{\beta c p}\, q\, \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$$

x = thickness

X_0 = radiation length of material

≈ 100 m for air ⇒ $x/X_0 \approx 0.01\%$ for 1cm of air

≈ 10 cm for silicon ⇒ $x/X_0 \approx 0.3\%$ for 300 μm of silicon

Multiple Coulomb Scattering

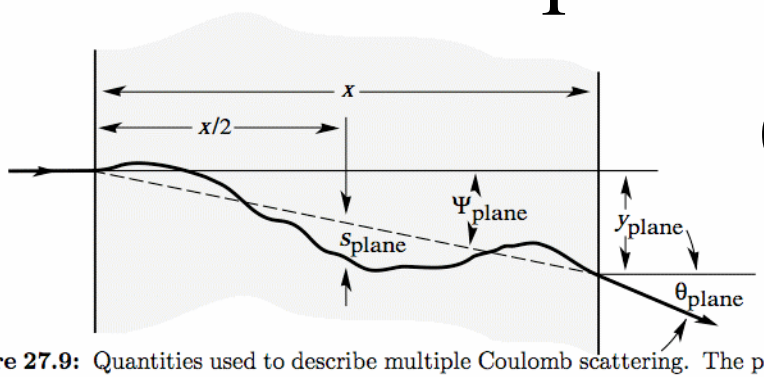


Figure 27.9: Quantities used to describe multiple Coulomb scattering. The particle is incident in the plane of the figure.

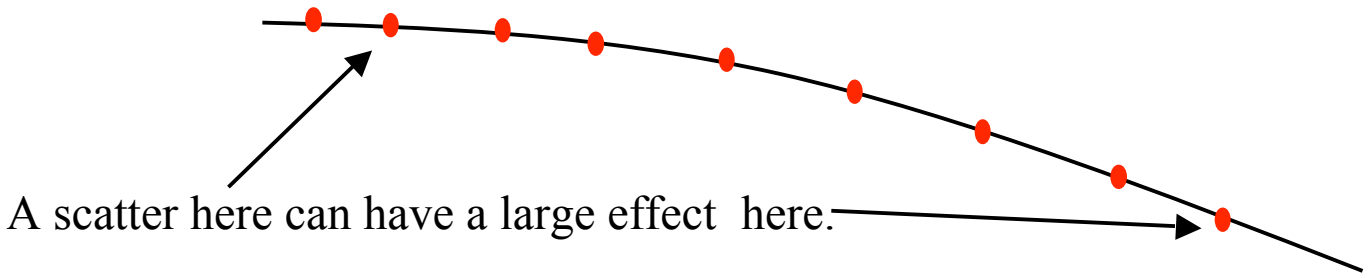
$$\theta_{\text{rms}} = \frac{13.6\ \text{MeV}}{\beta cp}\, q\, \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$$

$x/X_0 \approx 0.01\%$ for 1cm of air

$\Rightarrow$10 μm error / meter of projection at 1 GeV.

$x/X_0 \approx 0.3\%$ for 300 μm of silicon

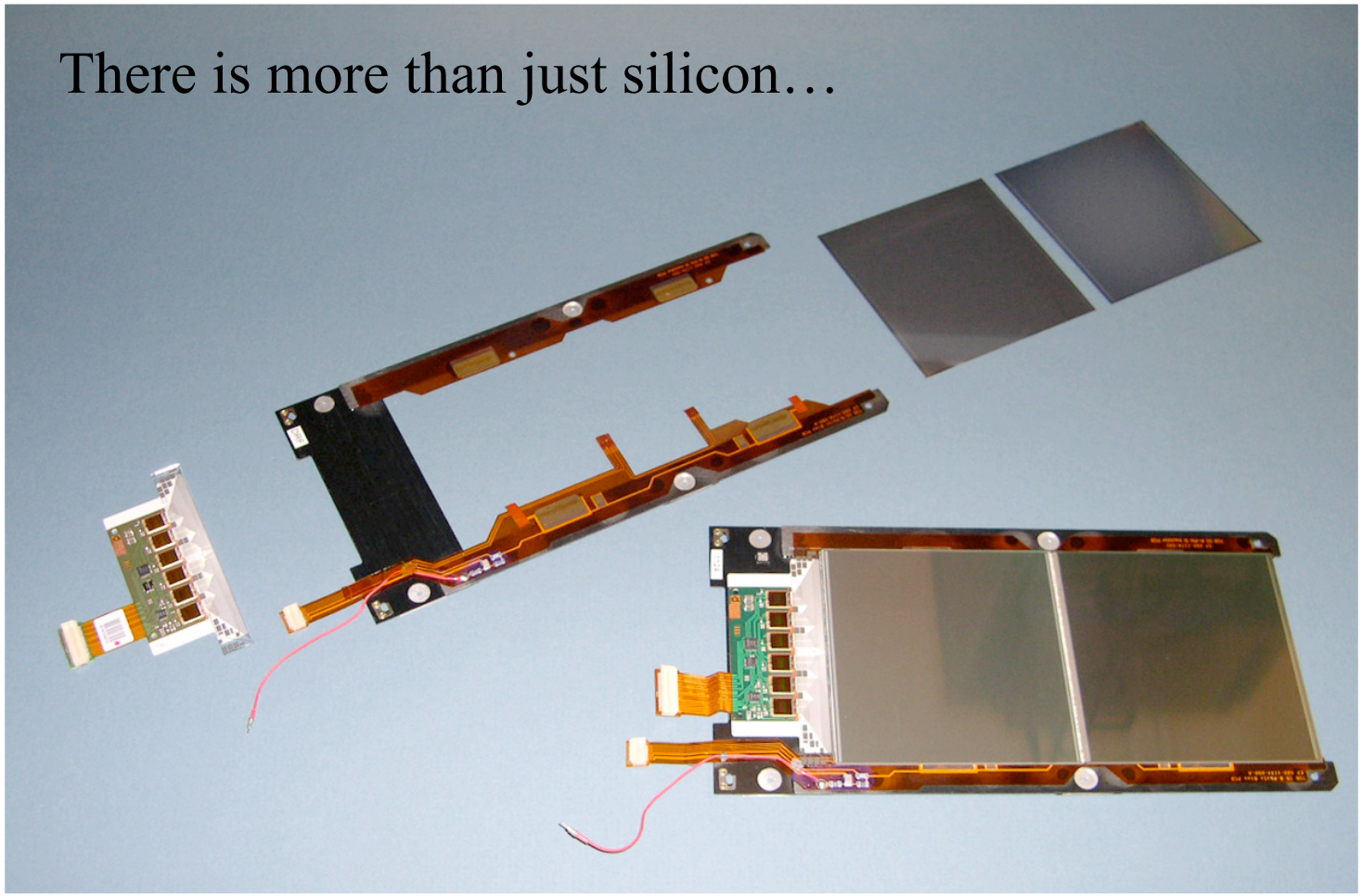
$\Rightarrow$ 600 μm error / meter of projection at 1 GeV.



Multiple Coulomb Scattering

$x/X_0 \approx 0.3\%$ for 300 μm of silicon

$\Rightarrow$ 600 μm error / meter of projection at 1 GeV.

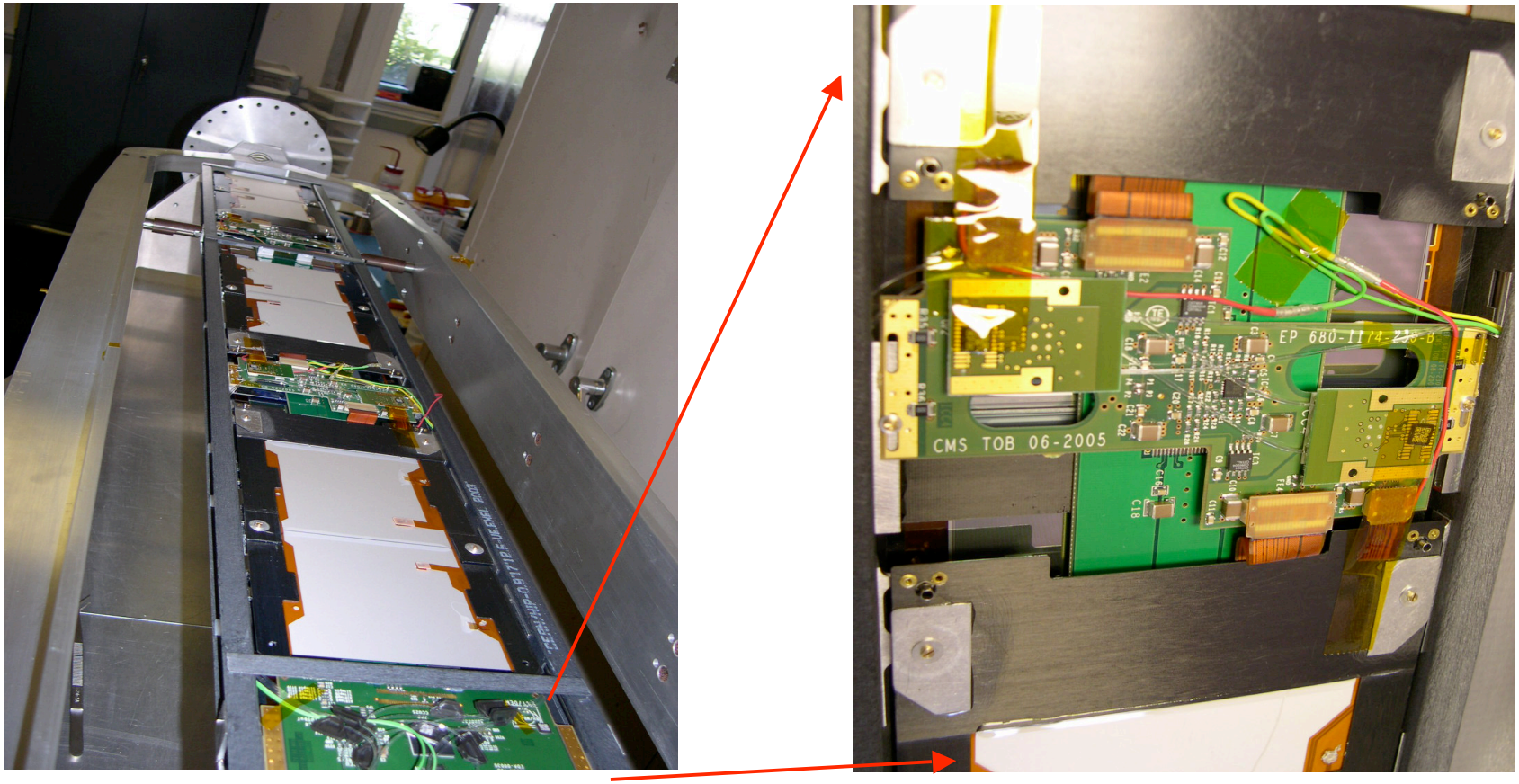


Multiple Coulomb Scattering

$x/X_0 \approx 0.3\%$ for 300 μm of silicon

$\Rightarrow$ 600 μm error / meter of projection at 1 GeV.

There is more than just silicon…



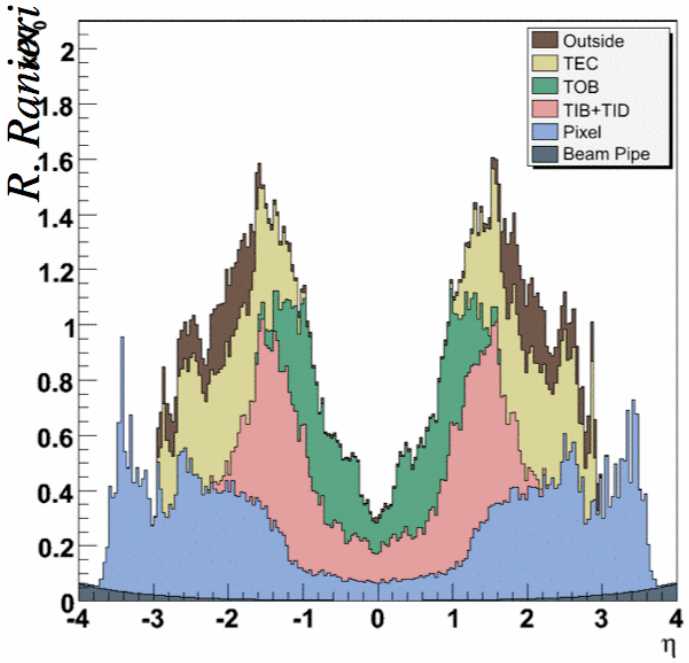
Multiple Coulomb Scattering

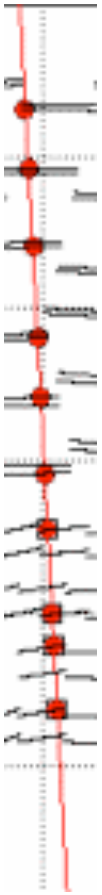
$x/X_0 \approx 0.3\%$ for 300 μm of silicon

$\Rightarrow$600 μm error / meter of projection at 1 GeV.

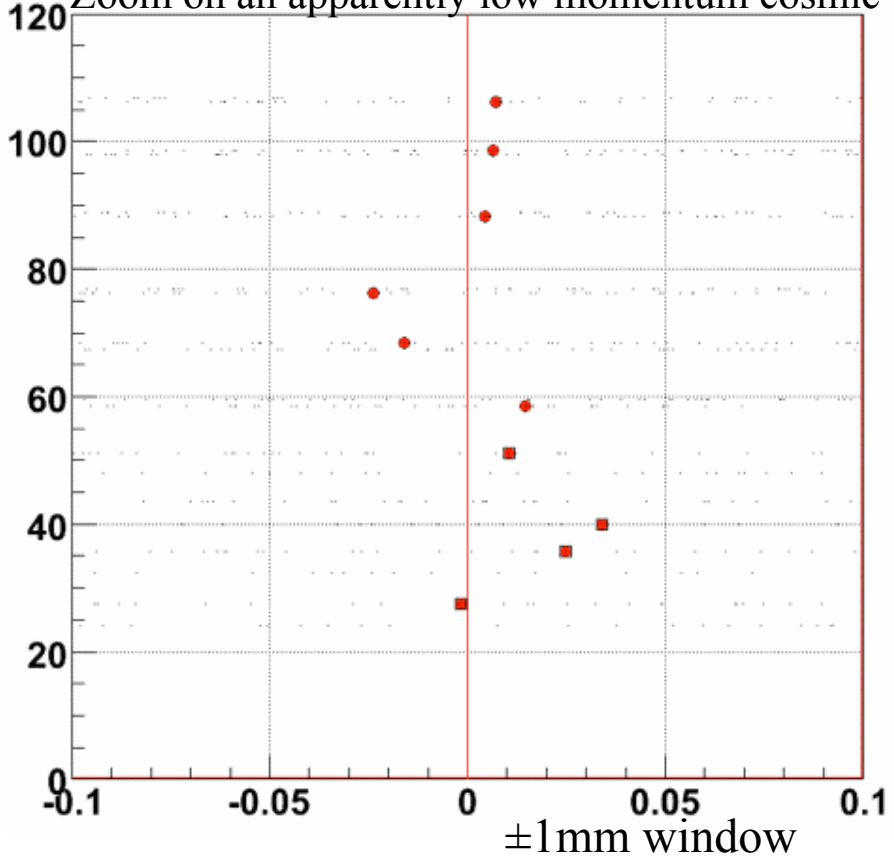
$x/X_0 \approx 2.5\%$ for each measurement layer

$\Rightarrow$ 2 mm error / meter of projection at 1 GeV.





Zoom on an apparently low momentum cosmic

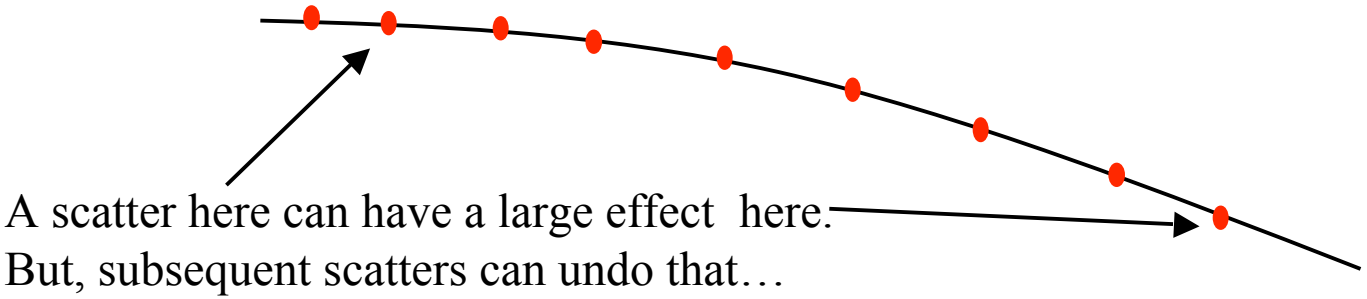


Scattering in each layer can have a large effect downstream, but could be "un-scattered" later.…

Fit as you go

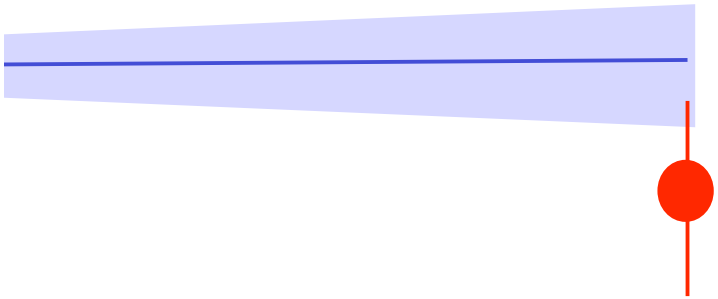
To include energy loss and MCS, we need to fit as we go, hit by hit.
After each hit is added, subtract dE/dx from p and add MCS to uncertainty.

But, each new hit partially measures the effect of any previous scattering…



Fit as you go

- Imagine that we begin with a track hypothesis, e.g., from a seed.
- It has parameters and uncertainties = 5 numbers and a 5x5 covariance matrix
- But I will simplify as one-dimensional, just x and σ_x.
- Project track (and its uncertainty) to the first hit.
- Test and improve the hypothesis with a new measurement.



$x_p \pm \sigma_p$ prediction

$x_h \pm \sigma_h$ hit

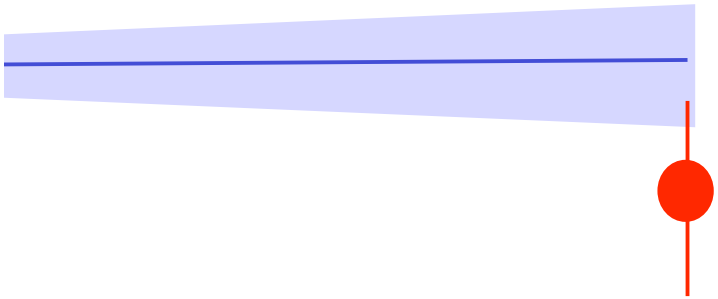
$$\text{New } x = \frac{(x_p / \sigma_p^2) + (x_h / \sigma_h^2)}{(1 / \sigma_p^2) + (1 / \sigma_h^2)} = \text{weighted average}$$

In 5D, it is just the matrix equivalent.

This is called a Kalman Filter, where the filter is just averaging the prediction & measurement.

Kalman Filter

- Ignores a poor prediction
- Ignores a poor measurement
- Naturally iterative
- Short projection may allow approximations



$x_p \pm \sigma_p$ prediction

$x_h \pm \sigma_h$ hit

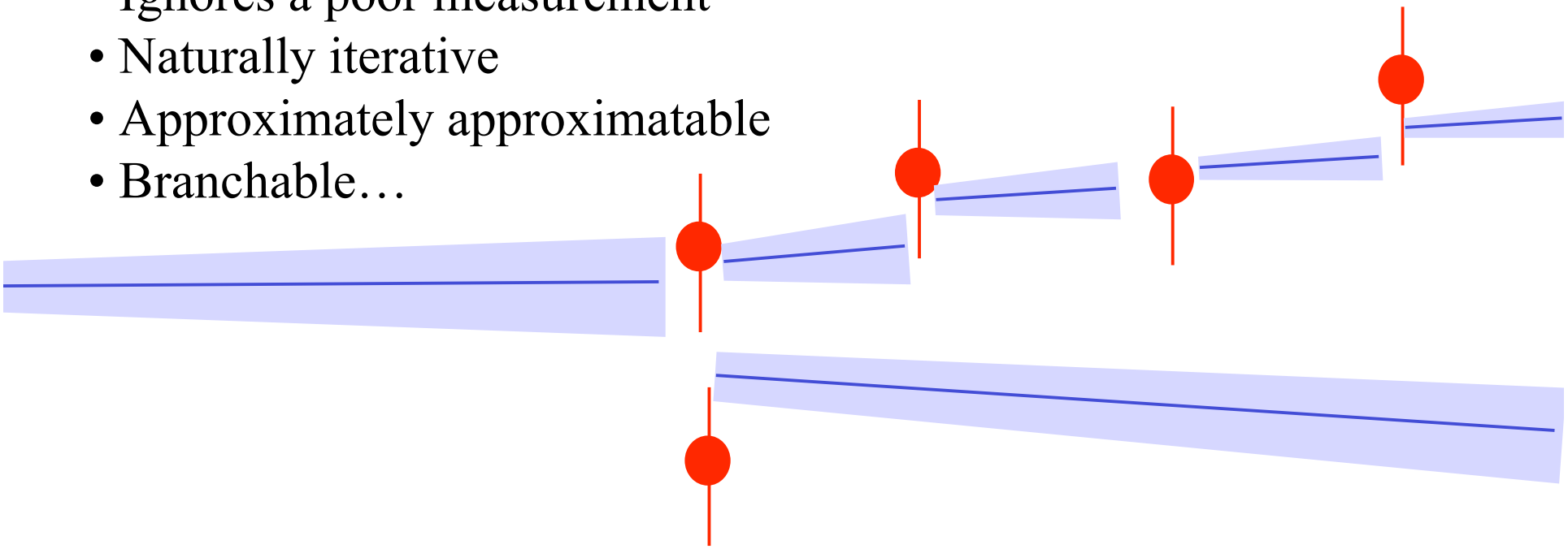
$$\text{New } x = \frac{(x_p / \sigma_p^2) + (x_h / \sigma_h^2)}{(1 / \sigma_p^2) + (1 / \sigma_h^2)} = \text{weighted average}$$

In 5D, it is just the matrix equivalent.

This is called a Kalman Filter, where the filter is just averaging the prediction & measurement.

Kalman Filter

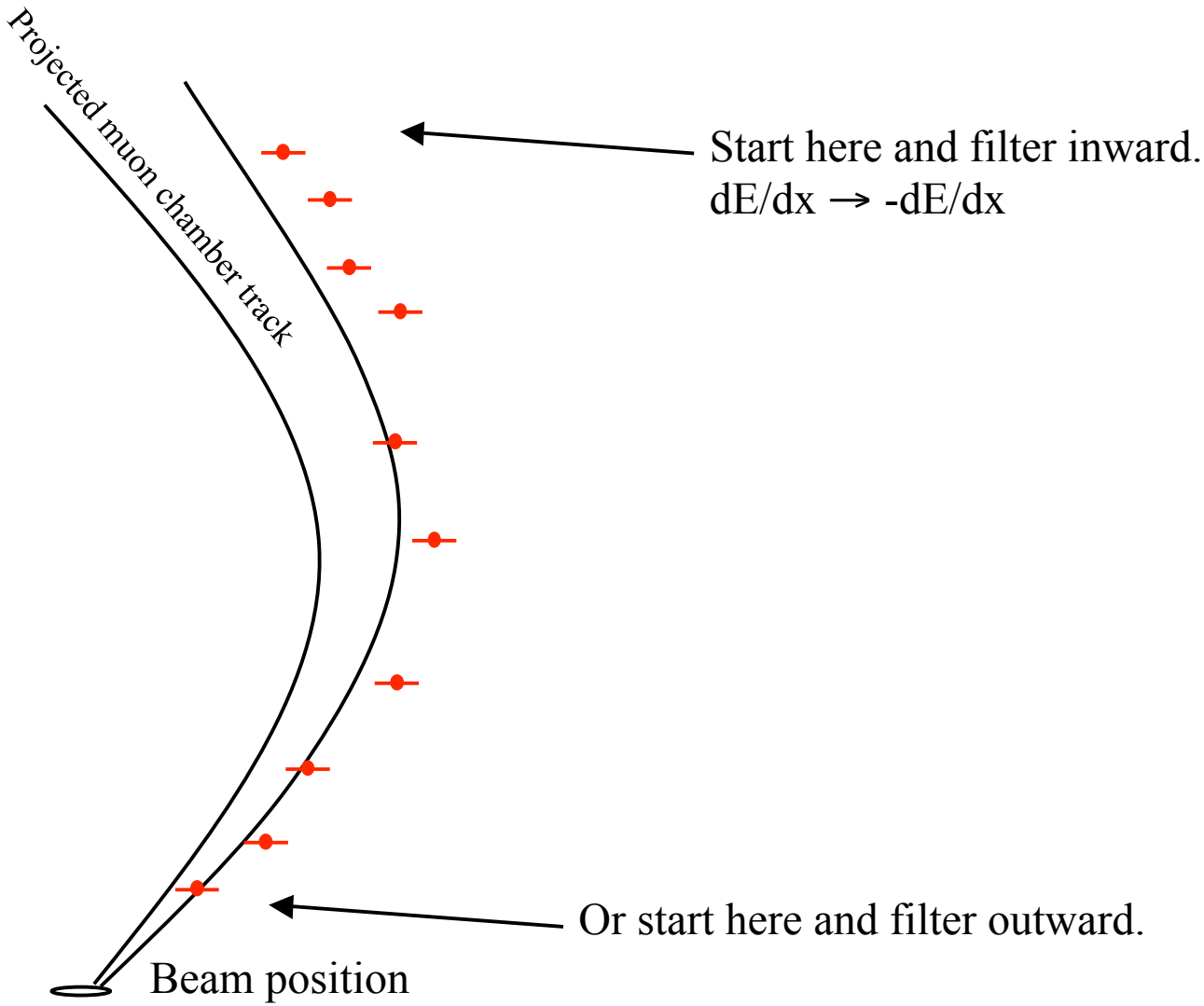
- Ignores a poor prediction
- Ignores a poor measurement
- Naturally iterative
- Approximately approximatable
- Branchable…



- Choose at the end which branch is best.

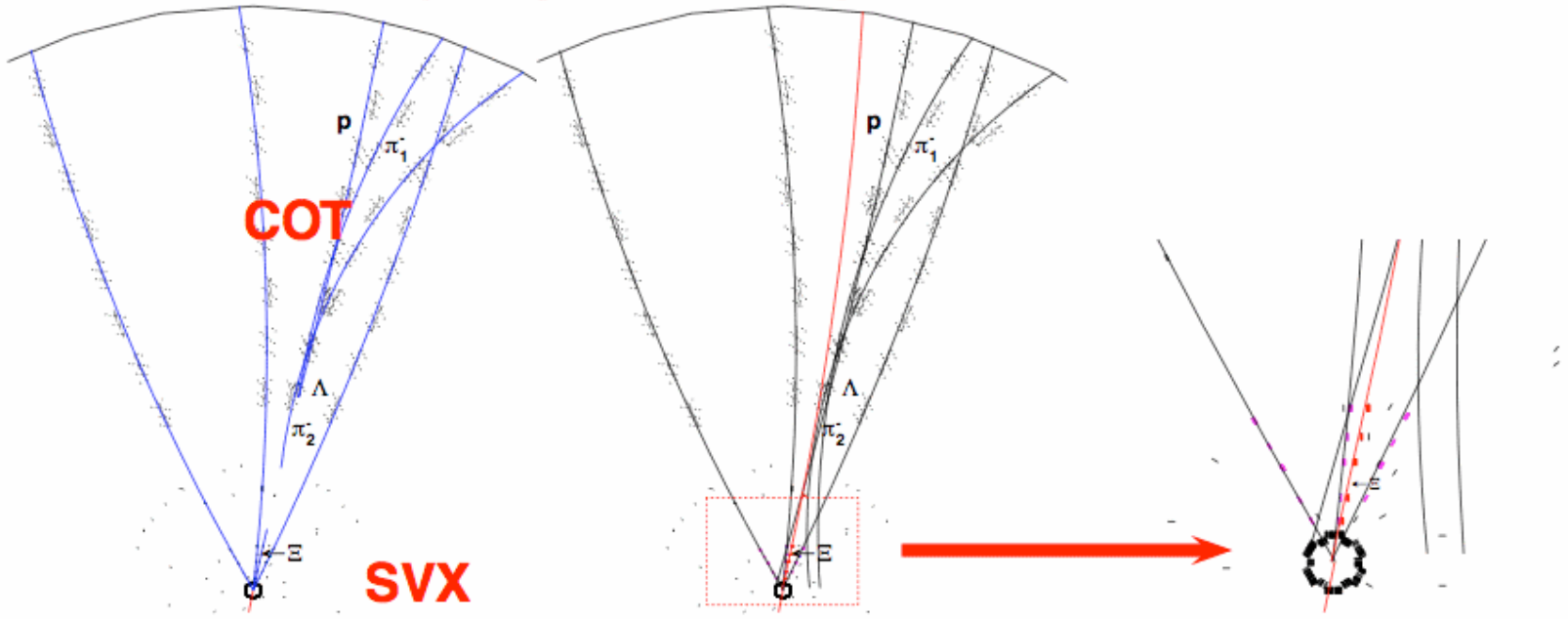
- Can in fact merge this into pattern recognition.

Muon seeded Kalman Filter



Cascade Tracked!

Event Display of generated Hyperons Tracked in Silicon



$$\Xi_b \to \Xi_c^+ \pi^- \quad \Xi_c^+ \to \Xi^- \pi^+ \pi^+ \quad \Xi^- \to \Lambda \pi_2^- \quad \Lambda \to p \pi_1^-$$

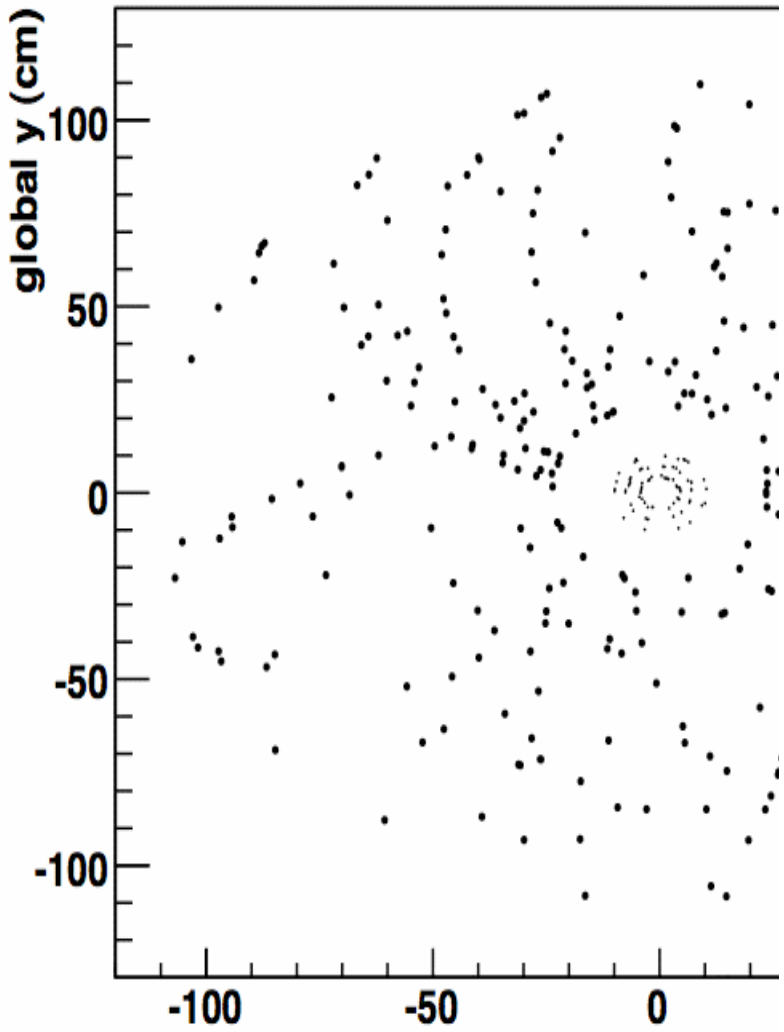
- Reduce random background - clean Ξ samples
- Improvement in Ξ impact parameter resolution

Possible Algorithms

Seed from other measurement, e.g., electron or muon

Seed from subset of hits.

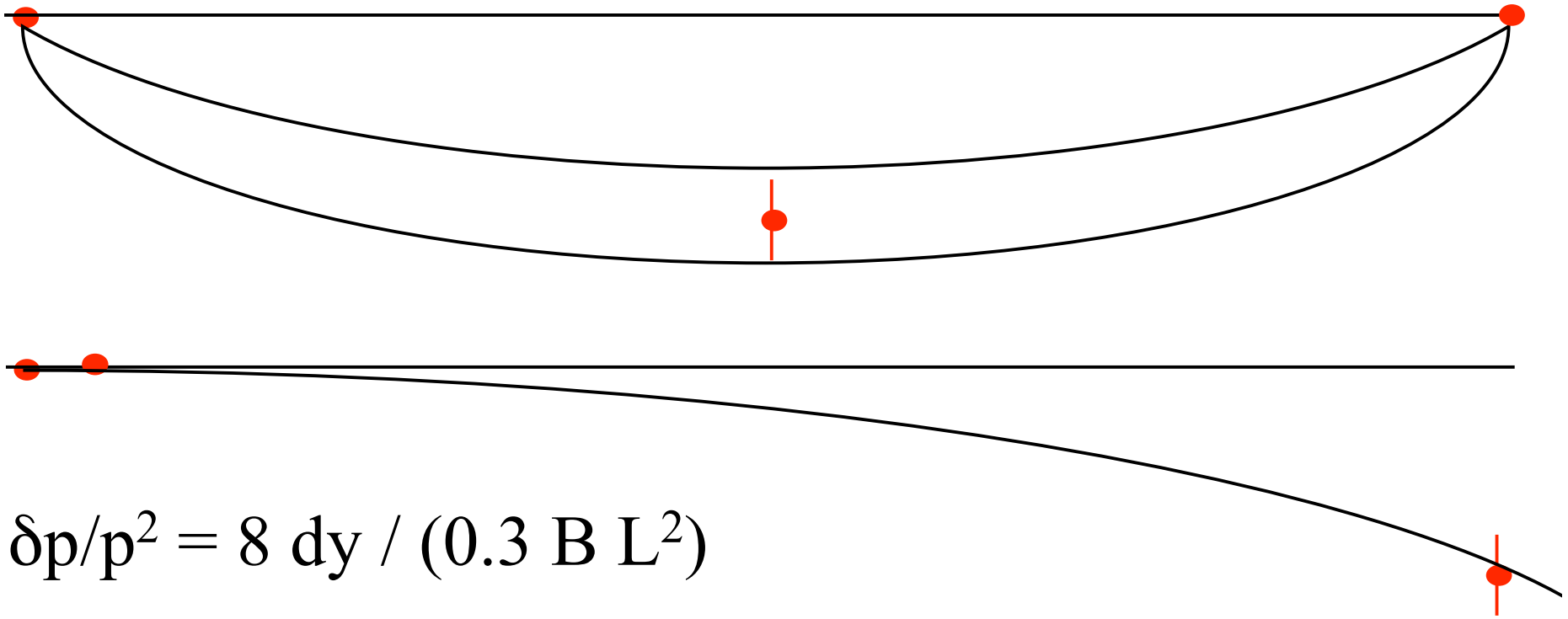
- Outer hits?
- Inner hits?
- Combinatorics of all hits
- Remove used hits



Track parameter resolution

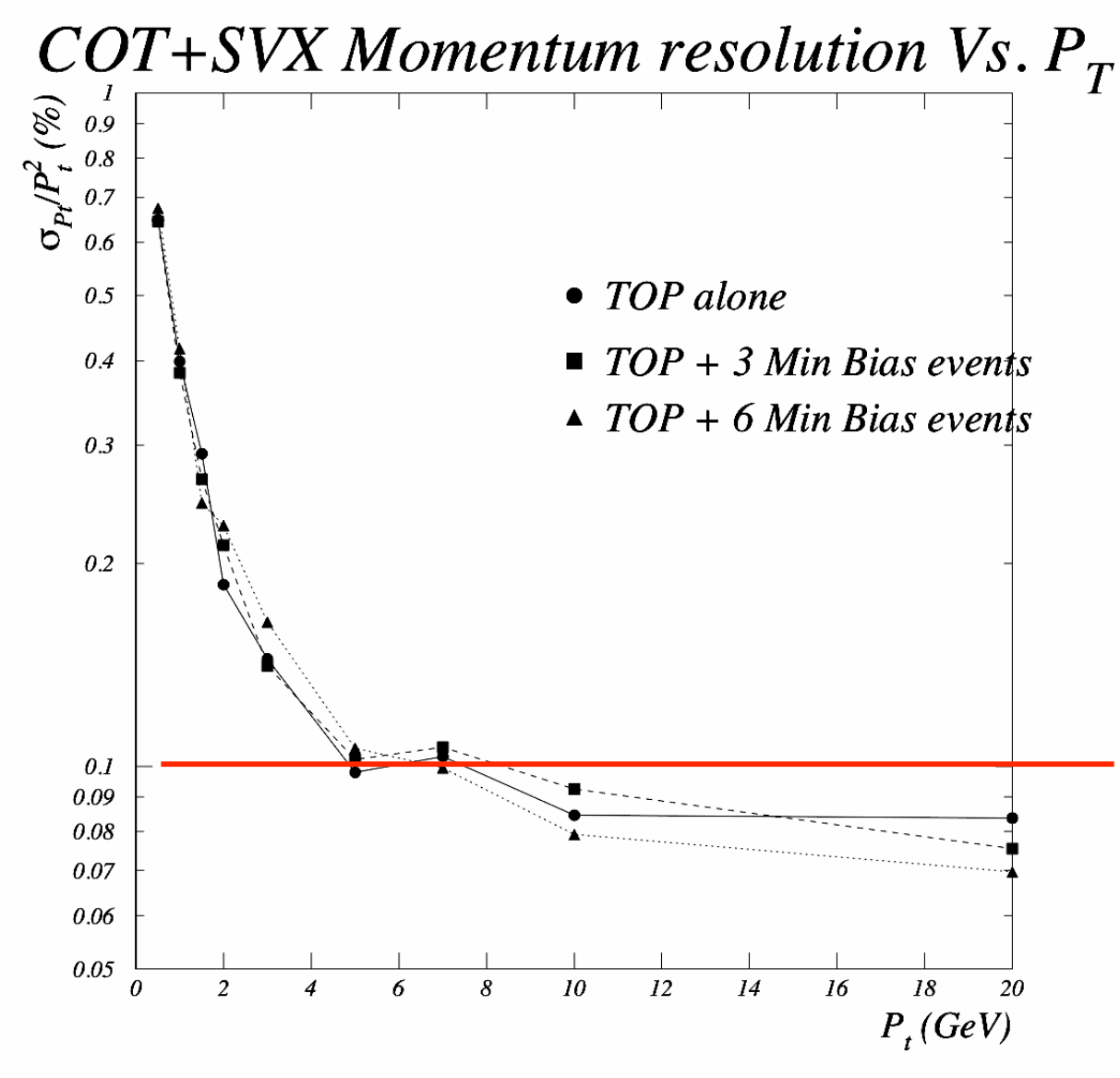
Momentum resolution ⇔ curvature resolution

Measuring the sagitta is driven by lever arm.



$\delta p/p^2 = 8\ dy\ /\ (0.3\ B\ L^2)$

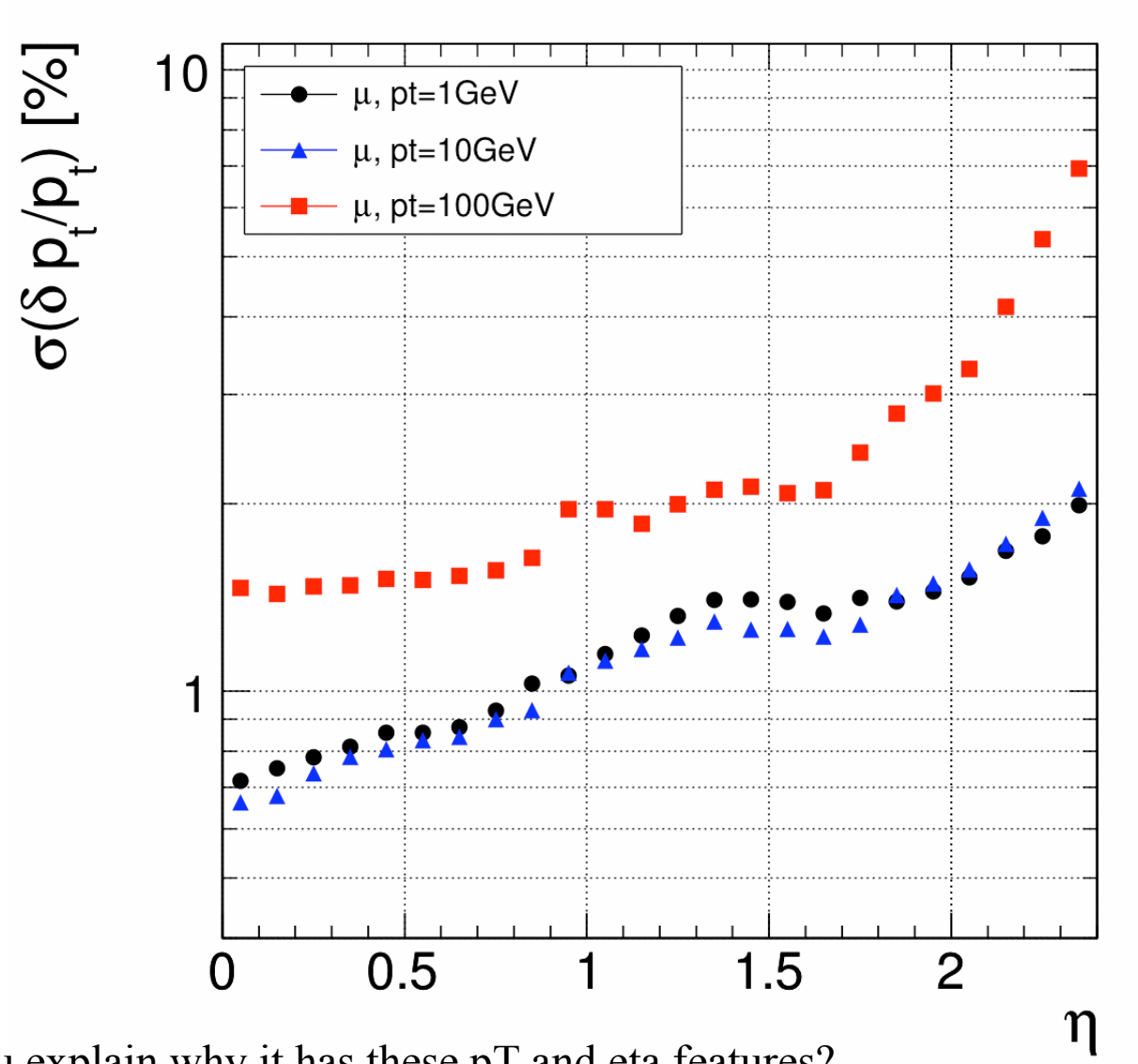
Momentum resolution



0.1% at
1 GeV

CDF TDR

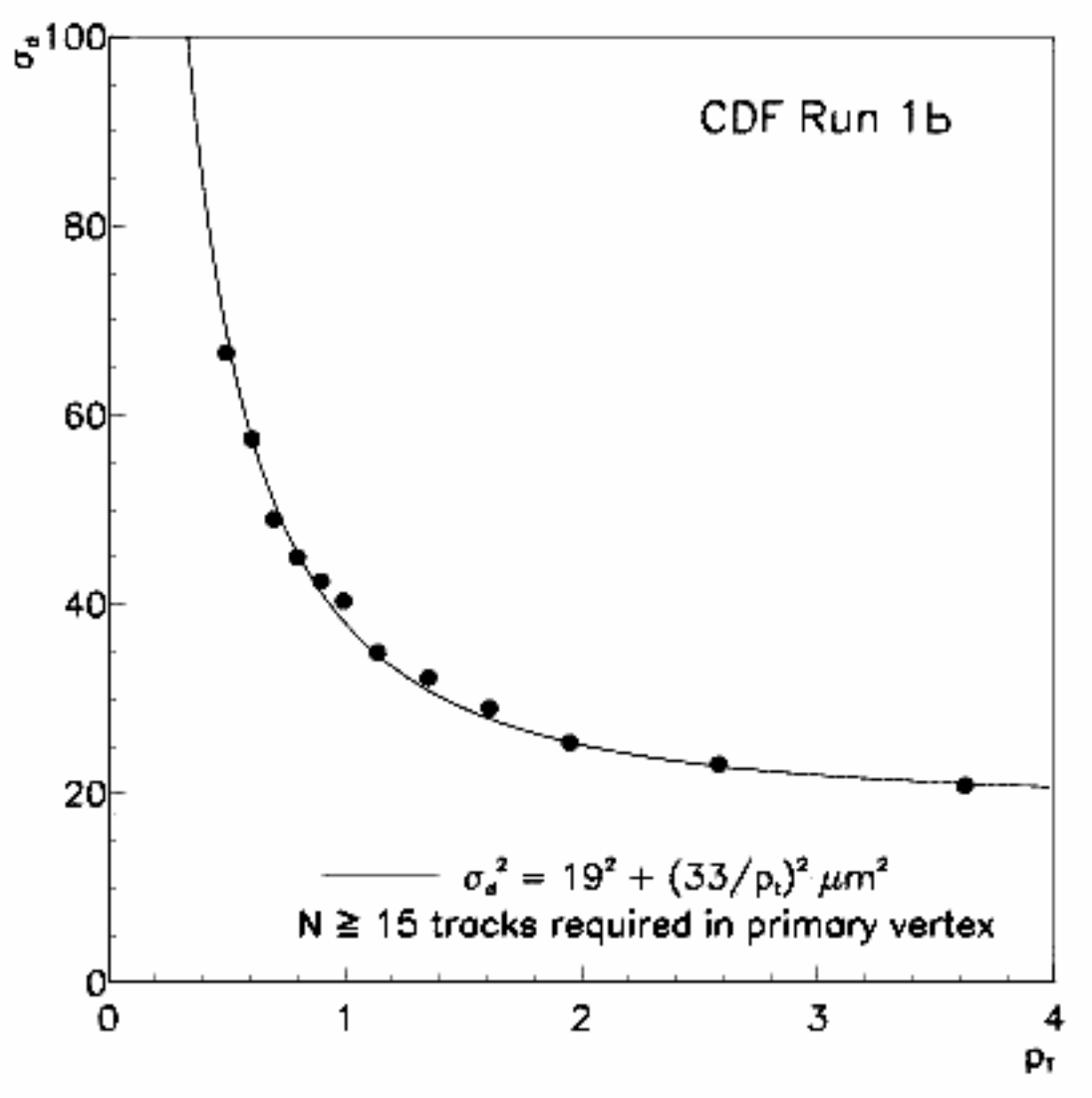
Momentum resolution



Check: Can you explain why it has these pT and eta features?

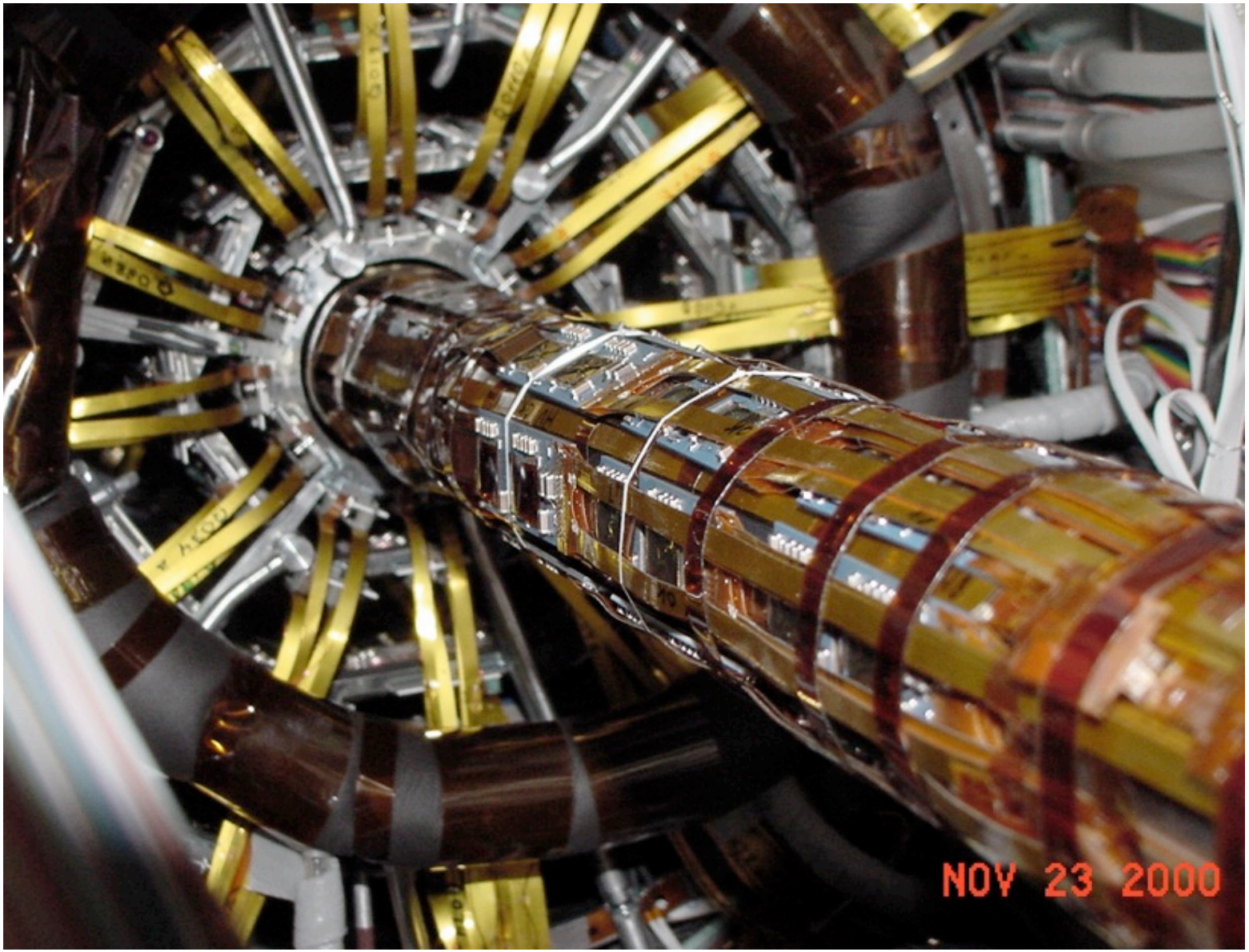
CMS TDR

Impact parameter resolution



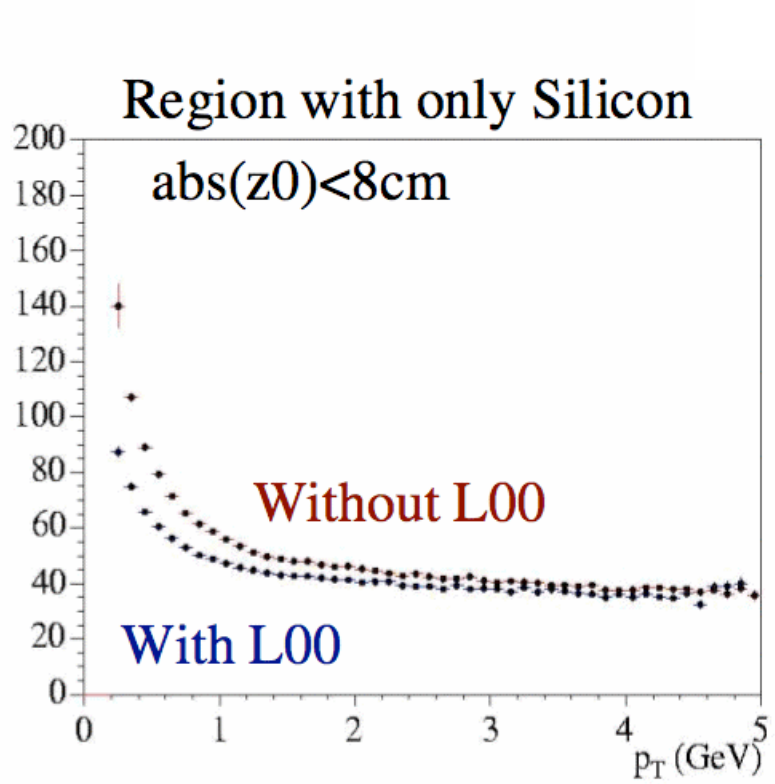
20 μm at high momentum, but MCS dominates at low momentum.

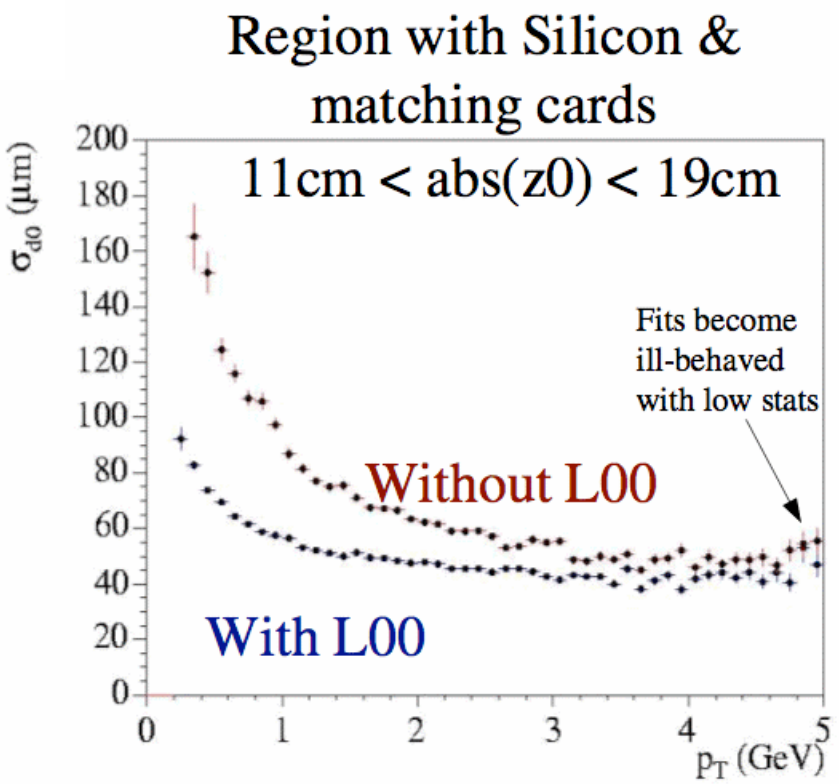
Impact parameter resolution

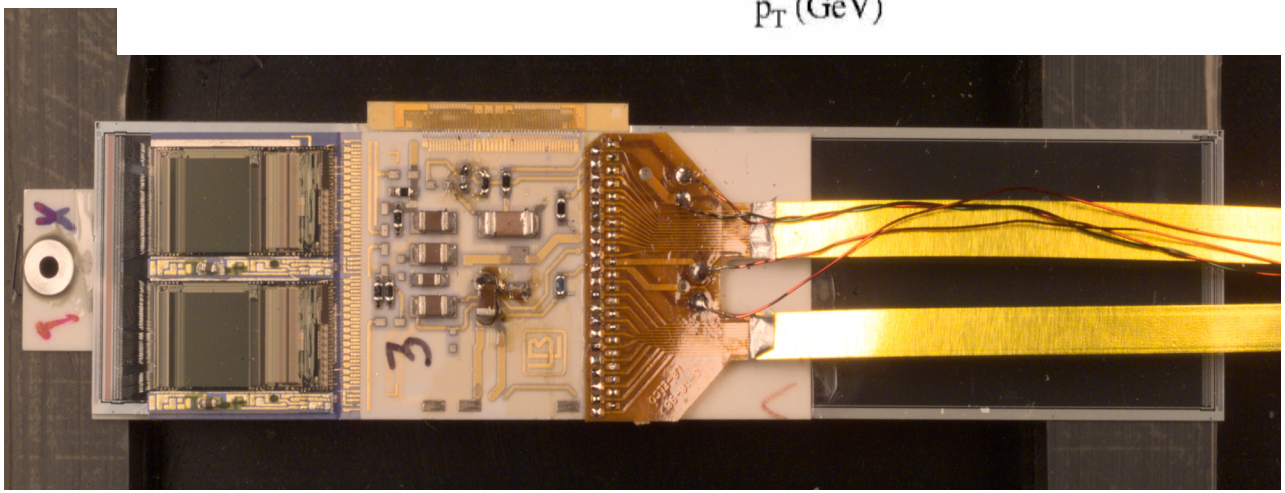


Aim for lowest radius and lowest mass.

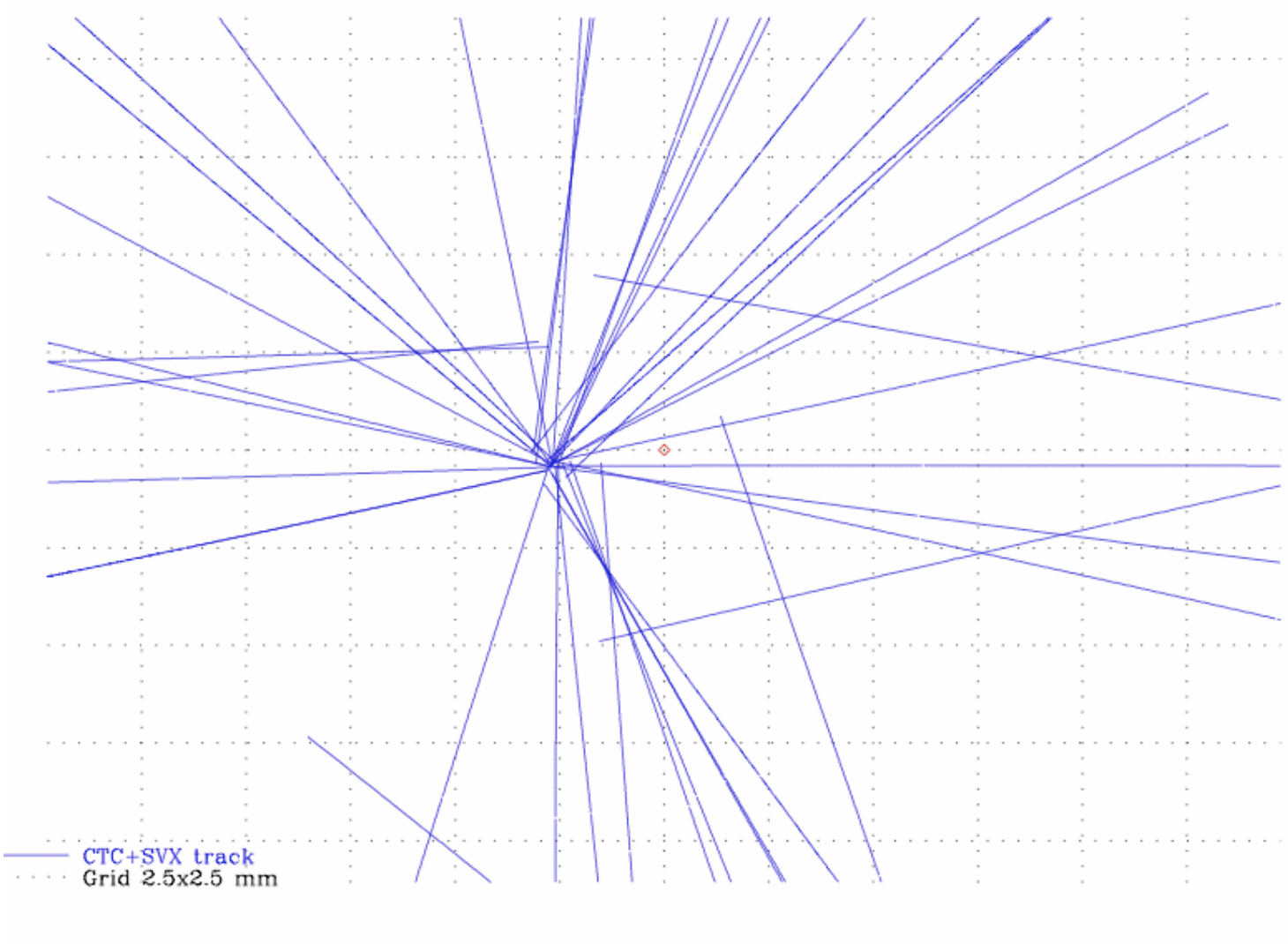
Impact parameter resolution





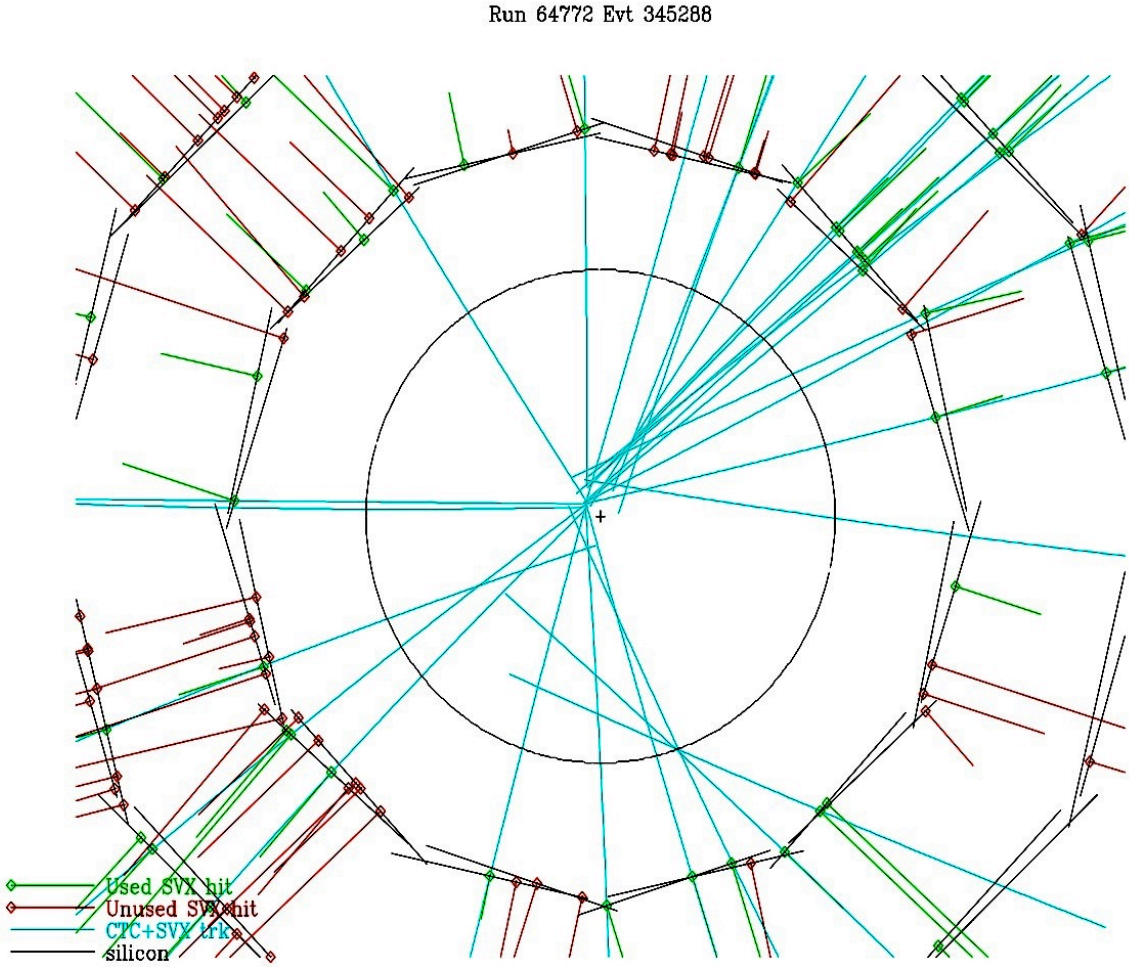


Vertexing



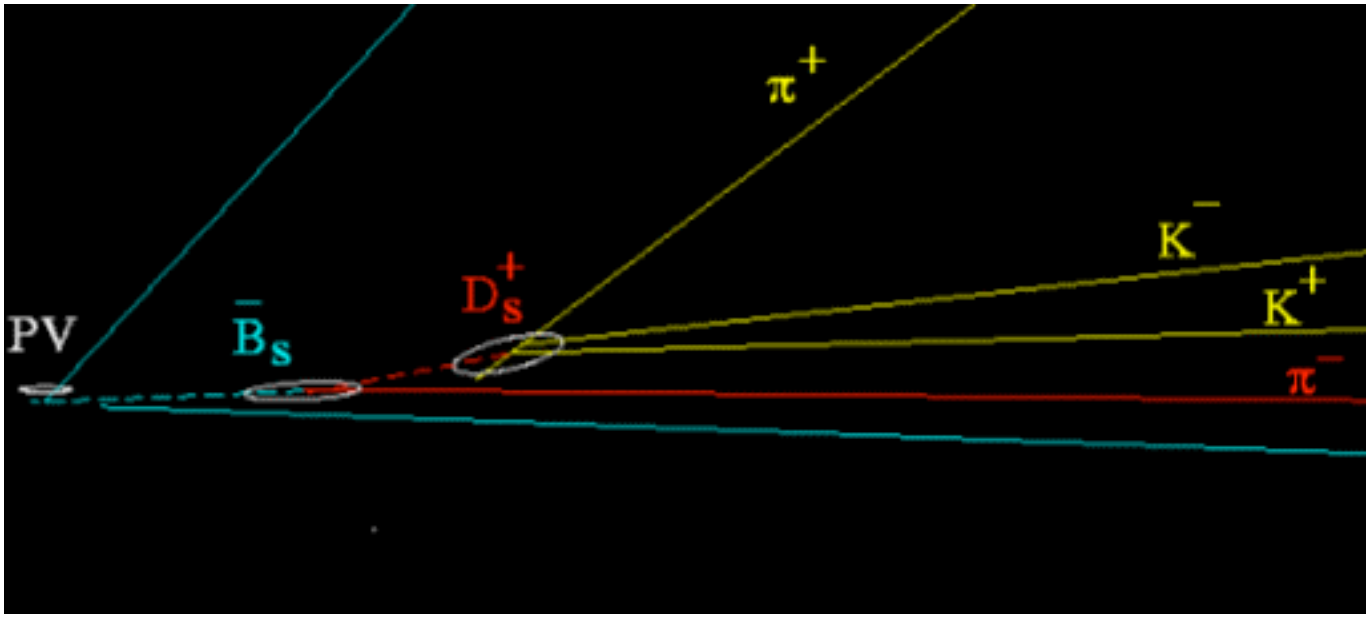
Find the vertex…

Vertexing



Find the vertex…

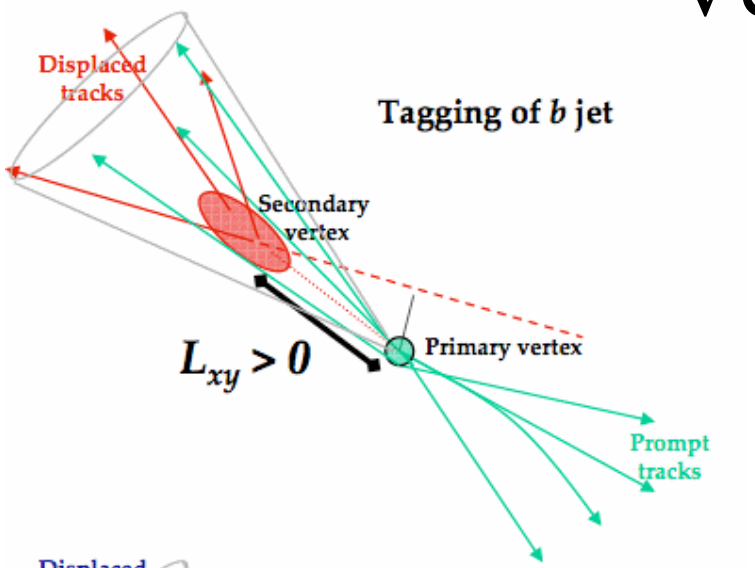
Vertexing

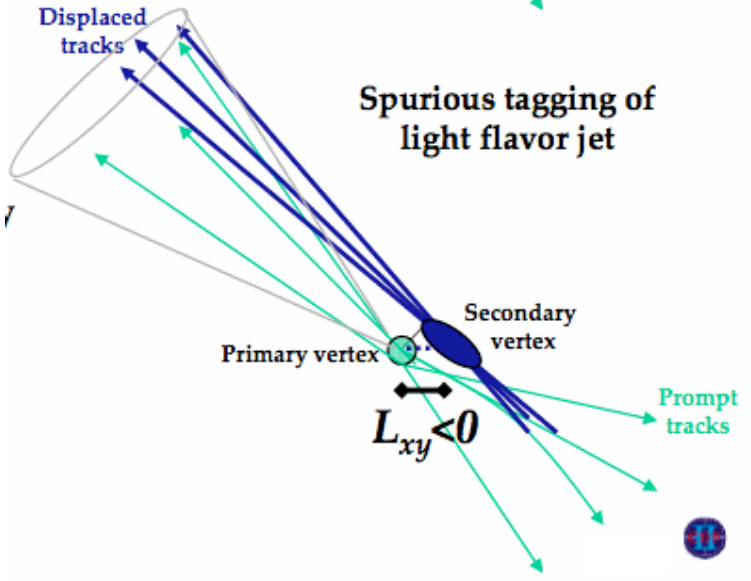


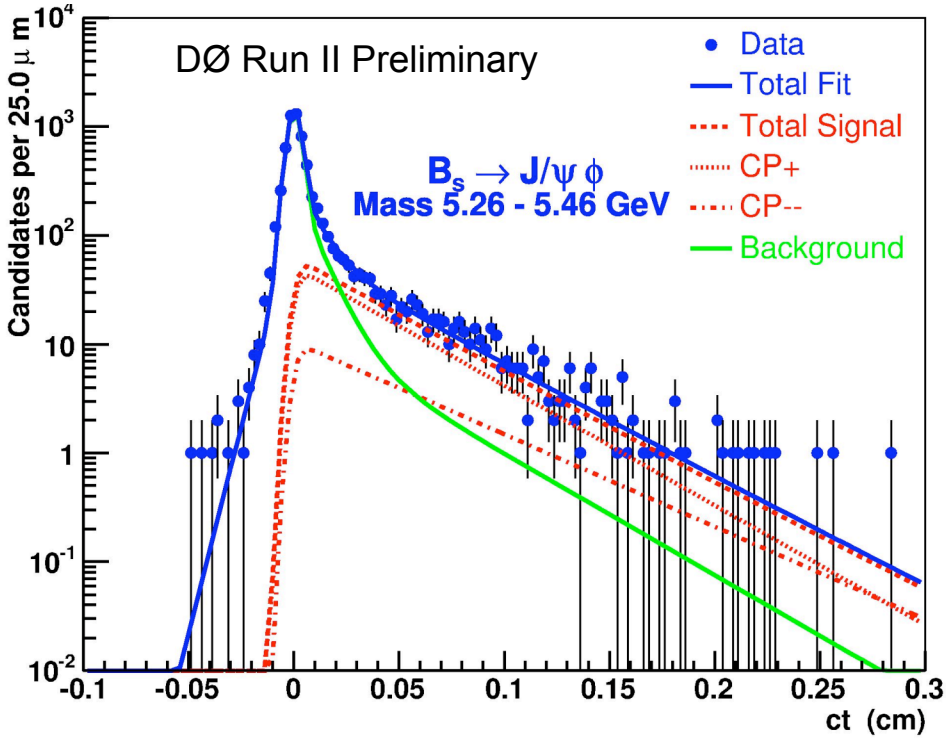
Aleph

Find the vertices…

Vertexing





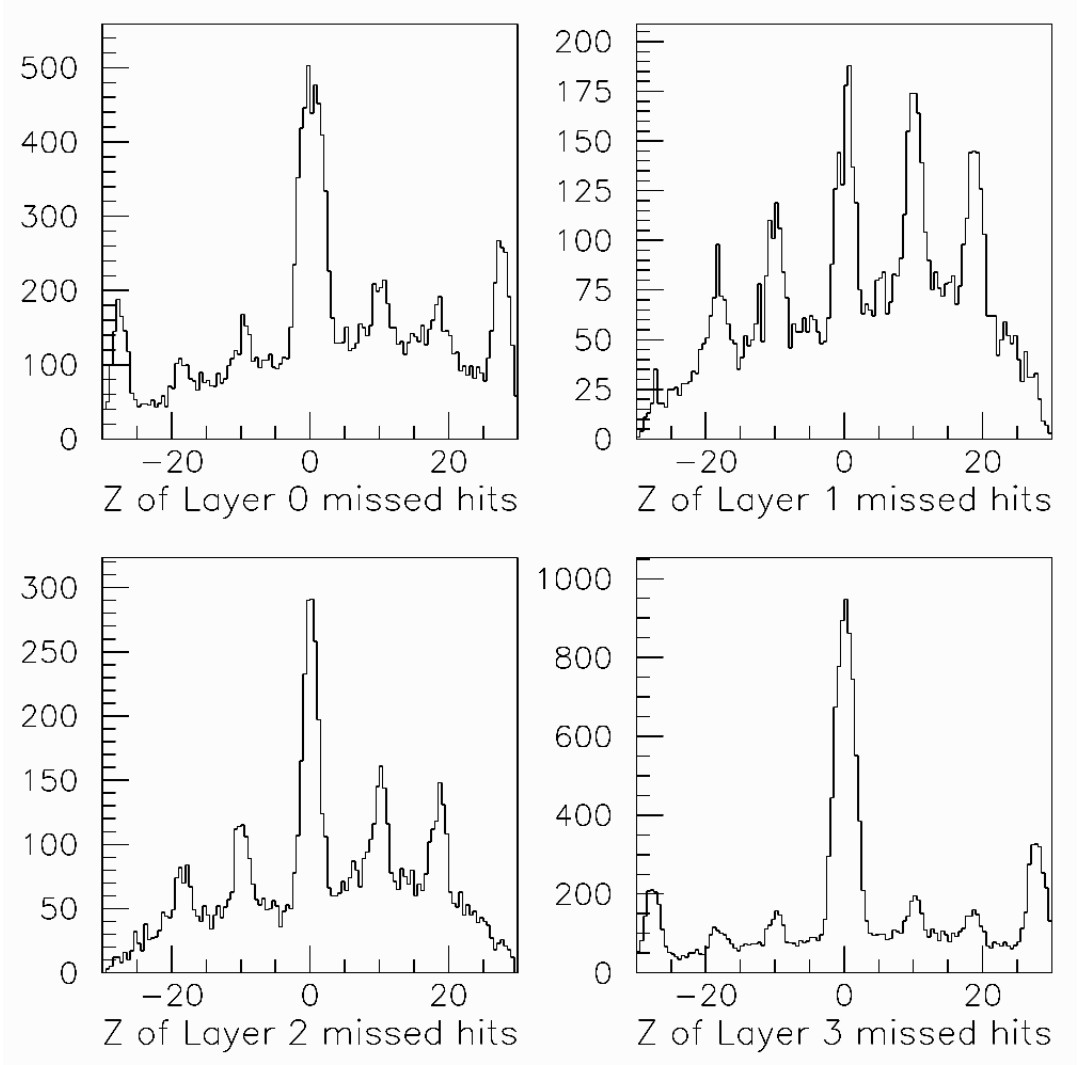


Find the vertices…

Track quality

Confirming track quality is important because one incorrect hit can have a huge effect!

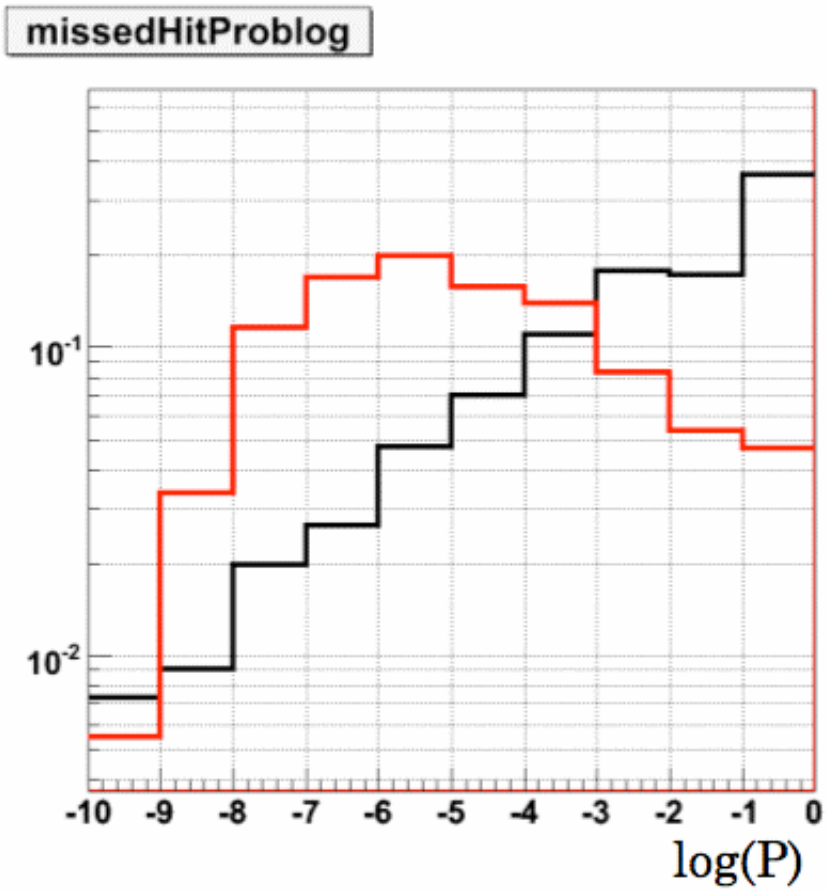
- A missed hit is as much a measurement as is a found hit.
- Charge is predictable

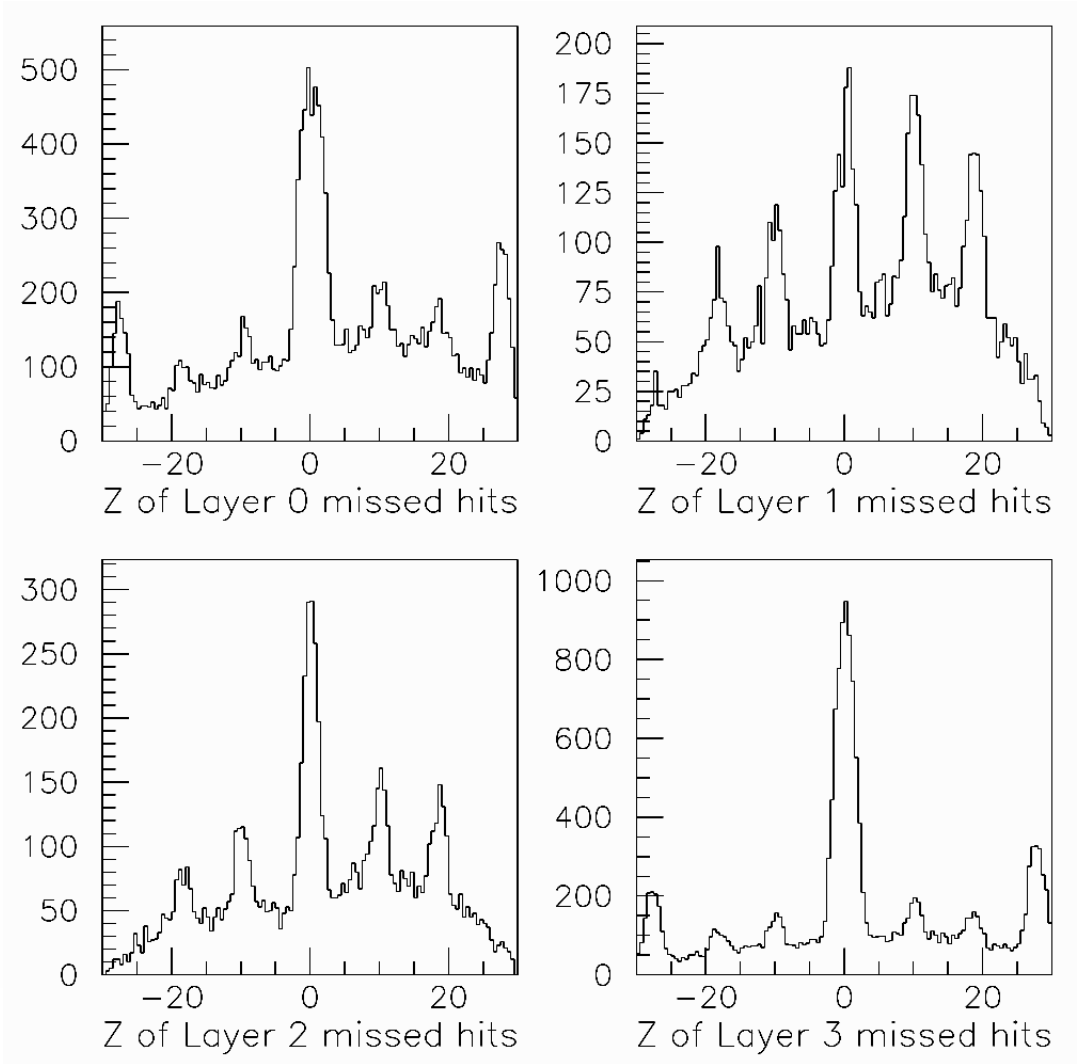


Track quality

Confirming track quality is important because one incorrect hit can have a huge effect!

- A missed hit is as much a measurement as is a found hit.
- Charge is predictable

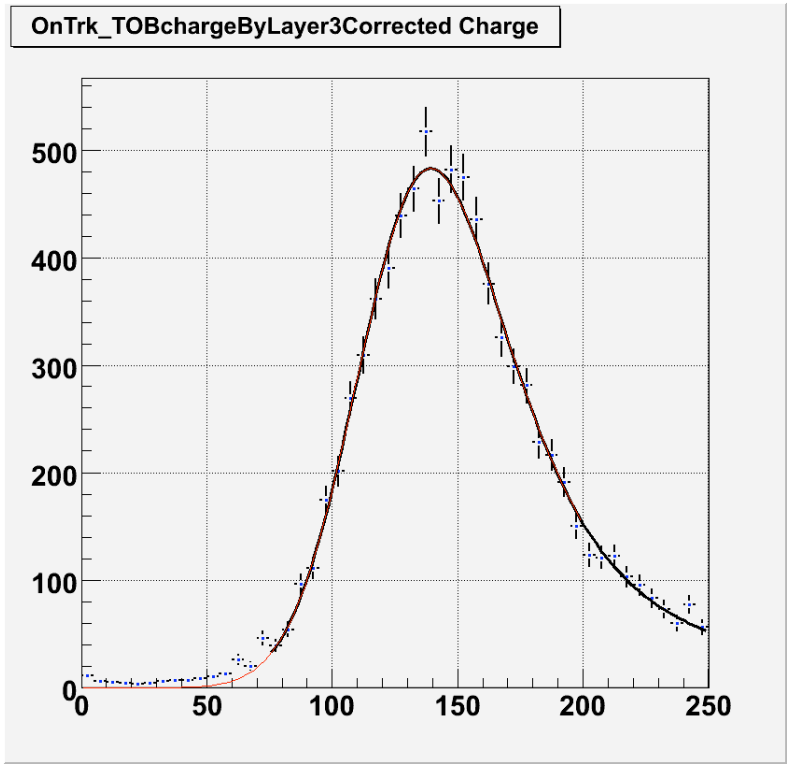


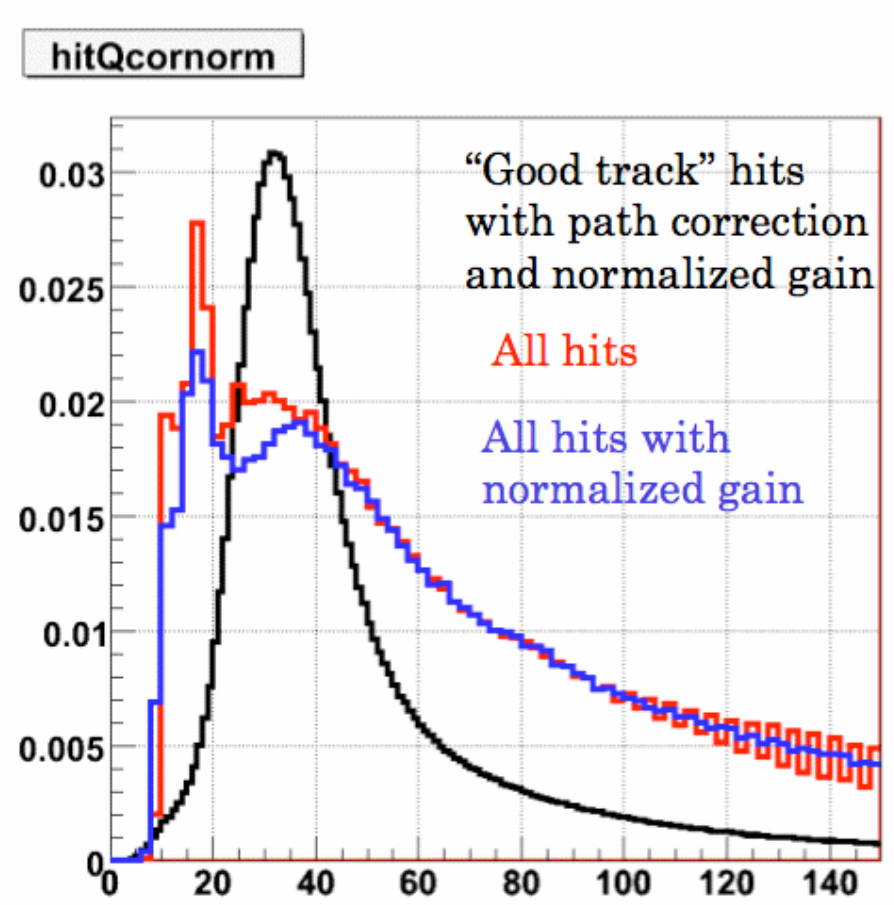


Track quality

Confirming track quality is important because one incorrect hit can have a huge effect!

- A missed hit is as much a measurement as is a found hit.
- Charge is predictable

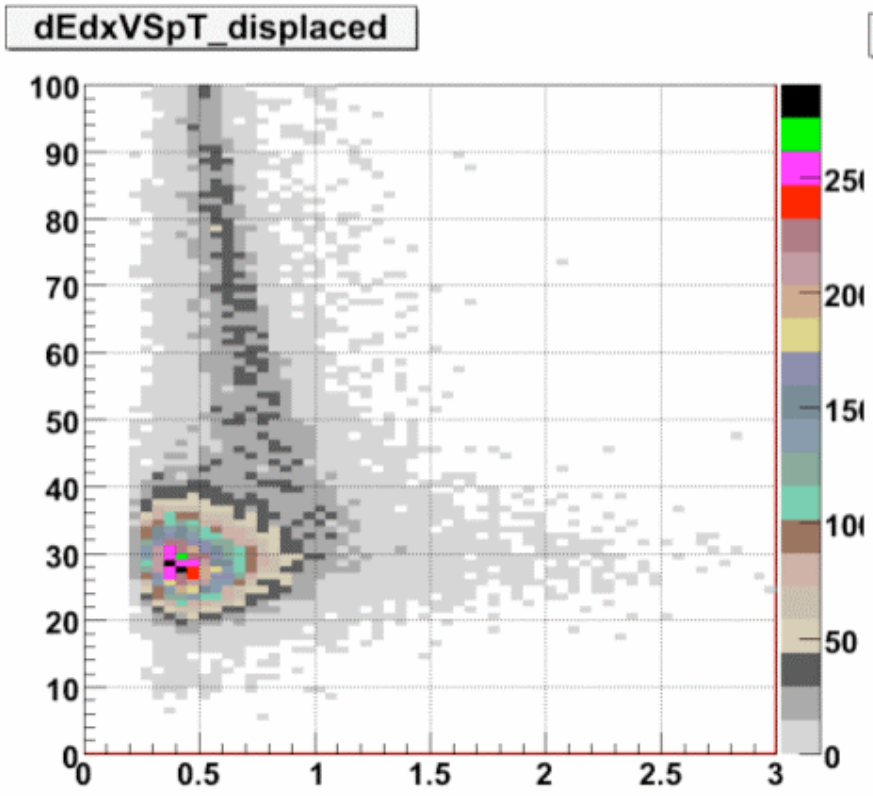


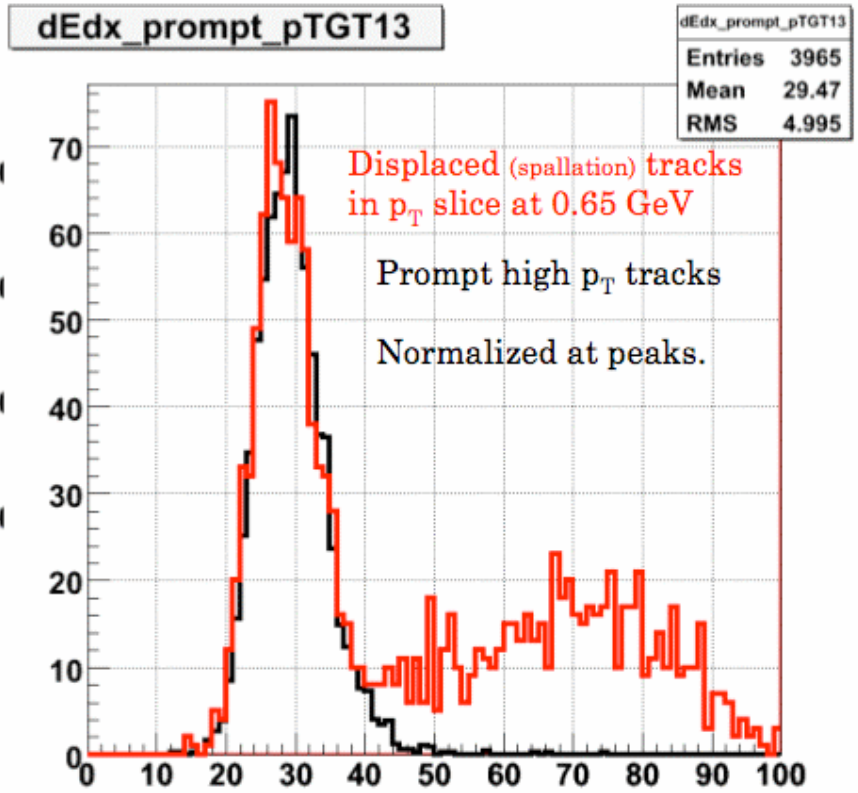


Track quality

Confirming track quality is important because one incorrect hit can have a huge effect!

- A missed hit is as much a measurement as is a found hit.
- Charge is predictable

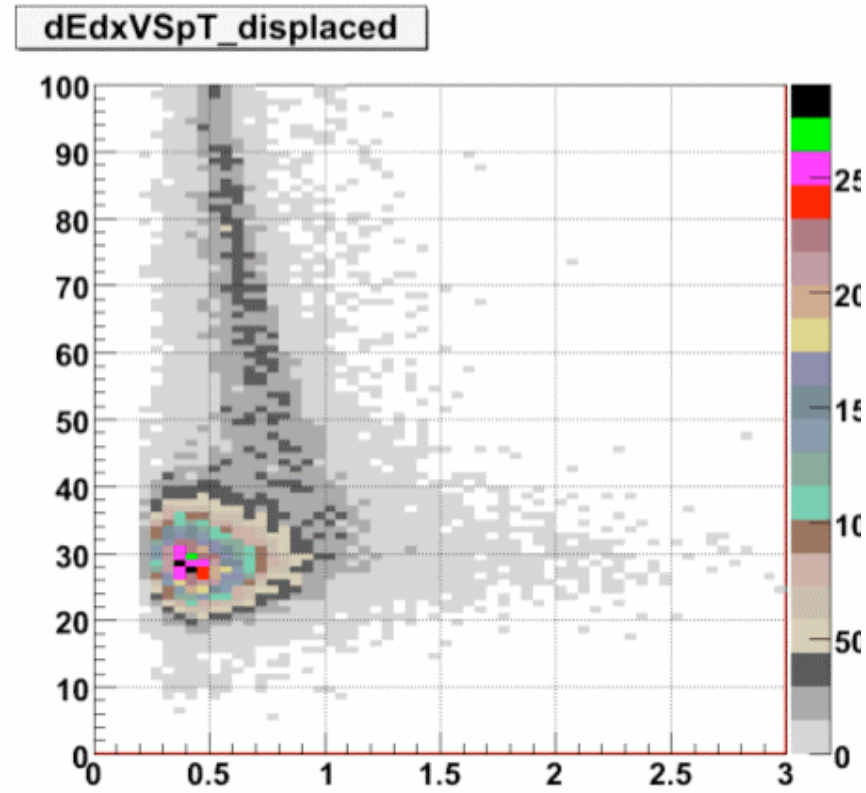


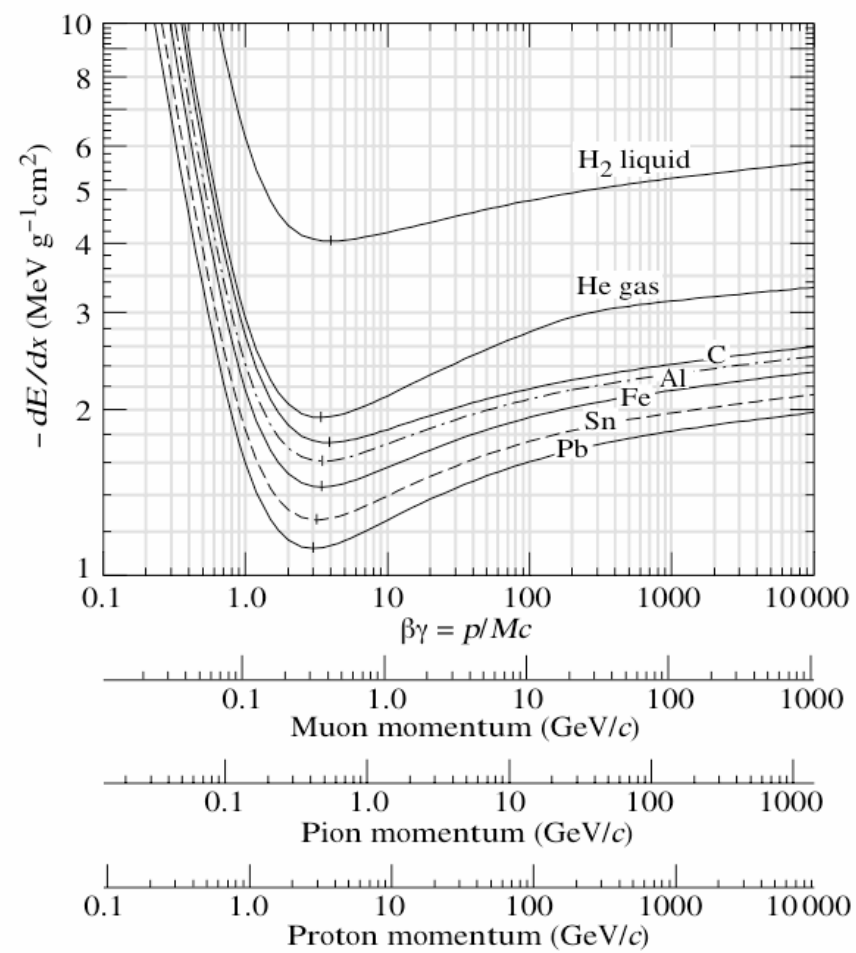


Track quality

Confirming track quality is important because one incorrect hit can have a huge effect!

- A missed hit is as much a measurement as is a found hit.
- Charge is predictable

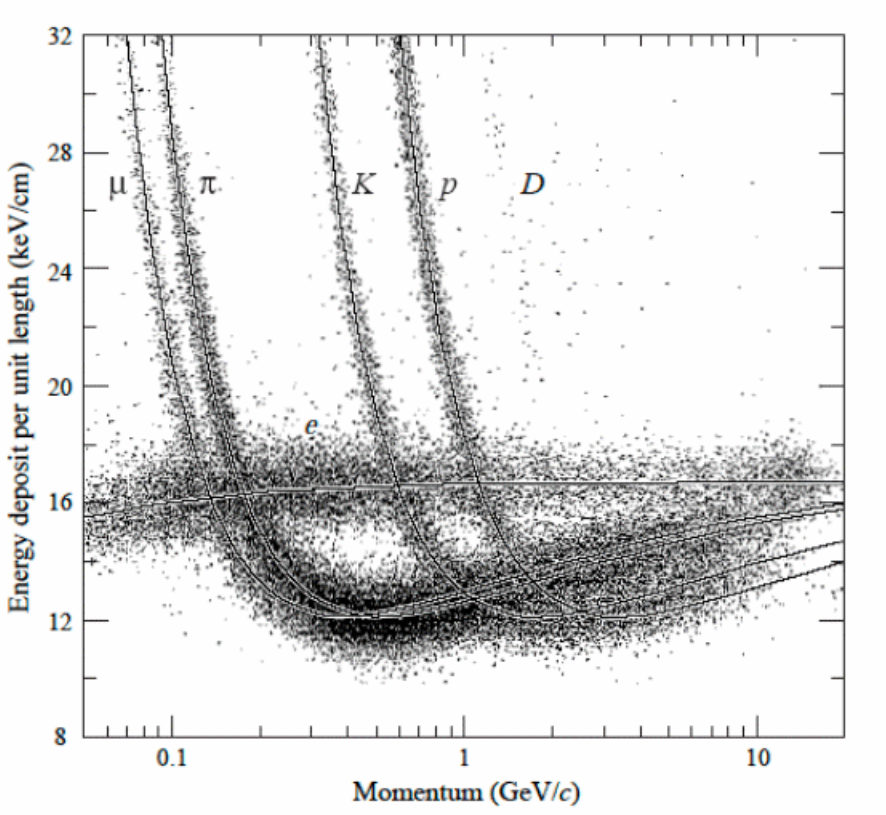




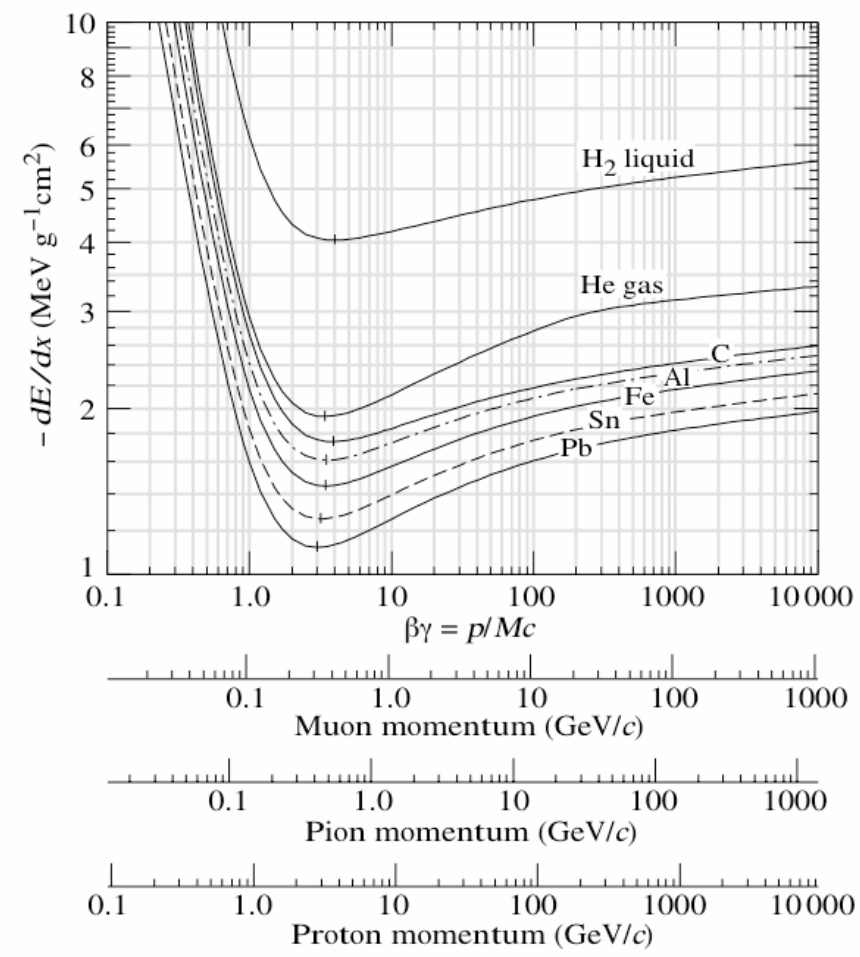
Track quality

Confirming track quality is important because one incorrect hit can have a huge effect!

- A missed hit is as much a measurement as is a found hit.
- Charge is predictable

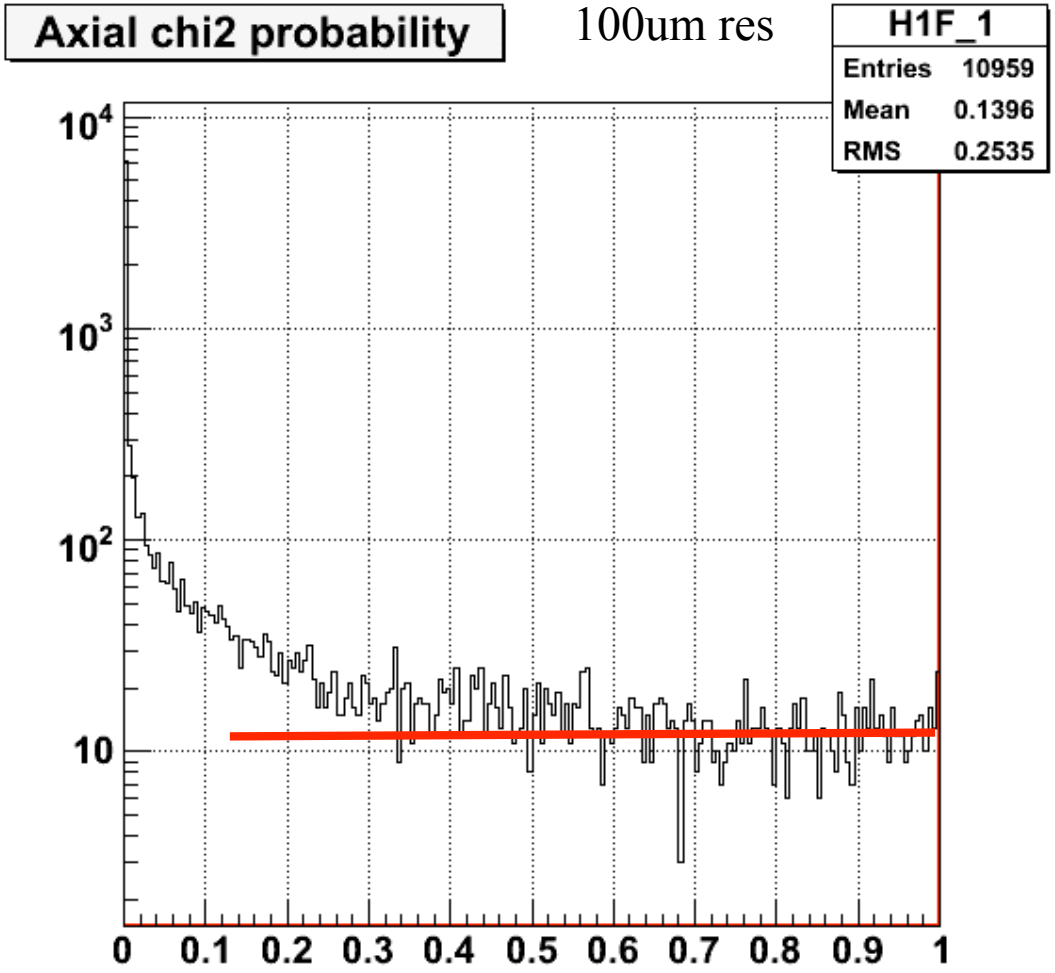


Opal (?)



χ^2 probability

If the errors are calculated correctly, the χ^2 probability should be reasonably flat.



This plot is from cosmics taken at CMS recently without a magnetic field.
Can you explain the very low probability entries?

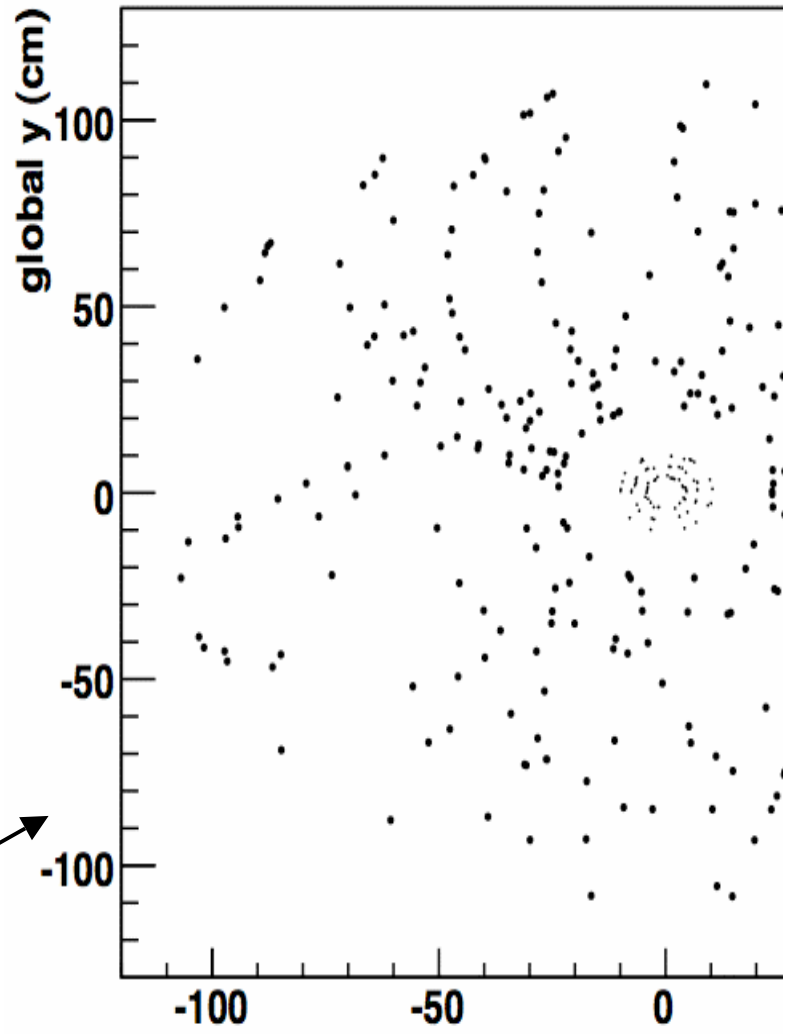
…Possible Algorithms…

Seed from other measurement, e.g., electron or muon

Seed from subset of hits.

- Outer hits?
- Inner hits?
- Combinatorics of all hits
- Remove used hits

Work pretty well for this.



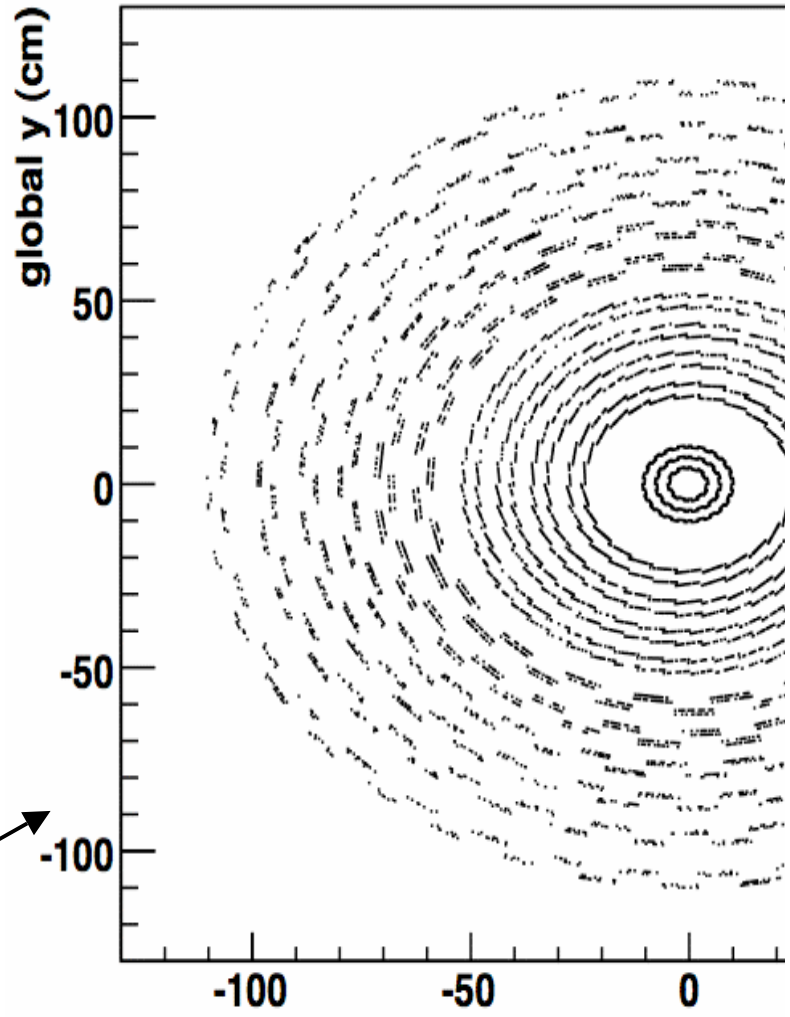
…Possible Algorithms…

Seed from other measurement, e.g., electron or muon

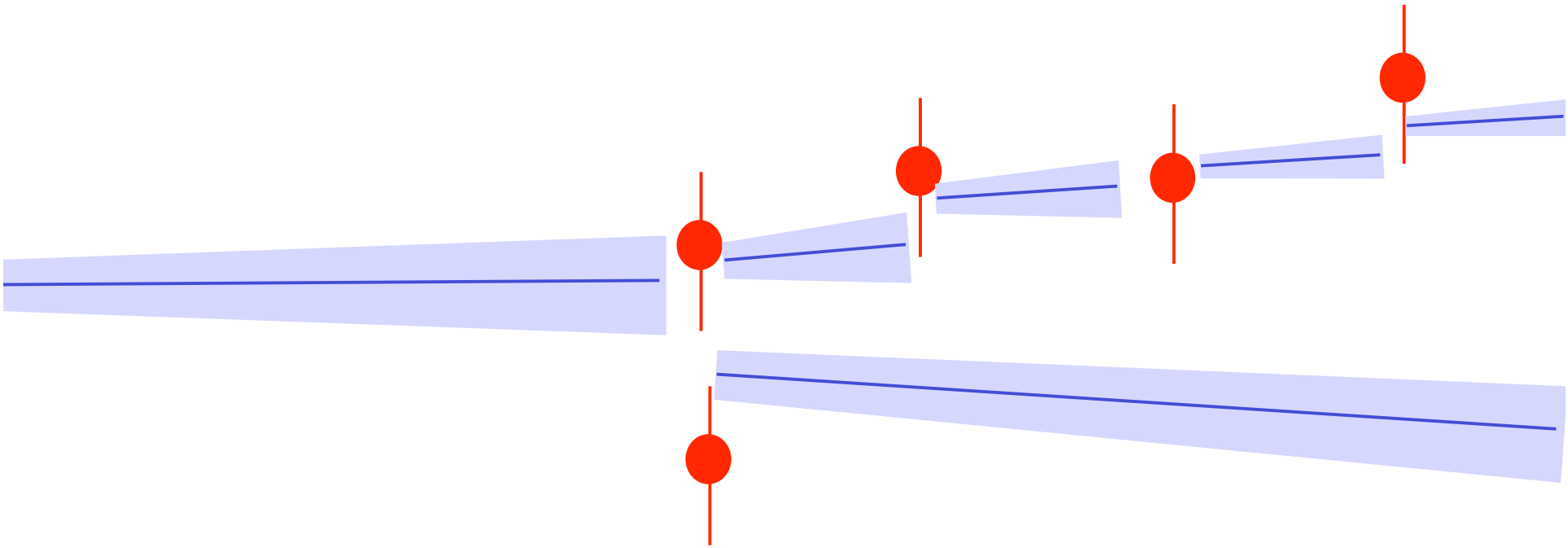
Seed from subset of hits.

- Outer hits?
- Inner hits?
- Combinatorics of all hits
- Remove used hits

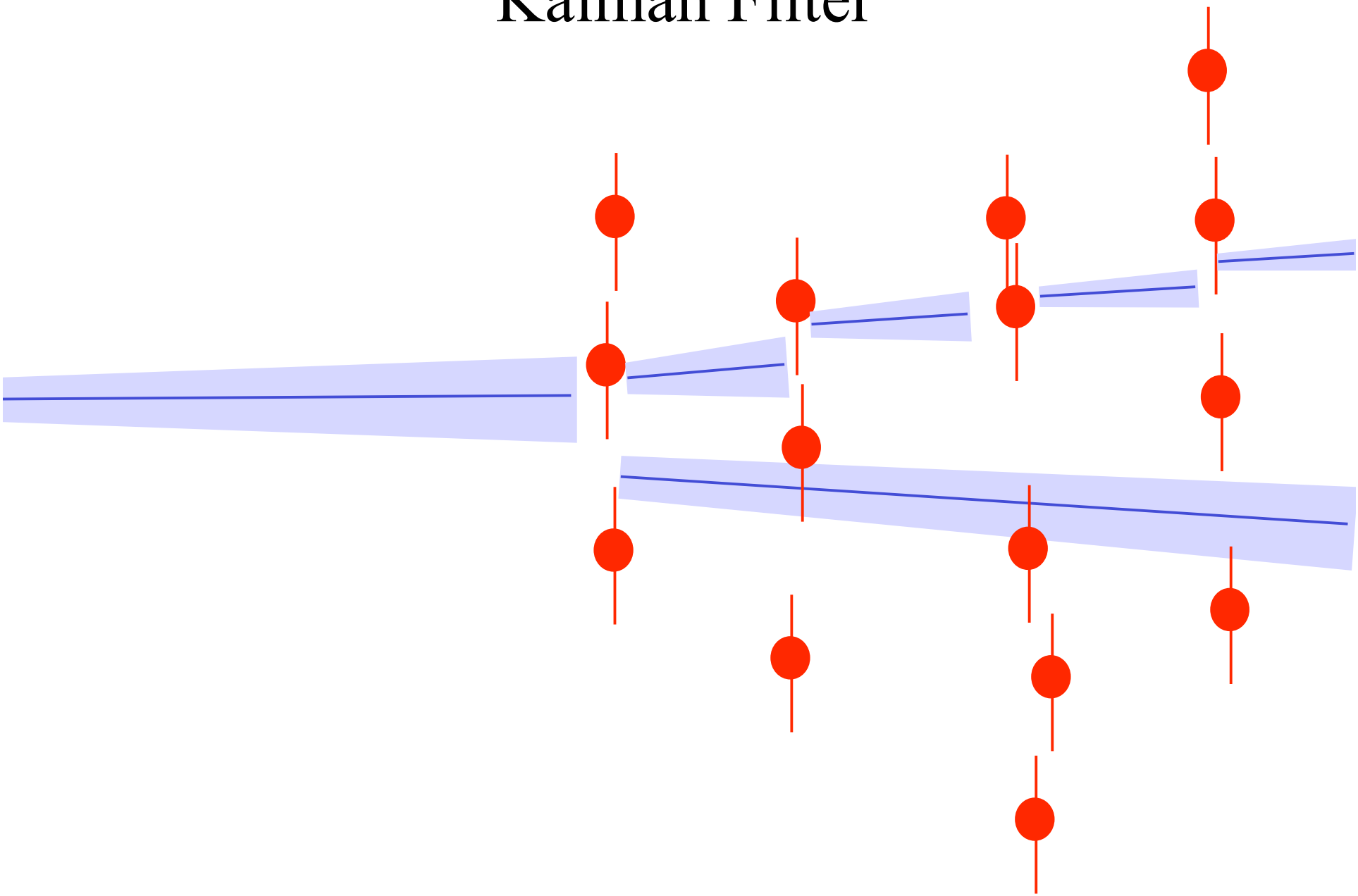
Challenging for this.



Kalman Filter

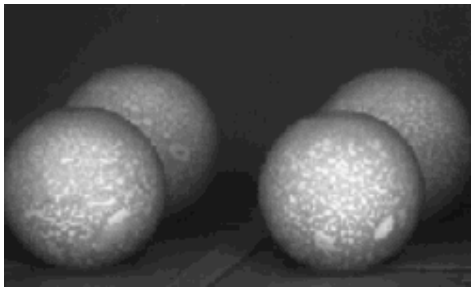


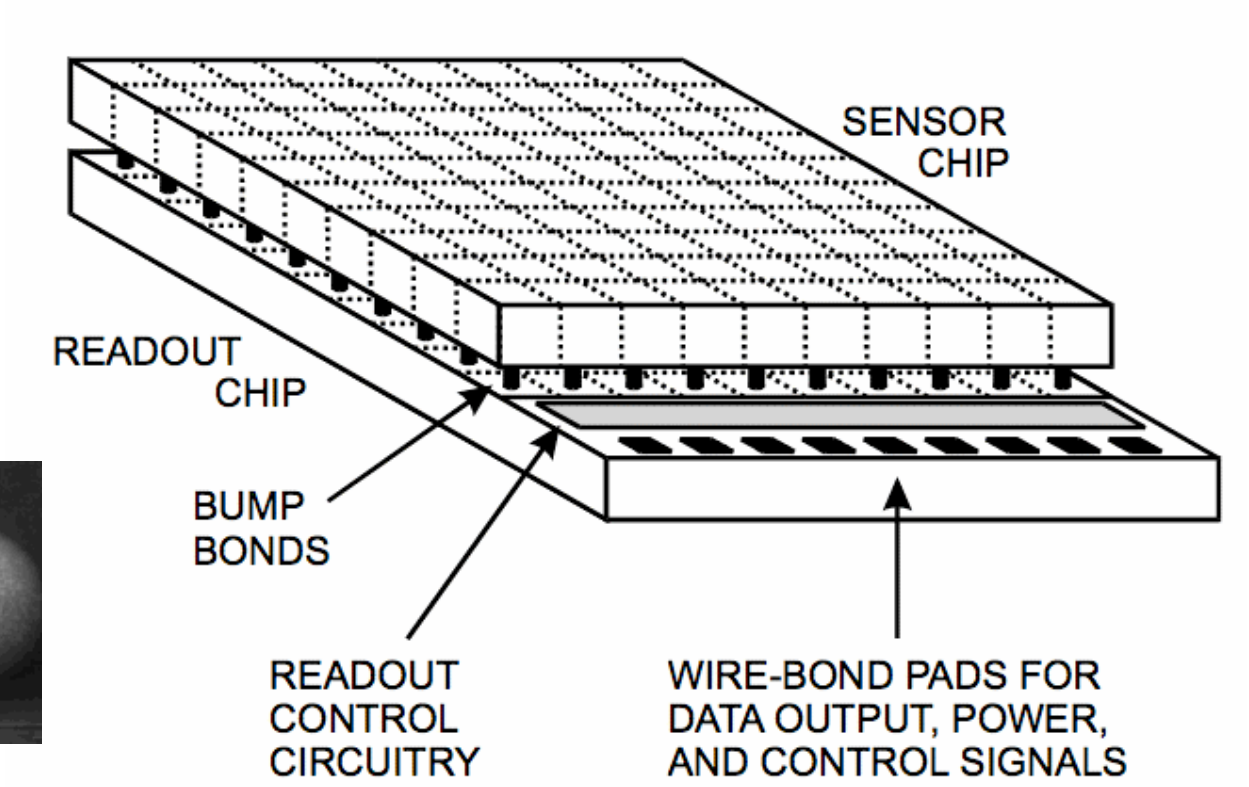
Kalman Filter



3D measurements: Pixels

Two precise and unambiguous measurements in the same sensor!

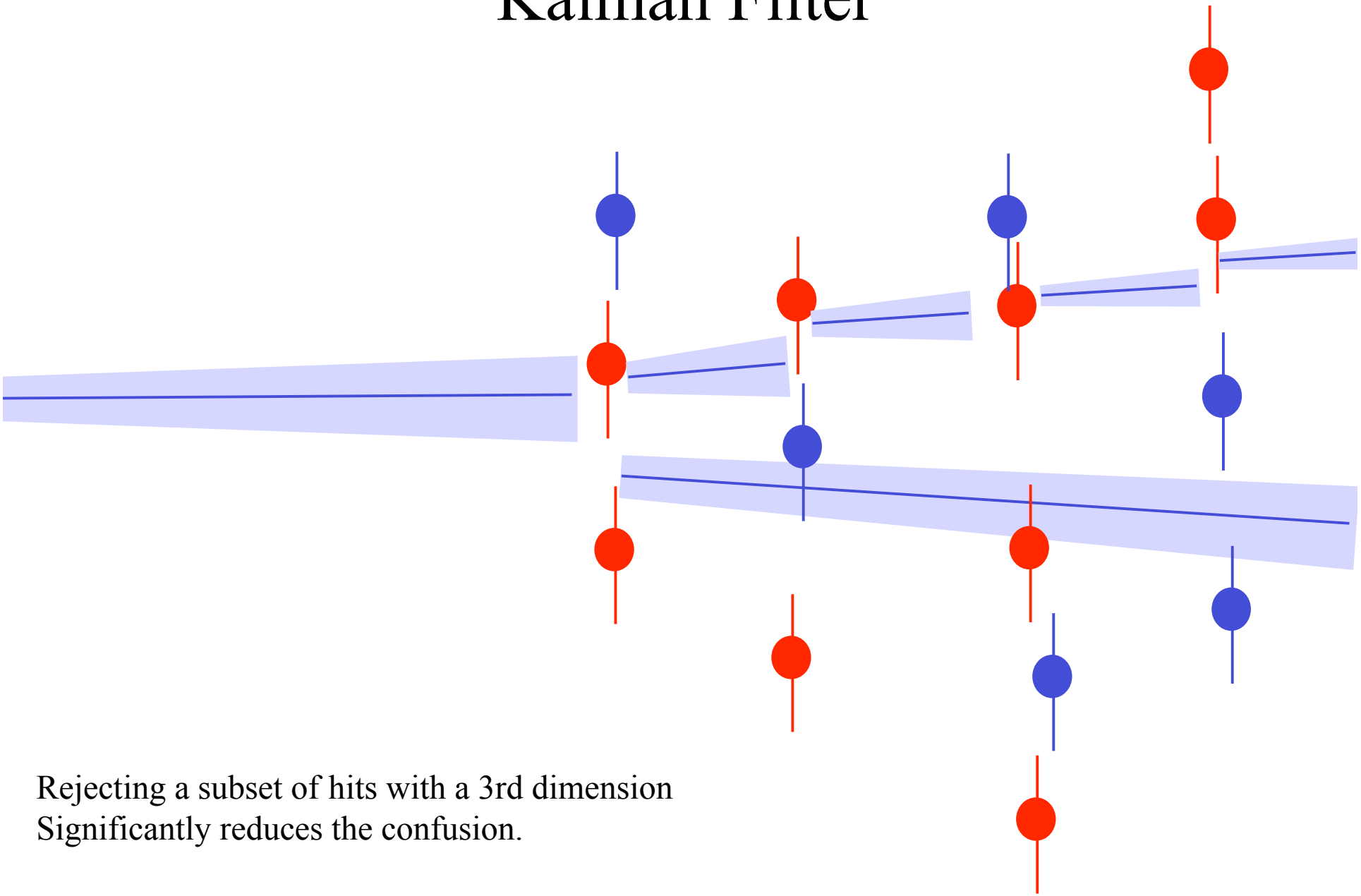




But, motivated more by granularity (for robust track finding) than by resolution…

Helmut Spieler, “Semiconductor Detector Systems”

Kalman Filter

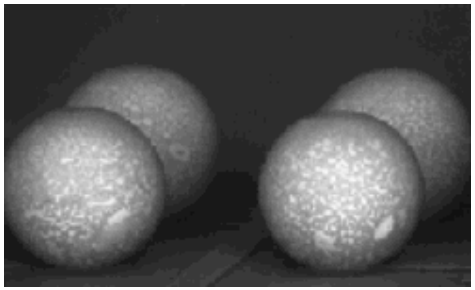


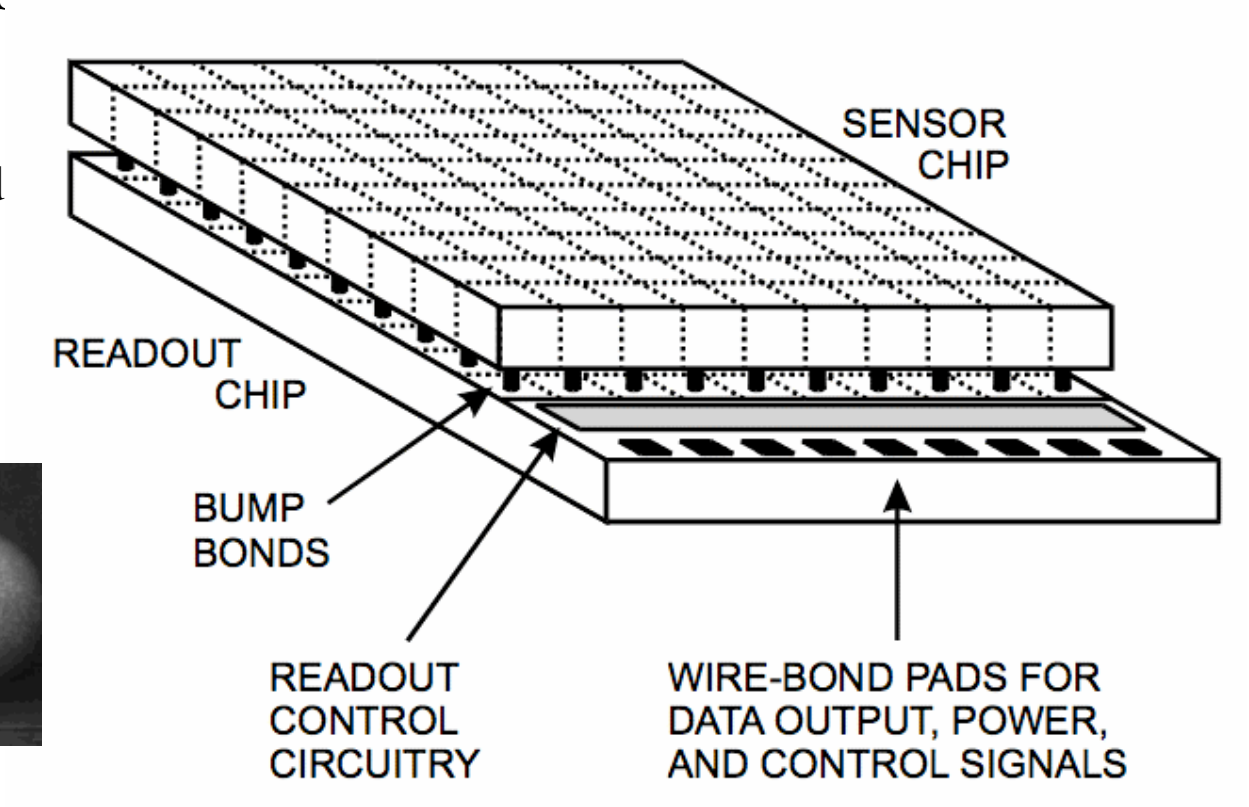
Rejecting a subset of hits with a 3rd dimension
Significantly reduces the confusion.

And pixels can give seeds that are precise in 3 dimensions.

Pixel challenges

The many connection challenge is squared.
Internal electronics
Cooling
Material
Mechanically hard





Helmut Spieler, “Semiconductor Detector Systems”

Pixel challenges

The many connection challenge is squared.

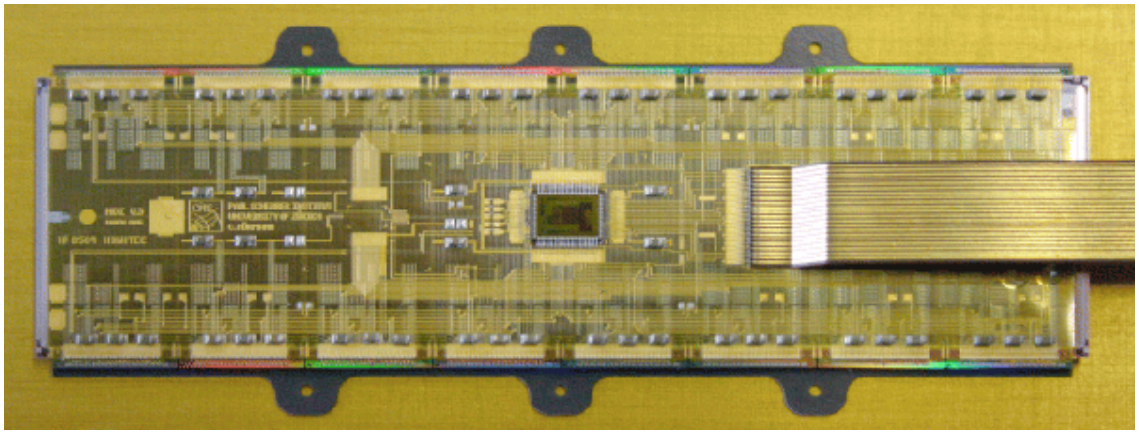
Internal electronics

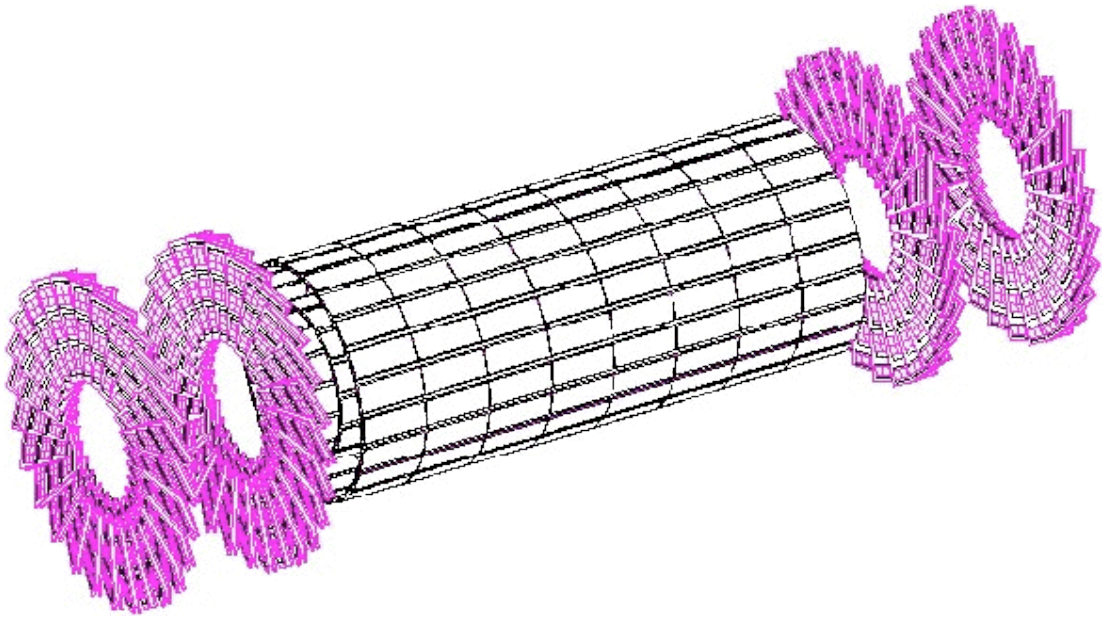
Cooling

Material

Mechanically hard

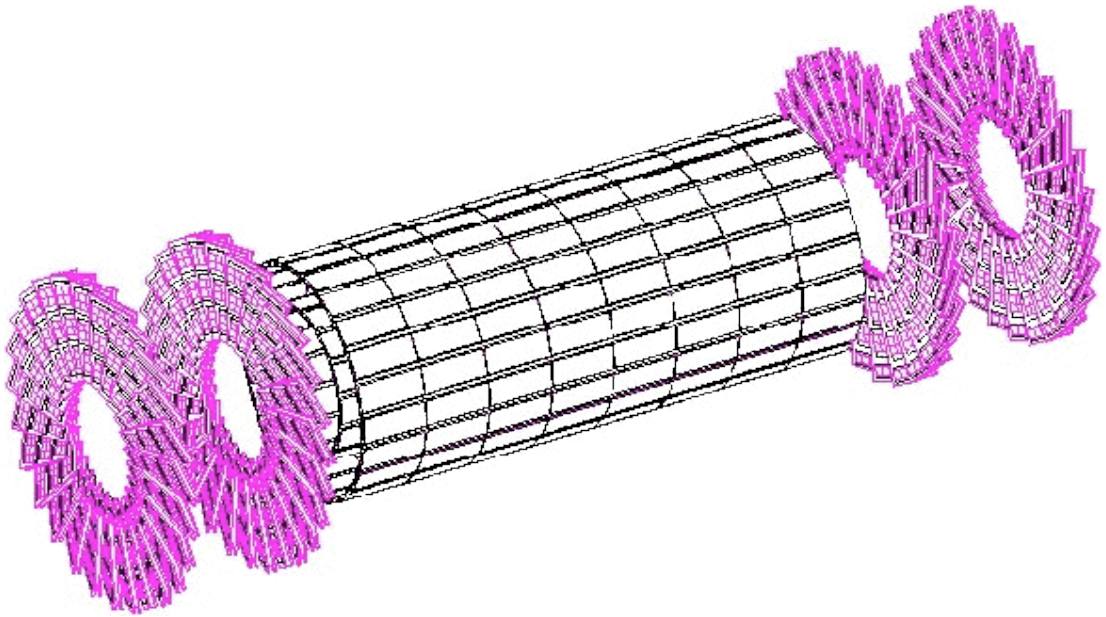
Complex

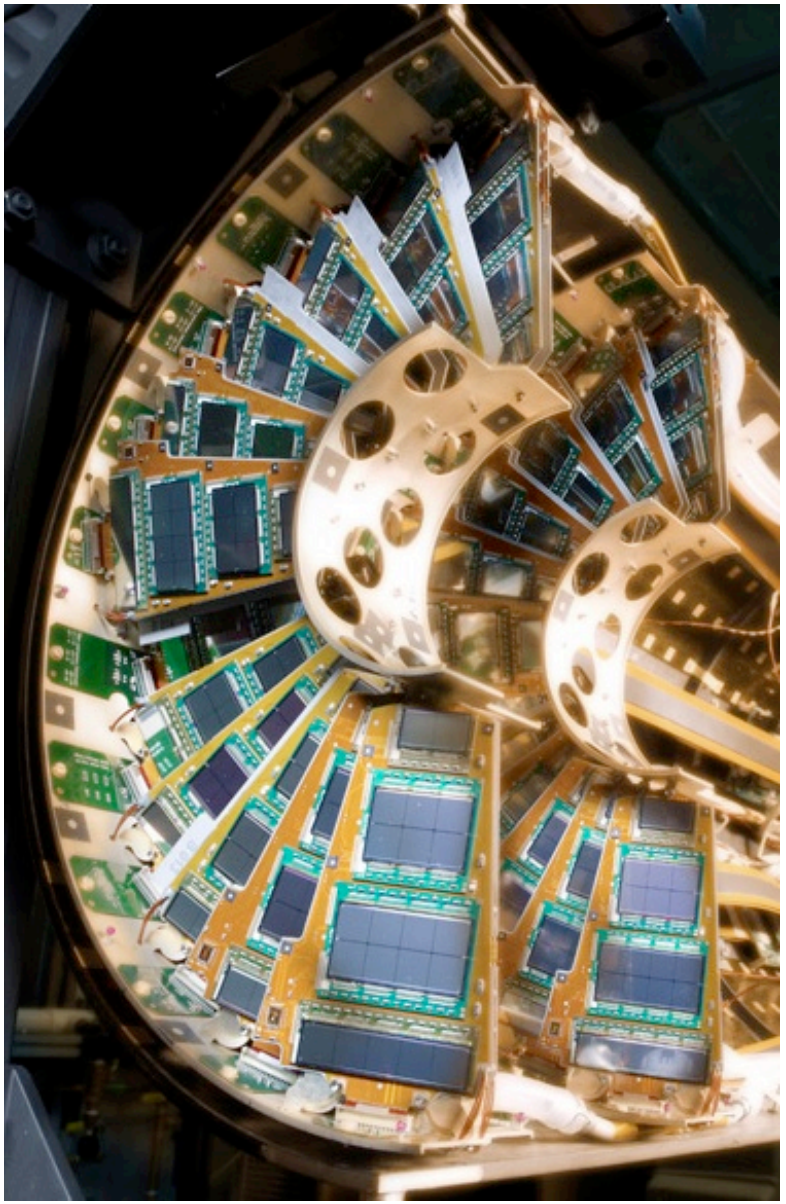




Pixel challenges

The many connection challenge is squared.
Internal electronics
Cooling
Material
Mechanically hard
Complex





Design Constraints

Large radius:
- Better momentum resolution
- Less confusion
- Cost scales as R^2.

Small radius:
- Betting impact parameter resolution
- More confusion

Large Magnetic field:
- Better momentum resolution
- Cost
- Curler confusion

Small hit resolution:
- Better track resolution
  - though MCS and mis-reco may dominate.
- Cost

Low mass:
- Better resolution at low momentum
- Mechanically difficult

Fine granularity (both dimensions):
- Less confusion
- Cost
- Complexity
- Material

Possible Designs

Wire chamber at high radius
- Large radius
- Low cost
- Low mass
- Many measurements

Silicon at small radius
- High precision for impact parameter
- Try to keep it light
- Enough layers to match seeds

Silicon strip detector at high radius
- Large radius
- Low-ish cost
- Low-ish mass
- Many-ish measurements

Pixels at small radius:
- High precision in both dimensions
- Enough layers to match (or make) seeds

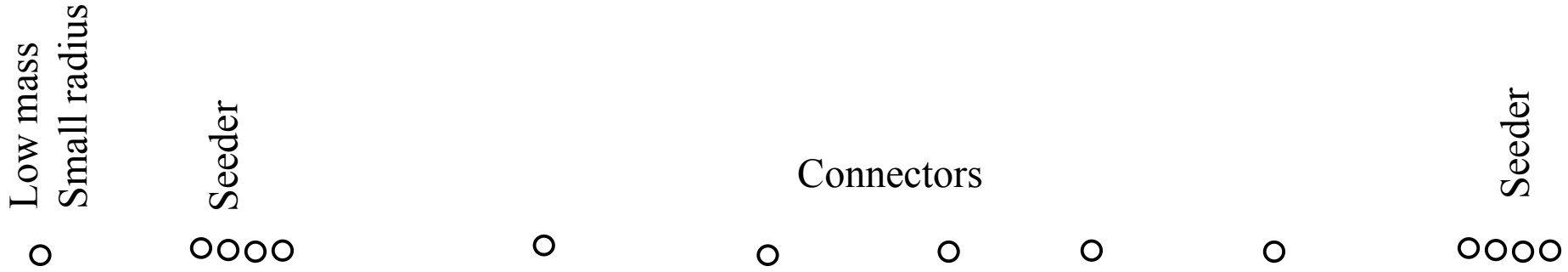
Possible Designs

Wire chamber at high radius
- Large radius
- Low cost
- Low mass
- Many measurements

Silicon strip detector at high radius
- Large radius
- Low-ish cost
- Low-ish mass
- Many-ish measurements

Silicon at small radius
- High precision for impact parameter
- Try to keep it light
- Enough layers to match seeds

Pixels at small radius:
- High precision in both dimensions
- Enough layers to match (or make) seeds



Please make it all pixels…or at least 1mm cell with 20 micron resolution
…for less than $50M and at 1% X0 per hit….and…

Next time…

What it takes to make it work

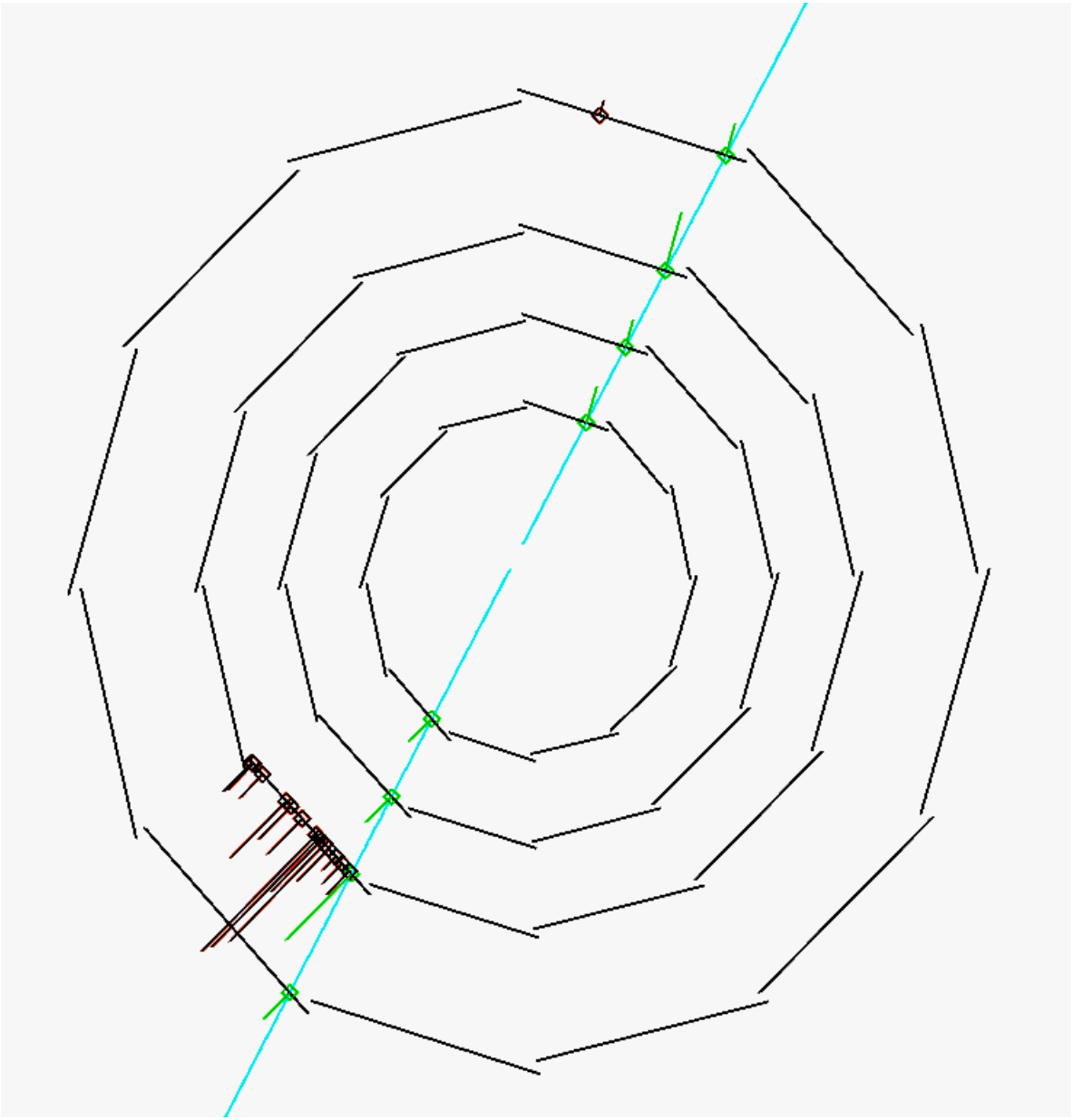
What will try to kill it

Bibliography

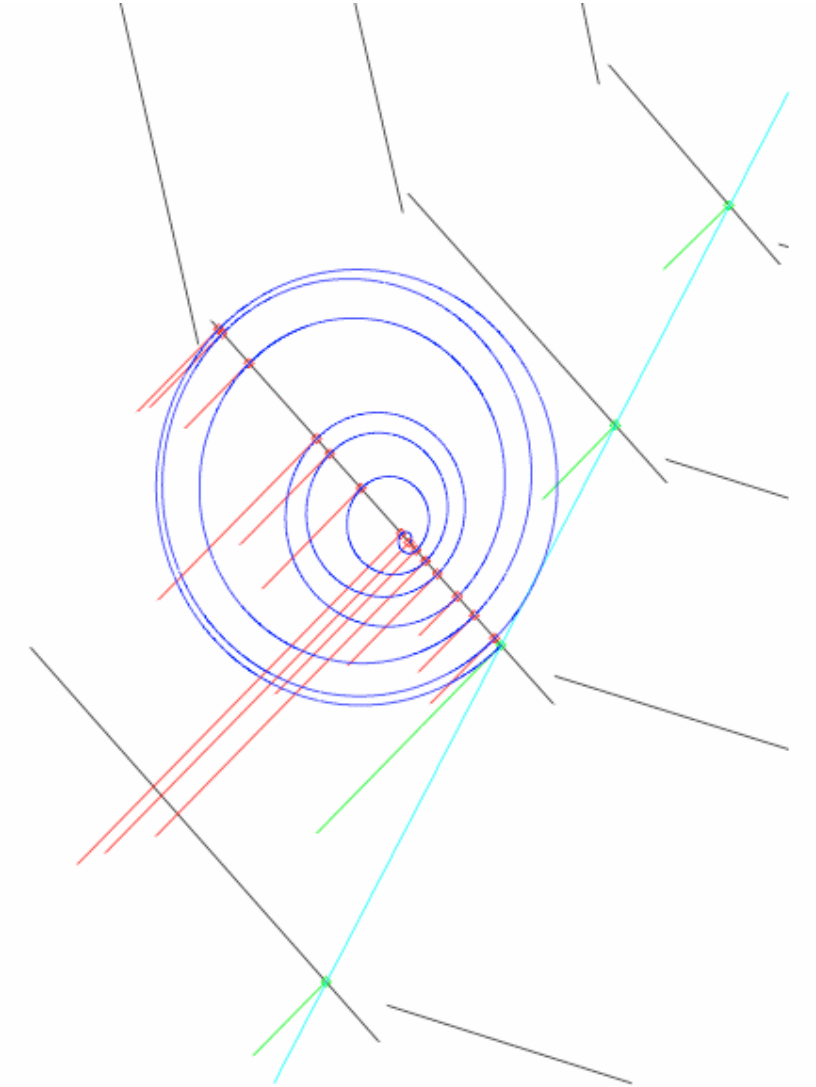
- Gino Bolla, UTeV seminar
- Guido Tonelli, Tracking at the LHC, SLAC Summer Institute 2006
- Aaron Dominguez, HCPSS06 Lecture on Tracking
- CMS Technical Design Report
- CDF Technical Design Report
- Daniella Bortoletto, 11th Vienna Conference on Instrumentation, Feb. 2007
- Helmut Spieler, “Semiconductor Detector Systems”
- Hans Wenzel, “Tracking in the SVX”, CDF Note 1790

Extraneous slides

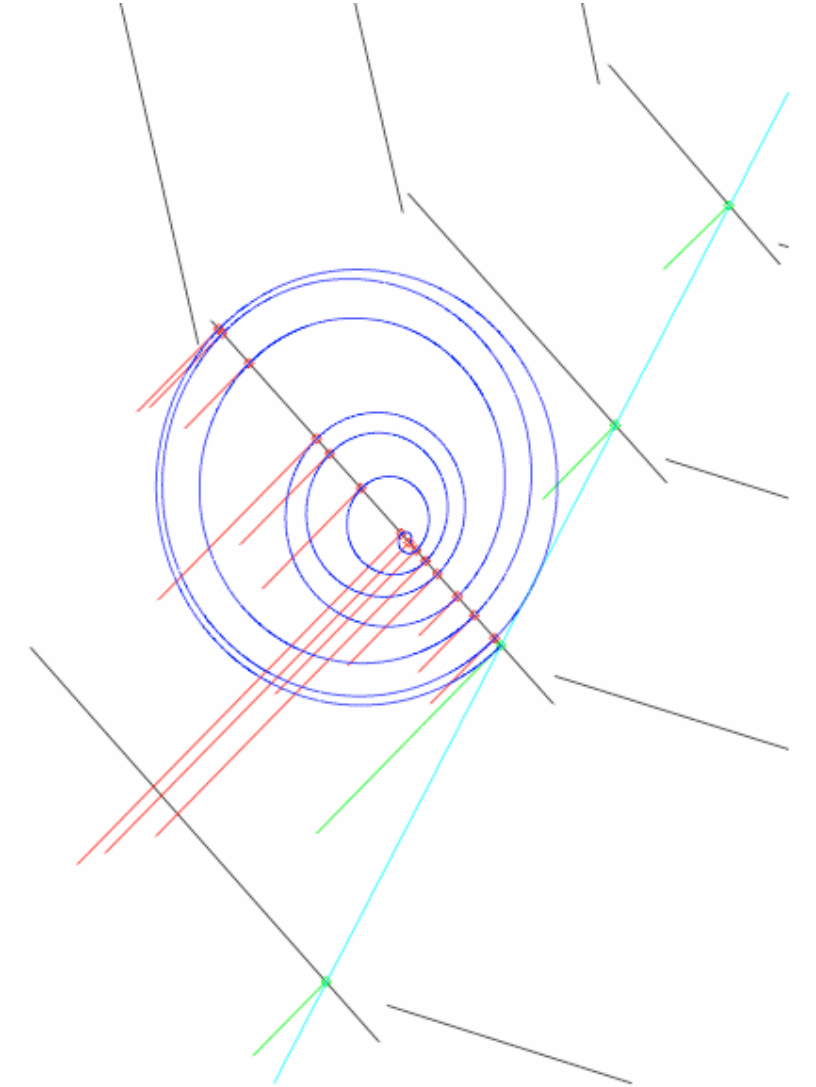
Today's game: What is this?



Today's game: What is this?



Today's game: What is this?



A delta ray kicked off a cosmic muon. Fit for path (explicitly using dE/dx) and get 5.4 MeV.