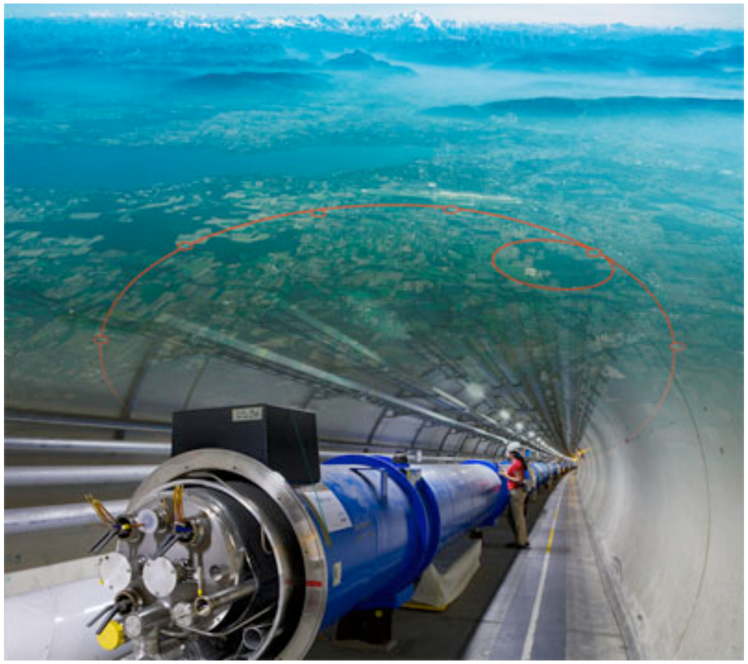
Strong Dynamics: Who needs a Higgs Boson?

R. Sekhar Chivukula
Michigan State University
August 21 & 22, 2008



2008 JOINT CERN-FERMILAB
HADRON COLLIDER PHYSICS
SUMMER SCHOOL

AUGUST 12 - 22, 2008
FERMILAB

The 3rd CERN-Fermilab hadron collider physics summer school will provide lectures aimed at preparing young experimental and theoretical researchers in the area of high energy particle physics for the coming discoveries at the Tevatron and the LHC.

List of Topics
- Electroweak Theory
- Quantum Chromodynamics
- Heavy Flavors
- Higgs
- Supersymmetry
- Extra Dimensions
- Strong Dynamics
- Calorimetry
- Muons
- Tracking
- Trigger / DAQ
- Data Analysis
- Statistics
- Accelerator Physics

Organizing Committee

International Advisory Committee

http://hcpss.fnal.gov/hcpss08

Where/When are we?

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CERN announces start-up date for LHC

PR06.08
07.08.2008

Geneva, 7 August 2008. CERN[1] has today announced that the first attempt to circulate a beam in the Large Hadron Collider (LHC) will be made on 10 September. This news comes as the cool down phase of commissioning CERN's new particle accelerator reaches a successful conclusion. Television coverage of the start-up will be made available through Eurovision.

The CERN Control Centre, from where the LHC will be operated.

The LHC is the world's most powerful particle accelerator, producing beams seven times

re intense
27-
ossible 30

Thursday 21 August 2008

top

09:00->*10:15* Strong Dynamics (Convener: Ian Low (*Northwestern University*))
Description:
Theory and phenomenology

09:00 Strong Dynamics - Lecture 1 (1h15') R. Sekhar Chivukula (*Michigan State University*)

10:15 Break

10:45->*12:00* Data Analysis (Convener: Tom Junk (*Fermilab*))
Description:
From triggers to analysis, background selections, different methods for searches vs. measurement, examples and cautionary tales, template methods, selecting analysis topics, Monte Carlo validation, systematics

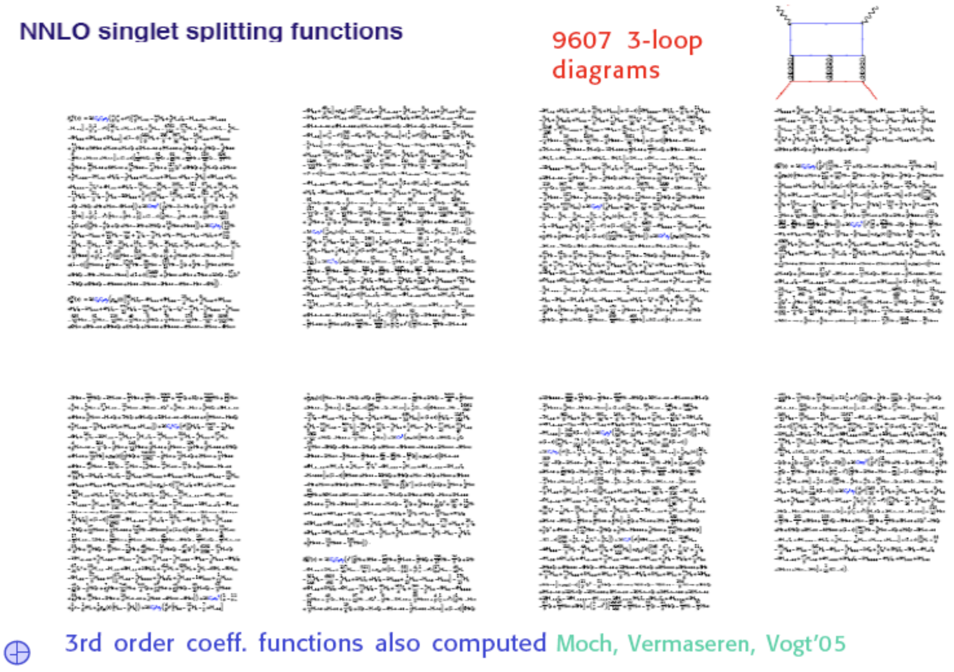
10:45 Data Analysis - Lecture 2 (1h15') Beate Heinemann (*University of California, Berkeley and LBNL*)

12:00 Lunch

12:30->*23:00* Trip to University of Chicago (Location: University of Chicago (---))

12:30 Buses depart Wilson Hall (can bring your lunch)
13:45 Arrive at Adler Planetarium
16:30 Depart Adler Planetarium
17:00 Arrive at University of Chicago (Ida Noyes Auditorium)
17:15 Welcome and Introductory Remarks (15') (Location: Ida Noyes Auditorium)
17:30 Connections between HEP and Cosmology (1h15') (Location: Ida Noyes Auditorium) Rocky Kolb (*University of Chicago*)
18:45 Cocktail Social
19:30 Banquet (Cloister Room)
21:00 Buses depart for Hotels

What I will try to avoid:



NNLO singlet splitting functions

9607 3-loop diagrams

3rd order coeff. functions also computed Moch, Vermaseren, Vogt'05

Your questions are welcome and encouraged at any time!!

Outline:
From Your Questions

- Why do we believe in field theory?
    - In gauge invariance?
- How can vector bosons have mass?

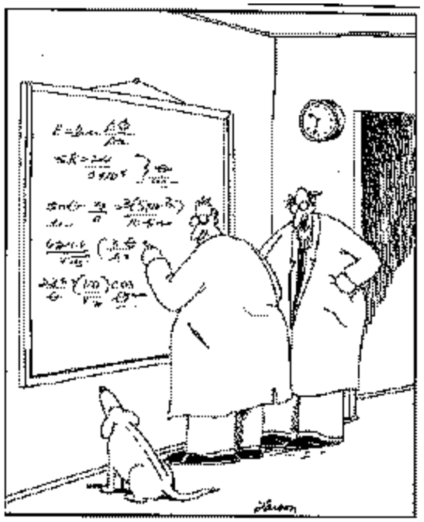
Lecture 1

- If there is no fundamental Higgs, then what?

Lecture 2

- What are experimental signatures of a non-standard EWSB sector? (3-site example)
- What about wild ideas?

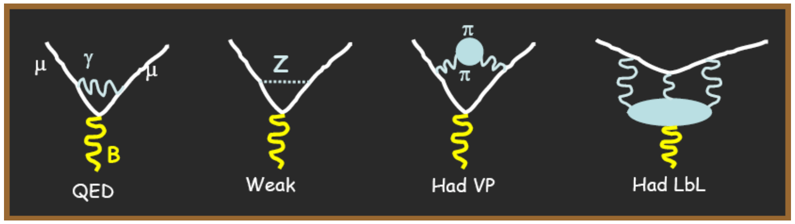
Why do we believe in (effective) field theory?



“They act so cute when they try to understand Quantum Field Theory”

QFT Reconciles QM with Relativity

A local, Lorentz-invariant, Hermitian, QFT with a finite number of fields yields a unitary, CPT-invariant, S-matrix satisfying cluster decomposition



à la Landau (e.g. superconductivity): the converse is also true!

“Any” S-matrix is derivable from a QFT

Example:
A Scalar Doublet...

Consider theory valid below UV cutoff Λ:

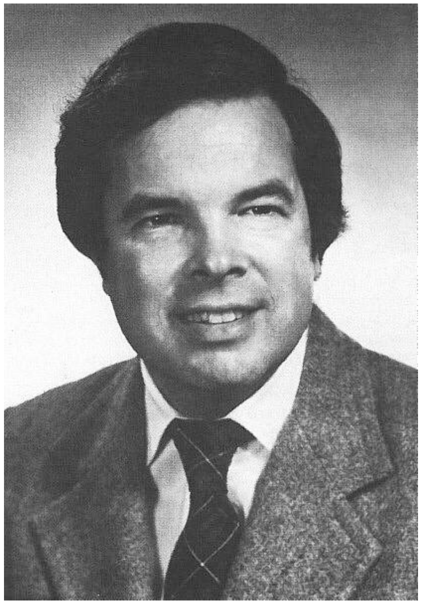
$$\mathcal{L}_\Lambda = \quad D^\mu \phi^\dagger D_\mu \phi + m^2(\Lambda)\phi^\dagger \phi + \frac{\lambda(\Lambda)}{4}(\phi^\dagger \phi)^2$$
$$+ \frac{\kappa(\Lambda)}{36\Lambda^2}(\phi^\dagger \phi)^3 + \ldots$$

(Note "scaling dimension" of operators)

- **That which is not forbidden is required**: includes all interactions consistent with **space-time, global, and gauge symmetries**.

Wilsonian Renormalization Group

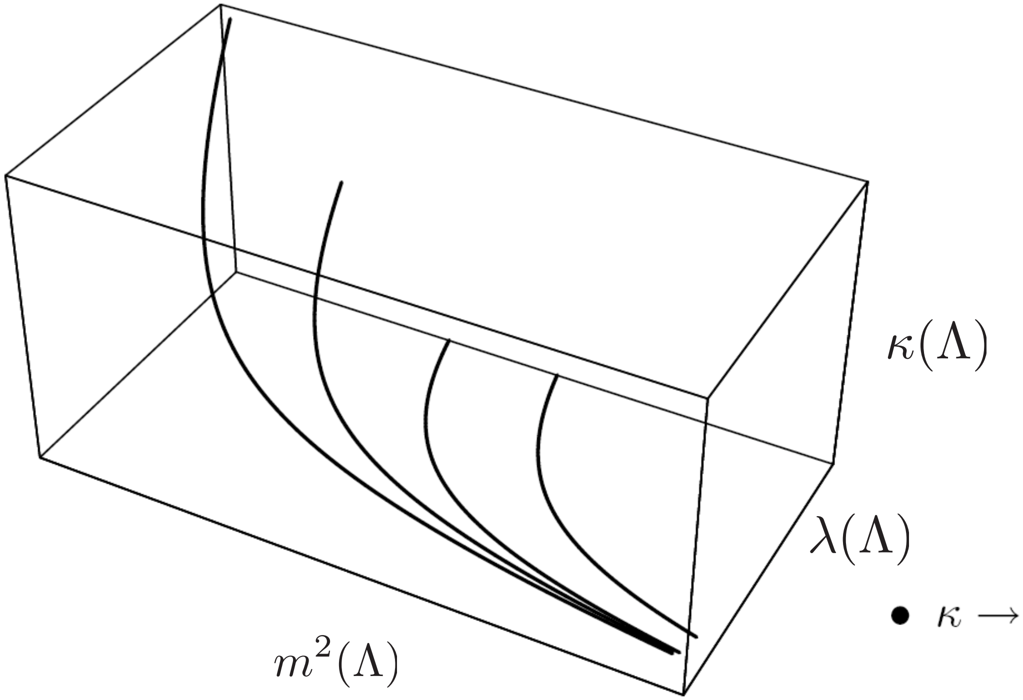
Integrate out states with $\Lambda' < k < \Lambda$:

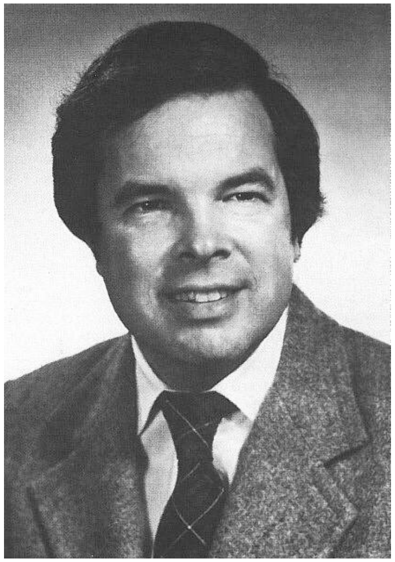


$$\begin{aligned} \mathcal{L}_\Lambda &\Rightarrow \mathcal{L}_{\Lambda'} \\ m^2(\Lambda) &\rightarrow m^2(\Lambda') \\ \lambda(\Lambda) &\rightarrow \lambda(\Lambda') \\ \kappa(\Lambda) &\rightarrow \kappa(\Lambda') \end{aligned}$$

Consider evolution of couplings in the IR-limit....

Wilsonian Renormalization Group





$$D^\mu\phi^\dagger D_\mu\phi + m^2(\Lambda)\phi^\dagger\phi + \frac{\lambda(\Lambda)}{4}(\phi^\dagger\phi)^2 + \frac{\kappa(\Lambda)}{36\Lambda^2}(\phi^\dagger\phi)^3 + \ldots$$

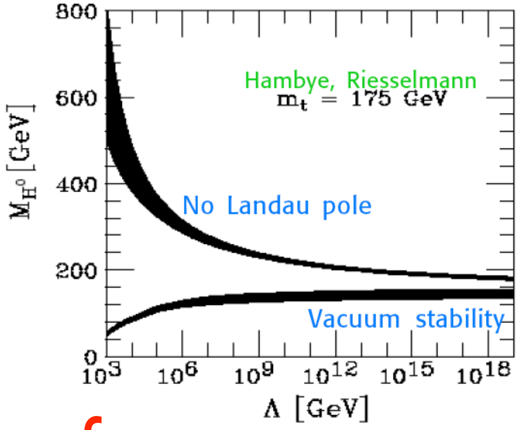
- $\kappa \to 0$ — "Renormalizability", if $m_H \ll \Lambda$.
- $m^2 \to \infty$ — Naturalness/Hierarchy Problem:

$$\frac{\Delta m^2(\Lambda)}{m^2(\Lambda)} \propto \frac{v^2}{\Lambda^2}$$

- $\lambda \to 0$ — Triviality ...

QFT *Reinterpreted*

- Lagrangian and S-matrix are expansions in p^2/Λ^2 - at any order, only a finite number of operators contribute.
- “Renormalizable” theories are a special case, with $\Lambda \rightarrow \infty$: S-matrix “exactly” calculable in terms of a few parameters.
- The Hierarchy problem is not a problem of principle, it is matter of (good) taste.
- Triviality and vacuum stability, on the other hand...



Elastic (2-body) Unitarity

$$S^\dagger S = \mathcal{I} \quad \Rightarrow \quad |s_l|^2 = 1$$

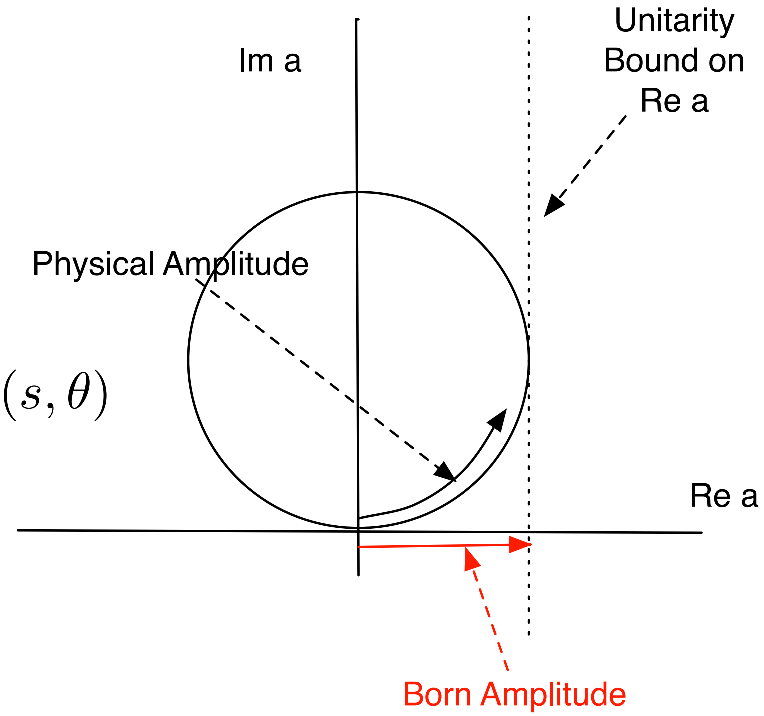
$$s_l = 1 + 2ia_l \quad \Rightarrow \quad \mathrm{Im}(a_l) = |a_l|^2$$

$$a_l = e^{i\delta_l} \sin\delta_l$$

$$a_l = \frac{1}{32(2)\pi} \left[\frac{4k^2}{s}\right]^{1/2} \int_{-1}^{+1} d\cos\theta \, P_l(\theta) \mathcal{M}(s,\theta)$$

Identical Particle Factor

Feynman Amplitude

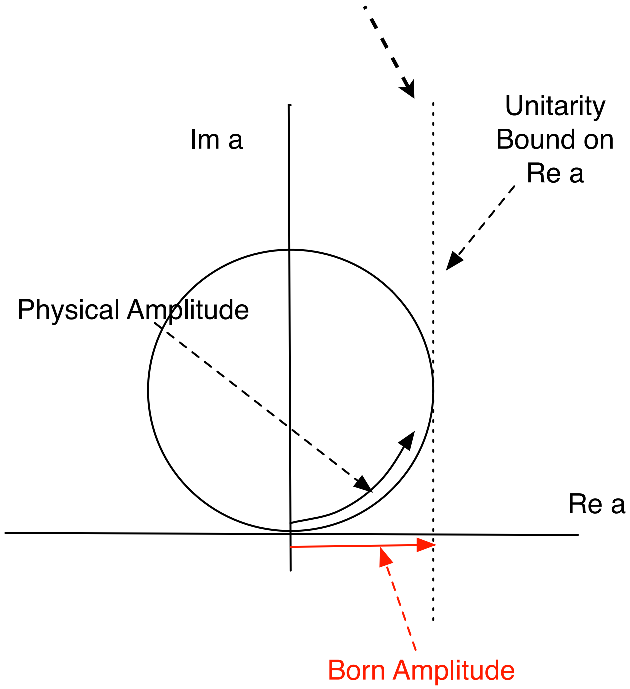


These formulae apply to the elastic scattering of pairs of particles of fixed helicity

Jacob and Wick, 1959

Limits of an Effective Theory

$\mathcal{L} = \sum_i \frac{\kappa_i(\Lambda)\mathcal{O}_i}{\Lambda^{d_i-4}}$ with $d_i>4$ leads to $\mathcal{M} \propto \frac{\kappa_i(\Lambda)p^{d_i-4}}{\Lambda^{d_i-4}}$



Amplitude "violates" unitarity at scale M, and the (perturbative) effective theory breaks down

M is the scale at which the description of the theory changes, e.g. the W instead of Fermi Theory

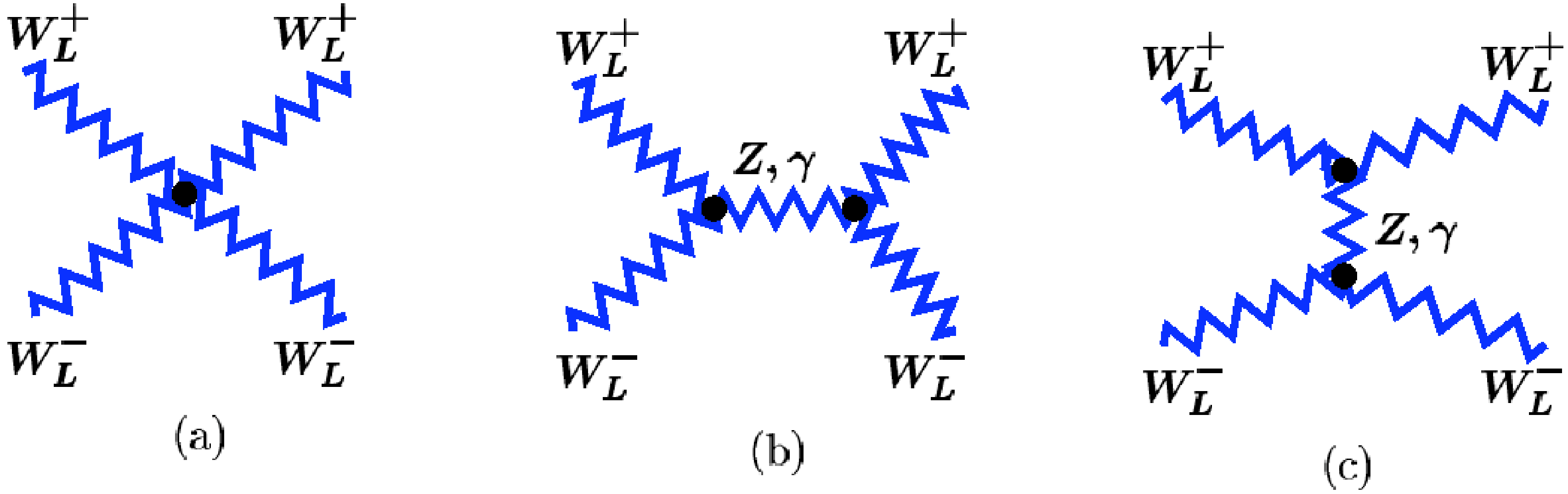
Gauge Invariance?

- The only consistent S-matrix for a spin-1 *massless* particle arises when it couples to a conserved current - e.g., like a gauge-boson! (Weinberg's theorem)
- Corollary: Given a spin-1 boson of mass m, the only theory consistent up to scale M *is*, in the limit m/M→0, a gauge theory.
- LEP I/II and Tevatron: SU(2) x U(1) gauge-invariance good to ~ few TeV! e.g.

$$\frac{(\varphi^\dagger D^\mu \varphi)^2}{M^2} \rightarrow \alpha T \text{ or } \Delta\rho$$

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^{0} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21581
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.377
Γ_W [GeV]	2.097 ± 0.048	2.092
m_t [GeV]	172.6 ± 1.4	172.8

SU(2) x U(1) @ E^4

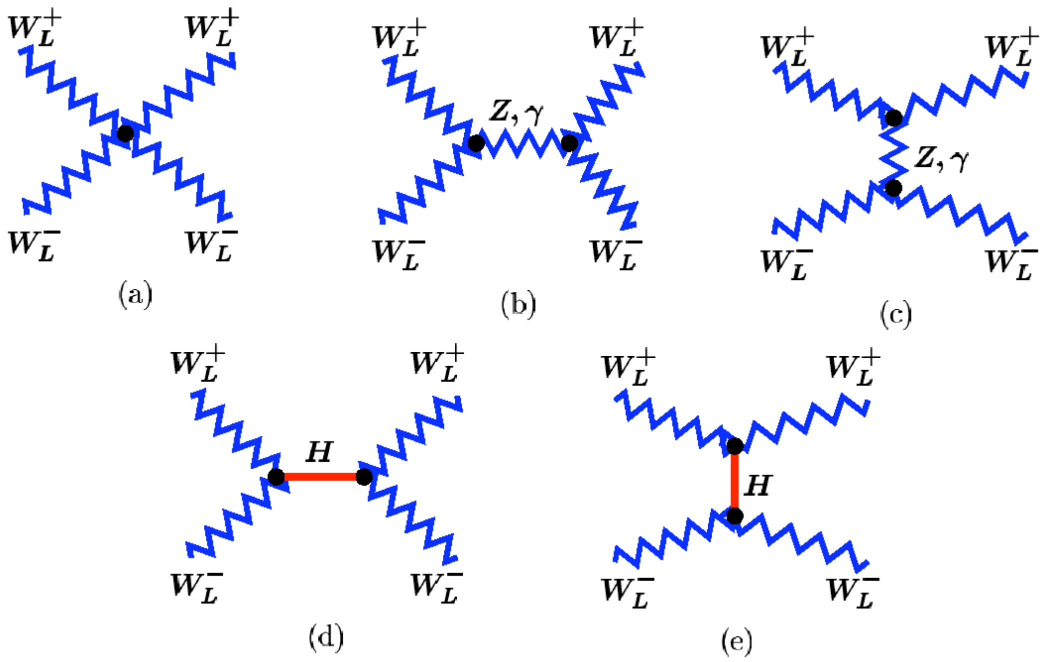


(a) (b) (c)

Graphs	$g^2 \frac{E^4}{m_w^4}$
(a)	$-3 + 6\cos\theta + \cos^2\theta$
(b)	$-4\cos\theta$
(c)	$+3 - 2\cos\theta - \cos^2\theta$
Sum	0

$$\epsilon_L^\mu(k) = \frac{k^\mu}{m_w} + \mathcal{O}\left(\frac{m_w}{E}\right)$$

SU(2) x U(1) @ E^2



Graphs	$g^2 \frac{E^2}{m_w^2}$
(a)	$+2 - 6\cos\theta$
(b)	$-\cos\theta$
(c)	$-\frac{3}{2} + \frac{15}{2}\cos\theta$
(d + e)	$-\frac{1}{2} - \frac{1}{2}\cos\theta$
Sum including (d+e)	**0**

- $\mathcal{O}(E^0) \Rightarrow$ 4d m_H bound: $m_H < \sqrt{16\pi/3}\, v \simeq 1.0$ TeV
- If no Higgs $\Rightarrow \mathcal{O}(E^2) \Rightarrow$ $E < \sqrt{4\pi}\, v \simeq 0.9$ TeV

Warnings[*]

- The QFT description of an S-matrix need *not* be unique, e.g. QCD and the χLagrangian, ADS/CFT.
- "Gauge Symmetries" are not symmetries: they are redundancies in our description.
- "Coupling constants" are not observables.
- "Fundamental" and "Composite" are in the eye of the *calculator* ... more important: strong or weak

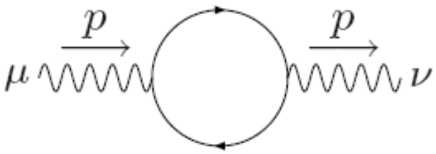
[*] Things you should know about QFT, but were afraid to ask.

What accounts for Vector Boson Mass Generation?

segue?
(no, Segway!)

The Higgs *Mechanism*

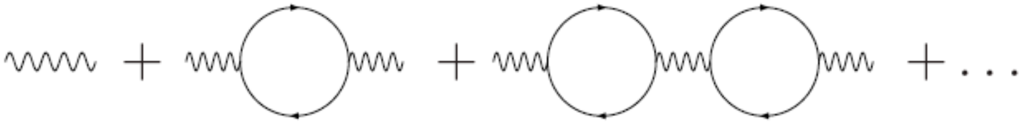
The polarization tensor $\Pi_{\mu\nu}(p)$ is defined as:



$$i\,\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2)$$

where the form of $\Pi_{\mu\nu}(p)$ is governed by gauge invariance, i.e. it satisfies $p^\mu \Pi_{\mu\nu}(p) = p^\nu \Pi_{\mu\nu}(p) = 0$.

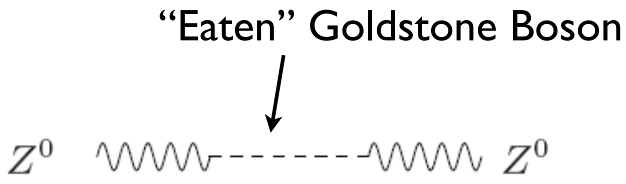
The renormalized propagator is the sum of a geometric series



$$= \frac{-i(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2})}{p^2[1+\Pi(p^2)]}$$

The pole at $p^2 = 0$ is shifted to a non-zero value if:

$$\Pi(p^2) \underset{p^2\to 0}{\simeq} \frac{-g^2 v^2}{p^2}\,.$$



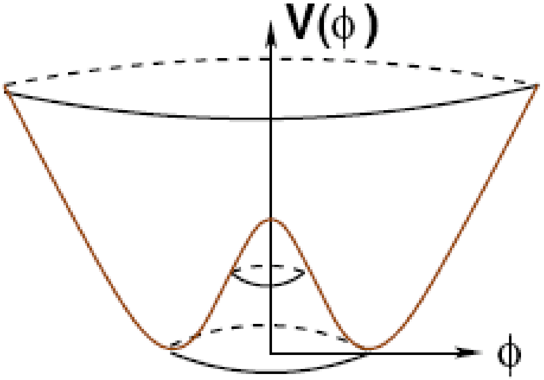
Then $p^2[1+\Pi(p^2)] = p^2 - g^2 v^2$, yielding a gauge boson mass of gv.

Haber

Trial answer: the SM with a Higgs

A Fundamental Scalar Doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad ,$$



with potential:

$$V(\phi) = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2$$

is employed both to break the electroweak symmetry and to generate masses for the fermions in the Standard Model.

Matrix Notation

Define $\tilde{\phi} = i\sigma_2 \phi^*$ and

$$\Phi = \left(\tilde{\phi} \ \phi \right) \quad \Rightarrow \quad \Phi^\dagger \Phi = \Phi \Phi^\dagger = (\phi^\dagger \phi) \mathcal{I} \ .$$

Under $SU(2)_L \times U(1)_Y$, $\Phi \to L \Phi R^\dagger$,

$$L = \exp\left(\frac{i w^a(x) \sigma^a}{2} \right) \ , \ R = \exp\left(\frac{i b(x) \sigma^3}{2} \right)$$

The Higgs-sector Lagrangian becomes

$$\frac{1}{2} \mathrm{Tr}\left(D^\mu \Phi D_\mu \Phi^\dagger \right) + \frac{\lambda}{4} \left(\mathrm{Tr}\left(\Phi \Phi^\dagger \right) - v^2 \right)^2 \ ,$$

$$D_\mu \Phi = \partial_\mu \Phi + \mathrm{i} g \mathbf{W}_\mu \Phi - \mathrm{i} \Phi g' \mathrm{B}_\mu \ .$$

The potential manifests the symmetry
$SU(2)_L \times SU(2)_R \to SU(2)_V$

A "Polar decomposition" of Φ

$$\Phi(x) = \frac{1}{\sqrt{2}}(H(x) + v)\Sigma(x) \ ,$$

$$\Sigma(x) = \exp(i\pi^a(x)\sigma^a/v) \ .$$

neatly separates the radial "Higgs boson" from the "pion" modes (Nambu-Goldstone Bosons).

By gauge choice, $\langle\Sigma\rangle = \mathcal{I}$.

Broken Symmetries $\Rightarrow$ Nambu-Goldstone Bosons

Gauge $SU(2)_W \times U(1)_Y$ $\Rightarrow$ Higgs Mechanism

$$\pi^{\pm}, \pi^0 \ \rightarrow \ W_L^{\pm}, Z_L$$

$$M_W = \frac{gv}{2} \rightarrow v \approx 250\text{GeV}$$

Custodial Symmetry: SU(2)V

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

Due to residual $SU(2)_V$ "custodial symmetry" for $g' \to 0$, the $SU(2)_L$ gauge bosons are degenerate.

This, plus $m_\gamma = 0$, tells us

$$M^2 = \frac{v^2}{2} \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix} ,$$

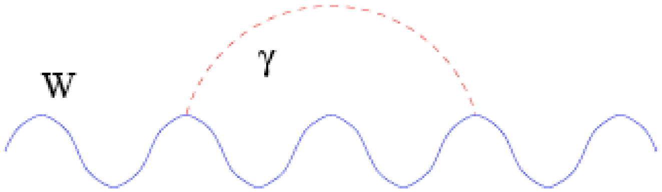
and hence

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \ .$$

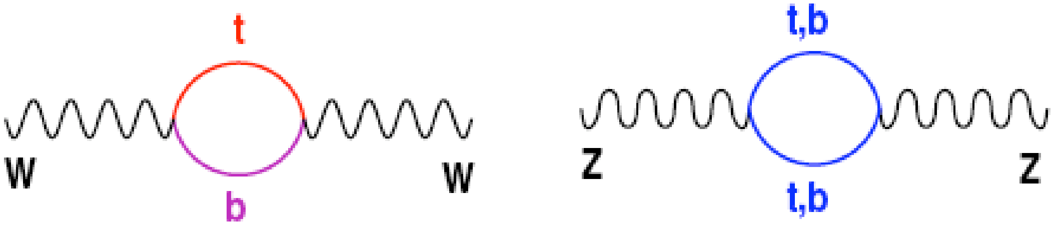
Violations of Custodial Symmetry

Conventionally, one speaks of $\Delta\rho \equiv \rho - 1$

Electromagnetism: $\mathcal{O}(\alpha)$ corrections to $\Delta\rho$ from



Yukawa Couplings: (i.e. mass differences) $\Delta\rho \approx \dfrac{3y_t^2}{32\pi^2}$



Experiment finds $|\Delta\rho| \leq 0.4\%$, which constrains physics beyond the SM.

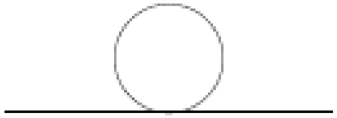
Custodial Symmetry is an important part of any theory of EWSB!

Problems with the Higgs Model

- No fundamental scalars observed in nature

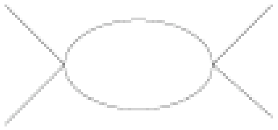
- No explanation of dynamics responsible for Electroweak Symmetry Breaking

- Hierarchy or Naturalness Problem



$$\Rightarrow \quad m_H^2 \propto \Lambda^2$$

- Triviality Problem...



$$\Rightarrow \beta = \frac{3\lambda^2}{2\pi^2} > 0 \qquad \lambda(\mu) < \frac{3}{2\pi^2 \log\frac{\Lambda}{\mu}}$$

A Fork in the Road...

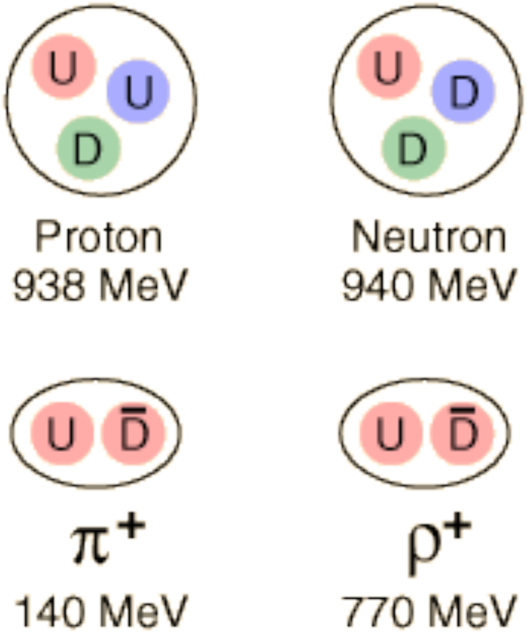
- Make the Higgs Natural: Supersymmetry (Martin, Haber)
- Make the Higgs Composite
  - Little Higgs
  - Twin Higgs
- Eliminate the Higgs
  - Technicolor
  - “Higgsless” Models

“When you come to a fork in the road, take it!”
— Yogi Berra

Technicolor:
Higgsless since 1976!

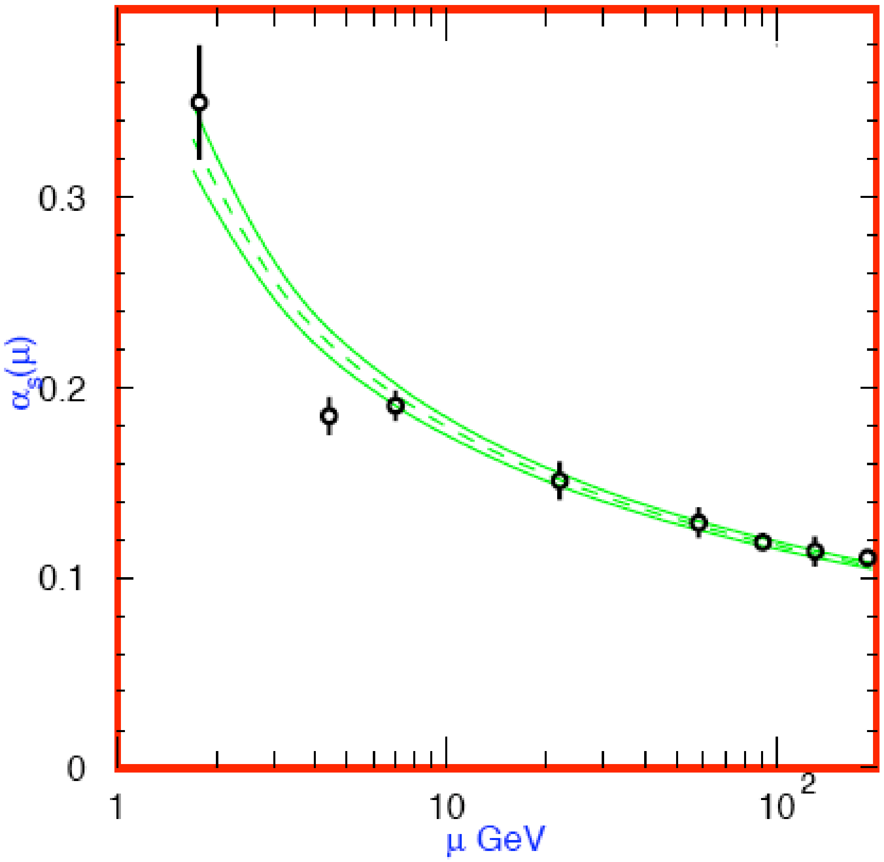
For a new approach to generating mass, we turn to the strong interactions (QCD) for inspiration

Consider the hadrons composed of up and down quarks:



Why is the pion so light?

Recall that the QCD coupling varies with energy scale, becoming strong at energies ~ 1 GeV



[coupling]^2

Energy

The strong-interaction (QCD) Lagrangian for the u and d quarks (neglecting their small masses)

$$\mathcal{L} = i\bar{u}_L \not{D} u_L + i\bar{d}_L \not{D} d_L + i\bar{u}_R \not{D} u_R + i\bar{d}_R \not{D} d_R$$

displays an $SU(2)_L$ x $SU(2)_R$ global ("chiral") symmetry

When the QCD coupling becomes strong

- $\langle \bar{q}_L q_R \rangle \neq 0$ breaks $SU(2)_L$ x $SU(2)_R$ → $SU(2)_{L+R}$
- pions $(\bar{q}_L q_R)$ are the associated Nambu-Goldstone bosons!

Bonus: from chiral to electroweak symmetry breaking

- u_L, d_L form weak doublet; u_R, d_R are weak singlets
- so $\langle \bar{q}_L q_R \rangle \neq 0$ also breaks electroweak symmetry
- could QCD pions be our composite Higgs bosons?

Not Quite:

- M_W = .5g F_π = 80 GeV requires F_π ~ 250 GeV
- $\langle \bar{q}_L q_R \rangle$ only supplies f_π~ 0.1 GeV
- need extra source of EW symmetry breaking

This line of reasoning inspired **Technicolor:**

introduce new gauge force with symmetry $SU(N)_{TC}$

force carriers are **techni**gluons, inspired by QCD gluons

add **techni**quarks carrying $SU(N)_{TC}$ charge:

matter particles inspired by QCD quarks

- e.g. $T_L = (U_L, D_L)$ forms a weak doublet

  U_R, D_R are weak singlets
- Lagrangian has familiar global (chiral)

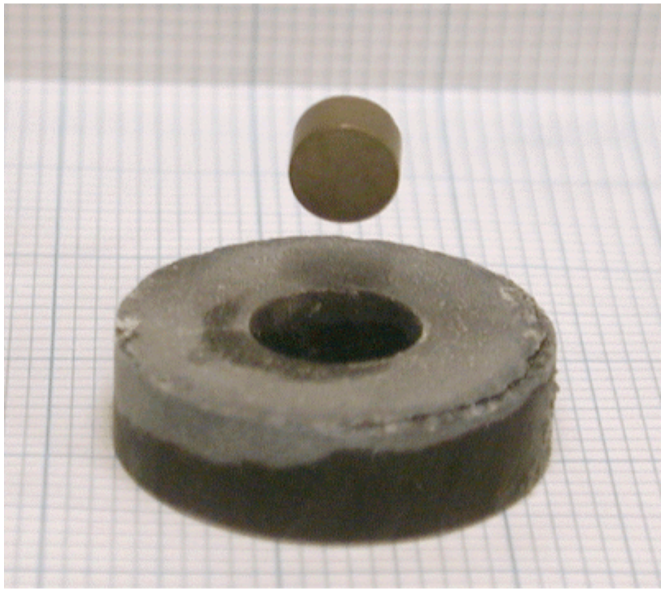
  symmetry $SU(2)_L$ x $SU(2)_R$

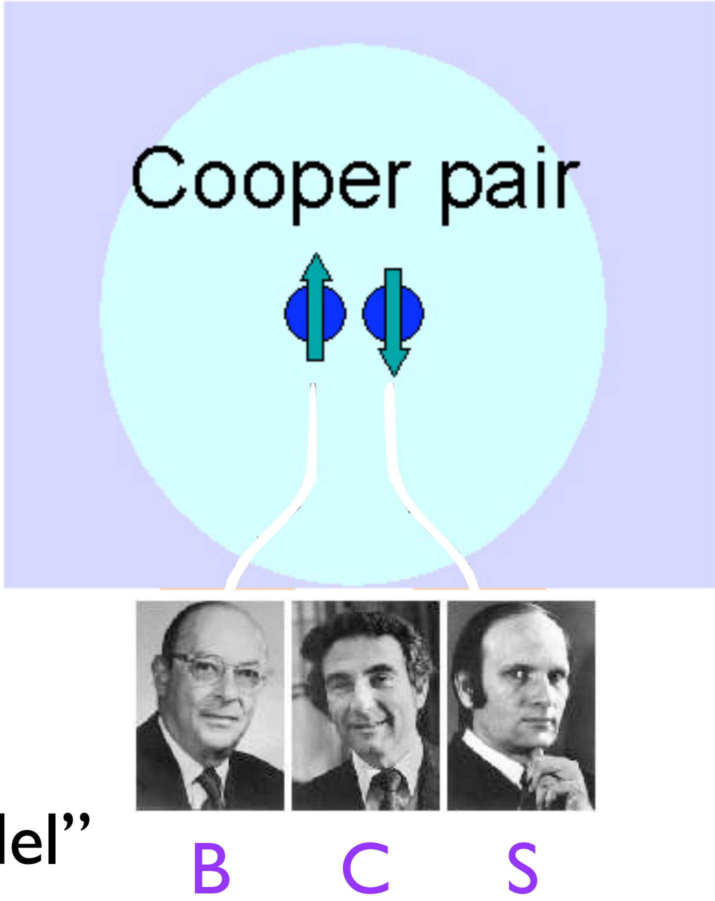
Susskind, Weinberg

If SU(N)$_{TC}$ force were stronger than QCD ... then spontaneous symmetry breaking and pion formation would happen at a higher energy scale... e.g.

- gauge coupling becomes large at $\Lambda_{TC} \approx 1000\,\mathrm{GeV}$
- $\langle T_L T_R \rangle \approx 250\,\mathrm{GeV}$ breaks electroweak symmetry
- `**techni**pions' Π_{TC} become the W_L, Z_L
- W and Z boson masses are the size seen in experiment!

(“Low-Energy” Analog)





$$\langle \phi^{++} \rangle \neq 0$$

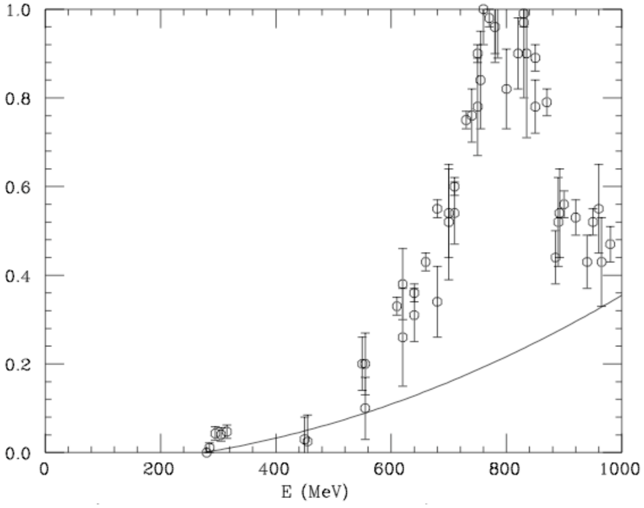
“Abelian Higgs Model”

Weinberg: “Superconductivity for Particular Theorists”

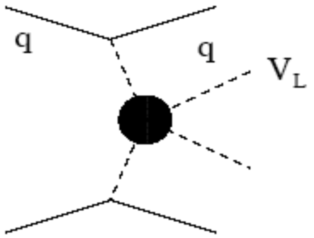
Classic TC @ LHC:

NB: unlike Higgs!!

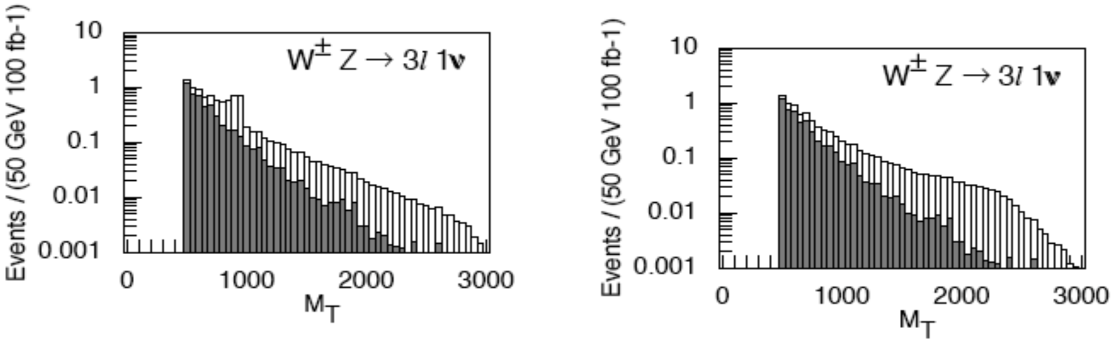
I=J=1 ππ scattering



Gauge-Boson Scattering:



For $M_{\rho_{TC}} = 1.0\ \mathrm{TeV}, 2.5\ \mathrm{TeV}$:

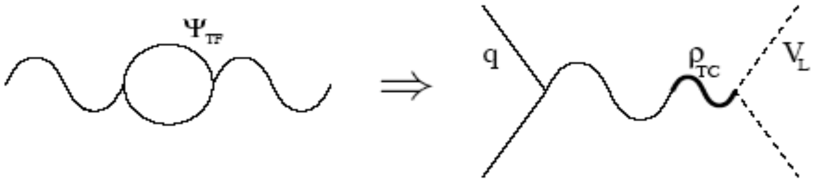


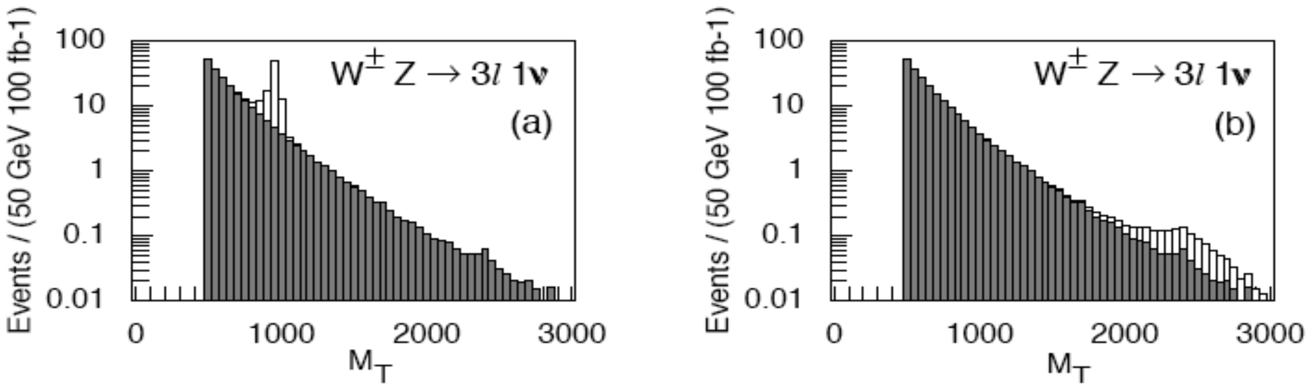
leptonic cuts	jet cuts
$\lvert y(\ell)\rvert < 2.5$	$E(j_{tag}) > 0.8$ TeV
$p_T(\ell) > 40$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
$p_T^{\mathrm{miss}} > 50$ GeV	$p_T(j_{tag}) > 40$ GeV
$p_T(Z) > \frac{1}{4} M_T$	$p_T(j_{veto}) > 60$ GeV
$M_T > 500$ GeV	$\lvert y(j_{veto})\rvert < 3.0$

*J. Bagger *et. al.*, `hep-ph/9306256, 9504426`

Classic TC @ LHC:

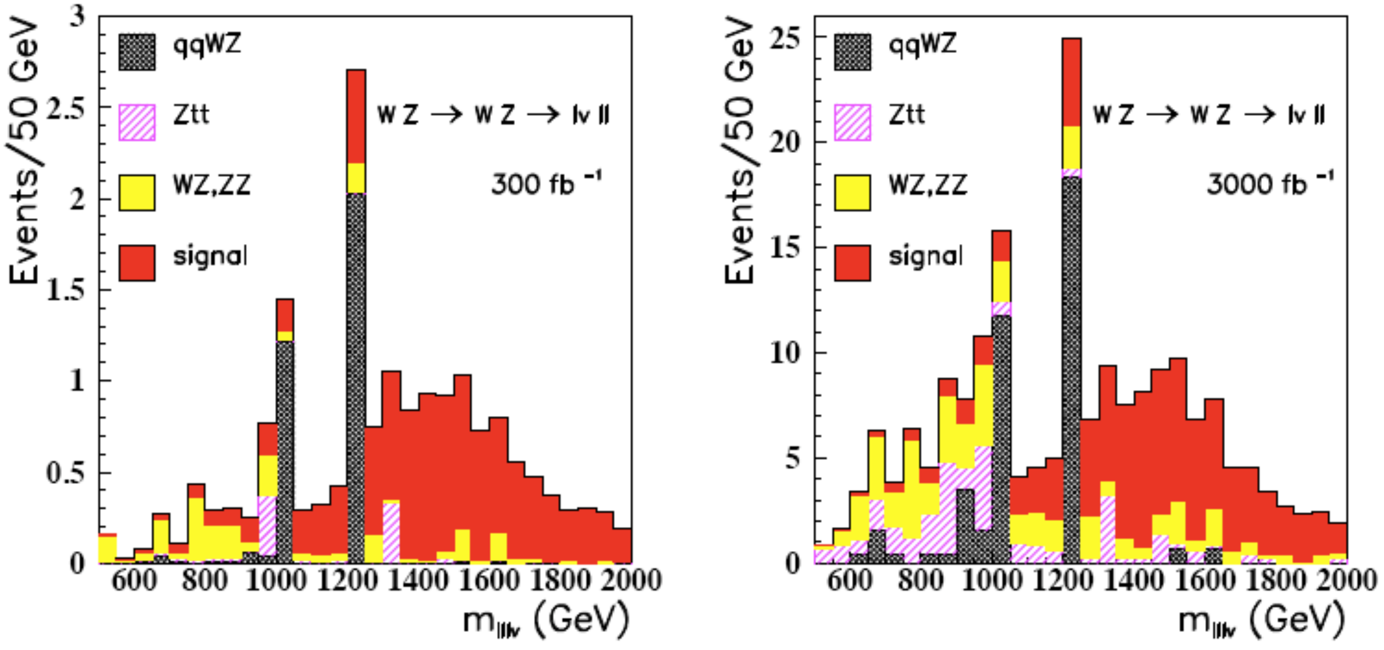
Gauge-Boson — Vector Meson Mixing:





*M. Golden, *et. al.*, `hep-ph/9511206`

WZ Scattering at SLHC



$$p_T(\ell_1) > 150 \text{ GeV}, \quad p_T(\ell_2) > 100 \text{ GeV}, \quad p_T(\ell_3) > 50 \text{ GeV}$$
$$|m(\ell_1\ell_2) - m_Z| < 10 \text{ GeV}$$
$$E_T^{miss} > 75 \text{ GeV}$$

+ forward jets

F. Gianotti, et. al., hep-ph/0204087

Any Questions?

What was good?

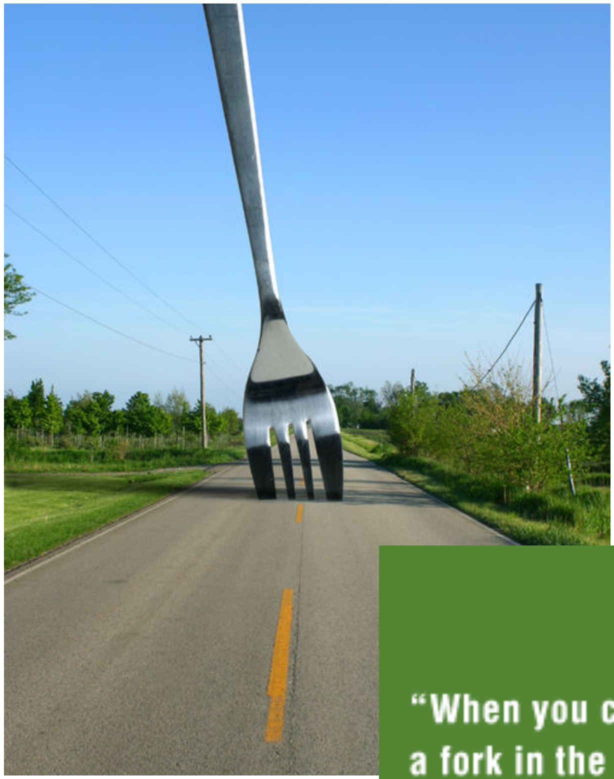
$+/\Delta$

What would you like next time?

Next time: Higgsless models, Composite Higgs, LHC signals of the 3-site model, wild ideas...

...we were following the TC fork...

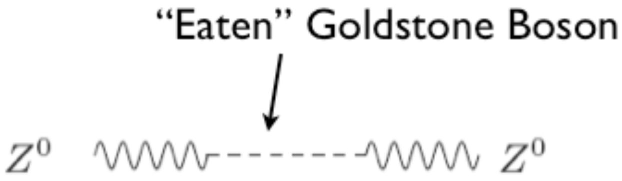
- Make the Higgs Natural: Supersymmetry (Martin, Haber)
- Make the Higgs Composite
  - Little Higgs
  - Twin Higgs
- Eliminate the Higgs
  - Technicolor
  - "Higgsless" Models



"When you come to
a fork in the road,
take it!"
— Yogi Berra

Higgs Mechanism:

$$\Pi(p^2) \underset{p^2 \to 0}{\simeq} \frac{-g^2 v^2}{p^2}$$



Custodial Symmetry:

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

$$M^2 = \frac{v^2}{2} \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix}$$

Technicolor Review:

Technicolor:

$SU(N_{TC})$ strong/confining theory,

$$\Psi_L = \begin{pmatrix} U \\ D \end{pmatrix}_L \qquad U_R, D_R$$

with massless fermions

$$\begin{aligned} \mathcal{L} \quad = \quad & \bar{U}_L i \not{D} U_L + \bar{U}_R i \not{D} U_R + \\ & \bar{D}_L i \not{D} D_L + \bar{D}_R i \not{D} D_R \end{aligned}$$

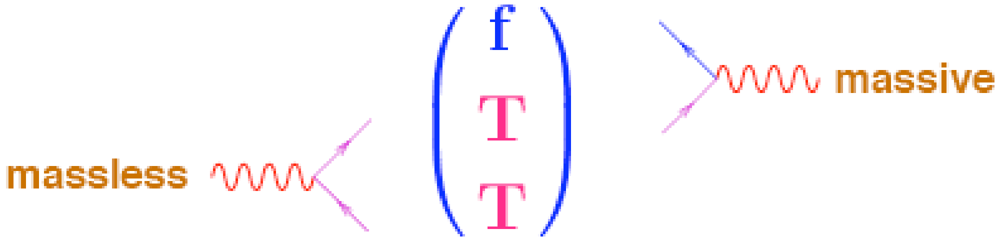
Like QCD in m_u, $m_d \to 0$ limit:

- Chiral $SU(2)_L \times SU(2)_R$ symmetry
- Dynamically broken $SU(2)_L \times SU(2)_R \to SU(2)_V$
- Pions: $\pi^{\pm}, \pi^0 \Leftrightarrow W_L^{\pm}, Z_L$

Fermion Masses

In extended technicolor (ETC) models, fermion masses arise because heavy gauge bosons couple the quarks and leptons to the condensing technifermions that break the EW symmetry

- larger ETC gauge group subsumes TC
- all fermions carry ETC charge
- ETC breaks to TC at scale $\mathrm{M} > \Lambda_{\mathrm{TC}}$.

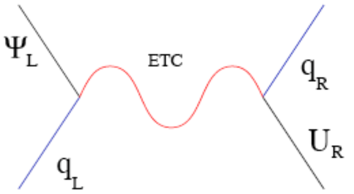


FCNC's?

*Dimpoulos & Susskind; Eichten & Lane

“Walking Technicolor”

Extended Technicolor Interactions — Connect chiral-symmetries of TFs to quarks & leptons.



$$\Rightarrow \frac{g_{ETC}^2}{M_{ETC}^2}(\overline{\Psi}_L U_R)(\overline{q}_R q_L)$$

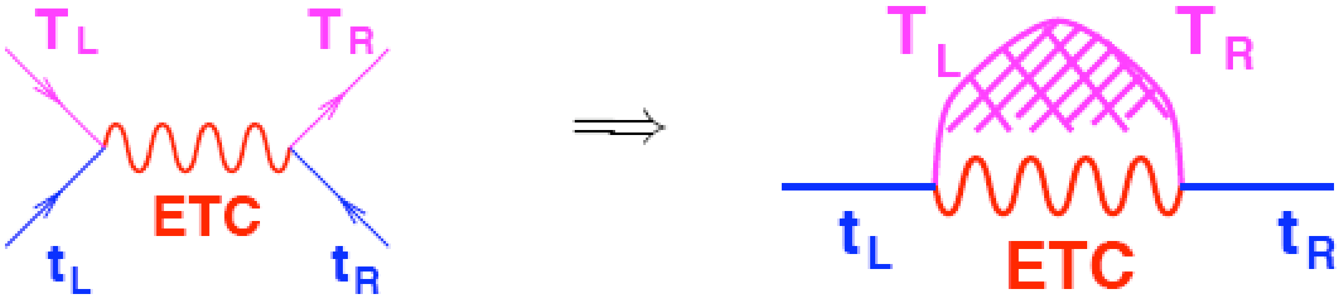
$$m_q \approx \frac{g_{ETC}^2}{M_{ETC}^2}\langle \overline{U} U\rangle_{ETC}$$

$$\langle \overline{U} U\rangle_{ETC} = \langle \overline{U} U\rangle_{TC} \exp\left(\int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu}\gamma_m(\mu)\right)$$

If $\beta_{TC}\sim 0$, we expect $\gamma_m \sim 1$, enhancing fermion masses.

A realistic (E)TC model will not be like QCD!

E.g. the top quark mass arises as follows:



and its size is $(\frac{g_{ETC}}{M_{ETC}})^2\langle\bar{T}T\rangle$ x (flavor-dependent factor)

Challenge: ETC must violate custodial symmetry to make $m_t >> m_b$. But how to keep this from causing additional large contributions to $\Delta\rho$?

Walking doesn't help here ... ??

TopColor Assisted Technicolor

If top feels a new strong interaction, a top-quark condensate $\langle \bar{t}t \rangle \neq 0$ is possible

$$G_{TC} \times \overset{(g_h > g_\ell)}{SU(3)_h \times SU(3)_\ell} \times SU(2)_W \times \overset{(g_h > g_\ell)}{U(1)_h \times U(1)_\ell}$$

$$\downarrow \quad M \gtrsim 1 \text{ TeV}$$

$$G_{TC} \times SU(3)_{QCD} \times SU(2)_W \times U(1)_Y$$

$$\downarrow \quad \Lambda_{TC} \sim 1 \text{ TeV}$$

$$G_{TC} \times SU(3)_{QCD} \times U(1)_{EM}$$

Phenomenology:
Topgluons &Z'

technicolor: provides most of EWSB
topcolor: provides most of m_t
hypercharge: keeps m_b small

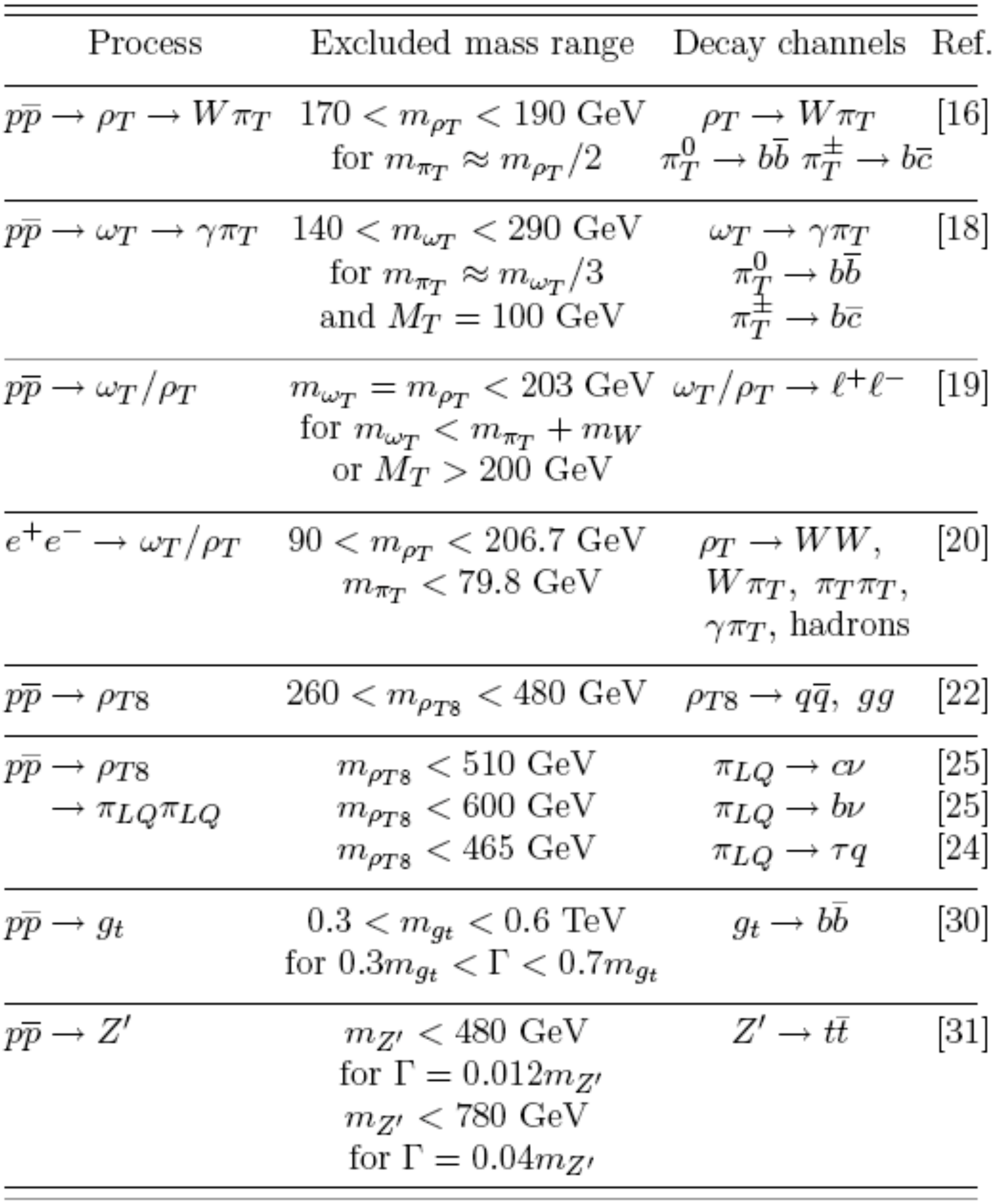
C.T. Hill

Technicolor Limits:

- Model Dependent
- Just Reaching interesting range!
- Run II & LHC will extend limits substantially

No Run II limits yet?

Narain, Womersley, RSC
PDG review



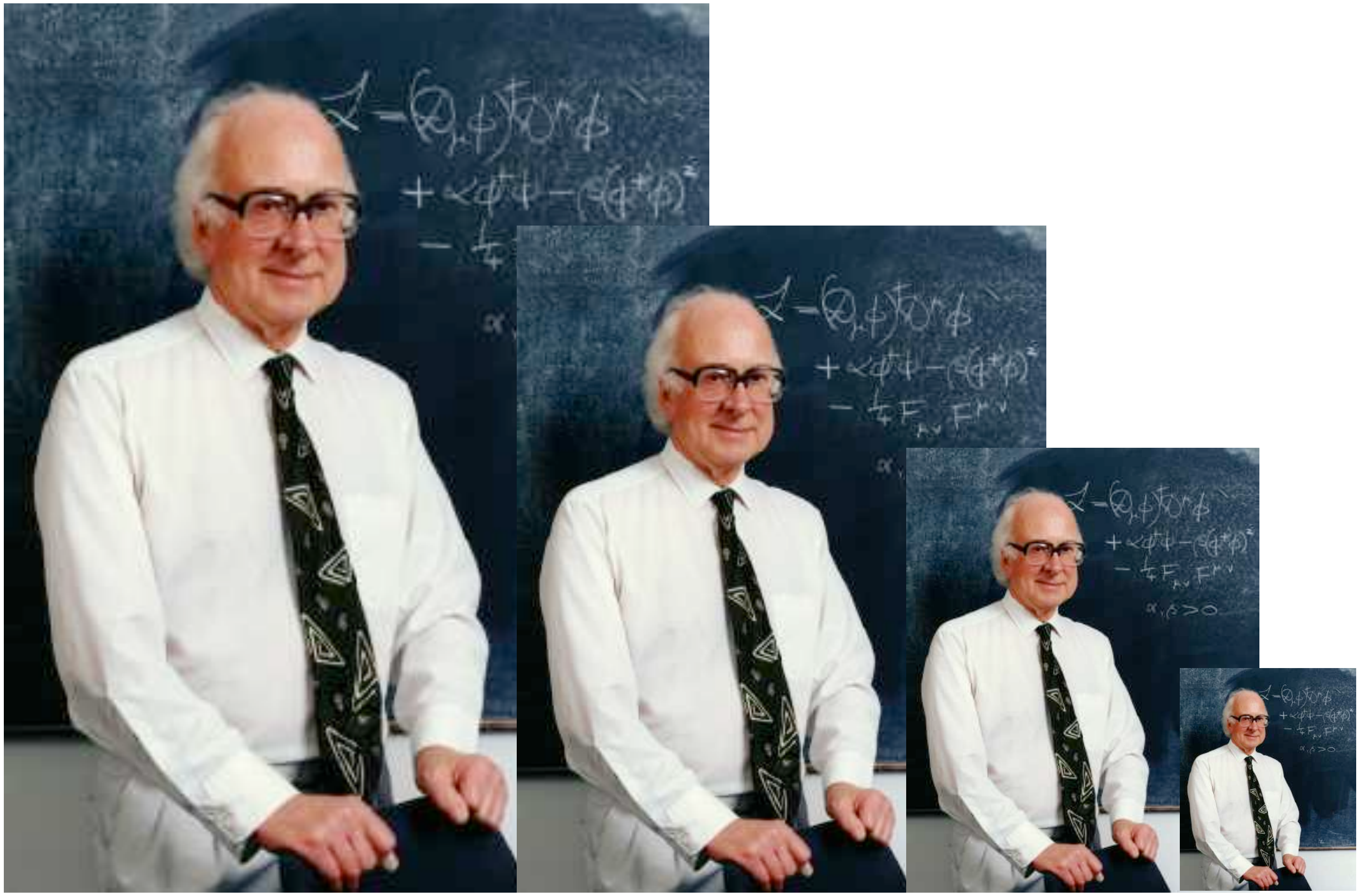
Process	Excluded mass range	Decay channels	Ref.
$p\bar{p} \to \rho_T \to W\pi_T$	$170 < m_{\rho_T} < 190$ GeV for $m_{\pi_T} \approx m_{\rho_T}/2$	$\rho_T \to W\pi_T$ $\pi_T^0 \to b\bar{b}$ $\pi_T^\pm \to b\bar{c}$	[16]
$p\bar{p} \to \omega_T \to \gamma\pi_T$	$140 < m_{\omega_T} < 290$ GeV for $m_{\pi_T} \approx m_{\omega_T}/3$ and $M_T = 100$ GeV	$\omega_T \to \gamma\pi_T$ $\pi_T^0 \to b\bar{b}$ $\pi_T^\pm \to b\bar{c}$	[18]
$p\bar{p} \to \omega_T/\rho_T$	$m_{\omega_T} = m_{\rho_T} < 203$ GeV for $m_{\omega_T} < m_{\pi_T} + m_W$ or $M_T > 200$ GeV	$\omega_T/\rho_T \to \ell^+\ell^-$	[19]
$e^+e^- \to \omega_T/\rho_T$	$90 < m_{\rho_T} < 206.7$ GeV $m_{\pi_T} < 79.8$ GeV	$\rho_T \to WW$, $W\pi_T$, $\pi_T\pi_T$, $\gamma\pi_T$, hadrons	[20]
$p\bar{p} \to \rho_{T8}$	$260 < m_{\rho_{T8}} < 480$ GeV	$\rho_{T8} \to q\bar{q}$, gg	[22]
$p\bar{p} \to \rho_{T8}$ $\to \pi_{LQ}\pi_{LQ}$	$m_{\rho_{T8}} < 510$ GeV	$\pi_{LQ} \to c\nu$	[25]
	$m_{\rho_{T8}} < 600$ GeV	$\pi_{LQ} \to b\nu$	[25]
	$m_{\rho_{T8}} < 465$ GeV	$\pi_{LQ} \to \tau q$	[24]
$p\bar{p} \to g_t$	$0.3 < m_{g_t} < 0.6$ TeV for $0.3 m_{g_t} < \Gamma < 0.7 m_{g_t}$	$g_t \to b\bar{b}$	[30]
$p\bar{p} \to Z'$	$m_{Z'} < 480$ GeV for $\Gamma = 0.012 m_{Z'}$ $m_{Z'} < 780$ GeV for $\Gamma = 0.04 m_{Z'}$	$Z' \to t\bar{t}$	[31]

What about the S-parameter?

Why are we still talking about technicolor?

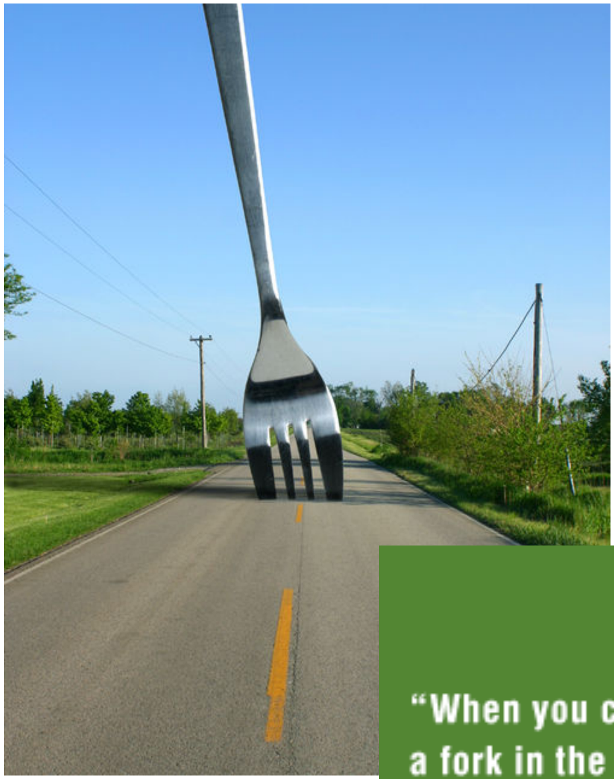
- Technicolor may be there
  - No "computations" of S in non-QCD like theories (S_{QCD}~0.5-1, a few too high)
- Technicolor has interesting experimental signatures
  - Complementary to other BSM theories
- AdS/CFT Correspondence:
  - Some 4D strongly-coupled theories "dual" to weakly-coupled 5D theories
  - New model building ideas
  - Address S parameter issues

Composite Higgs



A Fork in the Road...

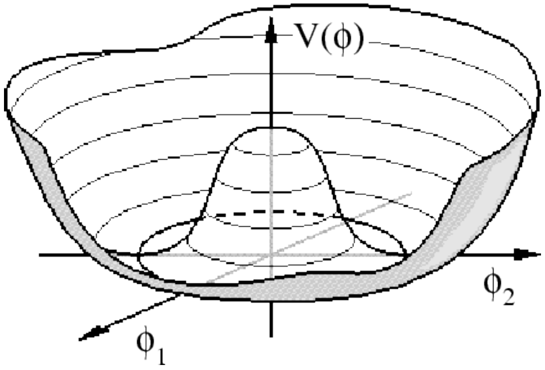
- Make the Higgs Natural: Supersymmetry (Martin, Haber)
- Make the Higgs Composite
  - Little Higgs
  - (Twin Higgs)
- Eliminate the Higgs
  - Technicolor
  - "Higgsless" Models



"When you come to a fork in the road, take it!"
— Yogi Berra

Composite Higgs

Higgs as (Pseudo-)Goldstone Boson:



Hard to do!

$$V(h) = \frac{Cg^2}{16\pi^2}\left(-\eta_2 f^2|h|^2 + \eta_4\frac{|h|^4}{2} + \ldots\right)$$

$g \ll 1$

Decay Constant

Yields: $\langle h\rangle^2 \simeq \frac{\eta_2}{\eta_4}f^2$

But, EWPT: $f > 4\ -\ 5\,\mathrm{TeV}$

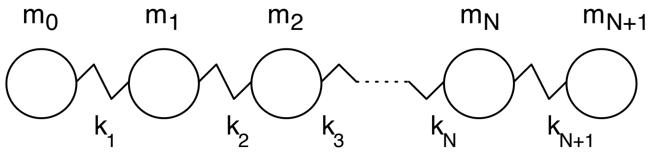
Must suppress η_2 without suppressing η_4

Georgi & Kaplan; Banks

Chacko et. al., hep-ph/0510273

The Little Higgs

Collective Symmetry Breaking:



For weak springs, masses at end very weakly coupled!

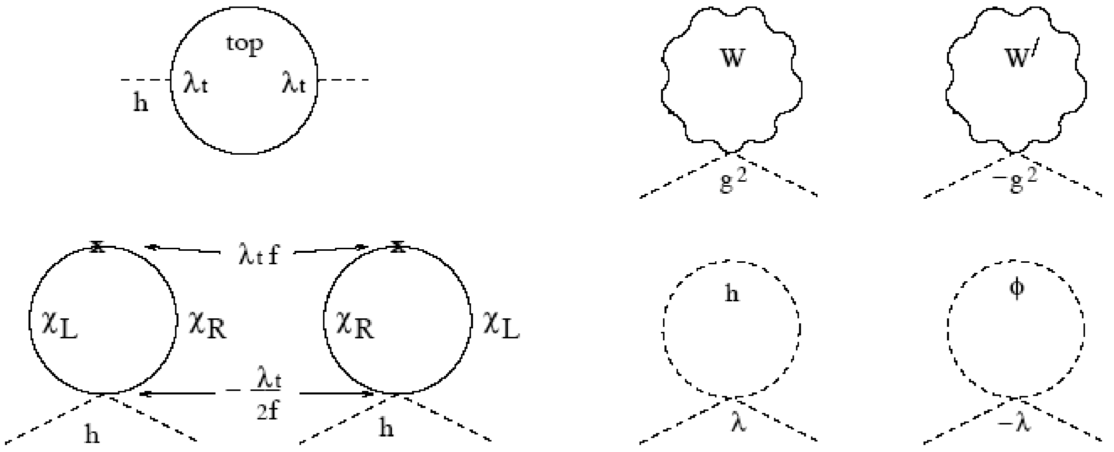
In practice: $\frac{\eta_2}{\eta_4} \simeq \frac{g^2}{16\pi^2}$ $m_h^2 \simeq \frac{g^2}{16\pi^2} f^2$

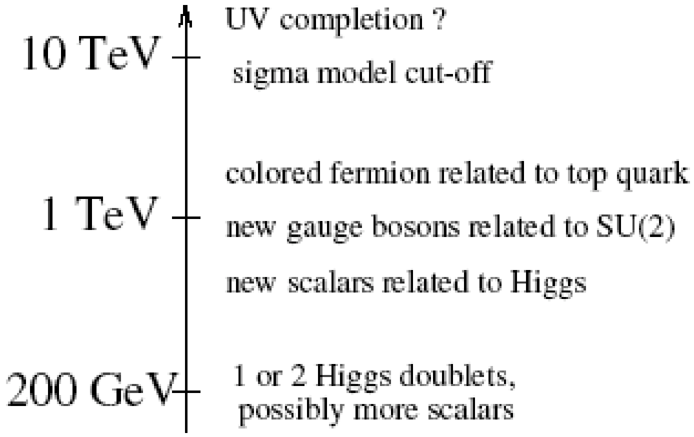
Global Symmetries	Gauge Symmetries	triplet	# Higgs
$SU(5)/SO(5)$	$[SU(2) \times U(1)]^2$	Yes	1
$SU(3)^8/SU(3)^4$	$SU(3) \times SU(2) \times U(1)$	Yes	2
$SU(6)/Sp(6)$	$[SU(2) \times U(1)]^2$	No	2
$SU(4)^4/SU(3)^4$	$SU(4) \times U(1)$	No	2
$SO(5)^8/SO(5)^4$	$SO(5) \times SU(2) \times U(1)$	Yes	2
$SU(9)/SU(8)$	$SU(3) \times U(1)$	No	2
$SO(9)/[SO(5) \times SO(4)]$	$SU(2)^3 \times U(1)$	Yes	1

Arkani-Hamed, Cohen, Georgi

Meade, hep-ph/0402036

Little Higgs : The Hierarchy



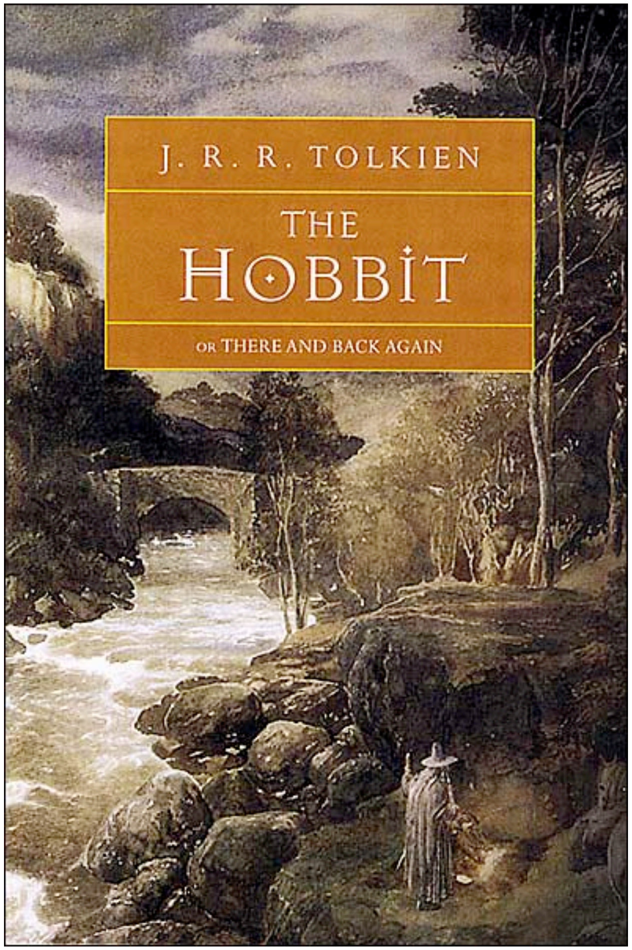


Cancellation of divergences by particles of same spin!

T-Parity: minimize Z-pole effects & DM

Schmaltz hep-ph/0210415

From Technicolor to Extra-Dimensions ... and Back Again: Higgsless Models

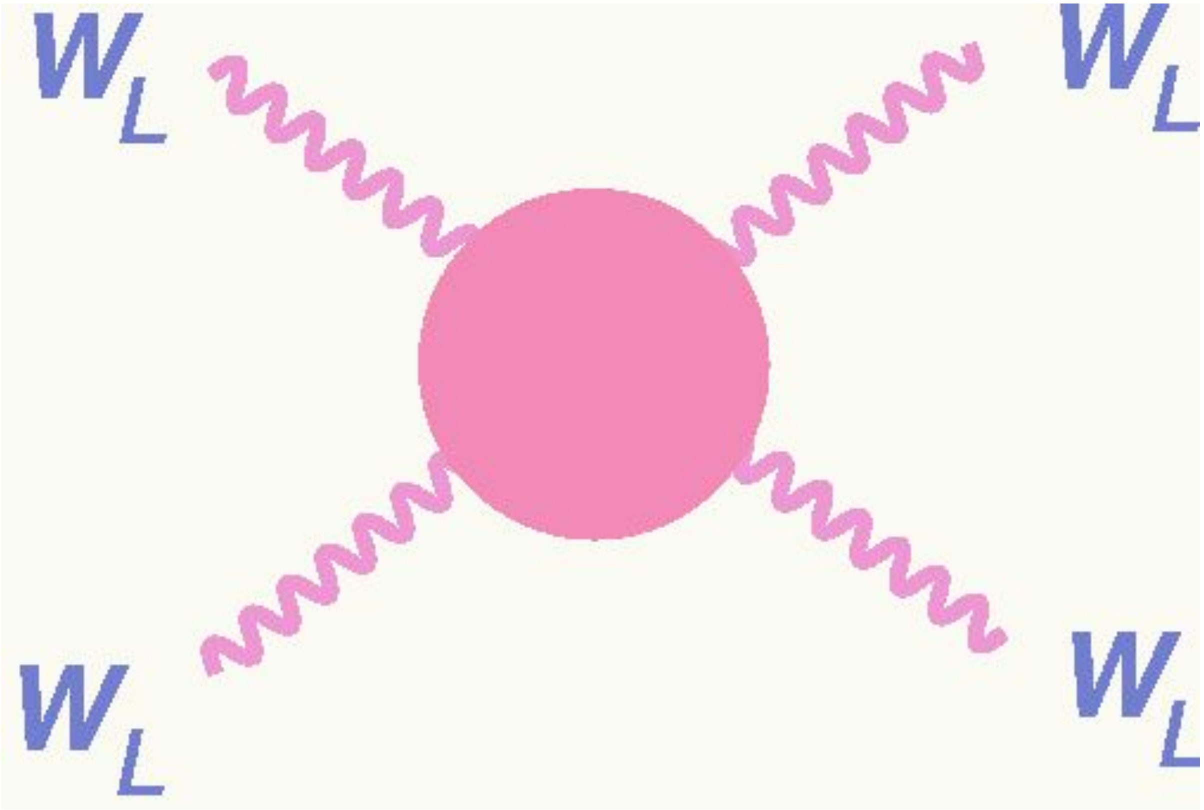

J. R. R. TOLKIEN

THE HOBBIT

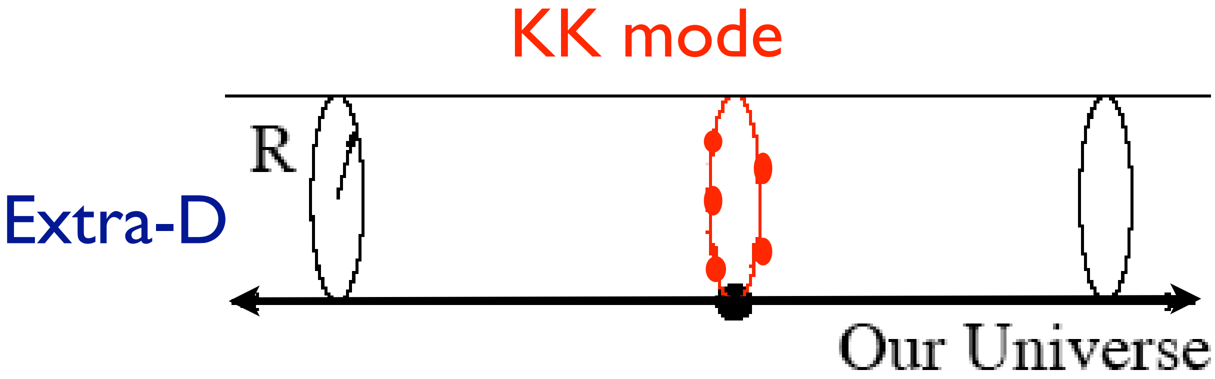
OR THERE AND BACK AGAIN

Can Extra-D be related to EWSB?

Consider Loss of Unitarity in



Extra-D Theories and Massive Vector Boson Scattering



Expand 5-D gauge bosons in eigenmodes: e.g. for S^1/Z_2:

$$\hat{A}^a_\mu = \frac{1}{\sqrt{\pi R}}\left[A^{a0}_\mu(x_\nu) + \sqrt{2}\sum_{n=1}^{\infty} A^{an}_\mu(x_\nu)\cos\left(\frac{n x_5}{R}\right)\right]$$

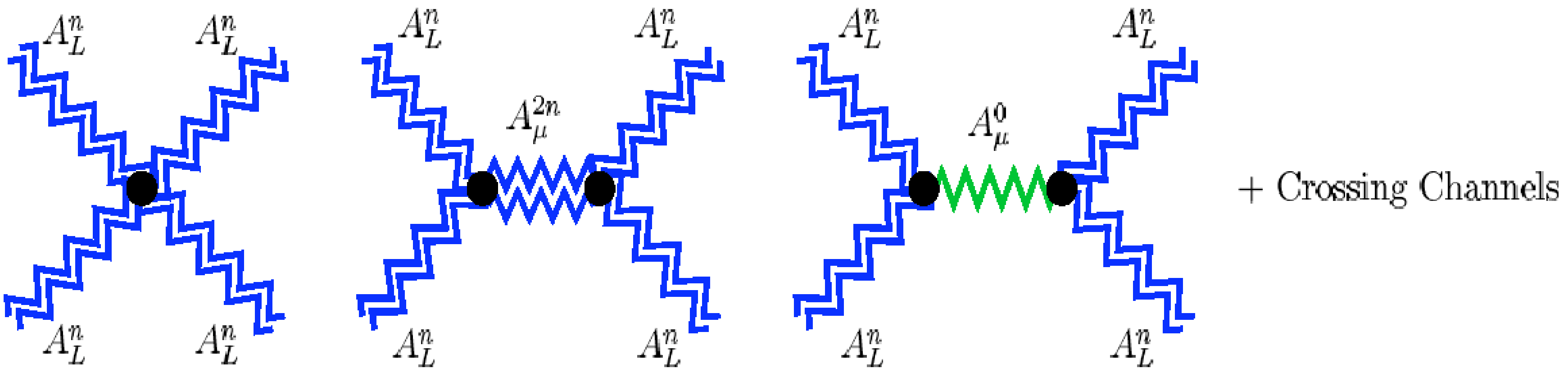
$$\hat{A}^a_5 = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} A^{an}_5(x_\nu)\sin\left(\frac{n x_5}{R}\right)$$

4-D gauge kinetic term contains

$$\frac{1}{2}\sum_{n=1}^{\infty}\left[M_n^2(A^{an}_\mu)^2 - 2M_nA^{an}_\mu\partial^\mu A^{an}_5 + (\partial_\mu A^{an}_5)^2\right]$$

i.e., $A^{an}_L \leftrightarrow A^{an}_5$

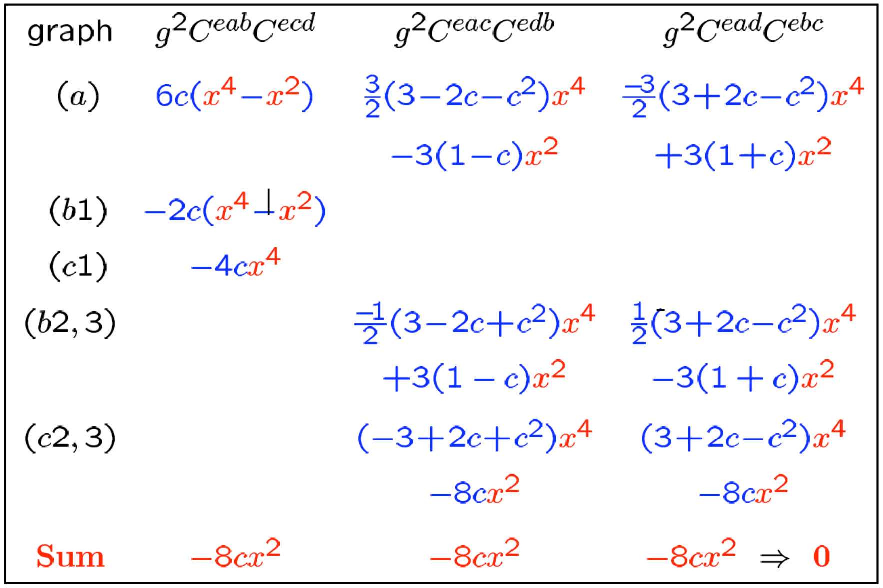
4-D KK Mode Scattering



(a) $(b1)$ $(c1)$ $(b2,b3)+(c2,c3)$

Cancellation of bad high-energy behavior through exchange of massive vector particles

Can we apply this to W and Z?



graph	$g^2C^{eab}C^{ecd}$	$g^2C^{eac}C^{edb}$	$g^2C^{ead}C^{ebc}$
(a)	$6c(x^4-x^2)$	$\frac{3}{2}(3-2c-c^2)x^4$ $-3(1-c)x^2$	$\frac{-3}{2}(3+2c-c^2)x^4$ $+3(1+c)x^2$
$(b1)$	$-2c(x^4+x^2)$		
$(c1)$	$-4cx^4$		
$(b2,3)$		$\frac{-1}{2}(3-2c+c^2)x^4$ $+3(1-c)x^2$	$\frac{1}{2}(3+2c-c^2)x^4$ $-3(1+c)x^2$
$(c2,3)$		$(-3+2c+c^2)x^4$ $-8cx^2$	$(3+2c-c^2)x^4$ $-8cx^2$
Sum	$-8cx^2$	$-8cx^2$	$-8cx^2 \Rightarrow \mathbf{0}$

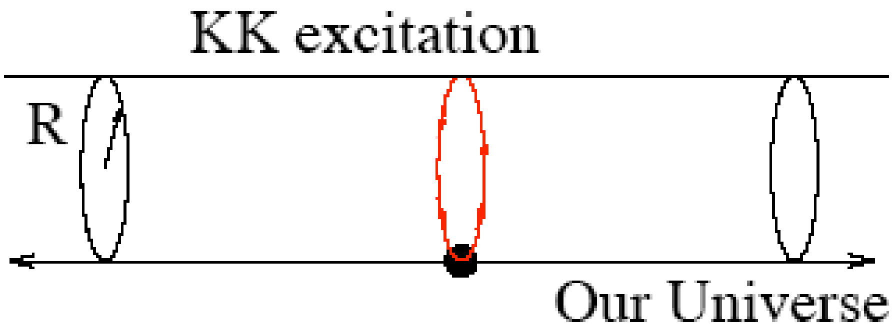
RSC, H.J. He, D. Dicus

Higgsless Models

- Can we use Extra-D/AdS-CFT in EWSB?
- Unitarize TeV-scale $W_L W_L$ scattering using vector bosons?
- If KK modes exist, $M_W << M_{KK}$!!
- Luckily, unitarization generalizes to a large class of 5-d manifolds and boundary conditions!

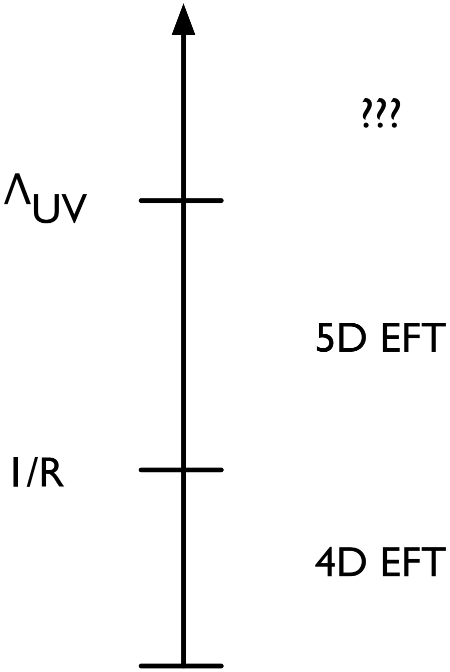
Csaki, Grojean, Murayama, Pilo, Terning

Energy Scales and Couplings



$$\widehat{A}^a_\mu = \frac{1}{\sqrt{\pi R}}\left[A^{a0}_\mu(x_\nu) + \sqrt{2}\sum_{n=1}^{\infty} A^{an}_\mu(x_\nu)\cos\left(\frac{n x_5}{R}\right)\right]$$

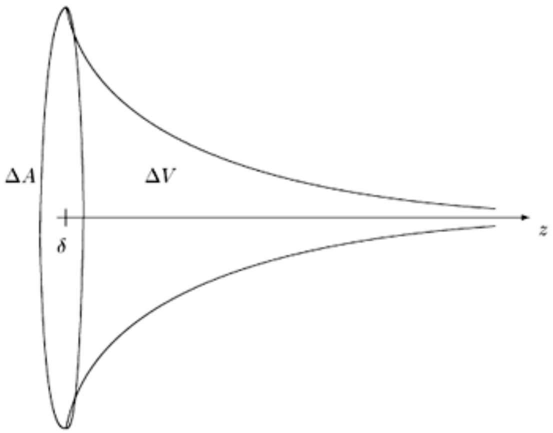
$$\widehat{A}^a_5 = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} A^{an}_5(x_\nu)\sin\left(\frac{n x_5}{R}\right)$$



$$g_4 = \frac{g_5}{\sqrt{\pi R}} \qquad M_n = \frac{n}{R} \qquad \boxed{\Lambda_{UV} \propto \frac{1}{g_5^2}}$$

AdS/CFT Duality

Conjecture: Equivalence of 5D theory in AdS and 4D CFT



UV ⟶ IR

$$ds^2 = \left(\frac{R}{z}\right)^2 \left[\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right]$$

$$R < z < R'$$

NB: Rescaling Invariance!

Strong evidence for N=4 SUSY YM string theory on AdS

Strongly-coupled CFT ⇔ Weakly-coupled 5D Theory!

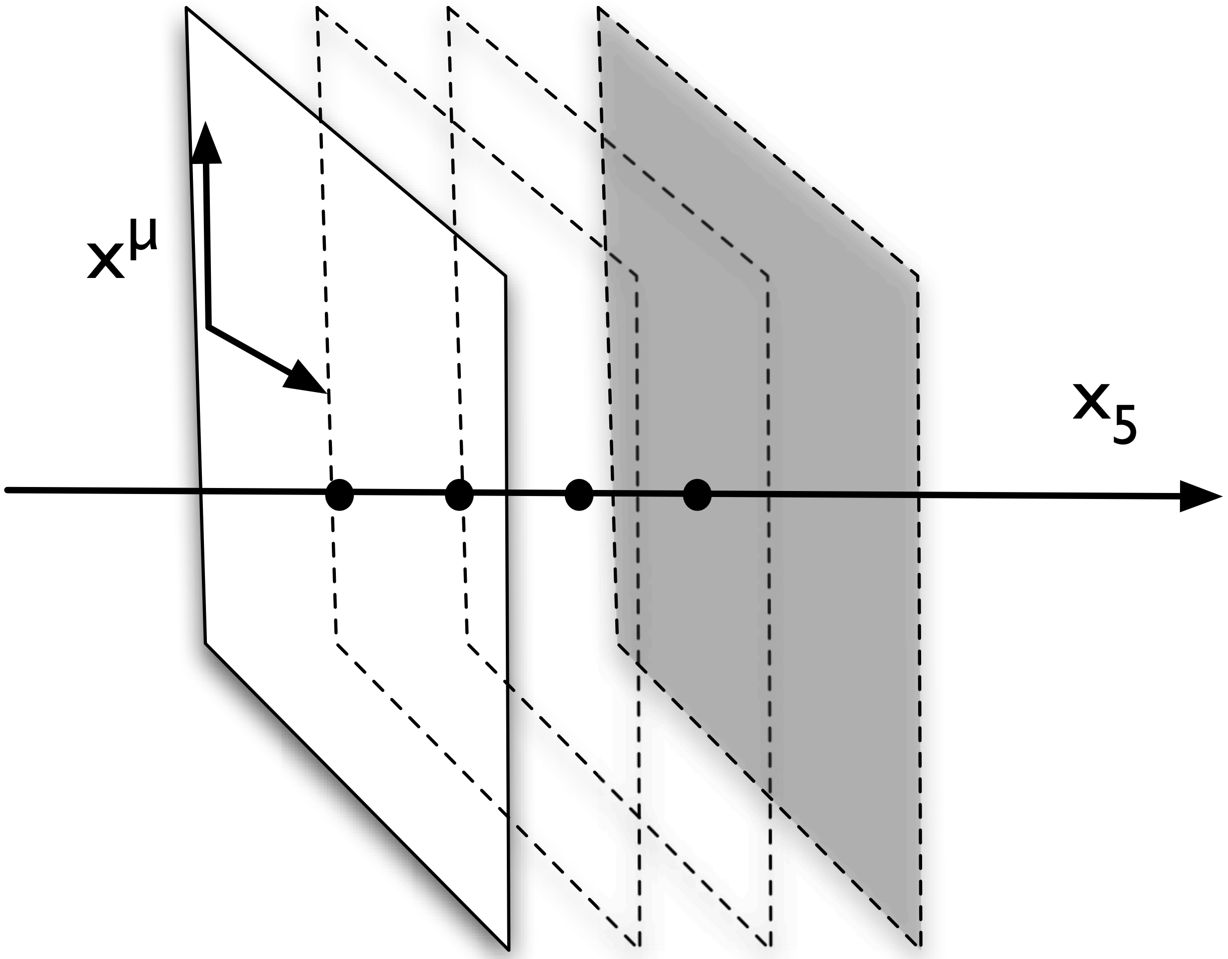
“Walking Technicolor” ⇔ Higgsless Models

Deconstruction

van Gogh

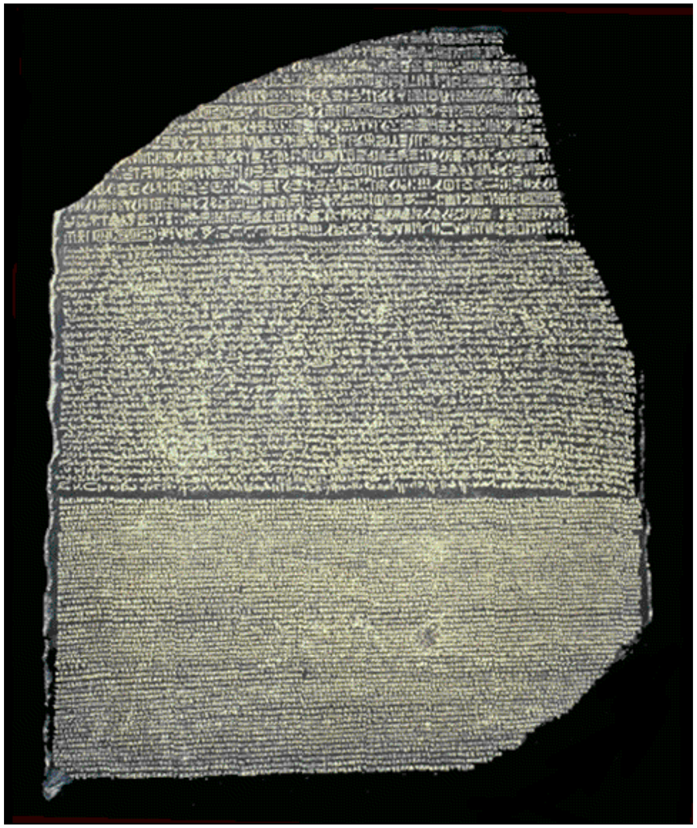
Wolff

Latticize Fifth Dimension



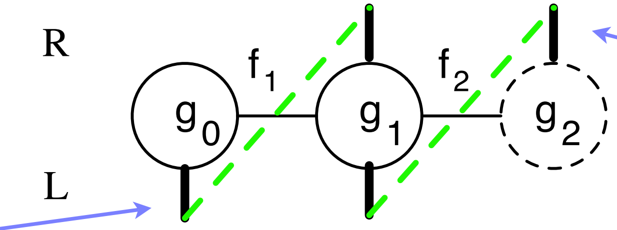
The 3-site Model:

General Higgsless Principles Translated into MonteCarlo



3-Site Model: fermion details

$SU(2) \times SU(2) \times U(1)$ $\qquad$ $g_0,\ g_2 \ll g_1$



RH Boundary Fermion

LH Boundary Fermion

“Bulk Fermion”

$$\mathcal{L}_f = \varepsilon_L M\, \bar{\psi}_{L0}\Sigma_1\psi_{R1} + M\,\bar{\psi}_{R1}\psi_{L1} + M\,\bar{\psi}_{L1}\Sigma_2 \begin{pmatrix}\varepsilon_{tR} & \\ & \varepsilon_{bR}\end{pmatrix}\begin{pmatrix} t_{R2}\\ b_{R2}\end{pmatrix} + h.c.$$

related to degree of delocalization

Fermion Structure Motivated by 5-D

Flavor Structure Identical to Standard Model

RSC, et. al., PRD74:075011 (2006)

3-Site Ideal Delocalization

General ideal delocalization condition $g_i(\psi_i^f)^2 = g_W v_i^w$

becomes $\dfrac{g_0(\psi^f_{L0})^2}{g_1(\psi^f_{L1})^2} = \dfrac{v_W^0}{v_W^1}$ in 3-site model

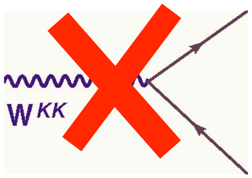
From W, fermion eigenvectors, solve for

$$\epsilon_L^2 \to (1+\epsilon_{fR}^2)^2\left[\frac{x^2}{2} + \left(\frac{1}{8} - \frac{\epsilon_{fR}^2}{2}\right)x^4 + \cdots\right]$$

$$x^2 \equiv \left(\frac{g_0}{g_1}\right)^2 \approx 4\left(\frac{M_W}{M'_W}\right)^2$$

For all but top, $\epsilon_{fR} \ll 1$ and $\varepsilon_L^2 = 2\left(\frac{M_W^2}{M_{W'}^2}\right) + 6\left(\frac{M_W^2}{M_{W'}^2}\right)^2 + \ldots$

insures W' and Z' are fermiophobic!

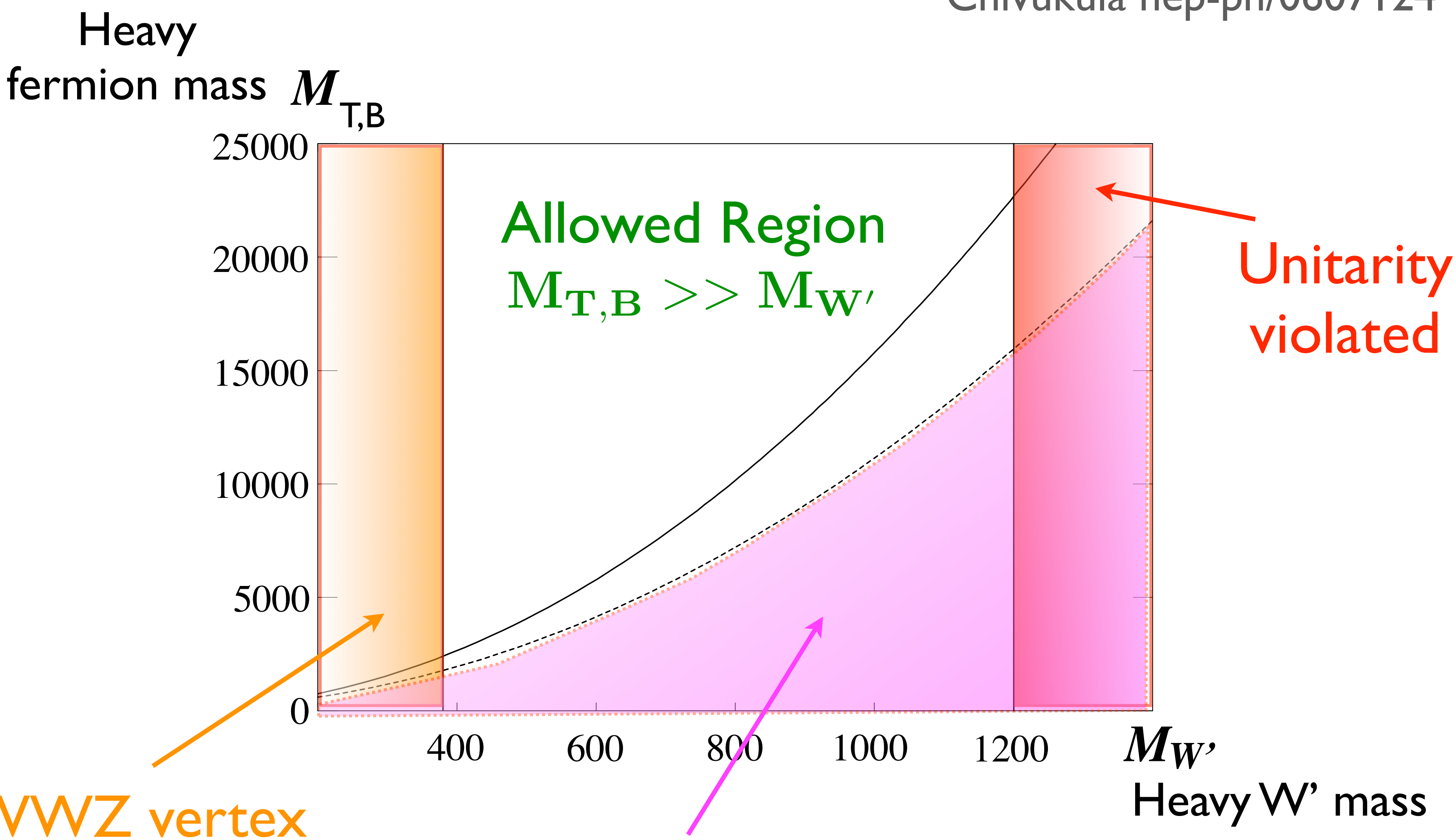


$$\hat{S} = \hat{T} = W = 0$$

$$Y = M_W^2(\Sigma_W - \Sigma_Z)$$

3-Site Parameter Space

Chivukula hep-ph/0607124



Unitarity violated

WWZ vertex visibly altered

1-loop fermionic EW precision corrections too large

3-Site LHC Phenomenology

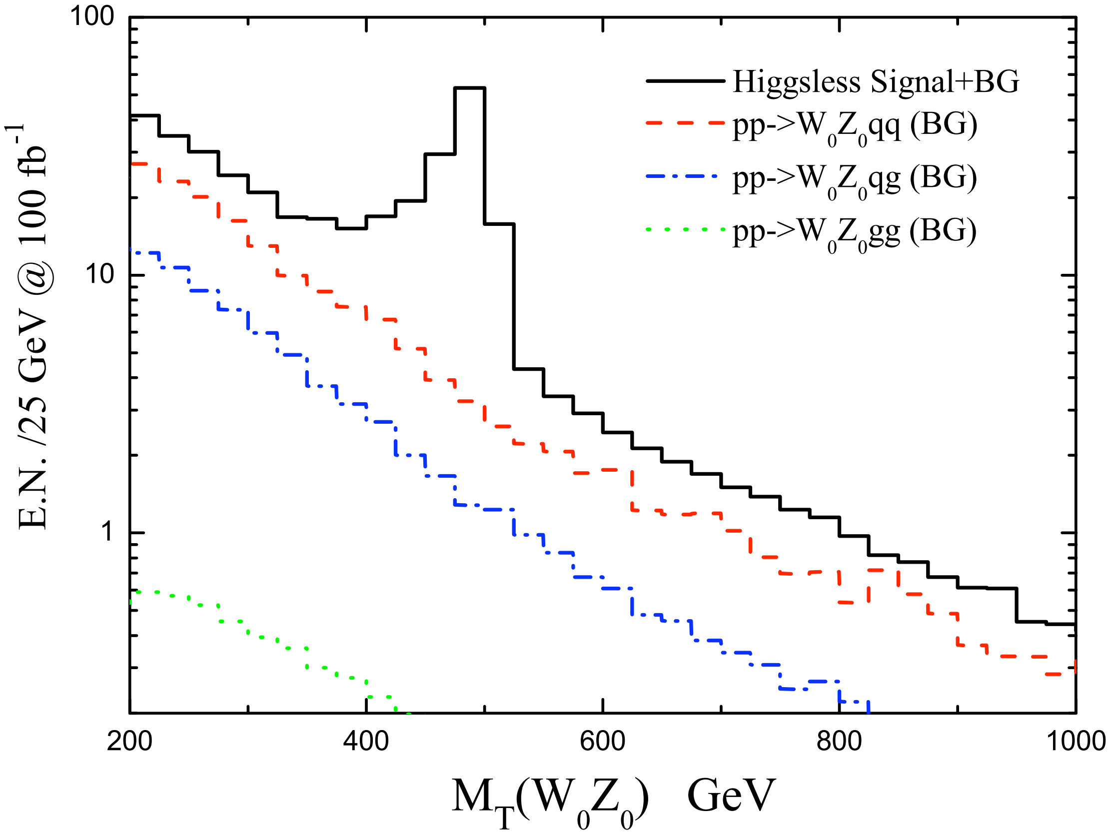
(calculations courtesy of
CalcHEP, MADGRAPH, and HANLIB)

H.J. He, et. al., PRD78: 031701(2008)

See also “Holographic TC” arXiv:0807.2465v1

Vector Boson Fusion (signal in WZjj channel)

$qq \to qqWZ$ 500 GeV W' boson



Background is
10x larger than
estimated in
Birkedal, Matchev &
Perelstein (2005)

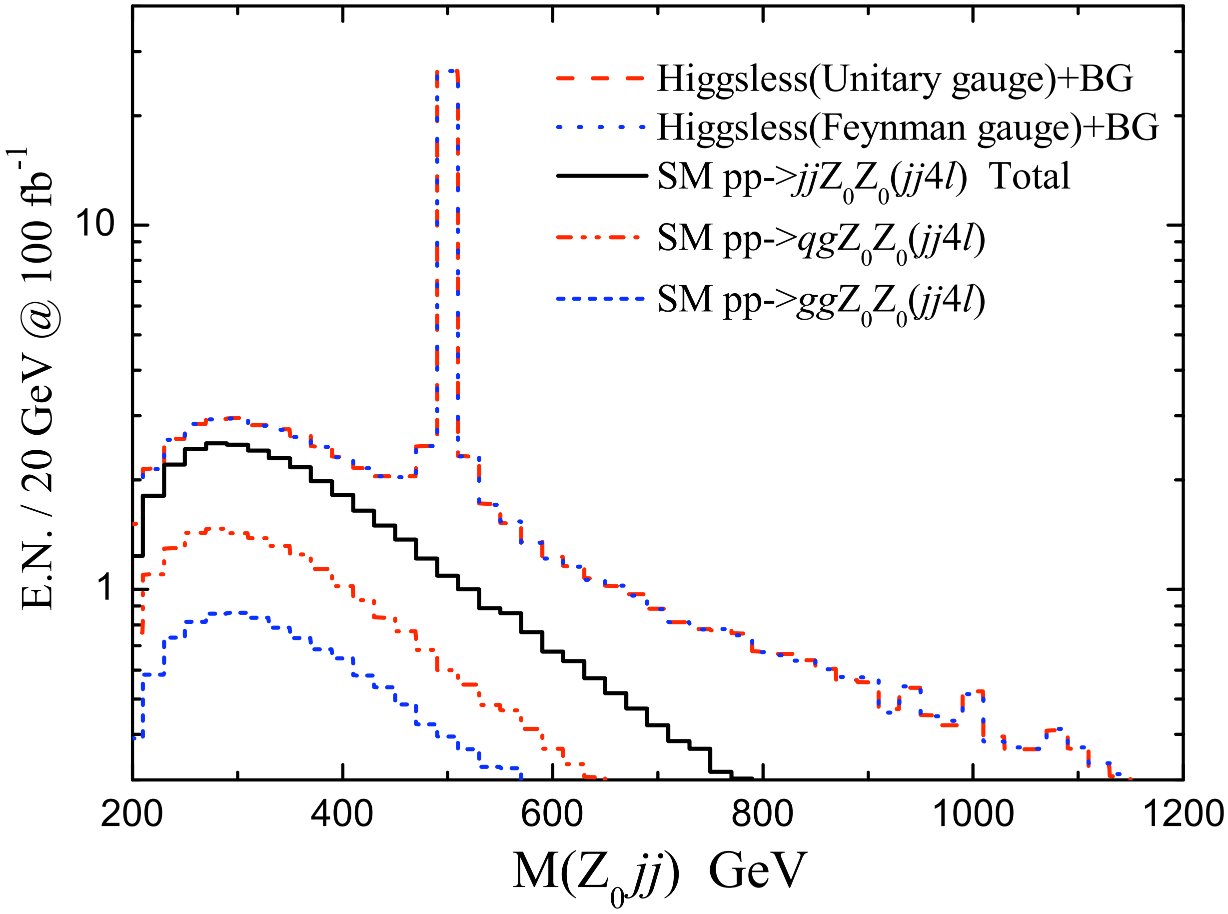
forward jet tag removes WZ background

$$E_j > 300\,\mathrm{GeV}\,, \qquad p_{Tj} > 30\,\mathrm{GeV}\,, \qquad |\eta_j| < 4.5\,, \qquad |\Delta\eta_{jj}| > 4$$

$$p_{T\ell} > 10\,\mathrm{GeV}\,, \qquad |\eta_\ell| < 2.5$$

Associated Production (signal in WZZ channel)

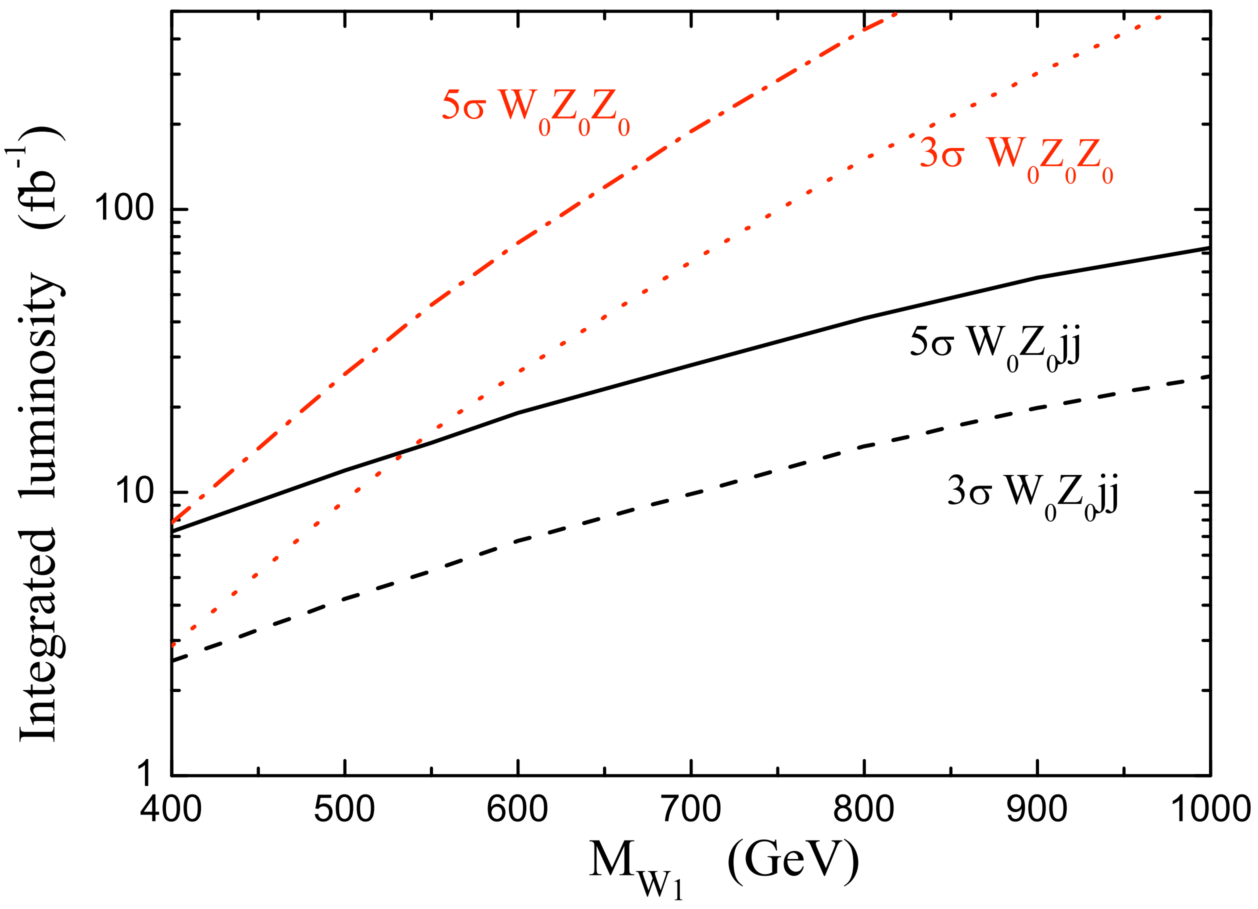
$pp \to W^* \to W'Z \to WZZ$ 500 GeV W' boson



$$M_{jj} = 80 \pm 15\,\text{GeV}, \qquad \Delta R(jj) < 1.5\,, \qquad \sum_Z p_T(Z) + \sum_j p_T(j) = \pm 15\ \text{GeV}.$$

$$p_{T\ell} > 10\,\text{GeV}, \qquad |\eta_\ell| < 2.5\,, \qquad p_{Tj} > 15\,\text{GeV}, \qquad |\eta_j| < 4.5\,.$$

Integrated LHC Luminosity required to discover W' in each channel



3-site Conclusions:

The 3-site model yields a viable effective Higgsless theory of electroweak symmetry breaking valid up to 1.5 - 2 TeV

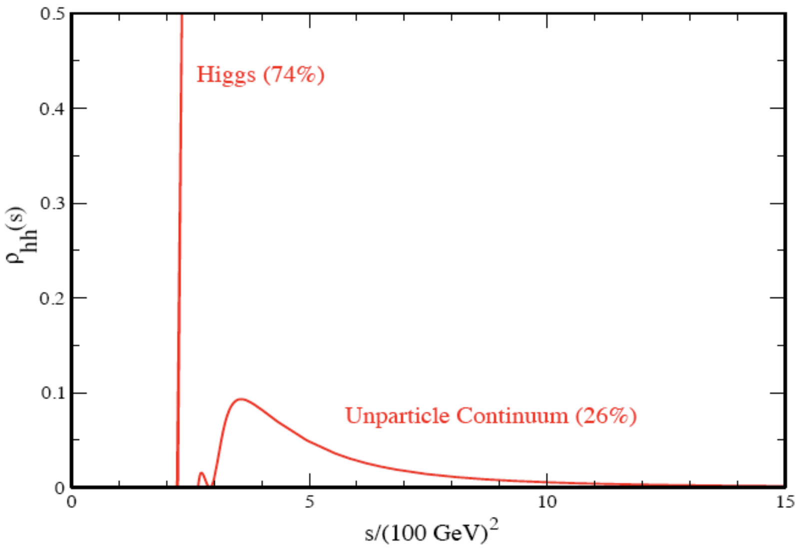
- incorporates / illustrates general principles [Higgsless models, deconstruction, ideal delocalization]
- accommodates flavor [e.g. heavy t quark]
- extra gauge bosons can be relatively light [since they are fermiophobic]
- W' and Z' promise clean multi-lepton signatures at LHC [gauge invariance is key to accurate calculation of rate]

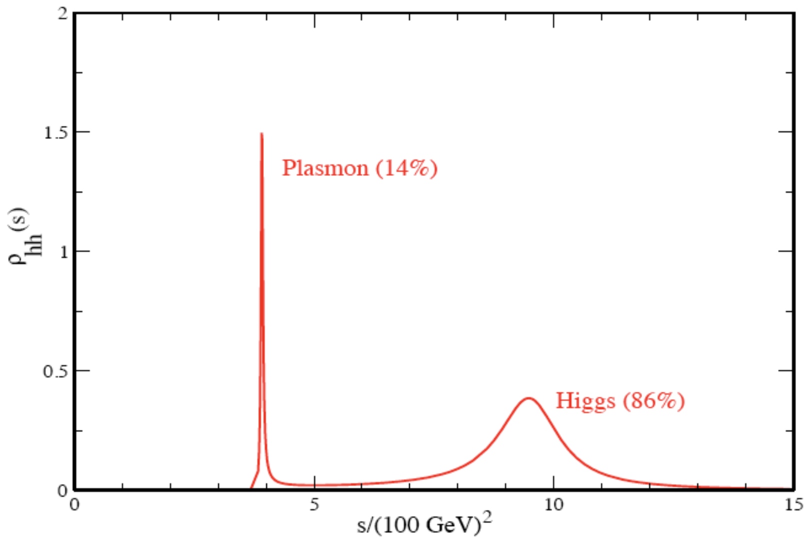
~~Crazy~~ Wild Ideas?

“Higgs” as portal to the unknown (review - J.Wells, arXiv: 0803.1243)

Higgs - Unparticle Mixing: $V_0 = m^2|H|^2 + \lambda|H|^4 + \kappa_U|H|^2\mathcal{O}_U$

New “conformally-invariant” sector





A. Delgado, et. al., arxiv: 0802.2680

Theory Summary

Theory	WW Scattering	Hierarchy Problem	"Calculable" @ LHC?	Precision EW	Λ_{UV}
Fundamental Higgs	I=J=0	YES!	✔	✔	1 TeV - M_{GUT}
SUSY	I=J=0	No	✔	✔	M_{GUT}?
Composite Higgs	I=J=0	No	✔	f > 5 TeV	50 TeV
Higgsless	I=J=1	No	✔	Ideal fermions	10 TeV
Technicolor	I=J=1	No	??	Non-QCD	few TeV

Conclusions

- What unitarizes WW scattering?
- Two new mechanisms, and one old, to address EWSB
  - Technicolor
  - Composite/Little/Twin Higgs
  - Higgsless Models
- All predict new TeV Scale particles, two new predict
  - Extended Electroweak Gauge Symmetries
  - Extended Fermion Sector
- Much Phenomenology Left to be done!

Observations

- Our standards have changed
  - We are content with a low-energy effective theory valid to ~ few TeV
  - This is a good thing in preparation for the LHC ...
- Fine-tuning is in the eye of the beholder
  - S=O(1) in QCD-like technicolor; experimental bound O(0.1) - hence need 10% fine-tuning?
  - Dynamics matters: Inflation makes fine-tuning of flatness problem irrelevant.

Some References

- PDG, “Strong Dynamics Review”, RSC, Narain, & Womersley
- Technicolor: Hill & Simmons, Phys.Rept.381:235-402,2003
- Little Higgs: Schmaltz, Ann.Rev.Nucl.Part.Sci 55:229-270,2005
- Little Higgs: Perelstein, Prog.Part.Nucl.Phys.58:247-291,2007
- Twin Higgs: Goh and Su, Phys.Rev.D75:075010,2007
- Higgsless Models: H. J. He, et. al., Phys.Rev.D78:031701,2008
- “Holographic TC”: Hirn, Martin, Sanz, arXiv:0807.2465

Backup Slides

Composite Higgs:

Global Symmetry Extended to Third Generation

- Top Yukawa Large and breaks chiral symmetries
- Extra singlet quarks added
- Top mass results from seesaw like mixing between doublet and singlet fermions
- EWSB: radiatively induced

Perelstein, hep-ph/0512128

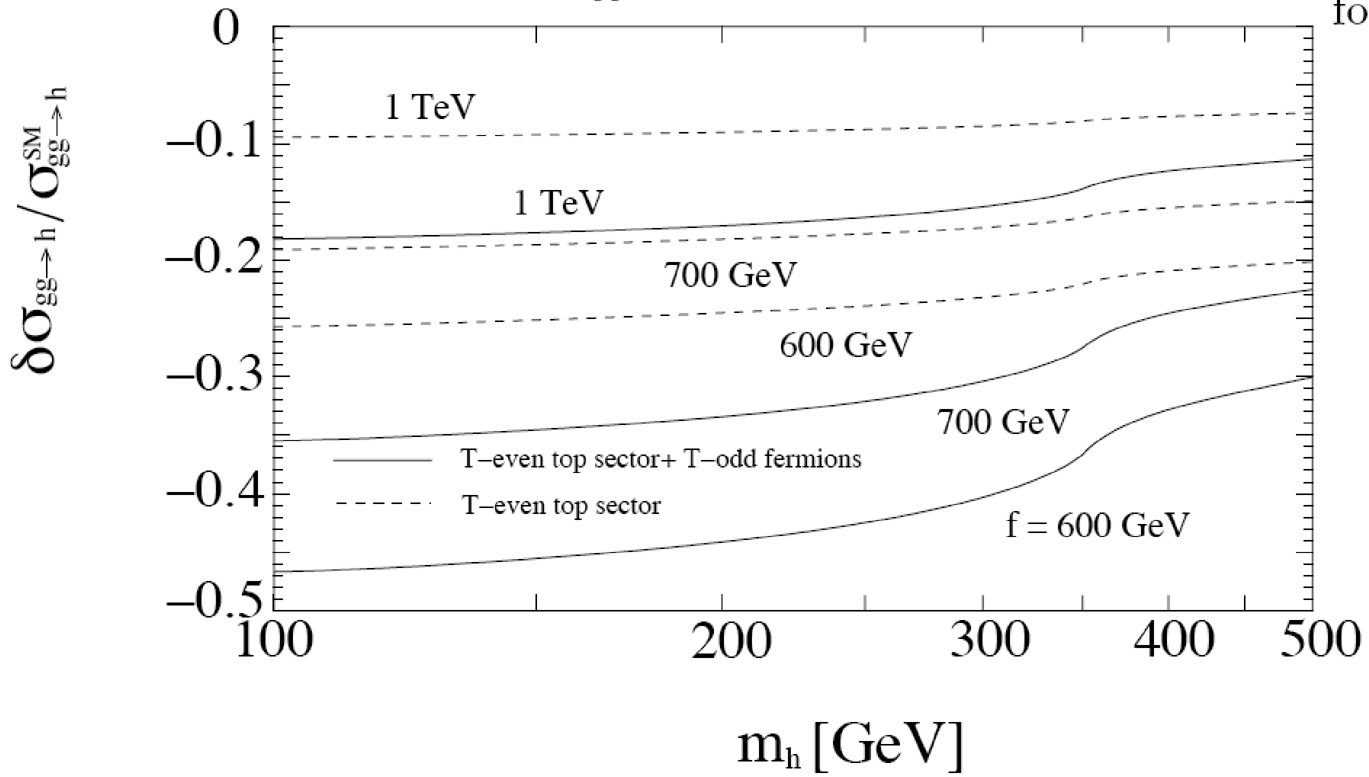
Composite Higgs:

Correction to Higgs production cross section via gluon fusion process

$$\frac{\delta\sigma_{gg\to h}}{\sigma^{\rm SM}_{gg\to h}} \quad (\text{where } \delta\sigma_{gg\to h} = \sigma^{\rm LH}_{gg\to h} - \sigma^{\rm SM}_{gg\to h})$$

$$\frac{\delta\sigma_{gg\to h}}{\sigma^{\rm SM}_{gg\to h}} \simeq -3\frac{v^2_{SM}}{f^2} \simeq \begin{cases} -37\% \text{ for } f = 700 \text{ GeV}, \\ -18\% \text{ for } f = 1000 \text{ GeV}. \end{cases}$$

for small m_h



The production cross section can be significantly suppressed

Chen, Tobe, Yuan

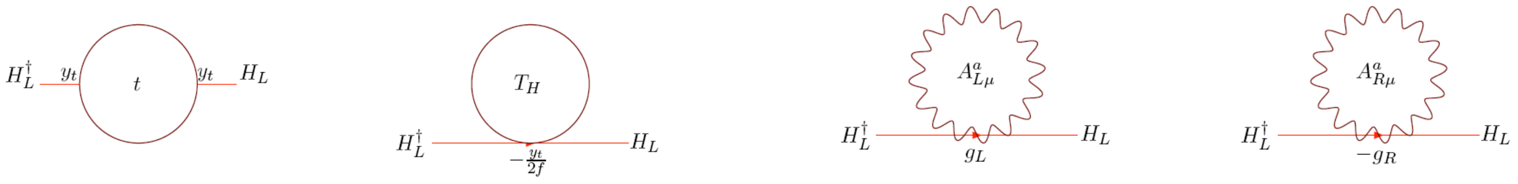
Twin Higgs

- Global SU(4) Symmetry, H in fundamental
  - $$V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$
  - <H>, SU(4) breaks to SU(3); 7 GBs
- Weakly Gauge SU(2)$_W$ x SU(2)$_H$, H=(H$_W$,H$_H$)
  - 3 GBs eaten, 4 remaining are "higgs"
  - $$\Delta V^{(2)} = \frac{9 g_A^2 \Lambda^2}{64\pi^2} H_W^\dagger H_W + \frac{9 g_B^2 \Lambda^2}{64\pi^2} H_H^\dagger H_H$$
- Z$_2$ symmetry: g$_A$=g$_B$
  - Accidental SU(4) symmetry of $\Delta V^{(2)}$
  - No mass generated for higgs boson to O(g^2)

Chacko, Go, and Harnick hep-ph/0506256

Twin Higgs (cont'd)

- Self-coupling $\Delta V^{(4)} \propto \frac{g^4}{16\pi^2} \log\left(\frac{\Lambda}{gf}\right) \left(|H_W|^4 + |H_H|^4\right)$
- Extend SU(4) global symmetry to top-quark sector
- EWSB: Radiatively induced
- Hierarchy : like Little Higgs



Goh, Argonne Workshop 2006

Energy Scales and Couplings with AdS/CFT

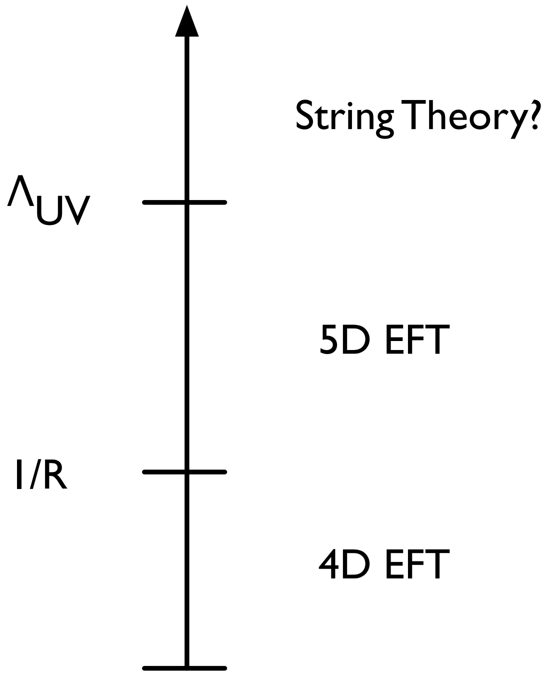
Quantum Corections in 5D KK Theory:

$$\mathcal{O}\left(\frac{g_4^2}{16\pi^2}\right) = \mathcal{O}\left(\frac{1}{N_{CFT}}\right)$$

$$g_4^2 = \frac{g_5^2}{R \log \frac{R'}{R}} \qquad M_{KK} \approx \frac{\pi}{4R'}$$

Naive Dimensional Analysis:

$$\Lambda_{NDA} = \frac{24\pi^3}{g_5^2} \simeq \frac{6\ N_{CFT}\ M_{KK}}{\log \frac{R'}{R}}$$



String Theory?

Λ_{UV}

5D EFT

1/R

4D EFT

How close are we to large-N?

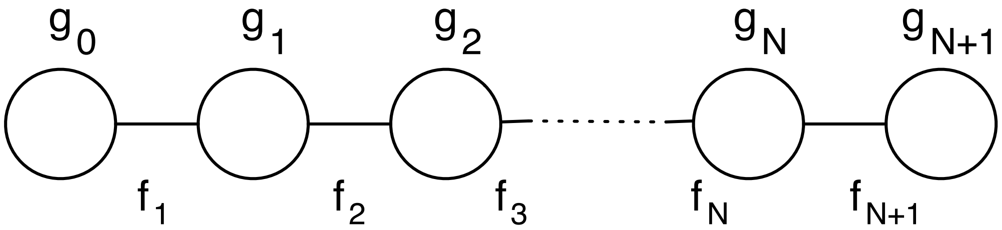
Recipe for a Higgsless Model:

- Choose “bulk” gauge group, location of fermions, and boundary conditions
- Choose $g(x_5)$
- Choose metric/manifold: $g_{MN}(x_5)$
- Calculate spectrum & eigenfunctions
- Calculate fermion couplings
- Compare to Standard Model: S, T, U, ...

Can we do better?

Yes, using deconstruction!

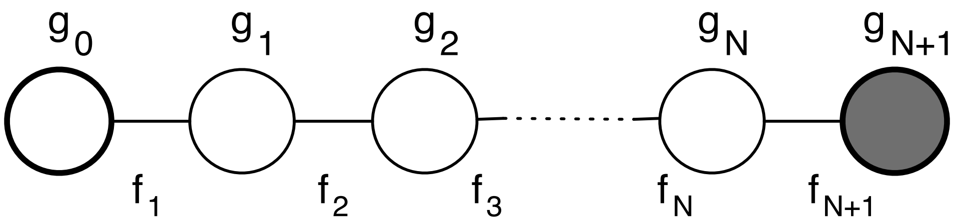
... Back Again: Deconstruction



- Discretize fifth dimension ↔
- 4D gauge group at each site ○
- Nonlinear sigma model link fields —
- To include warping: vary f_j
- For spatially dependent coupling: vary g_k
- Continuum Limit: take N → infinity
- Finite N, a 4D theory!

Arkani-Hamed, Georgi, Cohen & Hill, Pokorski, Wang

Deconstructed Higgsless Models

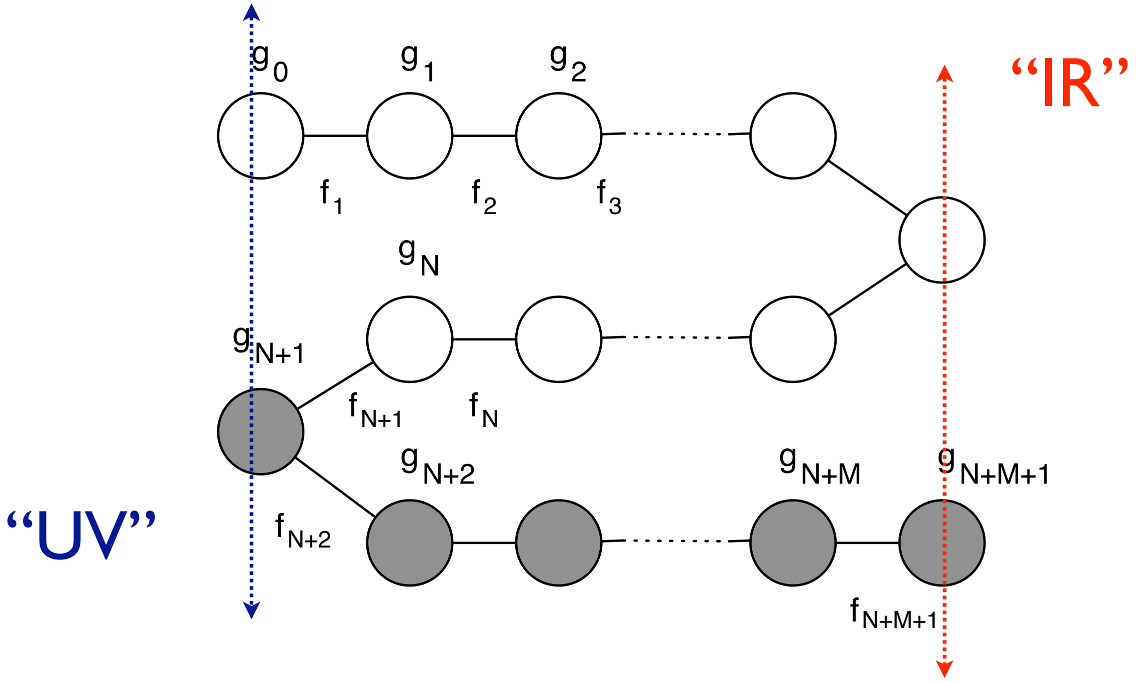


- $SU(2)^N \times U(1)$;   general f_j and g_k
- Fermions sit on “branes” [sites 0 and N+1]
- Many 4-D/5-D theories are limiting cases...
  study them all at once!
- e.g., N=1 equivalent to technicolor/one-Higgs

Foadi, *et. al.*   &   Chivukula *et. al.*

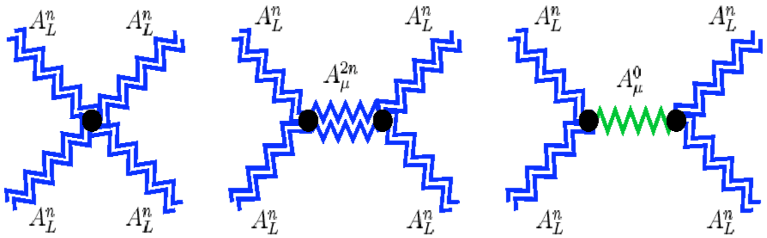
Generalizations

- by folding, represent SU(2) x SU(2) x U(1) in “bulk”
- modify fermions’ location (brane? bulk?)



Conflict of S & Unitarity

Heavy resonances must unitarize WW scattering (since there is no Higgs!)



This bounds lightest KK mode mass: $m_{Z_1} < \sqrt{8\pi}\, v$

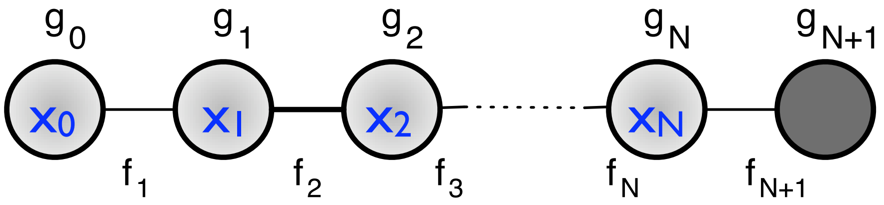
... and yields a value of the S-parameter that is

$$\alpha\, S \geq \frac{4 s_Z^2 c_Z^2 M_Z^2}{8\pi v^2} = \frac{\alpha}{2}$$

too large by a factor of a few!

Independent of warping or gauge couplings chosen...

A New Hope?



Since Higgsless models with localized fermions are not viable, look at:

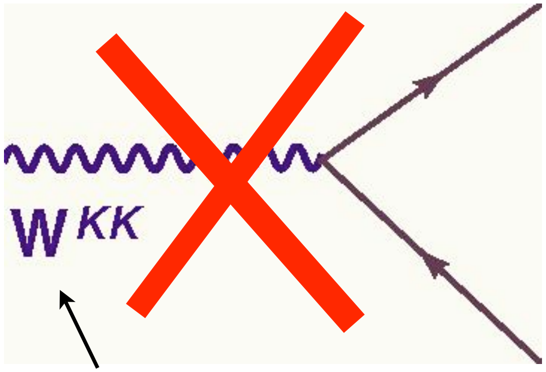
Delocalized Fermions, .i.e., mixing of "brane" and "bulk" modes

$$\mathcal{L}_f = \vec{J}^\mu_L \cdot \left(\sum_{i=0}^{N} x_i \vec{A}^i_\mu \right) + J^\mu_Y A^{N+1}_\mu$$

How will this affect precision EW observables?

Ideal Delocalization

- Choose delocalization related to W wavefunction: $g_i x_i \propto v_i^W$
- NB: $x_i = |\psi_f(i)|^2 > 0$
- W-wavefunction orthogonal to KK wavefunctions.
- No (tree-level) couplings to heavy modes!



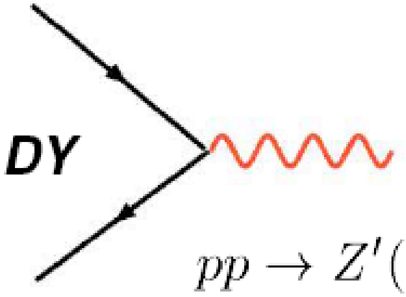
Mass Eigenstate

$$\hat{S} = \hat{T} = W = 0$$

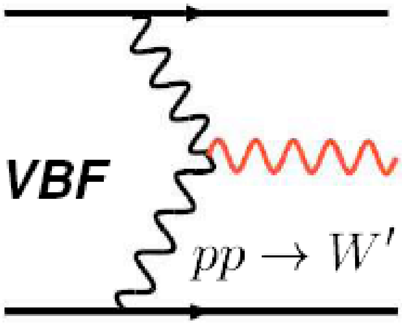
$$Y = M_W^2(\Sigma_W - \Sigma_Z)$$

RSC, HJH, MK, MT, EHS hep-ph/0504114

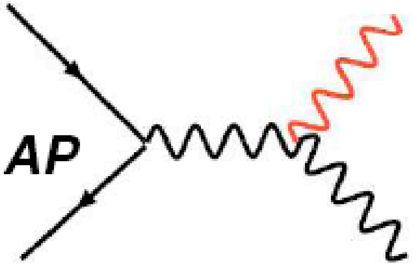
Production and Decay of W’ and Z’ bosons at LHC



$pp \to Z'(W')$

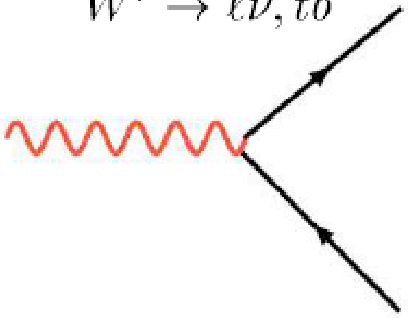


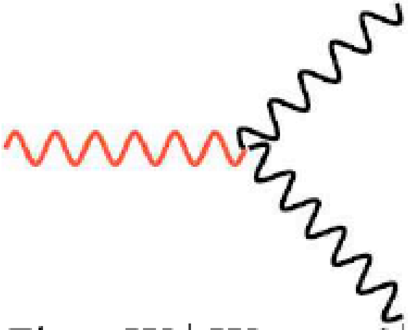
$pp \to W'(Z')j.$



$pp \to WZ', WW', \ldots$

$Z' \to \ell^+\ell^-, t\bar{t}$
$W' \to \ell\nu, tb$

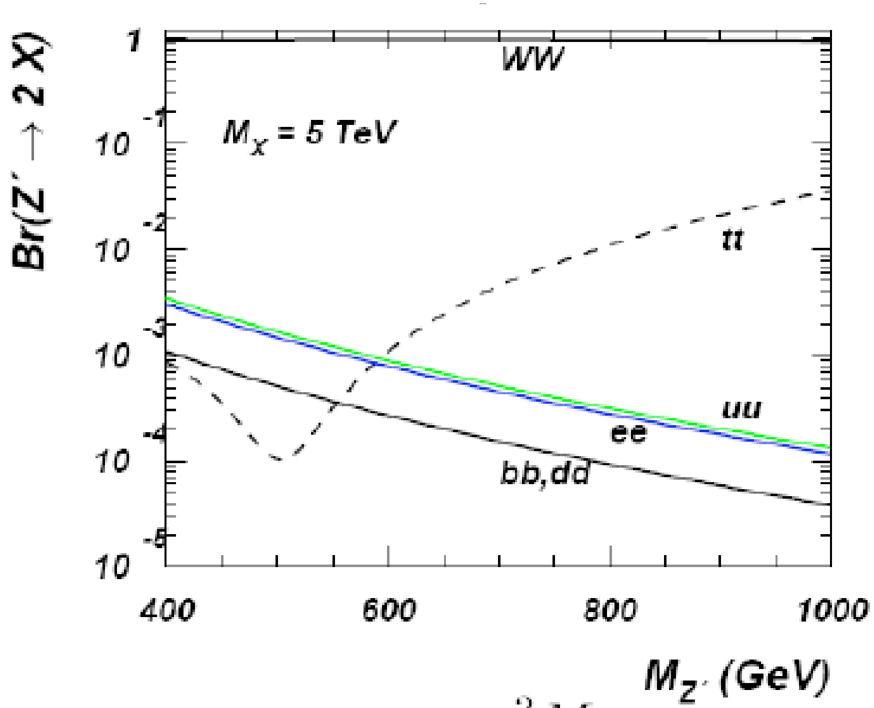


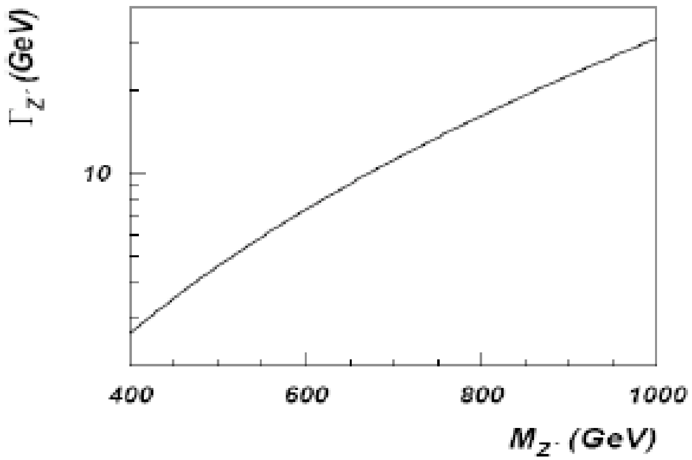


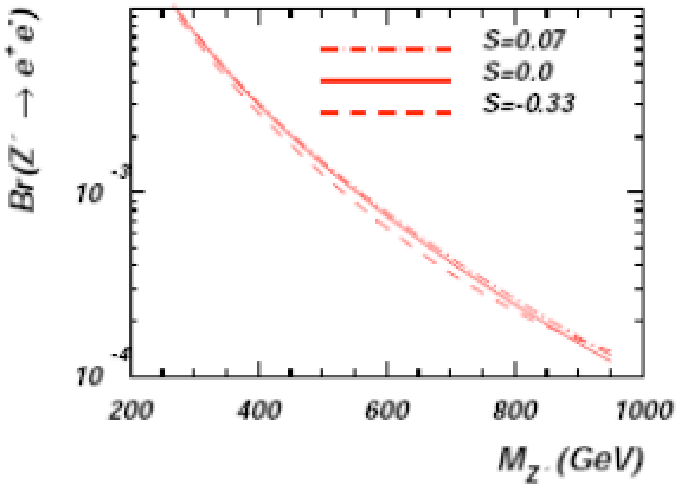
$Z' \to W^+W^- \to \ell^+\ell^-$
$W' \to WZ \to \ell\ell\ell\nu$

Z’ decay modes

- dominant decay is to gauge boson pairs
- BR to fermions not sensitive to deviation from ideal delocalization





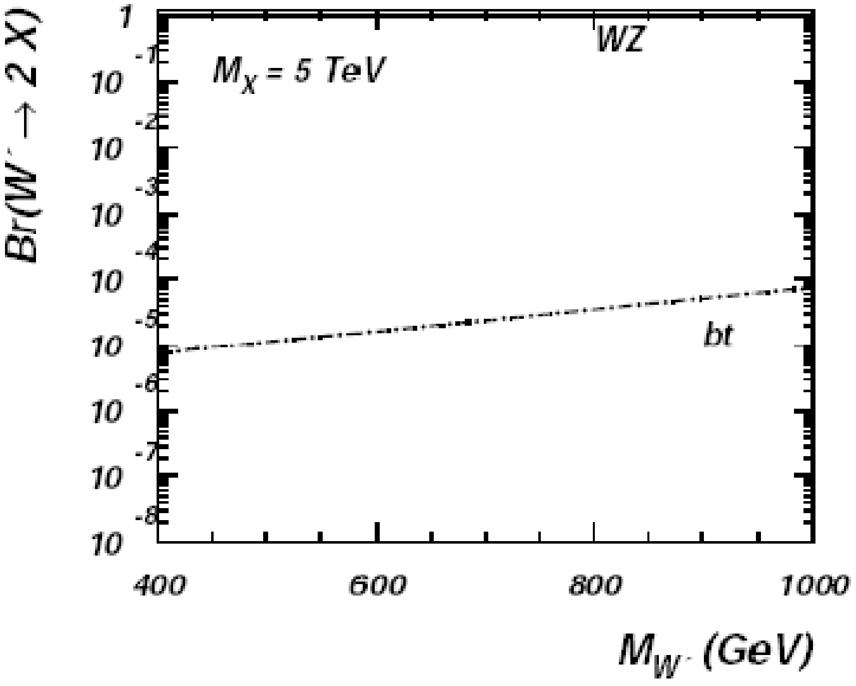


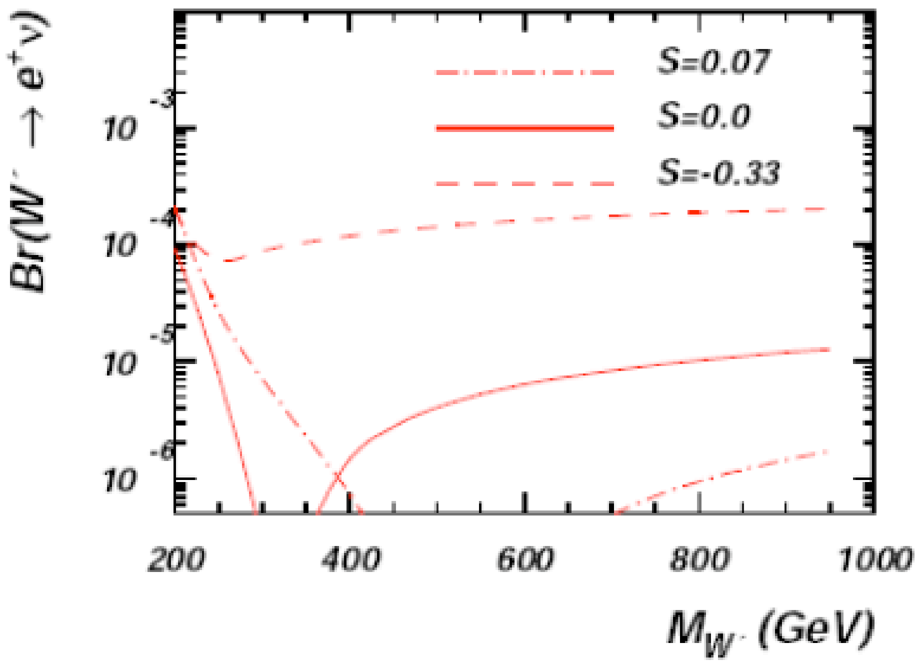
$$\Gamma(Z' \to W^+W^-) = \frac{e^2 M_{W'}}{192\pi \enclose{circle}{x^2} s_w^2}$$

$$\Gamma(Z' \to e^+e^-) = \frac{5e^2 M_{W'} \enclose{circle}{x^2} s_w^2}{384\pi c_w^4}$$

W’ decay modes

- dominant decay is, again, to gauge boson pairs
- BR to fermions is small -- but sensitive to delocalization

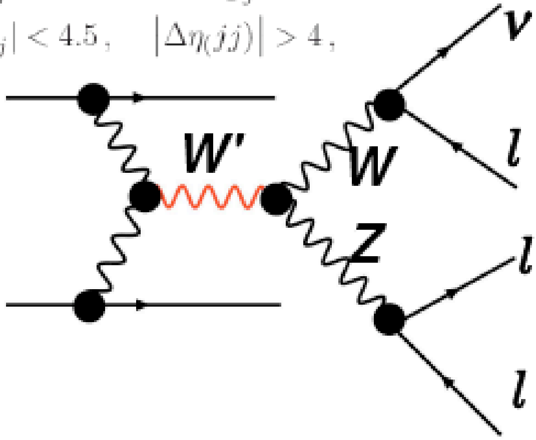


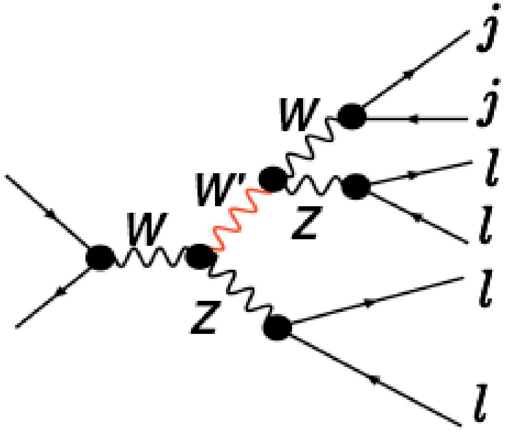


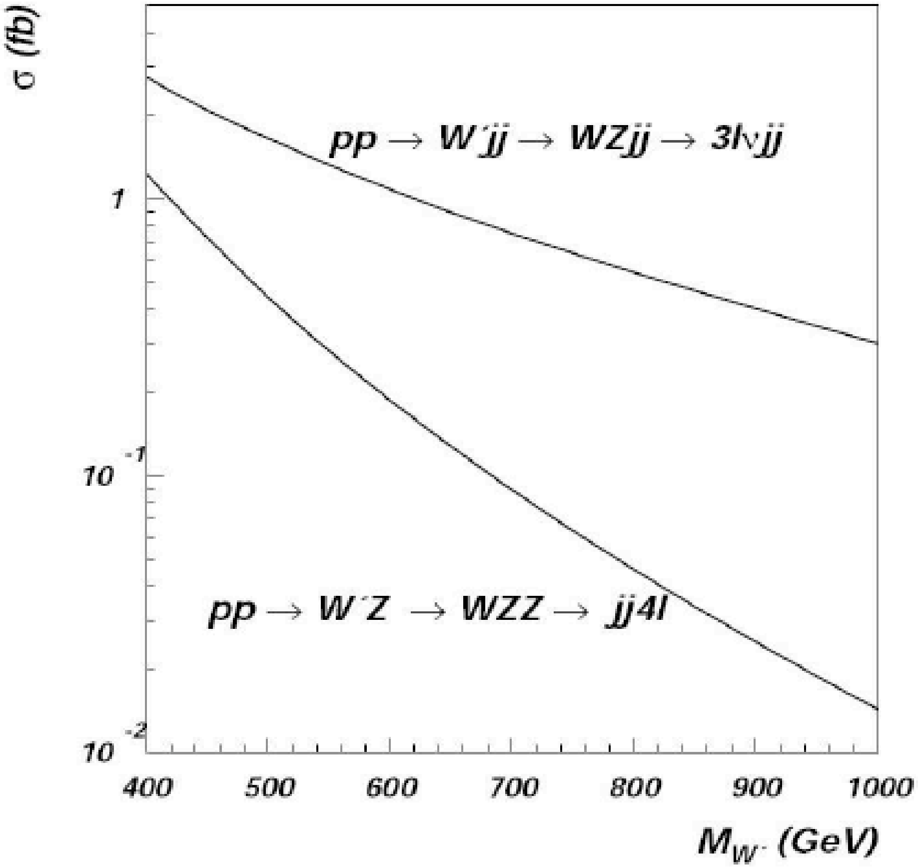
$$\Gamma(W' \to e^+e^-) = \frac{e^2 M_{W'} x^2 \left(1 - \frac{2\epsilon_L^2}{x^2}\right)^2}{192\pi s_w^2}$$

Vector Boson Fusion (WZ → W') and W'Z Associated Production promise large rates and clear signatures

$E_j > 300\,\mathrm{GeV}\,, \quad p_{Tj} > 30\,\mathrm{GeV}$
$|\eta_j| < 4.5\,, \quad |\Delta\eta(jj)| > 4\,,$





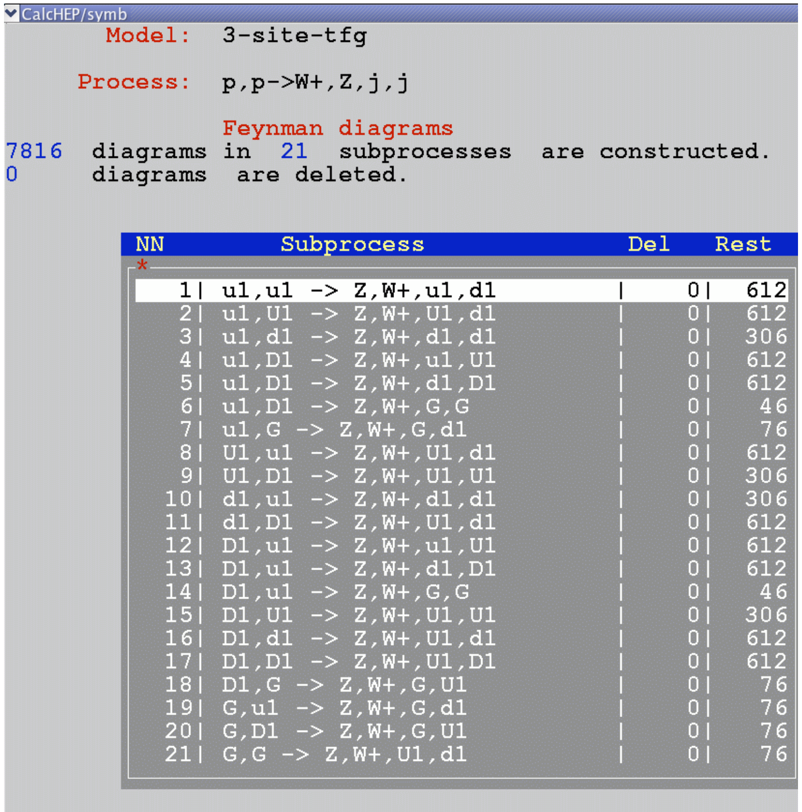


Example: CalcHEP
computation of $pp \to W^+Zjj$

MADGRAPH, and HANLIB also used

- ***No effective WZ approximation.***
- ***Complete set of signal and background diagrams including interference.***

in contrast with Birkedal, Matchev & Perelstein 2005



```
       Model:   3-site-tfg

     Process:   p,p->W+,Z,j,j

                Feynman diagrams
7816   diagrams in  21  subprocesses  are constructed.
0      diagrams  are deleted.

 NN         Subprocess              Del    Rest
 *
   1| u1,u1 -> Z,W+,u1,d1       |    0|  612
   2| u1,U1 -> Z,W+,U1,d1       |    0|  612
   3| u1,d1 -> Z,W+,d1,d1       |    0|  306
   4| u1,D1 -> Z,W+,u1,U1       |    0|  612
   5| u1,D1 -> Z,W+,d1,D1       |    0|  612
   6| u1,D1 -> Z,W+,G,G         |    0|   46
   7| u1,G -> Z,W+,G,d1         |    0|   76
   8| U1,u1 -> Z,W+,U1,d1       |    0|  612
   9| U1,D1 -> Z,W+,U1,U1       |    0|  306
  10| d1,u1 -> Z,W+,d1,d1       |    0|  306
  11| d1,D1 -> Z,W+,U1,d1       |    0|  612
  12| D1,u1 -> Z,W+,u1,U1       |    0|  612
  13| D1,u1 -> Z,W+,d1,D1       |    0|  612
  14| D1,u1 -> Z,W+,G,G         |    0|   46
  15| D1,U1 -> Z,W+,U1,U1       |    0|  306
  16| D1,d1 -> Z,W+,U1,d1       |    0|  612
  17| D1,D1 -> Z,W+,U1,D1       |    0|  612
  18| D1,G -> Z,W+,G,U1         |    0|   76
  19| G,u1 -> Z,W+,G,d1         |    0|   76
  20| G,D1 -> Z,W+,G,U1         |    0|   76
  21| G,G -> Z,W+,U1,d1         |    0|   76
```