The light-fermion contribution to the exact Higgs-gluon form factor in QCD

based on [arXiv:1907.06957]

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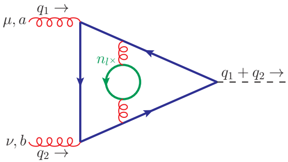
The *ggH* Form Factor



- tensor structure of the amplitude:

$$\mathcal{M}_{ggH}^{ab;\mu\nu} = \delta^{ab} \left[q_2^\mu q_1^\nu - (q_1 \cdot q_2) g^{\mu\nu} \right] C_{ggH}$$

- parameterization of the form factor:

$$C_{ggH} = \frac{1}{v} \frac{\alpha_s}{\pi} \left[C_{ggH}^{(0)} + \frac{\alpha_s}{\pi} C_{ggH}^{(1)} + \left(\frac{\alpha_s}{\pi} \right)^2 \left(C_{ggH}^{(2,0)} + \boxed{n_l C_{ggH}^{(2,1)}} \right) + \mathcal{O}(\alpha_s^3) \right]$$

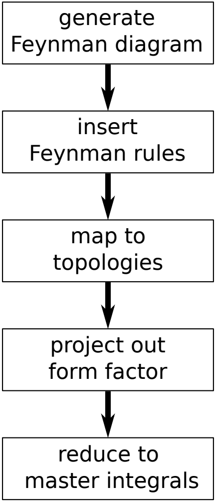
- n_l: number of massless quarks

Higgs Production via Gluon Fusion

- LO (1 loop) [Georgi et al. (1978); Rizzo (1980)]
- today known up to N^3LO in the heavy-top limit (effects of lighter quarks neglected) [Anastasiou et al. (2015), Mistlberger (2018)]
- full quark-mass dependence only available at NLO [Spira et al. (1995); Harlander,Kant (2005); Anastasiou et al. (2006)]
- at NNLO:
  - heavy-top limit [Harlander,Ozeren (2009);Pak et al. (2009)]
  - threshold expansion [Gröber et al. (2018)]

combined to Padé approximation [Davies et al. (2019)]

Toolchain



`qgraf` [Nogueira (1993)]

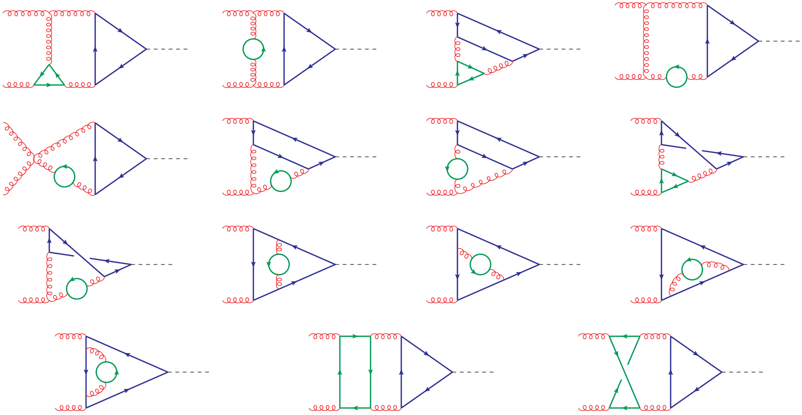
`q2e` [Harlander,Seidenstricker, Steinhauser (1998)]

`exp` [Harlander,Seidenstricker, Steinhauser (1998)]

custom-made `FORM`-Code

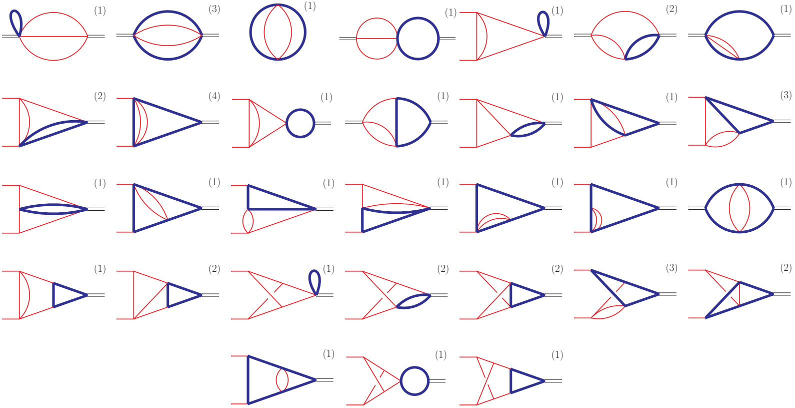
`Kira` [Maierhöfer,Usovitsch, Uwer (2017)]

Feynman Diagrams



60 diagrams

Master Integrals



46 master integrals

System of Coupled Differential Equations

- rescale master integrals to be dimensionless, i.e.
  - $m_q \to 1$
  - $q_1 \cdot q_2 \to 2\tau$ **only kinematic variable**
- let $\vec{f}(\tau, \epsilon) = (I_1, \cdots, I_{46})^T$ be our 46 master integrals.
  - $\epsilon = (4-d)/2$: regulator in DREG
  - $\tau = m_H^2/(4m_q^2)$: dimensionless kinematic variable

- $$\frac{\partial I_n}{\partial \tau} = \int \frac{\mathrm{d}^d k_1}{i\pi^{d/2}} \int \frac{\mathrm{d}^d k_2}{i\pi^{d/2}} \int \frac{\mathrm{d}^d k_3}{i\pi^{d/2}} \frac{\partial}{\partial \tau} \frac{1}{D_1^{a_1} \cdots D_{12}^{a_{12}}}$$

  ➡ linear combination of Feynman integrals of the same topology as I_n or subtopologies

➡ IBP reduction of $\frac{\partial \vec{f}}{\partial \tau}$ yields a closed system of coupled differential equations:

rational functions in τ and ϵ

$$\boxed{\frac{\partial \vec{f}(\tau, \epsilon)}{\partial \tau} = M(\tau, \epsilon) \vec{f}(\tau, \epsilon)}$$

Canonical Basis

- basis of master integral not unique
- $\vec{f}(x,\epsilon)$ is a canonical basis of master integrals if

$$\boxed{M(x,\epsilon) = \epsilon \sum_{x_j \in S} \frac{M^{(x_j)}}{x - x_j}}$$

(ϵ-form)
[Henn (2013)]

- $M^{(x_j)}$ depends neither on x nor on ϵ
- proper choice of kinematic variable x required:

$$x = \frac{\sqrt{1-1/\tau}-1}{\sqrt{1-1/\tau}+1} + i0$$

- only singularities at $x = -1, 0, 1$ appear

Solving Canonical Master Integrals

- differential equation in ϵ-form:

$$\frac{\partial \vec{f}(x,\epsilon)}{\partial x} = \epsilon \sum_{x_j \in S} \frac{M^{(x_j)}}{x - x_j} \vec{f}(x,\epsilon)$$

- set $\vec{f}(x,\epsilon) = U(x, x_0; \epsilon)\vec{f}(x_0,\epsilon)$
- ansatz, power series in ϵ: **boundary conditions**

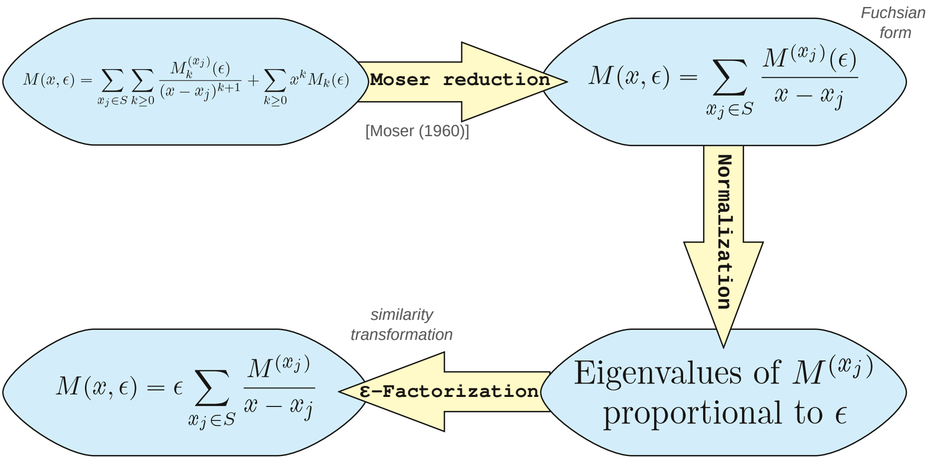
$$U(x, x_0; \epsilon) = \sum_{n=0}^{\infty} \epsilon^n U_n(x, x_0)$$

$$U_0(x, x_0) = \mathbb{1}$$

$$U_n(x, x_0) = \sum_{x_j \in S} M^{(x_j)} \int_{x_0}^{x} \frac{\mathrm{d}x'}{x' - x_j} U_{n-1}(x', x_0) \quad ; \quad n > 0$$

- evaluates to generalized polylogarithms (GPLs)

Lee's algorithm



[Lee (2014)]

C++-implementation: `epsilon` [MP (2017)]

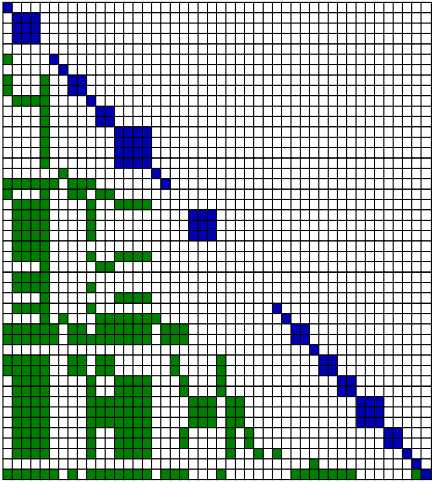
Block-Triangular Structure

- Unfortunately: a 46×46-matrix is way too large for Lee's algorithm
  - exploit block-triangular structure of $M(x, \epsilon)$

- order master integrals in $\vec{f}(x, \epsilon)$ by an increasing number of lines
  - block-triangular structure of $M(x, \epsilon)$
- transform all diagonal blocks into ϵ-form
- remove all higher order poles in x of off-diagonal blocks
  - global Fuchsian form, but only diagonal blocks in ϵ-form

runtime of `epsilon` $\approx$ 1 hour

Block-Triangular Structure

$M(x, \epsilon) =$ 

- □ entry is 0
- ■ entry in ϵ-form
- ■ entry is Fuchsian

Solving the Differential Equations

can we solve for the master integrals even if the differential equation is *not* in global ϵ-form?

- write the differential equation of a certain block as

Fuchsian form

$$\frac{\partial}{\partial x}\vec{f}(x,\epsilon) = \epsilon M(x)\vec{f}(x,\epsilon) + B(x,\epsilon)\vec{g}(x,\epsilon)$$

ϵ-form **already solved**

- solve homogeneous part as before, i.e. with an evolution operator $U(x, x_0; \epsilon)$ fulfilling

$$\frac{\partial U(x,x_0;\epsilon)}{\partial x} = \epsilon M(x) U(x,x_0;\epsilon)$$

- use variation of constants to include inhomogeneous part, i.e. set $\vec{f}(x,\epsilon) = U(x,x_0;\epsilon)\vec{k}(x,\epsilon)$:

$$\Rightarrow \quad U(x,x_0;\epsilon)\frac{\partial}{\partial x}\vec{k}(x,\epsilon) = B(x,\epsilon)\vec{g}(x,\epsilon)$$

Solving the Differential Equations

- integrate:

$$\vec{k}(x,\epsilon) = \int_{x_0}^{x} \mathrm{d}x' \, U(x_0, x'; \epsilon) B(x', \epsilon) \vec{g}(x', \epsilon) + \vec{f}(x_0; \epsilon)$$

- multiply by $U(x, x_0; \epsilon)$:

$$\vec{f}(x,\epsilon) = \int_{x_0}^{x} \mathrm{d}x' \, U(x, x'; \epsilon) B(x', \epsilon) \vec{g}(x', \epsilon) + U(x, x_0; \epsilon) \vec{f}(x_0; \epsilon)$$

boundary conditions

- remember:
  - $B(x,\epsilon)$ is Fuchsian (i.e. only simple poles in x)
  - $U(x, x_0; \epsilon)$ was solved solely in terms of generalized polylogarithms
  - $\vec{g}(x,\epsilon)$ are linear combinations of generalized polylogarithms
    - ➡ $\vec{f}(x,\epsilon)$ are linear combinations of generalized polylogarithms as well
- boundary conditions at $x_0 = 1$ (heavy-quark limit) via exp [Harlander, et al. (1998)]

Color Structure

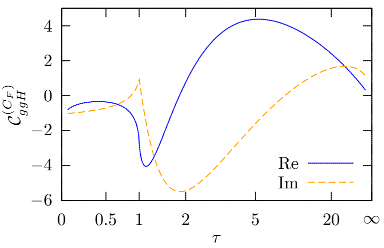
- color structure of $\mathcal{O}(n_l)$-contribution:

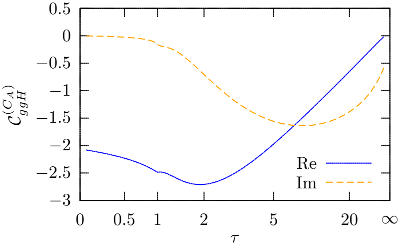
UV renormalized & IR subtracted

$$\tilde{C}_{ggH}^{(2,1)} = T_F^2 C_F \mathcal{C}_{ggH}^{(C_F)} + T_F^2 C_A \mathcal{C}_{ggH}^{(C_A)}$$

- C_F: fundamental Casimir eigenvalue (for QCD: $C_F = 4/3$)
- C_A: adjoint Casimir eigenvalue (for QCD: $C_A = 3$)
- T_F: trace normalization ($T_F = 1/2$)

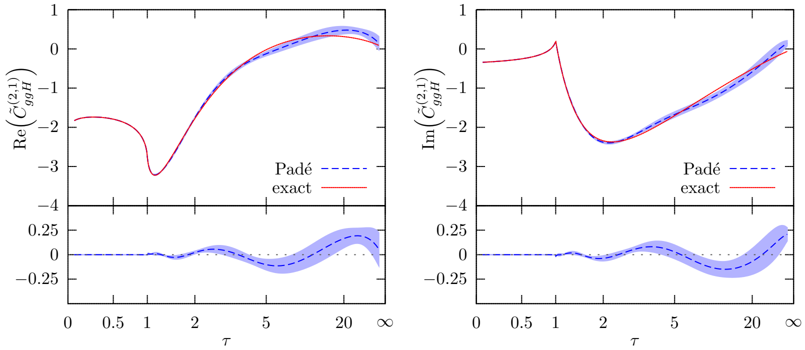
Results





- $$\tau = \frac{m_H^2}{4m_q^2}$$
- renormalization scale set to $\mu^2 = -m_H^2$

Comparison



- comparison to Padé approximation of [arXiv:1906.00982; Davies,Gröber,Maier,Rauh,Steinhauser]
- color factors set to QCD values ($C_F = 4/3,\ C_A = 3,\ T_F = 1/2$)
- lower panel: difference between Padé approximation and our result

Summary & Outlook

- the *ggH* form factor was, until recently, only known in the heavy-top limit at NNLO
- the calculation of the contribution to the three-loop *ggH* form factor containing a massless quark loop was presented
- the result was obtained using modern developments within the method of differential equations
- it agrees perfectly with recent Padé approximations

- it is known that the $\mathcal{O}(n_l^0)$-contribution contains elliptic master integrals
  - can the elliptic sector be expressed in terms of elliptic polylogarithms?
  - ... or expanded in a small quark mass?