Karlsruhe Institute of Technology

TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

NNLO real corrections to Higgs boson pair production in the large top mass limit

Florian Herren | 14.08.2019

based on [Davies, FH, Mishima, Steinhauser JHEP 1905 (2019) 157, [WIP]]

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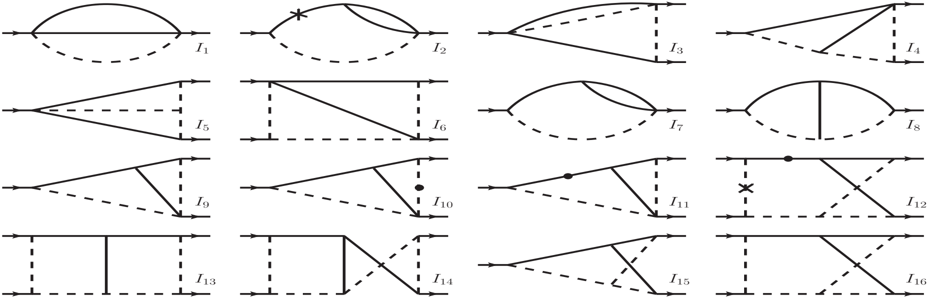
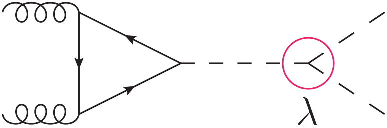
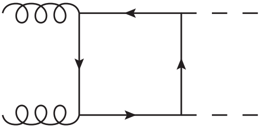
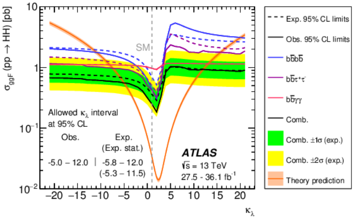


Why Higgs boson pair production?





- direct probe of cubic term of the scalar potential
- completely fixed in the SM

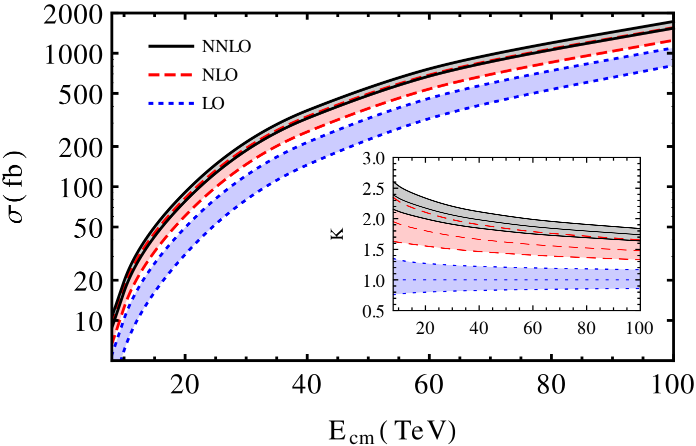


taken from arXiv:1906.02025 [hep-ex]

- current limit: $-5.0 < \kappa_\lambda < 12.0$
- HL-LHC: $0.5 < \kappa_\lambda < 2.4$ [WG2 Report '19]
- FCC-hh: 5% accuracy [FCC CDR '19]

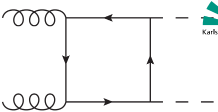
Why higher orders?

$pp \to HH + X$



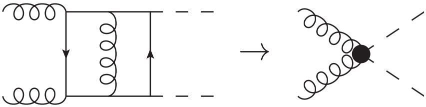
Hadronic c.o.m energy, taken from [de Florian, Mazzitelli '13]

Status at NLO



exact LO results known for 3 decades:

[Eboli, Marques, Novaes, Natale, '87],[Glover, van der Bij '88],[Plehn, Spira, Zerwas '96]



Various approximations used at NLO, mostly in the limit of (infinitely) heavy top quark:

- Born-improved HEFT [Dawson, Dittmaier, Spira '98]
- LME [Grigo, Hoff, Melnikov, Steinhauser '13], [Degrassi, Giardino, Gröber '16]
- $\mathrm{FT}_{\mathrm{approx}}, \mathrm{FT}'_{\mathrm{approx}}$ [Maltoni, Vryonidou, Zaro '14]
- LME + Threshold expansion [Gröber, Maier, Rauh '17]

Furthermore there exist approximations addressing other parts of the phase-space:

- High-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18 + '19],[Mishima '18]
- Small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]

Exact numerical calculations available, covering the whole phase-space

[Borowka, Greiner, Herinrich, Jones, Kerner, Schlenk, Zirke '16],

[Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '18]

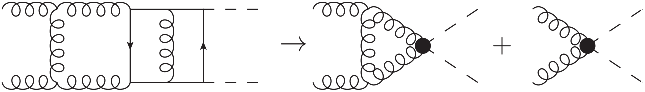
Recently have been combined with high-energy expansion

[Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann '19]

Status beyond NLO

The next step is to improve the situation at NNLO, here only few results exist:

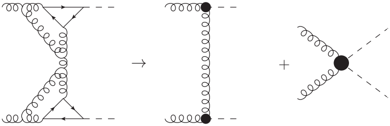
- HEFT [de Florian, Mazzitelli '13], [Grigo, Melnikov, Steinhauser '14]
- LME in SV-approximation [Grigo, Hoff, Steinhauser '14]
- $\mathrm{FT}_{\mathrm{approx}}$ [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]



$\rightarrow$ our goal is to improve upon the LME by also considering the real radiation

At N^{3}LO the basic building differing from single Higgs production recently have been computed for HEFT:

- Wilson coefficient C_{HH} [Spira '16],[Gerlach, FH, Steinhauser '18]
- 2 loop box-type diagrams [Banerjee, Borowka, Dhani, Gehrmann, Ravindran '18]



What are we computing?

- inclusive, partonic cross-section at NNLO

$$\sigma_{ij}\left(s, m_H, m_t\right) = \delta_{ig}\delta_{jg}\sigma^{(0)}_{gg} + \frac{\alpha_s}{\pi}\sigma^{(1)}_{ij} + \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^{(2)}_{ij} + \ldots$$

$$\sigma^{(2)}_{ij} = \underbrace{\delta_{ig}\delta_{jg}\sigma^{(2)}_{gg,\mathrm{virt}}}_{\text{[Grigo,Hoff,Steinhauser '14]}} + \underbrace{\sigma^{(k)}_{ij,\mathrm{real}}}_{\text{this work}}$$

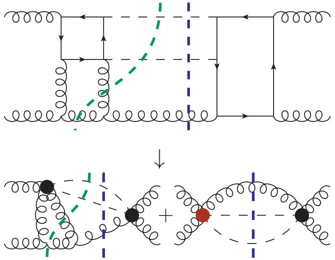
- so far only known in $m_t \to \infty$ limit, full computation not feasible
  $\to$ asymptotic expansion in large top quark mass

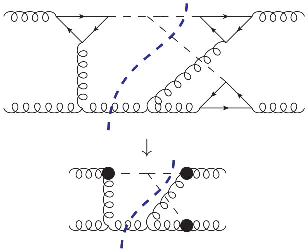
$$\sigma^{(l)}_{ij}\left(s, m_H, m_t\right) \approx \sum_k \rho^k \sigma^{(l),k}_{ij}\left(x\right)$$

$$\rho = m_H^2/m_t^2, \quad x = m_H^2/s$$

What are we computing?

We use reverse unitarity to express the real corrections through cuts of 5-loop diagrams with 2 masses in forward scattering kinematics.

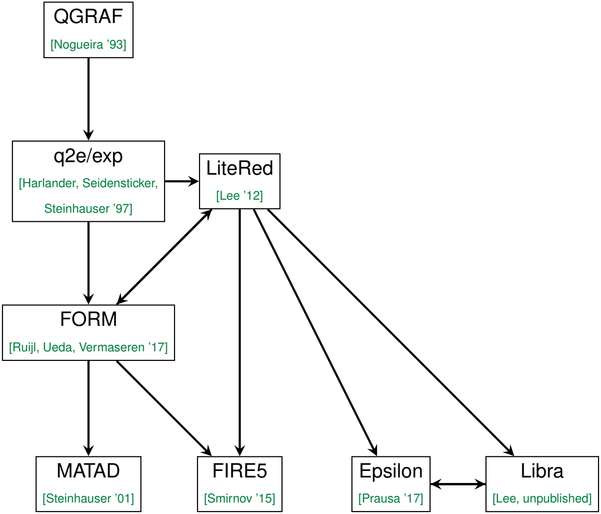




- 2 1-loop tadpoles + 3-loop phase space integral
- 1-loop tadpole + 2-loop tadpole + 2-loop phase space integral

- 3 1-loop tadpoles + 2-loop phase space integral

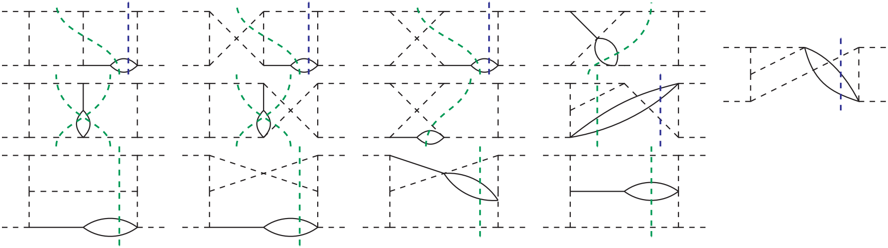
Setup


QGRAF
[Nogueira '93]
q2e/exp
[Harlander, Seidensticker, Steinhauser '97]
LiteRed
[Lee '12]
FORM
[Ruijl, Ueda, Vermaseren '17]
MATAD
[Steinhauser '01]
FIRE5
[Smirnov '15]
Epsilon
[Prausa '17]
Libra
[Lee, unpublished]

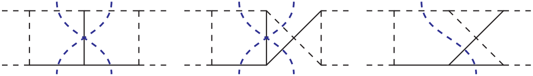
Families

We use LiteRed [Lee '12] for minimizing families, identifying common sectors and performing the reduction

We end up with 13 three-loop,



and 3 two-loop phase space integral families:



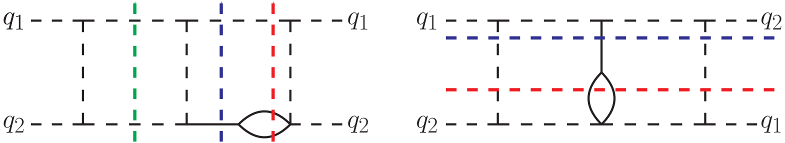
Differential equations

- 3 systems of differential equations
- We bring them to Fuchsian form using Epsilon [Prausa '17]:

$$\partial_x \vec{I} = \sum_i \frac{M^{(i)}(\epsilon)}{x - x_i} \vec{I} \qquad x = m_h^2/s$$

- They contain the letters:

$x_i =$	0	1	$\frac{1}{4}$	-1	$-\frac{1}{4}$	$e^{\pm i \frac{\pi}{3}}$	$-\frac{1}{3}$
2 loop, 3 particle	✓	✓	✓			✓	✓
3 loop, 3 particle	✓	✓	✓				
3 loop, 4 particle	✓	✓	✓	✓	✓		



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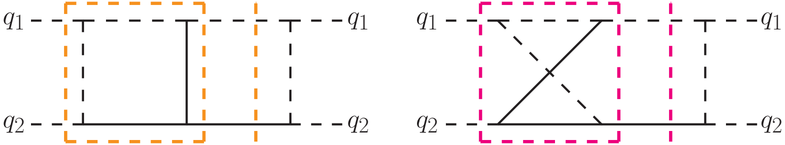

Differential equations

- We proceed to transform the differential equations to ϵ-form using Libra [Lee, unpublished]:

$$\partial_x \vec{I} = \sum_i \frac{M^{(i)}(\epsilon)}{x - x_i} \vec{I} \quad \rightarrow \quad \partial_x \vec{J} = \epsilon \sum_i \frac{\tilde{M}^{(i)}}{x - x_i} \vec{J}$$

- The solution to DEs in this form can be written as Goncharov Polylogarithms
- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

	$\sqrt{1-4x}$	$\sqrt{1+4x}$	$\sqrt{(1+3x)(1-x)}$
2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

$\rightarrow$ Need to find change of variable to transform square roots into rational functions

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2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

- In the DE for the three-loop, three-particle cuts only $\sqrt{1-4x}$ appears

$$x = \frac{y}{(1+y)^2} \qquad x \in (0, 1/4] \rightarrow y \in (0, 1]$$

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2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

- In the DE for the three-loop, four-particle cuts both $\sqrt{1-4x}$ and $\sqrt{1+4x}$ appear

$$x = \frac{t^4+1}{8t^2} \qquad x \in (0, 1/4] \rightarrow t = e^{i\phi} : \phi \in (1/4, 0]$$

Differential equations

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- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

	$\sqrt{1-4x}$	$\sqrt{1+4x}$	$\sqrt{(1+3x)(1-x)}$
2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

- In the DE for the two-loop, three-particle cuts $\sqrt{1-4x}$ and $\sqrt{(1+3x)(1-x)}$ appear
- Can not be rationalized together, however integrations over $\sqrt{(1+3x)(1-x)}$ do not contribute to cross-section

$$x = \frac{y}{(1+y)^2} \quad x \in (0, 1/4] \rightarrow y \in (0, 1]$$

Boundary conditions and δ-expansion

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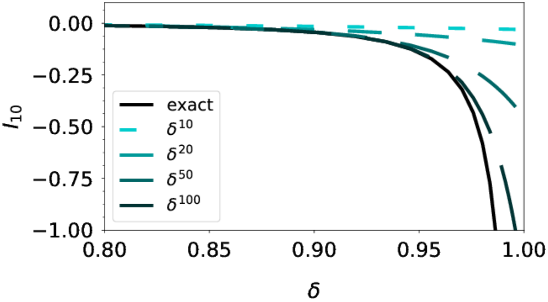
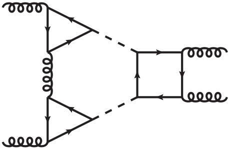
- Parametrize necessary phase-space integrals through $\delta = 1 - 4x$
- Expand around threshold $\delta \approx 0$
- Minimization and other manipulation of Goncharov Polylogarithms with PolyLogTools [Duhr, Dulat '19]
- We also obtained solutions to the DE with the ansatz

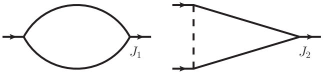
$$\vec{I} = \sum_{i,j,k} \vec{c}_{i,j,k} \epsilon^i \delta^{(2j+1)/2} \ln^k \delta$$



n_h^3: NLO



- only *gg* channel contributes
- already known in expansion around threshold [Grigo, Hoff, Melnikov, Steinhauser '13]
- recomputed exactly in $m_h^2/s \to 2$ MIs

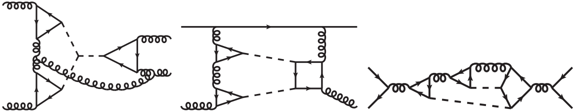


- collinear counterterm

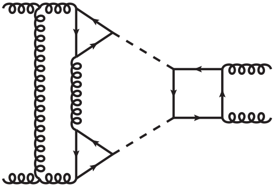
$$\sigma_{gg,\mathrm{coll}}^{(2),n_h^3} = 2\int_{1-\delta}^{1} \mathrm{d}z P_{gg}(z)\sigma_{gg}^{(1),n_h^3}(x/z)$$

n_h^3: NNLO

- three contributing channels gg, gq and $q\overline{q}$:

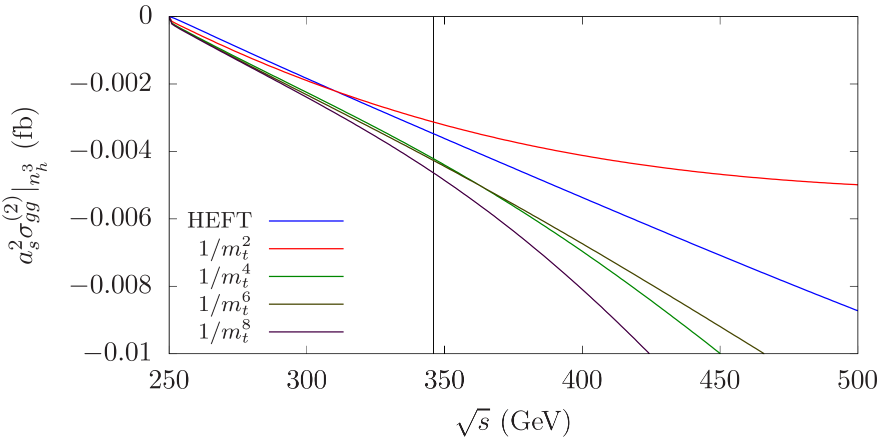


- expanded to $1/m_t^8$
- combine them with virtual corrections



Results

[Davies, FH, Mishima, Steinhauser JHEP 1905 (2019) 157]



- Convergence only below $2m_t$

Results

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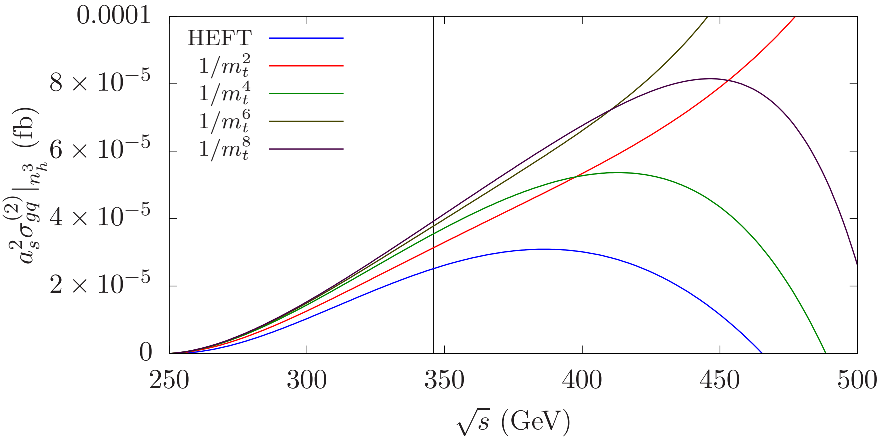

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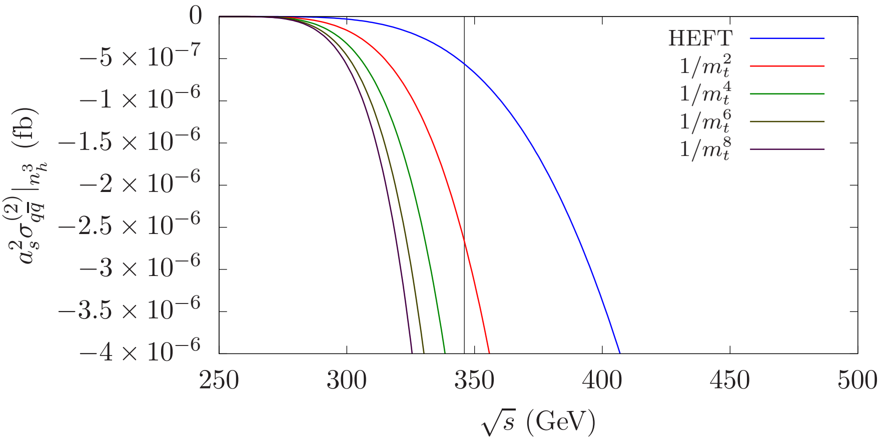
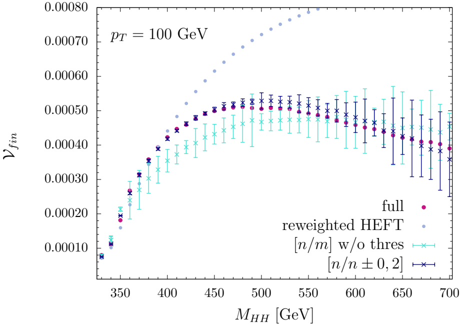


- No convergence, even below $2m_t$

Outlook

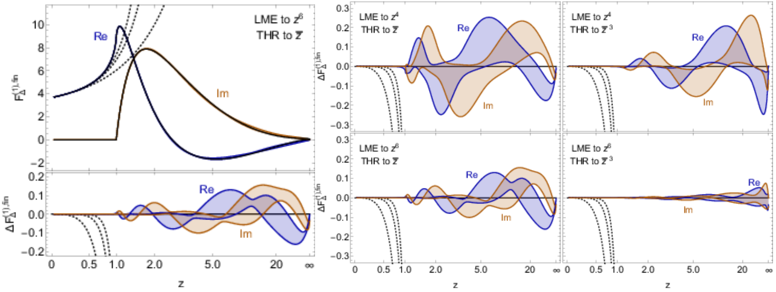
- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part:



taken from [Gröber, Maier, Rauh '17]

Outlook

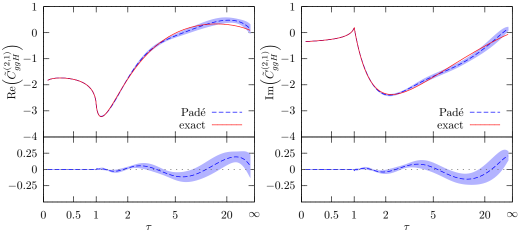
- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part
- Higher orders in LME crucial, e.g. for NLO *ggH* formfactor:



taken from [Davies, Gröber, Maier, Rauh, Steinhauser '19]

Outlook

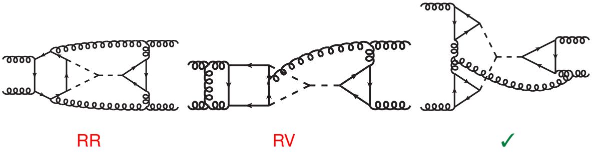
- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part
- Higher orders in LME crucial
- Recently contributions with light fermion loops at NNLO confirmed by analytic computation:



taken from [Harlander, Prausa, Usovitch '19]

- Exact computation already very challenging for $gg \to H$
- Expansions needed, in particular for $gg \to HH$

Conclusion



- Obtain top-mass suppressed terms at NNLO to Higgs boson pair production
- Computed phase-space MIs, both, in an expansion around threshold and exactly
  $\rightarrow$ rather generic to any pair-production process of massive particles
- Byproduct: NLO real corrections without expanding around threshold
- Work in progress: cross-sections for double real and remaining real-virtual contributions