

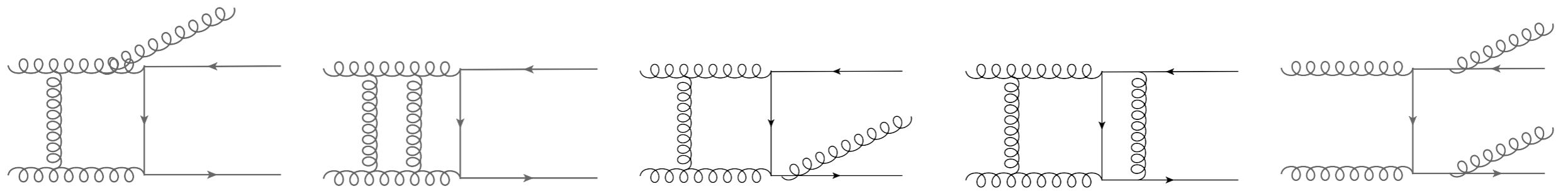
TOP-QUARK HADROPRODUCTION IN NNLO QCD

LoopFest XVIII, 14.08.19

Simone Devoto - University of Zürich



*In collaboration with:
S. Catani, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan*



CONTENTS

- Introduction;
- q_T subtraction formalism;
- q_T subtraction formalism for heavy quark production;
- Inclusive cross section;
- Differential distributions;
- Summary and outlook.

WHY TOP PAIR PRODUCTION AT NNLO?

Top quark production is of great importance at hadron colliders:

Standard Model Studies

- strong coupling with the Higgs boson;
- top mass is a fundamental parameter;
- standard candle at LHC.

BSM Studies

- possible window on new physics;
- background to new physics searches.

Top pair production is the main source of top events at LHC:

- 3 times larger than single top production;
- at LHC about 15 $t\bar{t}$ pairs produced per second .

QCD CORRECTIONS: THEORETICAL STATUS

Focusing on on-shell top pair production:

➤ NLO QCD:

- Total Cross Section [*P. Nason, S. Dawson, R. K. Ellis (1988)*], [...]
- Differential distribution [*M. L. Mangano, P. Nason and G. Ridolfi (1992)*], [...]

➤ NNLO QCD:

- Total Cross Section [*M. Czakon, P. Fiedler, A. Mitov (2013)*]
- Differential distributions [*M. Czakon, P. Fiedler and A. Mitov (2015)*], *M. Czakon, P. Fiedler, D. Heymes and A. Mitov (2016)*; *M. Czakon, D. Heymes and A. Mitov (2017)*]

➤ **NEW: NNLO QCD using qT subtraction**

[*S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 1901.04005*;
S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli: 1906.06535]

WHY TOP PAIR PRODUCTION AT NNLO... AGAIN?

NNLO QCD corrections for on shell $t\bar{t}$ production are known.


Why a new computation?



Very difficult computation, only one group able to complete it

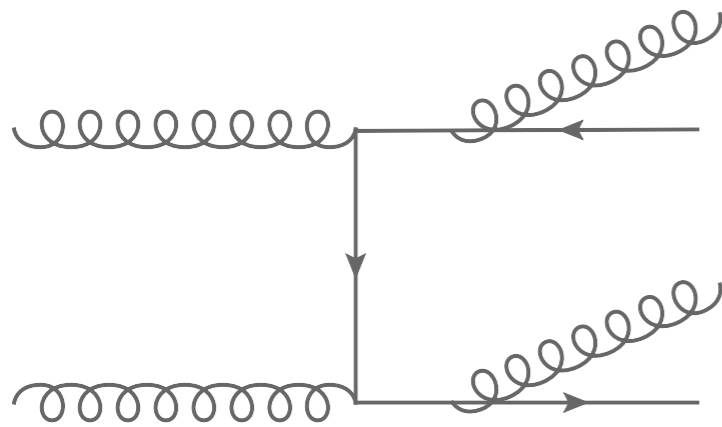
until now [Bärnreuther, Czakon, Mitov (2012); Czakon, Mitov (2012);
Czakon, Fiedler, Mitov (2013);
Czakon, Fiedler, Heymes, Mitov (2015, 2016); ...]

See talk by A. Mitov for recent developments!

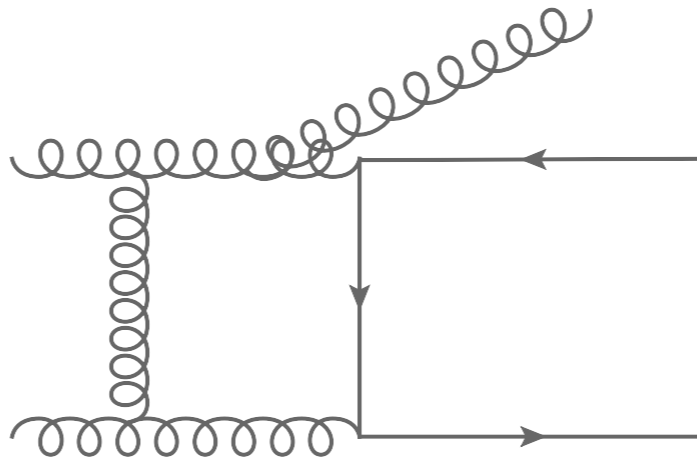
- 
- An independent check is always useful;
 - No public available NNLO generator yet.

TOP PAIR PRODUCTION AT NNLO - INGREDIENTS

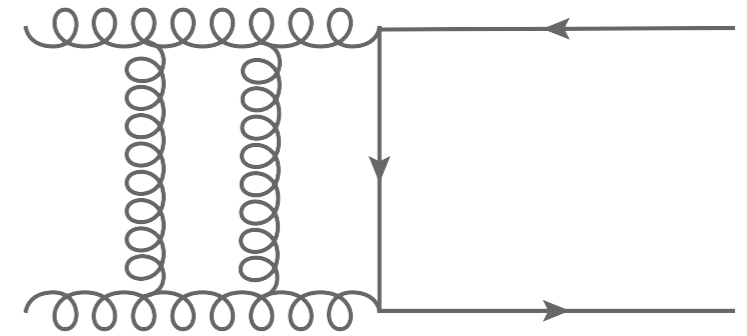
Double real



Real -virtual



Two loop virtual



Fast and stable evaluation with OPENLOOPS 2
[Cascioli et al. (2012), Buccioni et al. (2018), Buccioni et al. (2019)]

Numerically available
[Czakon (2008); Barnreuther et al. (2013)]

See talk by M. Zoller!

IR divergent

IR divergent

IR divergent

IR divergences cancel once all contributions are combined (KLN theorem) but they do not allow a straightforward implementation of numerical techniques.

We need a method to handle and cancel IR singularities

Q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini (2007)]

The q_T subtraction formalism is a method to handle and cancel IR divergences, originally developed for **colorless** final states.

$$d\sigma_{(N)NLO}^F = d\sigma_{(N)NLO}^F \Big|_{q_T=0} + d\sigma_{(N)NLO}^F \Big|_{q_T \neq 0}$$

q_t = transverse momentum of the system F

$$d\sigma_{(N)LO}^{F+jets}$$

$$d\sigma_{(N)NLO}^F = \mathcal{H}_{(N)NLO}^F \otimes d\sigma_{LO}^F + \left[d\sigma_{(N)LO}^{F+jets} - d\sigma_{(N)LO}^{CT} \right]$$

HARD COLLINEAR COEFFICIENT
Contains information on virtual corrections to the process.

Singularities for q_t ≠ 0 can be computed with NLO subtraction techniques.

Extra singularities of NNLO type associated to the q_t → 0 limit need additional subtraction. IR behaviour known from q_t resummation formalism allow us to construct a counterterm.
[J. C. Collins, D. E. Soper, G. Sterman (1985); G. Bozzi, S. Catani, D. de Florian, M. Grazzini (2005)]

MATRIX

[M. Grazzini, S. Kallweit,

M. Wiesemann: arXiv 1711.06631]

.....

*Computational framework which, implementing q_T subtraction, allows us to evaluate fully differential cross sections for a wide class of processes at hadron colliders **where the final state is a color singlet** in next-to-next-to-leading order (NNLO) QCD.*


```
[devoto:/mnt/runs2/devoto/MATRIX_v1.0.0] ./matrix
```



Version: 1.0.0
Reference: arXiv:1711.06631

Nov 2017

Munich -- the MULTI-chaNnel Integrator at swiss (CH) precision --
Automates q_T -subtraction and Resummation to Integrate X-sections



M. Grazzini (grazzini@physik.uzh.ch)
S. Kallweit (stefan.kallweit@cern.ch)
M. Wiesemann (maris.wiesemann@cern.ch)

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-----  
process_id || process || description  
-----  
pph21 >> p p --> H >> on-shell Higgs production  
ppz01 >> p p --> Z >> on-shell Z production  
ppw01 >> p p --> W^- >> on-shell W- production with CKM  
ppwx01 >> p p --> W^+ >> on-shell W+ production with CKM  
ppeex02 >> p p --> e^- e^+ >> Z production with decay  
ppnenex02 >> p p --> v_e^- v_e^+ >> Z production with decay  
ppenex02 >> p p --> e^- v_e^+ >> W- production with decay and CKM  
ppexne02 >> p p --> e^+ v_e^- >> W+ production with decay and CKM  
ppaa02 >> p p --> gamma gamma >> gamma gamma production  
ppeexa03 >> p p --> e^- e^+ gamma >> Z gamma production with decay  
ppnenexa03 >> p p --> v_e^- v_e^+ gamma >> Z gamma production with decay  
ppenexa03 >> p p --> e^- v_e^+ gamma >> W- gamma production with decay  
ppexnea03 >> p p --> e^+ v_e^- gamma >> W+ gamma production with decay  
ppzz02 >> p p --> Z Z >> on-shell ZZ production  
ppwxw02 >> p p --> W^+ W^- >> on-shell WW production  
ppemexmx04 >> p p --> e^- mu^- e^+ mu^+ >> ZZ production with decay  
ppeexex04 >> p p --> e^- e^- e^+ e^+ >> ZZ production with decay  
ppeexnmnm04 >> p p --> e^- e^+ v_mu^- v_mu^+ >> ZZ production with decay  
ppemxnmnex04 >> p p --> e^- mu^+ v_mu^- v_e^+ >> WW production with decay  
ppeexnenex04 >> p p --> e^- e^+ v_e^- v_e^+ >> ZZ/WW production with decay  
ppemexnm04 >> p p --> e^- mu^- e^+ v_mu^+ >> W-Z production with decay  
ppeexnex04 >> p p --> e^- e^- e^+ v_e^+ >> W-Z production with decay  
ppeexmxnm04 >> p p --> e^- e^+ mu^+ v_mu^- >> W+Z production with decay  
ppeexexne04 >> p p --> e^- e^+ e^+ v_e^- >> W+Z production with decay  
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| pph21 | >> | p p --> H | >> | on-shell Higgs production |
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| ppw01 | >> | p p --> W^- | >> | on-shell W- production with CKM |
| ppwx01 | >> | p p --> W^+ | >> | on-shell W+ production with CKM |
| ppeex02 | >> | p p --> e^- e^+ | >> | Z production with decay |
| ppnenex02 | >> | p p --> v_e^- v_e^+ | >> | Z production with decay |
| ppenex02 | >> | p p --> e^- v_e^+ | >> | W- production with decay and CKM |
| ppexne02 | >> | p p --> e^+ v_e^- | >> | W+ production with decay and CKM |
| ppaa02 | >> | p p --> gamma gamma | >> | gamma gamma production |
| ppeexa03 | >> | p p --> e^- e^+ gamma | >> | Z gamma production with decay |
| ppnenexa03 | >> | p p --> v_e^- v_e^+ gamma | >> | Z gamma production with decay |
| ppenexa03 | >> | p p --> e^- v_e^+ gamma | >> | W- gamma production with decay |
| ppexnea03 | >> | p p --> e^+ v_e^- gamma | >> | W+ gamma production with decay |
| ppzz02 | >> | p p --> Z Z | >> | on-shell ZZ production |
| ppwxw02 | >> | p p --> W^+ W^- | >> | on-shell WW production |
| ppemexmx04 | >> | p p --> e^- mu^- e^+ mu^+ | >> | ZZ production with decay |
| ppeeexex04 | >> | p p --> e^- e^- e^+ e^+ | >> | ZZ production with decay |
| ppeexnmnx04 | >> | p p --> e^- e^+ v_mu^- v_mu^+ | >> | ZZ production with decay |
| ppemxnmnex04 | >> | p p --> e^- mu^+ v_mu^- v_e^+ | >> | WW production with decay |
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| ppeeexnex04 | >> | p p --> e^- e^- e^+ v_e^+ | >> | W-Z production with decay |
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MATRIX

[M.Grazzini, S. Kallweit,

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- Munich (S. Kallweit);
- OpenLoops2 (F. Buccioni, J.N. Lang, J.Lindert, P. Maierhofer, S. Pozzorini, H. Zhang, M. F. Zoller);
- TDHPL, GiNaC, VVAMP, ...

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| ppeex02 | p p --> e^- e^+ | Z production with decay |
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| ppenexa03 | p p --> e^- v_e^+ gamma | W- gamma production with decay |
| ppexnea03 | p p --> e^+ v_e^- gamma | W+ gamma production with decay |
| ppzz02 | p p --> Z Z | on-shell ZZ production |
| ppwxw02 | p p --> W^+ W^- | on-shell WW production |
| ppemexmx04 | p p --> e^- mu^- e^+ mu^+ | ZZ production with decay |
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MATRIX

[M.Grazzini, S. Kallweit,
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WHAT ABOUT TOP PAIR PRODUCTION?

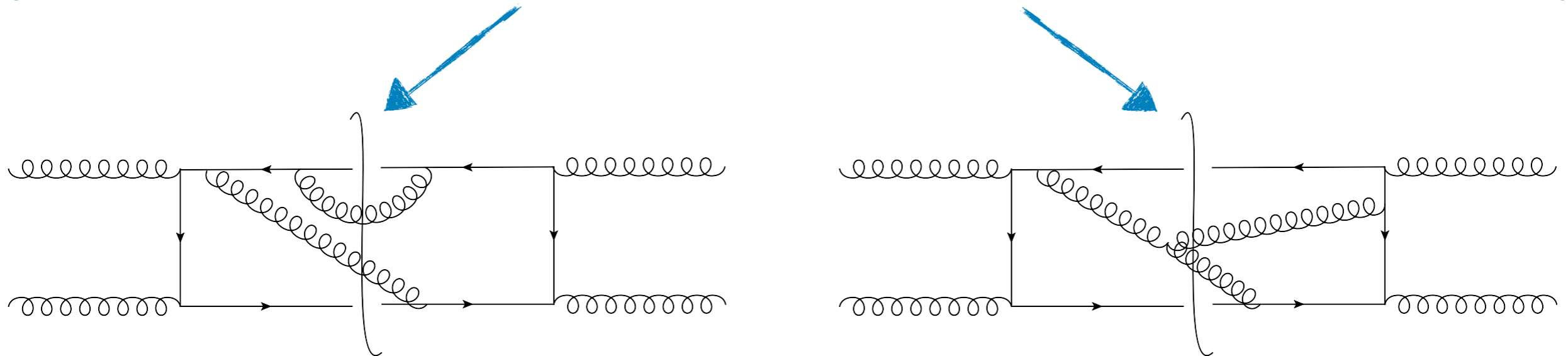
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- TDHPL, GiNaC, VVAMP, ...

Q_T SUBTRACTION FOR COLORFUL MASSIVE FINAL STATES

With the inclusion of extra contributions, q_T subtraction formalism can be extended to **massive colored** final states.

- *Successfully applied to top pair production at NLO and NNLO considering only off-diagonal channels. [R. Bonciani, S. Catani, M. Grazzini, H. Sargsyan, A. Torre (2015)]*

The extension to colored final state requires the inclusion of additional final-state soft singularities.



... and so on...

Q_T SUBTRACTION FOR COLORFUL MASSIVE FINAL STATES

- What was missing to fully implement q_T subtraction at NNLO for top pair production?

$$d\sigma_{NNLO}^{t\bar{t}} = \mathcal{H}_{NNLO}^{t\bar{t}} \otimes d\sigma_{LO}^{t\bar{t}} + \left[d\sigma_{NLO}^{t\bar{t}+jets} - d\sigma_{NNLO}^{CT} \right]$$

HARD COLLINEAR COEFFICIENT



Computable with NLO subtraction techniques.

IR behaviour known from studies in q_t resummation
[A. Ferroglia, M. Neubert, B. D. Pecjak, L. L. Yang (2009)];
[Hai Tao Li, Chong Sheng Li, Ding Yu Shao, Li Lin Yang, Hua Xing Zu (2013)];
[S. Catani, M. Grazzini, A. Torre (2014)]

Contains the integrations of the additional final-state soft singularities.

We recently completed their computation. [S. Catani, SD, M. Grazzini, J. Mazzitelli, in preparation.
See also R. Angeles-Martinez, M. Czakon, S. Sapeta (2018)]

HARD COLLINEAR COEFFICIENT

Colourless subtraction operator

Specified up to order α_s^2 in [S.Catani, L. Cieri, D. De Florian, G. Ferrera, M. Grazzini (2013)].

Colourless final state

$$\mathcal{H} \propto \langle \tilde{M} | \tilde{M} \rangle$$

$$|\tilde{M}\rangle = (1 - I_C) |M\rangle$$

All loop renormalised virtual amplitude

Colourful final state

Additional soft radiative factor

$$\mathcal{H} \propto \langle \tilde{M} | \Delta | \tilde{M} \rangle$$

- We are only interested in the low q_T behaviour → soft limit;
- We need to integrate the **soft current** in the case of:
 - Double gluon emission; [S.Catani, M.Grazzini (1999); M. Czakon (2011)]
 - Light quark pair production; [S.Catani, M.Grazzini (1999)]
 - Gluon emission at one loop. [I. Bierenbaum, M. Czakon, A. Mitov (2011); M. Czakon, A Mitov (2019)]

HARD COLLINEAR COEFFICIENT – NLO

[S. Catani, M. Grazzini,
A. Torre: arXiv 1408:4564]

Example: $t\bar{t}$ production at **NLO** within q_T subtraction formalism:

Computation of the soft contribution



Integration of a suitably subtracted soft current.

$$\int \frac{d^n k}{2\pi^{n-1}} \delta_+(k) \left| J_{sub}(k) \right|^2 e^{i\vec{b} \cdot \vec{k}_T}$$

► J_{sub} is the soft current after a proper subtraction of the (known) colourless contribution:

$$\left| J_{sub}(k) \right|^2 = \sum_{j=3,4} \frac{m_j^2}{(p_j \cdot k)^2} \mathbf{T}_j^2 + \frac{2 p_3 \cdot p_4}{p_3 \cdot k p_4 \cdot k} \mathbf{T}_3 \cdot \mathbf{T}_4 + \sum_{\substack{i=1,2 \\ j=3,4}} \frac{2}{p_i \cdot k} \left(\frac{p_i \cdot p_j}{p_j \cdot k} - \frac{p_1 \cdot p_2}{(p_1 + p_2) \cdot k} \right) \mathbf{T}_i \cdot \mathbf{T}_j$$

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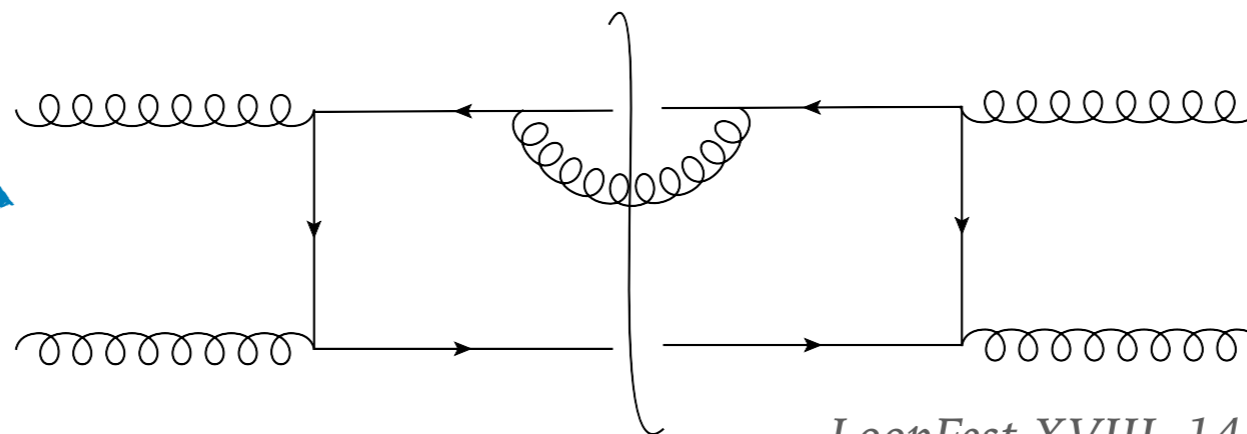
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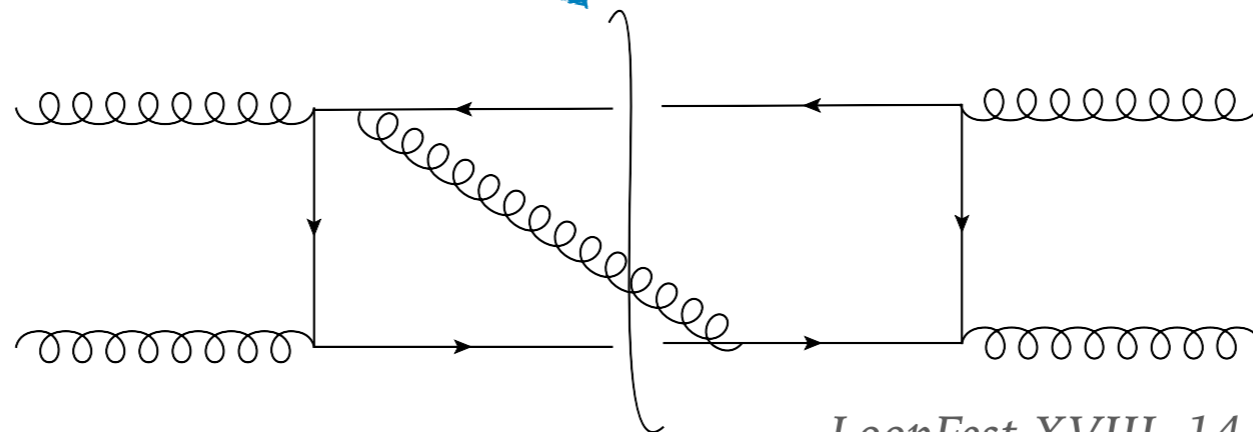
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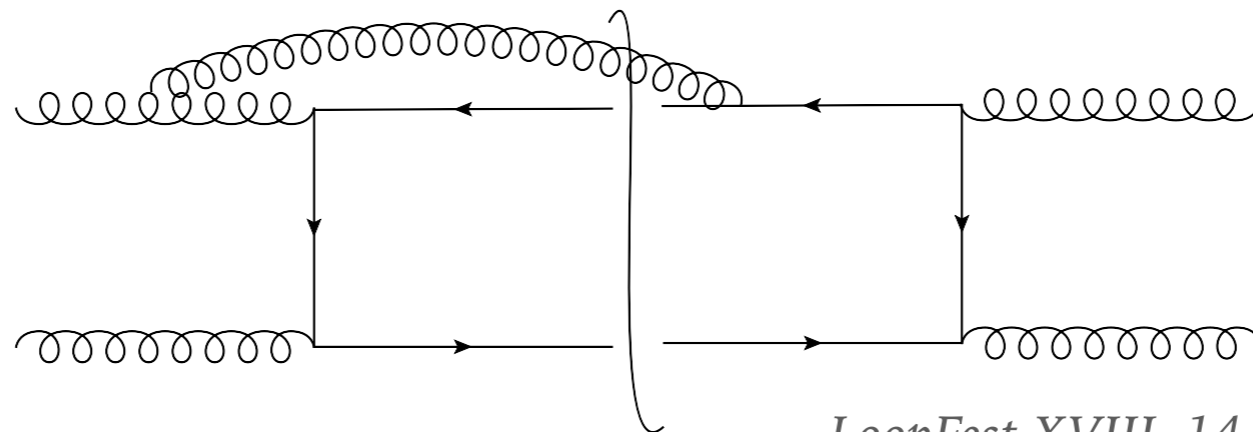
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HARD COLLINEAR COEFFICIENT – NNLO

[S. Catani, SD, M. Grazzini, J. Mazzitelli, in preparation.]

NNLO: the soft current is more complicated. Contributions from:

► **Light quark pair production:**

$$\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1) \delta_+(k_2) \left| J_{sub}^{NNLO(q\bar{q})}(k_1, k_2) \right|^2 e^{i\vec{b} \cdot (\vec{k}_{T1} + \vec{k}_{T2})}$$

[S. Catani, M. Grazzini (1999)]

► **Double gluon emission:**

$$\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1) \delta_+(k_2) \left| J_{sub}^{NNLO(gg)}(k_1, k_2) \right|^2 e^{i\vec{b} \cdot (\vec{k}_{T1} + \vec{k}_{T2})}$$

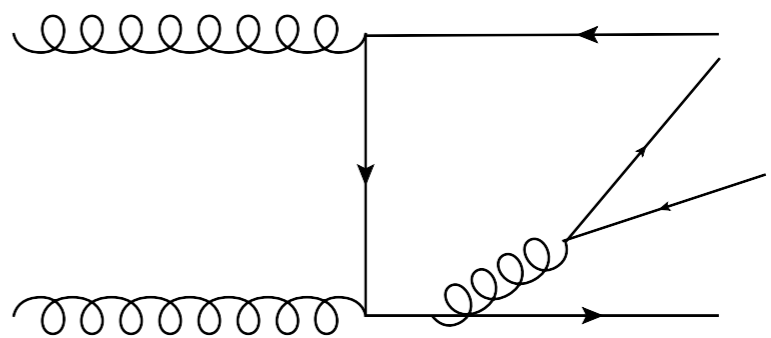
[S. Catani, M. Grazzini (1999); M. Czakon (2011)]

► **One gluon emission at 1 loop:**

$$\int \frac{d^n k}{2\pi^{n-1}} \delta_+(k) \left| J_{sub}^{NNLO(1L)}(k) \right|^2 e^{i\vec{b} \cdot \vec{k}_T}$$

[I. Bierenbaum, M. Czakon, A. Mitov (2011)]

$$\left| J_{sub}^{NNLO(q\bar{q})}(k_1, k_2) \right|^2 = \frac{J_{\mu,sub}^{NLO}(k_1 + k_2) \Pi^{\mu\nu}(k_1, k_2) J_{\nu,sub}^{NLO}(k_1 + k_2)}{(k_1 \cdot k_2)^2}$$

$$\Pi^{\mu\nu}(k_1, k_2) = \frac{T_R(-g^{\mu\nu} k_1 \cdot k_2 + k_1^\mu k_2^\nu + k_1^\nu k_2^\mu)}{(k_1 \cdot k_2)^2}$$


HARD COLLINEAR COEFFICIENT – NNLO

[S. Catani, SD, M. Grazzini, J. Mazzitelli, in preparation.]

NNLO: the soft current is more complicated. Contributions from:

- Light quark pair production:

$$\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1) \delta_+(k_2) \left| J_{sub}^{NNLO(q\bar{q})}(k_1, k_2) \right|^2 e^{i\vec{b} \cdot (\vec{k}_{T1} + \vec{k}_{T2})}$$

[S. Catani, M. Grazzini (1999)]

- Double gluon emission:

$$\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1) \delta_+(k_2) \left| J_{sub}^{NNLO(gg)}(k_1, k_2) \right|^2 e^{i\vec{b} \cdot (\vec{k}_{T1} + \vec{k}_{T2})}$$

[S. Catani, M. Grazzini (1999); M. Czakon (2011)]

- One gluon emission at 1 loop:

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[I. Bierenbaum, M. Czakon, A. Mitov (2011)]

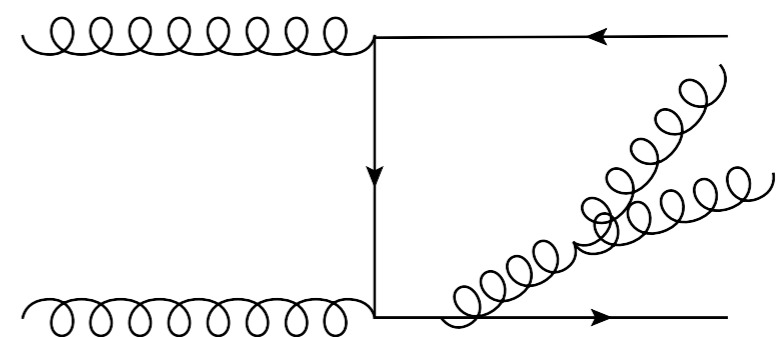
$$\left| J_{sub}^{NNLO(gg)}(k_1, k_2) \right|^2 = \frac{1}{2} \{ \mathbf{J}^2(k_1), \mathbf{J}^2(k_2) \} - C_a \sum_{i,j=1}^n \mathbf{T}_i \cdot \mathbf{T}_j \mathcal{S}_{ij}(k_1, k_2)$$

Iteration of NLO results

New part

$$\mathcal{S}_{ij}(k_1, k_2) = \mathcal{S}_{ij}^{m=0}(k_1, k_2)$$

$$+ \left(m_i^2 \mathcal{S}_{ij}^{m \neq 0}(k_1, k_2) + m_j^2 \mathcal{S}_{ji}^{m \neq 0}(k_1, k_2) \right)$$



HARD COLLINEAR COEFFICIENT – NNLO

[S. Catani, SD, M. Grazzini, J. Mazzitelli, in preparation.]

NNLO: the soft current is more complicated. Contributions from:

► Light quark pair production:

$$\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1) \delta_+(k_2) \left| J_{sub}^{NNLO(q\bar{q})}(k_1, k_2) \right|^2 e^{i\vec{b} \cdot (\vec{k}_{T1} + \vec{k}_{T2})}$$

[S. Catani, M. Grazzini (1999)]

► Double gluon emission:

$$\int \frac{d^n k_1}{2\pi^{n-1}} \frac{d^n k_2}{2\pi^{n-1}} \delta_+(k_1) \delta_+(k_2) \left| J_{sub}^{NNLO(gg)}(k_1, k_2) \right|^2 e^{i\vec{b} \cdot (\vec{k}_{T1} + \vec{k}_{T2})}$$

[S. Catani, M. Grazzini (1999); M. Czakon (2011)]

► **One gluon emission at 1 loop:**

$$\int \frac{d^n k}{2\pi^{n-1}} \delta_+(k) \left| J_{sub}^{NNLO(1L)}(k) \right|^2 e^{i\vec{b} \cdot \vec{k}_T}$$

[I. Bierenbaum, M. Czakon, A. Mitov (2011)]

$$\left| J_{sub}^{NNLO(1L)}(k) \right|^2 = \langle M_a^{(0)}(n+1; k) | M_a^{(1)}(n+1; k) \rangle + c.c.$$

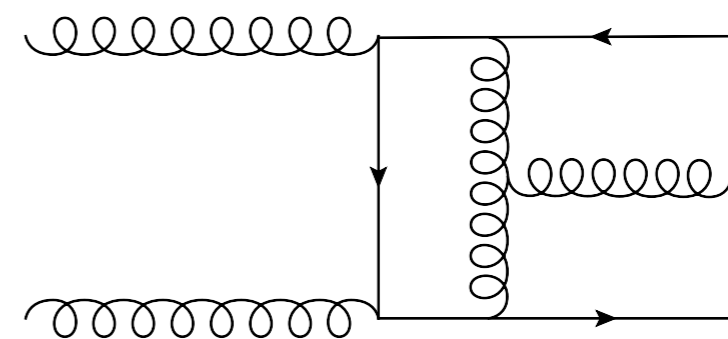
$$\langle M_a^{(0)}(n+1; k) | M_a^{(1)}(n+1; k) \rangle + c.c. = -4\pi\alpha_S\mu^{2\epsilon}$$

$$\times \left\{ 2C_A \sum_{i \neq j=1}^n (e_{ij} - e_{ii}) R_{ij} \langle M^{(0)}(n) | T_i \cdot T_j | M^{(0)}(n) \rangle \right.$$

$$- 4\pi \sum_{i \neq j \neq k=1}^n e_{ik} I_{ij} \langle M^{(0)}(n) | f^{abc} T_i^a T_j^b T_k^c | M^{(0)}(n) \rangle$$

$$+ \left(\sum_{i \neq j=1}^n e_{ij} \langle M^{(0)}(n) | T_i \cdot T_j | M^{(1)}(n) \rangle + c.c. \right)$$

$$\left. + \left(\sum_{i=1}^n \mathcal{C}_i e_{ii} \langle M^{(0)}(n) | M^{(1)}(n) \rangle + c.c. \right) \right\}$$

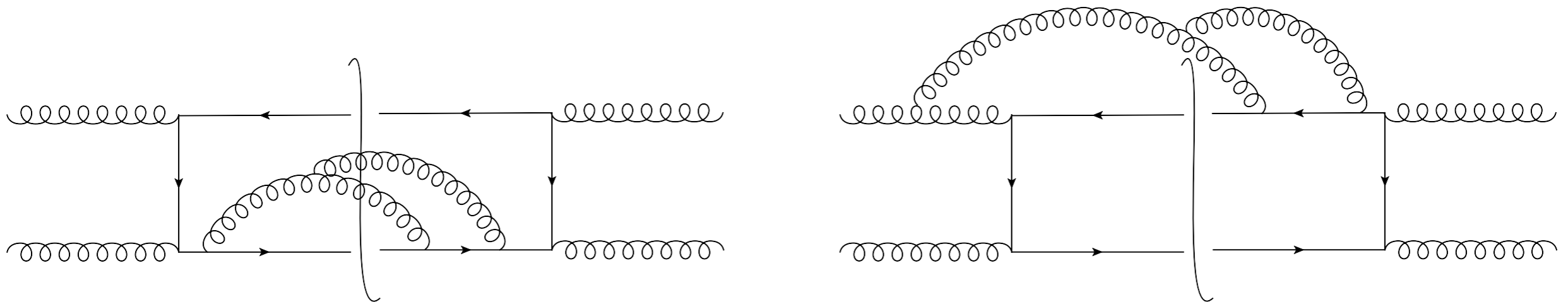


HARD COLLINEAR COEFFICIENT - NNLO

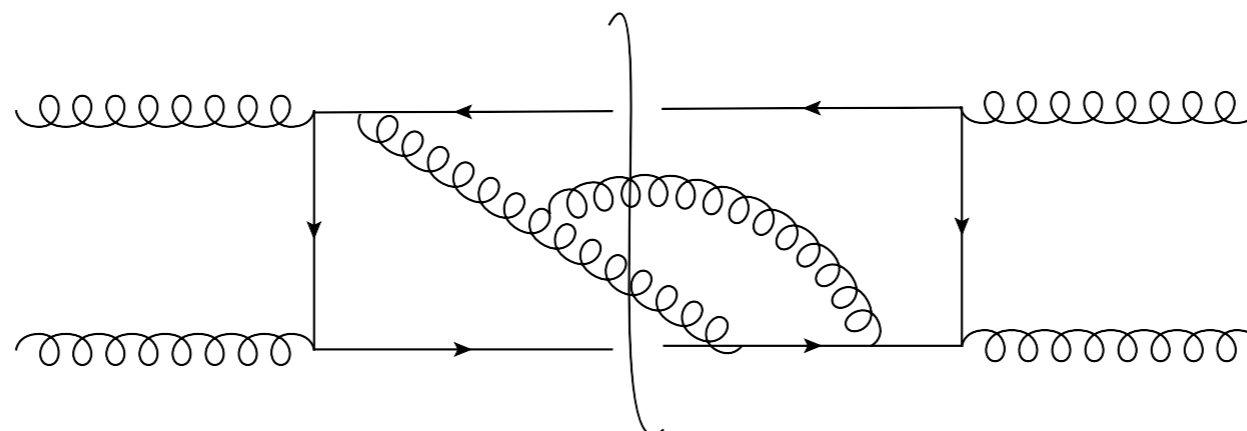
[S. Catani, SD, M. Grazzini,
J. Mazzitelli, in preparation.]

We computed all the needed integrals:

- Analytic expression for $\mathbf{T}_i\mathbf{T}_j, \mathbf{T}_j\mathbf{T}_j$ contributions:

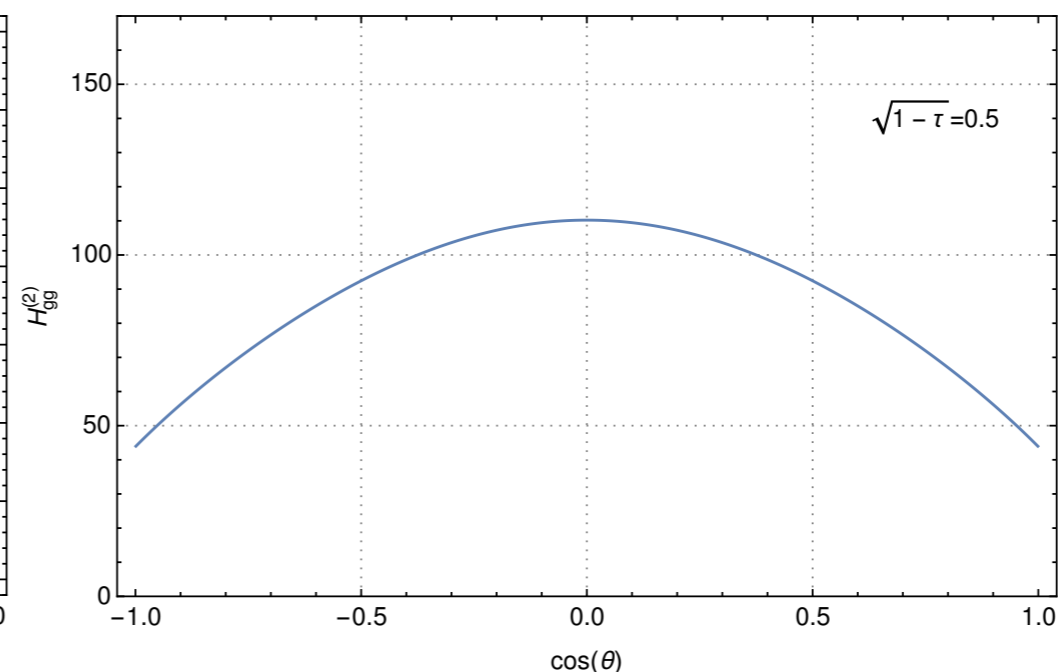
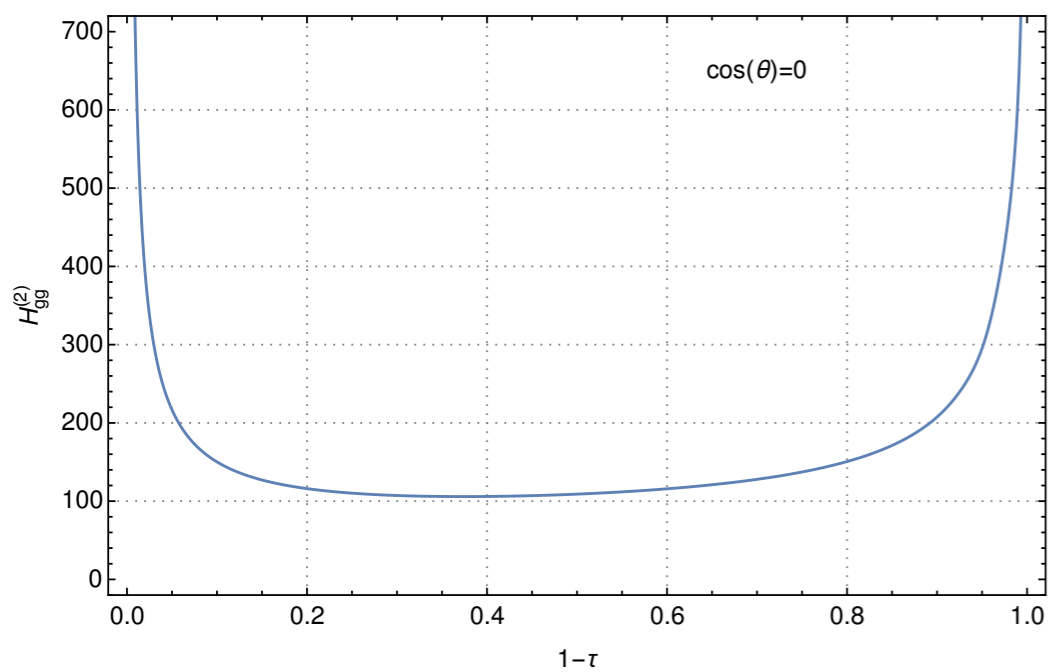
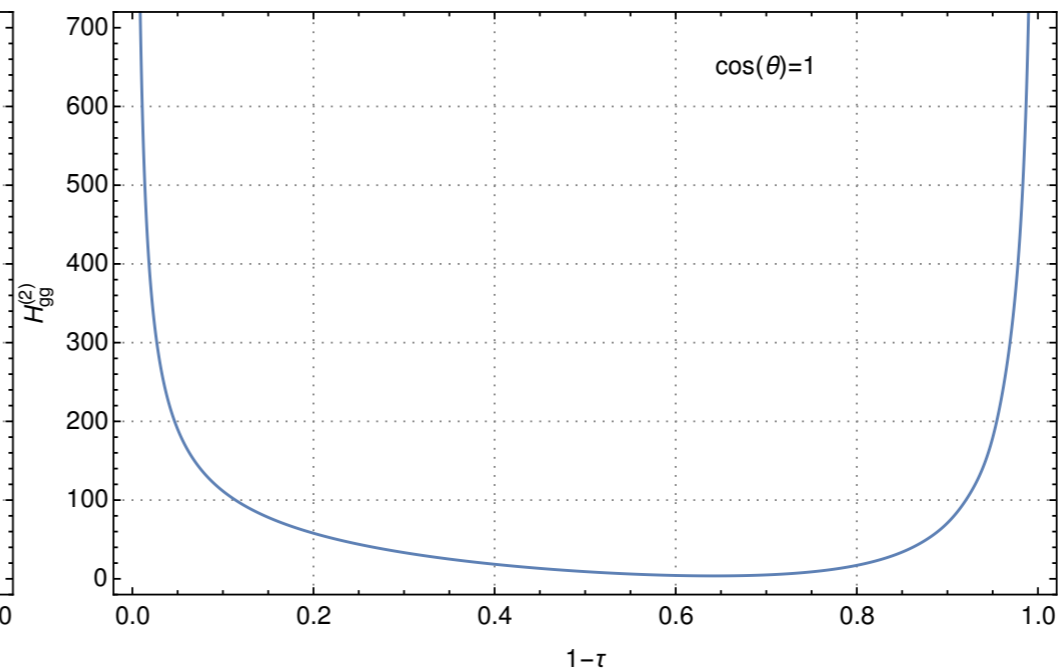
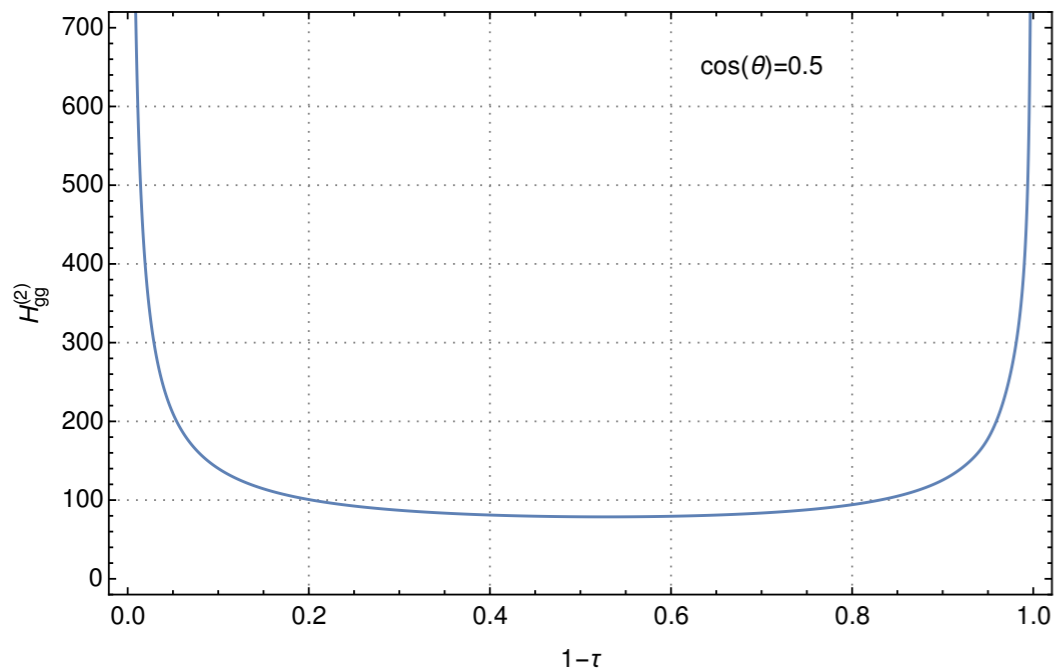


- Numerical expression for some pieces of the $\mathbf{T}_3\mathbf{T}_4$ contribution:



HARD COLLINEAR COEFFICIENT - NNLO

[S. Catani, SD, M. Grazzini,
J. Mazzitelli, in preparation.]



$$H_{gg}^{(2)}$$

$$\tau = 4m_t^2/s$$

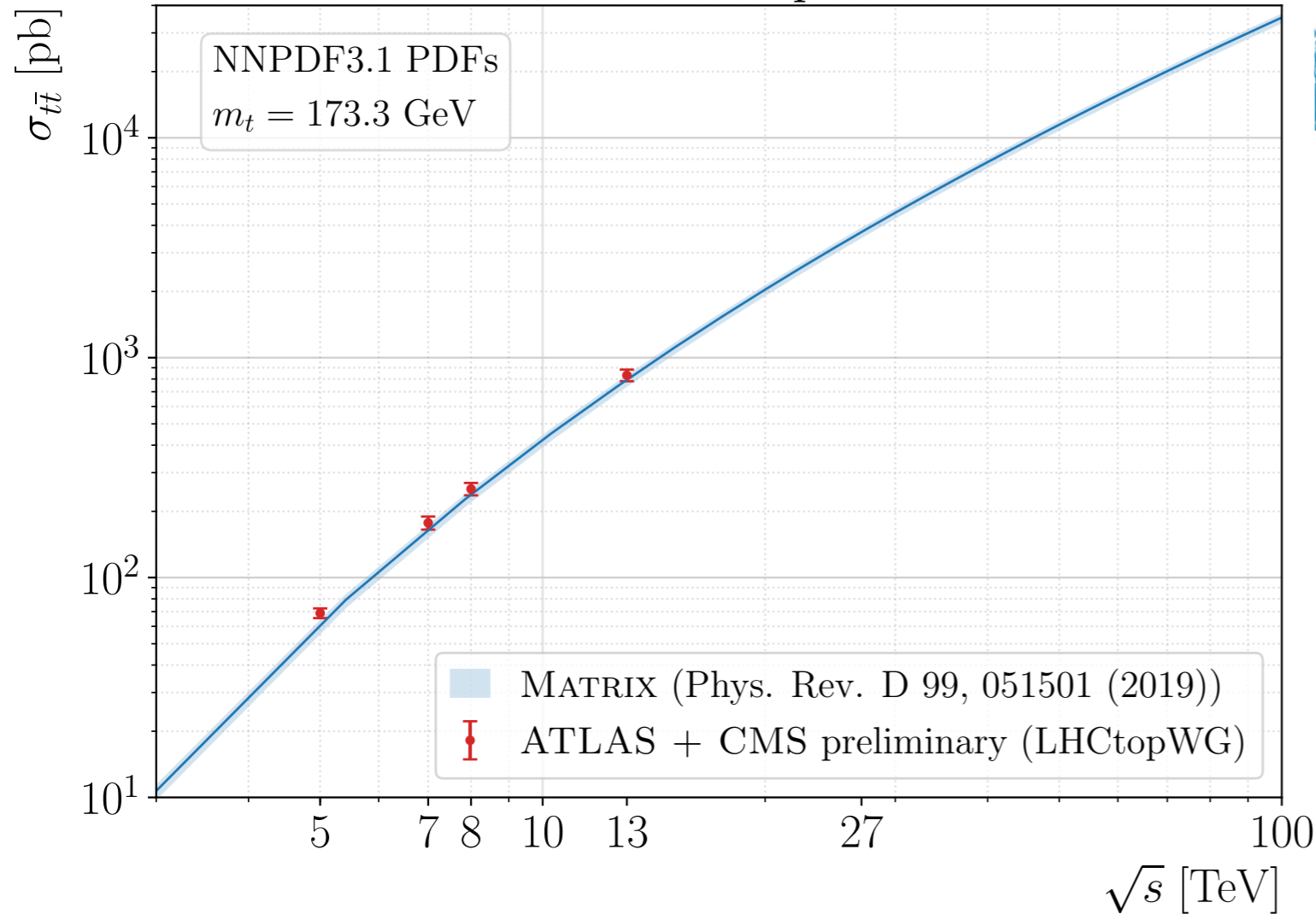
$\cos \theta$
scattering angle

The completion of this calculation allowed the implementation of top pair production in the **MATRIX** framework!

INCLUSIVE CROSS SECTION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan (2019)]

NNLO $t\bar{t}$ cross section computed with MATRIX



Per-mille accuracy
in ~ 1000 CPU days

Excellent agreement
with TOP++!

| σ_{NNLO} [pb] | MATRIX | TOP++ |
|-----------------------------|---|---|
| 8 TeV | 238.5(2) ^{+3.9%} _{-6.3%} | 238.6 ^{+4%} _{-6.3%} |
| 13 TeV | 794.0(8) ^{+3.5%} _{-5.7%} | 794.0 ^{+3.5%} _{-5.7%} |
| 100 TeV | 35215(74) ^{+2.8%} _{-4.7%} | 35216 ^{+2.9%} _{-4.8%} |

Statistical + systematic
uncertainties

Scale uncertainties

$$\mu_0 = m_t$$

$$\frac{1}{2}\mu_0 < \mu_F, \mu_R < 2\mu_0 \quad \frac{1}{2} < \frac{\mu_F}{\mu_R} < 2$$

DIFFERENTIAL DISTRIBUTIONS

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli (2019)]

- We computed single and double differential distributions;
- We compared with recent measurements from CMS in the leptons+jet channels [CMS-TOP-17-002].

Renormalization and factorization scales, μ_R and μ_F , should be chosen of the order of the characteristic hard scale:

| Hard scale | |
|----------------------------------|----------------|
| Total cross section | m_t |
| Rapidity distribution | m_t |
| Invariant mass distribution | $m_{t\bar{t}}$ |
| Transverse momentum distribution | m_T |

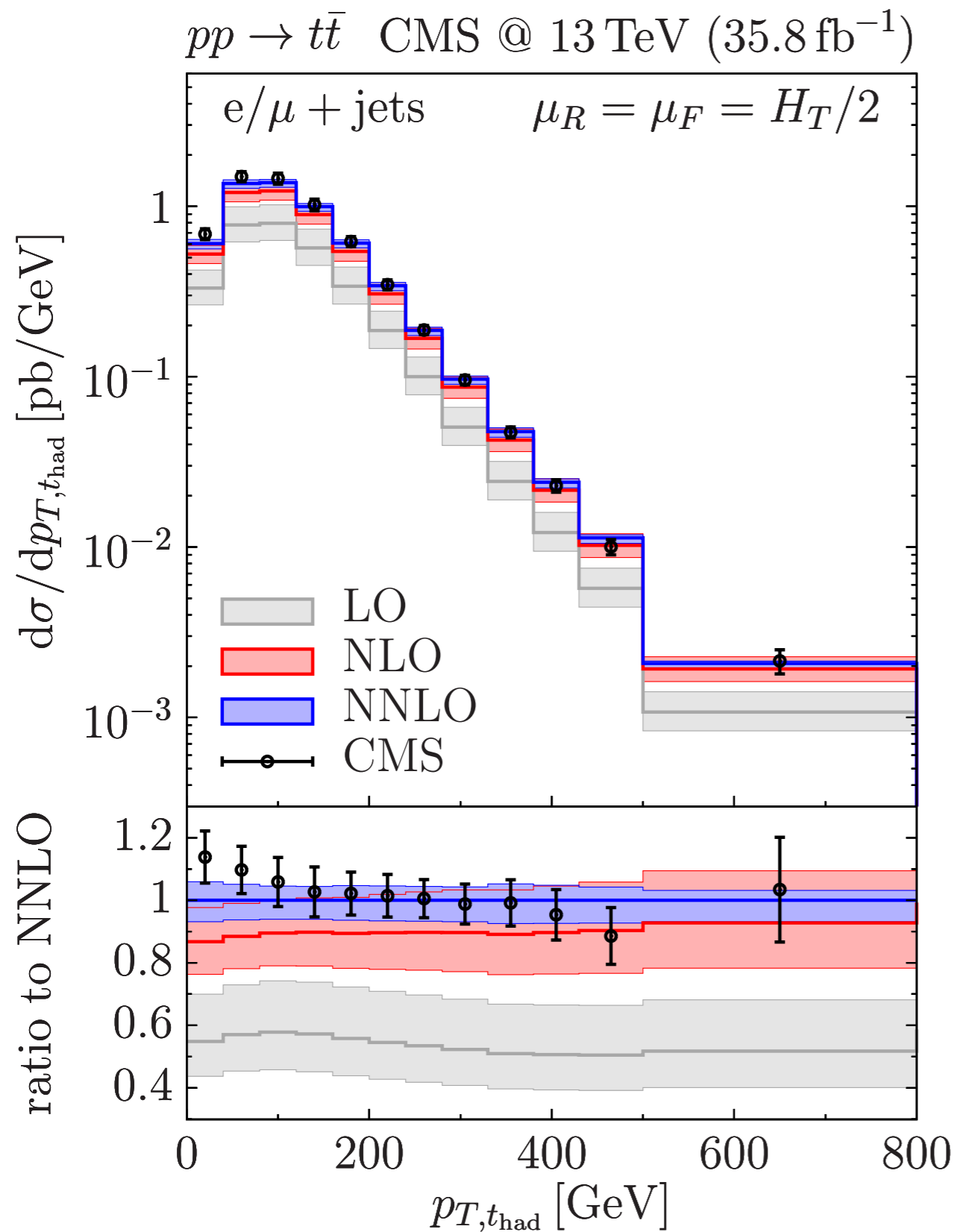
The dynamical scale
 $\mu_0 = \frac{1}{2}H_T = \frac{1}{2}(m_{T,t} + m_{T,\bar{t}})$ is a good approximation of all these scales.

The comparison with CMS is performed:

- Without cuts (extrapolation to parton level in the inclusive phase space);
- Multiplying our predictions by 0.438 (semileptonic BR of the $t\bar{t}$ pair) times 2/3 (only electron and muons).

SINGLE DIFFERENTIAL DISTRIBUTIONS

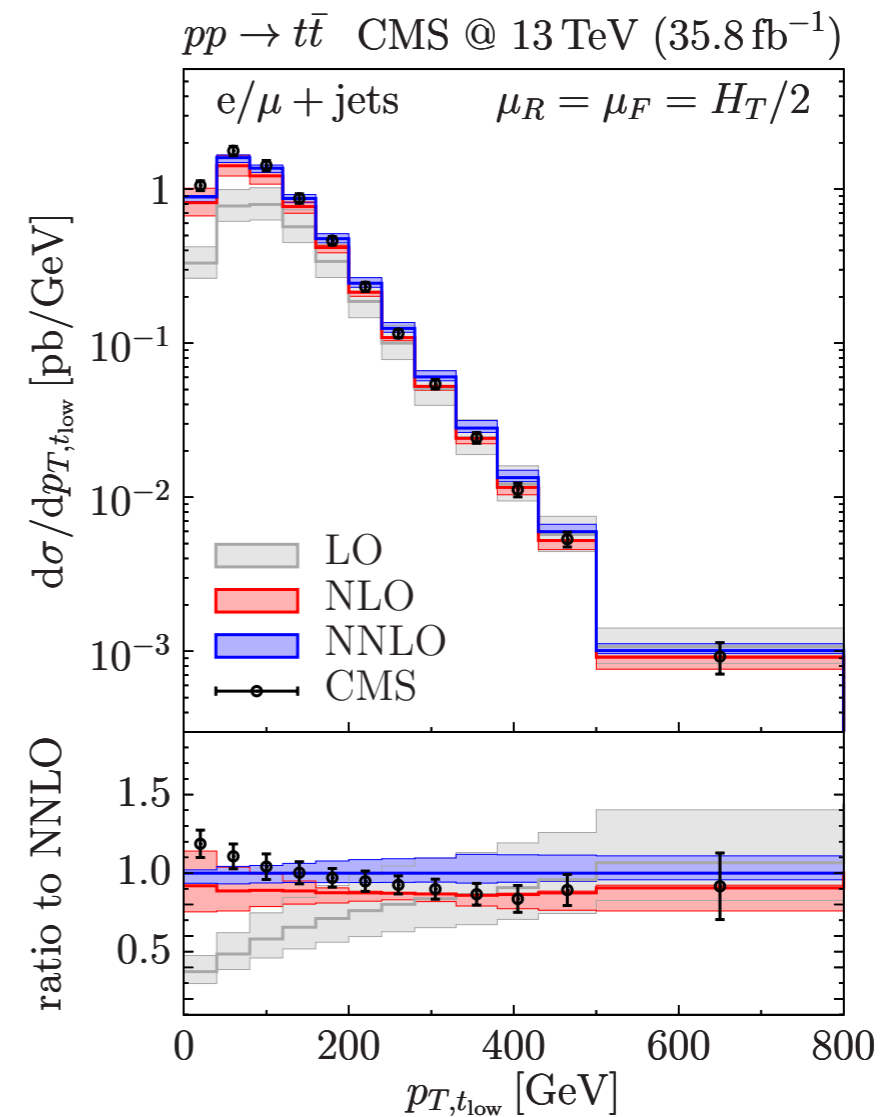
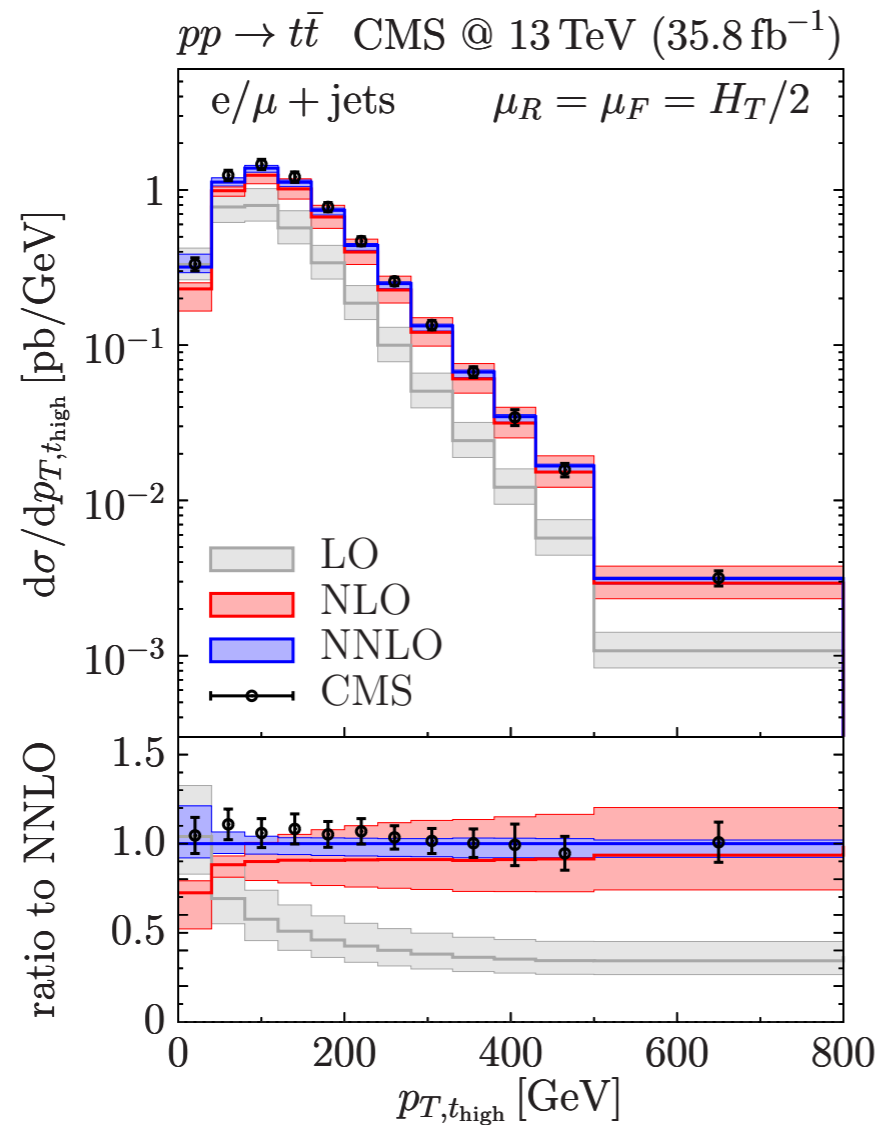
[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli (2019)]



- LO and NLO bands do not overlap (consistent with total cross sections),
- NLO and NNLO bands overlap, suggesting convergence of the perturbative expansion;
- Measured distribution is slightly softer than the theoretical prediction, as already observed in several analyses;
- Data and theory are consistent within uncertainties.

SINGLE DIFFERENTIAL DISTRIBUTIONS

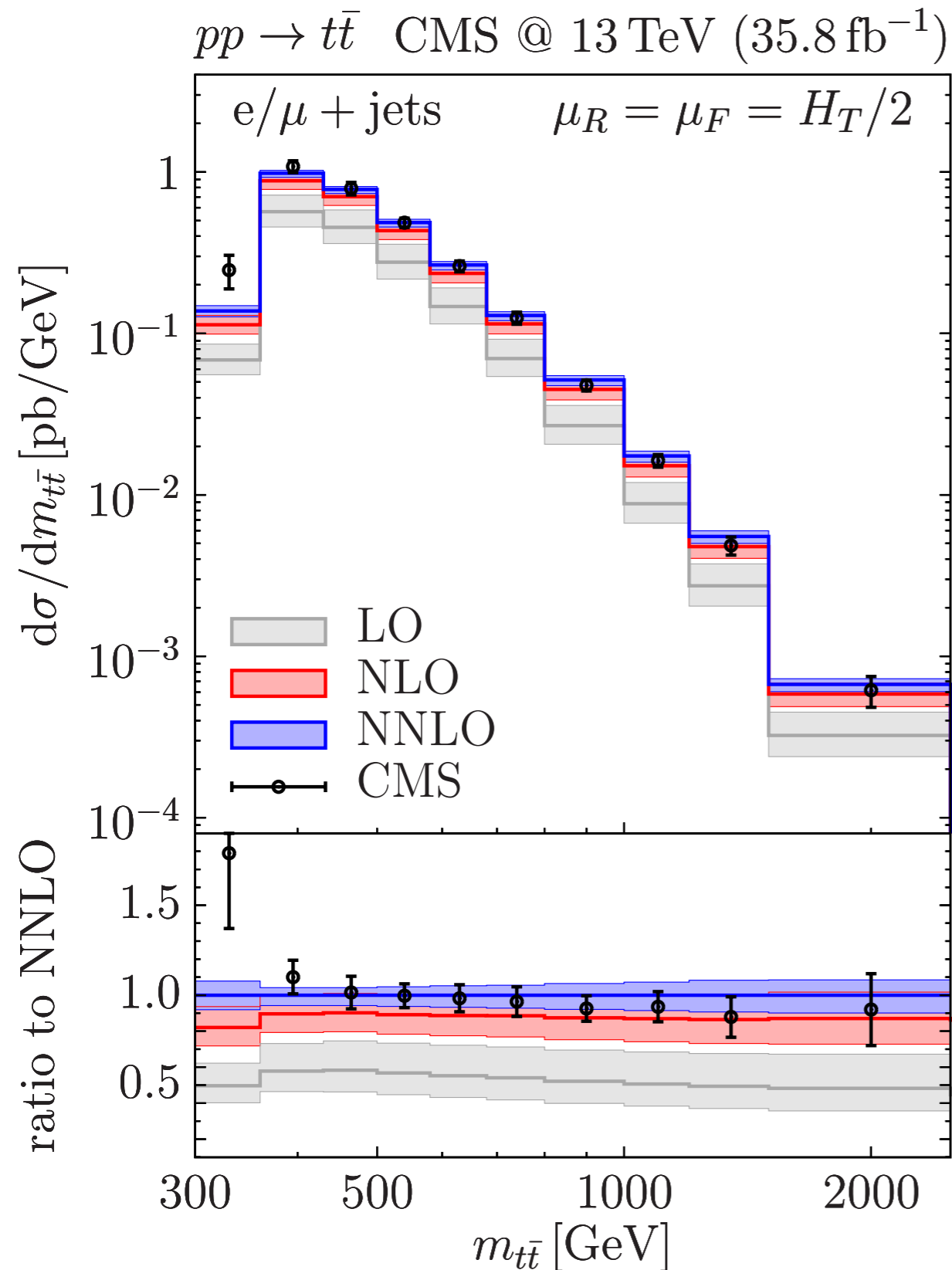
[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli (2019)]



- Higher order corrections have big effect on the shape:
 - $p_{T,thigh}$: at small p_T the p_T of the pair is forced to be small;
 - $p_{T,tlow}$: the effect is spread over the entire p_T region.

SINGLE DIFFERENTIAL DISTRIBUTIONS

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli (2019)]



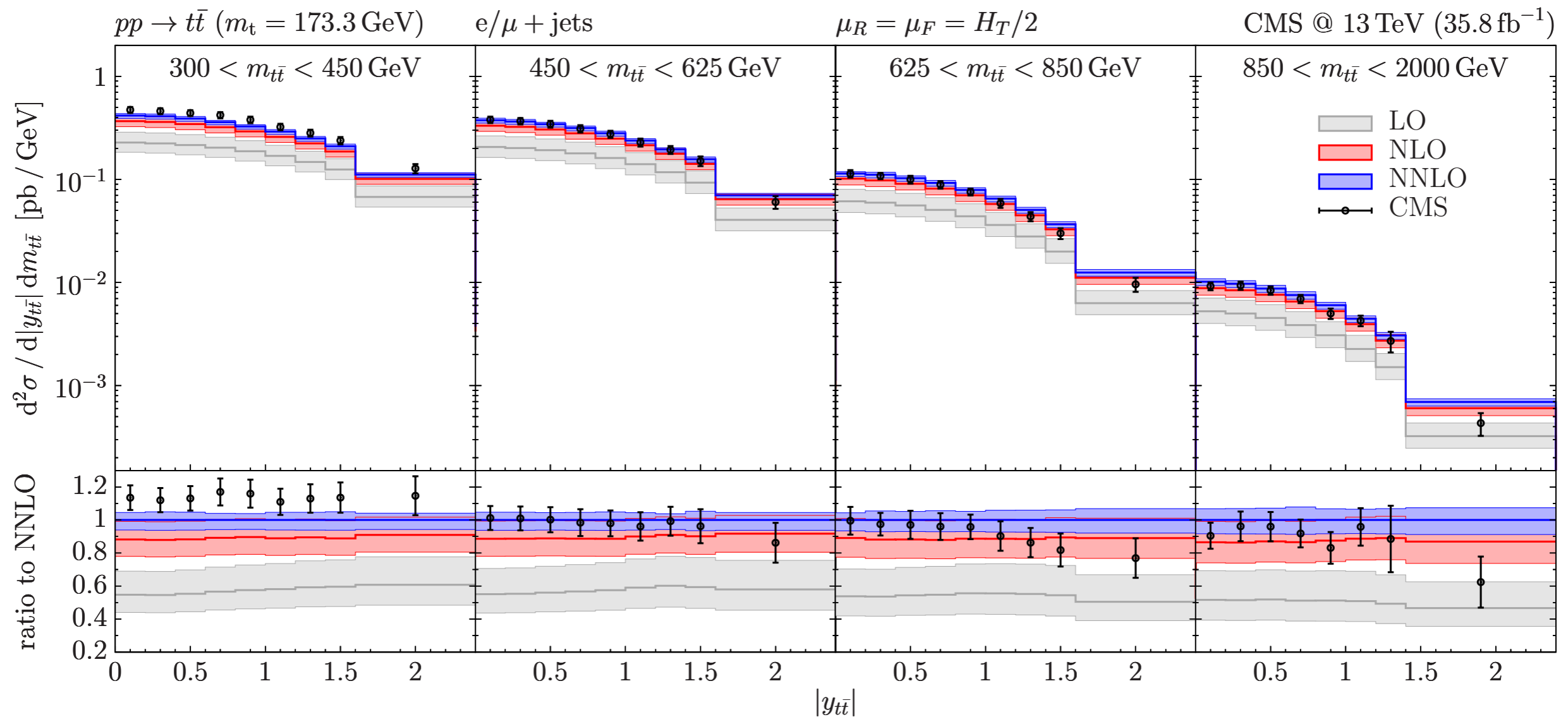
- Good convergence of the perturbative expansion;
- Good agreement with the data, neglecting the first bin.

Possible causes:

- Threshold region: issues in experimental extrapolation?
- Smaller top mass?

DOUBLE DIFFERENTIAL DISTRIBUTIONS

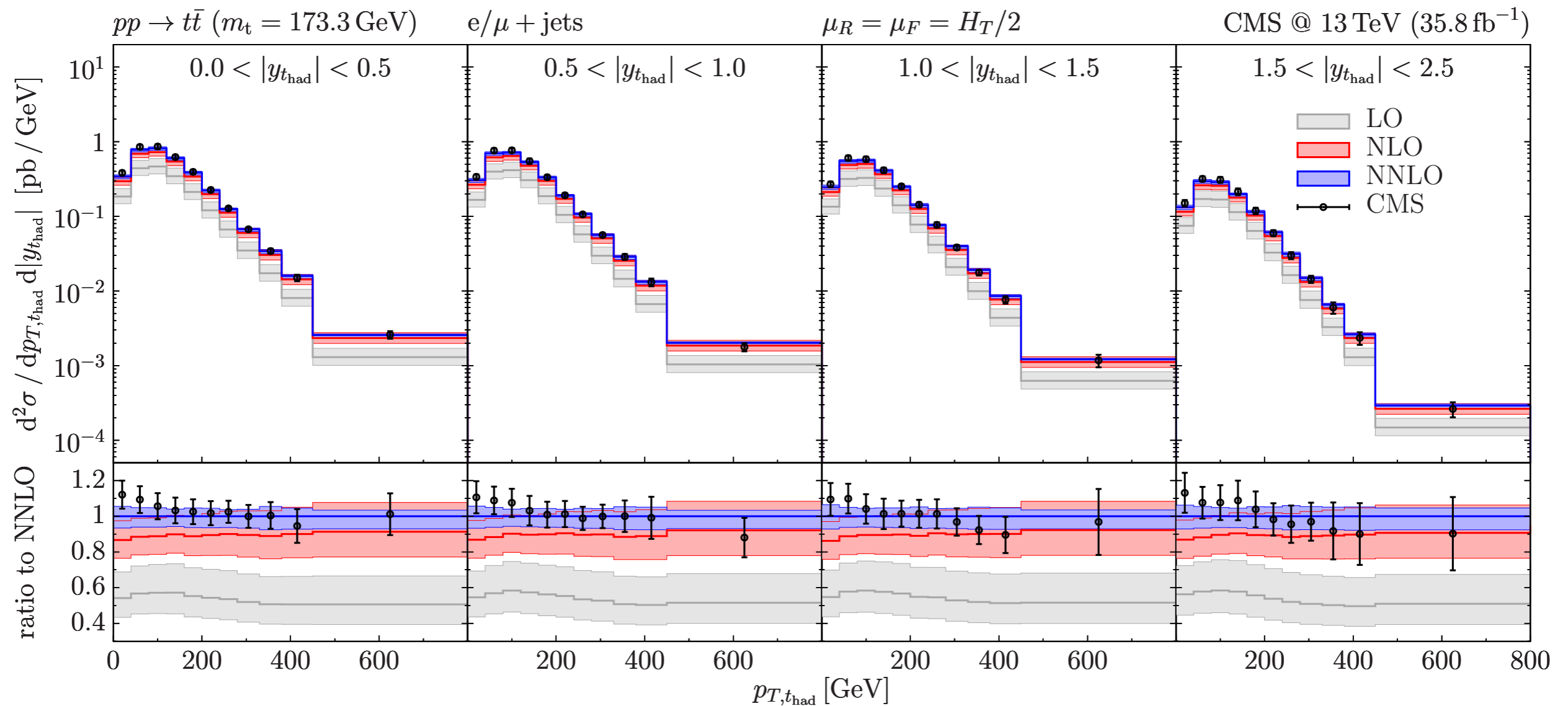
[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli (2019)]



- First bin in the $m_{t\bar{t}}$ distribution overshoots again the theoretical prediction, smaller effect due to larger bin size.
- Relatively uniform impact of radiative corrections in both variables.

DOUBLE DIFFERENTIAL DISTRIBUTIONS

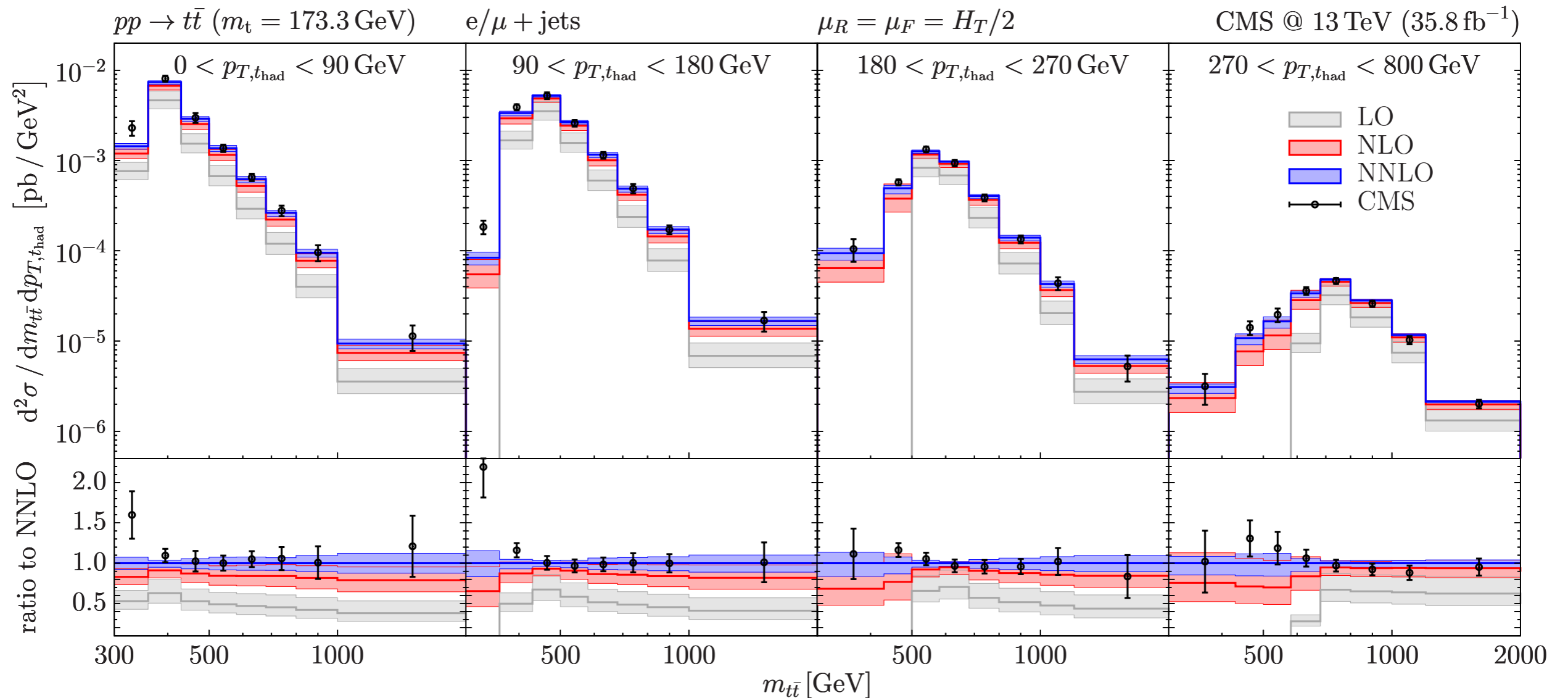
[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli (2019)]



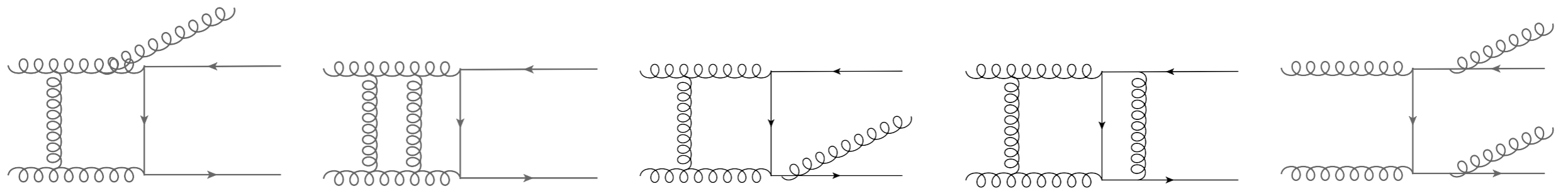
► As for single differential distributions, p_T data softer than NNLO.

DOUBLE DIFFERENTIAL DISTRIBUTIONS

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli (2019)]

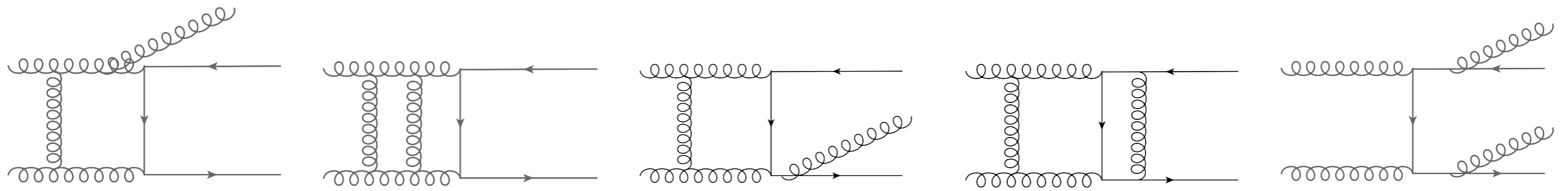


- Kinematical boundary at LO: $m_{t\bar{t}} > 2m_{T\text{min}}$;
- Below the threshold, NLO (NNLO) is effectively LO (NLO) → larger uncertainties;
- NNLO nicely describes the data except near threshold.



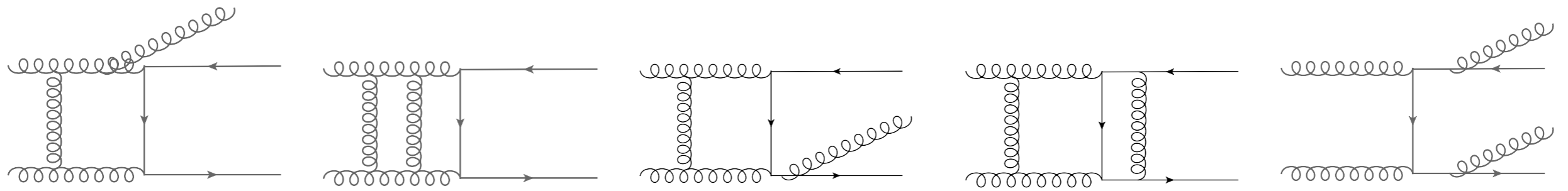
SUMMARY & OUTLOOK

- We have presented a new computation for top-quark production at NNLO;
- First complete application of q_T -subtraction formalism for massive colourful final state at NNLO;
- The process has been implemented into the MATRIX framework;
- Results for NNLO inclusive and multi-differential cross section: NNLO differential distributions in 1000-2000 CPU days.



SUMMARY & OUTLOOK

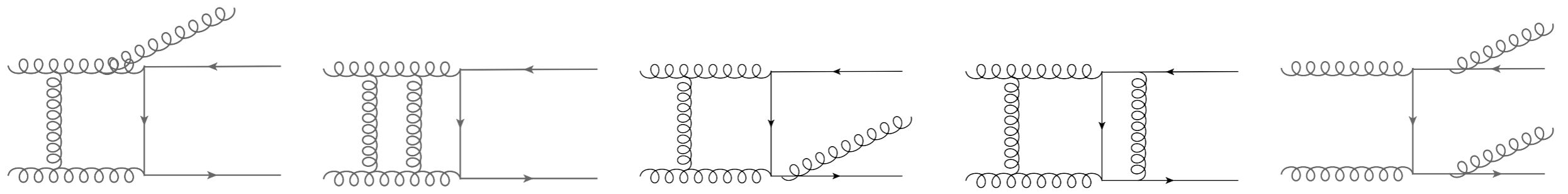
- New public MATRIX release with the inclusion of $t\bar{t}$ production;
- Improve NNLO QCD:
 - NLO EW?
 - Inclusion of top decays?
- Extend to different processes:
 - $b\bar{b}$ production?
 - $t\bar{t}$ + colorless?



SUMMARY & OUTLOOK

- New public MATRIX release with the inclusion of $t\bar{t}$ production;
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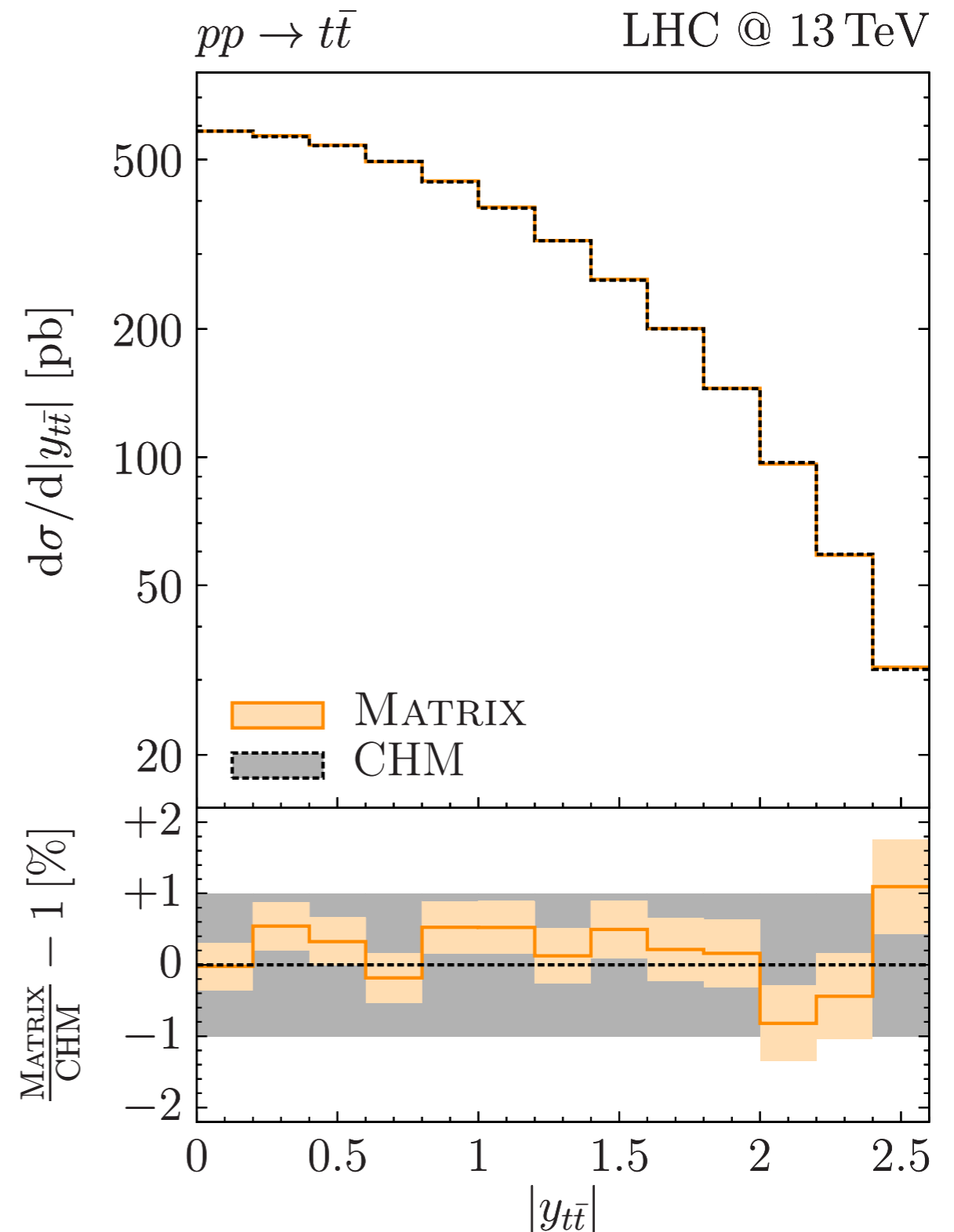
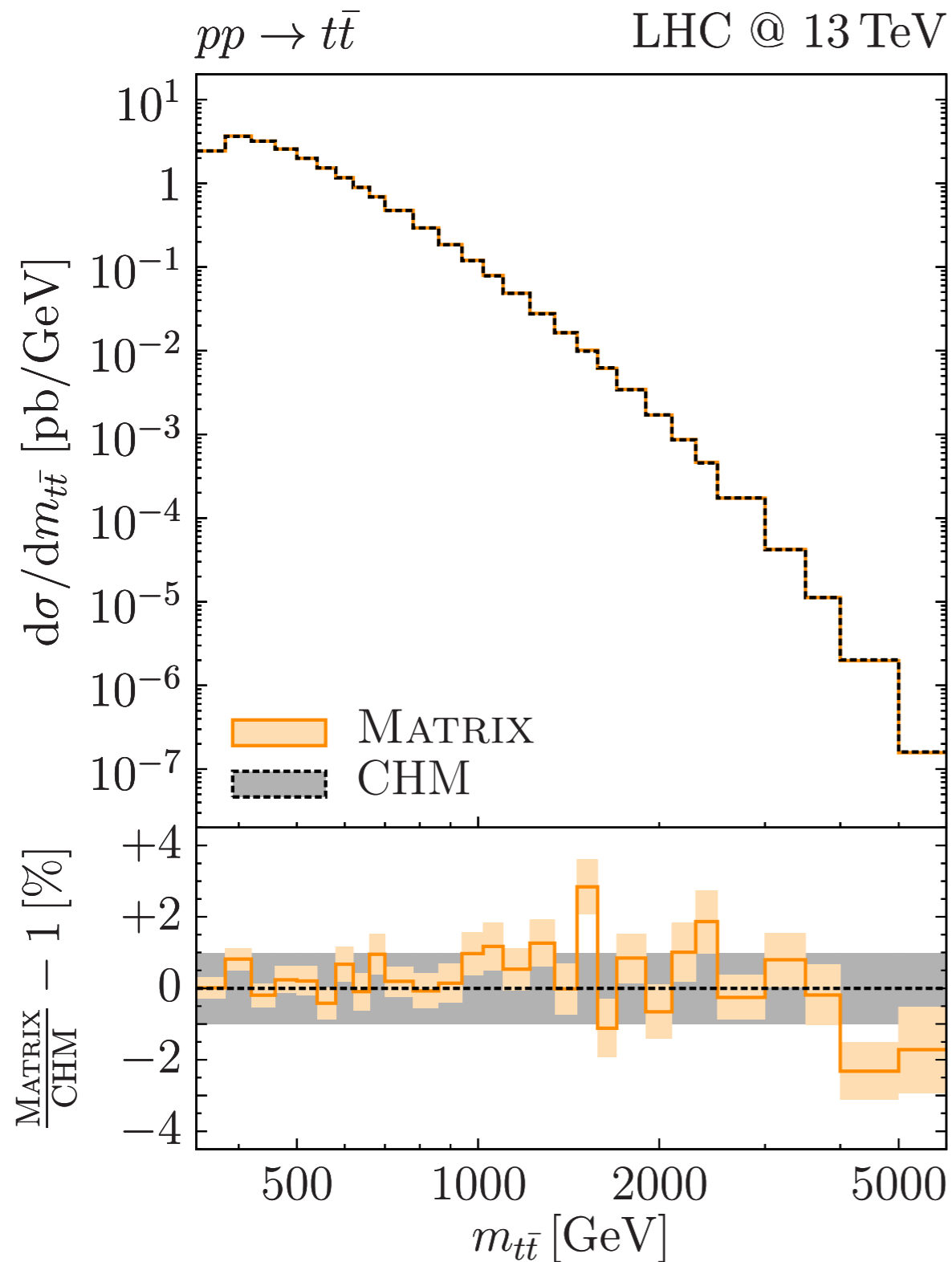
THANKS!



BACKUP SLIDES

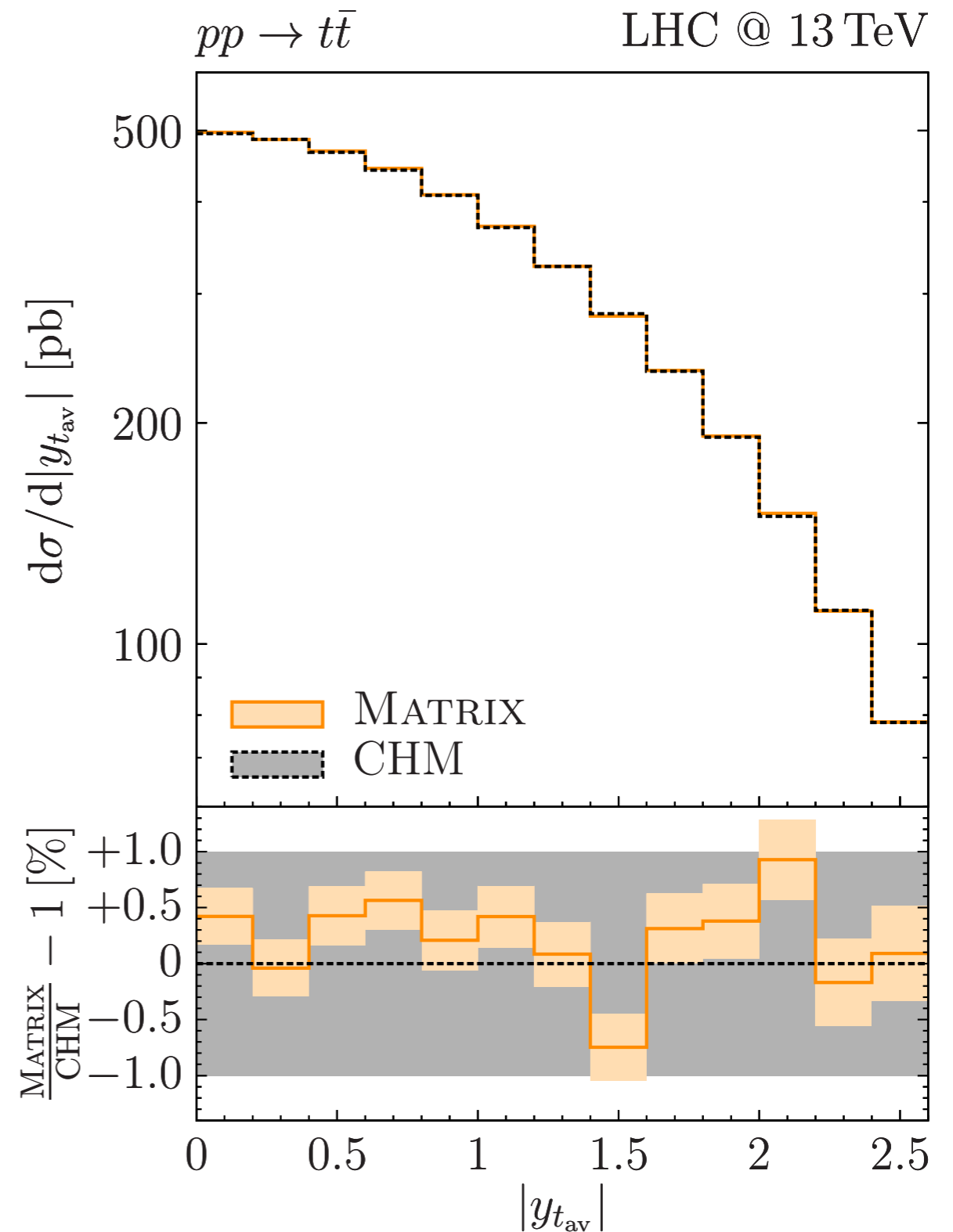
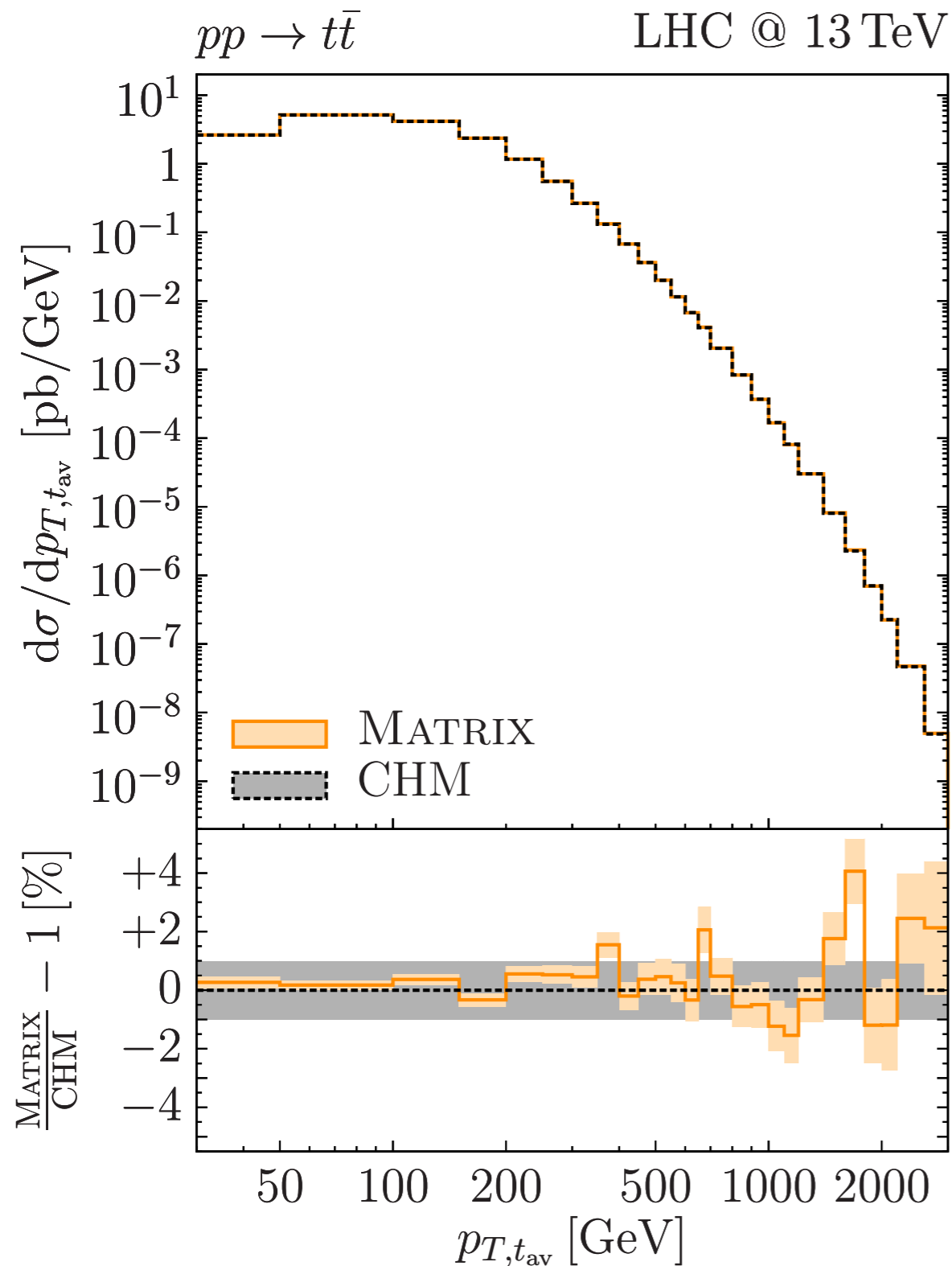
COMPARISON TO EXISTING RESULTS

CHM: [M. Czakon, D. Heymes,
A. Mitov (2017)]

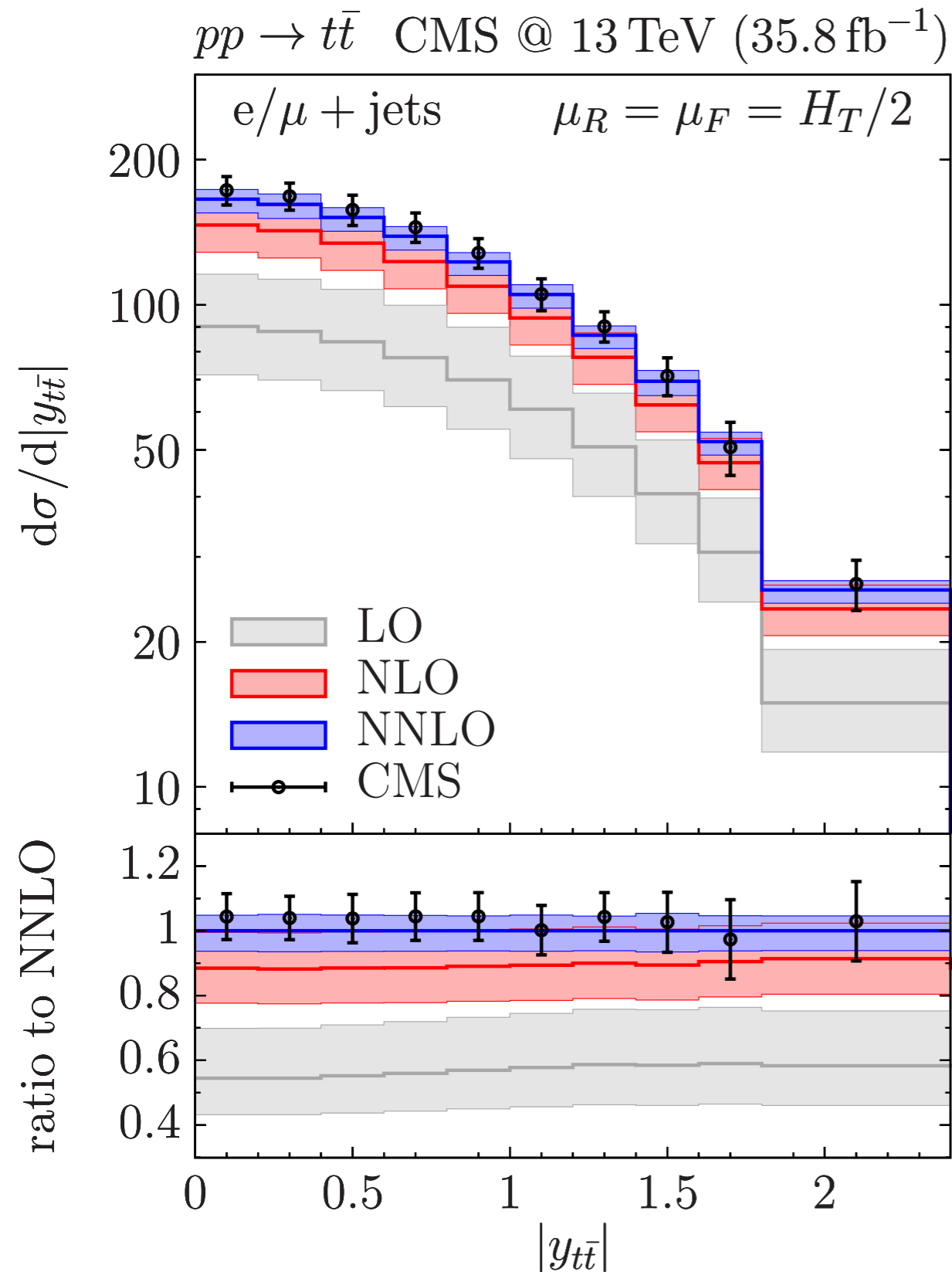


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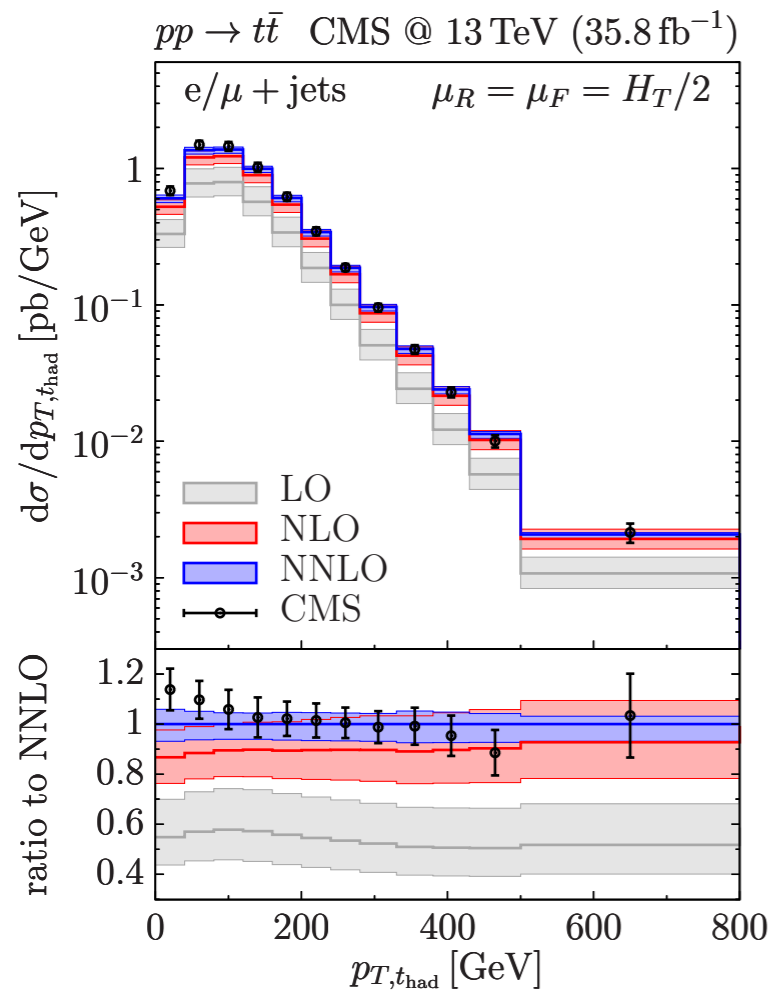
MORE DISTRIBUTIONS



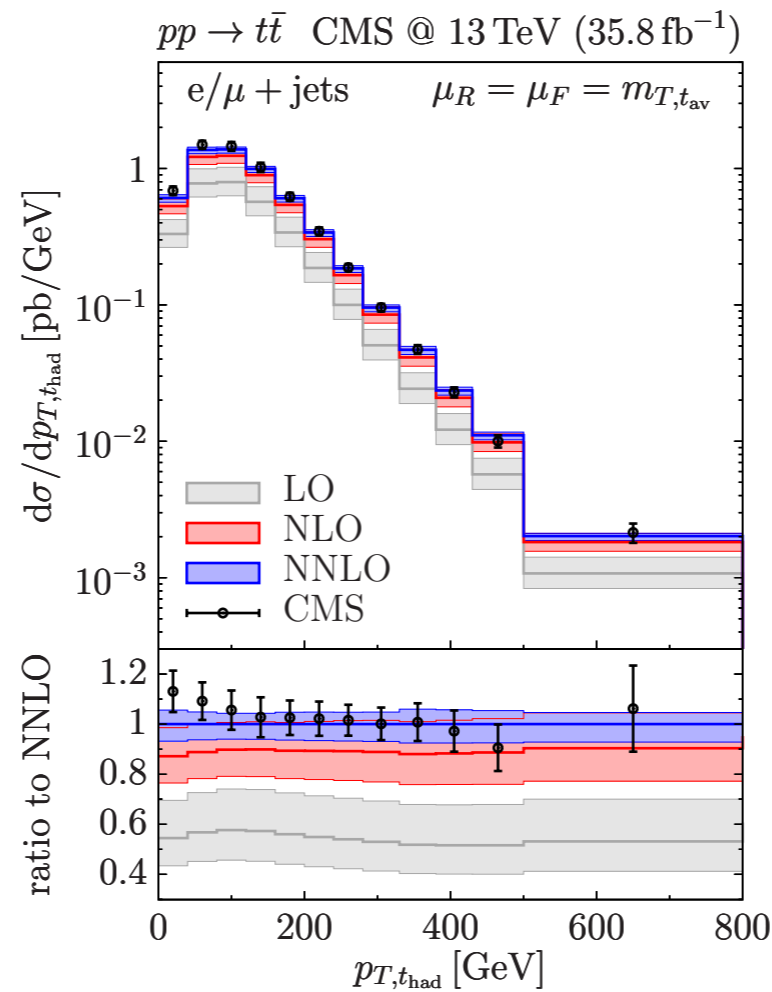
- LO and NLO bands do not overlap (consistent with total cross sections),
- NLO and NNLO bands overlap, suggesting convergence of the perturbative expansion;
- Good agreement data-theory.

DIFFERENT SCALE CHOICES

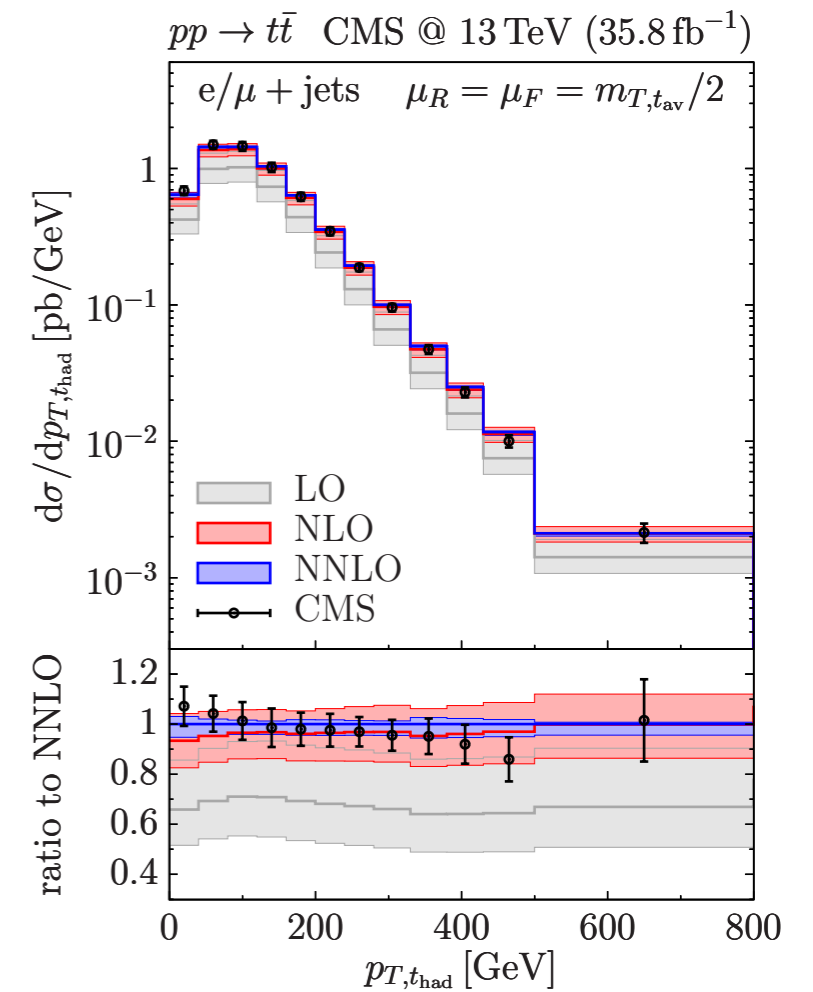
$$\mu_R = \mu_F = H_T/2$$



$$\mu_R = \mu_F = m_{T,t_{\text{av}}}$$



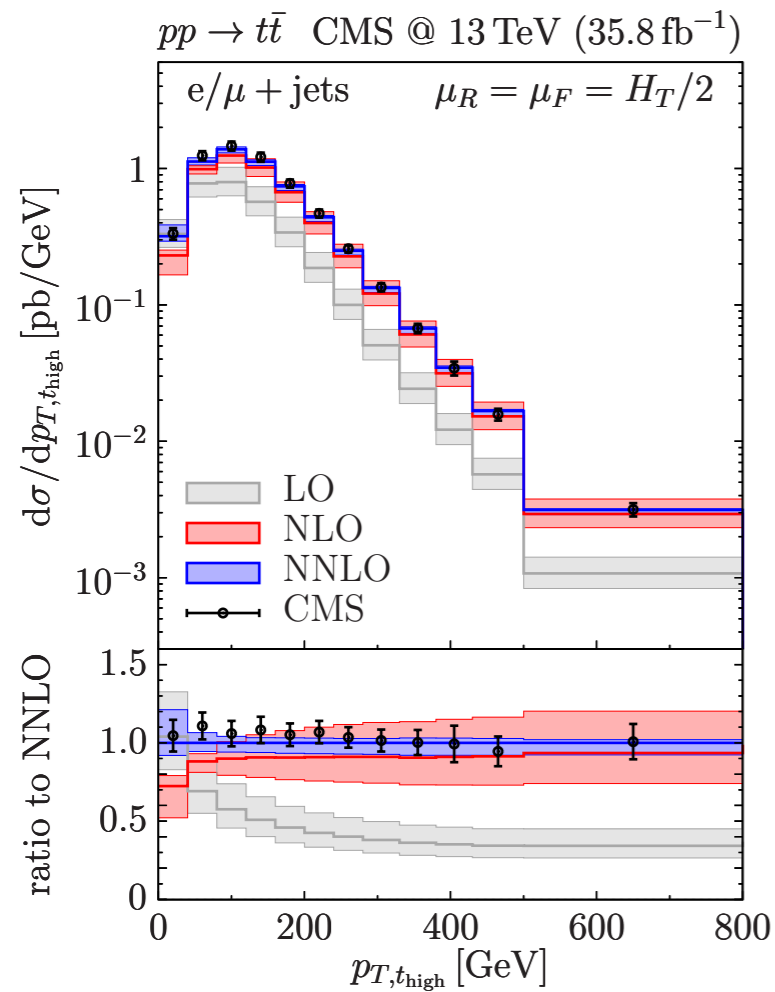
$$\mu_R = \mu_F = m_{T,t_{\text{av}}}/2$$



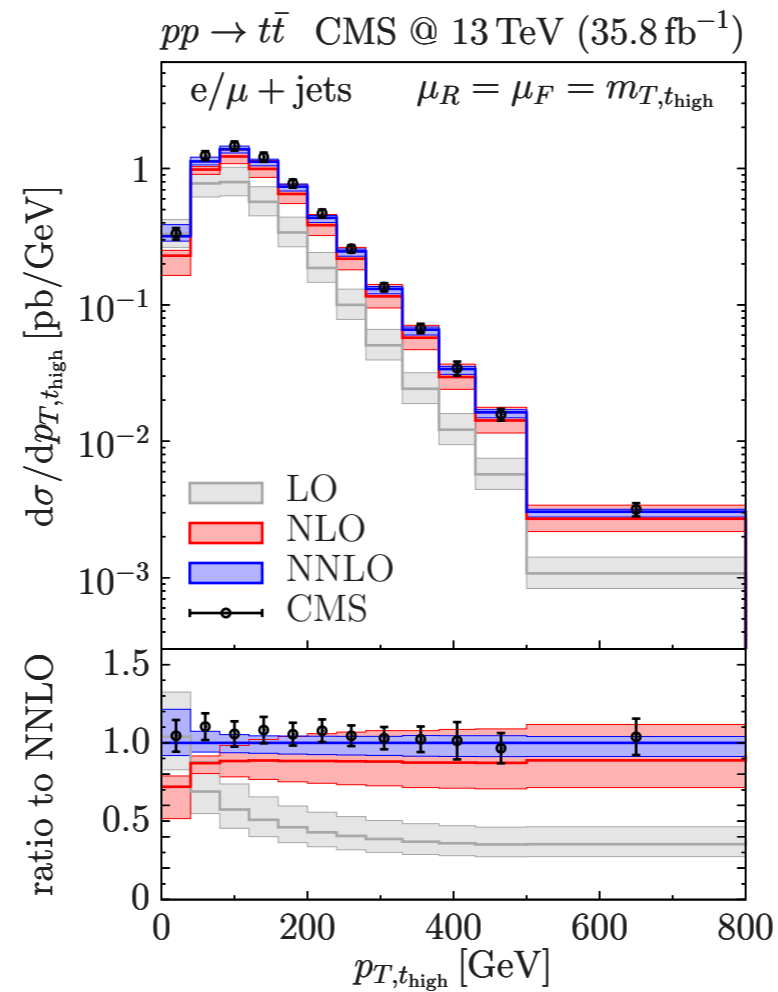
$p_{T,t_{\text{had}}}$ distributions

DIFFERENT SCALE CHOICES

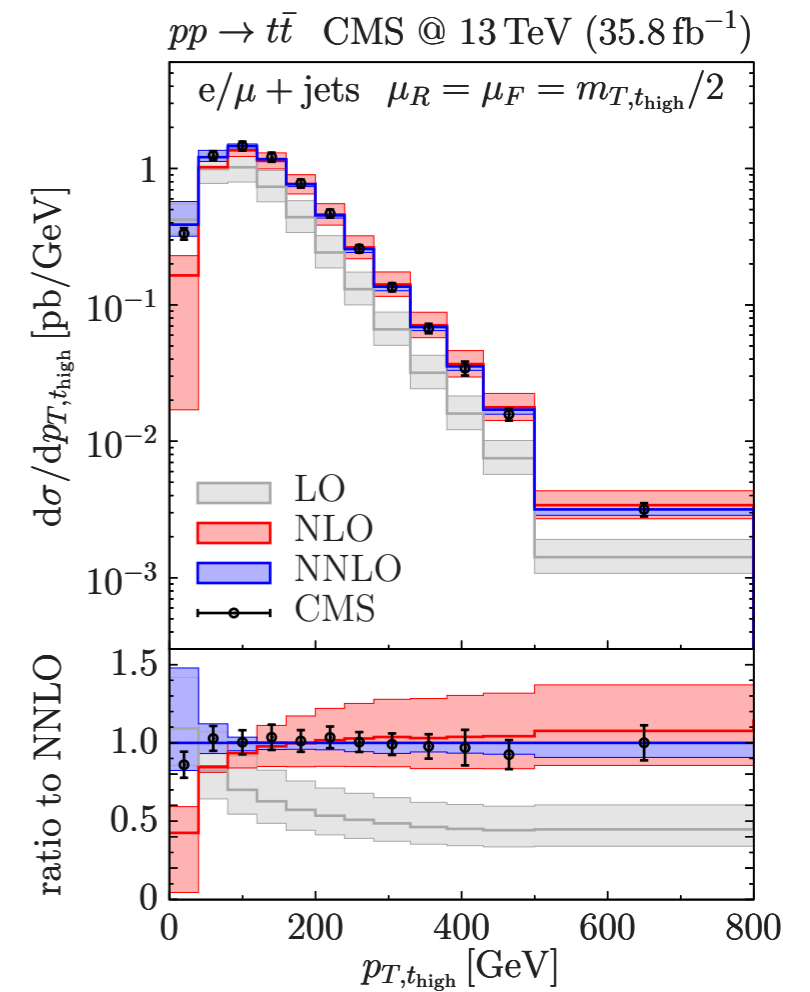
$$\mu_R = \mu_F = H_T/2$$



$$\mu_R = \mu_F = m_{T,t_{\text{high}}}$$



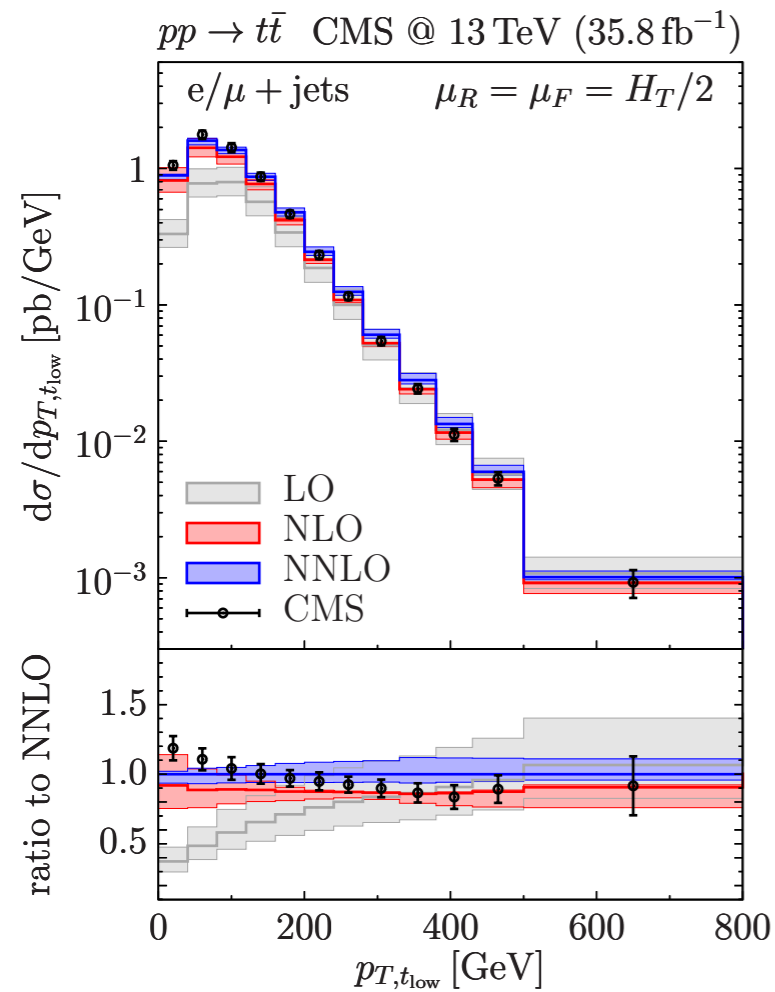
$$\mu_R = \mu_F = m_{T,t_{\text{high}}}/2$$



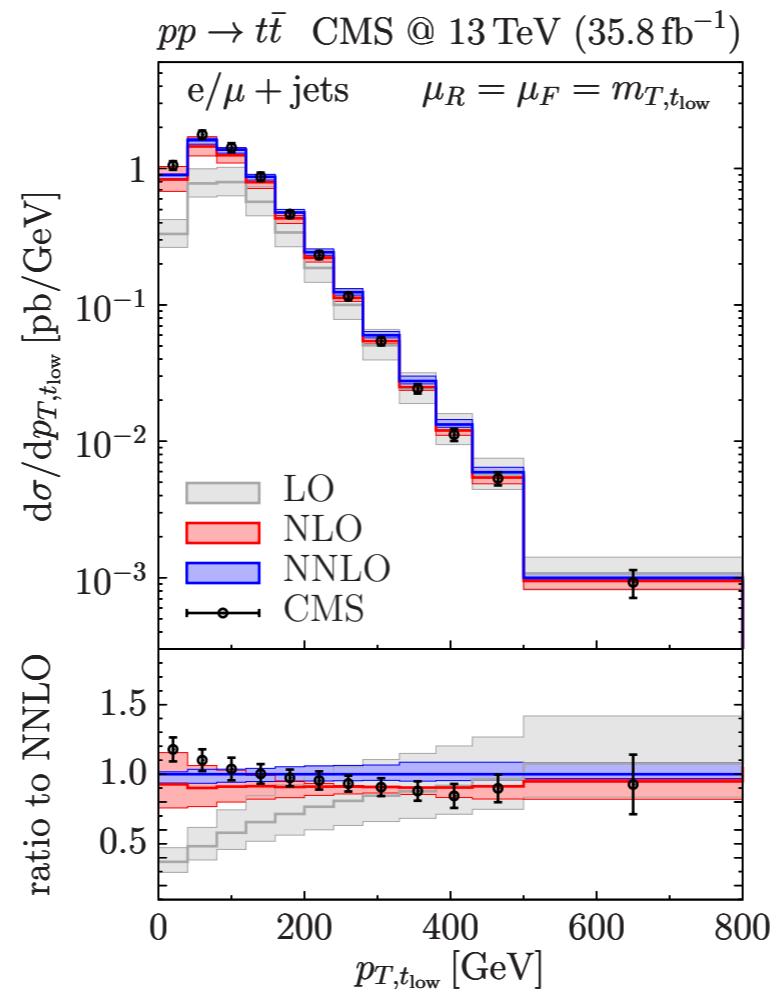
$p_{T,t_{\text{high}}}$ distributions

DIFFERENT SCALE CHOICES

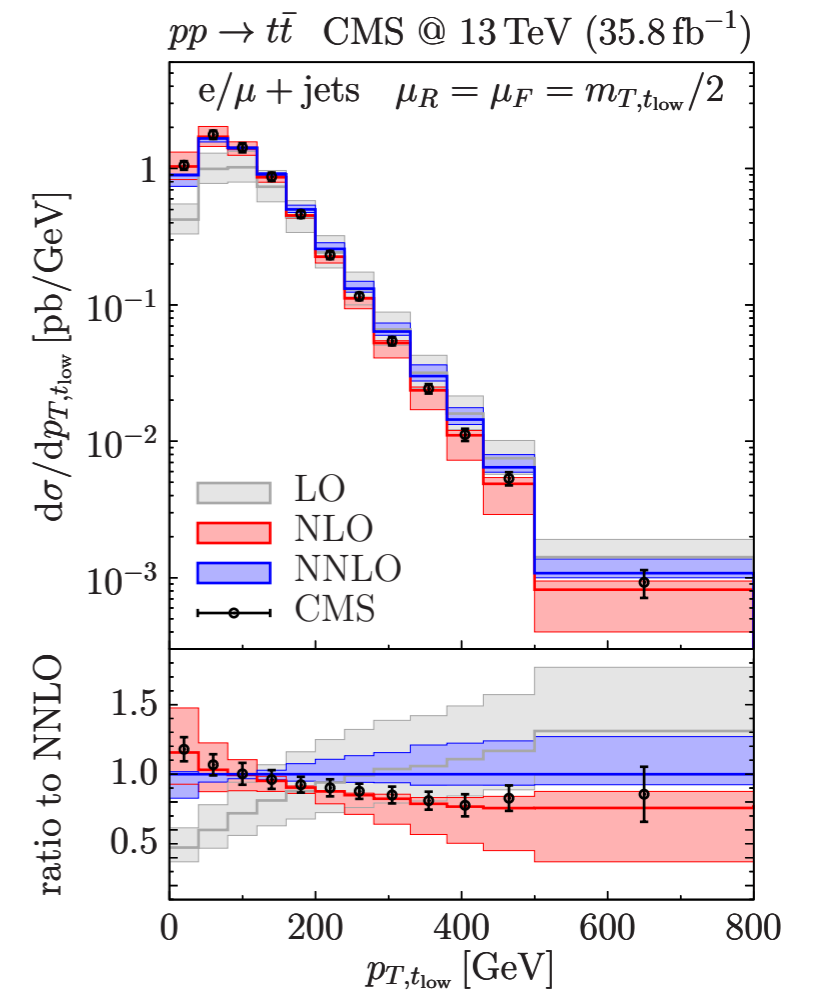
$$\mu_R = \mu_F = H_T/2$$



$$\mu_R = \mu_F = m_{T,t_{\text{low}}}$$



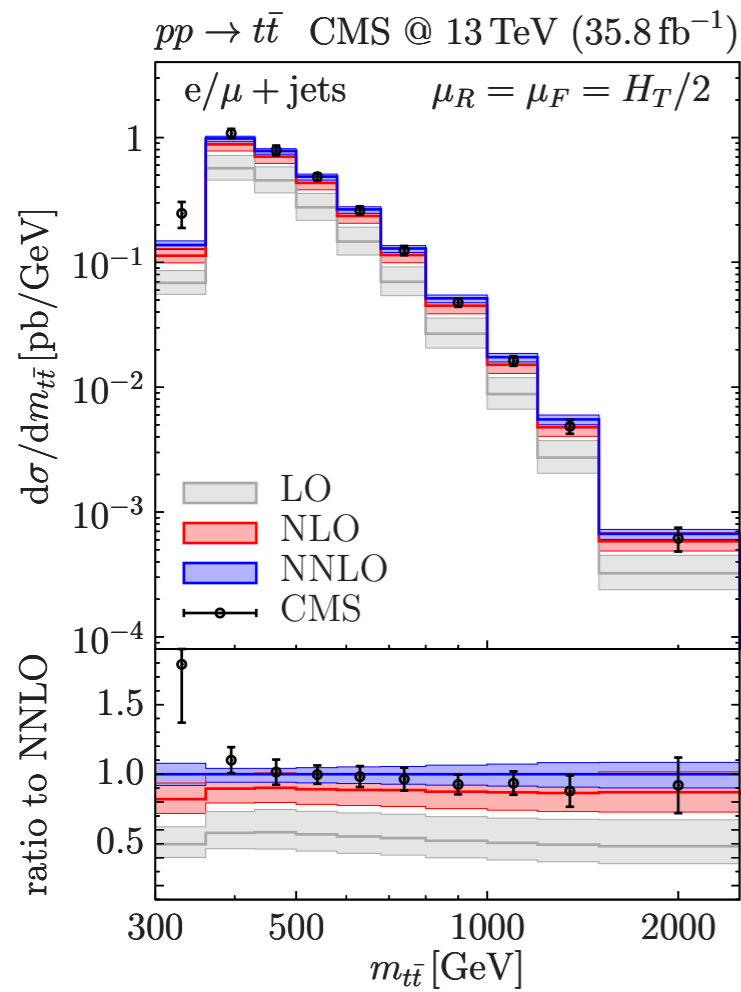
$$\mu_R = \mu_F = m_{T,t_{\text{low}}}/2$$



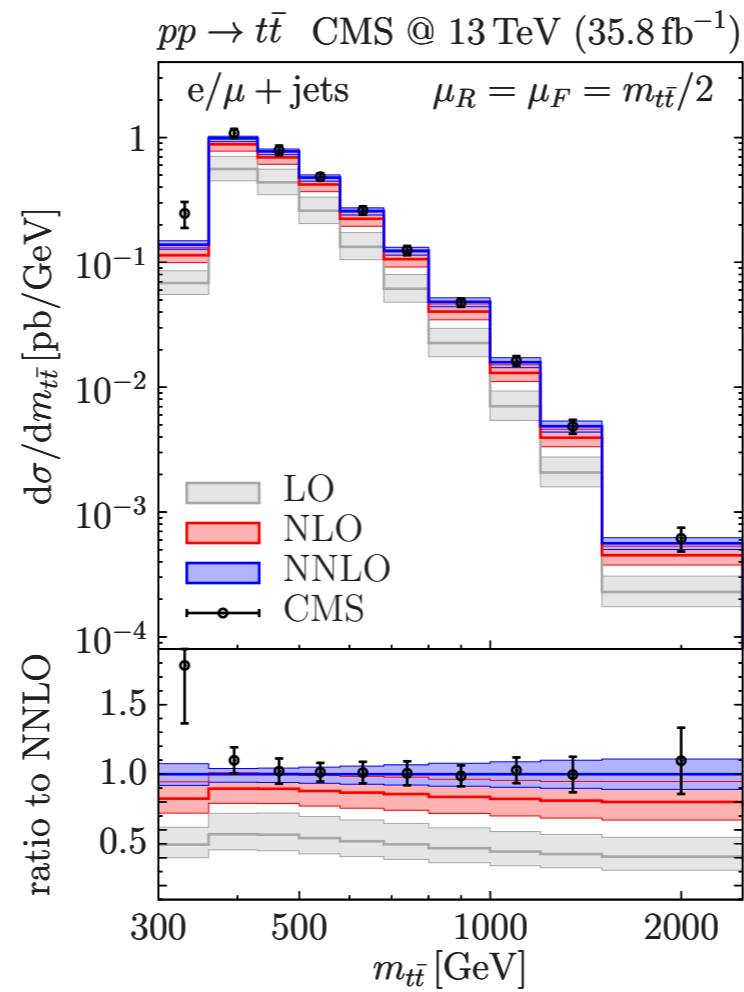
$p_{T,t_{\text{low}}}$ distributions

DIFFERENT SCALE CHOICES

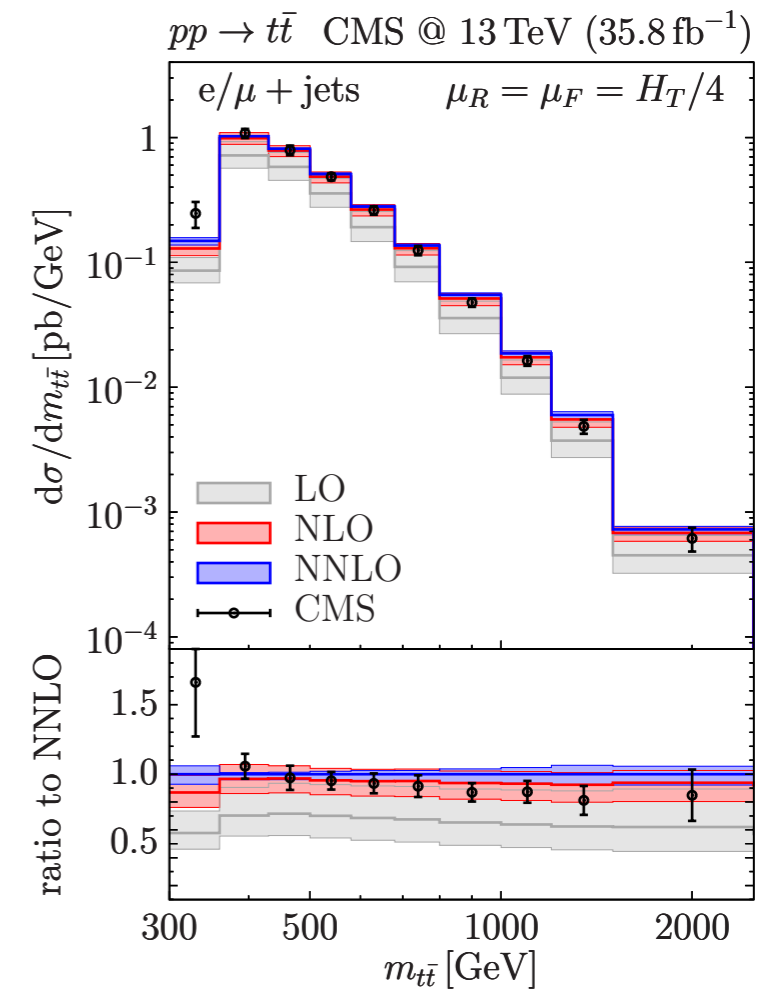
$$\mu_R = \mu_F = H_T/2$$



$$\mu_R = \mu_F = m_{t\bar{t}}$$

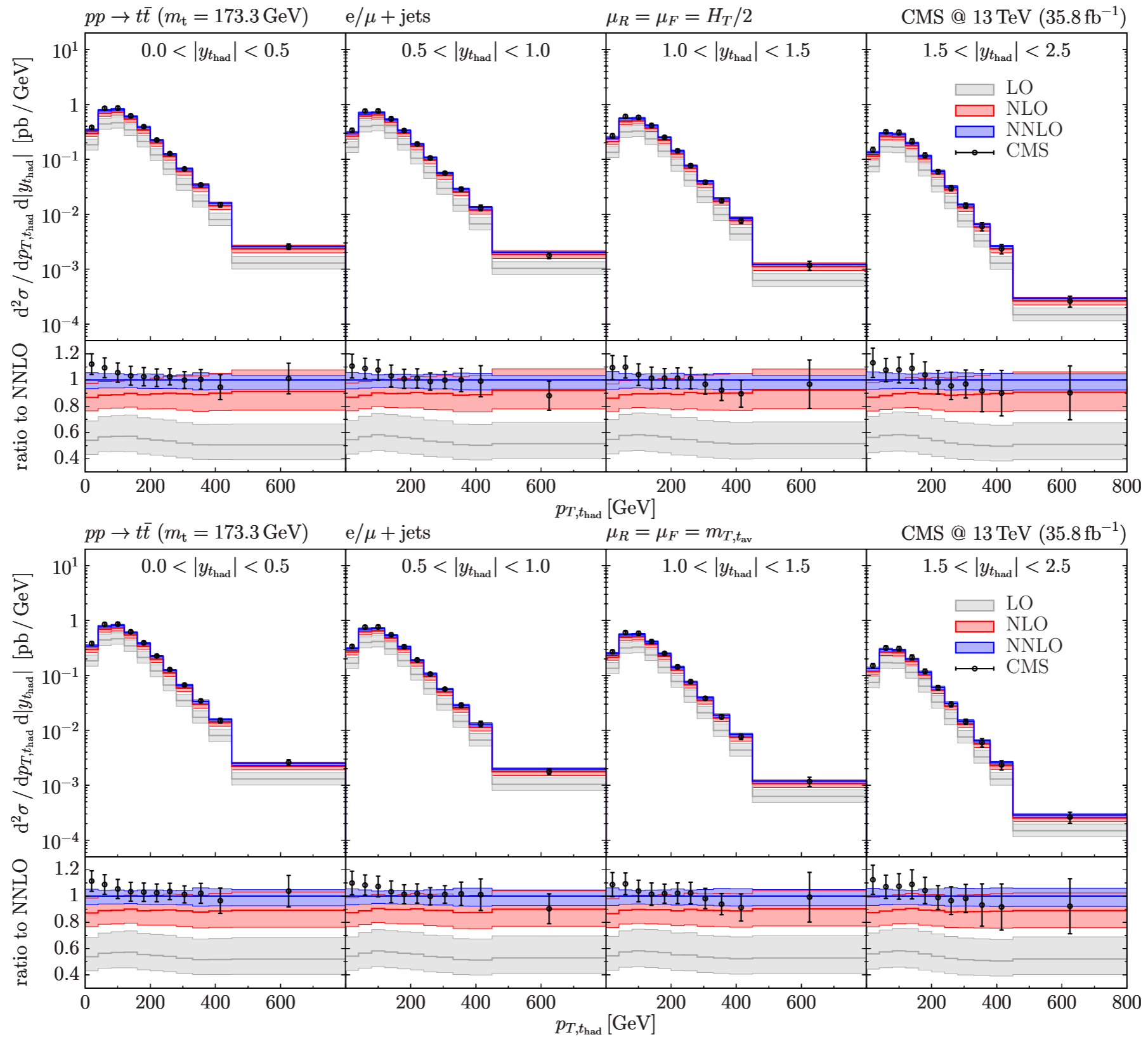


$$\mu_R = \mu_F = H_T/4$$

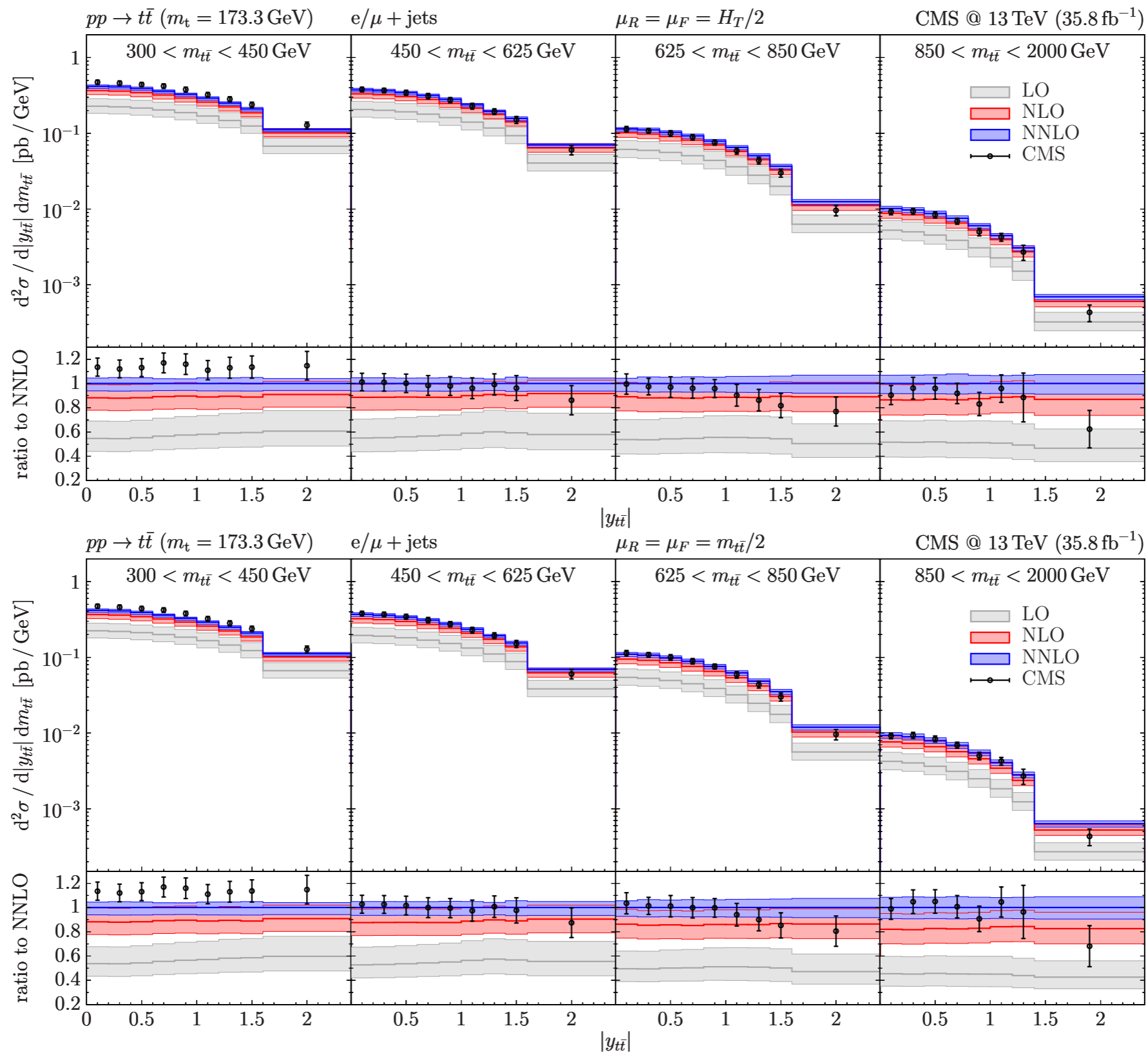


$m_{t\bar{t}}$ distributions

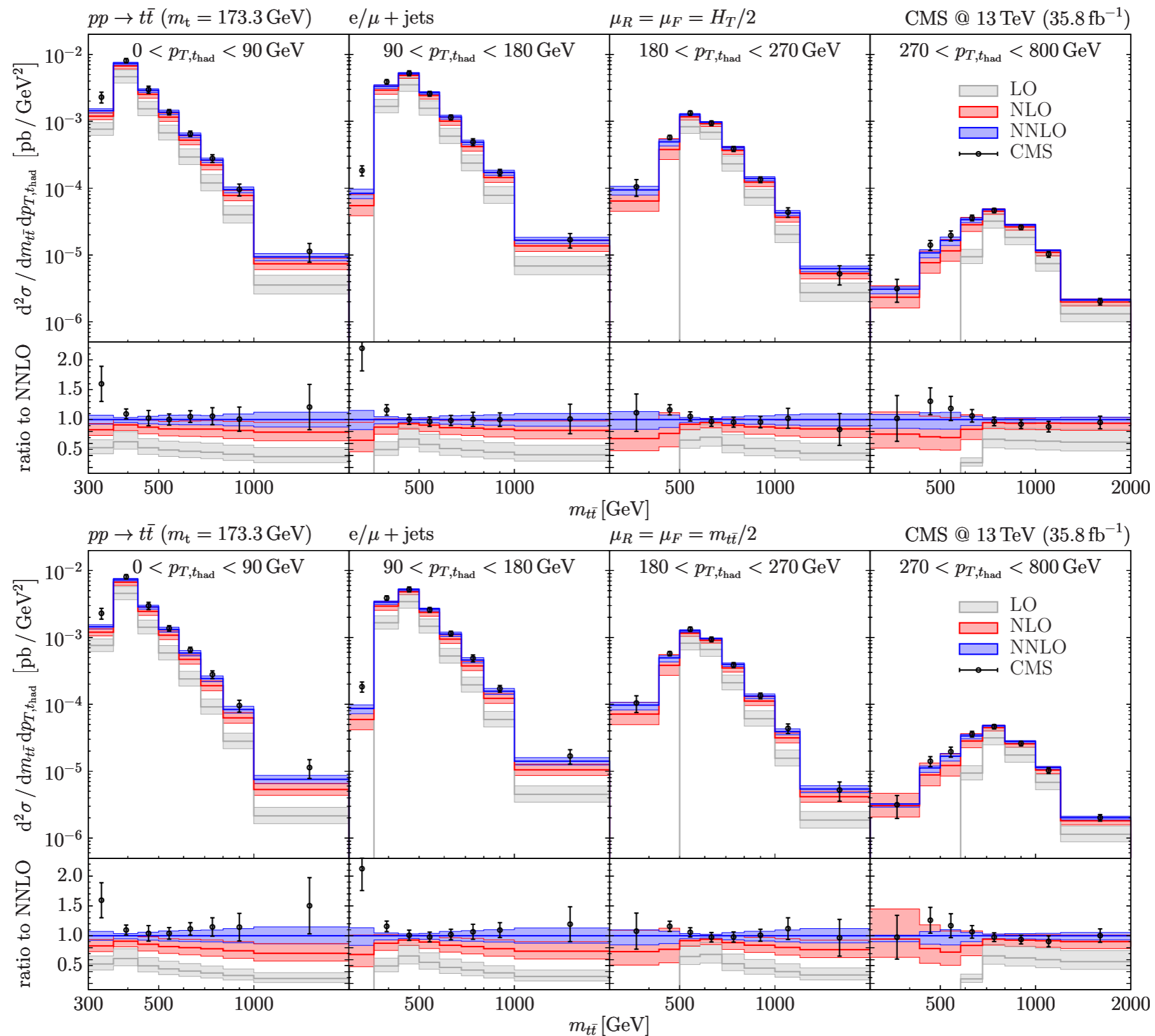
DIFFERENT SCALE CHOICES



DIFFERENT SCALE CHOICES



DIFFERENT SCALE CHOICES



INCLUSIVE CROSS SECTION

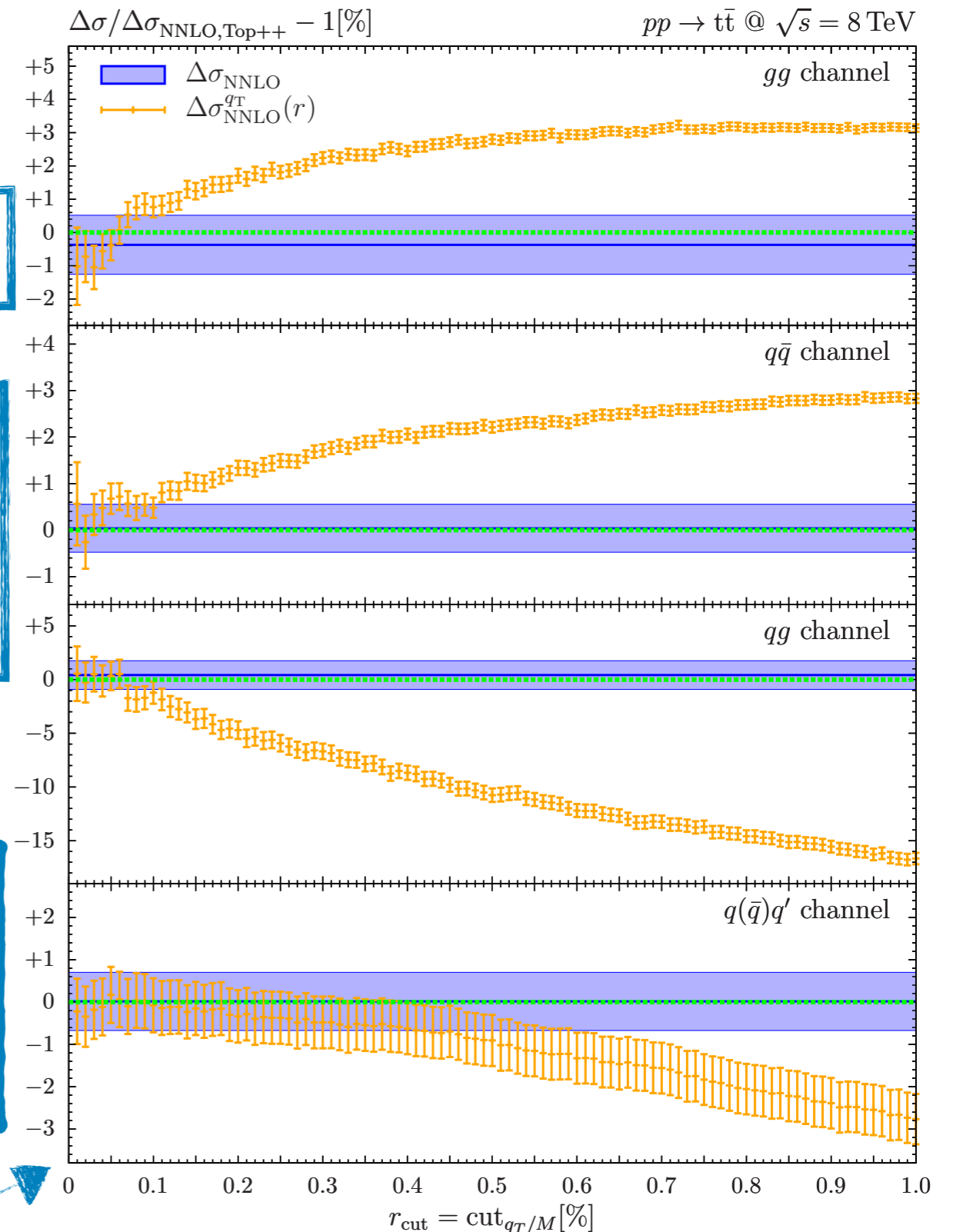
[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, H. Sargsyan (2019)]

$$d\sigma_{NNLO}^F = \mathcal{H}_{NNLO}^F \otimes d\sigma_{LO}^F + \left[d\sigma_{NLO}^{F+jets} - d\sigma_{NLO}^{CT} \right]$$

Separately divergent.

In practice, q_T subtraction implemented as a slicing method, introducing a cutoff $r_{cut} = Q/M$ and performing the limit $r_{cut} \rightarrow 0$.

Quality of the $q_T \rightarrow 0$ extrapolation can be understood looking at the r_{cut} dependence



DOUBLE GLUON EMISSION

[S. Catani, M. Grazzini: arXiv:9908523;
M. Czakon: arXiv:1101.0642]

$$\begin{aligned}
 \mathcal{S}_{ij}^{m=0}(q_1, q_2) = & \frac{(1 - \epsilon)}{(q_1 \cdot q_2)^2} \frac{p_i \cdot q_1 p_j \cdot q_2 + p_i \cdot q_2 p_j \cdot q_1}{p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \\
 & - \frac{(p_i \cdot p_j)^2}{2 p_i \cdot q_1 p_j \cdot q_2 p_i \cdot q_2 p_j \cdot q_1} \left[2 - \frac{p_i \cdot q_1 p_j \cdot q_2 + p_i \cdot q_2 p_j \cdot q_1}{p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \right] \\
 & + \frac{p_i \cdot p_j}{2 q_1 \cdot q_2} \left[\frac{2}{p_i \cdot q_1 p_j \cdot q_2} + \frac{2}{p_j \cdot q_1 p_i \cdot q_2} - \frac{1}{p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \right. \\
 & \left. \times \left(4 + \frac{(p_i \cdot q_1 p_j \cdot q_2 + p_i \cdot q_2 p_j \cdot q_1)^2}{p_i \cdot q_1 p_j \cdot q_2 p_i \cdot q_2 p_j \cdot q_1} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S}_{ij}^{m \neq 0}(q_1, q_2) = & - \frac{1}{4 q_1 \cdot q_2 p_i \cdot q_1 p_i \cdot q_2} + \frac{p_i \cdot p_j p_j \cdot (q_1 + q_2)}{2 p_i \cdot q_1 p_j \cdot q_2 p_i \cdot q_2 p_j \cdot q_1 p_i \cdot (q_1 + q_2)} \\
 & - \frac{1}{2 q_1 \cdot q_2 p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)} \left(\frac{(p_j \cdot q_1)^2}{p_i \cdot q_1 p_j \cdot q_2} + \frac{(p_j \cdot q_2)^2}{p_i \cdot q_2 p_j \cdot q_1} \right)
 \end{aligned}$$

DOUBLE GLUON EMISSION

[S. Catani, M. Grazzini: arXiv:9908523;
M. Czakon: arXiv:1101.0642]

$$\tilde{\mathcal{S}}_{ij}^{m=0}(q_1, q_2) = -\frac{(p_i \cdot p_j)^2}{2(p_i \cdot k)(p_j \cdot k)} \left(\frac{2}{(p_i \cdot q_1)(p_j \cdot q_1)} + \frac{1}{(p_i \cdot q_1)(p_j \cdot q_2)} \right) + \frac{(p_i \cdot p_j)}{k^2} \frac{2}{(p_i \cdot q_1)(p_j \cdot q_2)}$$

$$-\frac{(p_i \cdot p_j)}{2k^2(p_i \cdot k)(p_j \cdot k)} \frac{((p_i \cdot q_1)(p_j \cdot q_2) - (p_i \cdot q_2)(p_j \cdot q_1))^2}{(p_i \cdot q_1)(p_j \cdot q_2)(p_i \cdot q_2)(p_j \cdot q_1)} + (1 \leftrightarrow 2)$$

$$\tilde{\mathcal{S}}_{ij}^{m \neq 0}(q_1, q_2) = \frac{(p_i \cdot p_j)}{2(p_i \cdot k)^2} \left(\frac{1}{(p_i \cdot q_1)(p_j \cdot q_1)} + \frac{1}{(p_i \cdot q_1)(p_j \cdot q_2)} \right)$$

$$-\frac{1}{k^2(p_i \cdot k)} \frac{1}{(p_i \cdot q_1)} \left(\frac{(p_j \cdot q_1)^2}{(p_j \cdot k)(p_j \cdot q_2)} - \frac{(p_i \cdot q_1)^2}{(p_i \cdot k)(p_i \cdot q_2)} \right) + (1 \leftrightarrow 2)$$

$$k = q_1 + q_2$$

ONE GLUON EMISSION AT 1 LOOP

[I. Bierenbaum, M.Czakon, A. Mitov: arXiv:1107.4384;
M. Czakon, A.Mitov: arXiv:1804.02069]

$$\left| J_{sub}^{NNLO(1L)}(k) \right|^2 = \langle M_a^{(0)}(n+1; k) | M_a^{(1)}(n+1; k) \rangle + c.c.$$

$$\langle M_a^{(0)}(n+1; k) | M_a^{(1)}(n+1; k) \rangle + c.c. = -4\pi\alpha_S\mu^{2\epsilon}$$

$$\times \left\{ 2C_A \sum_{i \neq j=1}^n (e_{ij} - e_{ii}) R_{ij} \langle M^{(0)}(n) | T_i \cdot T_j | M^{(0)}(n) \rangle - 4\pi \sum_{i \neq j \neq k=1}^n e_{ik} I_{ij} \langle M^{(0)}(n) | f^{abc} T_i^a T_j^b T_k^c | M^{(0)}(n) \rangle \right.$$

The expansion in ϵ of R_{ij} , I_{ij} can be found in [I. Bierenbaum, M.Czakon, A. Mitov: arXiv:1107.4384].
Simplified expressions recently published in [M. Czakon, A.Mitov: arXiv:1804.02069].

$$+ \left(\sum_{i \neq j=1}^n e_{ij} \langle M^{(0)}(n) | T_i \cdot T_j | M^{(1)}(n) \rangle + c.c. \right) + \left(\sum_{i=1}^n \mathcal{C}_i e_{ii} \langle M^{(0)}(n) | M^{(1)}(n) \rangle + c.c. \right) \Bigg\}$$

NLO-like contributions

$$e_{ij} = \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)}$$

$$e_{ii} = \frac{m_i^2}{(p_i \cdot k)^2}$$