Mixed QCD-Electroweak Corrections at $O(\alpha\alpha_s)$ to Drell-Yan Production of W and Z bosons

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in collaboration with

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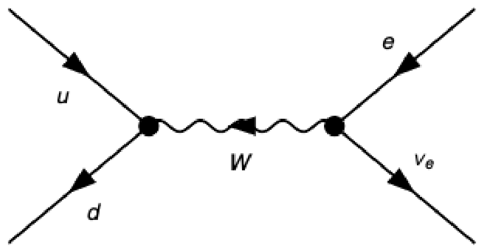
Outline

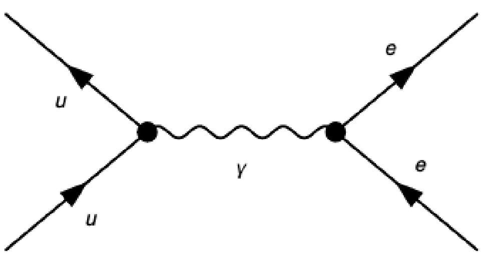
- Motivation
- Mixed QCD-EW corrections to DY processes
- Double Virtual corrections ($O(\alpha\alpha_s)$) to W and Z production
- Real-Virtual Corrections
- Outlook

W and Z production at the LHC via Drell-Yan Processes

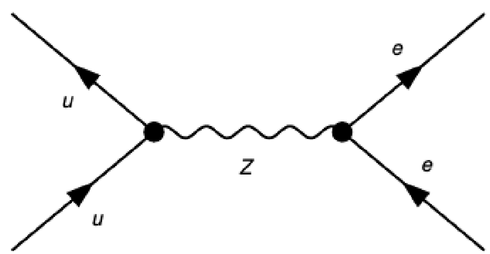


T1 C1 N1

Charged Current



T1 C1 N1



T1 C2 N2

Neutral Current

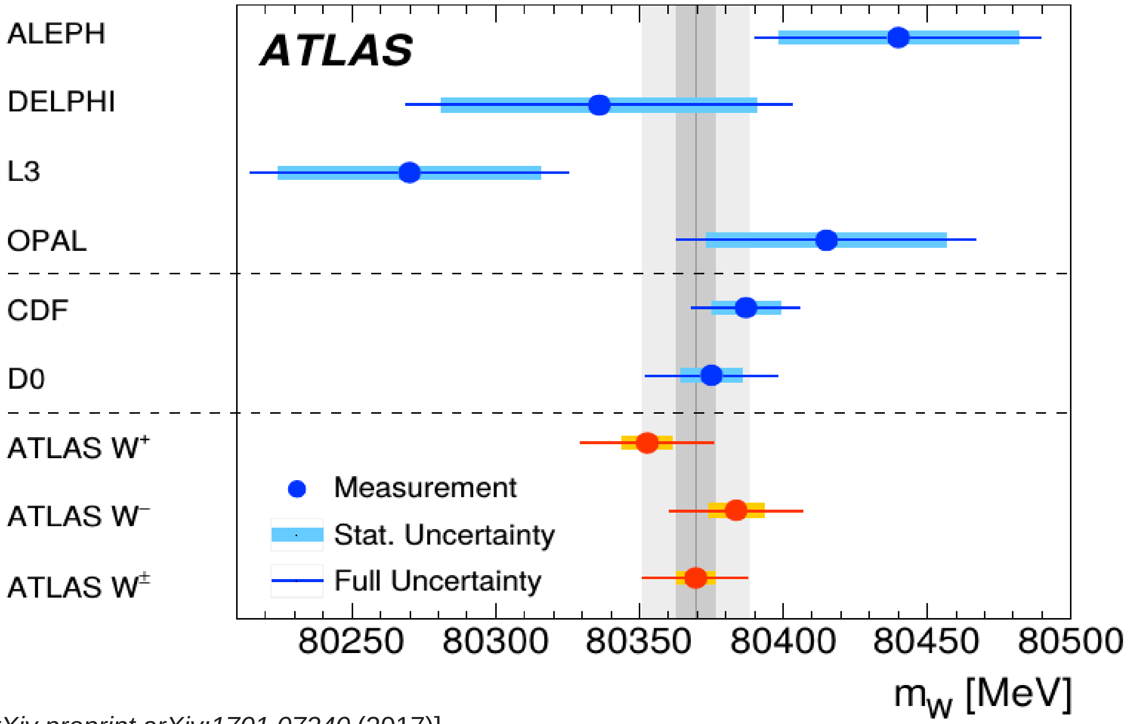
W and Z production at the LHC

- Big cross section and clean experimental signature; which allows precise test of SM interactions.
- Allows to determine important parameters in electroweak sector, eg. W boson mass and $sin^2\theta^l_{eff}$. Together with top and Higgs masses, it provides constraint for the validity of SM.
- Important in search for new physics, eg. W' and Z' resonances.
- Important for constraining Parton Distribution Function (PDF), for detector calibration and determination of collider luminosity.

ATLAS Report on W mass (January 2017)

Currently at the LHC M_W is extracted from M_T and P_T of the $l\nu$ in W boson production.

$$m_W = 80370 \pm 7\,(\text{stat.}) \pm 11\,(\text{exp. syst.}) \pm 14\,(\text{mod. syst.})\ \text{MeV}$$
$$= 80370 \pm 19\ \text{MeV},$$



[ATLAS Collaboration. *arXiv preprint arXiv:1701.07240* (2017)]

W mass (Theory)

- W mass can be extracted from combining EW precision observables with accurate theoretical predictions.
- Given precise values of fine structure constant, Fermi constant, Z mass, recent measured values of top and Higgs, SM prediction of W mass (as a result of global electroweak fit) is:

M. Baak et al.
GFITTER

$$m_W = 80358 \pm 8 \text{ MeV}$$

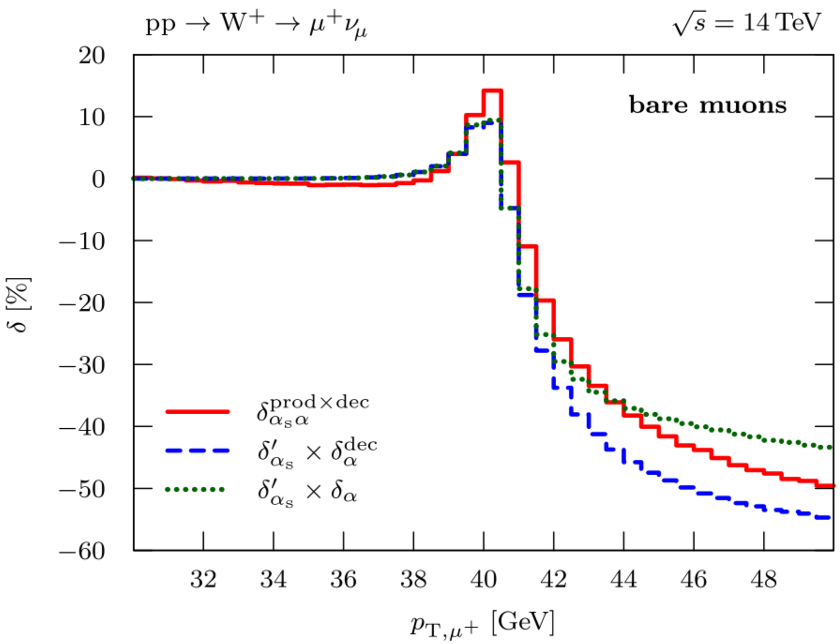
J. de Blas et al.
HEPFIT

$$m_W = 80362 \pm 8 \text{ MeV}$$

Why $O(\alpha\alpha_s)$ corrections ?

- Including NLO (Next to Leading order) QED final state radiative effects to W production cross sections makes W mass to shift about 100 MeV - 200 MeV.
- Including final state multiple photon radiation to all orders includes an additional shift of order -10% of the $O(\alpha)$.
- Fixed order QCD effects are known up to NNLO.
- Given that QED effects are so large one also needs to control Mixed EW-QCD corrections at $O(\alpha\alpha_s)$ when aiming for 10 MeV precision.
- Different subsets of corrections became available in the past years in codes that simulate QCD or purely EW effects.
- Combination of QCD and EW effect is important step to develop next MC programs for DY processes at the LHC. Preliminary studies of approximation to $O(\alpha\alpha_s)$ has shown that these effects are not negligible.

Mixed QCD-EW correction in pole approximation



$$\delta_\alpha \equiv \frac{\Delta\sigma^{\mathrm{NLO_{ew}}}}{\sigma^0}$$

$$\delta_{\alpha_s\alpha}^{\mathrm{prod\times dec}} \equiv \frac{\Delta\sigma^{\mathrm{NNLO_{s\otimes ew}}}_{\mathrm{prod\times dec}}}{\sigma^{\mathrm{LO}}}$$

$$\delta'_{\alpha_s} \equiv \frac{\Delta\sigma^{\mathrm{NLO_s}}}{\sigma^{\mathrm{LO}}}$$

$$\sigma^{\mathrm{NNLO_{s\otimes ew}}} = \sigma^0 + \Delta\sigma^{\mathrm{NLO_s}} + \Delta\sigma^{\mathrm{NLO_{ew}}} + \Delta\sigma^{\mathrm{NNLO_{s\otimes ew}}}_{\mathrm{prod\times dec}}$$

$$\sigma^{\mathrm{NNLO_{s\otimes ew}}}_{\mathrm{naive\ fact}} = \sigma^{\mathrm{NLO_s}}(1+\delta_\alpha) = \sigma^0 + \Delta\sigma^{\mathrm{NLO_s}} + \Delta\sigma^{\mathrm{NLO_{ew}}} + \Delta\sigma^{\mathrm{NLO_s}}\,\delta_\alpha$$

[Huss et. al. 2016]

Mixed QCD-EW correction enhancement at higher energy

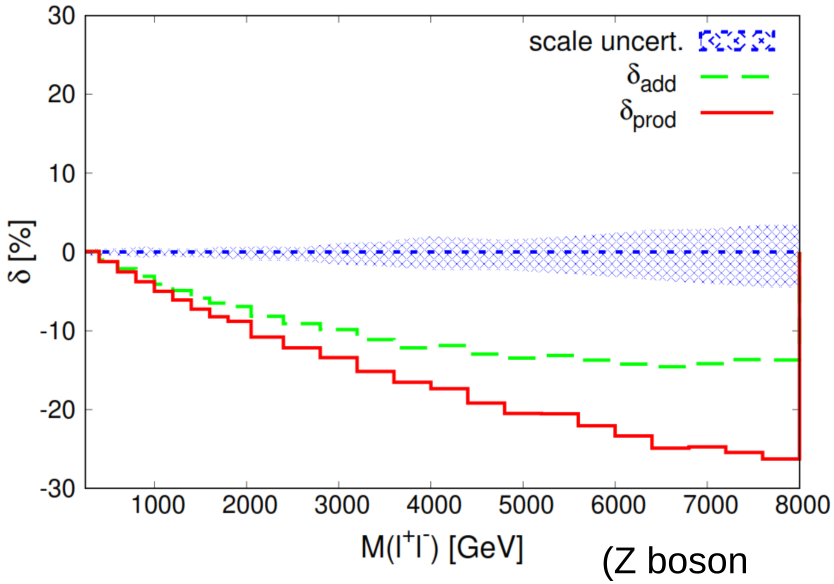
[Campbell, John M., Doreen Wackeroth, and Jia Zhou. "Study of weak corrections to Drell-Yan, top-quark pair, and dijet production at high energies with MCFM." *Physical Review D* 94.9 (2016): 093009]

$$\sigma_{QCD+\mathrm{wk}} = \sigma_{(N)NLO\,QCD} + \sigma_{\mathrm{wk}}$$

$$\sigma_{QCD\times\mathrm{wk}} = \sigma_{(N)NLO\,QCD}\left(1 + \frac{\sigma_{\mathrm{wk}}}{\sigma_{LO}}\right)$$

$$\delta_{\mathrm{add}} = \frac{\sigma_{QCD+\mathrm{wk}} - \sigma_{(N)NLO\,QCD}}{\sigma_{(N)NLO\,QCD}} = \frac{\sigma_{\mathrm{wk}}}{\sigma_{(N)NLO\,QCD}}$$

$$\delta_{\mathrm{prod}} = \frac{\sigma_{QCD\times\mathrm{wk}} - \sigma_{(N)NLO\,QCD}}{\sigma_{(N)NLO\,QCD}} = \frac{\sigma_{\mathrm{wk}}}{\sigma_{LO}}$$



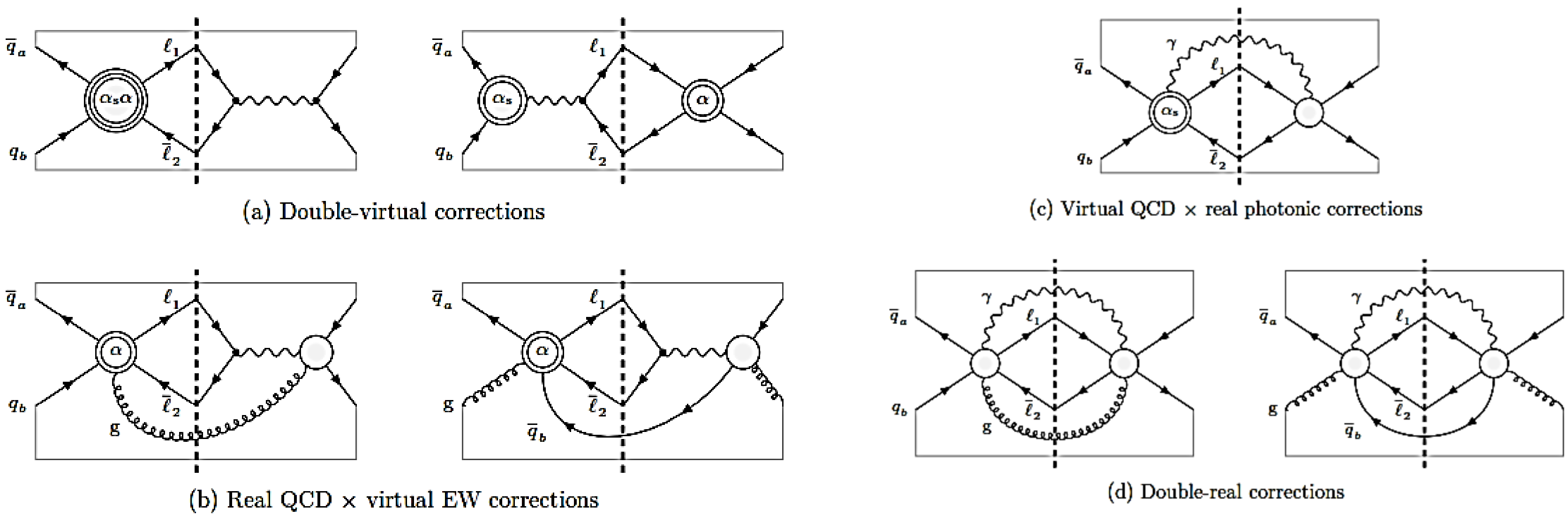
(Z boson invariant Mass)

Examples of $O(\alpha\alpha_s)$ calculations for W and Z production

- NNLO QCD and QED corrections [Hamberg et al ’91],[Anastasiou et al ‘03,’04],[Melnikov,Petriello ‘06],[Stefano et al ’07]
- NNLO Mixed QCD-EW corrections to decay of W & Z boson. [Kuhn et al ‘96],[Kara ‘13]
- NNLO Mixed QCD-EW corrections to Z production form factors [Kotikov et al ‘08]
- NNLO QCD-QED virtual corrections to lepton pair production. [Kilgore et al ‘12]
- NNLO Mixed QCD-EW virtual corrections to DY production of W and Z bosons [Bonciani ‘11]
- Double real contribution to total cross section for on-shell single gauge boson production. [Bonciani et al 2016]
- NNLO Mixed QCD-EW corrections adopting pole approximation. [Dittmaier, Huss, Schwinn ‘14,’16]
- QCD×QED [O($\alpha\alpha_s$)] mixed and QED2 [O(α^2)] corrections to the production of an on-shell Z boson [Florian, Ignacio 2018]
- To do : Complete NNLO Mixed QCD-EW corrections for W and Z production in a fully flexible Monte Carlo program.

Structure of the fixed order prediction

$$d\sigma = d\sigma_{LO} + \alpha d\sigma_{\alpha} + \alpha^2 d\sigma_{\alpha^2} + \ldots + \alpha_s d\sigma_{\alpha_s} + \alpha_s^2 d\sigma_{\alpha_s^2} + \ldots + \alpha\alpha_s d\sigma_{\alpha\alpha_s} + \alpha\alpha_s^2 d\sigma_{\alpha\alpha_s^2} + \ldots$$



(a) Double-virtual corrections

(c) Virtual QCD × real photonic corrections

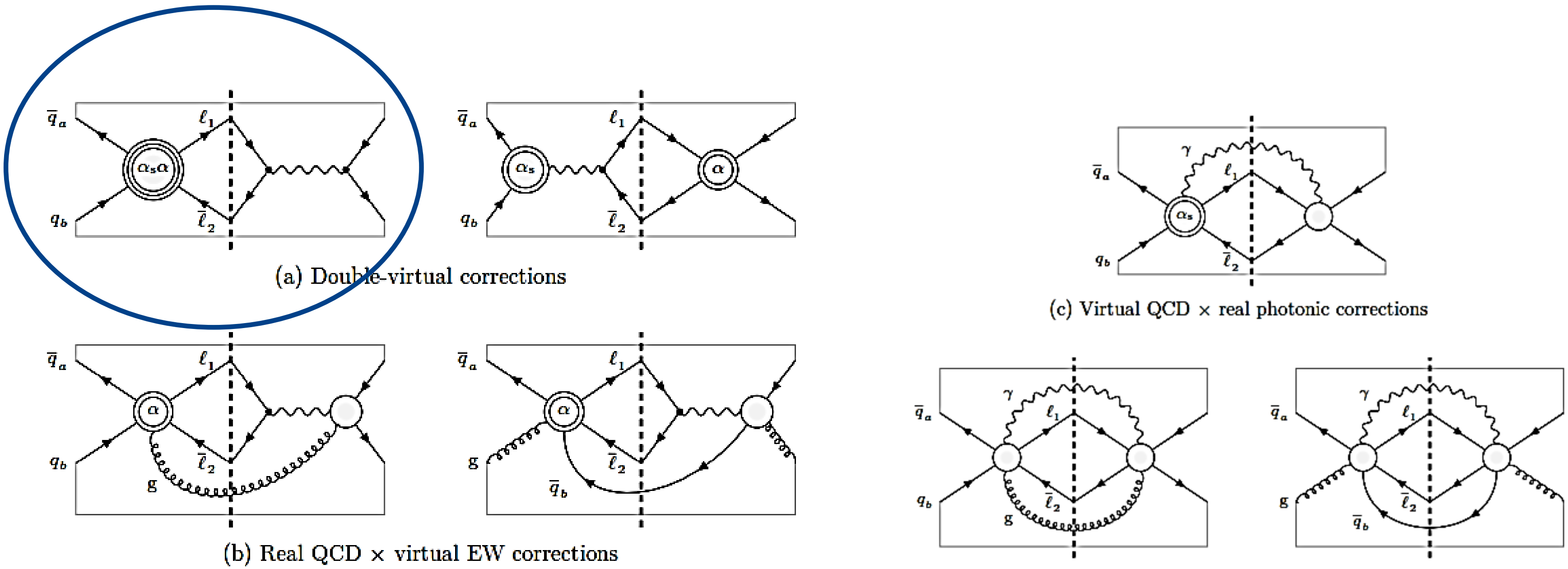
(b) Real QCD × virtual EW corrections

(d) Double-real corrections

[Alexander Huss '14]

Structure of the fixed order prediction

$$d\sigma = d\sigma_{LO} + \alpha d\sigma_{\alpha} + \alpha^2 d\sigma_{\alpha^2} + \ldots + \alpha_s d\sigma_{\alpha_s} + \alpha_s^2 d\sigma_{\alpha_s^2} + \ldots + \alpha\alpha_s d\sigma_{\alpha\alpha_s} + \alpha\alpha_s^2 d\sigma_{\alpha\alpha_s^2} + \ldots$$



(a) Double-virtual corrections

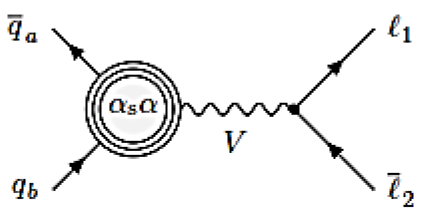
(b) Real QCD × virtual EW corrections

(c) Virtual QCD × real photonic corrections

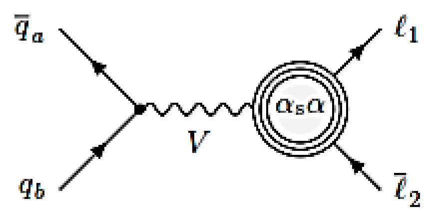
(d) Double-real corrections

[Alexander Huss '14]

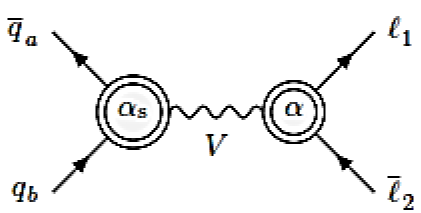
Born interfered double virtual corrections at $O(\alpha\alpha_s)$



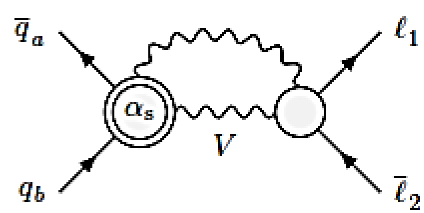
(a) Factorizable "initial–initial" corrections



(b) Factorizable "final–final" corrections



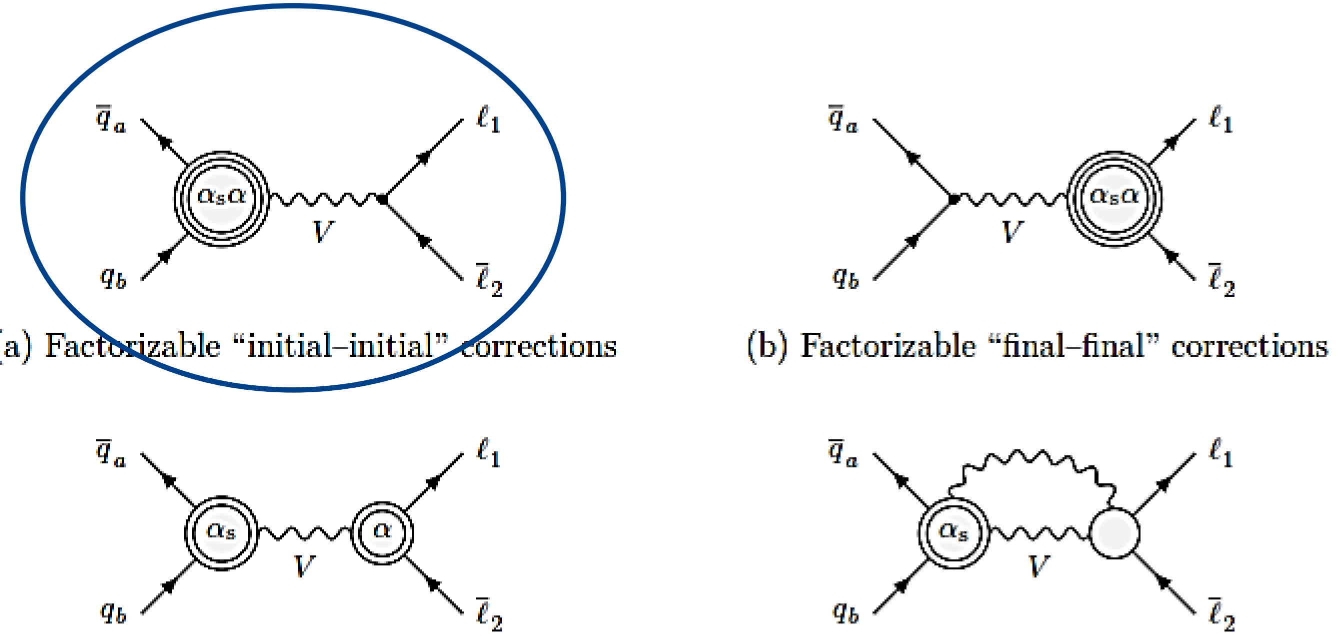
(c) Factorizable "initial–final" corrections



(d) Non-factorizable corrections

[Alexander Huss '14]

Born interfered double virtual corrections at $O(\alpha\alpha_s)$



(a) Factorizable "initial–initial" corrections

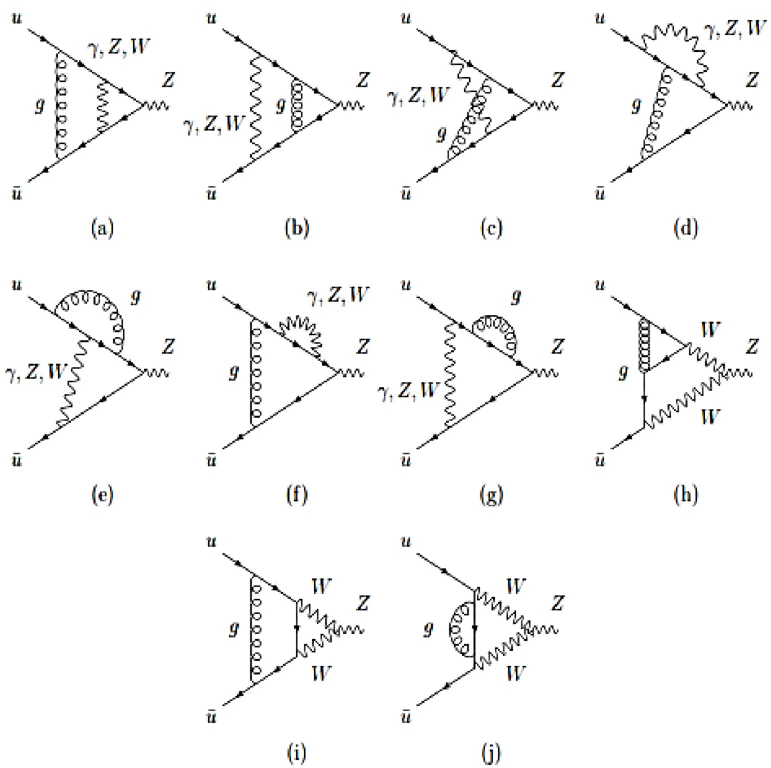
(b) Factorizable "final–final" corrections

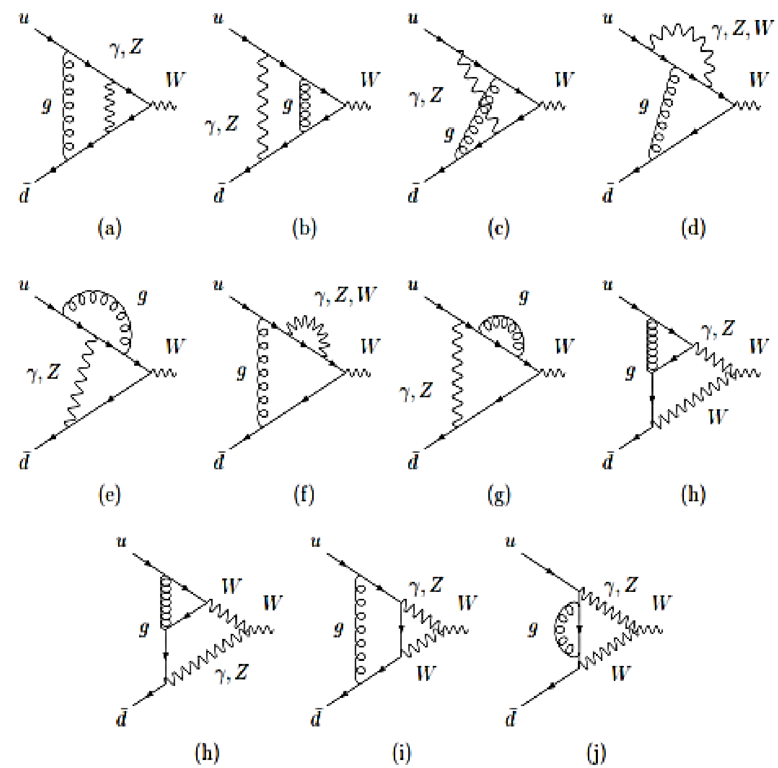
(c) Factorizable "initial–final" corrections

(d) Non-factorizable corrections

[Alexander Huss '14]

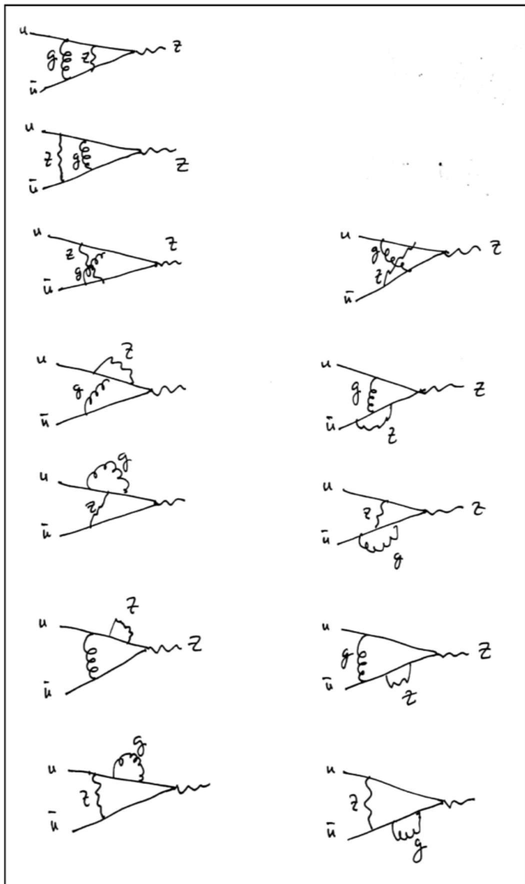
Double virtual corrections to W and Z boson production at $O(\alpha\alpha_s)$





[Bonciani ’11]

Double virtual corrections to W and Z boson production at $O(\alpha\alpha_s)$



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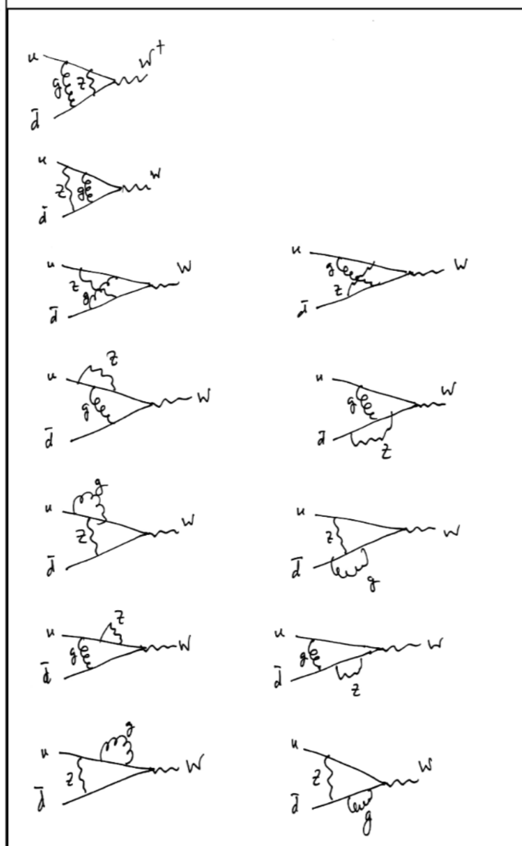

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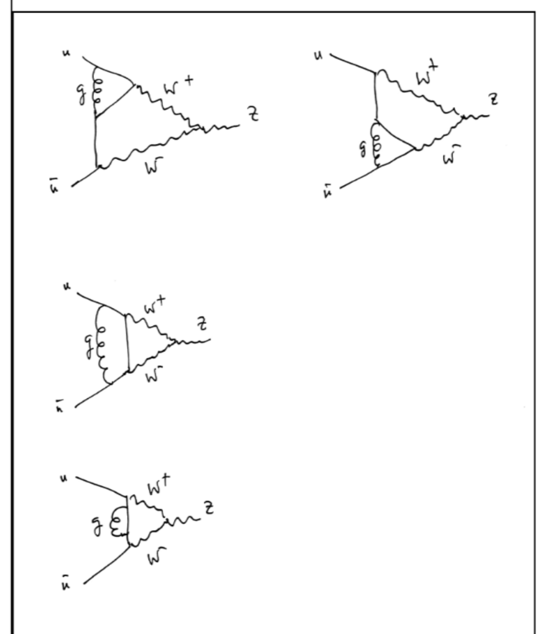
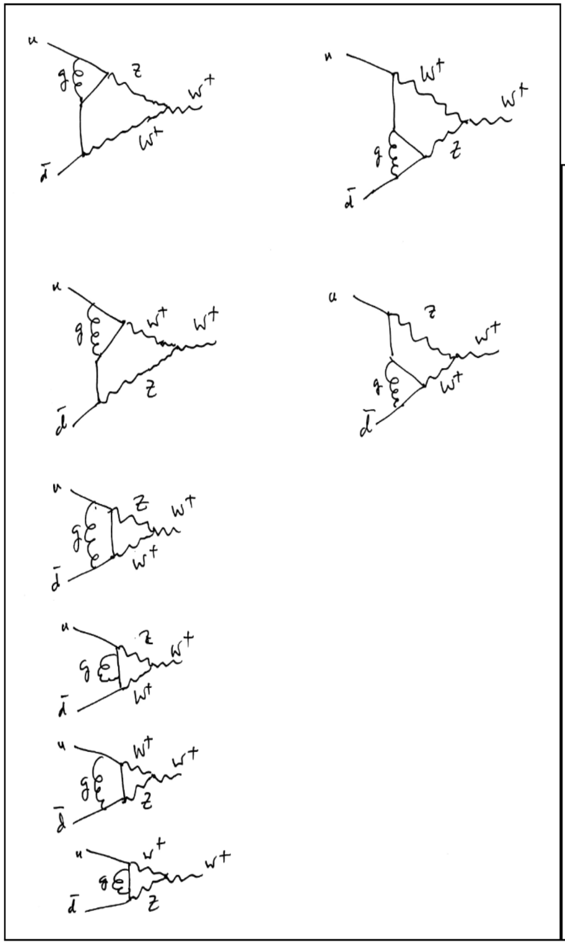


Set 3

Double virtual corrections to W and Z boson production at $O(\alpha\alpha_s)$



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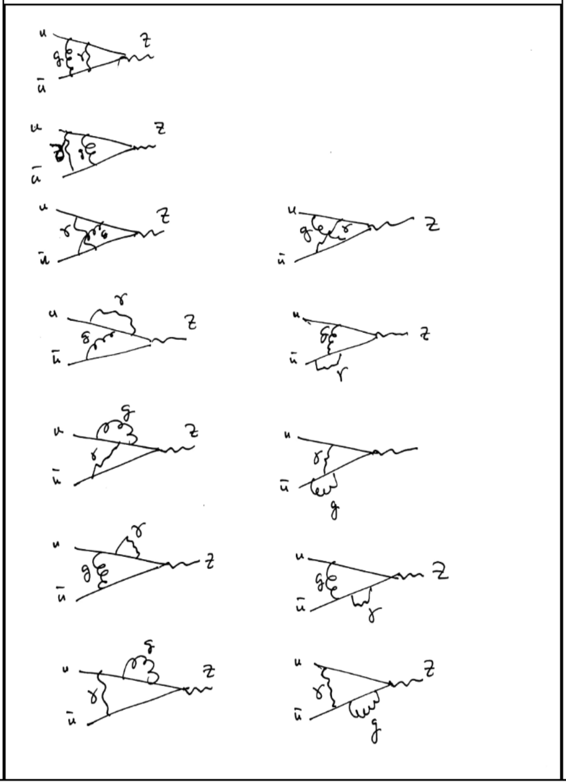

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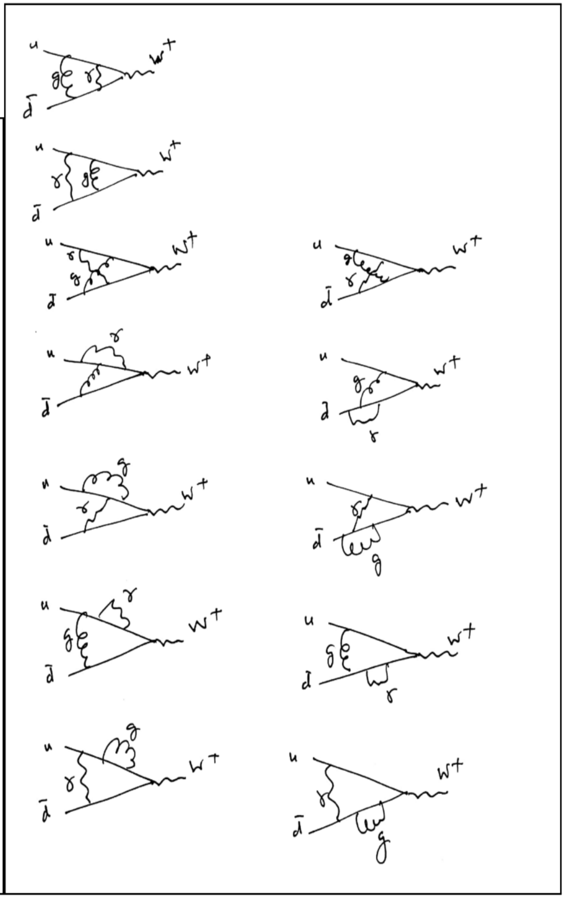

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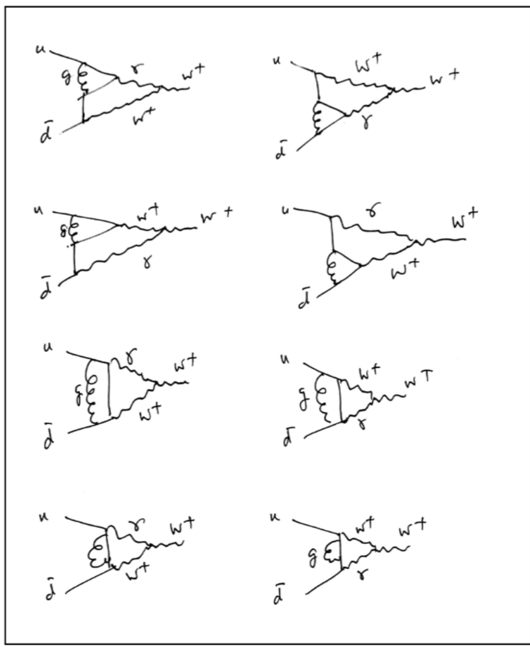
Double virtual corrections to W and Z boson production at $O(\alpha\alpha_s)$



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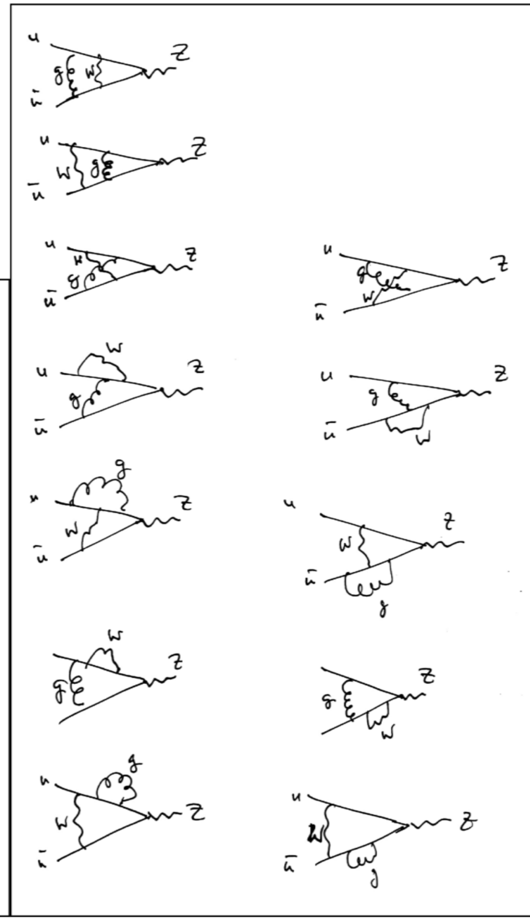

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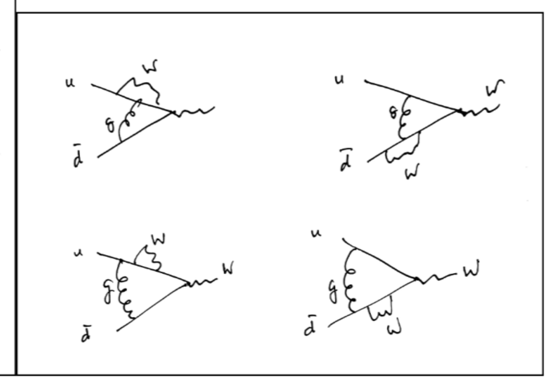

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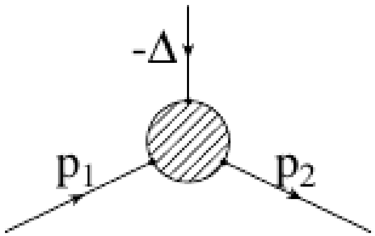
Form Factors



Most general Matrix element of the vector and axial vector current between spin ½ fermions:

$$\langle \alpha_f | M_\mu | \alpha_i \rangle = \bar{u}_f(p_2) \Big[F_1(t)\gamma_\mu - \frac{i}{2m} F_2(t)\sigma_{\mu\nu}\Delta^\nu + \frac{1}{m} F_3(t)\Delta_\mu$$

$$+\gamma_5 \left(G_1(t)\gamma_\mu - \frac{i}{2m} G_2(t)\sigma_{\mu\nu}\Delta^\nu + \frac{1}{m} G_3(t)\Delta_\mu \right) \Big] u_i(p_1)$$

with $\Delta = p_1 - p_2$ and $t = \Delta^2$.

[Czarnecki, Krause '96]

Form Factors

Matrix Element for vertex type diagram:

$$i\mathcal{M} = \bar{v}\Gamma^{\mu} u\, \varepsilon_{\mu}$$

Vertex function:

$$\Gamma^{\mu} = F_1(q^2)\gamma^{\mu} - \frac{i}{2m}F_2(q^2)\sigma^{\mu\nu}q_{\nu} + \frac{1}{m}F_3(q^2)q^{\mu} + \gamma^5\left(G_1(q^2)\gamma^{\mu} - \frac{i}{2m}G_2(q^2)\sigma^{\mu\nu}q_{\nu} + \frac{1}{m}G_3(q^2)q^{\mu}\right)$$

where, $q = p_1 + p_2$ $\qquad s \equiv (p_1 + p_2)^2$

Projector for form factors :

$$P^{\mu}_{f,i} = (m + \not{p}_1)\left(f_{1,i}\gamma^{\mu} - \frac{f_{2,i}(p_1 - p_2)^{\mu}}{2m} - \frac{f_{3,i}(p_1 + p_2)^{\mu}}{m}\right)(m - \not{p}_2)$$

$$P^{\mu}_{g,i} = (m + \not{p}_1)\gamma^5\left(g_{1,i}\gamma^{\mu} - \frac{g_{2,i}(p_1 - p_2)^{\mu}}{2m} - \frac{g_{3,i}(p_1 + p_2)^{\mu}}{m}\right)(m - \not{p}_2)$$

Form Factors

Vertex for massless fermions:

$$\Gamma^{\mu} = F_1(q^2)\gamma^{\mu} + \gamma^5 G_1(q^2)\gamma^{\mu}$$

$$F_1 = Tr(P^{\mu}_{f,1}\Gamma^{\mu}) \text{ and } G_1 = Tr(P^{\mu}_{g,1}\Gamma^{\mu})$$

Condition to extract F_1 :

$$f_{1,1} = \frac{2m^2}{(D-2)s(4m^2-s)}$$

$$f_{2,1} = \frac{2(Dm^2 s + 4m^4 - 2m^2 s)}{(D-2)s(4m^2-s)^2}$$

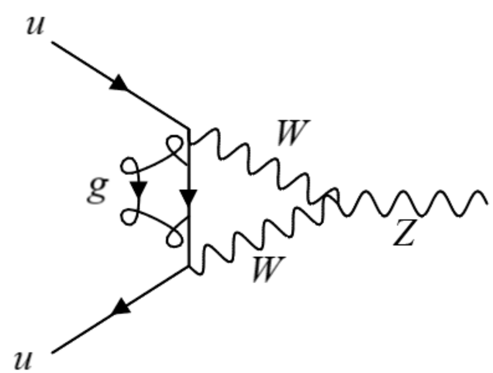
$$f_{3,1} = 0$$

$$g_{1,1} = 0$$

$$g_{2,1} = \frac{2m^2}{s(4m^2-s)}$$

$$g_{3,1} = 0$$

A sample extraction of form factor



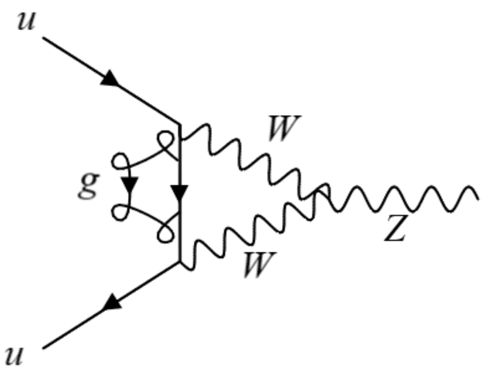
Numerator: (ignoring color factor and coupling constant)

$$-\gamma^{mu}.\bar{\gamma}^{7}.(\gamma\cdot(p1-k2)+m).\gamma^{lam}.(\gamma\cdot(-k1-k2+p1)+m).\gamma^{lam}.(\gamma\cdot(p1-k2)+m).(\gamma\cdot(-k2+2\,p1+2\,p2)).\bar{\gamma}^{7}-$$
$$(\gamma\cdot(-k2-p1-p2)).\bar{\gamma}^{7}.(\gamma\cdot(p1-k2)+m).\gamma^{lam}.(\gamma\cdot(-k1-k2+p1)+m).\gamma^{lam}.(\gamma\cdot(p1-k2)+m).\gamma^{mu}.\bar{\gamma}^{7}-$$
$$(2\,k2-p1-p2)^{mu}\,\gamma^{sig}.\bar{\gamma}^{7}.(\gamma\cdot(p1-k2)+m).\gamma^{lam}.(\gamma\cdot(-k1-k2+p1)+m).\gamma^{lam}.(\gamma\cdot(p1-k2)+m).\gamma^{sig}.\bar{\gamma}^{7}$$

Projector:

$$(m+\gamma\cdot p1).\left(g1(i)\,\gamma^{mu}-\frac{g2(i)\left(\frac{1}{2}(p1-p2)^{mu}\right)}{m}-\frac{g3(i)\,(p1+p2)^{mu}}{m}\right).(m-\gamma\cdot p2)$$

A sample extraction of form factor

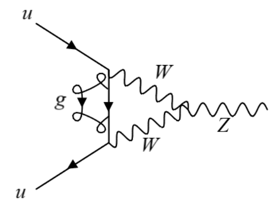


After taking the trace we will be left with sum of numerous Lorentz invariant functions. Each of them have to be integrated over the loop momenta. The number of integrals can easily exceed hundreds.

But these integrals are not independent. Most of them can be written as a linear combination of a few integrals, which we call Master Integrals. By using the Integration by Parts (IBP) identity we can figure out these relations.

Preliminary Result

Formfactor F_1



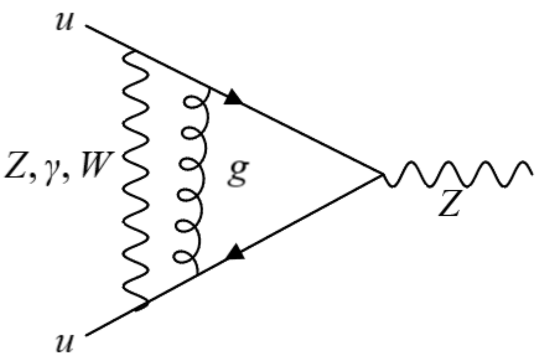
After taking the trace and performing IBP reduction, we are left with handful of Master Integrals which are taken from available literature. [Bonciani et al ‘03,’04], [Bonciani, Di Vita, Mastrolia, Schubert ’16]. Finally the F_1 Formfactor is following:

$$F_{1_{example}}(x) = C_c\Bigg[\frac{1}{\epsilon^2}\left[\frac{3}{4}\right]+\frac{1}{\epsilon}\Bigg[\left(\frac{\sqrt{x(x+4)}}{2x}-\frac{\sqrt{x(x+4)}}{x^2}\right)H(-r,x)+\left(\frac{2}{x^2}-\frac{4}{x}\right)H(-r,-r,x)+\frac{1}{x}+\frac{9}{8}\Bigg]+\left(\frac{7\sqrt{x(x+4)}}{4x}-\frac{9\sqrt{x(x+4)}}{2x^2}\right)H(-r,x)+\left(\frac{\sqrt{x(x+4)}}{x^2}-\frac{\sqrt{x(x+4)}}{2x}\right)H(-4,-r,x)+\left(\frac{5}{x^2}-\frac{4}{x}-\frac{1}{2}\right)H(-r,-r,x)+\left(\frac{2}{x^2}-\frac{4}{x}\right)H(0,-r,-r,x)+\left(\frac{4}{x}-\frac{2}{x^2}\right)H(-r,-4,-r,x)+\frac{3\zeta(2)}{2}+\frac{11}{2x}+\frac{19}{16}\Bigg]$$

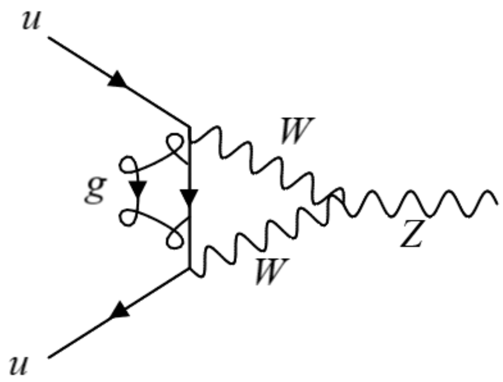
Prefactor: $C_c = N^2 C_2^2 \frac{ie^3 c_w C_F g_s^2}{s_w^3}$

Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_2 = \left(\frac{\mu^2}{M_W^2}\right)^{\epsilon}$, $D = 4-2\epsilon$, $x = -\frac{s}{M_W^2}$

Numerical Evaluation



Master integrals for diagrams with maximum 1 internal mass can be written in terms of HPLs (Harmonic Polylogarithms). Many packages or Libraries are available to evaluate them.



Master integrals for diagrams with 2 internal mass are written in terms of GHPLs (Generalized Harmonic Polylogarithms). One way to evaluate them is to convert them to GPLs (Goncharov Polylogs) with linear weights [Bonciani et al ‘10].

GPL, HPL, GHPL

GPL $$G(a_1, a_2, \ldots, a_n; z) = \int_0^z \frac{\mathrm{d}t}{t-a} G(a_2, \ldots, a_n; z)$$

$G(z) = G(;z) = 1$ and $a_i \in \mathbf{C}$ are some chosen constants and z is a complex variable.

$$G(\vec{0}_n, z) = \frac{1}{n} \log^n(z)$$

HPL $$H(\vec{a}; z) = (-1)^p G(\vec{a}; z)$$ $a_i \in -1, 0, 1$

GHPL $$G(-r, \vec{a}; z) = \int_0^z \frac{\mathrm{d}t}{\sqrt{t(4-t)}} G(\vec{a}; t)$$

$$z = \frac{(1-\xi)^2}{\xi}$$

$$\xi = \frac{\sqrt{4+z} - \sqrt{z}}{\sqrt{4+z} + \sqrt{z}}$$

Replacing t by $(1-\eta)^2/\eta$

$$G(-r, \vec{a}; z) = -\int_1^\xi \frac{\mathrm{d}\eta}{\eta} G(\vec{a}; \frac{(1-\eta)^2}{\eta})$$

Software Tools

Graph Generation: QGRAF (P. Nogueira), FeynArt (Thomas Hahn et al.), DIANA (M. Tentiyukov) etc

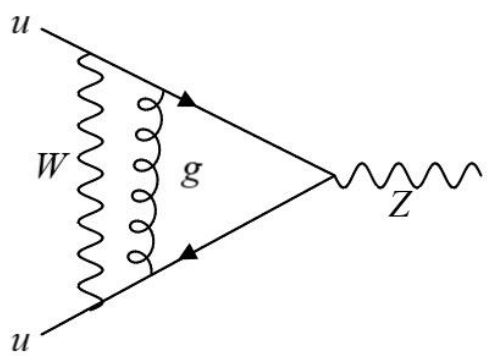
Amplitude and Trace Calculation: FORM (Jos Vermaseren), FormCalc (Thomas Hahn), FeynCalc (R. Mertig et al.) etc

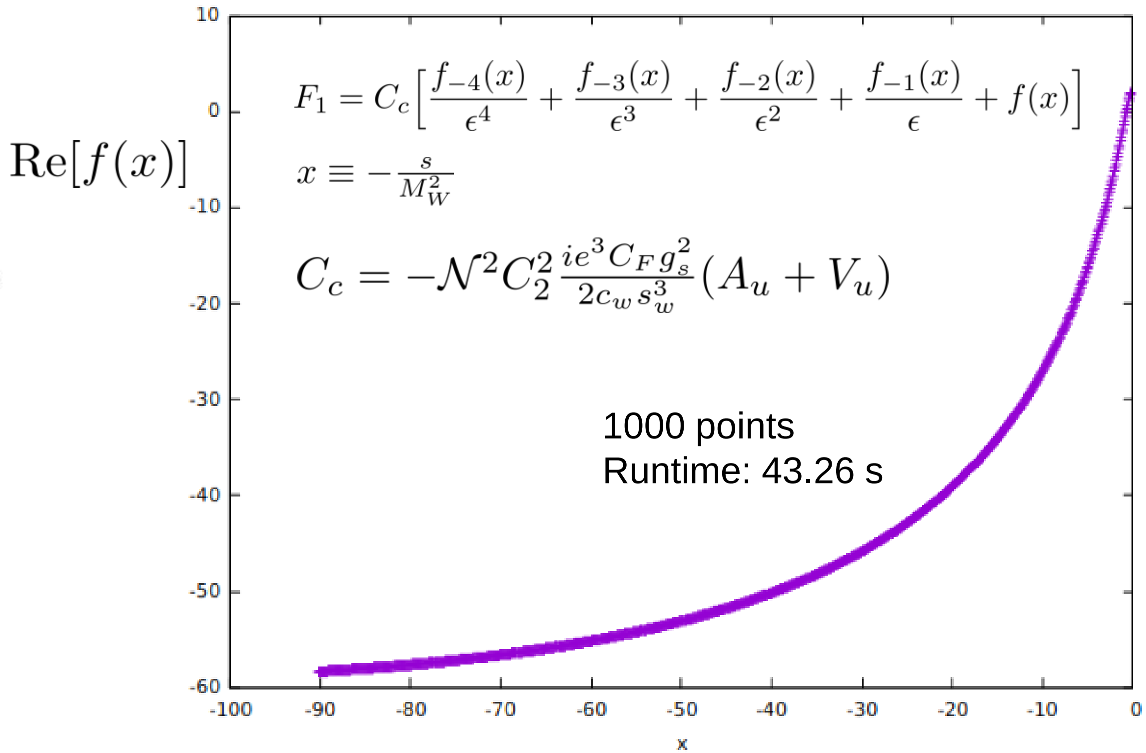
IBP Reduction: FIRE (Smirnov), LiteRed (R. Lee), REDUZE2 (von Manteuffel et. al), AIR, CRUSHER, Kira, Finred etc

Numerical Evaluation of HPL & GPL: Chaplin (Duhr et al.), GiNaC C++ Library (Vollinga et al.), HPL Mathematica Package (Maitre) etc

Behavior of the finite part of F_1 in physical region

Preliminary Result



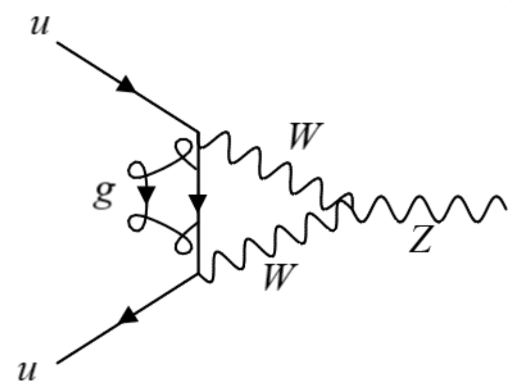


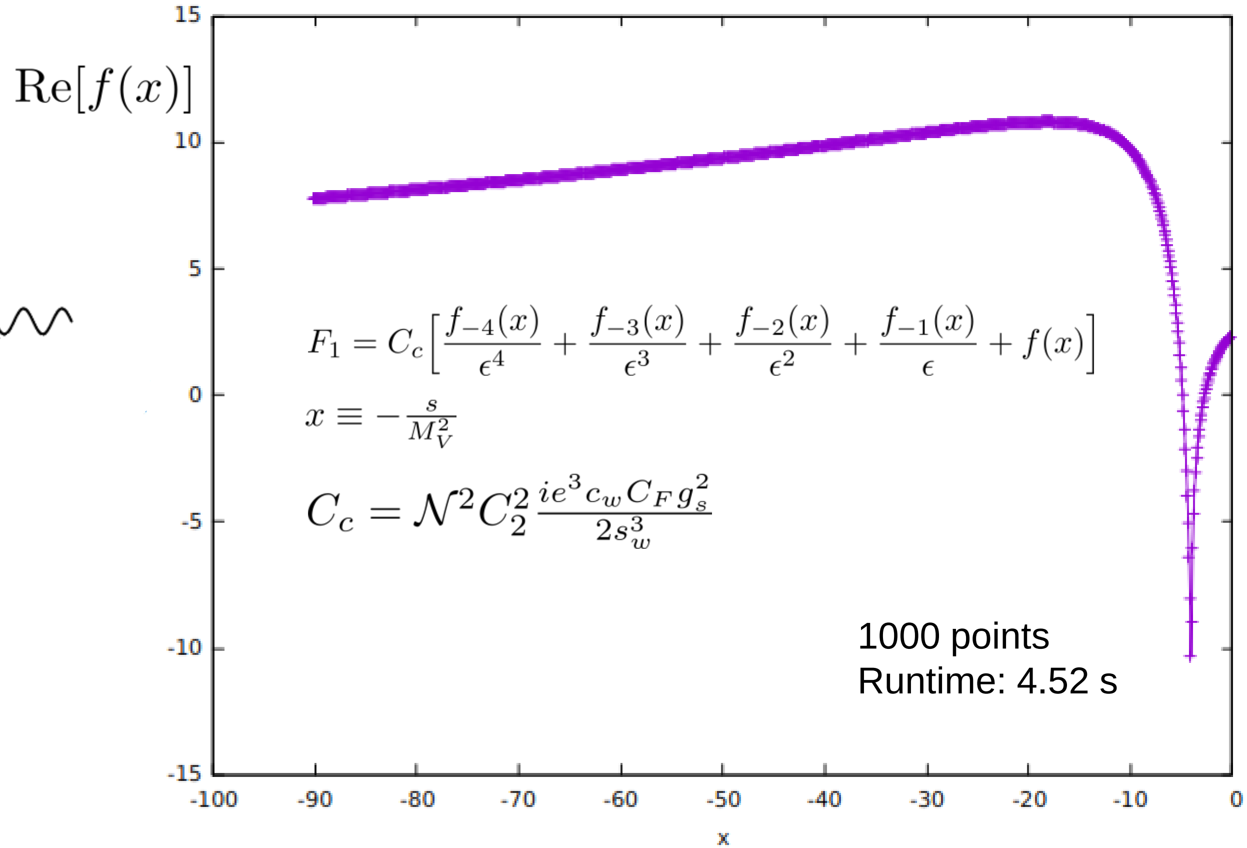
Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_2 = \left(\frac{\mu^2}{M_W^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$, $x = -\frac{s}{M_W^2}$

<3.70 GHz Single Core
Xenon Processor
31.3 GiB>

Behavior of the finite part of F_1 in physical region

Preliminary Result

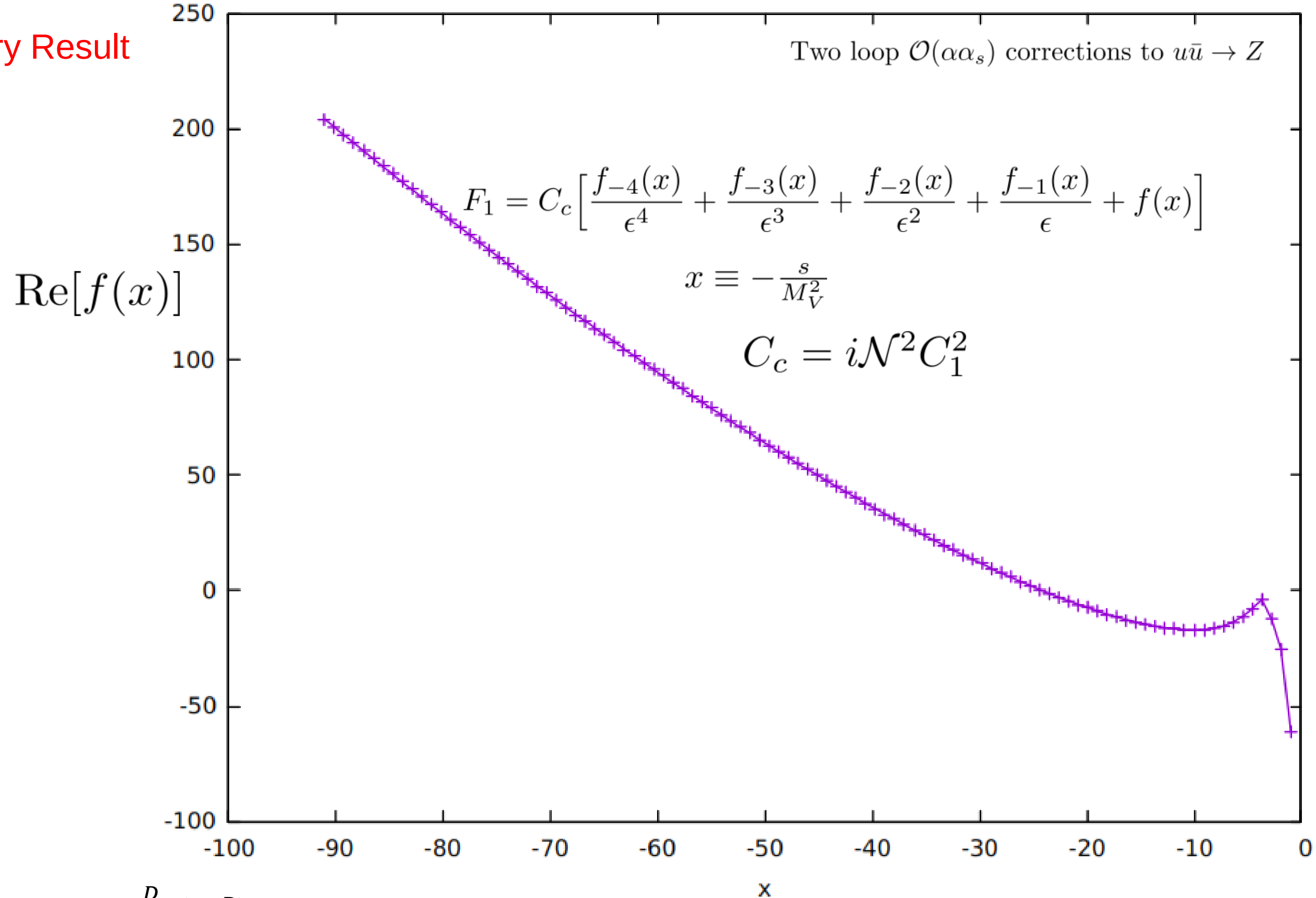




Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_2 = \left(\frac{\mu^2}{M_W^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$, $x = -\frac{s}{M_W^2}$

Total Z boson Production form factor (QCD×EW)

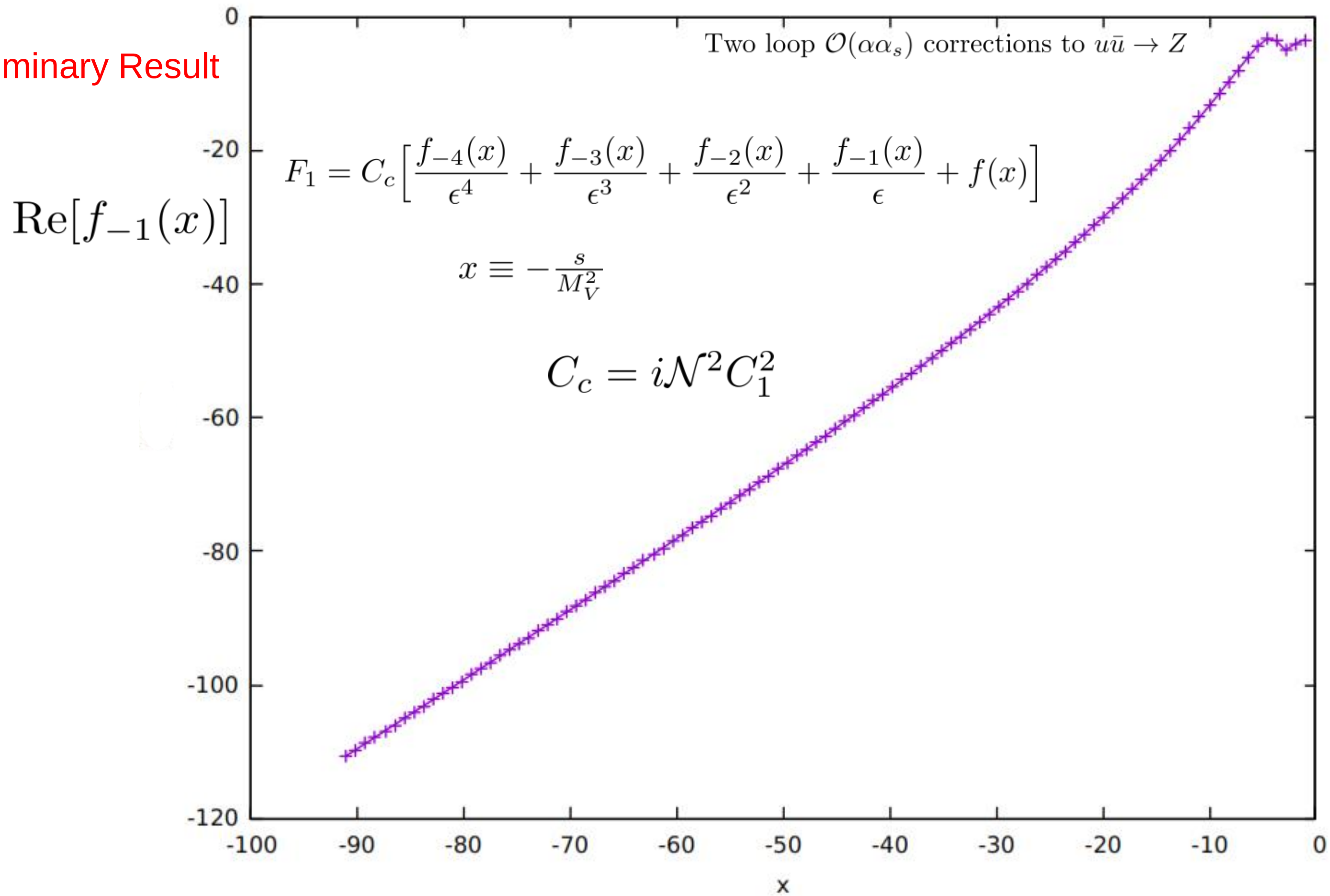
Preliminary Result



Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_1 = \left(\frac{\mu^2}{M_Z^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$

Total Z boson Production form factor (QCD×EW)

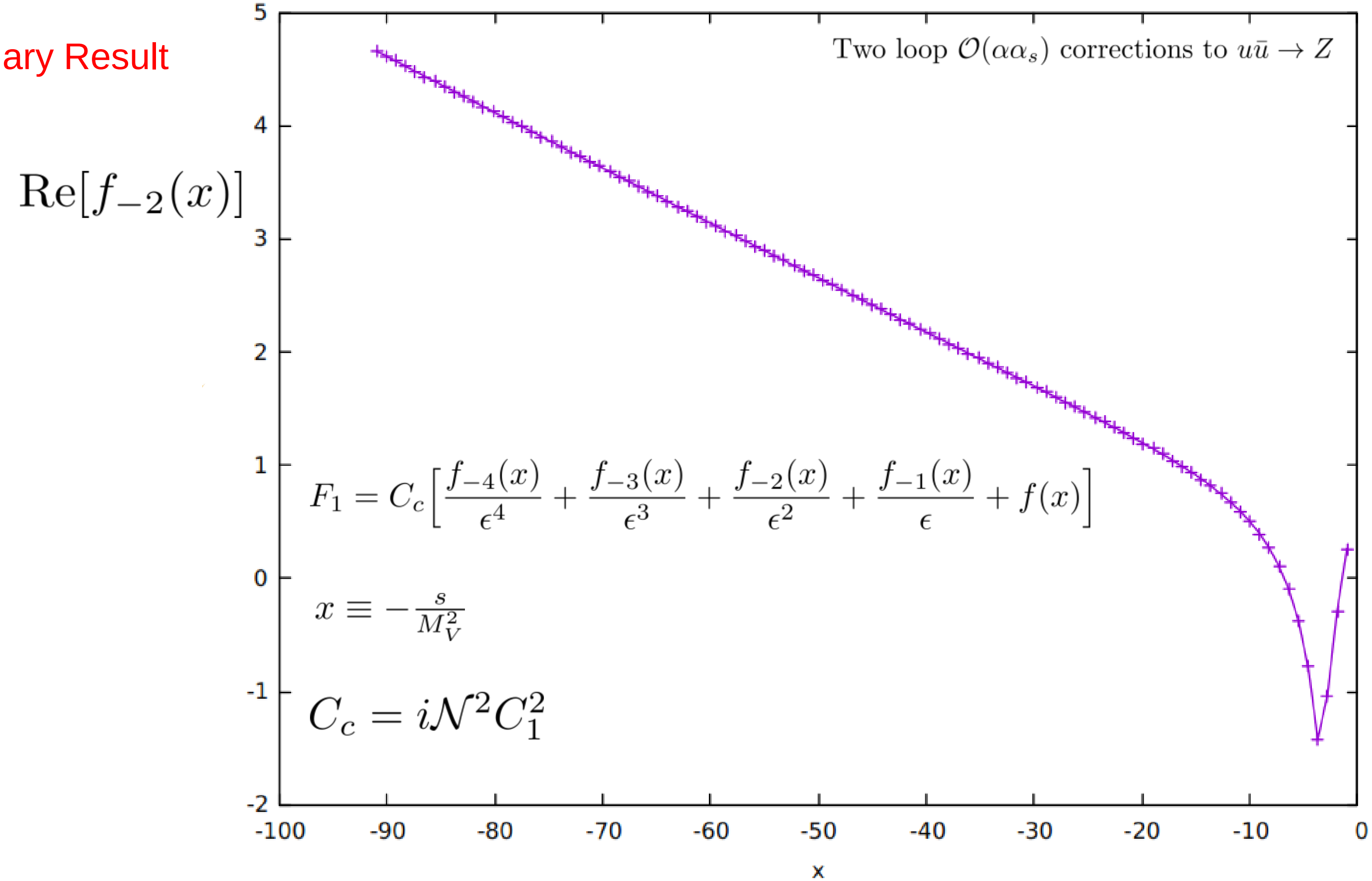
Preliminary Result



Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_1 = \left(\frac{\mu^2}{M_Z^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$

Total Z boson Production form factor (QCD×EW)

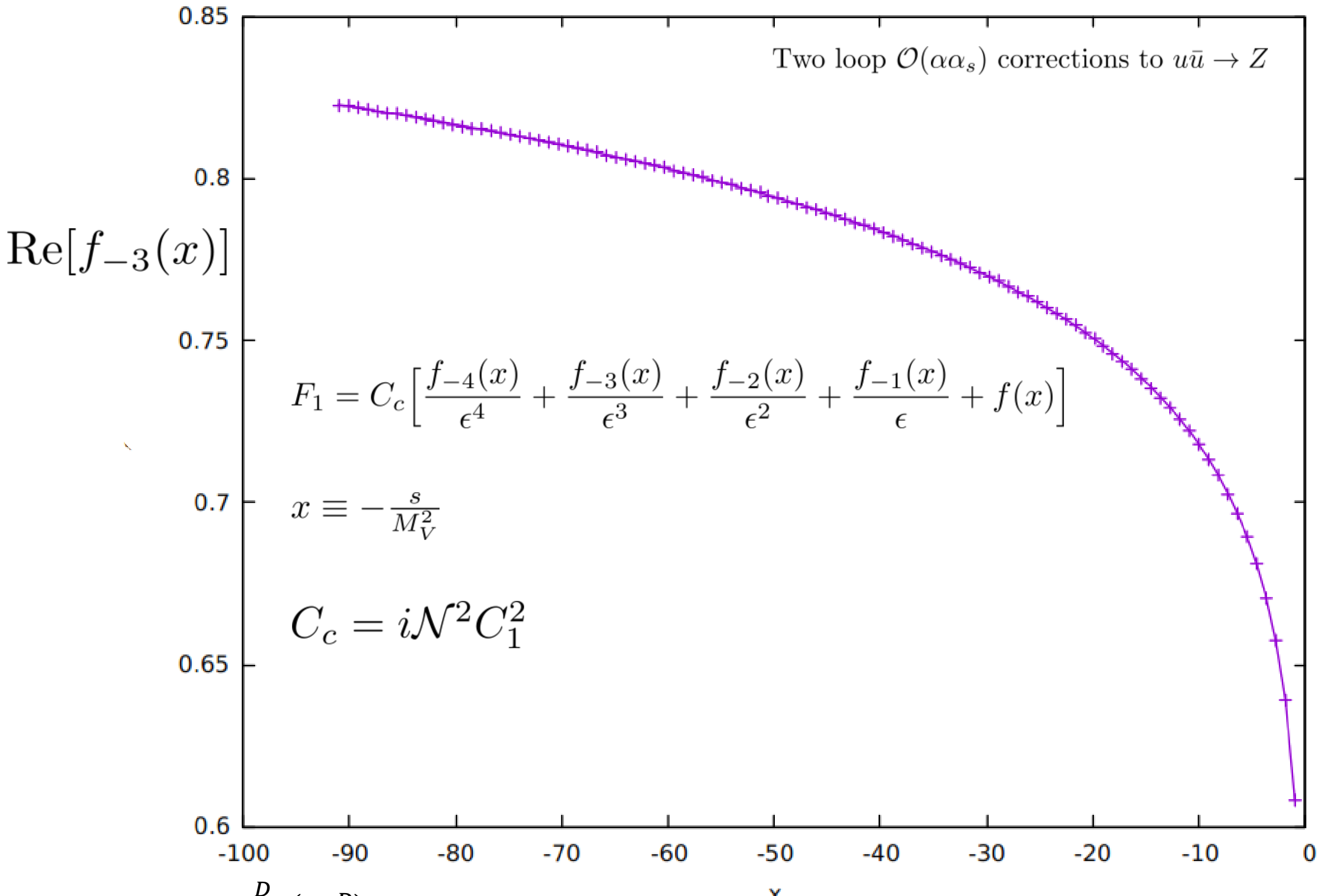
Preliminary Result



Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_1 = \left(\frac{\mu^2}{M_Z^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$

Total Z boson Production form factor (QCD×EW)

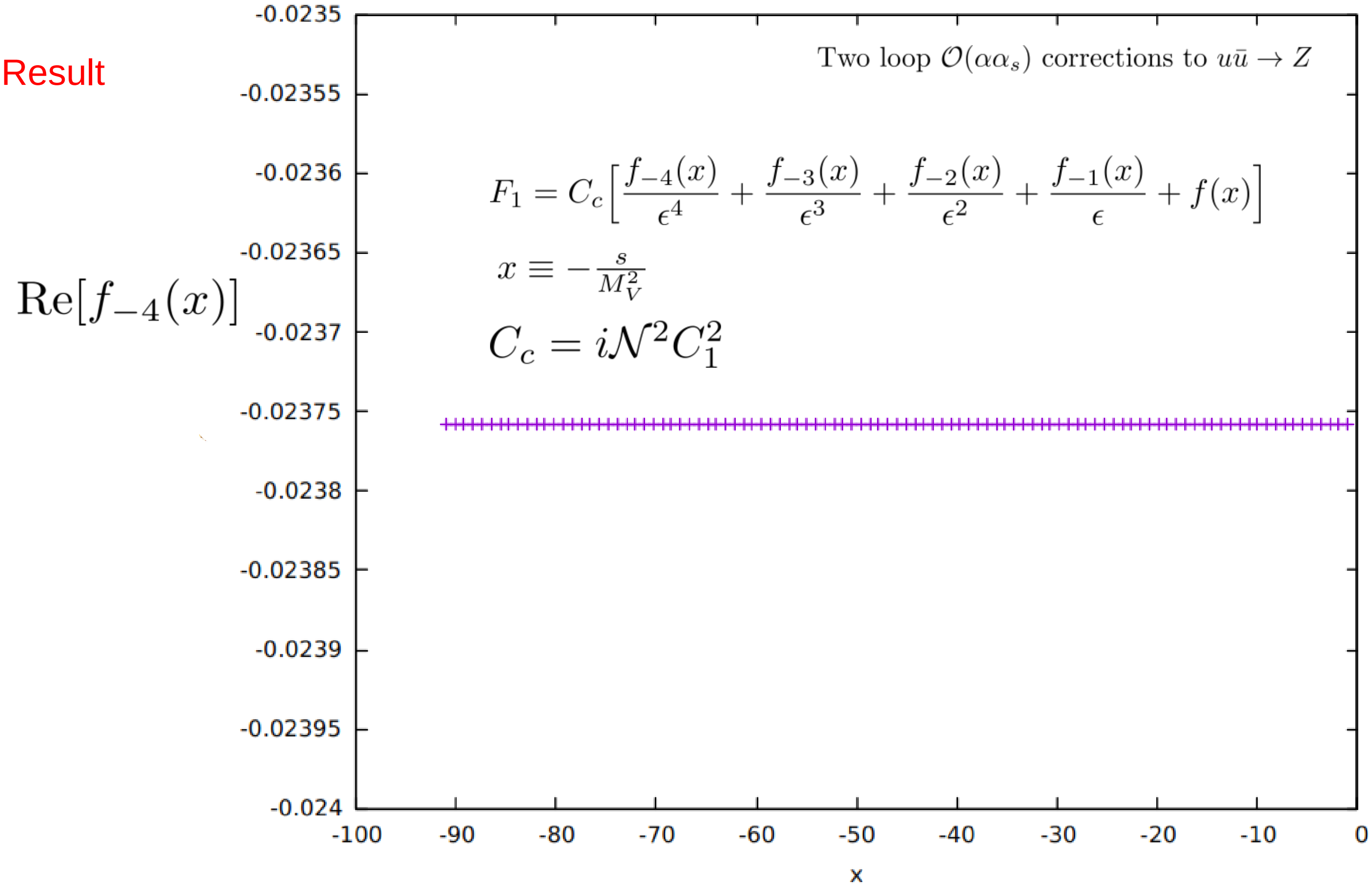
Preliminary Result



Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_1 = \left(\frac{\mu^2}{M_Z^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$

Total Z boson Production form factor (QCD×EW)

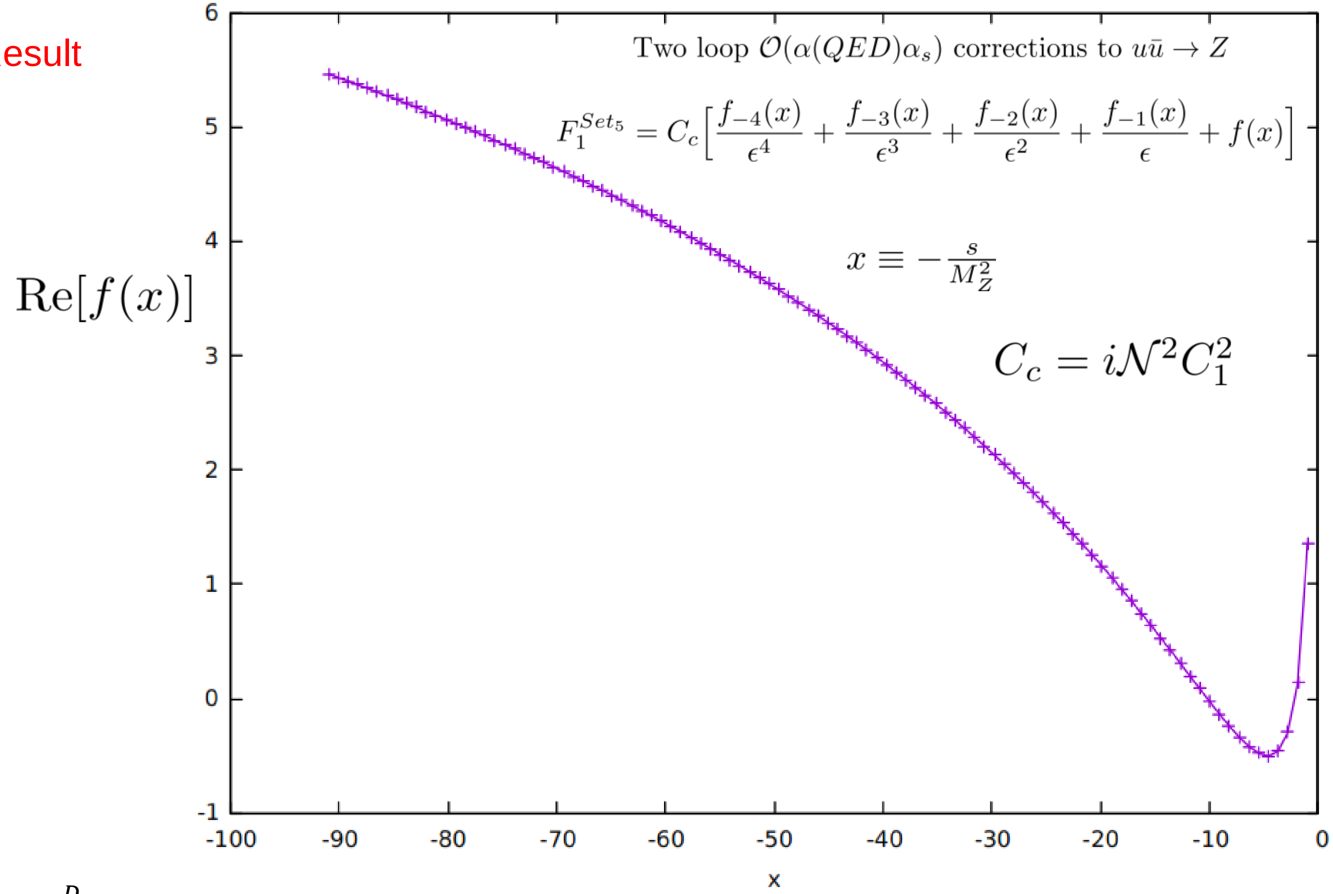
Preliminary Result



Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_1 = \left(\frac{\mu^2}{M_Z^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$

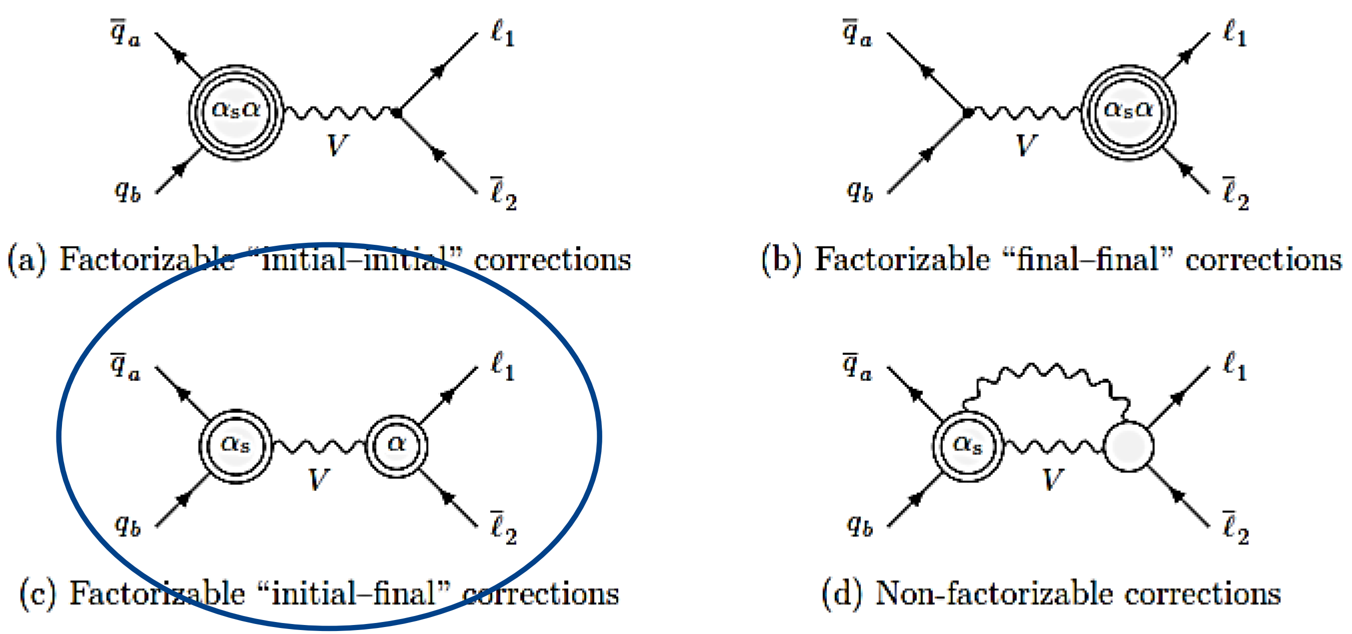
Total Z boson Production form factor (QCD×QED)

Preliminary Result



Where $N = \frac{i\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{(2\pi)^D}$, $C_1 = \left(\frac{\mu^2}{M_Z^2}\right)^{\epsilon}$, $D = 4 - 2\epsilon$

Initial Final type corrections at $O(\alpha\alpha_s)$



(a) Factorizable "initial–initial" corrections

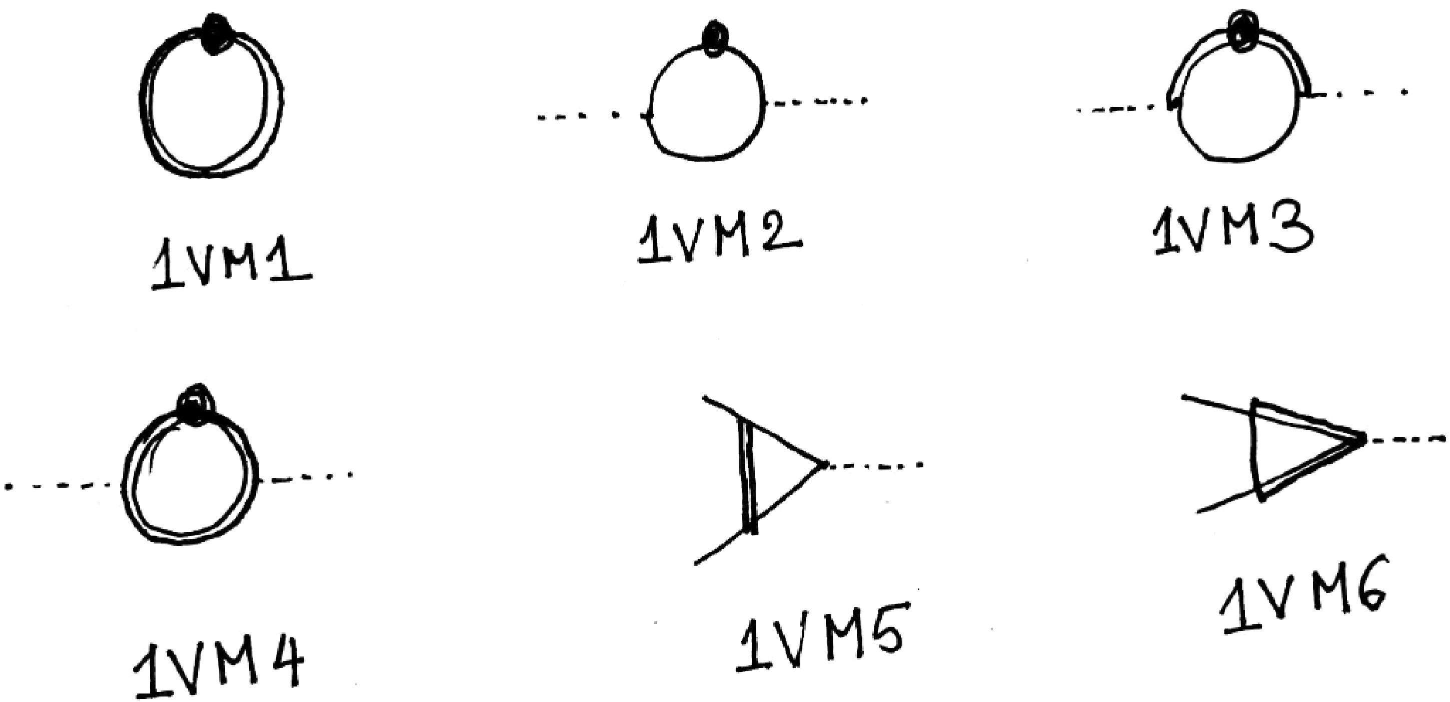
(b) Factorizable "final–final" corrections

(c) Factorizable "initial–final" corrections

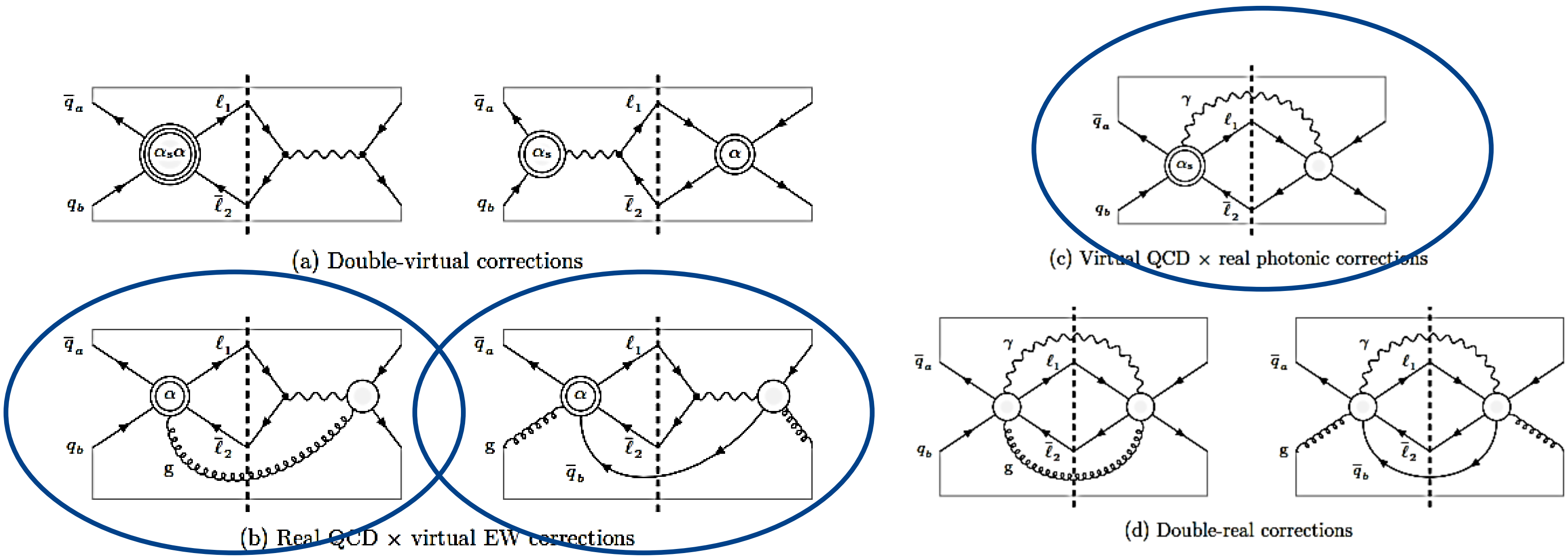
(d) Non-factorizable corrections

[Alexander Huss '14]

Master integrals for Initial Final type corrections at $O(\alpha\alpha_s)$



Real Virtual Corrections



(a) Double-virtual corrections

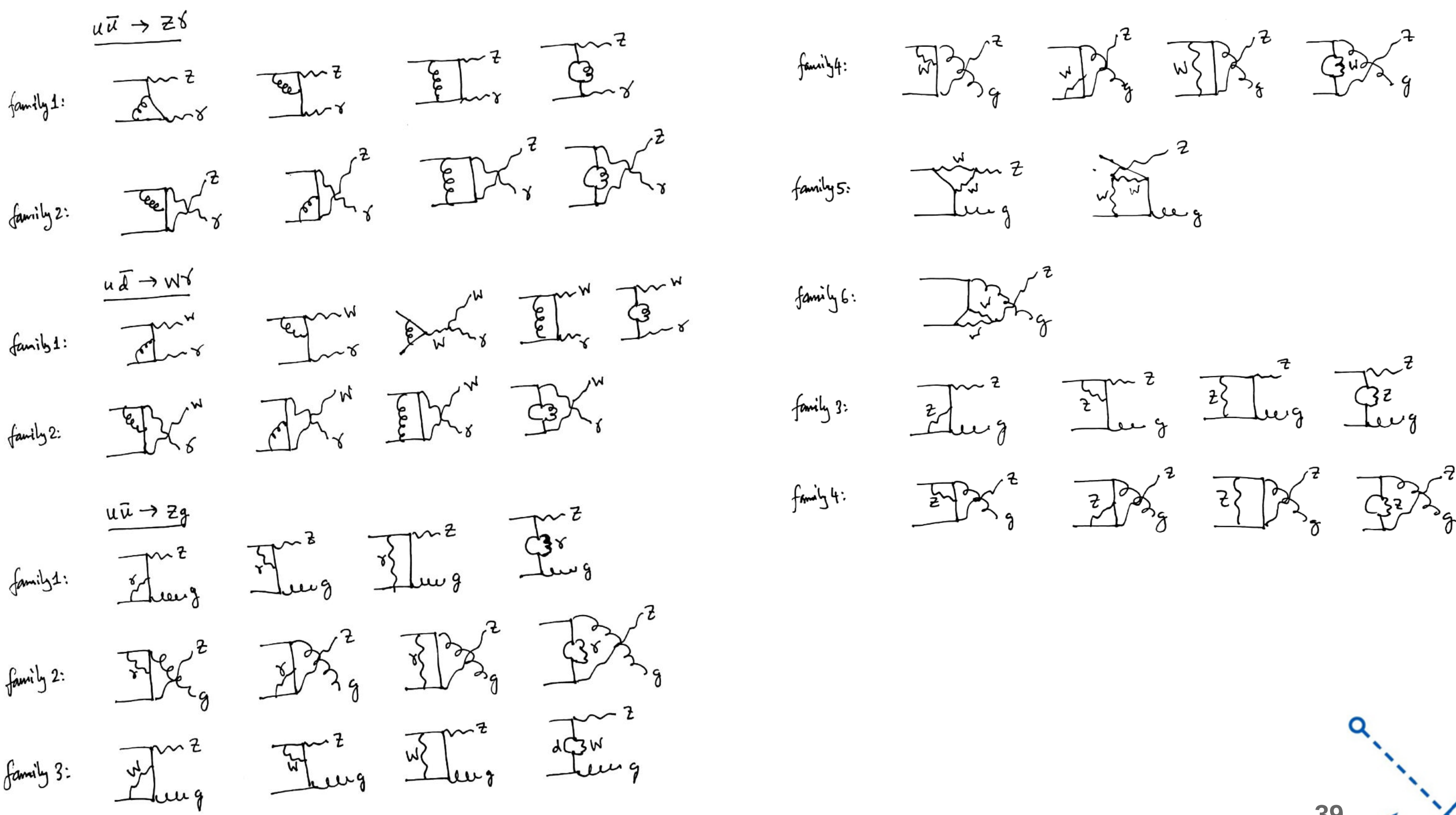
(b) Real QCD × virtual EW corrections

(c) Virtual QCD × real photonic corrections

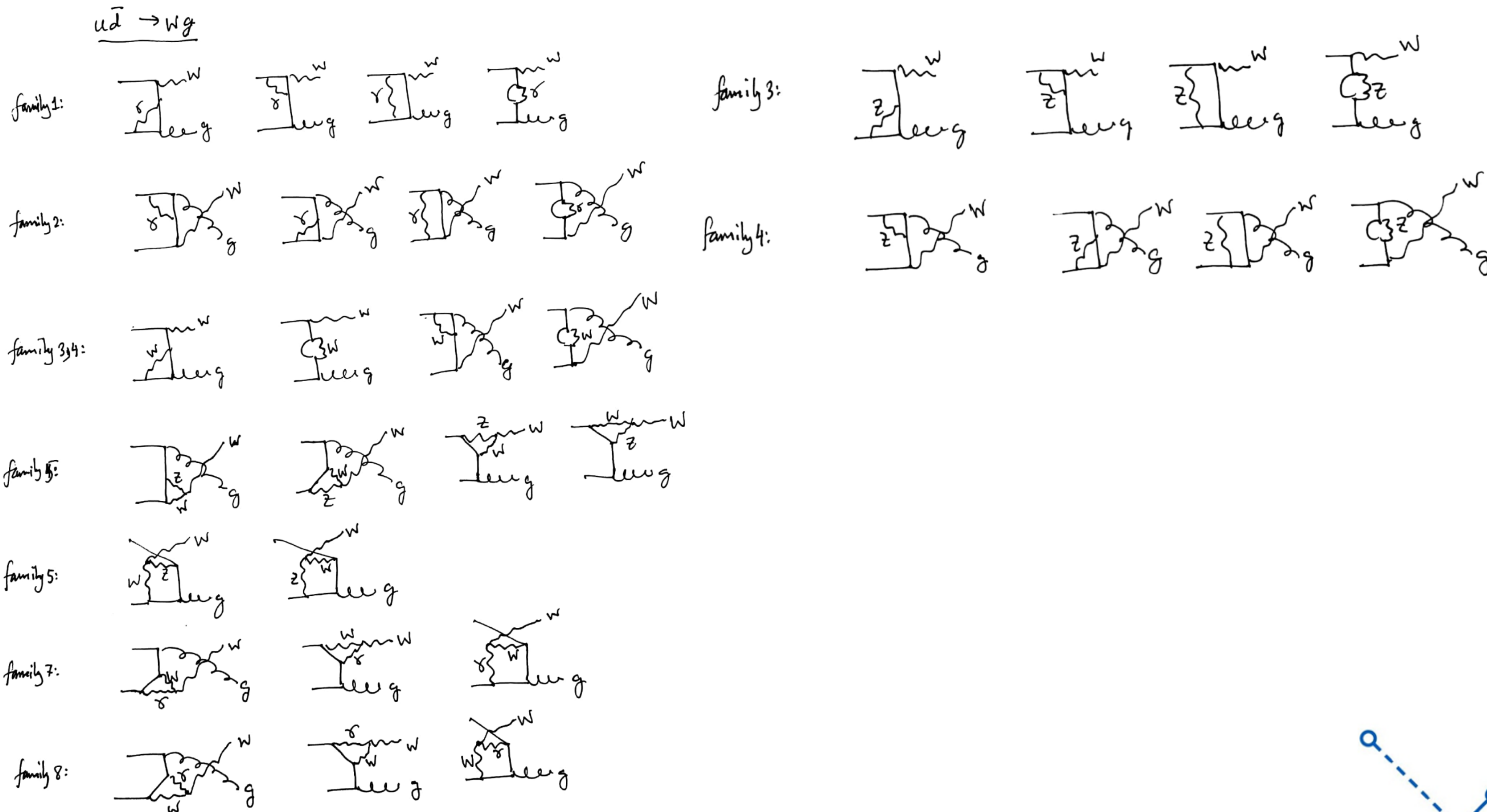
(d) Double-real corrections

[Alexander Huss '14]

$q\bar{q}'$ channel Feynman diagrams for Real-Virtual Corrections to W/Z production at $O(\alpha\alpha_s)$



$q\bar{q}'$ channel Feynman diagrams for Real-Virtual Corrections to W/Z production at $O(\alpha\alpha_s)$



$q\bar{q}'$ channel Feynman diagrams for Real-Virtual Corrections to W/Z production at $O(\alpha\alpha_s)$

Preliminary Result

Input Parameters:

$s = 2M_Z^2$

$t = -(s - M_Z^2)$

$M_W = 80.379 GeV$

$M_Z = 91.1876 GeV$

$\alpha = \frac{1}{137.036}$

$\alpha_s = 0.118$

$\cos\theta_W = \frac{M_W}{M_Z}$

$$\overline{\sum} Re\left(\mathcal{M}_{LO}^{*}\mathcal{M}_{1loop}\right)_{u\bar{u}\to Z\gamma} = \left(\frac{4\pi^2\mu_R^2}{M_Z^2}\right)^{\varepsilon}\left(0.00114547 - \frac{0.000646274}{\varepsilon^2} - 0.0030633\varepsilon + \frac{0.00137741}{\varepsilon}\right)$$

$$\overline{\sum} Re\left(\mathcal{M}_{LO}^{*}\mathcal{M}_{1loop}\right)_{u\bar{d}\to W\gamma} = \left(\frac{4\pi^2\mu_R^2}{M_W^2}\right)^{\varepsilon}\left(0.0000790054 - \frac{0.0000442685}{\varepsilon^2} - 0.000277506\varepsilon + \frac{0.000112031}{\varepsilon}\right)$$

$$\overline{\sum} Re\left(\mathcal{M}_{LO}^{*}\mathcal{M}_{1loop}\right)_{u\bar{u}\to Zg} = \left(\frac{4\pi^2\mu_R^2}{M_Z^2}\right)^{\varepsilon}\left(-0.0254516 - \frac{0.000646274}{\varepsilon^2} + 0.0412328\varepsilon + \frac{0.00932825}{\varepsilon}\right)$$

$$\overline{\sum} Re\left(\mathcal{M}_{LO}^{*}\mathcal{M}_{1loop}\right)_{u\bar{d}\to Wg} = \left(\frac{4\pi^2\mu_R^2}{M_W^2}\right)^{\varepsilon}\left(1.62495 - \frac{0.0000739862}{\varepsilon^2} + 40.2357\varepsilon + \frac{0.0659907}{\varepsilon}\right)$$

$q\bar{q}'$ channel Feynman diagrams for Real-Virtual Corrections to W/Z production at $O(\alpha\alpha_s)$

Preliminary Result

$$\overline{\sum} Re\left(\mathcal{M}_{LO}^{*}\mathcal{M}_{1loop}\right)_{u\bar{u}(\bar{d})\to Z(W^{+})\gamma} = \frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu_R^2}{M_V^2}\right)^{\varepsilon} \left(\overline{\sum}|\mathcal{M}_{LO}|_0^2\left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}(-3 + 2\gamma_E + 2\log\left(\frac{s}{M_V^2}\right))\right] - \overline{\sum}|\mathcal{M}_{LO}|_{\varepsilon}^2 \frac{2}{\varepsilon} + \mathcal{O}(\varepsilon^0)\right)$$

$$|\mathcal{M}_{LO}|^2 = |\mathcal{M}_{LO}|_0^2 + \varepsilon|\mathcal{M}_{LO}|_{\varepsilon}^2 + \mathcal{O}(\varepsilon^2)$$

$$\overline{\sum}|\mathcal{M}_{LO}|_0^2 = \frac{4(4\pi\alpha)^2 C_V}{27tu}(t^2 + u^2 + 2M_V^2 s)$$

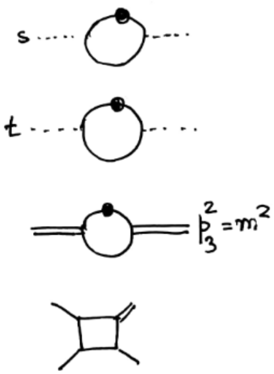
$$\overline{\sum}|\mathcal{M}_{LO}|_{\varepsilon}^2 = -\frac{8(4\pi\alpha)^2 C_V}{27tu}(t^2 + u^2 + tu + M_V^2 s)$$

$$C_Z = \frac{9 - 24sw^2 + 32sw^4}{36s_w^2 c_w^2}$$

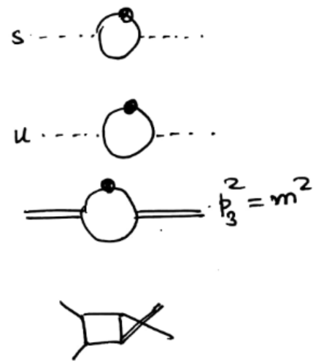
$$C_W = \frac{(2t-u)^2}{8(t+u)^2 s_w^2}$$

[J Ohnemus ’93]

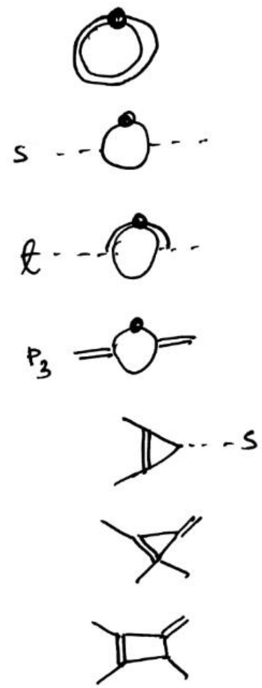
Master Integrals for Real-Virtual Corrections to W/Z production at $O(\alpha\alpha_s)$



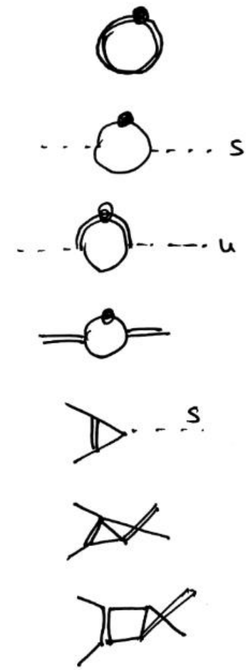
family1



family2

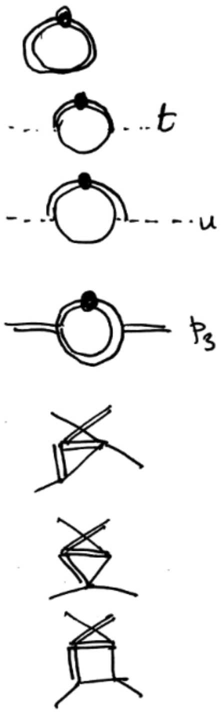


family3

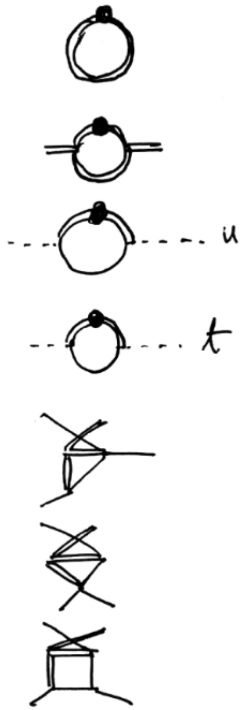


family4

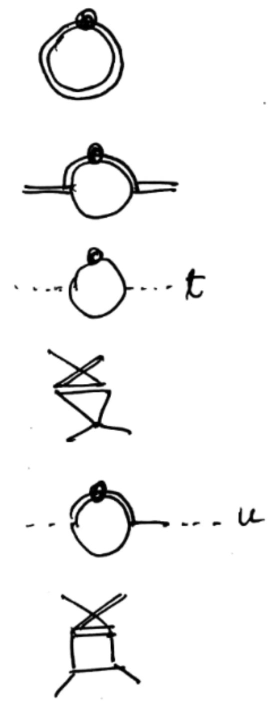
Master Integrals for Real-Virtual Corrections to W/Z production at $O(\alpha\alpha_s)$



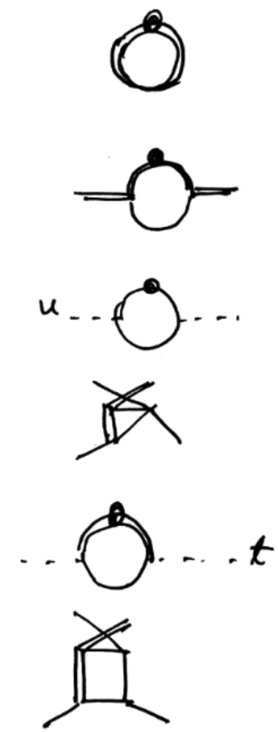
family5



family6



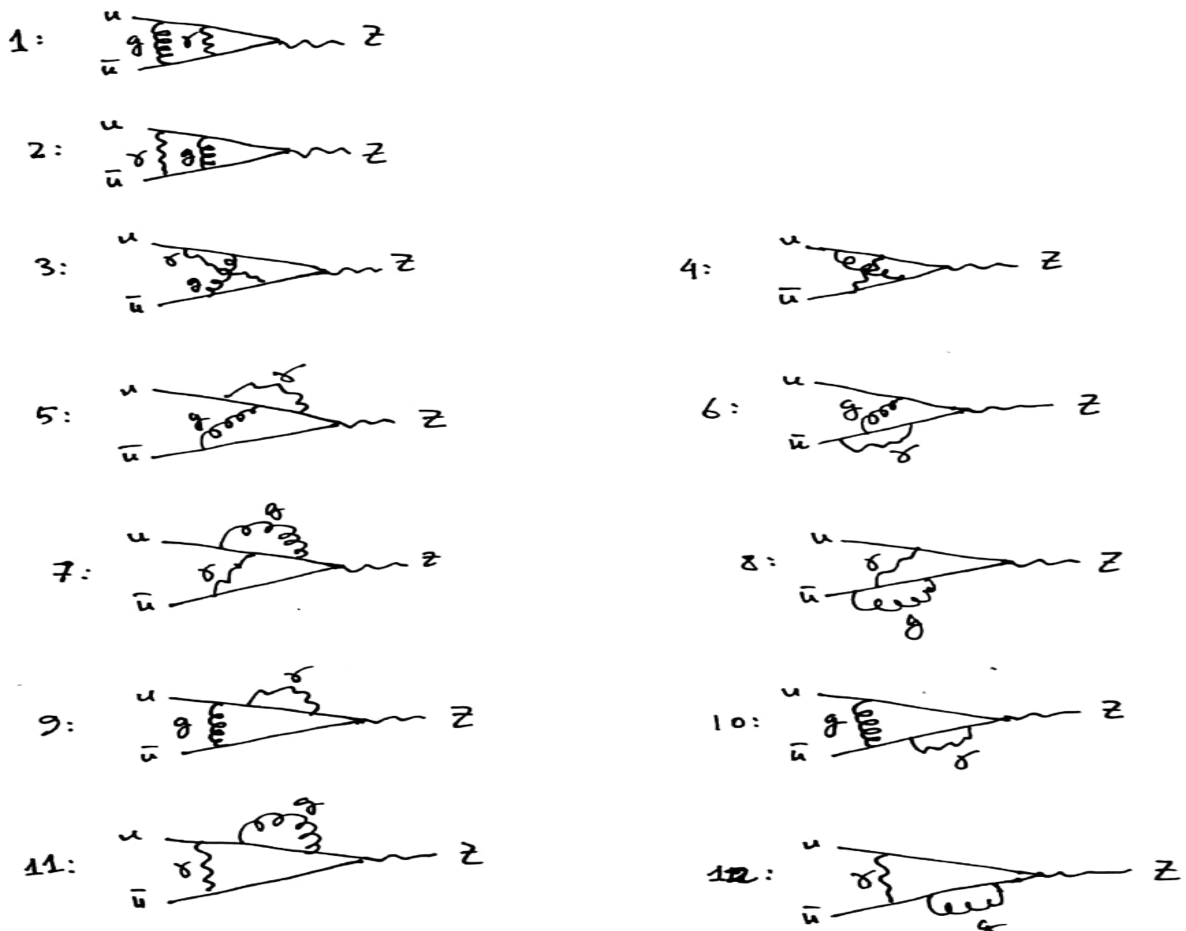
family7



family8

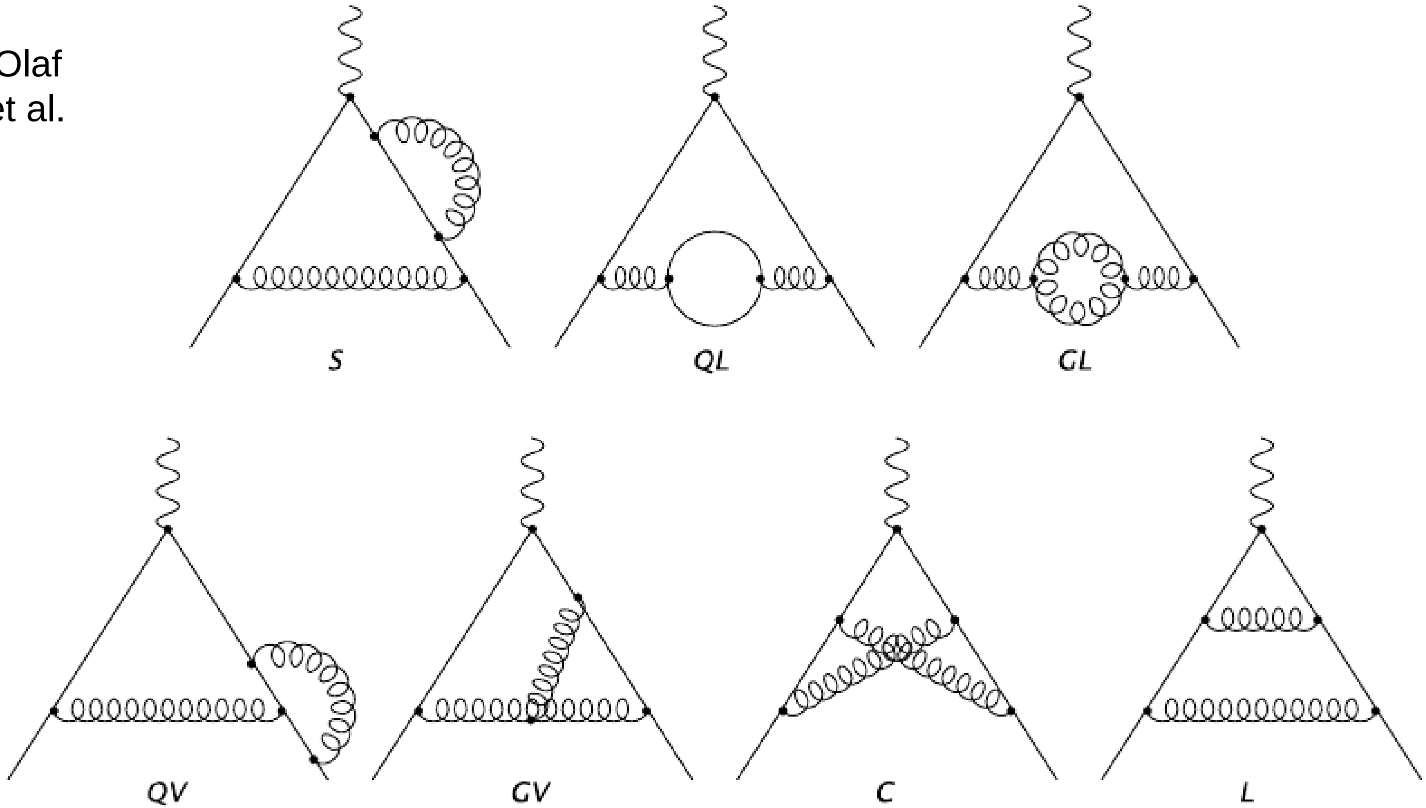
University at Buffalo
College of Arts and Sciences

QCD QED Correction to Z boson production



2 loop photon-quark-antiquark QCD vertex form factor

(Sven-Olaf Moch et al. 2005)



2 loop photon-quark-antiquark QCD vertex form factor

Expected:
(Sven-Olaf Moch et al. 2005)

$$S = C_F^2\left\{\frac{1}{\epsilon^3} + \frac{7}{2}\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{53}{4} - \zeta_2\right) + \frac{355}{8} - \frac{7}{2}\zeta_2 - \frac{32}{3}\zeta_3 + \right.$$
$$+\epsilon\left(\frac{2281}{16} - \frac{53}{4}\zeta_2 - \frac{112}{3}\zeta_3 - \frac{57}{10}\zeta_2^2\right)$$
$$\left. + \epsilon^2\left(\frac{14299}{32} - \frac{355}{8}\zeta_2 - \frac{424}{3}\zeta_3 - \frac{399}{20}\zeta_2^2 + \frac{32}{3}\zeta_2\zeta_3 - \frac{272}{5}\zeta_5\right)\right\}$$

Calculated (ignoring coupling terms):

$$\frac{1}{\mathrm{eps}^3} + \frac{7}{2\,\mathrm{eps}^2} + \frac{\frac{53}{4} - \frac{\pi^2}{6}}{\mathrm{eps}} + \left(-10\,\zeta(3) + \frac{355}{8} - \frac{7\pi^2}{12} + \frac{\psi^{(2)}(1)}{3}\right) + O(\mathrm{eps}^1)$$

Polygamma :

$$\psi_n(z) = (-1)^{n+1}\, n!\left[\zeta(n+1) - H_{z-1}^{(n+1)}\right]$$

Harmonic Number:

$$H_n = \sum_{k=1}^{n}\frac{1}{k}$$

2 loop photon-quark-antiquark QCD vertex form factor

Expected:

$$QL = C_F n_f \left\{ \frac{1}{3}\frac{1}{\epsilon^3} + \frac{14}{9}\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{353}{54} + \frac{1}{3}\zeta_2\right) + \frac{7541}{324} + \frac{14}{9}\zeta_2 - \frac{26}{9}\zeta_3 + \right.$$
$$+ \epsilon\left(\frac{150125}{1944} + \frac{353}{54}\zeta_2 - \frac{364}{27}\zeta_3 - \frac{41}{30}\zeta_2^2\right) +$$
$$\left. + \epsilon^2\left(\frac{2877653}{11664} + \frac{7541}{324}\zeta_2 - \frac{4589}{81}\zeta_3 - \frac{287}{45}\zeta_2^2 - \frac{26}{9}\zeta_2\zeta_3 - \frac{242}{15}\zeta_5\right)\right\}$$

Calculated (ignoring color factors):

$$\frac{2}{3\,\text{eps}^3} + \frac{28}{9\,\text{eps}^2} + \frac{353 + 3\pi^2}{27\,\text{eps}} + \frac{1}{162}\left(-864\,\zeta(3) + 7541 + 84\,\pi^2 + 36\,\psi^{(2)}(1)\right) + O\left(\text{eps}^1\right)$$

2 loop photon-quark-antiquark QCD vertex form factor

Expected:

$$QV = C_F(C_F - \frac{C_A}{2})\Big\{-\frac{1}{\epsilon^3} - \frac{1}{\epsilon^2}\Big(\frac{11}{2} - 2\zeta_2\Big) + \frac{1}{\epsilon}\Big(-\frac{109}{4} + 10\zeta_2 + 2\zeta_3\Big) - \\ -\frac{911}{8} + \frac{91}{2}\zeta_2 + \frac{59}{3}\zeta_3 + \frac{8}{5}\zeta_2^2 + \\ + \epsilon\Big(-\frac{6957}{16} + \frac{689}{4}\zeta_2 + \frac{296}{3}\zeta_3 + \frac{129}{10}\zeta_2^2 - \frac{58}{3}\zeta_2\zeta_3 + 6\zeta_5\Big) + \\ + \epsilon^2\Big(-\frac{49639}{32} + \frac{4843}{8}\zeta_2 + \frac{1307}{3}\zeta_3 + \frac{1267}{20}\zeta_2^2 - \frac{293}{3}\zeta_2\zeta_3 + \\ + \frac{407}{5}\zeta_5 - \frac{281}{35}\zeta_2^3 - \frac{58}{3}\zeta_3^2\Big)\Big\},$$

Calculated (ignoring color factors):

$$-\frac{1}{\text{eps}^3} + \frac{\frac{\pi^2}{3} - \frac{11}{2}}{\text{eps}^2} + \frac{2\,\zeta(3) - \frac{109}{4} + \frac{5\,\pi^2}{3}}{\text{eps}} + \left(19\,\zeta(3) - \frac{911}{8} + \frac{91\,\pi^2}{12} + \frac{2\,\pi^4}{45} - \frac{\psi^{(2)}(1)}{3}\right) + O(\text{eps}^1)$$

2 loop photon-quark-antiquark QCD vertex form factor

Expected:

$$C = C_F\left(C_F - \frac{C_A}{2}\right)\Bigg\{\frac{1}{\epsilon^4} + \frac{4}{\epsilon^3} + \frac{1}{\epsilon^2}(16 - 7\zeta_2) + \frac{1}{\epsilon}\left(58 - 16\zeta_2 - \frac{122}{3}\zeta_3\right) + \\ + 204 - 58\zeta_2 - \frac{380}{3}\zeta_3 - \frac{53}{2}\zeta_2^2 + \\ + \epsilon\left(697 - 181\zeta_2 - \frac{1646}{3}\zeta_3 - \frac{402}{5}\zeta_2^2 + \frac{326}{3}\zeta_2\zeta_3 - \frac{842}{5}\zeta_5\right) + \\ + \epsilon^2\left(\frac{4631}{2} - \frac{1141}{2}\zeta_2 - \frac{6293}{3}\zeta_3 - \frac{1744}{5}\zeta_2^2 + \frac{836}{3}\zeta_2\zeta_3 - \\ - \frac{2708}{5}\zeta_5 + \frac{1399}{70}\zeta_2^3 + \frac{4274}{9}\zeta_3^2\right)\Bigg\},$$

Calculated (ignoring color factors):

$$\frac{1}{\mathrm{eps}^4} + \frac{4}{\mathrm{eps}^3} + \frac{16 - \frac{7\pi^2}{6}}{\mathrm{eps}^2} + \frac{-40\,\zeta(3) + 58 - \frac{8\pi^2}{3} + \frac{\psi^{(2)}(1)}{3}}{\mathrm{eps}} + \left(-124\,\zeta(3) + 204 - \frac{29\,\pi^2}{3} - \frac{53\,\pi^4}{72} + \frac{4\,\psi^{(2)}(1)}{3}\right) + O(\mathrm{eps}^1)$$

2 loop photon-quark-antiquark QCD vertex form factor

Expected:

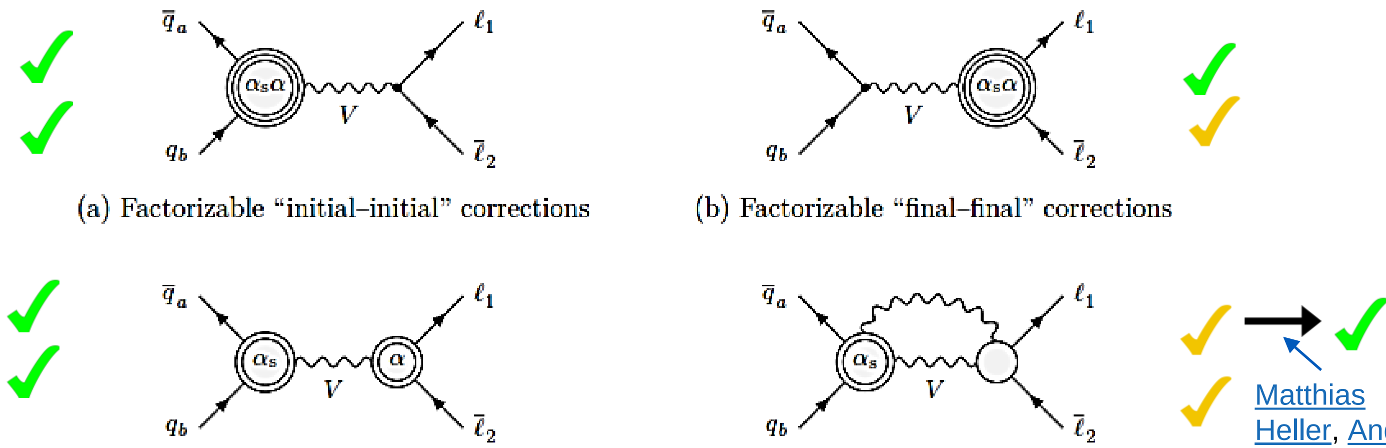
$$L = C_F^2 \Bigg\{ \frac{1}{\epsilon^4} + \frac{2}{\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{17}{2} + \zeta_2\right) + \frac{1}{\epsilon}\left(\frac{101}{4} - 2\zeta_2 + \frac{46}{3}\zeta_3\right) + \\ + \frac{631}{8} - \frac{35}{2}\zeta_2 + \frac{152}{3}\zeta_3 + \frac{103}{10}\zeta_2^2 \\ + \epsilon\left(\frac{3941}{16} - \frac{335}{4}\zeta_2 + \frac{439}{3}\zeta_3 + \frac{159}{5}\zeta_2^2 - \frac{98}{3}\zeta_2\zeta_3 + \frac{598}{5}\zeta_5\right) + \\ + \epsilon^2\left(\frac{24495}{32} - \frac{2573}{8}\zeta_2 + \frac{2065}{6}\zeta_3 + \frac{1839}{20}\zeta_2^2 - \frac{152}{3}\zeta_2\zeta_3 + \\ + \frac{1976}{5}\zeta_5 + \frac{2847}{70}\zeta_2^3 - \frac{1318}{9}\zeta_3^2\right)\Bigg\}.$$

Calculated (ignoring color factors):

$$\frac{1}{\text{eps}^4} + \frac{2}{\text{eps}^3} + \frac{51 + \pi^2}{6\,\text{eps}^2} + \frac{192\,\zeta(3) + 303 - 4\,\pi^2 + 4\,\psi^{(2)}(1)}{12\,\text{eps}} + \\ \left(52\,\zeta(3) + \frac{631}{8} - \frac{35\,\pi^2}{12} + \frac{103\,\pi^4}{360} + \frac{2\,\psi^{(2)}(1)}{3}\right) + O\left(\text{eps}^1\right)$$

Outlook

Full DY QCD-EW Virtual Corrections



(a) Factorizable "initial–initial" corrections

(b) Factorizable "final–final" corrections

(c) Factorizable "initial–final" corrections

(d) Non-factorizable corrections

Matthias Heller, Andreas von Manteuffel, Robert M. Schabinger, 2019

- Matrix elements of Real-Virtual, Double Real and double virtual pieces for the on shell vector boson production via DY mechanism are collected. They are ready to be implemented in an in-house Monte Carlo.
- Master integrals keeping the lepton mass in the final state up to logarithmic terms can play an important role for the full calculation . With our available technology we can evaluate them in terms of GPLs. The work is in progress.

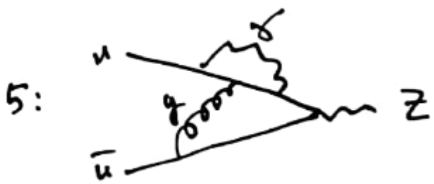
- Thank You

EXTRA SLIDES

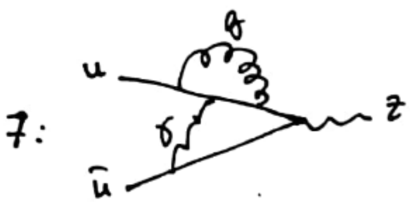
Consistency Checks:

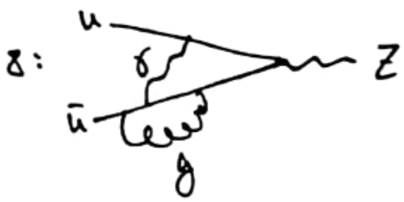
$$F_1 = G_1$$

$$F_2 = F_3 = G_2 = G_3 = 0$$

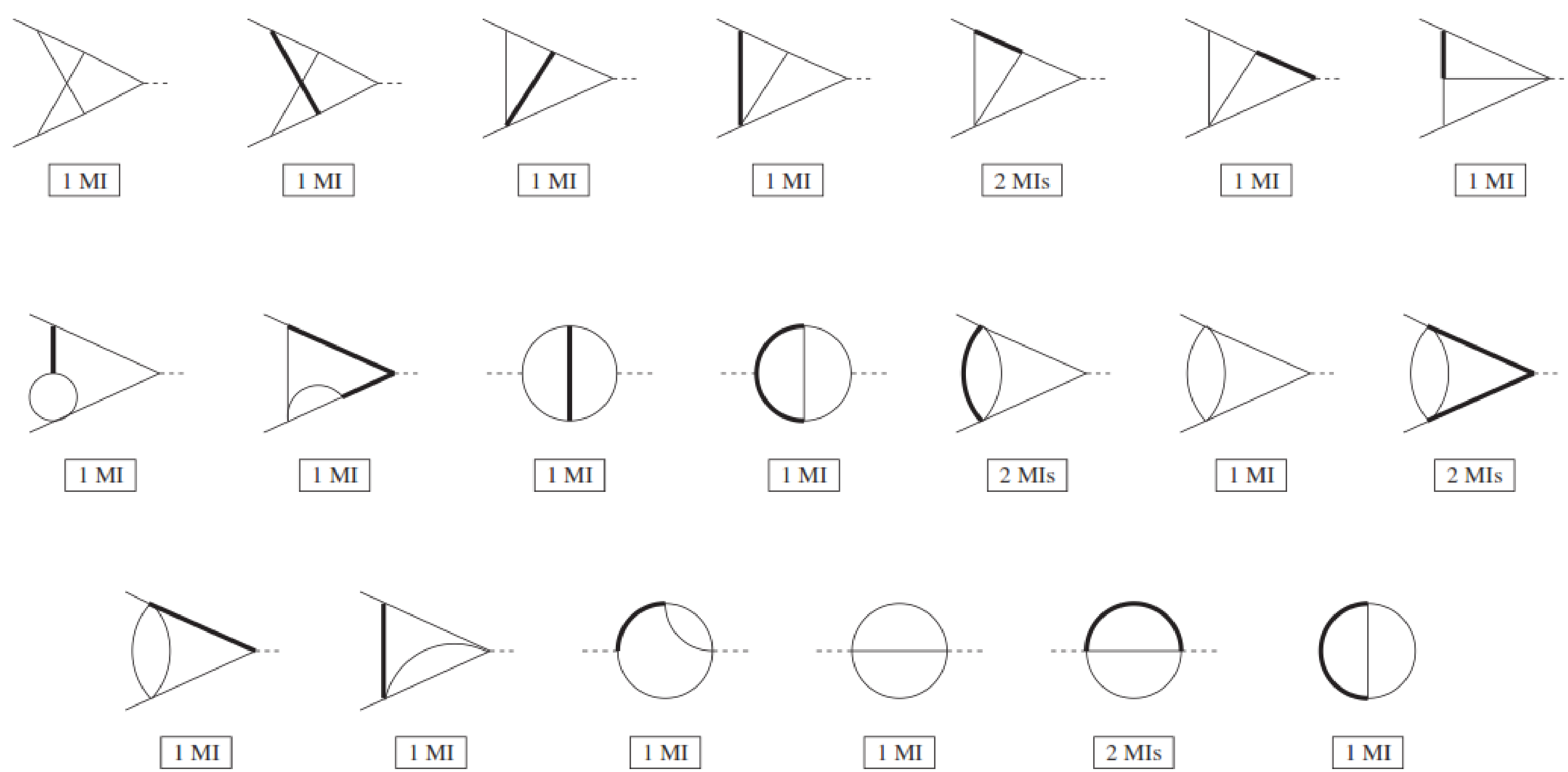


Should be equal to each other





All the Master Integrals are checked against SecDec both in Eucledian and physical regions.


1 MI
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2 MIs
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2 MIs
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1 MI
1 MI
2 MIs
1 MI

GHPL

$$g(-r;x) = \frac{1}{\sqrt{x(x+4)}},$$
$$g(w;x) = \frac{1}{x-w}, \quad \text{with } w \in \{-4,-1,0\}.$$

$$G(0;x) = \log(x),$$
$$G(-r;x) = \int_0^x \frac{dt}{\sqrt{t(t+4)}} = -\log\left(\frac{\sqrt{x+4}-\sqrt{x}}{\sqrt{x+4}+\sqrt{x}}\right),$$
$$G(w;x) = \int_0^x \frac{dt}{t-w} = \log(x-w) - \log(-w), \quad \text{with } w \in \{-4,-1\},$$

$$G(a,\mathbf{w};x) = \int_0^x dt\, g(a;t)\, G(\mathbf{w};t),$$

$$G(\mathbf{0}_w;x) = \frac{1}{w!}\log^w(x).$$

GHPL

$$x = \frac{(1-\xi)^2}{\xi}\,, \quad \xi = \frac{\sqrt{x+4}-\sqrt{x}}{\sqrt{x+4}+\sqrt{x}}\,,$$

$$\int_0^x dt = \int_1^\xi \frac{(\eta+1)(\eta-1)}{\eta^2}\,d\eta\,,$$

$$\begin{aligned}
g(-r;t) &= \frac{1}{\sqrt{t(t+4)}} = -\frac{\eta}{(\eta+1)(\eta-1)}\,,\\
g(-4;t) &= \frac{1}{t+4} = \frac{\eta}{(\eta+1)^2}\,,\\
g(-1;t) &= \frac{1}{t+1} = \frac{\eta}{(\eta-c)(\eta-\bar{c})}\,,\\
g(0;t) &= \frac{1}{t} = \frac{\eta}{(\eta-1)^2}\,,
\end{aligned}$$

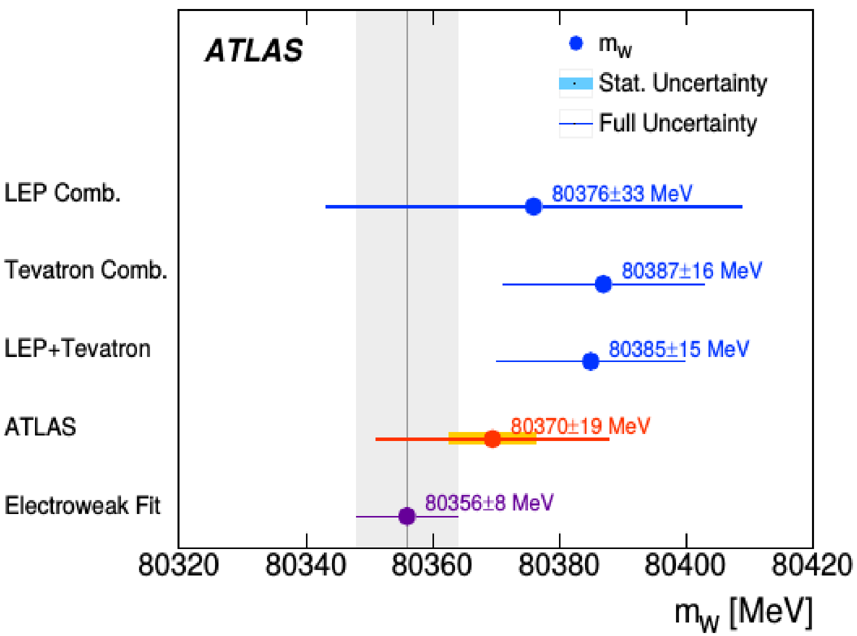
$$c = \frac{1+i\sqrt{3}}{2} = e^{i\frac{\pi}{3}}\,, \quad \bar{c} = \frac{1-i\sqrt{3}}{2} = e^{-i\frac{\pi}{3}}\,,$$

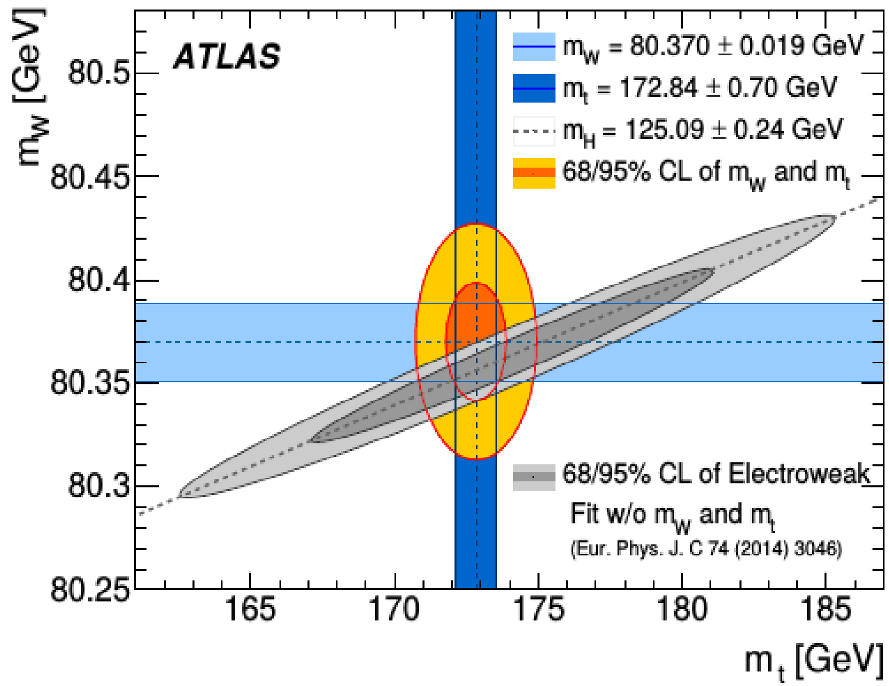
$$\begin{aligned}
G(-r,\mathbf{w};x) &= \int_0^x dt\, g(-r;t)\, G(\mathbf{w};t) = -\int_1^\xi d\eta\, \frac{1}{\eta}\, G(\mathbf{w};t(\eta))\,,\\
G(-4,\mathbf{w};x) &= \int_0^x dt\, g(-4;t)\, G(\mathbf{w};t) = \int_1^\xi d\eta \left(-\frac{1}{\eta}+\frac{2}{\eta+1}\right) G(\mathbf{w};t(\eta))
\end{aligned}$$

$$G(0;x) = \log(x) = 2\log(1-\xi) - \log(\xi) = 2G(1;\xi) - G(0;\xi)$$

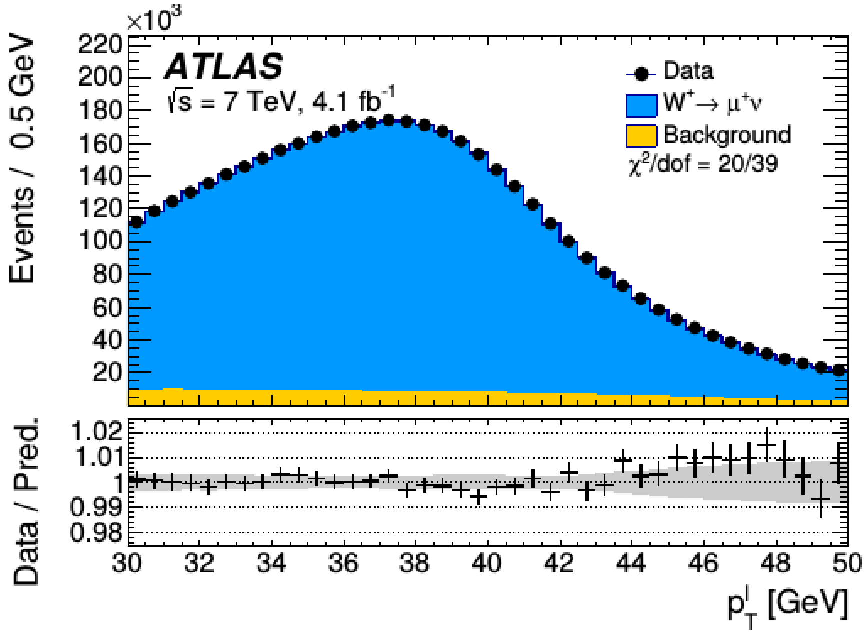
$$\begin{aligned}
G(-r;x) &= -\int_1^\xi \frac{d\eta}{\eta} = -G(0;\xi)\,,\\
G(-4;x) &= \int_1^\xi d\eta \left(-\frac{1}{\eta}+\frac{2}{\eta+1}\right) = -2\log(2) + 2G(-1;\xi) - G(0;\xi)\,,\\
G(-1;x) &= \int_1^\xi d\eta \left(-\frac{1}{\eta}+\frac{1}{\eta-c}+\frac{1}{\eta-\bar{c}}\right)\,,\\
&= -G(c;1) - G(\bar{c};1) + G(c;\xi) + G(\bar{c};\xi) - G(0;\xi)\,.
\end{aligned}$$

ATLAS Report on W mass

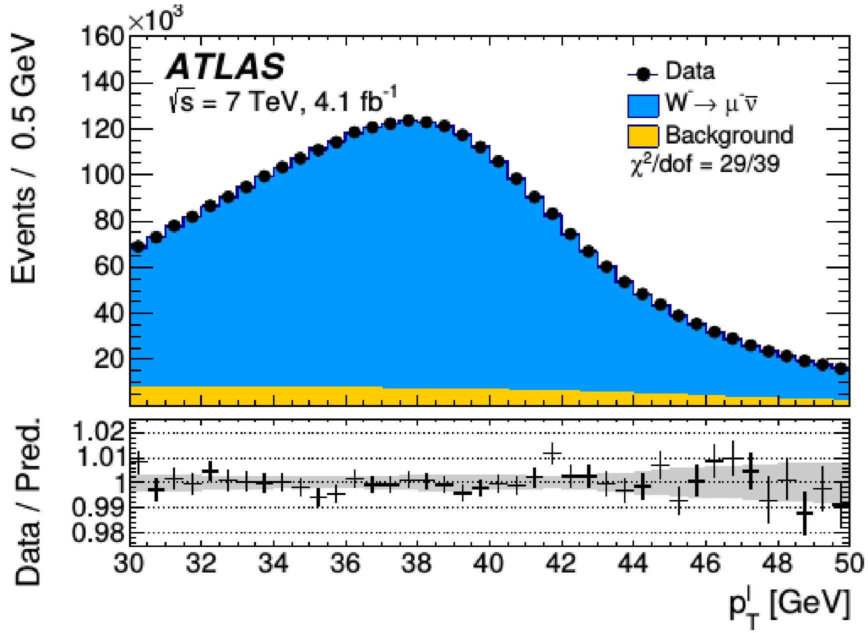




Lepton transverse momenta distribution

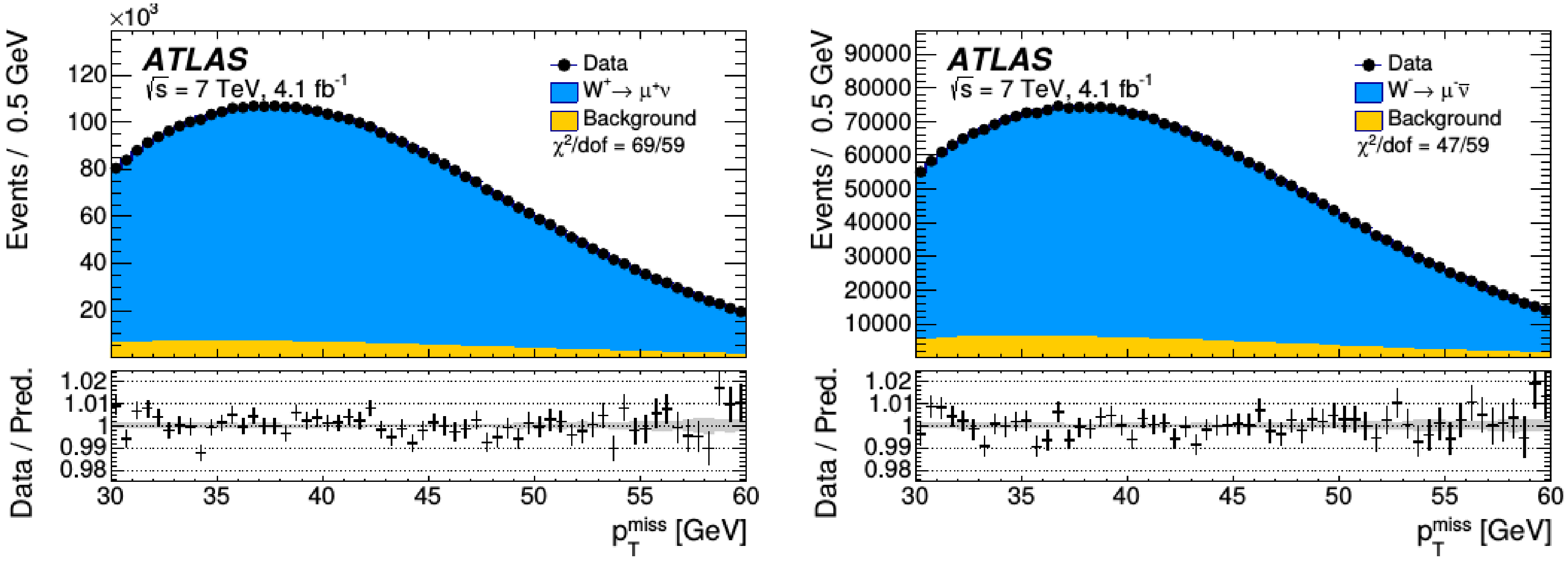


(a)



(b)

Missing transverse momenta distribution



(e) (f)

EXTRA SLIDES

Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
M_H [GeV]$^{(\circ)}$	125.14 ± 0.24	yes	125.14 ± 0.24	93^{+25}_{-21}	93^{+24}_{-20}
M_W [GeV]	80.385 ± 0.015	–	80.364 ± 0.007	80.358 ± 0.008	80.358 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1880 ± 0.0021	91.200 ± 0.011	91.2000 ± 0.010
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4950 ± 0.0014	2.4946 ± 0.0016	2.4945 ± 0.0016
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.484 ± 0.015	41.475 ± 0.016	41.474 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.743 ± 0.017	20.722 ± 0.026	20.721 ± 0.026
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01626 ± 0.0001	0.01625 ± 0.0001	0.01625 ± 0.0001
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1472 ± 0.0005	0.1472 ± 0.0005	0.1472 ± 0.0004
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	0.23150 ± 0.00006	0.23149 ± 0.00007	0.23150 ± 0.00005
A_c	0.670 ± 0.027	–	0.6680 ± 0.00022	0.6680 ± 0.00022	0.6680 ± 0.00016
A_b	0.923 ± 0.020	–	0.93463 ± 0.00004	0.93463 ± 0.00004	0.93463 ± 0.00003
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	0.0738 ± 0.0003	0.0738 ± 0.0003	0.0738 ± 0.0002
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	0.1032 ± 0.0004	0.1034 ± 0.0004	0.1033 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	$0.17226^{+0.00009}_{-0.00008}$	0.17226 ± 0.00008	0.17226 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21578 ± 0.00011	0.21577 ± 0.00011	0.21577 ± 0.00004
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
m_t [GeV]	173.34 ± 0.76	yes	$173.81 \pm 0.85^{(\triangledown)}$	$177.0^{+2.3}_{-2.4}{}^{(\triangledown)}$	177.0 ± 2.3
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{(\dagger\triangle)}$	2757 ± 10	yes	2756 ± 10	2723 ± 44	2722 ± 42
$\alpha_s(M_Z^2)$	–	yes	0.1196 ± 0.0030	0.1196 ± 0.0030	0.1196 ± 0.0028

- There is no unique way to handle $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in DR
- In D-dimensions, the relations

$$\{\gamma^5, \gamma^\mu\} = 0$$

and

$$\mathrm{tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) \neq 0$$

cannot be simultaneously satisfied.

- In other words, there is a conflict between the anticommutativity of γ^5 and the cyclicity property of Dirac traces that involve and odd number of γ^5

[Chanowitz et al., 1979]
[Jegerlehner, 2001]

Issues with Gamma5

Resolution:

- We can stick to anticommuting $\gamma 5$ in D dimension. Its fine as long as we have only traces with even number of $\gamma 5$. This is called Naive dimensional regularization.
- We can put additional prescriptions to compute trace. For example Kreimer's prescription [Kreimer, 1990] or Larin prescription [Larin et al., 1993].
- Or, we can accept $\gamma 5$ is a purely 4 dimensional object and does not anticommute with D dimensional Dirac matrices. ['t Hooft and Veltman, 1972]

Larin-Gorishny-Akyeampong-Delburgo prescription allows one to use anticommuting γ^5 in D-dimensions but compute the chiral traces, such, that the result is expected to be equivalent with the BMHV scheme, if we have only one axial-vector current. The prescription is essentially

- Anticommute γ^5 to the right inside the trace
- Replace $\gamma^\mu \gamma^5$ with $-\frac{i}{6}\varepsilon^{\mu\alpha\beta\sigma}\gamma^\alpha\gamma^\beta\gamma^\sigma$
- Treat $\varepsilon^{\mu\alpha\beta\sigma}$ as if it were D-dimensional, i.e. $\varepsilon^{\mu\alpha\beta\sigma}\varepsilon_{\mu\alpha\beta\sigma} = -D(D^3 - 6D^2 + 11D - 6)$ instead of -24.