ETH zürich

Generalized unitarity and numerical ansatzes for 2-loop 5-point amplitudes

LoopFest XVIII Conference, Fermilab

Mao Zeng

Aug 13, 2019

Institute for Theoretical Physics, ETH Zürich

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- Background - precision QCD at high multiplicity

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    - Differential equations
    - Final amplitudes
- Conclusions and Outlook

Nonplanar 2-to-3 amplitudes

Phys. Rev. Lett. 122, 121603 (2019),
Samuel Abreu, Lance Dixon, Enrico Herrmann, Ben Page, MZ

JHEP 03 (2019) 123,
Samuel Abreu, Lance Dixon, Enrico Herrmann, Ben Page, MZ

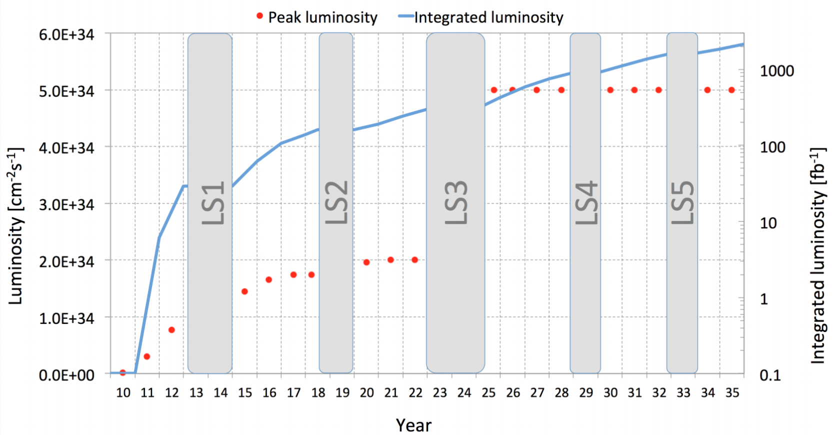
Nonplanar 2-to-3 differential equations

JHEP 1901 (2019) 006, Samuel Abreu, Ben Page, MZ

JHEP 1706 (2017) 121, MZ

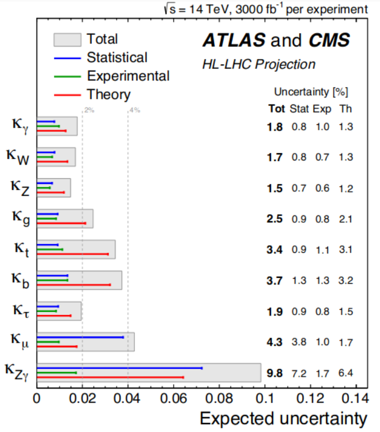
HL-LHC PERFORMANCE PROJECTION

$\sim$150 fb^{-1} from LHC Run 2 (2015-2018). High-Luminosity LHC ($\sim$2026-2035) promises 3000 fb^{-1}.



HL-LHC PERFORMANCE PROJECTION

Percent-level accuracy for most SM measurements, e.g. Higgs coupling strengths.

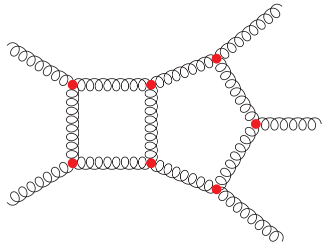


- Projection assumes 50% reduction of theoretical uncertainty for many channels.
- Only possible with **NNLO QCD** calculations.

HL-LHC Workshop Report, R. Abdul Khalek *et al.*

$2 \to 3$ PROCESSES AT THE LHC

- $p + p \to 3j$: α_s determination, testing QCD up to TeV range.



CMS measurements of 3-jet / 2-jet cross section ratio:

$\alpha_s(M_Z) = 0.1148 \pm 0.0014(\text{exp.}) \pm 0.0018(\text{PDF}) \pm 0.0050\,(\text{theory})$

[Chatrchyan *et al.*, Eur. Phys. J. C 73 (2013) 2604]

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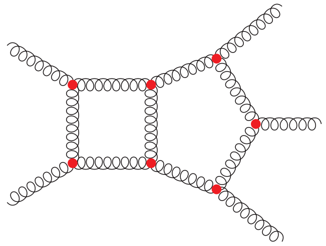

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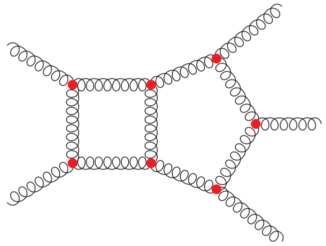

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- Other processes in Les Houches 2017 wishlist [arXiv:1803.07977]: $pp \to W/Z/H + 2j$, $pp \to t\bar{t} + j$...

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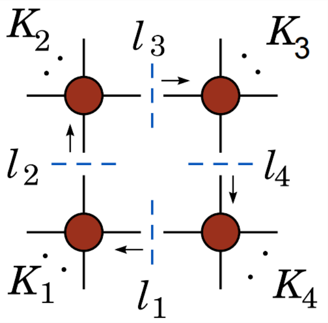
  Degree-d polynomial in n variables: $\binom{d+n}{n}$ terms.
- **Master integrals**: analytic / numerical evaluation
- **Our approach:** Ansatz for analytic results. Fit using numerical data (unitarity cuts, numerical IBP).

  Finite fields & rational reconstruction: [von Manteuffel, Schabinger, '14, Peraro '16, Klappert, Lange '19, Peraro '19 ...]

Cutting with *complex momenta*: $\ell_1^2 = \ell_2^2 = \cdots = \ell_N^2 = 0$,

Local integrand factorization: $\mathcal{I}(\ell_\mu)\big|_{\text{cut}} = \prod \mathcal{A}_{\text{tree}}\big|_{\text{cut}}$.

[Bern, Dunbar, Kosower, ’94; Bern, Dixon, Dunbar, Dosower, ’95; Britto, Cachazo, Feng ’05] [Zvi Bern’s talk]



arXiv:0803.4180

- Constraint: factorization on cuts fixes integrand up *contact terms*.
- Merging cuts fix all terms.
- Highly efficient, “NLO revolution”. [Ellis *et al.* ’07; Giele *et al.* ’08; Berger *et al.* ’08...]

NUMERICAL UNITARITY: TWO LOOPS?

[Abreu, Febres Cordero, Ita, Jacquier, Page, MZ, PRL 2017]

- Philosophy: (1) Ansatz from *analytic properties*, (2) fit parameters from *numerical computation*

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- Ansatz of integrand: Master integrals plus spurious / surface terms that integrate to 0. [talk: Samuel Abreu]

$$\mathcal{I} = \sum_{\text{topologies}} \frac{1}{\prod_j \rho_j} \left(\sum_{j \in \text{masters}} c_j \mathcal{N}_j + \sum_{k \in \text{surface}} c_k \mathcal{N}_k \right)$$

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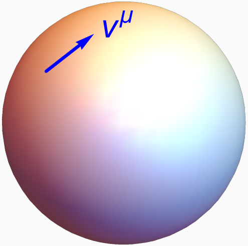
- **Line 1 problem:** what will integrate to zero?
- Answer: integration by parts (IBP) identities. from $\ell^\mu \to \ell^\mu + \delta \cdot v^\mu$, *but breaks unitarity...*

MARRYING IBP AND UNITARITY

Cut surface $\rho_i = q_i^2 - m_i^2 = 0$



IBP identities from *polynomial tangent vectors* $\ell^\mu \to \ell^\mu + \delta \cdot v^\mu$,

$$v^\mu \frac{\partial}{\partial \ell^\mu} \rho_i = f_i \, \rho_i.$$ [Gluza, Kajda, Kosower]

1 loop: rotation vectors for spheres.

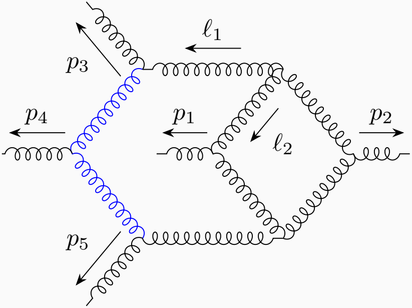
≥ 2 **loops:** *Very difficult* to solve!

Improvement: change ℓ_i^μ to Cutkosky / Baikov variables $\rho_1, \ldots, \rho_n, \rho_{n+1}, \ldots, \rho_{n+m}$. [Ita, 2015; Larsen, Zhang, 2015]

Efficient hybrid algorithm: momentum space ansatz for v^μ, but solve in new variables. [Abreu, Febres Cordero, Ita, Page, MZ, '17; *Laplace expansion:* Böhm, Georgoudis, Larsen, Schulze, Zhang, '17]

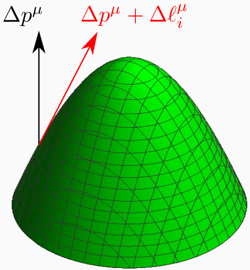
EXTENSION TO DIFFERENTIAL EQUATIONS

Nonplanar needed for 3-jet production beyond large N_c.



Need master integrals! Powerful method: differential equations w.r.t. external p^μ.

Speed up construction of DEs using generalized unitarity: [MZ, '17; Larsen, Zhang, '17; Abreu, Page, MZ, '18].



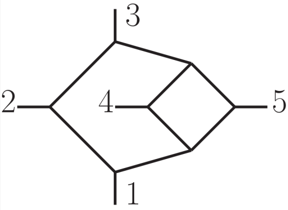
Find compensating vector $\Delta \ell_i^\mu$, or choose momentum routing and cut "undotted" lines.

ANSATZ FOR DIFFERENTIAL EQUATIONS

Input 1: pure master integrals $\mathcal{I}_i$ with only *logarithmic singularities*, from e.g. integrand of $\mathcal{N} = 4$ SYM. [Bern *et al.* '15]

Input 2: singularity locations encoded in *symbol alphabet* W_j, from permutations of planar alphabet [Chicherin, Henn, Mitev, '17] or generally from **analytic (near-) maximal cut DEs.**

$$\frac{\partial}{\partial x}\vec{\mathcal{I}} = \sum_j \left(\epsilon\, \mathbf{M}^{(j)} \cdot \vec{\mathcal{I}}\right) \frac{\partial}{\partial x} \log W_j \,. \quad \text{[Henn, '13]}$$



$W_1 = (p_1 + p_2)^2 = s_{12},\ W_2 = (p_2 + p_3)^2 = s_{23}$

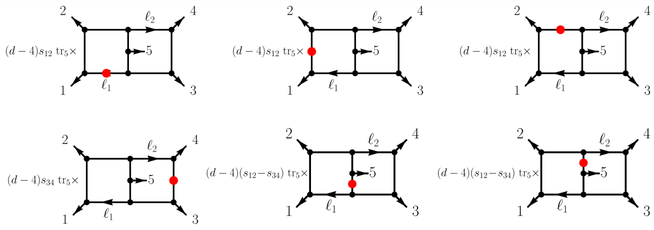
$W_6 = s_{12} - s_{23}, \ldots, W_{31} = \det(p_1^\mu, p_2^\mu, p_3^\mu, p_4^\mu)$

Analytic DEs from **numerical** IBP at **31 points**. Timing: < 24 CPU hours!

REMAINING NONPLANAR TOPOLOGY: DOUBLE PENTAGON

Symbols for all 2-loop 5-point masters & $\mathcal{N} = 4$ SYM amplitude [Abreu, Dixon, Herrmann, Page, MZ, PRL 2018]

- Parity-even (under $\mathrm{tr}_5 \to -\mathrm{tr}_5$) MIs from $\mathcal{N} = 4$ SYM.
- Parity-odd MIs from $(6 - 2\epsilon)$ dimensions.



Result in alternative basis: [talk by Simone Zoia] [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia, PRL 2018]

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- Both IBP and *assembly* of amplitude can be heavy.

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- Warmup from low tensor rank: $\mathcal{N} = 4$ SYM (rank 1), $\mathcal{N} = 8$ SUGRA (rank 2). Future: QCD (rank ≥ 5).

RATIONAL FACTORS FOR $\mathcal{N} = 4$ SYM AMPLITUDE

- Parke-Taylor MHV tree amplitude, e.g. $(g^- g^- g^+ g^+ g^+ \dots)$

$$\mathcal{A}^{\text{tree}}_{\text{MHV}}(1, 2, \dots, n) = \text{PT}[1, 2, \dots, n] = \frac{\delta^{(8)}(\mathcal{Q})}{\langle 12 \rangle \langle 23 \rangle \dots \langle n-1\, 1 \rangle}$$

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- Planar MHV loop amplitudes - leading singularity $\propto \mathcal{A}^{\text{tree}}_{\text{MHV}}$, no new tensor structures! [Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka, 2012]

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- Nonplanar MHV loop amplitudes - leading singularities consists of permutations $\text{PT}[\sigma(1), \sigma(2), \ldots, \sigma(n)]$. [Arkani-Hamed, Bourjaily, Cachazo, Postnikov, Trnka, 2014]

2-LOOP 5-POINT AMPLITUDE OF $\mathcal{N} = 4$ SYM

[Abreu, Dixon, Herrmann, Page, MZ, arXiv:1812.08941, PRL] See also: [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia, 1812.11057, PRL]

- Color-stripped amps. = rational prefactors $\times$ pure GPLs

$$A^{\mathrm{ST}}[12345] = \mathrm{PT}[12345]\, \mathrm{M}^{\mathrm{BDS}}_{(2)},$$
$$A^{\mathrm{DT}}[15|234] = \sum_{\sigma(234)\in S_3} \mathrm{PT}\,[1\sigma_2\sigma_3\sigma_4 5]\, g^{\mathrm{DT}}_{\sigma_2\sigma_3\sigma_4}\,.$$

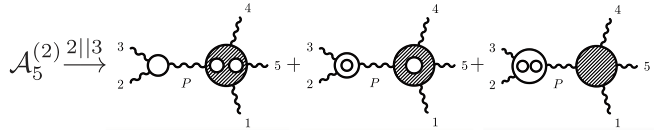
- Need numerical IBP reduction of known integrand [Carrasco, Johansson, 2011] at **6 phase space points**.

- First analytic 2-loop 5-point amplitude for any gauge theory beyond large N_c. **Uniformly transcendental.**

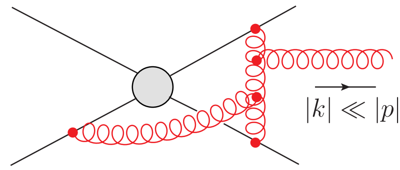
CHECKS FOR $\mathcal{N} = 4$ SYM AMPLITUDE

- Matches universal IR poles. [Catani, 1998; Bern, Dixon, Kosower, 2004; Aybat, Dixon, Sterman, 2006]

- Matches expected collinear limits. [Bern, Dixon, Kosower '04]



- Matches two-loop soft limits. [Dixon, Zhu, *et al.*, *in progress*]



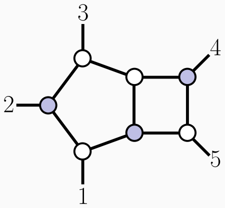
2-LOOP 5-POINT AMPLITUDE OF $\mathcal{N} = 8$ SUGRA

[Abreu, Dixon, Herrmann, Page, MZ, arXiv:1901.08563], **See also:**
[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia 1901.05932]

- BCJ double copy, Gravity = YM^2. [Bern, Carrasco, Johansson, 2008]

$$\text{MHV:} \quad \mathcal{A}_3 = \frac{\delta^8(\mathcal{Q})}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}, \quad \mathcal{M}_3 = \frac{\delta^{16}(Q)}{\langle 12\rangle^2\langle 23\rangle^2\langle 31\rangle^2}$$

- 45 rational factors from square of $\mathcal{N} = 4$! $\text{LS}_{\text{SUGRA}} = \text{LS}^2_{\text{SYM}} \times \mathcal{J}$.



Alternative computations of leading singularities:
Herrmann, Trnka, 2016
Heslop, Lipstein, 2016

Numerical IBP at 45 points sufficient

- **Space of rational factors** simplifies QCD calculation [Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov, '19] [Samuel Abreu's talk]

CONCLUSIONS AND OUTLOOK

- Lessons from modern scattering amplitude methods: generalized unitary + numerical ansatz.
- Applied to all stages of calculation: integrand, IBP, DEs, assembly of amplitude.
- Efficient construction of multi-scale canonical DEs
  - pure integrals & alphabet from cut information
  - Fit matrix of rational numbers from numerical IBP.
  - **Future applications:** 2-loop MIs for $pp \to W/Z/H + 2j$.
- SUSY amplitudes useful warmup for **QCD** $pp \to 3j$, $pp \to 3\gamma$ beyond large N_c: re-use MIs & IBPs in numerical unitarity.

Thank you!