ON GG->ZZ AMPLITUDE AT TWO LOOPS WITH FULL TOP-MASS DEPENDENCE

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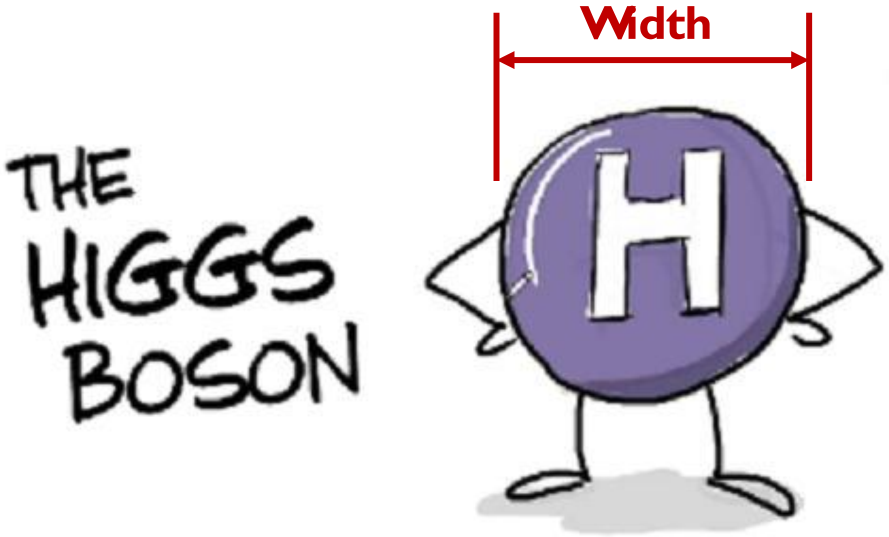
(WORK IN COLLABORATION WITH ANDREAS VON MANTEUFFEL)

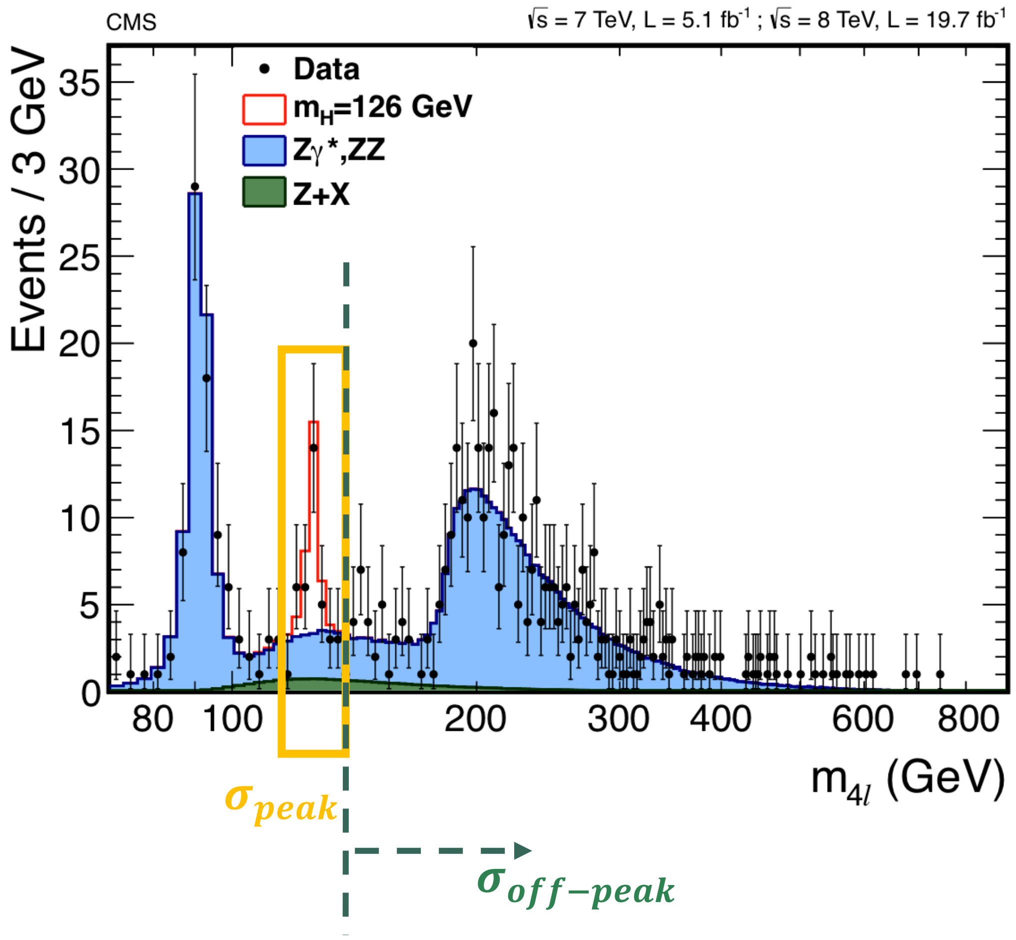
LOOPFEST - 13/08/2019

HIGGS WIDTH



- Higgs width predicted in SM : $\Gamma_H \sim 4.1$ MeV
- Important measurement. Deviation from SM value $\Rightarrow$ New Physics
- Too small to be measured at LHC. Detector resolution $\sim O(1)$ GeV
- Constrain using off-shell production. Proposed by F. Caola & K. Melnikov (arxiv:1307.4935)

HIGGS WIDTH



- $H \rightarrow ZZ^* \rightarrow 4l$

$$\frac{d\sigma}{dM_{4l}^2} \sim \frac{g_{Hgg}^2 g_{HZZ}^2}{\left(M_{4l}^2 - m_H^2\right)^2 + m_H^2 \Gamma_H^2}$$

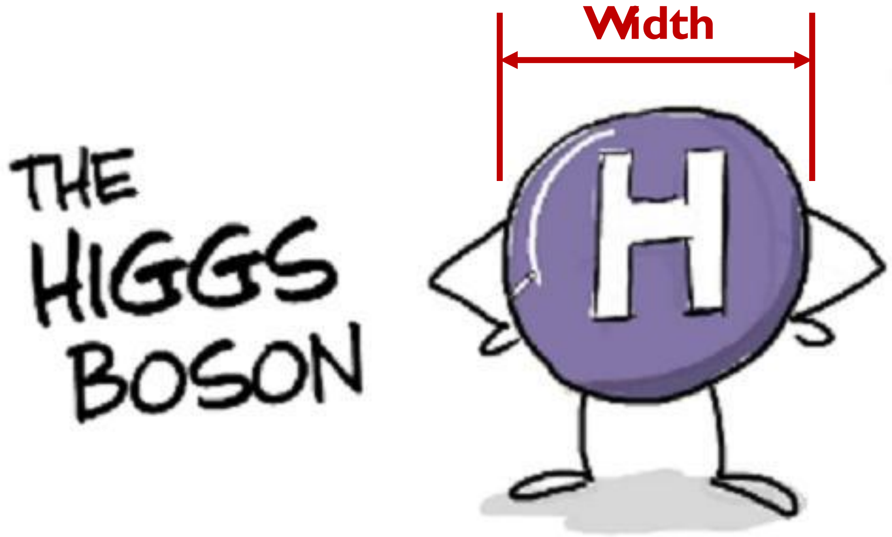
- Assume g_{Hgg}, g_{HZZ} scale linearly with ξ while Γ_H scales as ξ^4
- On-peak : $\sigma_{peak} \sim \frac{(\xi^2 g_{Hgg}^2)(\xi^2 g_{HZZ}^2)}{\xi^4 \Gamma_H} = \frac{g_{Hgg}^2 g_{HZZ}^2}{\Gamma_H}$

unchanged

- Off-peak : $\sigma_{off-peak} \sim \xi^4 g_{Hgg}^2 g_{HZZ}^2$

scales

HIGGS WIDTH



<u>Current status :</u>

- CMS : ZZ channel - $\Gamma_H < 22$ MeV at 95% confidence level (arxiv:1405.3455)
- CMS: Combined WW & ZZ analysis - $\Gamma_H < 13$ MeV at 95% confidence level (arxiv:1605.02329)
- ATLAS: ZZ channel - $\Gamma_H < 14.4$ MeV at 95% confidence level (arxiv:1808.01191)
- CMS: $\mathbf{3.2^{+2.8}_{-2.2}}$ **MeV** from combined analysis gg->VV (arxiv:1901.00174)
- Direct constraints : CMS combined $H \rightarrow ZZ^* \rightarrow 4l \Rightarrow \Gamma_H < 1.1$ GeV (arxiv:1706.09936)

HIGGS & ZZ PRODUCTION

$gg \to H(\to ZZ)$

- $gg \to H$ exact result known at NLO : M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas (arXiv:hep-ph/9504378)
- $gg \to H$ known at N3LO with infinite top mass approximation :

  C. Anastasiou et al (arXiv:1503.06056)

  B. Mistlberger (arXiv:1802.00833)

$gg \to ZZ$

- $gg \to ZZ$ exact result known at LO :

  E. N. Glover and J. J. van der Bij

  https://doi.org/10.1016/0550-3213(89)90262-9
- $gg \to ZZ$ NLO amplitude with massless quarks :

  A. von Manteuffel and L. Tancredi (arxiv:1503.08835)

  F. Caola, J. Henn, K. Melnikov, A. Smirnov & V. Smirnov (arxiv:1503.08759)
- $gg \to ZZ$ at NLO with expansion around heavy top limit

  F. Caola, M. Dowling, K. Melnikov, R. Röntsch, L. Tancredi (arxiv:1605.04610)
- NLO corrections to $gg \to ZZ$ around heavy top mass limit with Pade' approximants

  J. Campbell, R. Ellis, M. Czakon, S. Kirchner (arxiv:1605.01380)
- Top quark mass effects in $gg \to ZZ$ at 2-loops and off-shell Higgs interference : R. Gröber, A. Maier, T. Rauh (arxiv:1908.04061)

$gg \to ZZ$

Importance of gg->ZZ :

- $\mathcal{O}$(10%) correction from off-shell production, Higgs-continuum interference very important (N. Kauer & G. Passarino, arxiv:1206.4803)
- gg -> ZZ @ LO very substantial to pp -> ZZ @ NNLO ~ 60% of the full NNLO correction, due to the large gg luminosity at the LHC (F. Cascioli, T. Gehrmann, M. Grazzini, S. Kallweit et al, arxiv:1405.2219)
- Expectation of large NLO K-factor : $\mathcal{O}$(40%-90%) increase from LO to NLO (F. Caola, K. Melnikov, R. Röntsch, L. Tancredi, arxiv:1509.06734)

$gg \to ZZ$

Limitations :

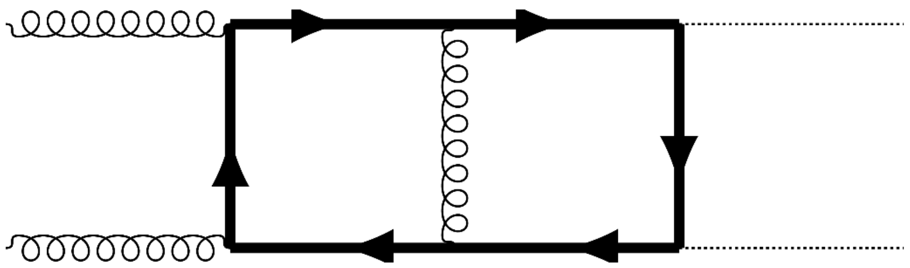
- Heavy top expansion breaks down around top quark threshold
- Equivalance theorem : At high energies, Longitudinal modes of gauge bosons $\Rightarrow$ Goldstone bosons (coupling proportional to the mass of the fermion)
- Contribution from top quark loops at high invariant mass very significant

➢ Need an NLO calculation with full top mass dependence

$gg \to ZZ$

Similar calculations:

$gg \to HH$



- Same topologies
- Higgs is a scalar : rank 2 Lorentz tensor; rank 4 in ZZ production
- State of the art calculation done using purely numerical methods by S. Borowka, N. Greiner, G. Heinrich et al (arxiv:1608.04798)
- Incomplete reductions for the non-planar topologies, computed very difficult integrals numerically
- Using finite integrals very beneficial

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^\mu \, p_j^\nu \, p_k^\rho \, p_l^\lambda \, A_{ijkl} \; + \ldots$$

Virtual correction

- Solve linear system of equations to relate the 'form factors' to the original amplitude

- Use **Integration By Parts** identities to reduce the number of integrals to a basis set

New methods

- Rotate the basis integrals to a set of **finite integrals** ⇒ Much better behaved numerically

New methods

- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements)

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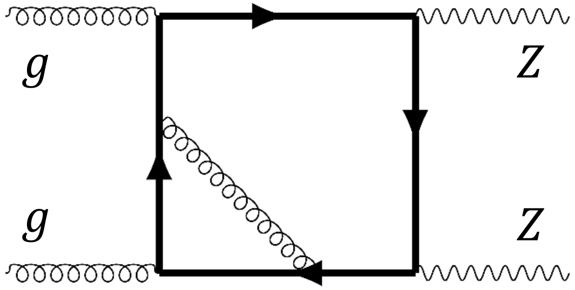
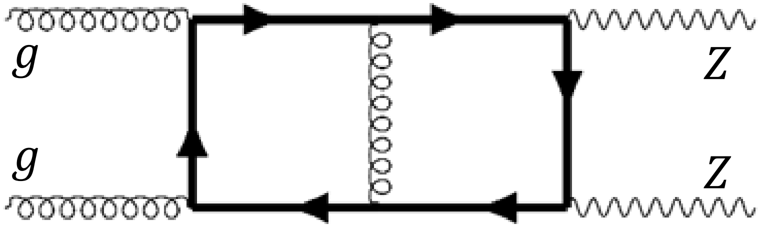
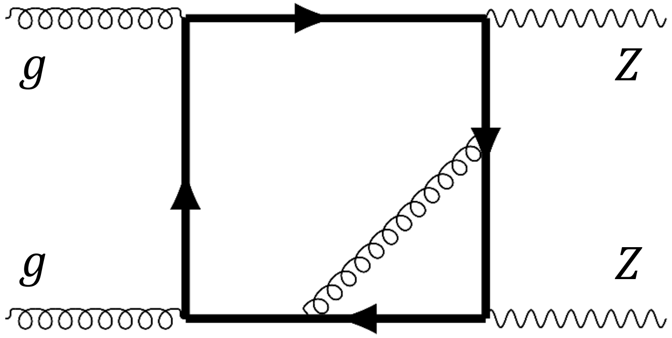
SETUP

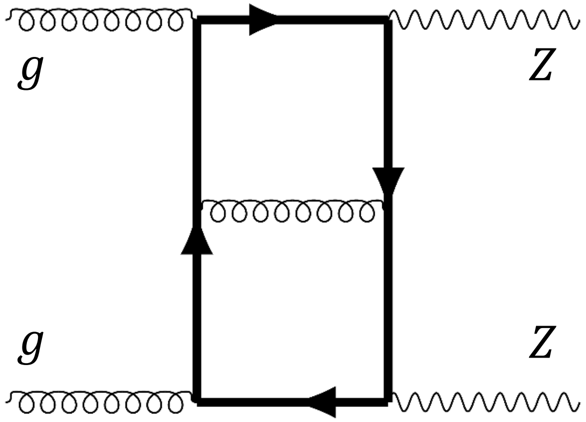
- 166 Diagrams in total
- 48 diagrams vanish due to colour structure
- 4 scales : m_t^2, m_Z^2, s, t (and **d**)
- Consider on-shell Z bosons
- Need 4 different sets of propagators to cover all topologies : Integral families A, B, C, D

INTEGRAL FAMILIES & TOPOLOGIES



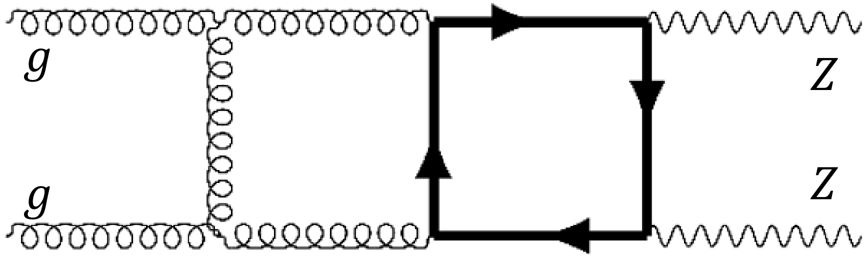


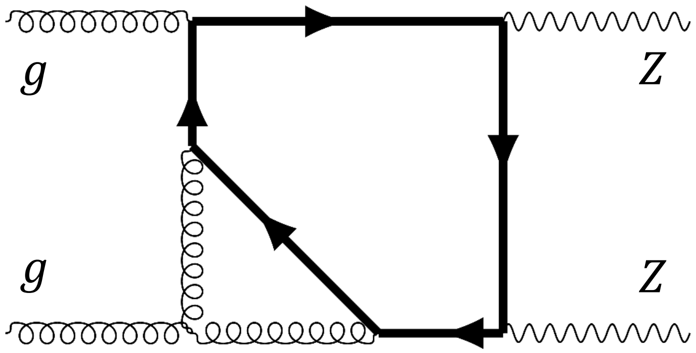




A

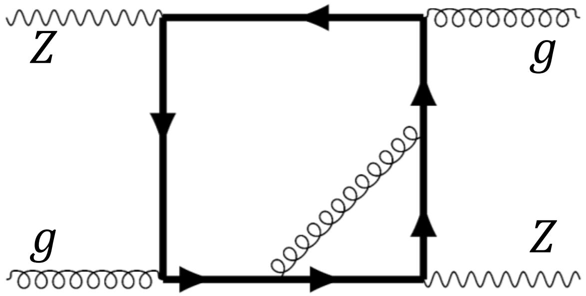
INTEGRAL FAMILIES & TOPOLOGIES

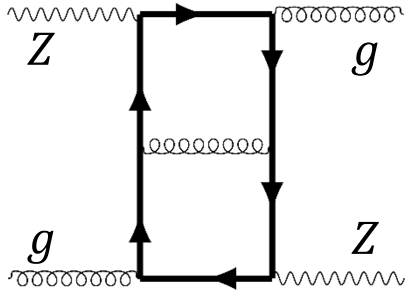


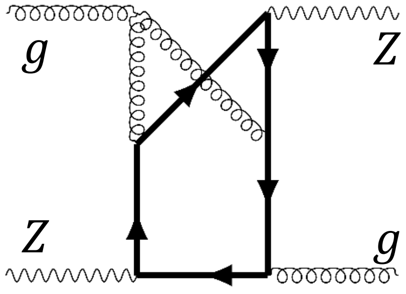


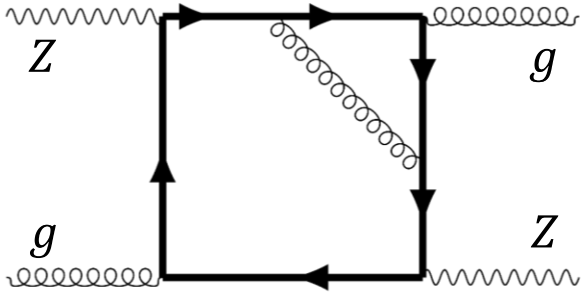
B

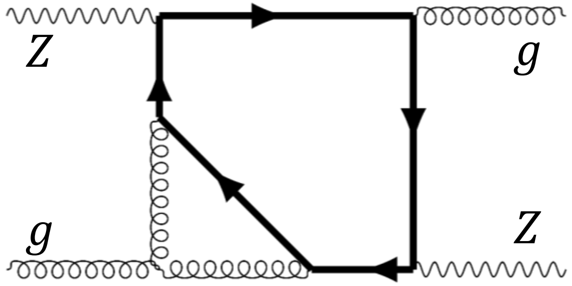
INTEGRAL FAMILIES & TOPOLOGIES





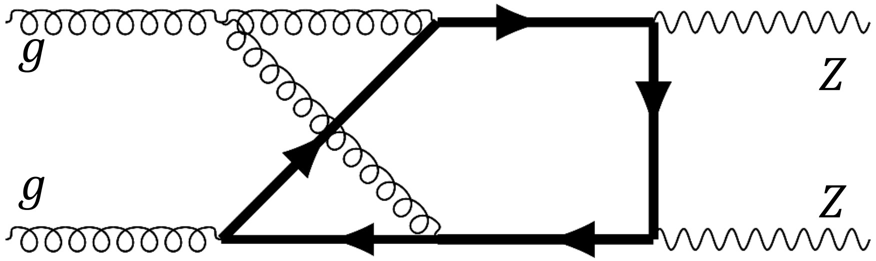


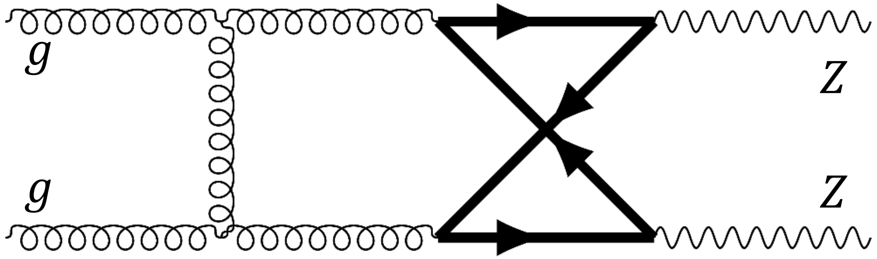




C

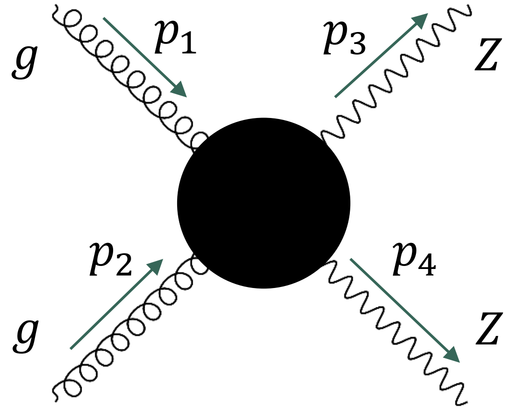
INTEGRAL FAMILIES & TOPOLOGIES





D

SETUP



- Can decompose the amplitude into 138 tensor structures
- Use transversality of gluons and gauge freedom to eliminate most of these:

$$\epsilon_1 . p_1 = \epsilon_2 . p_2 = 0 \quad \& \quad \epsilon_1 . p_2 = \epsilon_2 . p_1 = \epsilon_3 . p_3 = \epsilon_4 . p_4 = 0$$

- 20 tensor structures left :

$$= A_1 g^{\mu\nu} g^{\rho\lambda} + A_2 g^{\mu\rho} g^{\nu\lambda} + A_3 g^{\mu\lambda} g^{\nu\rho} + (A_{1,1,1} - A_{1,1,3}) g^{\mu\nu} {p_1}^{\rho} {p_1}^{\lambda} + (A_{1,1,2} - A_{1,1,3}) g^{\mu\nu} {p_1}^{\rho} {p_2}^{\lambda} + (A_{1,2,1} - A_{1,2,3}) g^{\mu\nu} {p_2}^{\rho} {p_1}^{\lambda} + (A_{1,2,2} - A_{1,2,3}) g^{\mu\nu} {p_2}^{\rho} {p_2}^{\lambda} + (A_{2,3,1} - A_{2,1,3}) g^{\mu\rho} {p_3}^{\nu} {p_1}^{\lambda} + (A_{2,3,2} - A_{2,1,3}) g^{\mu\rho} {p_3}^{\nu} {p_2}^{\lambda} + A_{3,1,3} g^{\mu\lambda} {p_1}^{\rho} {p_3}^{\nu} + A_{3,2,3} g^{\mu\lambda} {p_2}^{\rho} {p_3}^{\nu} + (A_{4,3,1} - A_{4,3,3}) g^{\rho\nu} {p_3}^{\mu} {p_1}^{\lambda} + (A_{4,3,2} - A_{4,3,3}) g^{\rho\nu} {p_3}^{\mu} {p_2}^{\lambda} + A_{5,3,3} g^{\rho\lambda} {p_3}^{\mu} {p_3}^{\nu} + A_{6,1,3} g^{\lambda\nu} {p_1}^{\rho} {p_3}^{\mu} + A_{6,2,3} g^{\lambda\nu} {p_2}^{\rho} {p_3}^{\mu} + (A_{3,3,1,1} - A_{3,3,1,3}) {p_3}^{\mu} {p_3}^{\nu} {p_1}^{\rho} {p_1}^{\lambda} + (A_{3,3,1,2} - A_{3,3,1,3}) {p_3}^{\mu} {p_3}^{\nu} {p_1}^{\rho} {p_2}^{\lambda} + (A_{3,3,2,1} - A_{3,3,2,3}) {p_3}^{\mu} {p_3}^{\nu} {p_2}^{\rho} {p_1}^{\lambda} + +(A_{3,3,2,2} - A_{3,3,2,3}) {p_3}^{\mu} {p_3}^{\nu} {p_2}^{\rho} {p_2}^{\lambda}$$

SETUP

- Contract with each of the 20 tensor structures to relate form factors to the amplitude :

$$A_i = \mathcal{A}_{\mu\nu\rho\lambda} * P_i^{\mu\nu\rho\lambda}$$

$$= \sum_{j=1}^{20} A_j * T_{j,\mu\nu\rho\lambda} * P_i^{\mu\nu\rho\lambda}$$

Solve : $T_{j,\mu\nu\rho\lambda} * P_i^{\mu\nu\rho\lambda} = \delta_{ij}$ to obtain $P_i^{\mu\nu\rho\lambda}$

- Total size of unreduced form factors : 2.8*20 GB, with the largest being ~50 MB
- Intermediate expressions in tens of gigabytes
- FORM code to perform the contraction and bringing the amplitude into the desired form
- Total of 29540 unreduced integrals; 281 master integrals

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^{\mu}\, p_j^{\nu}\, p_k^{\rho}\, p_l^{\lambda}\, A_{ijkl} + ..$$

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INTEGRATION BY PARTS REDUCTION

- General scalar Feynman integral with **L-loops** and **N-edges** :

$$\mathrm{I}(\mathrm{a}_1..\mathrm{a}_\mathrm{N}) = \int d^D k_1..d^D k_L \prod_{i=1}^{N} \frac{1}{(q_i^2 - m_i^2)^{a_i}}$$

Work in dimensional regularization to regulate the Ultraviolet/Infrared divergences appearing in the amplitude

$$D = 4 - 2\epsilon$$

p_i : *External momenta*
k_i : *Loop momenta*
q_i : *Momentum of the edge i*
m_i: *Mass of the edge i*
a_i : *Exponent of the propagator for the edge i*

INTEGRATION BY PARTS REDUCTION

- Integration by part identity:

$$0 = \int d^D k_1 .. d^D k_L \boxed{\frac{\partial}{\partial k_\mu} v_\mu \left(\prod_{i=1}^{N} \frac{1}{(q_i^2 - m_i^2)^{a_i}} \right)}$$

$v = \{p_i, k_i\}$

p_i : *External momenta*
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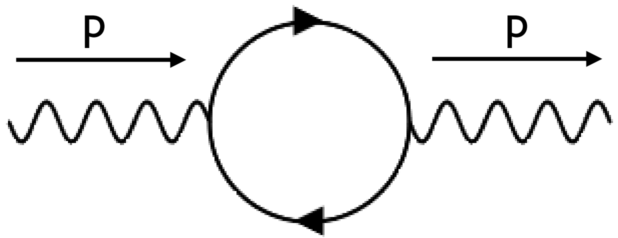
INTEGRATION BY PARTS REDUCTION



$$I(a_1, a_2) = \int d^D k \frac{1}{(k^2 - m^2)^{a_1}\left((k+p)^2 - m^2\right)^{a_2}}$$

IBP relations :

$$(D - 2a_1 - a_2)I(a_1, a_2) - 2a_1 m^2 \boxed{I(a_1+1, a_2)} - a_2(2m^2 - p^2)\boxed{I(a_1, a_2+1)} - a_2 I(a_1 - 1, a_2 + 1) = 0$$

$$(a_1 - a_2)I(a_1, a_2) + a_1 p^2 \boxed{I(a_1+1, a_2)} - a_1 I(a_1+1, a_2-1) + a_2 I(a_1-1, a_2+1) - a_2 p^2 \boxed{I(a_1, a_2+1)} = 0$$

- Integrals with doubled propagators don't usually appear in amplitudes
- Significantly larger system to reduce

Avoiding doubled propagators :

- Generating vectors using Groebner basis : J. Kluza, K. Kajda & D. Kosower (arxiv:1009.0472)
- Linear algebra based approach : R.Schabinger (arxiv:1111.4220)
- Differential geometry: Y. Zhang (arxiv:1408.4004)

INTEGRATION BY PARTS REDUCTION

- General scalar Feynman integral in Baikov representation (arxiv: hep-ph/9603267) with **L-loops** and **N-edges** :

$$\mathrm{I}(\mathrm{a}_1..\mathrm{a}_\mathrm{N}) = CU^{(D-L-E-1)/2}\int dz_1..dz_N \frac{1}{\prod_{i=1}^{N} {z_i}^{a_i}} P^{(D-L-E-1)/2}$$

z_i : *Baikov parameters*
P : *Baikov polynomial (depends on z_i in general)*
a_i : *Exponent of the propagator for the edge i*
C : *Constant from integrating over the solid angles*
U : *From the jacobian of transformation*

INTEGRATION BY PARTS REDUCTION

- IBPs in Baikov representation :

$$0 = \int dz_1..dz_N \sum_{i=1}^{N} \frac{\partial}{\partial z_i}\left(f_i(z_1,..,z_N)\, P^{(D-L-E-1)/2} \frac{1}{z_1^{a_1}..z_N^{a_N}} \right)$$

$$0 = \int dz_1..dz_N \sum_{i=1}^{N} \left(\frac{\partial f_i}{\partial z_i} + \boxed{\frac{D-L-E-1}{2P} f_i \frac{\partial P}{\partial z_i}} - \boxed{\frac{a_i f_i}{z_i}} \right) P^{(D-L-E-1)/2}$$

Dimension shifting term　Dots (doubled propagators)

- Impose following constraints :
  - No dimension shift –
  - No 'Doubled' propagators –

$$\sum_{i=1}^{N} f_i \frac{\partial P}{\partial z_i} + g\, P = 0$$

$$f_i \sim z_i$$

'Syzygy' constraints

SYZYGIES

$$\sum_{i=1}^{N} f_i \frac{\partial P}{\partial z_i} + g\, P = 0$$

- Explicit solutions known, pointed out by J. Boehm, A. Georgoudis, K. J. Larsen, H. Schoenemann, Y. Zhang (arxiv:1712.09737) in Baikov representation
- In momentum space representation : S. Abreu, F. Febres Cordero, H. Ita, B. Page and M. Zeng (arxiv:1712.03946) using SINGULAR
- Polynomials of degree 1 in z_i and kinematic invariants
- Very easy to construct

$$f_i \sim z_i$$

- $f_i's$ proportional to z_i to avoid doubled propagators

How to combines the two constraints?

- Original strategy : Use $f_i = b_i z_i$ and substitute in the no dimension shift syzygy; solve the syzygy explicitly : K. Larsen & Y. Zhang (arxiv:1511.01071)
- Use Groebner bases methods to find the intersection between the sets of polynomials satisfying these two constraints : J. Boehm, A. Georgoudis, K. J. Larsen, H. Schoenemann & Y. Zhang (arxiv:1805.01873)
- Our method : Use explicit solutions for the no dimension shift syzygy to construct solutions also satisfying $f_i \sim z_i$

SYZYGIES

Singular   https://www.singular.uni-kl.de/

- State-of-the-Art Public code for computer algebra; lot more powerful than Mathematica for such purposes
- Can almost use out of the box
- Provides all solutions to the syzygies
- Slow :
  - 6-line sectors already extremely challenging
  - Solutions for 7-line sectors still unfeasible

New custom syzygy solver

- Custom implementation based on linear algebra to solve the syzygies
- Reduce the problem to row-reduction of a matrix - Use **Finred** for row-reduction
- Solutions only up to a requested 'degree' of polynomial
- Feasible for complicated topologies at high tensor ranks

$gg \to ZZ$ at 2-loops

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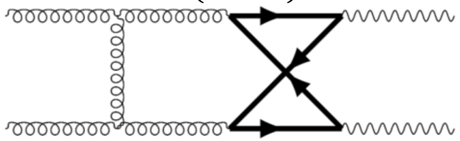
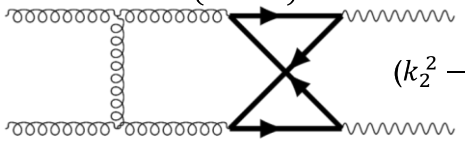
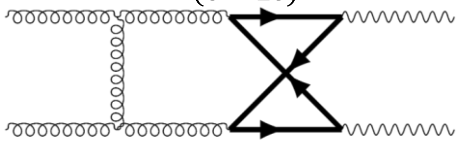
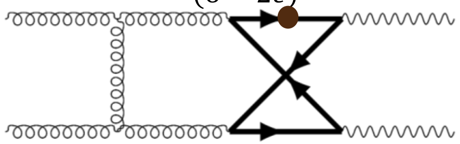
- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements

FINITE INTEGRALS

- Why use finite integrals?
  - Much better behaved numerically
  - Pole structure of the amplitude explicit
  - Often require fewer orders in epsilon

- How to get finite integrals?
  - Existence of a finite basis : A. von Manteuffel, E. Panzer & R. Schabinger arxiv:1411.7392
  - Reduze can generate finite integrals for any sector
  - Usually involves **dots** and **dimension shifts**
  - Finite integrals using dimension shifts first pointed out in arxiv:hep-ph/9212237

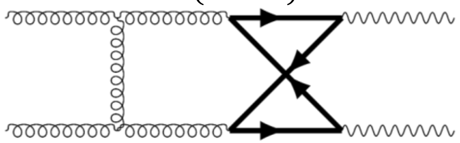
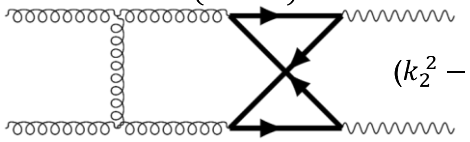
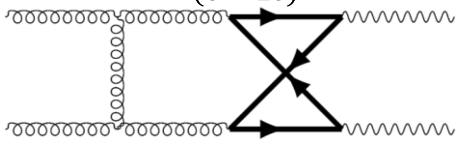
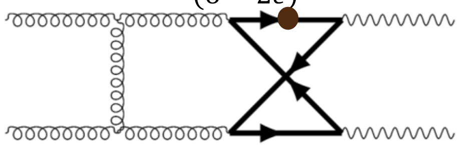
Integral	Rel.Err. (ϵ^0)	Timing(s)
$(4-2\epsilon)$ 	~1	123
$(4-2\epsilon)$  $(k_2^2 - m_t^2)$	~5*10^-1	272
$(6-2\epsilon)$ 	~8*10^-4	81
$(6-2\epsilon)$ 	~2*10^-3	135

Numbers generated using pySecDec

FINITE INTEGRALS

- Current prescription for finite integrals not enough
  - Not fast enough convergence
  - Reductions to such integrals very hard often e.g. integrals with up to **4 dots** required for computing the reductions to dimension shifted integrals

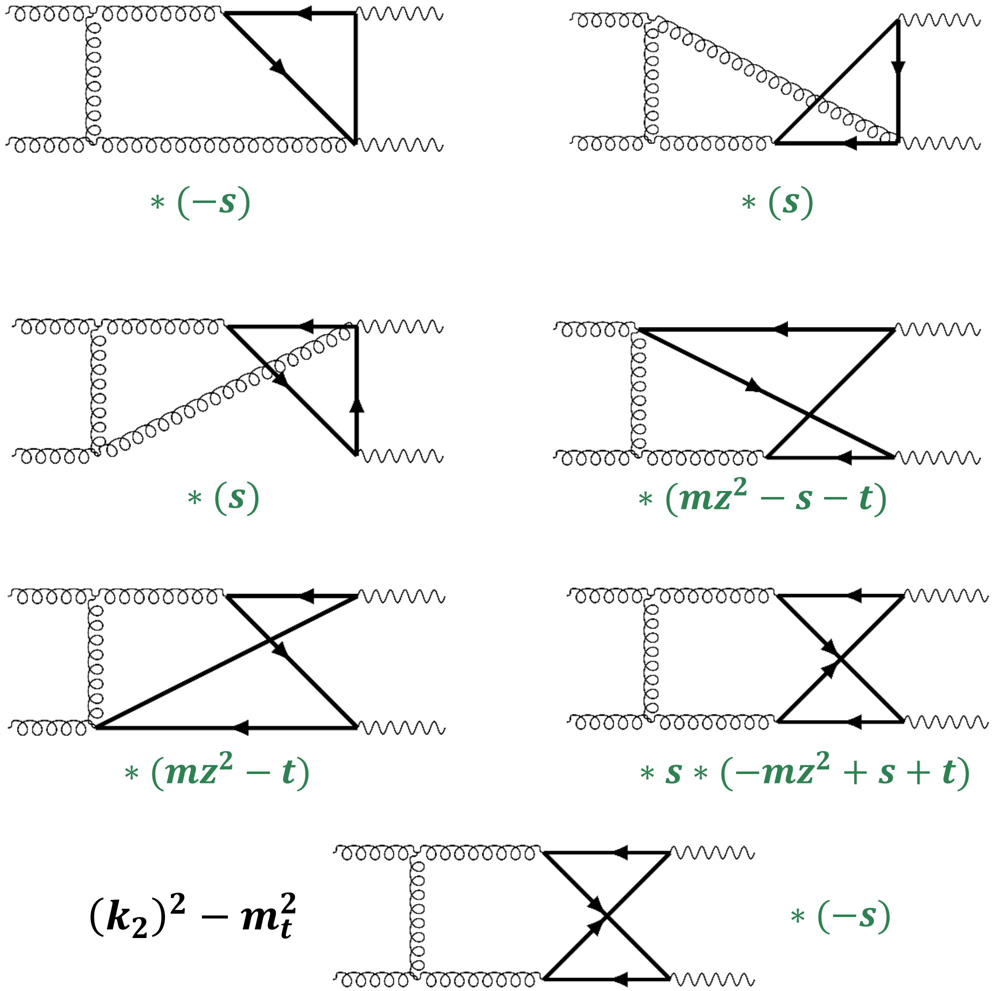
- Instead, use **linear combinations of divergent integrals** to produce finite integrals

Integral	Rel.Err. (ϵ^0)	Timing(s)
$(4-2\epsilon)$ 	~1	123
$(4-2\epsilon)$  $(k_2^2 - m_t^2)$	~5*10^-1	272
$(6-2\epsilon)$ 	~8*10^-4	81
$(6-2\epsilon)$ 	~2*10^-3	135

Numbers generated using pySecDec

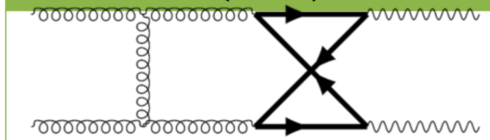
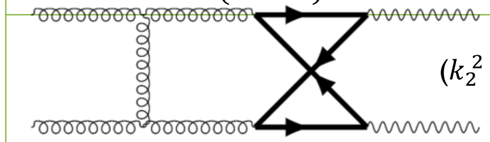
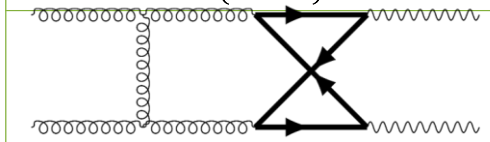
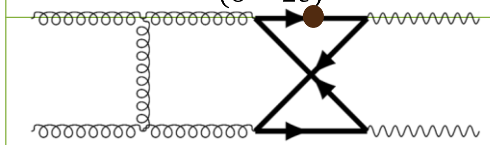
FINITE INTEGRALS

- Instead, use **linear combinations of divergent integrals** to produce finite integrals
- Usually involve integrals with numerators, and subsector integrals
- Very successful with higher-line topologies; linear combinations involving even tensor rank 3 integrals



FINITE INTEGRALS

- Advantages:
  - Can write a custom integrator to evaluate such integrals much faster than available public codes : Initial tests suggest huge potential
  - Use integrals already appearing in the amplitude, often even as master integrals
  - Avoid computing reductions beyond those required for the amplitude
- In practice, need a mixture with conventional finite integrals (with dots and dimension shifts), especially for lower sectors

Integral	Rel.Err. (ϵ^0)	Timing(s)
$(4-2\epsilon)$ 	~1	123
$(4-2\epsilon)$  $(k_2^2 - m_t^2)$	~5*10^-1	272
$(6-2\epsilon)$ 	~8*10^-4	81
$(6-2\epsilon)$ 	~2*10^-3	135
Linear combination	~2*10^-3	125

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^{\mu}\, p_j^{\nu}\, p_k^{\rho}\, p_l^{\lambda}\, A_{ijkl} + ..$$

- Solve linear system of equations to relate the 'form factors' to the original Feynman integral

- Use **Integration By Parts** identities to reduce the number of integrals to a basis set

- Rotate the basis integrals to a set of **finite integrals** ⇒ Much better behaved numerically

- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements)

CONCLUSIONS

- Higher order calculations ever more important; need precision in theoretical predictions to match LHC data
- Great progress in the field of multiloop calculations
- Method of syzygies to construct smaller IBP systems very powerful
  - Can construct syzygies of other types, depending on the requirement
- Reductions for the $gg \to ZZ$ (on-shell) amplitude to master integrals available
- Reductions for off-shell Z-bosons still extremely challenging
- Exciting new method to construct finite integrals