

# Using a nonlocal dispersive-optical-model to generate ingredients for $\nu$ -A cross sections

Mack C. Atkinson

Washington University in St. Louis

Fermilab 2019

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- The structure of large nuclei can be determined using a nonlocal dispersive optical model (DOM)
- Experimental data is used to constrain the DOM
- The  $(e, e' p)$  reaction can be described using the DOM
- This can be extended to different leptonic probes
- In particular, a DOM analysis of  $^{40}\text{Ar}$  (relevant for DUNE) is underway

# Single-Particle Propagator and the Dyson Equation

$$G_{\ell j}(r, r'; E) = \sum_m \frac{\langle \Psi_0^A | a_{r\ell j} | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_{r'\ell j}^\dagger | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} \\ + \sum_n \frac{\langle \Psi_0^A | a_{r'\ell j}^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_{r\ell j} | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta}$$

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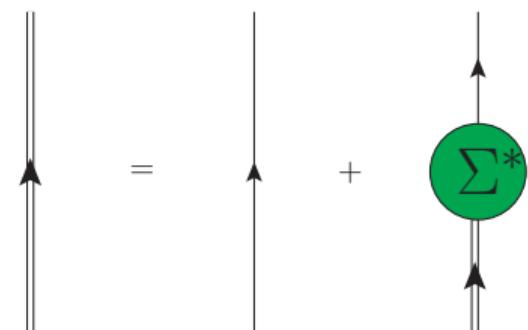
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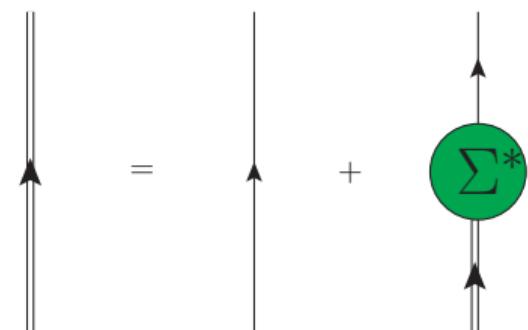
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- Perturbation expansion of  $G$  leads to the Dyson equation
- If the irreducible self-energy ( $\Sigma^*$ ) is known, then so is  $G$



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$$\begin{aligned} Re\Sigma_{\ell j}(r, r'; E) = & Re\Sigma_{\ell j}(r, r'; \epsilon_F) - \frac{1}{\pi}(\epsilon_F - E)\mathcal{P} \int_{\epsilon_T^+}^{\infty} dE' Im\Sigma_{\ell j}(r, r'; E') \left[ \frac{1}{E - E'} - \frac{1}{\epsilon_F - E'} \right] \\ & + \frac{1}{\pi}(\epsilon_F - E)\mathcal{P} \int_{-\infty}^{\epsilon_T^-} dE' Im\Sigma_{\ell j}(r, r'; E') \left[ \frac{1}{E - E'} - \frac{1}{\epsilon_F - E'} \right] \end{aligned}$$

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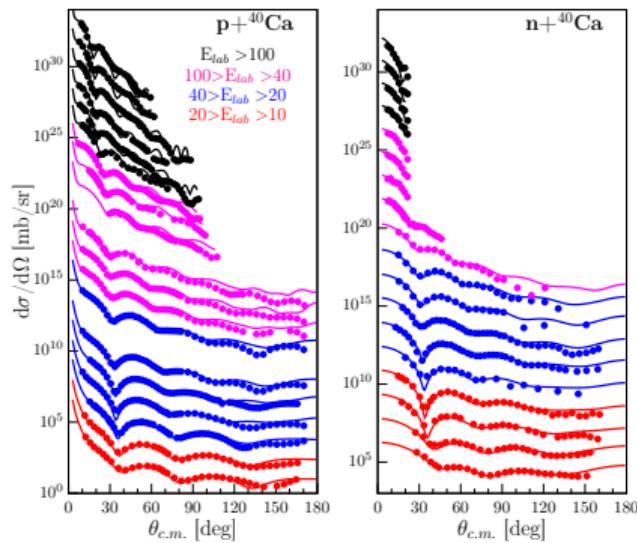
- This constraint ensures bound and scattering quantities are simultaneously described

## Fitting the Self-energy ( $^{40}\text{Ca}$ )

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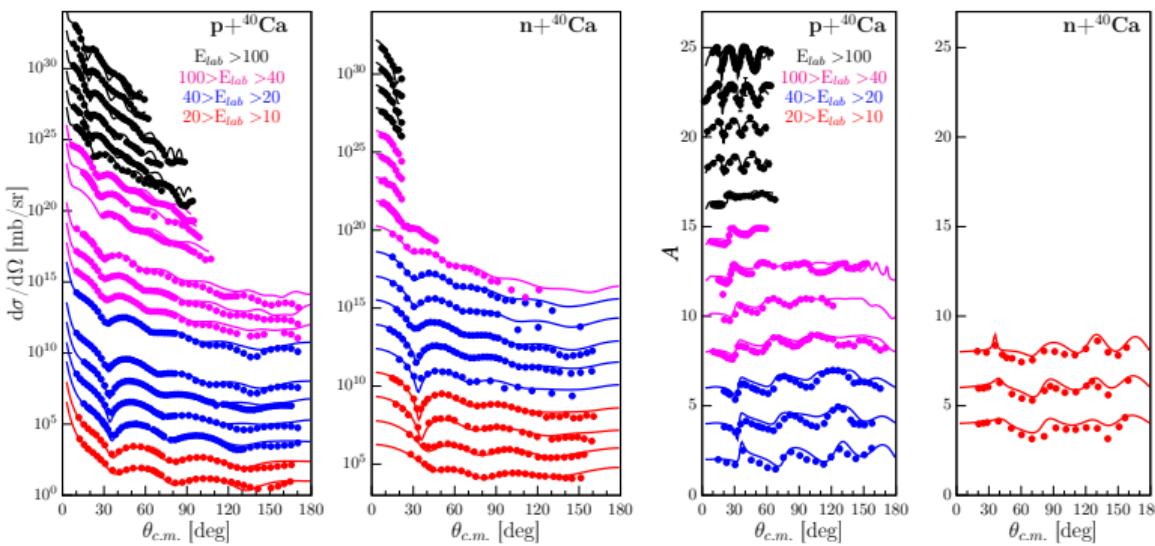
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Data: J.M. Mueller et al. *Phys. Rev. C*, **83** 064605, 2011

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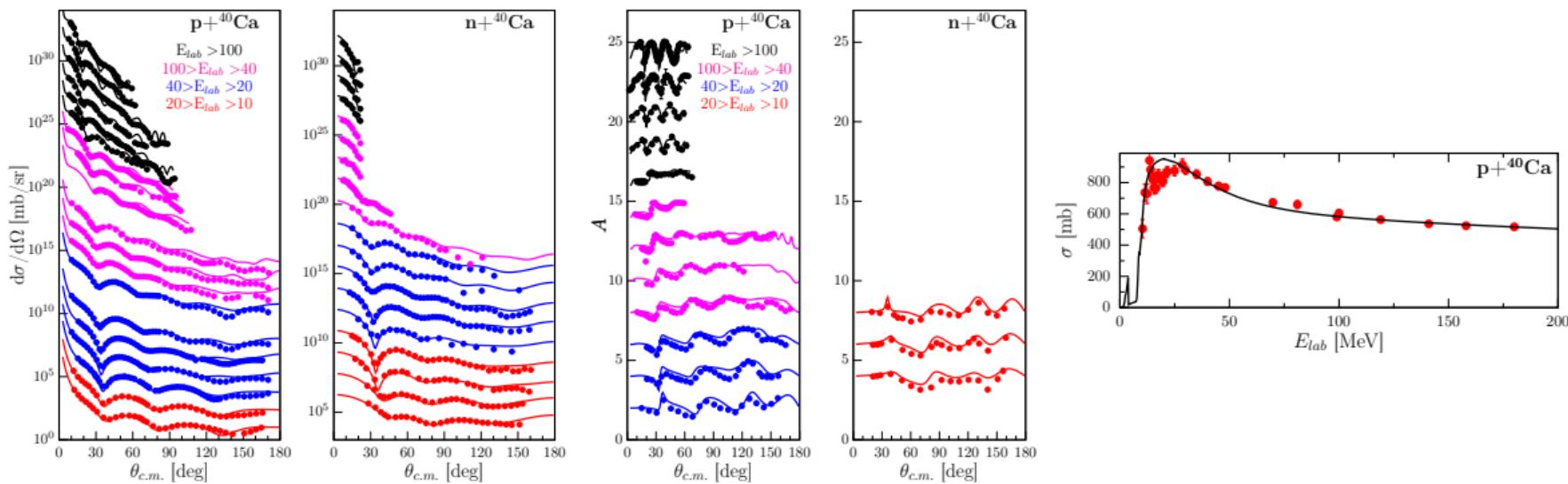
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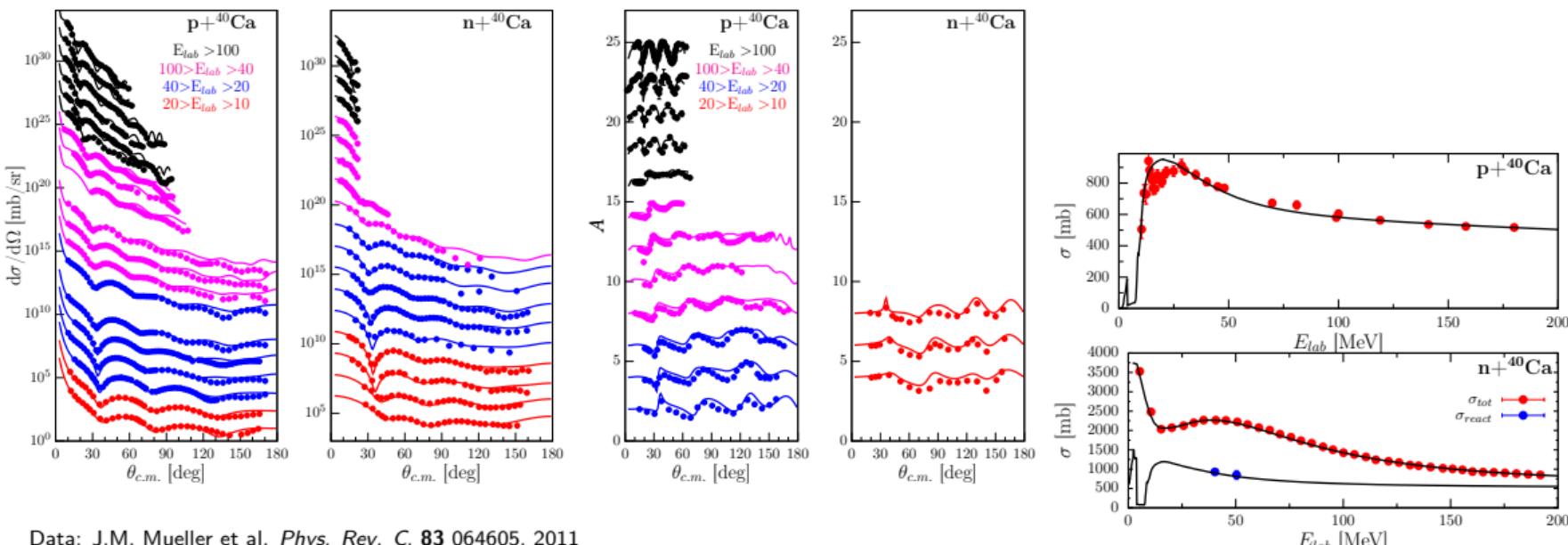
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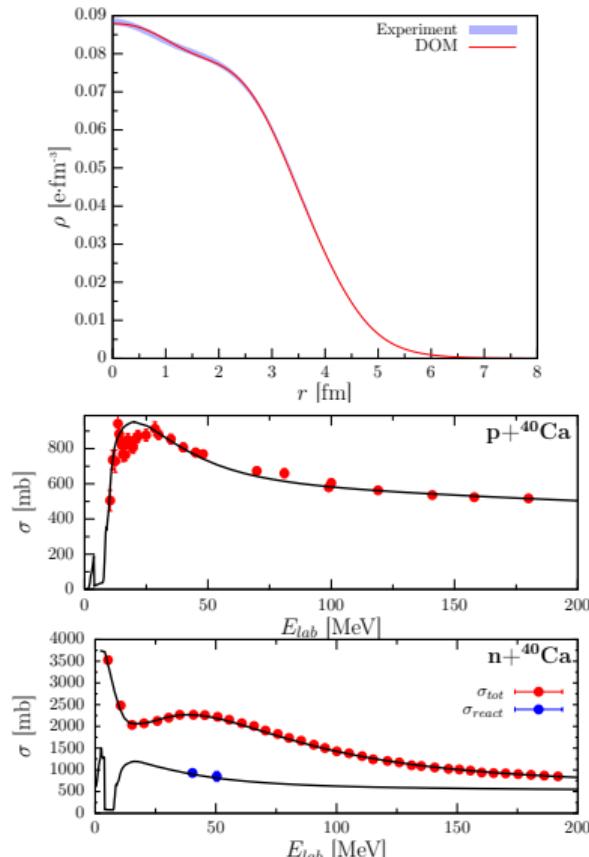
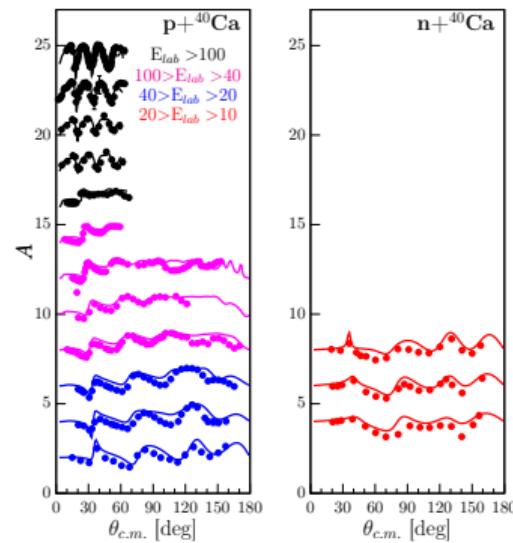
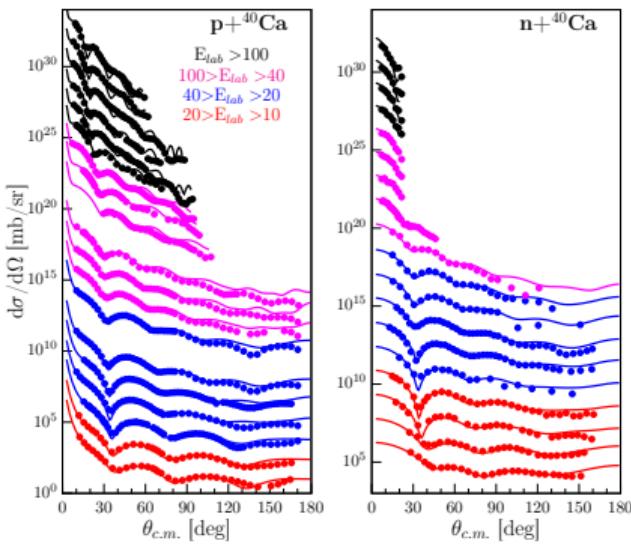
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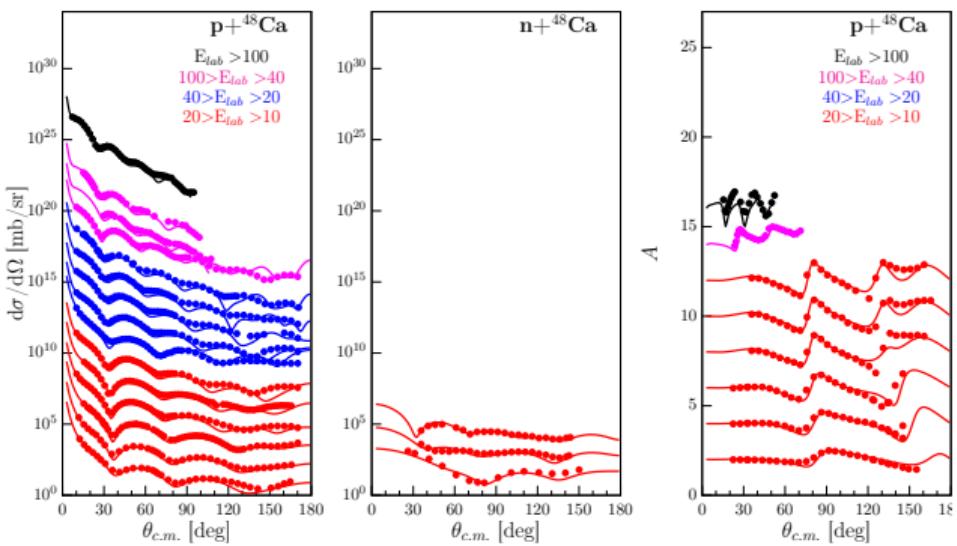
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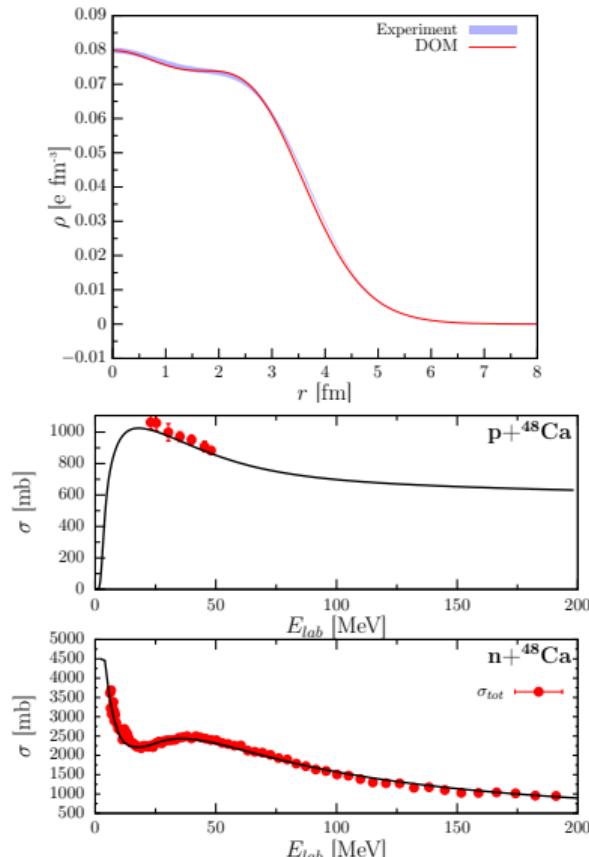
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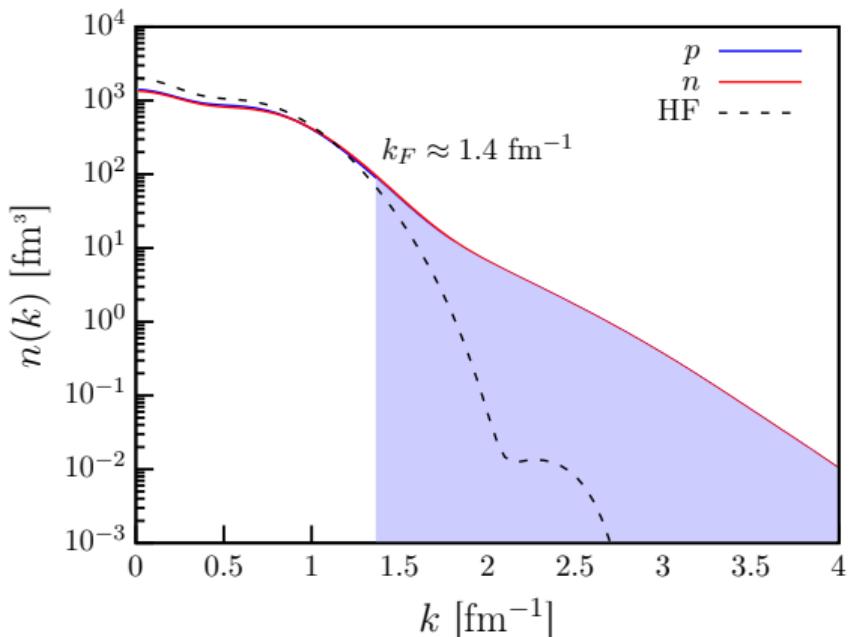
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# Momentum Distributions

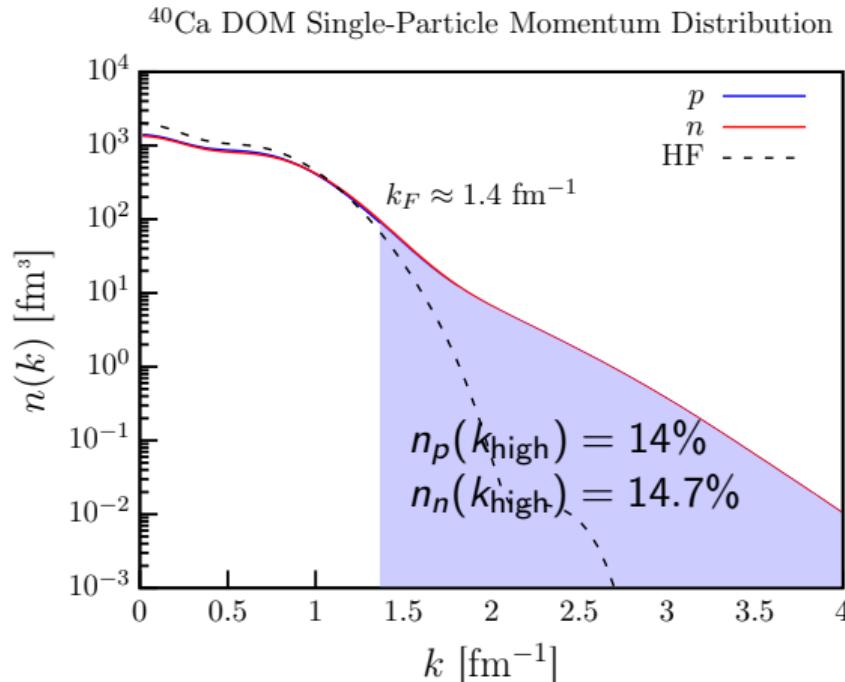
$$n(\mathbf{k}) = \int d^3r \int d^3r' e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \rho(\mathbf{r}, \mathbf{r}')$$

$^{40}\text{Ca}$  DOM Single-Particle Momentum Distribution



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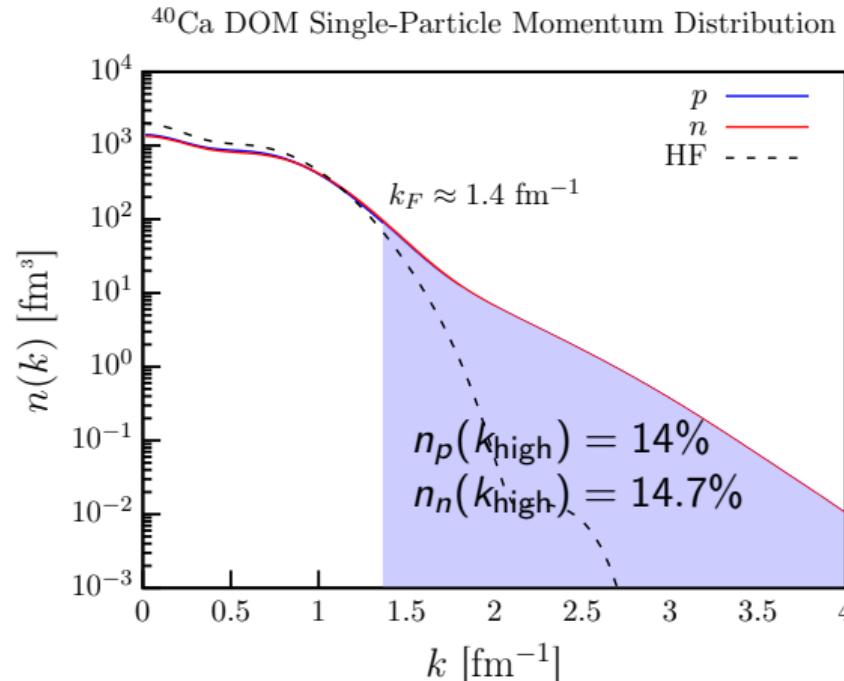
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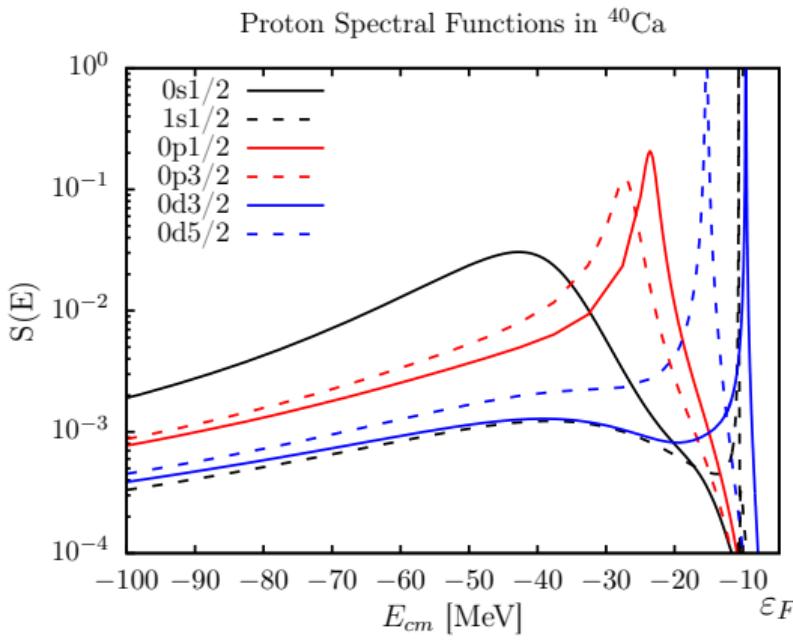
- Short-range correlations (SRC) responsible for this high-momentum content

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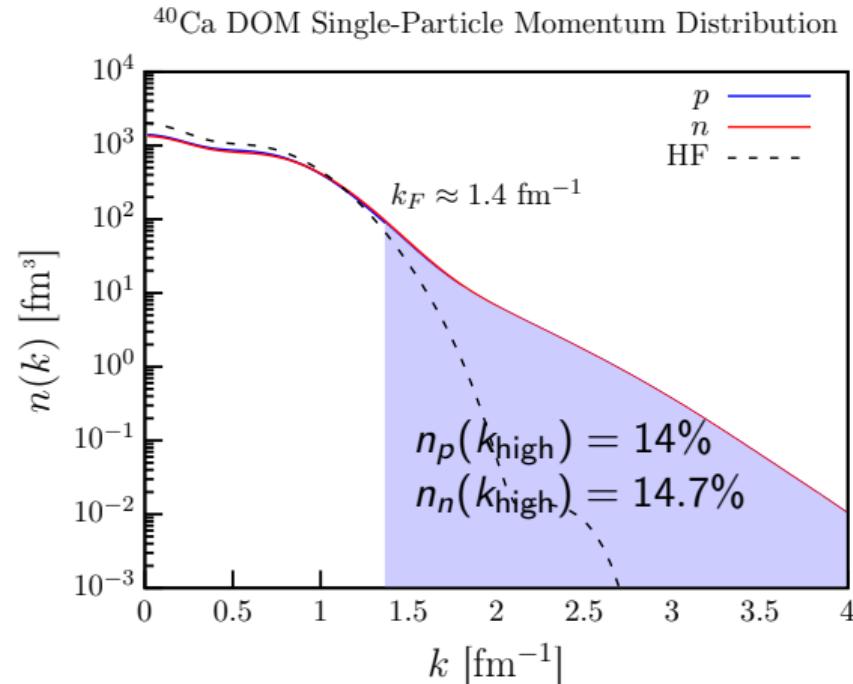


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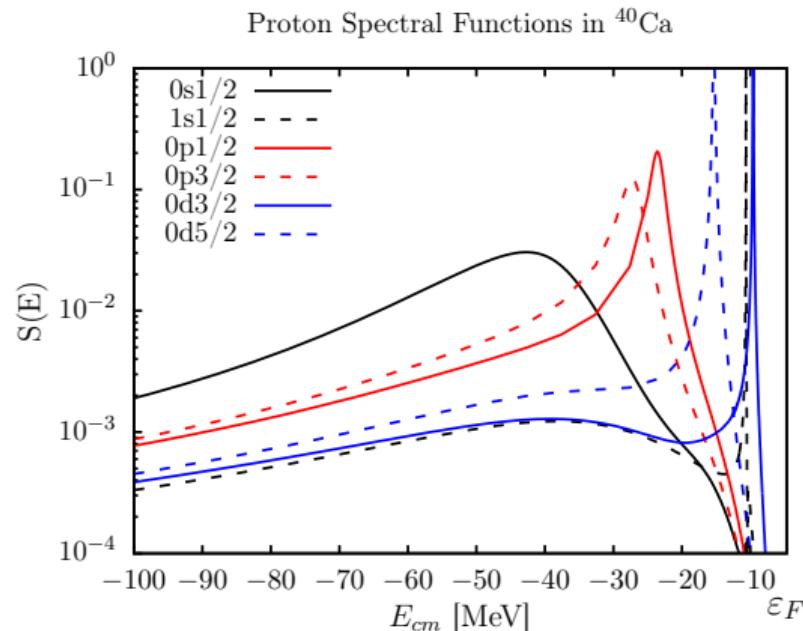


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# Constraints for High-Momentum Content

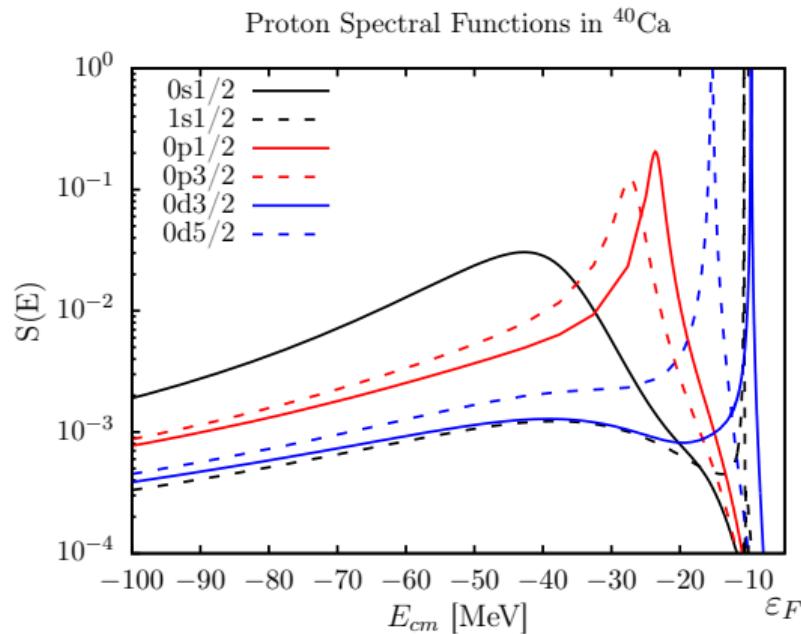
$$S^h(\alpha, \beta; E) = \frac{1}{\pi} \text{Im}\{G(\alpha, \beta; E)\} \quad S^h(E) = \sum_{\alpha} S(\alpha, \alpha; E)$$



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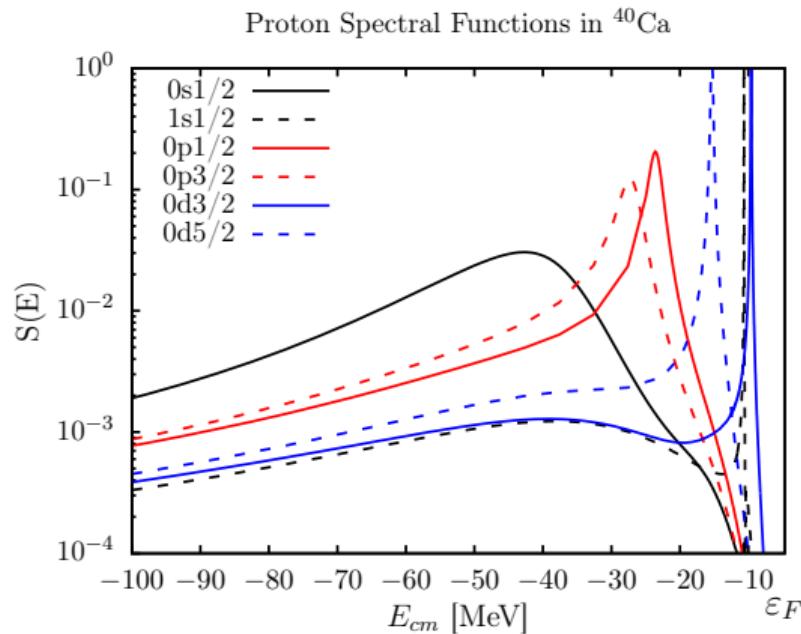
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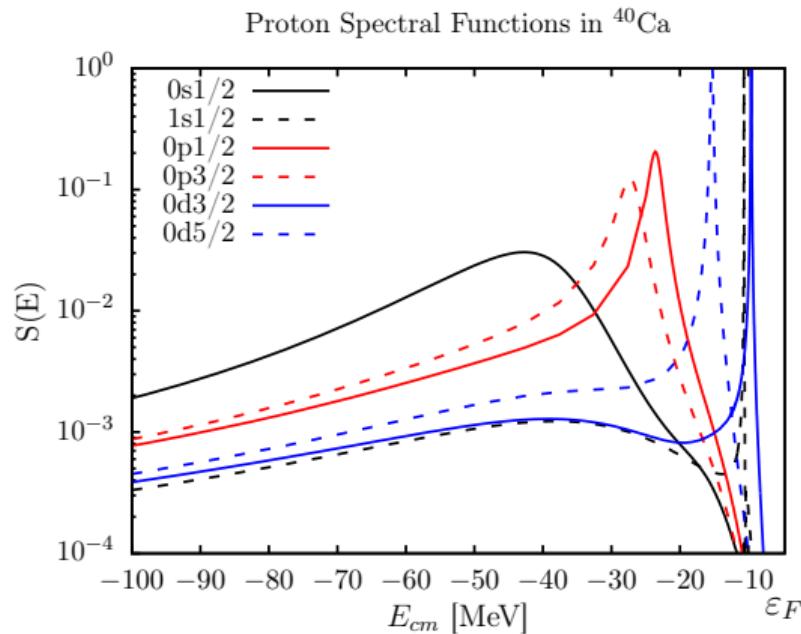


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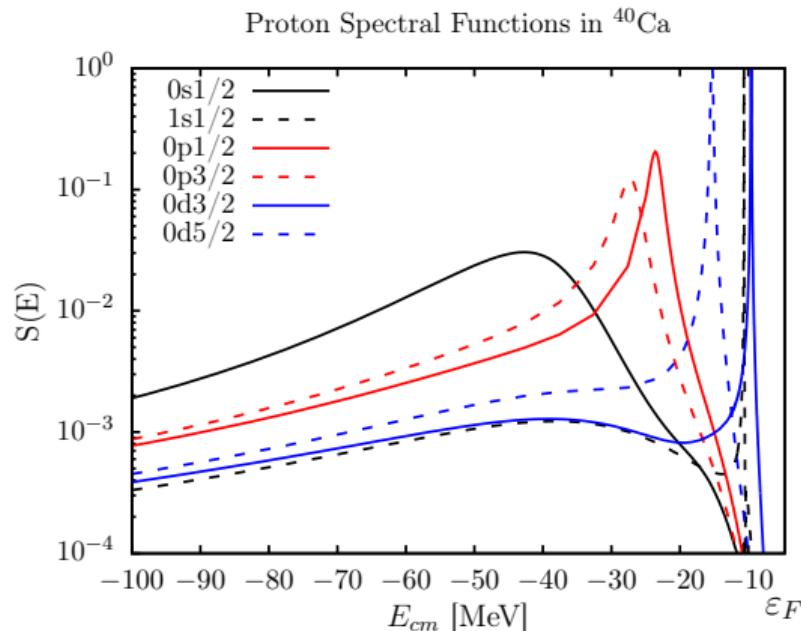
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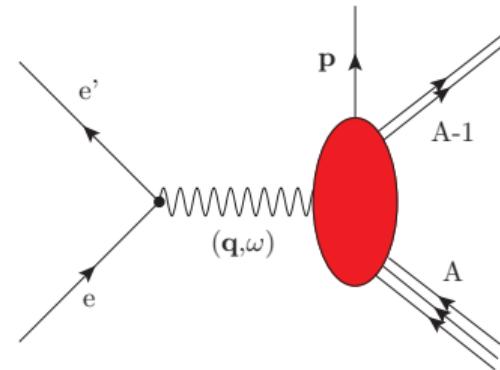
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	N	Z	DOM $E_0^A/A$	Exp. $E_0^A/A$
$^{40}\text{Ca}$	19.9	19.8	-8.49	-8.55
$^{48}\text{Ca}$	27.9	19.9	-8.7	-8.66

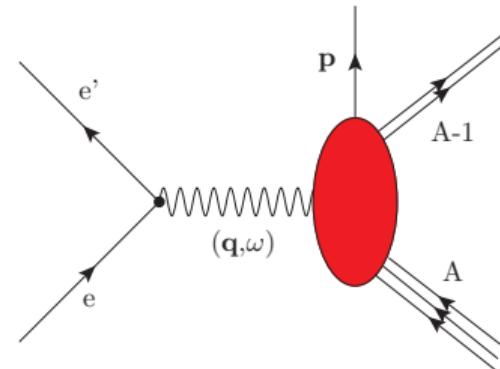


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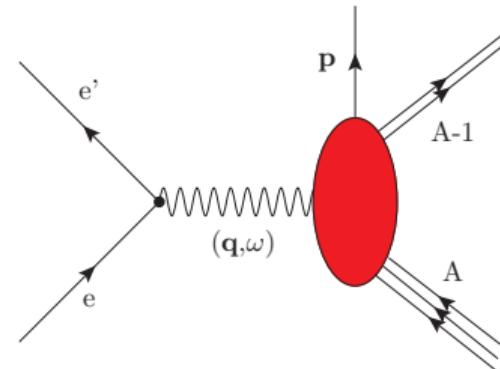
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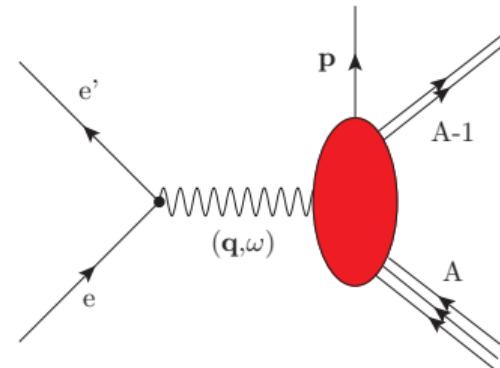


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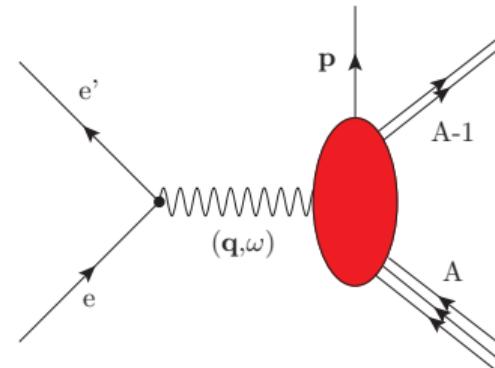


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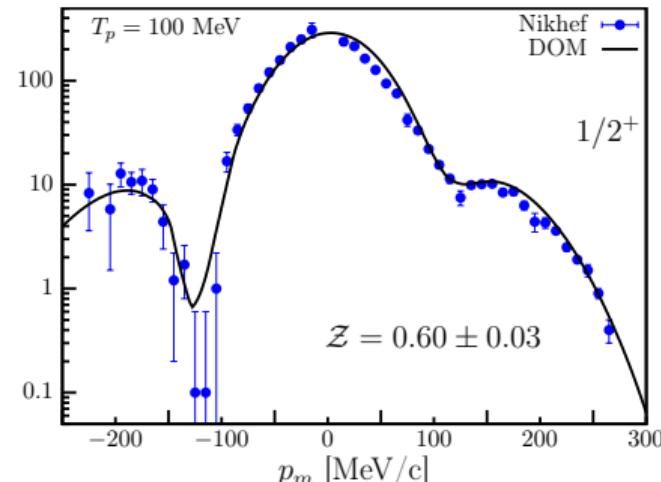
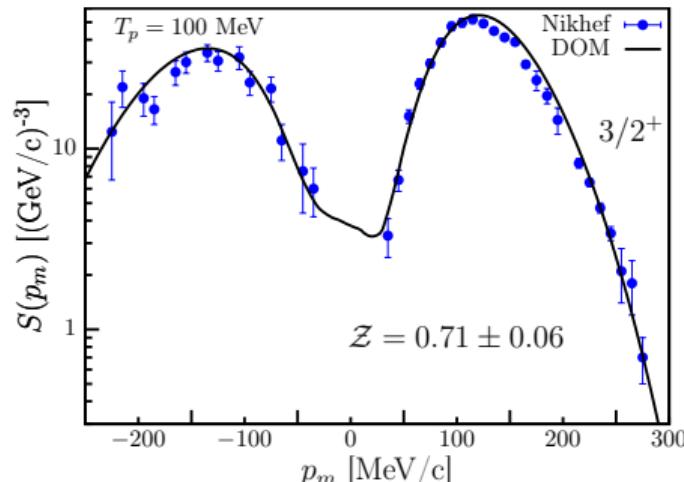
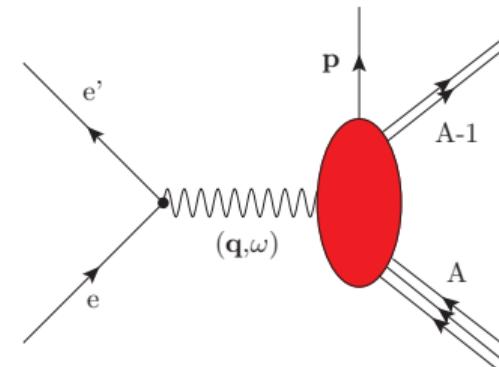


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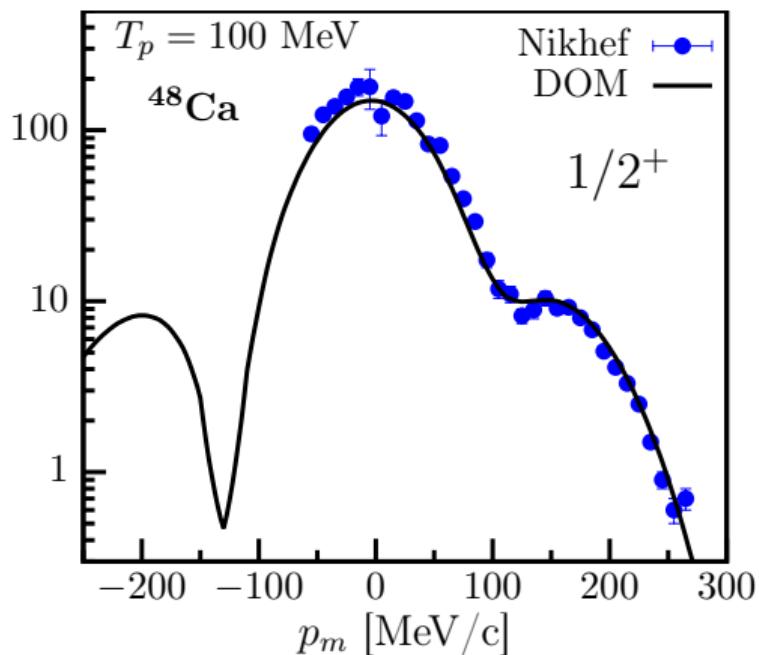
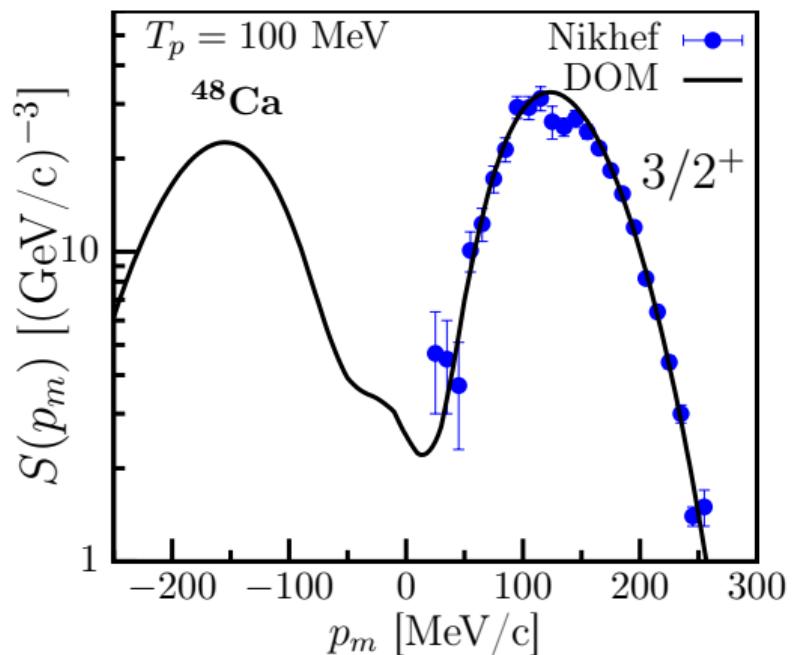
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# $^{48}\text{Ca}(e,e'p)^{47}\text{K}$ Momentum Distribution



Data: G. J. Kramer *et. al.*, Nucl. Phys. A, **679**, 267 (2001)

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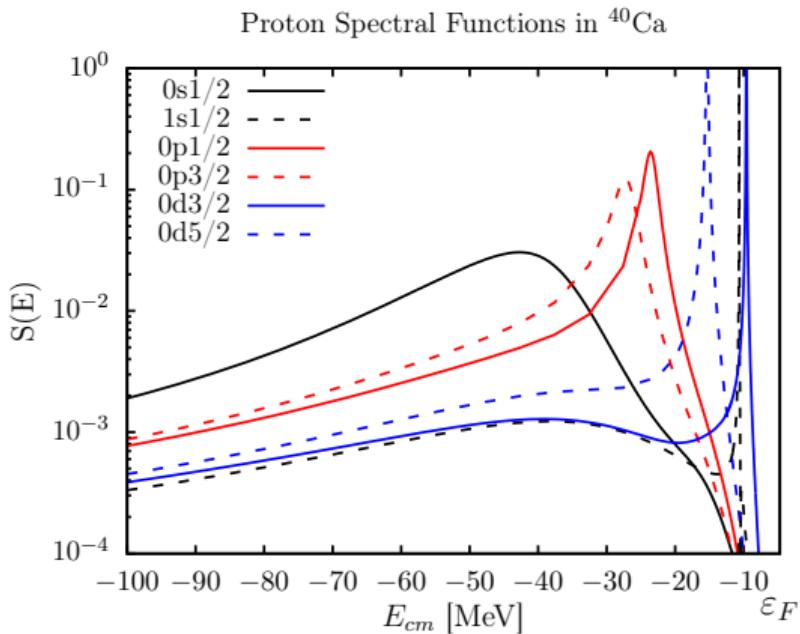
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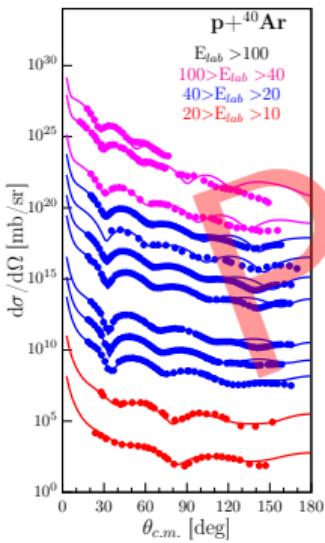
$$W^{\mu\nu} = \sum_f \langle \Psi_0 | \hat{J}^{\mu\dagger} | f \rangle \langle f | \hat{J}^\nu | \Psi_0 \rangle$$

- $W^{\mu\nu}$  contains  $S(\mathbf{p}, \mathbf{p}'; E)$

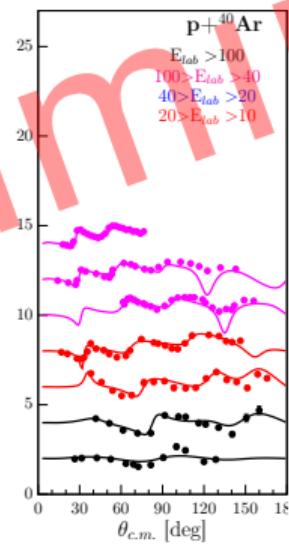
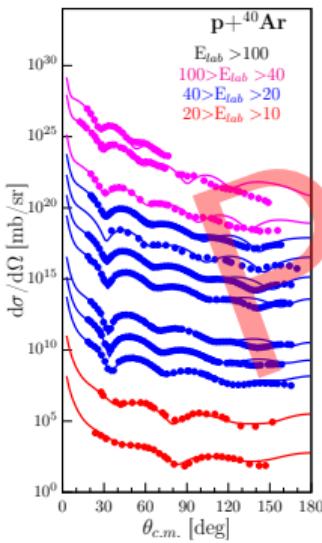


# Preliminary Fit of $^{40}\text{Ar}$

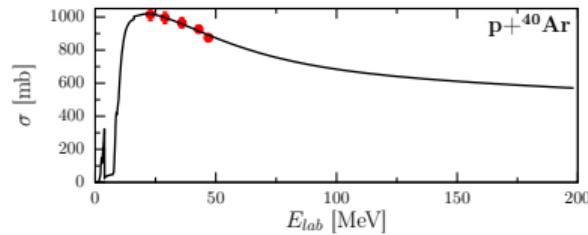
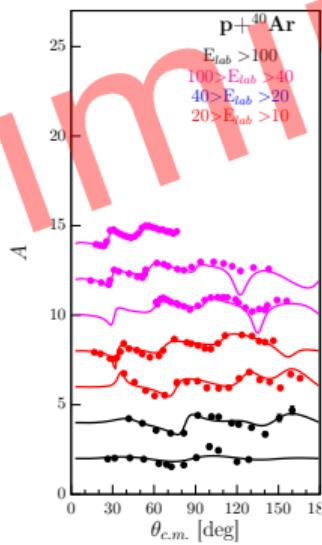
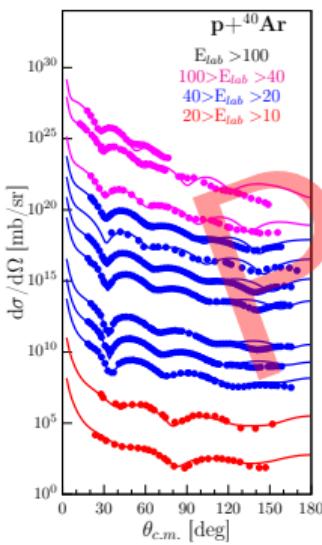
preliminary



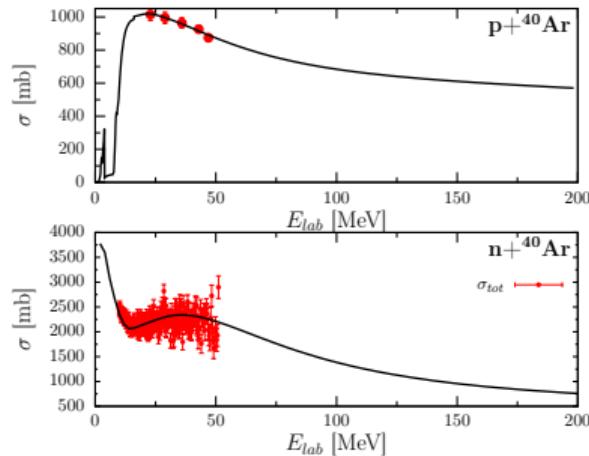
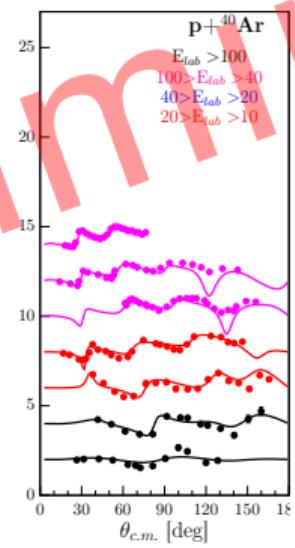
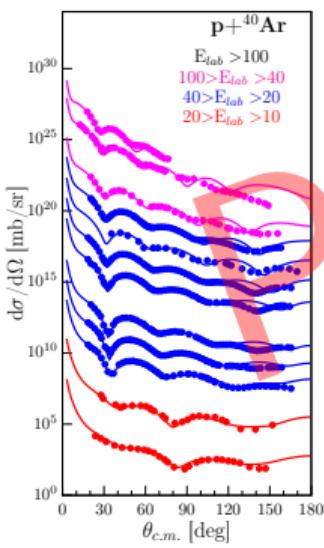
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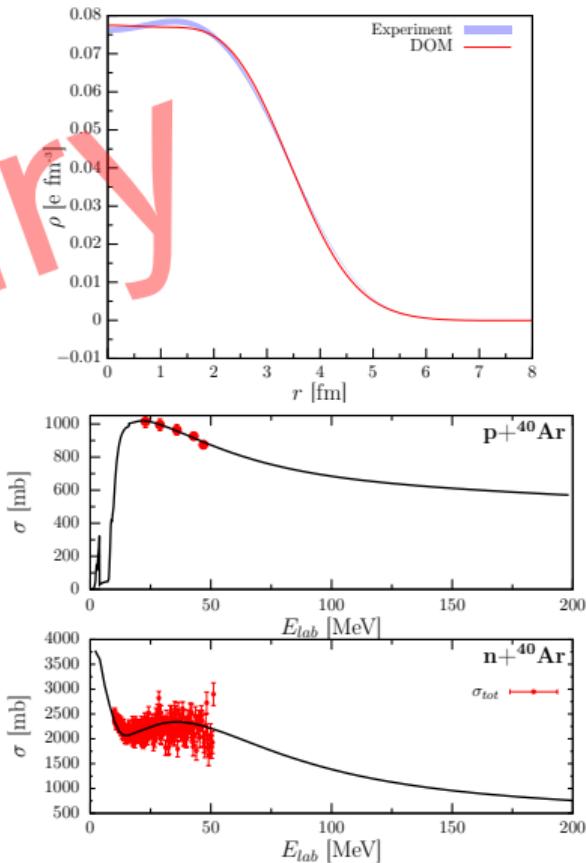
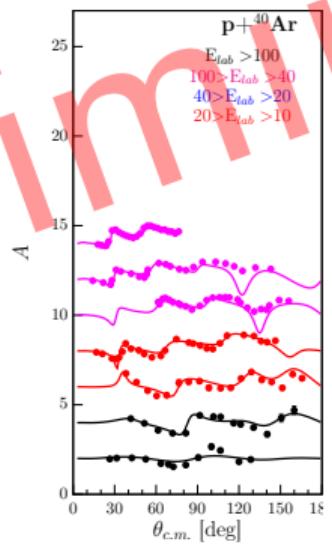
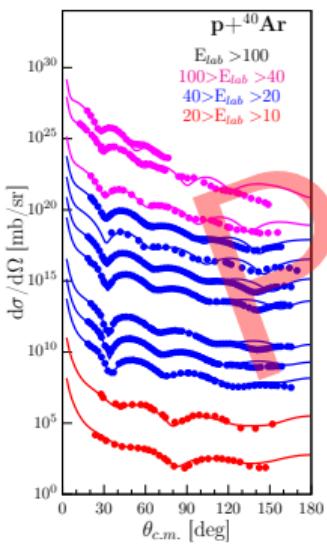
# Preliminary Fit of $^{40}\text{Ar}$



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## Conclusions and Outlook

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- DOM analysis of  $^{40}\text{Ar}$  is underway

# Thanks

- Willem Dickhoff - Advisor
- Robert Charity - DOM and data for DOM
- Henk Blok -  $(e, e'p)$  data at Nikhef
- Louk Lapikás -  $(e, e'p)$  data at Nikhef
- Carlotta Giusti - DWEEPY Code
- Hossein Mahzoon - DOM
- Lee Sobotka - Data for DOM



# Spectroscopic factor, Occupation, and Depletion

- No imaginary component of  $\Sigma^*$  around  $\epsilon_F$

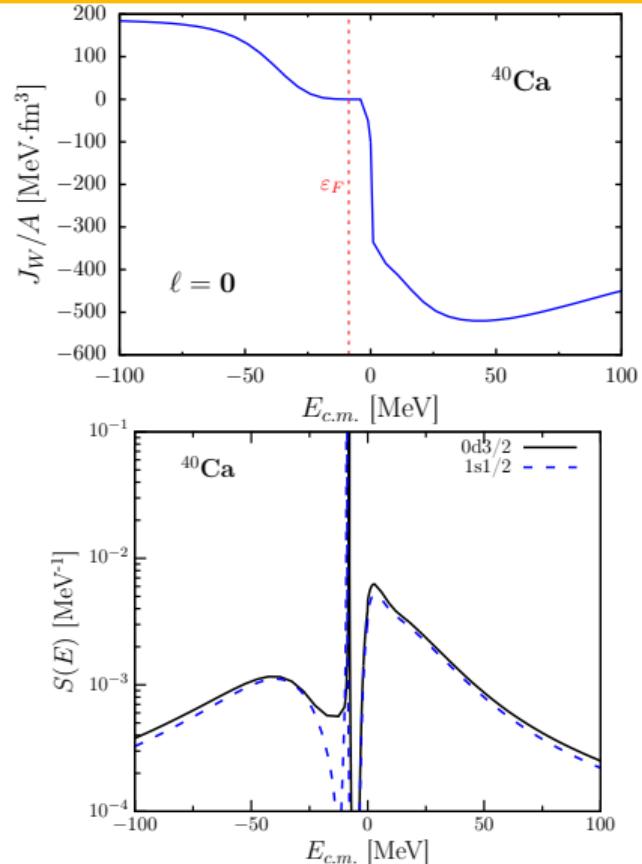
$$J_W^\ell(E) = (4\pi)^2 \int_0^\infty dr r^2 \int_0^\infty dr' r'^2 \text{Im}\{\Sigma_\ell^*(r, r'; E)\}$$

- Spectroscopic factor for states near  $\epsilon_F$  is well defined from  $\Sigma^*$

$$\mathcal{Z} = \left( 1 - \frac{\partial \Sigma^*(\alpha_{qh}, \alpha_{qh}; E)}{\partial E} \Big|_{\epsilon} \right)^{-1}$$

$$n_{n\ell j} = \int_{-\infty}^{\epsilon_f} dE S_{n\ell j}^h(E) \quad d_{n\ell j} = \int_{\epsilon_f}^\infty dE S_{n\ell j}^p(E)$$

Orbital	$\mathcal{Z}$	$n_{n\ell j}$	$d_{n\ell j}$
$0d\frac{3}{2}$	0.71	0.80	0.17
$1s\frac{1}{2}$	0.60	0.82	0.15



# Backup

- “Smearing” of self-energy poles inflates  $\mathcal{Z}$
- Renormalize with experimental excitation energy spectrum

$$\frac{\mathcal{Z}_F^{\text{DOM}}}{\int dE S^{\text{DOM}}(E)} = \frac{\mathcal{Z}_F^{\text{exp}}}{\int dE S^{\text{exp}}(E)}$$

