Parton Showers and Final State Interactions

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Nuclear and Particle Theory for Accelerator and Neutrino Experiments

9-11 May 2019

Outline

- Parton Shower Basics and Similarities to Final State Interactions
- Advancements in Parton Shower accuracy
- Subleading Color
- High Order Corrections

Pieces of an Event

Factorization of an event:

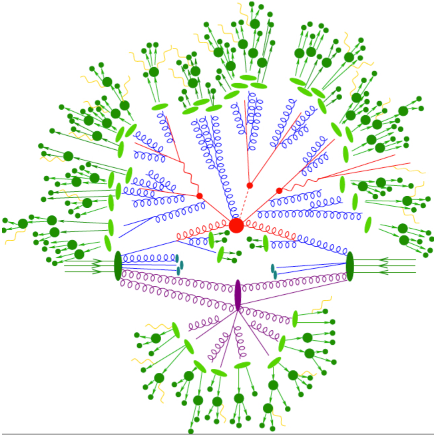


Image from the Sherpa Authors

Pieces of an Event

Factorization of an event:

- Hard Process (Neutrino-nucleon interaction)

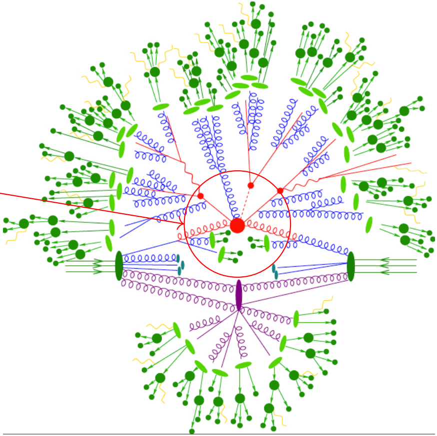


Image from the Sherpa Authors

Pieces of an Event

Factorization of an event:

- Hard Process (Neutrino-nucleon interaction)
- Parton Shower (Final State Interactions)

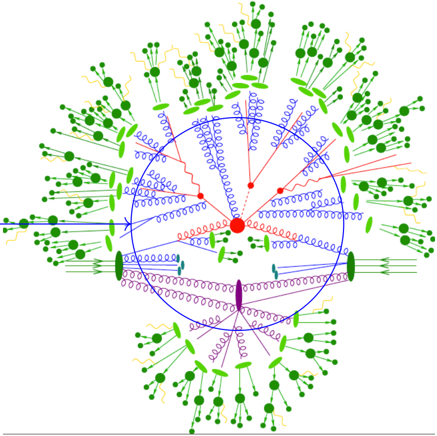


Image from the Sherpa Authors

Pieces of an Event

Factorization of an event:

- Hard Process (Neutrino-nucleon interaction)
- Parton Shower (Final State Interactions)
- Multiple Parton Interations (Final State Interactions)

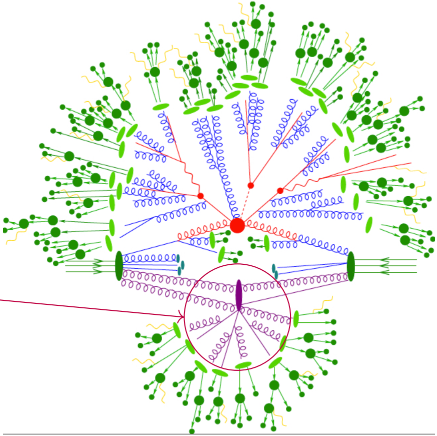


Image from the Sherpa Authors

Pieces of an Event

Factorization of an event:

- Hard Process (Neutrino-nucleon interaction)
- Parton Shower (Final State Interactions)
- Multiple Parton Interations (Final State Interactions)
- Hadronization (N/A)

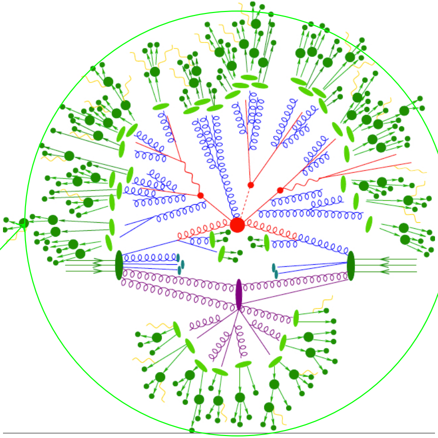


Image from the Sherpa Authors

Parton Shower Basics

Evolution Equation

$$\frac{df_a(x,t)}{d\ln t} = \sum_{b=q,g} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \left[P_{ab}(z)\right]_+ f_b\left(\frac{x}{z}, t\right)$$

- $f_a(x,t)$ is the obervable being evolved
- $P_{ab}(z)$ is the evolution kernel
- Solve using Markovian Monte-Carlo algorithms
- Treat P_{ab} as a probability
- Virtual corrections defined at kinematic endpoints by $+$ prescription

Parallels to FSI

FSI Evolution (According to Alessandro)

$$\begin{aligned}\langle p|e^{iHt}|k\rangle &\to \int dp_1 \dots dp_N \langle p|p_N\rangle\langle p_N|e^{iH\Delta t}|p_{N-1}\rangle \dots \langle p_1|e^{iH\Delta t}|k\rangle \\ &\to \int dp_1 \dots dp_N |\langle p|p_N\rangle|^2|\langle p_N|e^{iH\Delta t}|p_{N-1}\rangle|^2 \dots |\langle p_1|e^{iH\Delta t}|k\rangle|^2\end{aligned}$$

- Can recast into a similar form as a parton shower
- In both cases, currently both neglect interference effects
- Large effort to include these effects in parton showers, may help with ideas for cascade models

Coherent Branching

Consider the process: $e^+e^- \rightarrow q\bar{q}g$

$$d\sigma_3 = d\sigma_2 \frac{dw}{w} \frac{d\Omega}{2\pi} C_F W^g_{q\bar{q}}$$

$$W_{q\bar{q}} = \frac{1 - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})\,(1 - \cos\theta_{\bar{q}g})}$$

$$W^{(q)}_{q\bar{q}} = \frac{1}{2}\left(W_{q\bar{q}} + \frac{1}{1 - \cos\theta_{qg}} - \frac{1}{1 - \cos\theta_{\bar{q}g}}\right)$$

This leads to a radiating "color dipole":

$$\mathbf{W}_{ij} = -\mathbf{T}_i \cdot \mathbf{T}_j W_{ij}$$

For a three parton final state:

$$\mathbf{W}_{ijk} = \frac{1}{2}\left[\mathbf{T}_i^2\left(W_{ij} + W_{ik} - W_{jk}\right) + \mathbf{T}_j^2\left(W_{jk} + W_{ij} - W_{ik}\right) + \mathbf{T}_k^2\left(W_{ik} + W_{jk} - W_{ij}\right)\right]$$

When i and j are close, they form a combined system l:

$$\mathbf{W}_{ijk} \approx \mathbf{T}_i^2 W^{(i)}_{ij} + \mathbf{T}_j^2 W^{(j)}_{ij} + \mathbf{T}_k^2 W^{(k)}_{lk} + \mathbf{T}_l^2 W^{(l)}_{lk}\Theta(\theta_{lg} - \theta_{ij})$$

Dipole Showers

- Splitting Kernels: Catani-Seymour Dipole Kernels
- Evolution and Splitting Parameters: $k_\perp$ and momentum fraction (z)
- Construction of Kinematics: Recoil scheme, reverse of Catani-Seymour mappings

Catani-Seymour Dipole Kernels

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle\mathcal{M}_n| \frac{\mathbf{T}_{\tilde{k}}\cdot\mathbf{T}_{\tilde{ij}}}{\mathbf{T}^2_{\tilde{ij}}} \quad \mathcal{V}_{\tilde{ij},\tilde{k}} \quad |\mathcal{M}_n\rangle,$$

Catani-Seymour Dipole Kernels

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle\mathcal{M}_n| \boxed{\frac{\mathbf{T}_{\tilde{k}}\cdot\mathbf{T}_{\tilde{ij}}}{\mathbf{T}_{\tilde{ij}}^2}} \; \mathcal{V}_{\tilde{ij},\tilde{k}} \; |\mathcal{M}_n\rangle,$$

- Color factor

Catani-Seymour Dipole Kernels

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle\mathcal{M}_n| \frac{\mathbf{T}_{\tilde{k}}\cdot\mathbf{T}_{\tilde{ij}}}{\mathbf{T}^2_{\tilde{ij}}} \mathcal{V}_{\tilde{ij},\tilde{k}} |\mathcal{M}_n\rangle,$$

- Color factor
- Kinematic factor

Catani-Seymour Dipole Kernels

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle\mathcal{M}_n| \frac{\mathbf{T}_{\tilde{k}} \cdot \mathbf{T}_{\tilde{ij}}}{\mathbf{T}_{\tilde{ij}}^2} \mathcal{V}_{\tilde{ij},\tilde{k}} |\mathcal{M}_n\rangle,$$

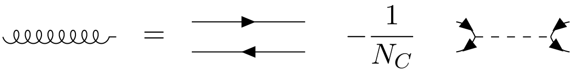
- Color factor
- Kinematic factor
- Conservation of Color:

$$\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}} \mathbf{T}_{\tilde{k}} \cdot \mathbf{T}_{\tilde{ij}} = -\mathbf{T}_{\tilde{ij}}^2$$

Color Flow

Gluon Propagator can be written as:

$$\langle (\mathcal{A}_\mu)^{i_1}_{j_1} (\mathcal{A}_\mu)^{i_2}_{j_2} \rangle \propto \delta^{i_1}_{j_2}\delta^{i_2}_{j_1} - \frac{1}{N_C}\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}.$$



Taking $N_C \to \infty$ limit reduces the Catani-Seymour Dipoles to:

$$|\mathcal{M}_{n+1}|^2 \approx \sum^{n}_{\tilde{ij},\tilde{k}\in \mathsf{LC}} \langle \mathcal{M}_n | \mathcal{V}_{\tilde{ij},\tilde{k}} | \mathcal{M}_n \rangle$$

Can capture dominate subleading color in parton showers by leaving $C_F = \frac{4}{3}$.

Algorithm: Overview

- **Goal**: Exponentiate Catani-Seymour exactly:
  $$-\sum_{k,ij}\langle\mathcal{M}'|\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}\mathcal{V}_{ij,k}|\mathcal{M}\rangle$$
- First Accept/Reject is traditional LC Shower $(P^{\mathrm{LC}}/P^{\mathrm{over}})$
- If below t_{FC}^{cut}, done
- Second Accept/Reject is Color factor accept reject:
  $$-\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}$$
- Randomly choose a specific color flow with probability:
  $$\frac{|t_k^\alpha\cdot t_{ij}^\beta|}{\sum_{\alpha,\beta}|t_k^\alpha\cdot t_{ij}^\beta|}$$
- Correct Accept/Reject with a weighting factor:
  $$\frac{t_k^\alpha\cdot t_{ij}^\beta}{|t_k^\alpha\cdot t_{ij}^\beta|}\frac{\sum_{\alpha,\beta}|t_k^\alpha\cdot t_{ij}^\beta|}{\sum_{\alpha,\beta}t_k^\alpha\cdot t_{ij}^\beta}$$

Algorithm: Accept-Reject

- Accept-Reject Probaility is given by: $\frac{P(t)}{G(t)}$, $P(t)$ is the function to exponentiate, and $G(t)$ is an "overestimate".
- Probability of one acceptance after n rejections is:

$$\mathcal{P}_1^{(n)}(t,t') = \frac{P(t)}{G(t)} G(t) \exp\left(-\int_t^{t_1} d\bar{t} G(\bar{t})\right) \\ \times \prod_{i=1}^{n} \left(\int_{t_{i-1}}^{t_{i+1}} dt_i \frac{G(t_i) - P(t_i)}{G(t_i)} G(t_i) \exp\left(-\int_{t_i}^{t_{i+1}} d\bar{t} G(\bar{t})\right)\right) \\ t_{n+1} = t'$$

- What if $P(t) < 0$?
- What if $\frac{P(t)}{G(t)} > 1$?

Algorithm: Accept-Reject

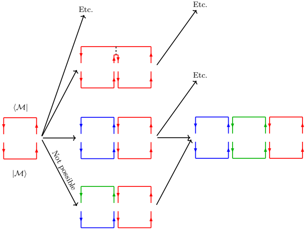
1. $P(t) < 0$:

$$\mathcal{P}_1^{(n)}(t,t') = \frac{P(t)}{G(t)} H(t) \exp\left(-\int_t^{t_1} d\bar{t} H(\bar{t})\right)$$
$$\times \prod_{i=1}^{n} \left(\int_{t_{i-1}}^{t_{i+1}} dt_i \frac{G(t_i) - P(t_i)}{G(t_i)} H(t_i) \exp\left(-\int_{t_i}^{t_{i+1}} d\bar{t} H(\bar{t})\right)\right)$$

   Apply event weight: $\dfrac{G(t)}{H(t)} \prod_{i=1}^{n} \dfrac{G(t_i)}{H(t_i)} \dfrac{H(t_i) - P(t_i)}{G(t_i) - P(t_i)}$

2. $\dfrac{P(t)}{G(t)} > 1$:
   Take $H(t) \to cP(t)$, where c is some arbitrary value greater than 1. Then apply the above equations.

Color Flow Sampling



- 36 possible configurations at each emission
- "color weight" for each configuration given by:

$$P_{\alpha\beta} = |\langle \mathcal{M}' | \mathbf{t}_k^{\alpha} \cdot \mathbf{t}_{ij}^{\beta} | \mathcal{M} \rangle|$$

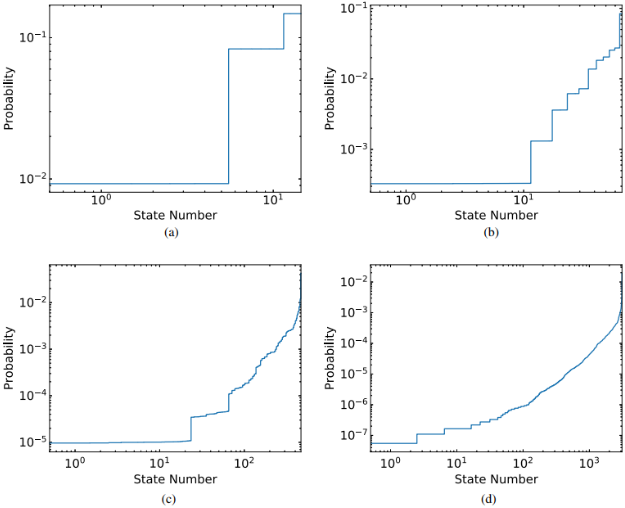
- Choose one color structure with probability:

$$P = \frac{P_{\alpha\beta}}{\sum_{\alpha,\beta} P_{\alpha\beta}}$$

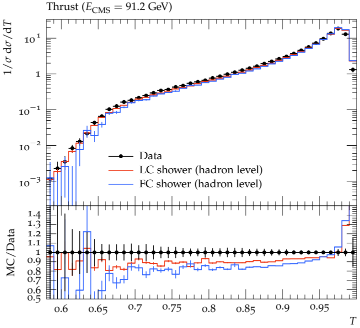
- Probabilities from top down: $\frac{1}{12}, \; \frac{1}{4}, \; 0$
- Correct overall sign:

$$\frac{g_{col}}{h_{col}} = \frac{\mathbf{t}_k^{\alpha} \cdot \mathbf{t}_{ij}^{\beta}}{|\mathbf{t}_k^{\alpha} \cdot \mathbf{t}_{ij}^{\beta}|} \frac{\sum_{\alpha,\beta} |\mathbf{t}_k^{\alpha} \cdot \mathbf{t}_{ij}^{\beta}|}{\sum_{\alpha,\beta} \mathbf{t}_k^{\alpha} \cdot \mathbf{t}_{ij}^{\beta}}.$$

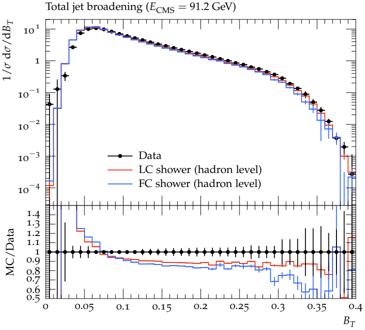
State Probabilities



Results:



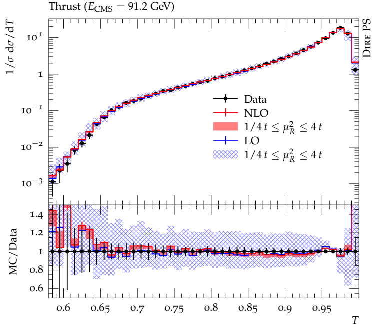
Thrust as measured by ALEPH



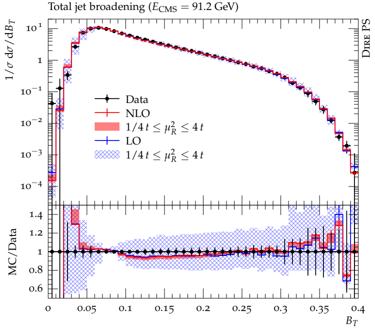
Total jet broadening as measured by ALEPH

Higher Order Corrections and Matrix Element Corrections

- Automatically capture cusp anomalous dimension into Sudakov factor
- Matrix Element Corrections allow to adjust total cross-section



Thrust as measured by ALEPH



Total jet broadening as measured by ALEPH

[Höche, Prestel, et. al.: 1705.00742, 1705.00982, 1805.03757]

Conclusions

Conclusions:

- Steps toward a Full Color Parton Shower
- Able to handle any number of emissions (additional cutoff for stability)
- Matching to Hadronization when including "singlet" gluons

Future Steps:

- Match to $\mathcal{O}\left(\alpha_s^2\right)$ corrections to recover Γ_2
- Extend to initial-state splittings
- Study interplay of sub-leading color, kinematics, spin, and higher-order corrections

Color Flow

- Separate gluon into "nonet" $(\delta^{i1}_{j2}\delta^{i_2}_{j1})$ and "singlet" $(\delta^{i1}_{j1}\delta^{i_1}_{j_1})$ components
- Non-orthogonal and overcomplete basis
- Easy to compute numerically with
- Represented by color-anticolor pairs, denoting flow of color from one leg to another
- Decompose color operators in color flow basis

	λ_i	$\overline{\lambda}_i$
Quark	$\sqrt{T_R}$	0
Antiquark	0	$\sqrt{T_R}$
Gluon	$\sqrt{T_R}$	$\sqrt{T_R}$

$$|\sigma\rangle = \left| \begin{array}{cccc} 1 & 2 & \dots & n \\ \overline{\sigma}(1) & \overline{\sigma}(2) & \dots & \overline{\sigma}(n) \end{array} \right\rangle = \delta^{c1}_{\overline{c}_{\sigma(1)}} \delta^{c2}_{\overline{c}_{\sigma(2)}} \dots \delta^{c_n}_{\overline{c}_{\sigma(n)}}$$

$$\mathbf{T}_i = \lambda_i \mathbf{t}_{c_i} - \overline{\lambda}_i \overline{\mathbf{t}}_{\overline{c}_i} - \frac{1}{N}\left(\lambda_i - \overline{\lambda}_i\right)\mathbf{s},$$

[Ángeles Martínez, De Angelis, Forshaw, Plätzer, Seymour: 2018 (arxiv:1802.08531)]

Algorithm: Splitting Kernels

- Work in explicit $N_C = 3$
- Break splitting kernels up based on color and splitting type
- Increases number of splitting kernels by ≈ 6
- P_{gq} is tricky:
    - Determined by color conservation
    - Tricky to determine spectator

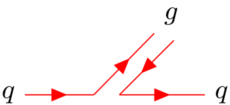
$$P_{qq}^{(9)}(x) = \lambda_i \mathbf{t}_{c_i} \frac{P_{qq}(x)}{C_F},$$

$$P_{qq}^{(1)}(x) = -\frac{\lambda_i}{N_C} \mathbf{s} \frac{P_{qq}(x)}{C_F},$$

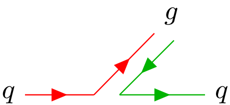
$$P_{gg}^{(+)} = \lambda_i \mathbf{t}_{c_i} \frac{P_{gg}(x)}{C_A},$$

$$P_{gg}^{(-)} = -\overline{\lambda}_i \overline{\mathbf{t}}_{\bar{c}_i} \frac{P_{gg}(x)}{C_A}$$

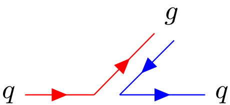
Example Splitting Kernels: Graphically



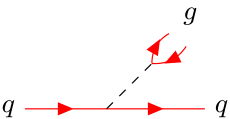
(a) $P_{rr}^{(9)}$



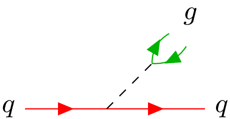
(b) $P_{rg}^{(9)}$



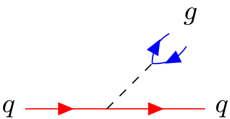
(c) $P_{rb}^{(9)}$



(d) $P_{rr}^{(1)}$

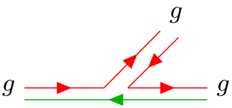


(e) $P_{rg}^{(1)}$

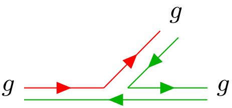


(f) $P_{rb}^{(1)}$

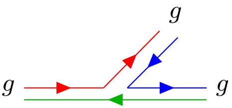
Example Splitting Kernels: Graphically



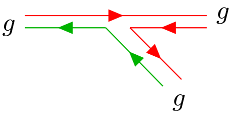
(g) $P_{rr}^{(+)}$



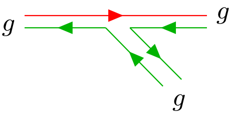
(h) $P_{rg}^{(+)}$



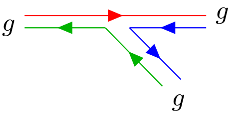
(i) $P_{rb}^{(+)}$



(j) $P_{gr}^{(-)}$

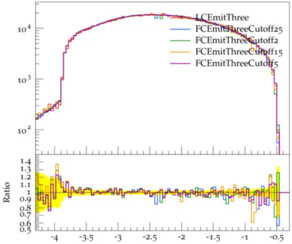


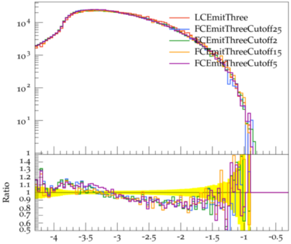
(k) $P_{gg}^{(-)}$

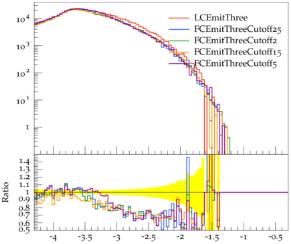


(l) $P_{gb}^{(-)}$

Backup: Cutoff Dependence







Results

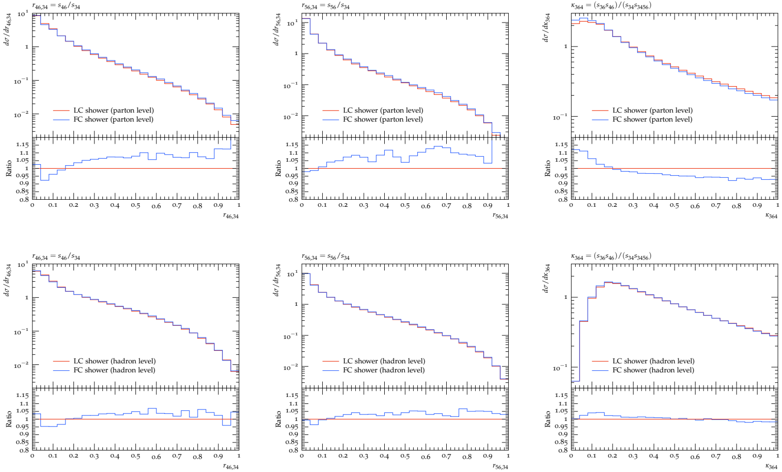
Parameters

- No inclusion of Γ_2
- DIRE PYTHIA hadronization tune
- $\alpha_s\left(M_Z\right) = 0.125$

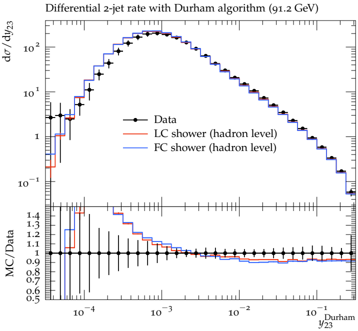
Validation Invariants:

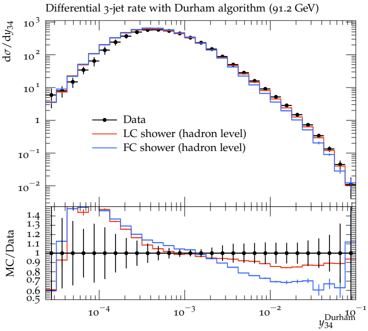
- $\frac{s_{46}}{s_{34}}$
- $\frac{s_{56}}{s_{34}}$
- $\kappa_{364} = \frac{s_{36}s_{64}}{s_{34}s_{3456}}$

Validation


$r_{46,34} = s_{46}/s_{34}$
$r_{56,34} = s_{56}/s_{34}$
$\kappa_{364} = (s_{36}s_{46})/(s_{34}s_{3456})$
$d\sigma/dr_{46,34}$
$d\sigma/dr_{56,34}$
$d\sigma/d\kappa_{364}$
LC shower (parton level)
FC shower (parton level)
LC shower (hadron level)
FC shower (hadron level)
Ratio
$r_{46,34}$
$r_{56,34}$
κ_{364}

Results:





Jet separation between two- and three-jet configurations in the Durham algorithm, as measured by OPAL

Jet separation between three- and four-jet configurations in the Durham algorithm, as measured by OPAL