Ab-initio calculations of neutrino-nucleus interactions

Nuclear and Particle Theory for Accelerator Neutrino Experiments, May 9–11, 2019, Fermilab

Alessandro Lovato

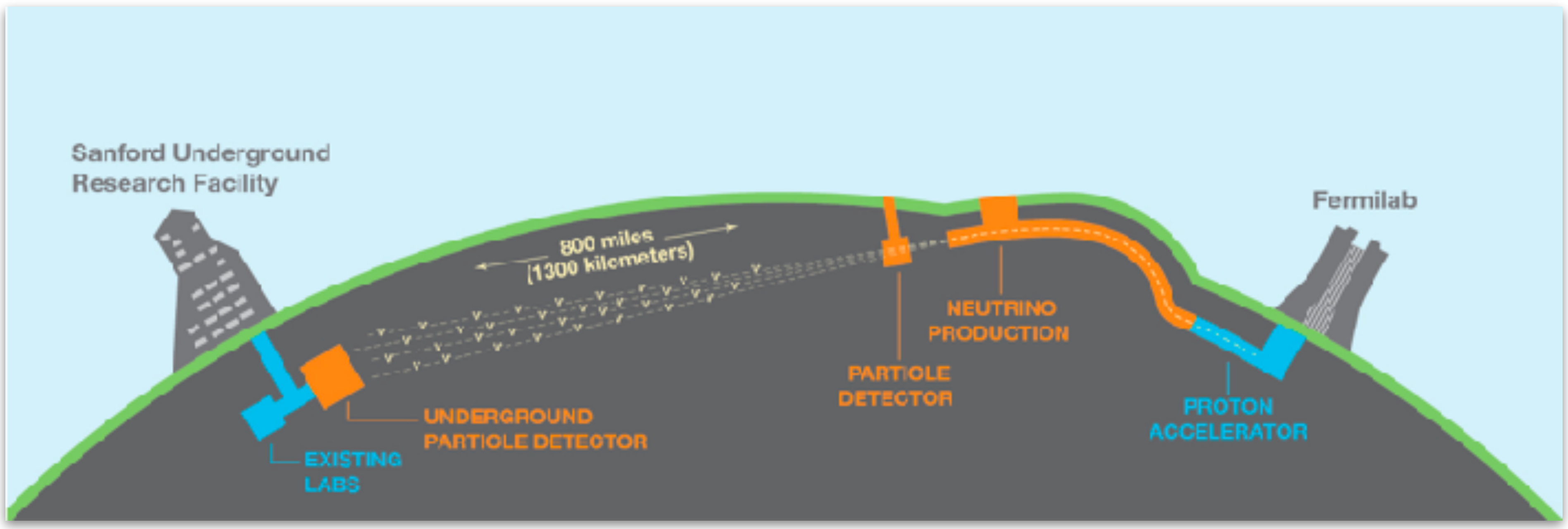
INFN Istituto Nazionale di Fisica Nucleare

Argonne NATIONAL LABORATORY

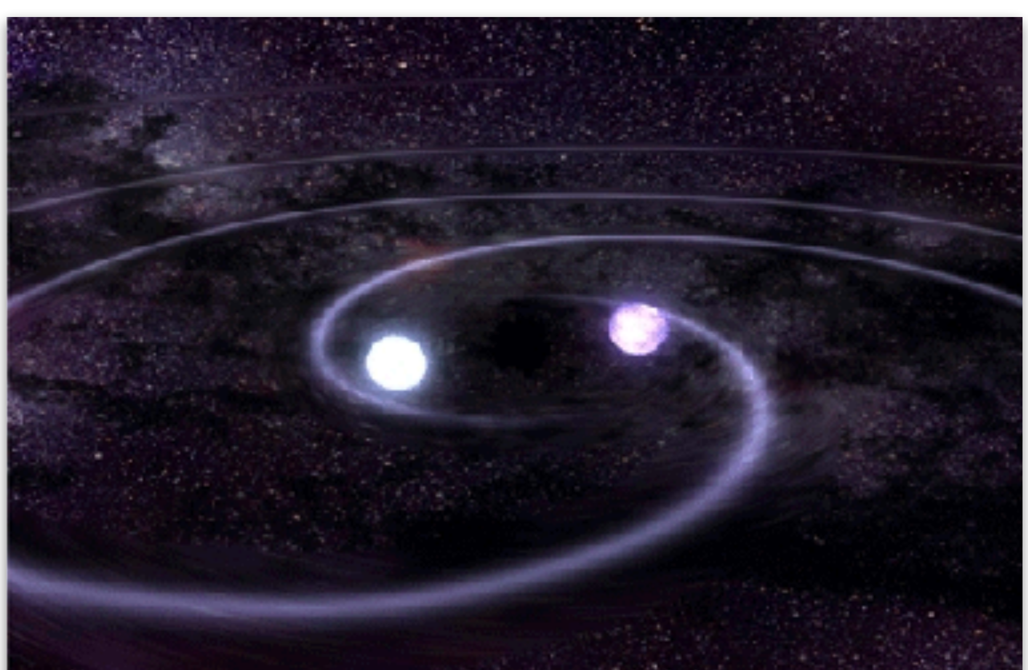
In collaboration with:
O. Benhar, J. Carlson, S. Gandolfi, W. Leidemann, G. Orlandini, S. Pieper, N. Rocco,
R. Schiavilla, B. Wiringa

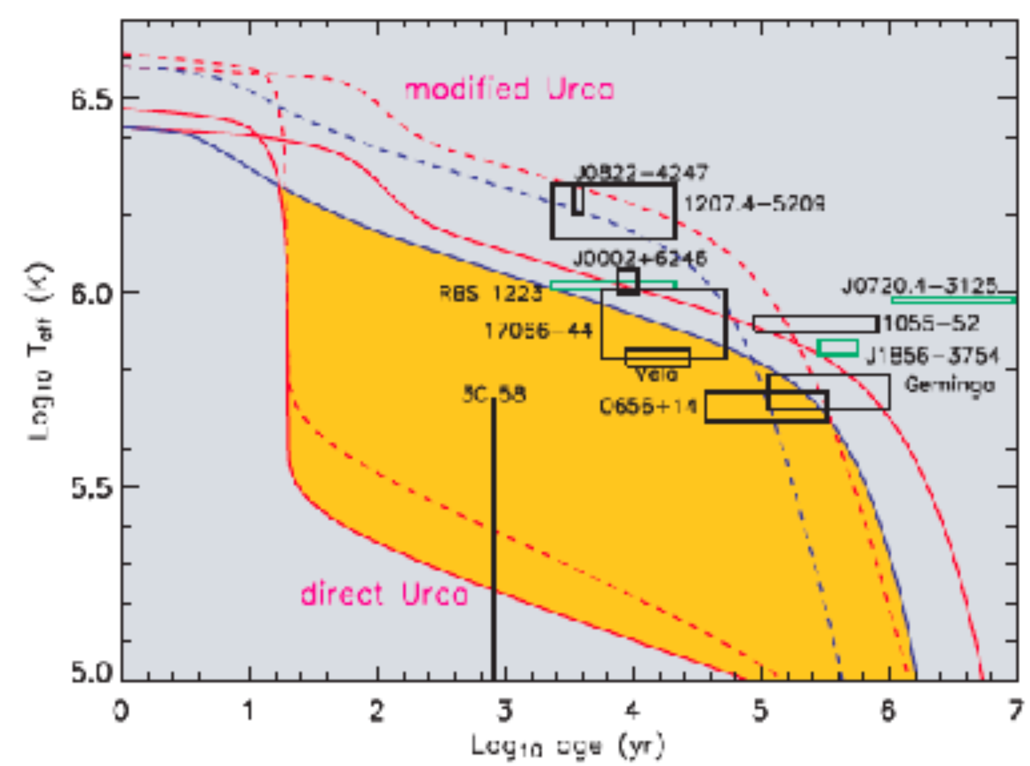
(Some of the) Challenges for nuclear theory

- Neutrino-oscillation parameters, **including δ_{CP}**, will be accurately measured

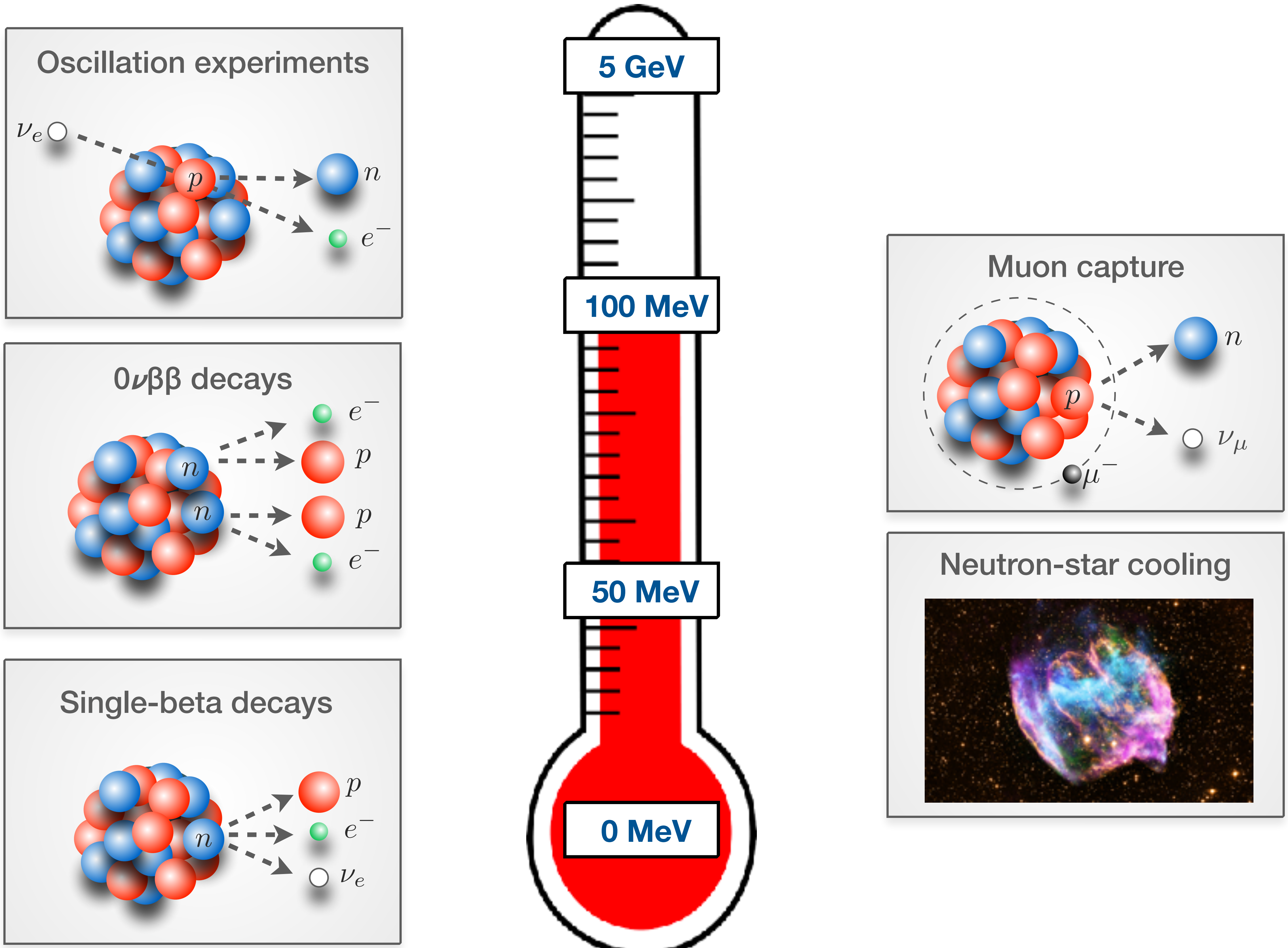


- Nuclear dynamics determines the **structure** of neutron stars, imprinted in gravitational-wave signals, and their **cooling** via the emission of neutrinos





Momentum scales at play with neutrinos



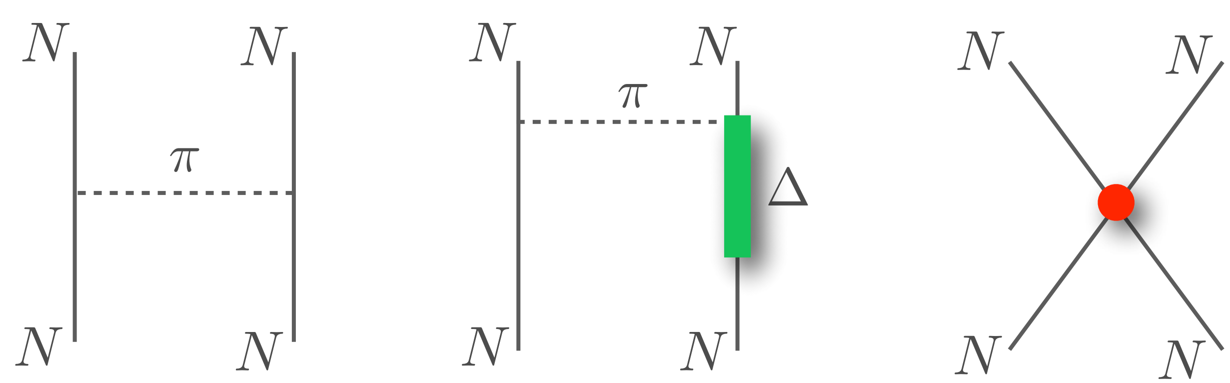
Nuclear Hamiltonian

- **Nuclear microscopic approaches** are based on the non relativistic hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

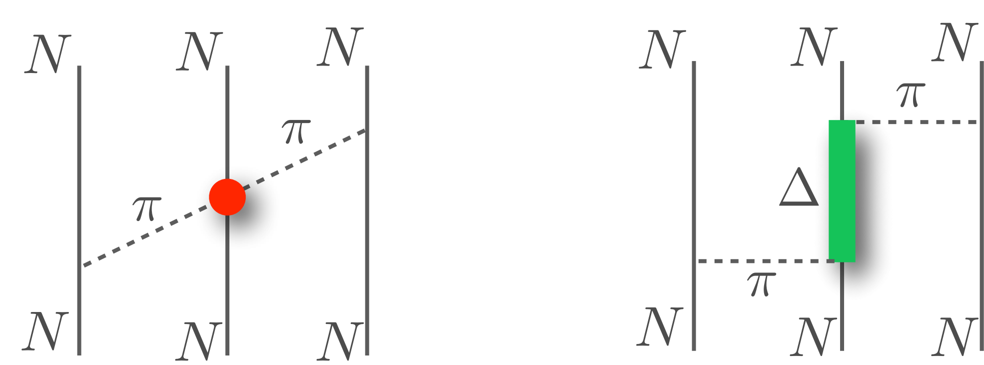
- **Argonne v_{18}** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O^p_{ij}$$



- **Three-nucleon interactions** effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

$$V^{3N}_{ijk} = A^{PW}_{2\pi} O^{2\pi,PW}_{ijk} + A^{SW}_{2\pi} O^{2\pi,SW}_{ijk} + A^{\Delta R}_{3\pi} O^{3\pi,\Delta R}_{ijk} + A^R O^R_{ijk}$$

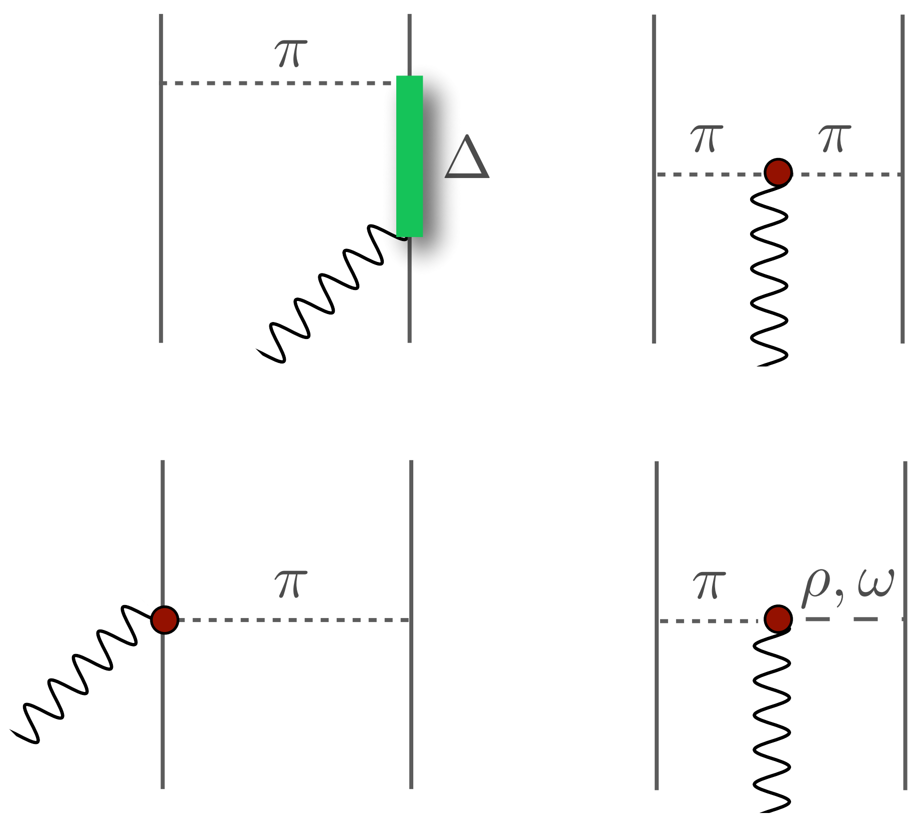


Electroweak currents

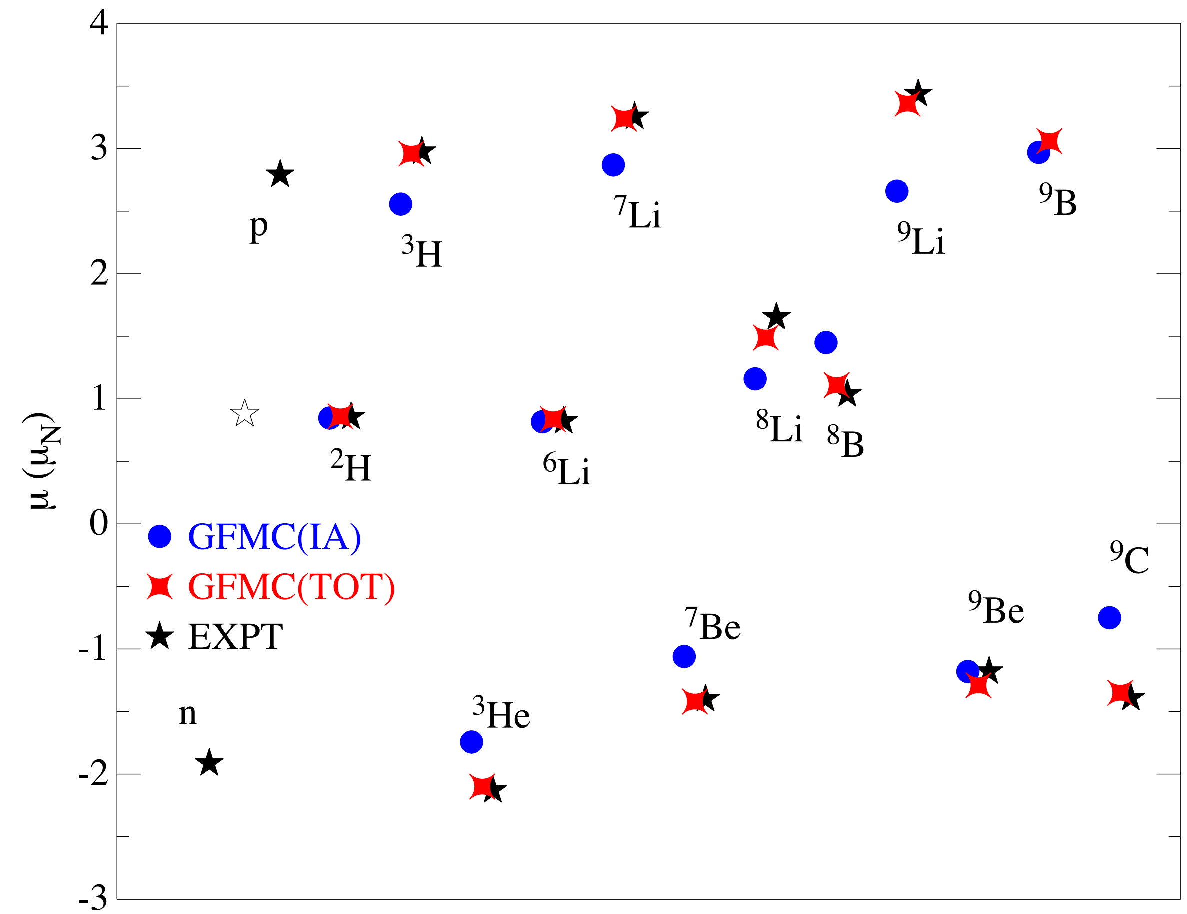
The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.



- They are essential for low-momentum and low-energy transfer transitions.



S. Pastore at al., PRC **87**, 035503 (2013)

Variational Monte Carlo

A fundamental step towards a first-principle description of atomic nuclei is the solution of the **many-body Schrödinger equation**

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$

- In VMC, one assumes a form for the trial wave function and optimizes its variational parameters

$$E_T = \langle\Psi_T|H|\Psi_T\rangle \geq E_0$$

- The **short-range behavior** of the trial wave function is modeled by Jastrow-like correlations

$$\Psi_T = \left(1 + \sum_{i<j<k} F_{ijk}\right)\left(\mathcal{S}\prod_{i<j} F_{ij}\right)\Phi_A(J, M, T, T_z)$$

- They reflect the **spin-isospin dependence** of the two- three-nucleon interactions

$$F_{ij} \simeq \sum_p f^p(r_{ij})O^p_{ij} \qquad F_{ijk} = \sum_x \epsilon_x V^x_{ijk}(\tilde{r}_{ij}, \tilde{r}_{ik}, \tilde{r}_{jk})$$

Green's function Monte Carlo

- GFMC overcomes the limitations of the variational wave-function by using an **imaginary-time projection technique**

- Any trial wave function can be expanded in the complete set of eigenstates of the the hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

which implies

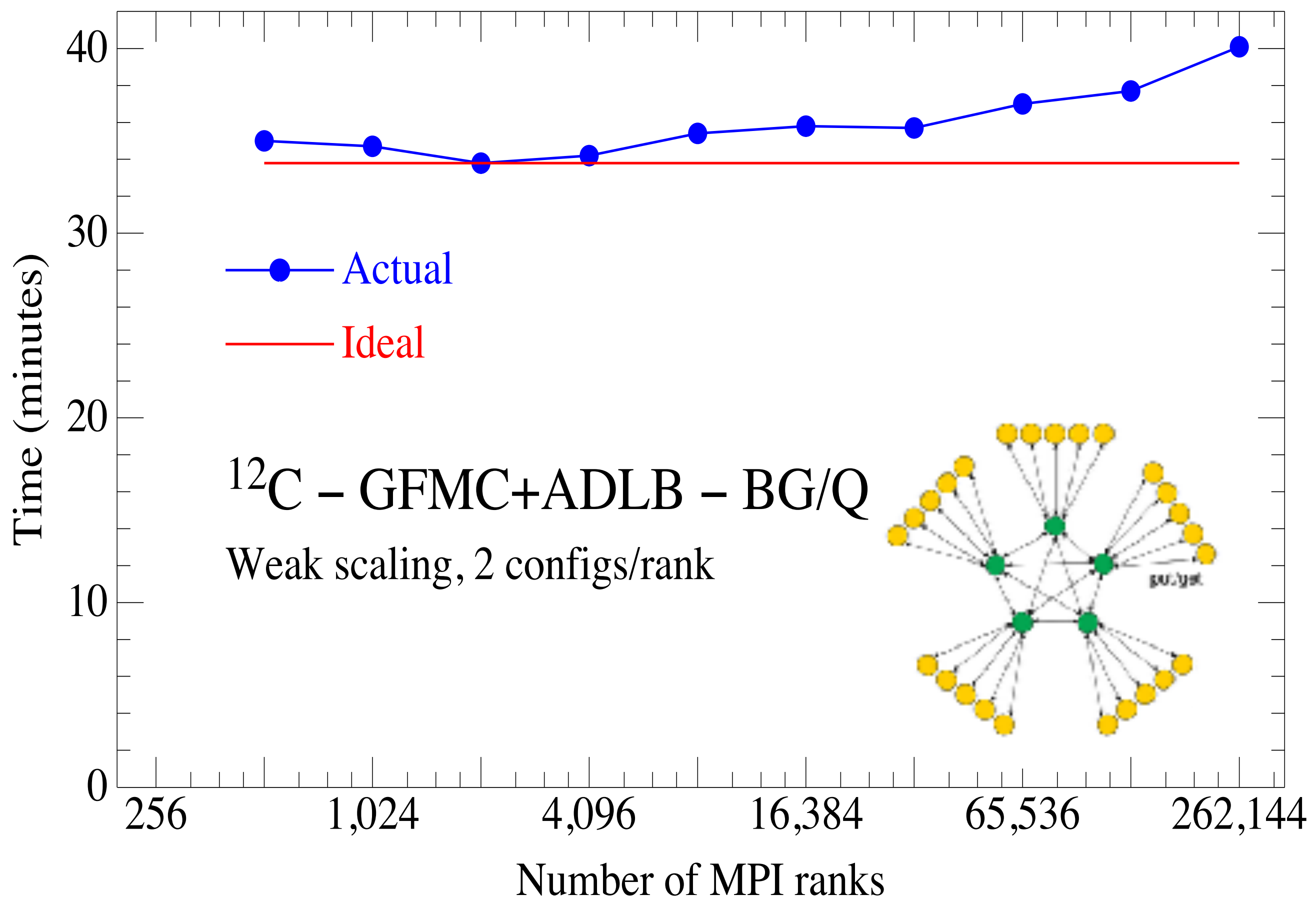
$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

GFMC projects out the exact lowest-energy state, provided the trial wave function it is not orthogonal to the ground state.

Special care needs to be taken to deal with the **fermion-sign problem**

Exploiting leadership-class computers

- GFMC has steadily undergone development to **take advantage of each new generation of parallel machine** and was one of the first to deliver new scientific results each time.

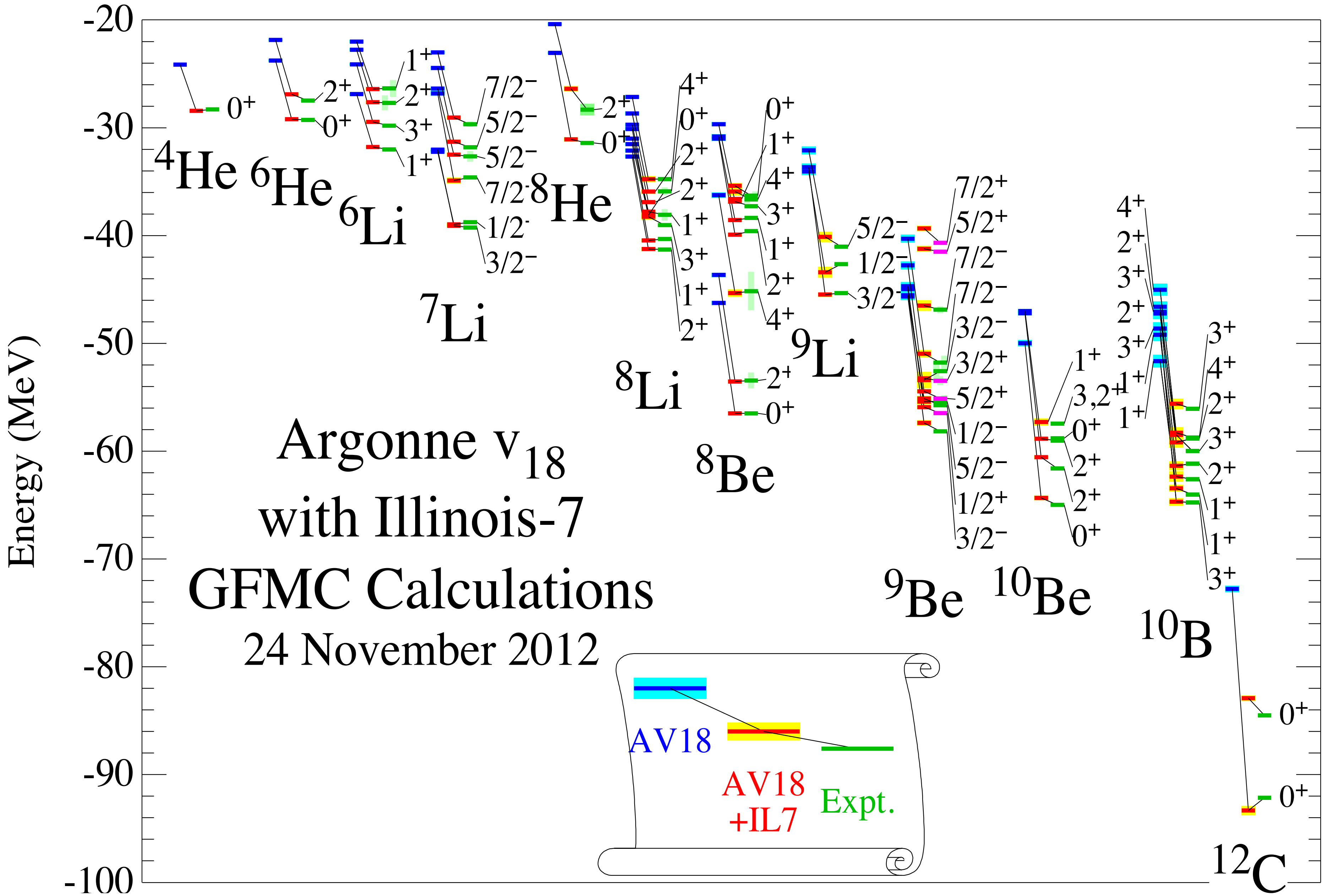


U.S. DEPARTMENT OF ENERGY
INCITE
LEADERSHIP COMPUTING

SciDAC
Scientific Discovery through Advanced Computing

Green's function Monte Carlo

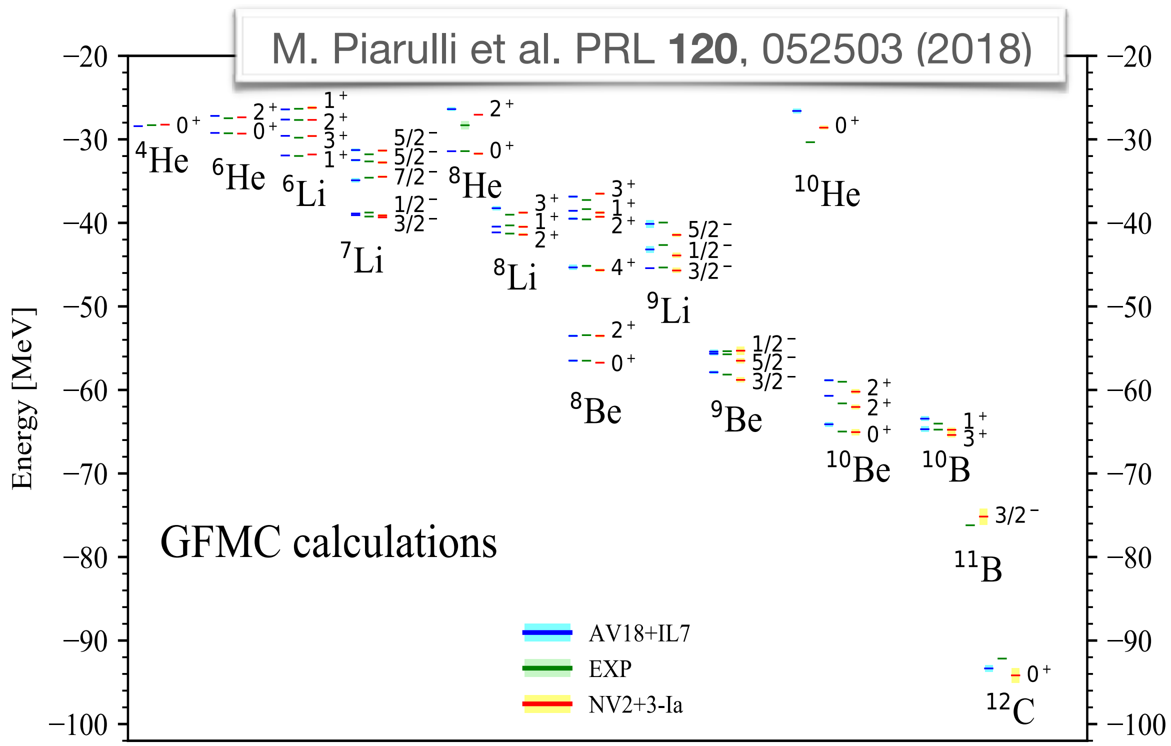
- GFMC is suitable to solve the **spectrum of A ≤ 12 nuclei with ~1% accuracy**

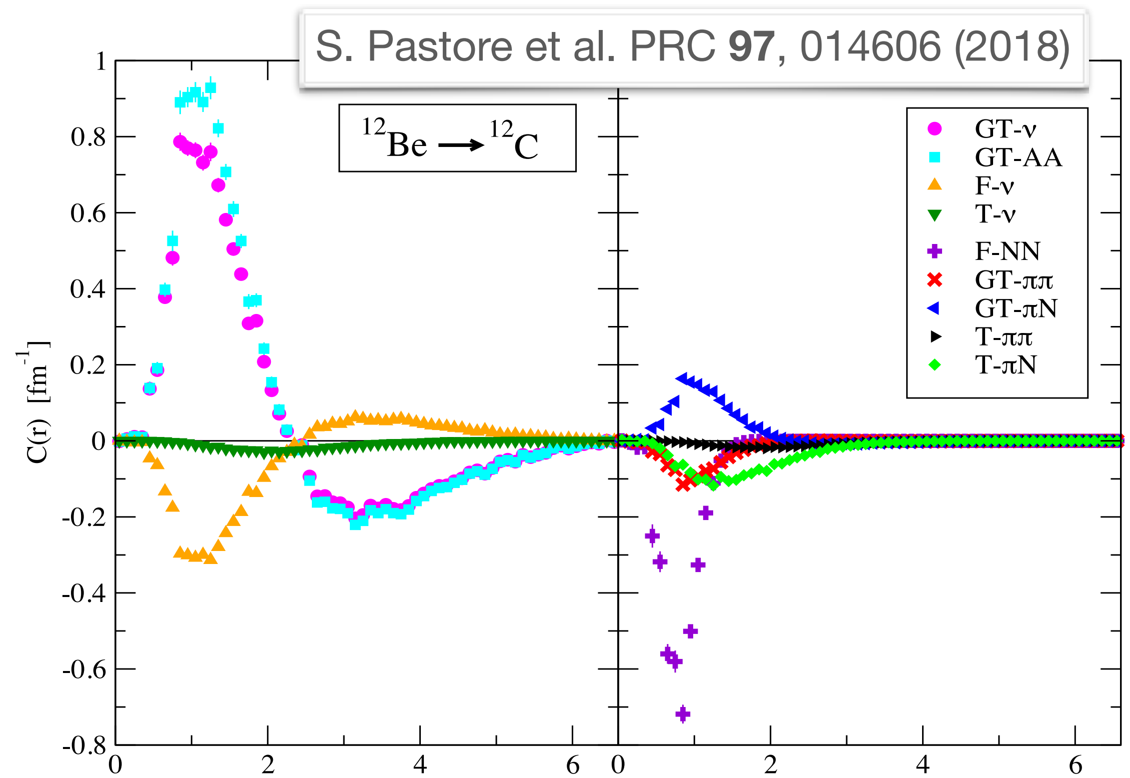


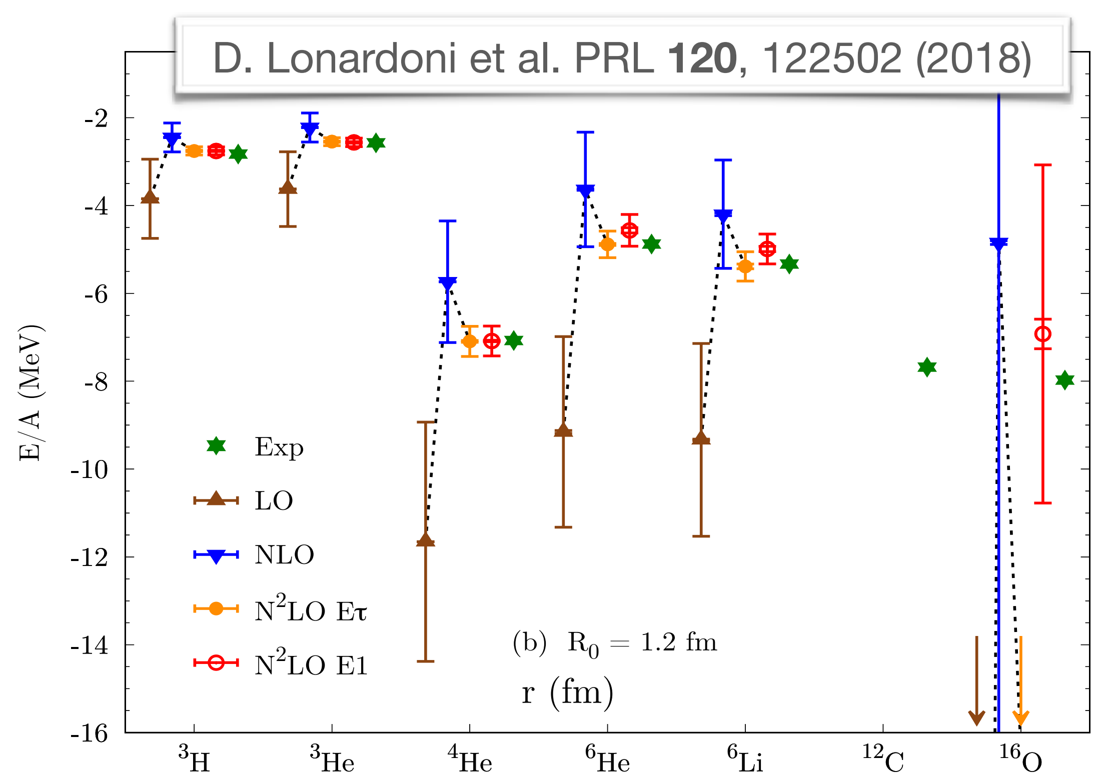
J. Carlson et al. RMP **87**, 1067 (2015)

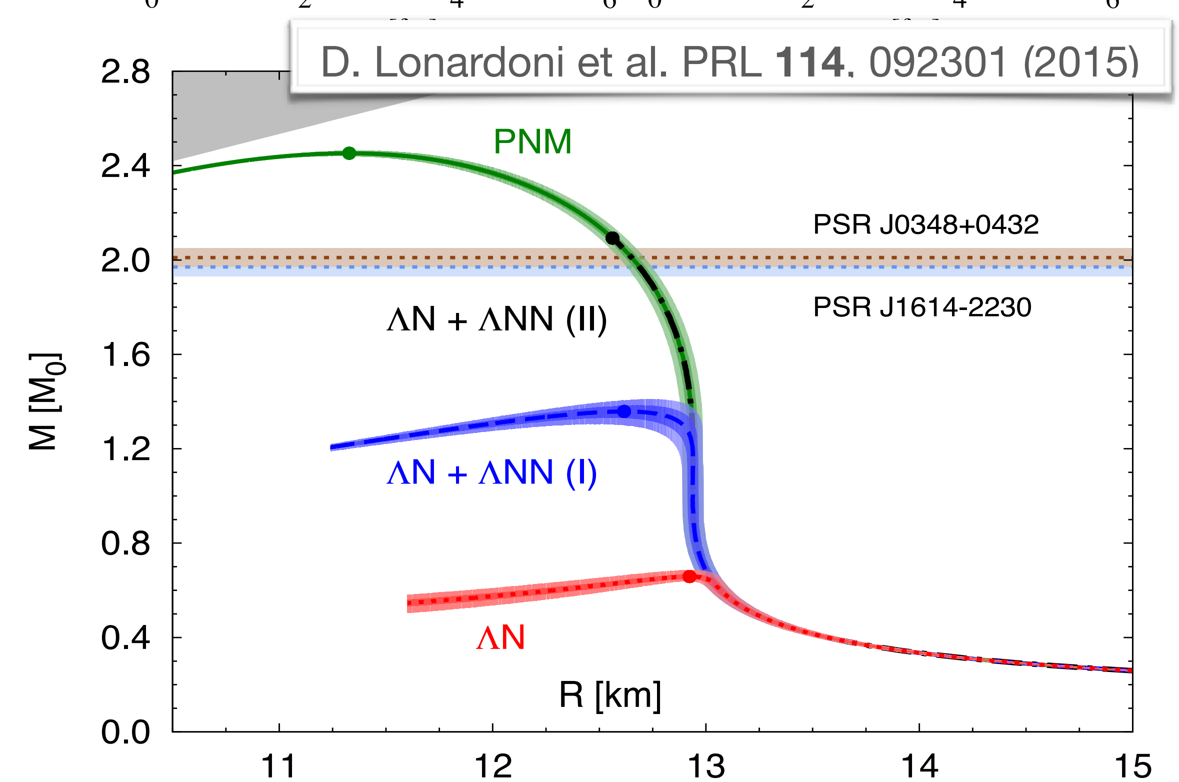
The *basic model* of nuclear Physics

Realistic interactions and currents **+** Nuclear ab-initio methods



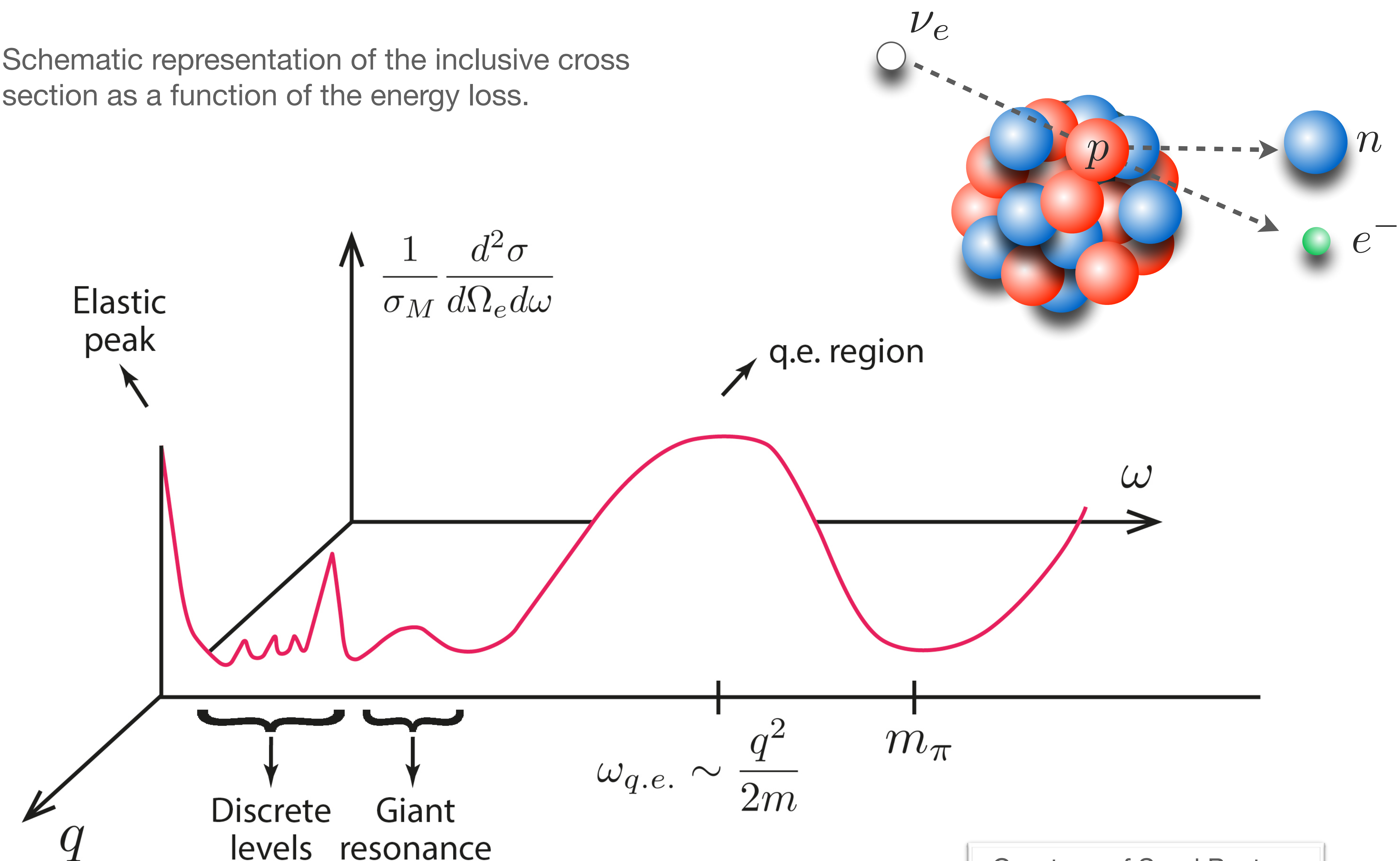






Lepton-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.



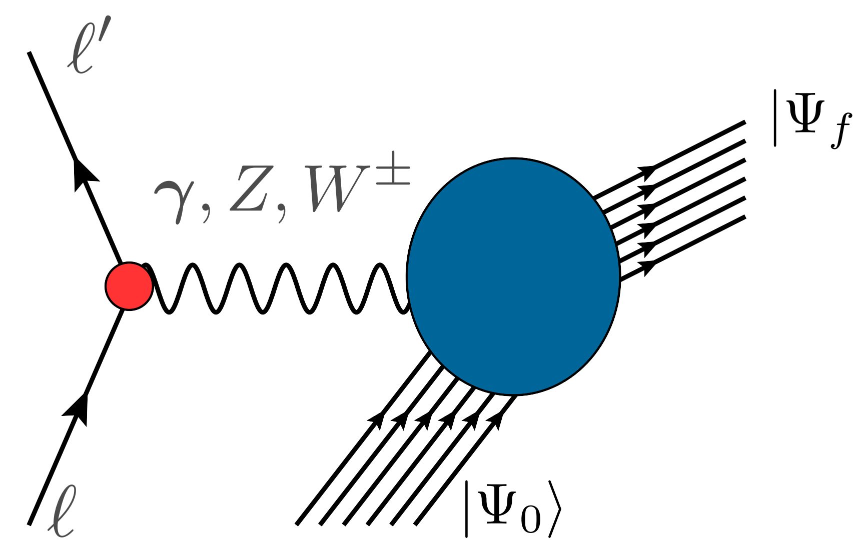
Courtesy of Saori Pastore

Lepton-nucleus scattering

The lepton-nucleus inclusive cross section can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'}d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$

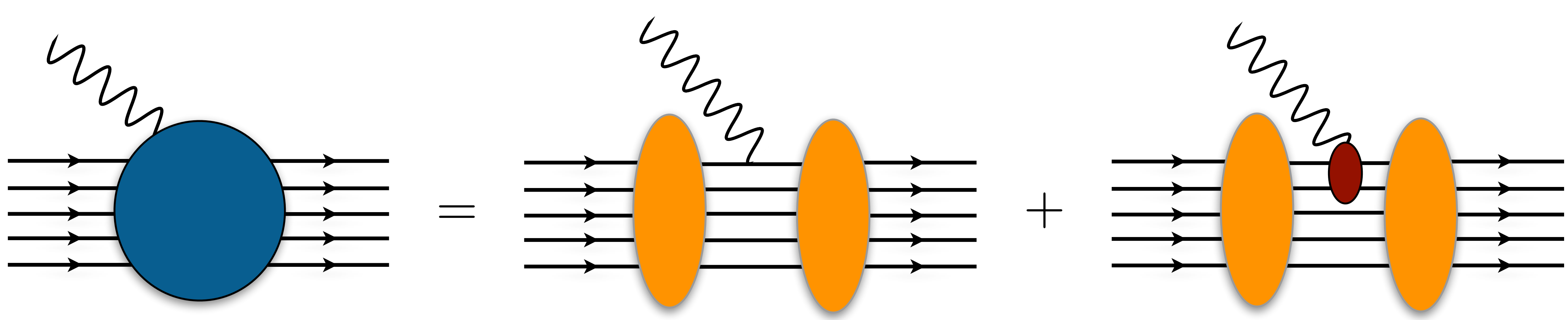
In electron scattering only the longitudinal and transverse responses contribute



- The response functions contain all information on the structure and dynamics of the target

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

- They include initial-state correlations, final state correlations and two-body currents



Inclusive responses at moderate momentum transfers

Moderate momentum-transfer regime

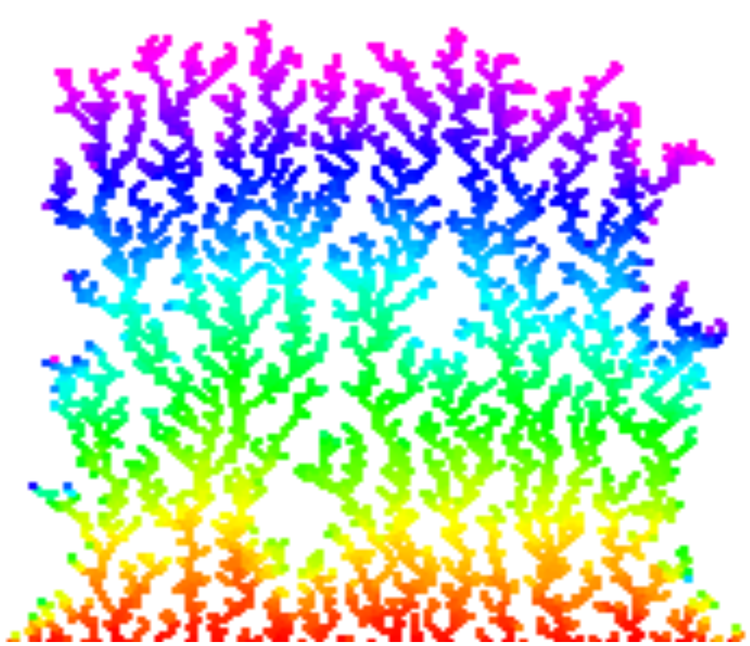
- Both initial and final states are eigenstates of the nuclear Hamiltonian. Relativistic corrections are included in the current operators

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \qquad H|\Psi_f\rangle = E_f|\Psi_f\rangle$$

- The energy dependence of the response functions can be inferred from their Laplace transforms

$$E_{\alpha\beta}(\tau,\mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega,\mathbf{q})$$

- Using the completeness of the final states, the Euclidean responses are expressed in terms of ground-state expectation values that are computed within **Green's function Monte Carlo**

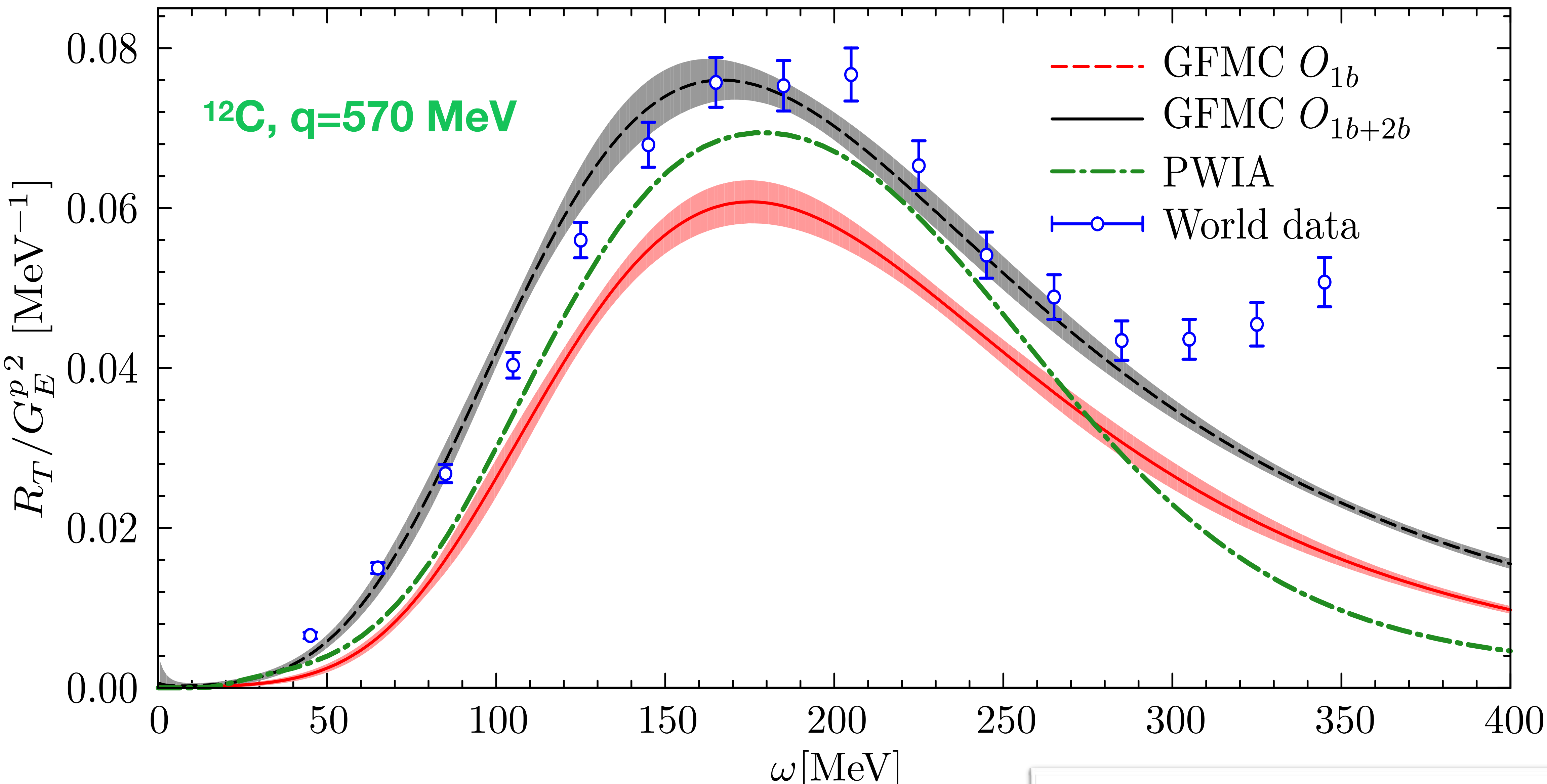


$$E_{\alpha\beta}(\tau,\mathbf{q}) = \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$$

Analogous techniques are used in Lattice QCD and condensed matter Physics

^{12}C electromagnetic response

- We computed the electromagnetic Euclidean response of ^{12}C
- Good agreement with the experimental data once two-body currents are accounted for
- Need to include relativistic corrections in the kinematics



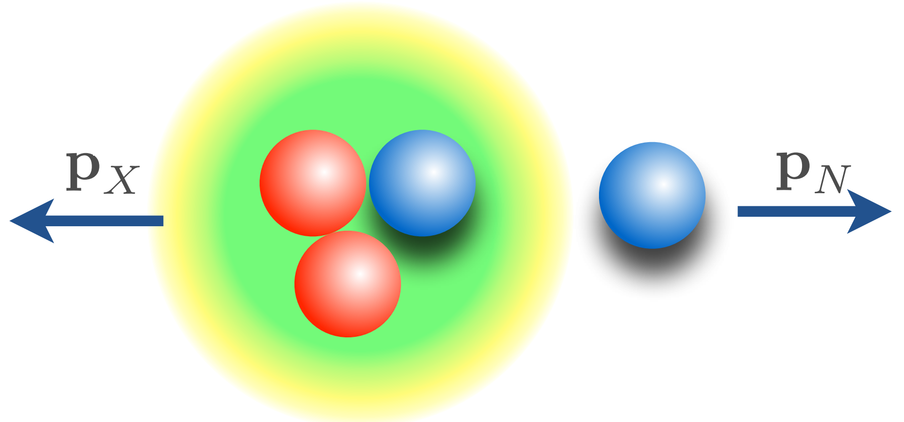
AL et al. PRL **117** 082501 (2016)

Relativistic kinematics in GFMC

- To determine the relativistic corrections in the kinematics, we use a **two-body fragment model**

$$\mathbf{p}_f = \mu\left(\frac{\mathbf{p}_N}{m} - \frac{\mathbf{p}_X}{M_X}\right) \qquad \mu = \frac{mM_X}{m + M_X}$$

$$\mathbf{P}_f = \mathbf{p}_N + \mathbf{p}_X \qquad M_X = (A-1)m + \epsilon_0^{A-1}$$



- The relative momentum is derived in a relativistic fashion

$$E_f = \sqrt{m^2 + [\mathbf{p}_f + (\mu/M_X)\mathbf{P}_f]^2} + \sqrt{M_X^2 + [\mathbf{p}_f - (\mu/m)\mathbf{P}_f]^2}$$

$$E_f = \omega + E_i$$

- And it is used as input in the non relativistic kinetic energy

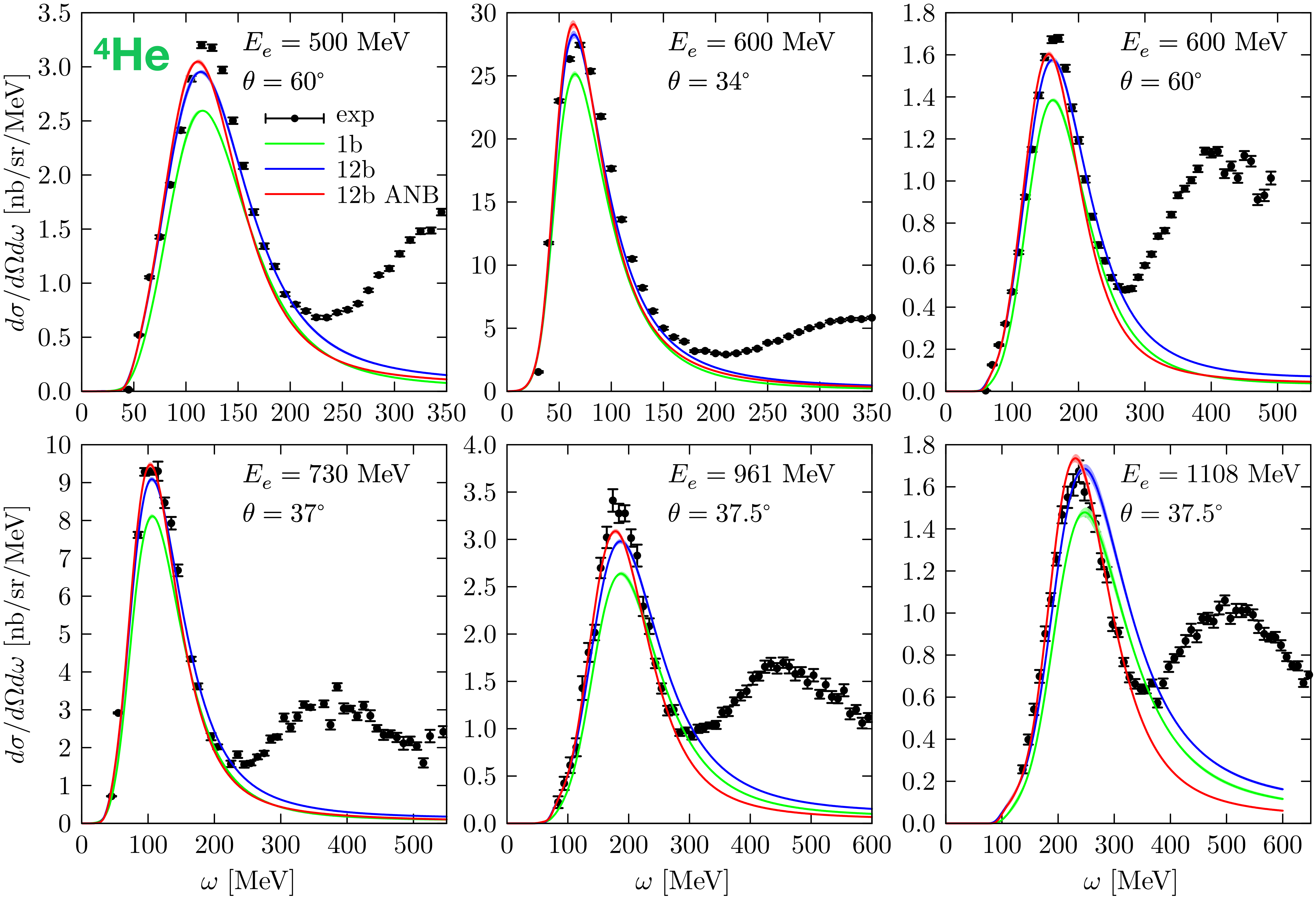
$$\epsilon_f = \frac{\mathbf{p}_f^2}{2\mu} + \epsilon_0^{A-1}$$

- The energy-conserving delta function reads

$$\delta(\omega - E_f(\epsilon_f) + E_0) = \left(\frac{\partial E_f(\epsilon_f)}{\partial \epsilon_f}\right)^{-1} \delta\left(\epsilon_f - \frac{\mathbf{p}_f^2}{2\mu} - \epsilon_0^{A-1}\right)$$

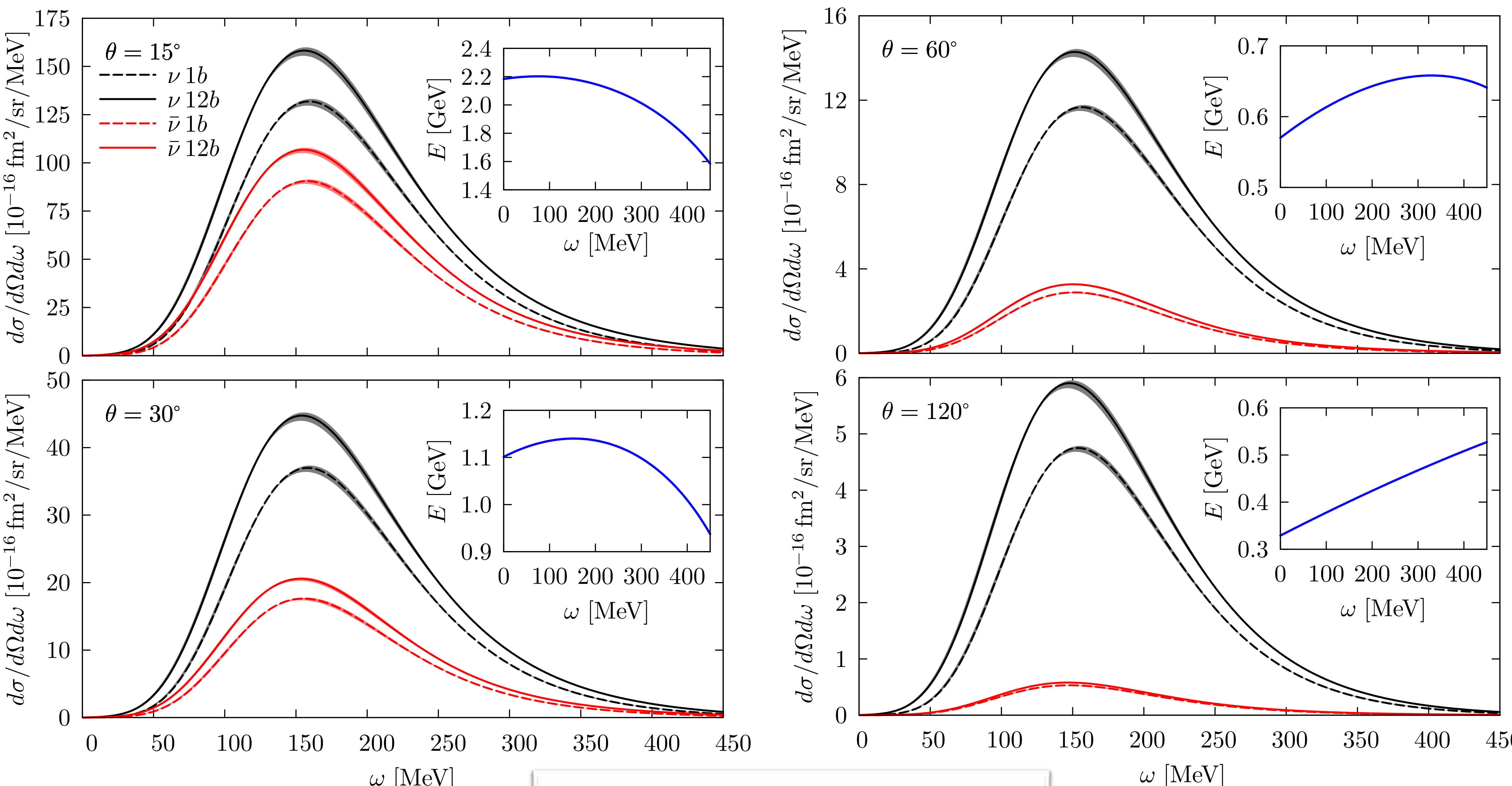
N. Rocco et al. PRC **97** 055501(2018)

^{4}He electromagnetic cross sections



^{12}C neutral-current cross-section

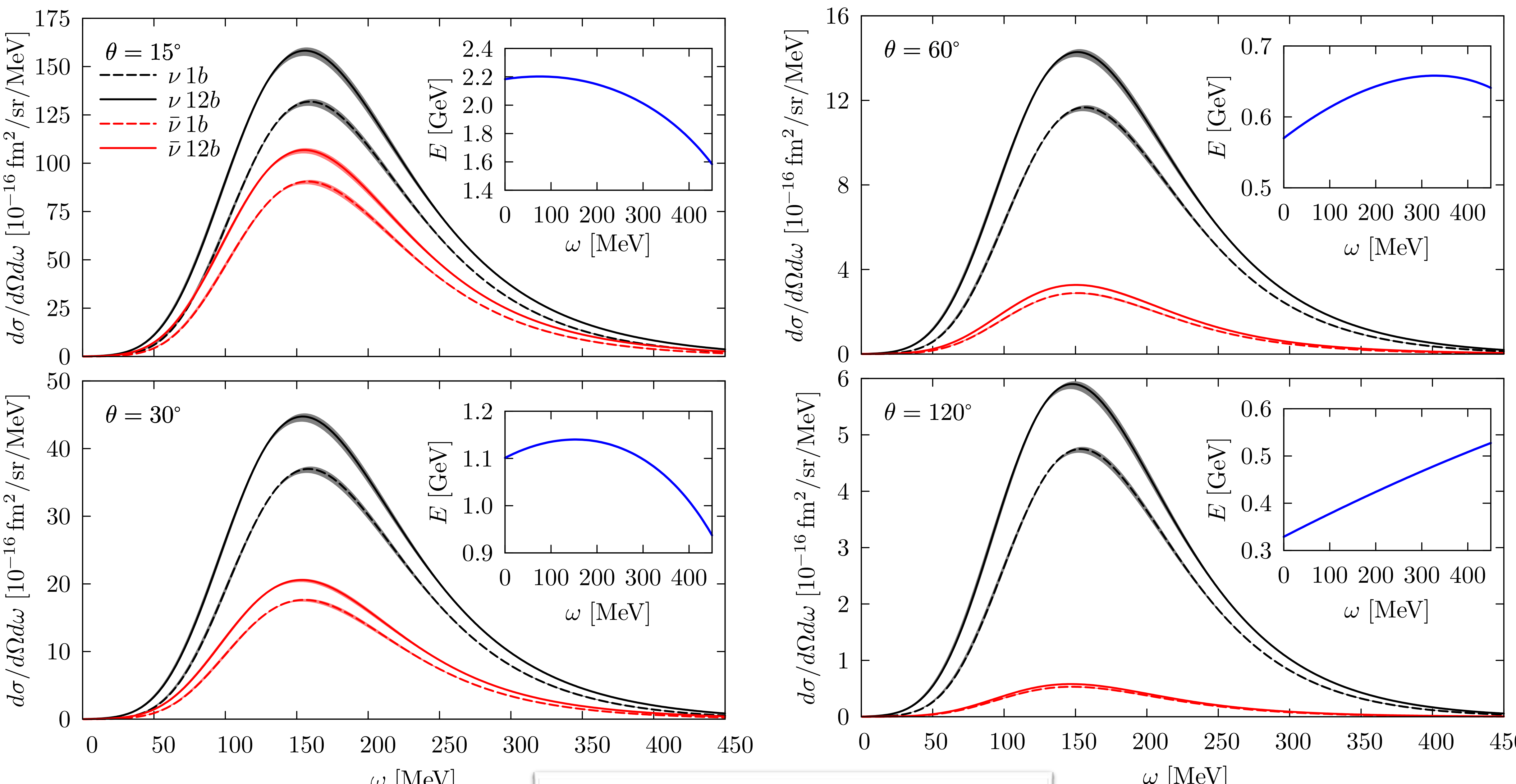
- We computed the neutrino and anti-neutrino differential cross sections for a fixed value of the three-momentum transfer as function of the energy transfer for a number of scattering angles



AL et al. PRC **97** 022502 (2018)

^{12}C neutral-current cross-section

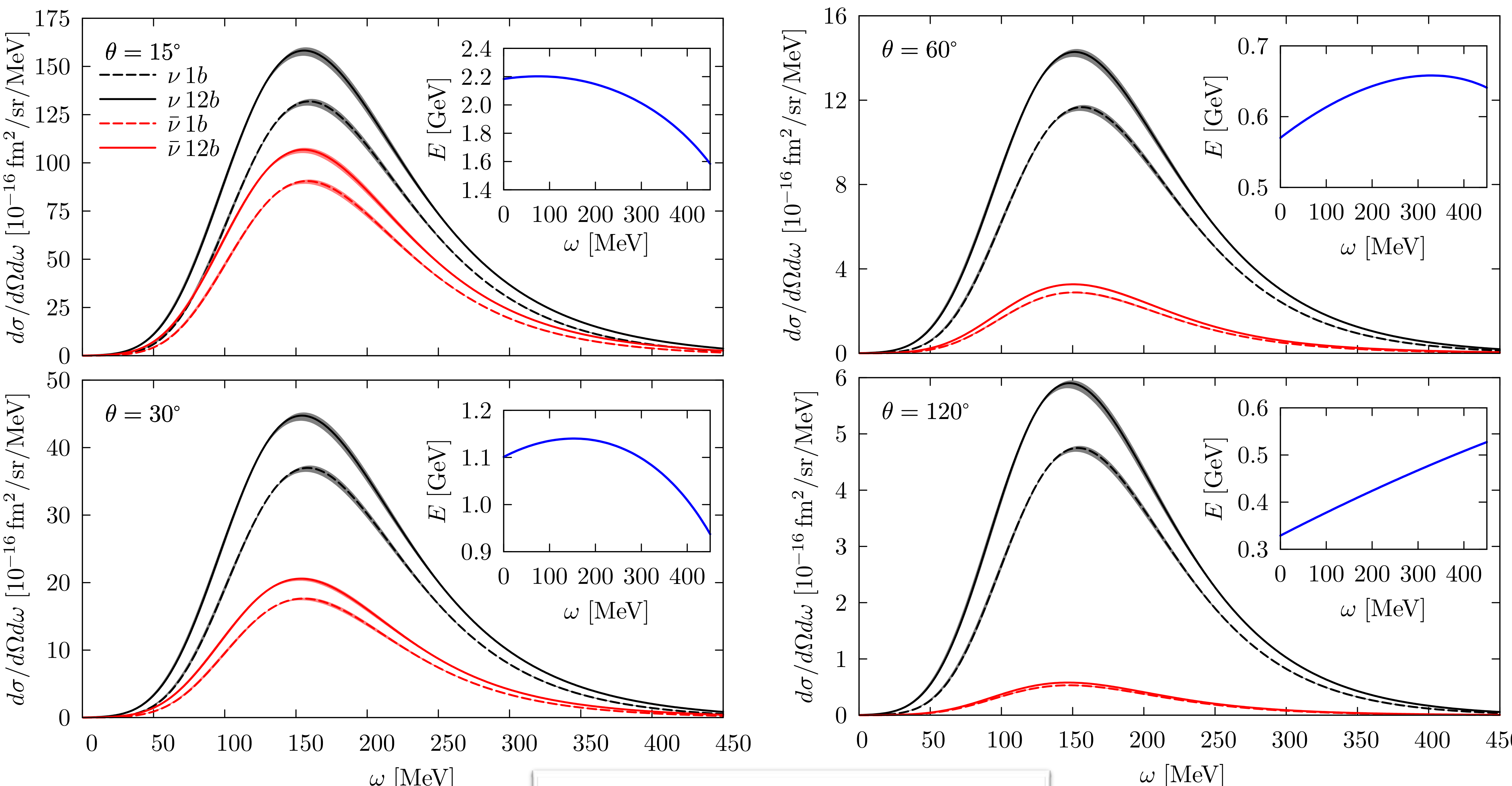
- The anti-neutrino cross section decreases rapidly relative to the neutrino cross section as the scattering angle changes from the forward to the backward hemisphere



AL et al. PRC **97** 022502 (2018)

^{12}C neutral-current cross-section

- For this same reason, two-body current contributions are smaller for the antineutrino than for the neutrino cross section

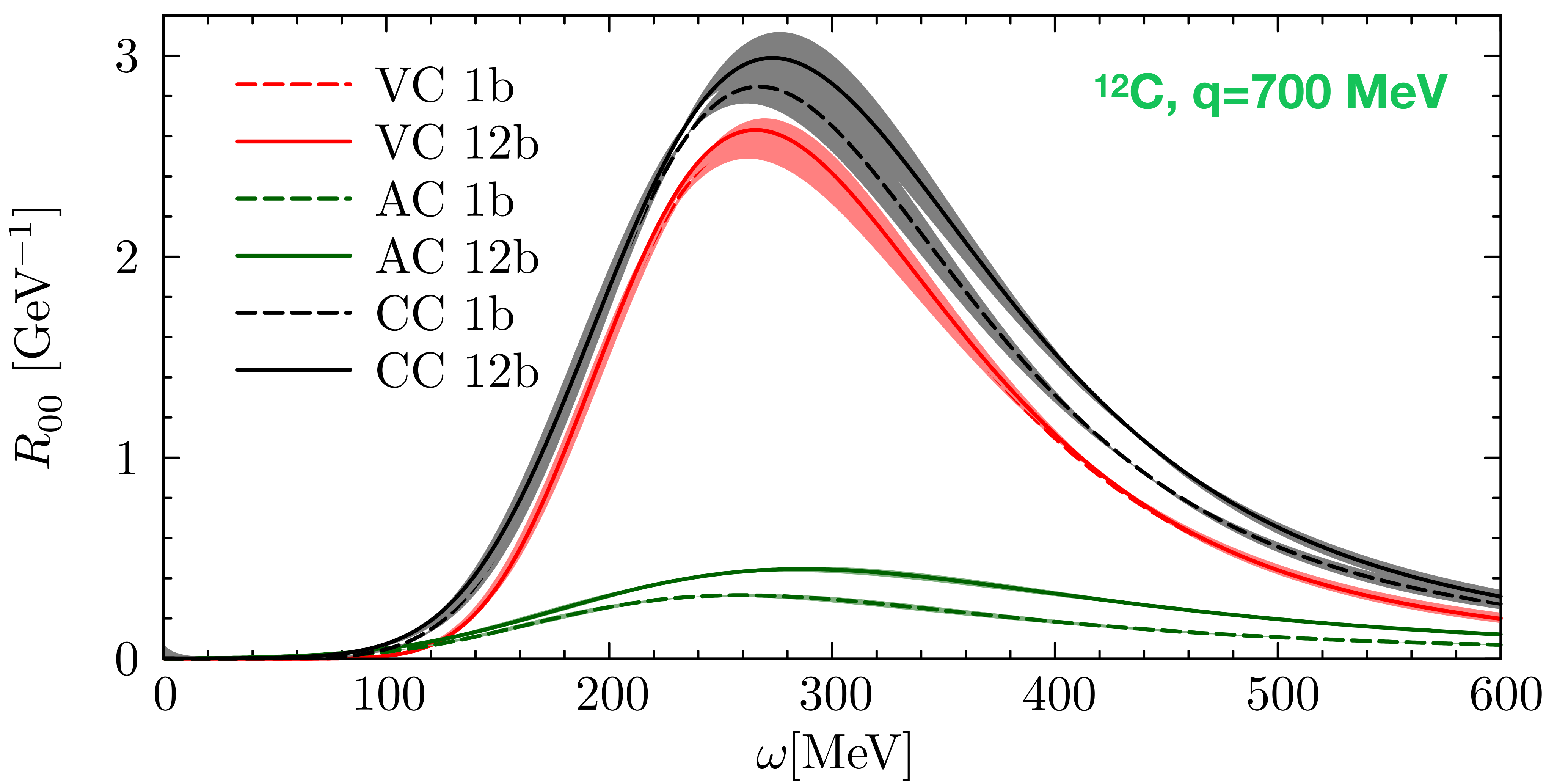


AL et al. PRC **97** 022502 (2018)

^{12}C charged-current responses

- We recently computed the charged-current response function of ^{12}C

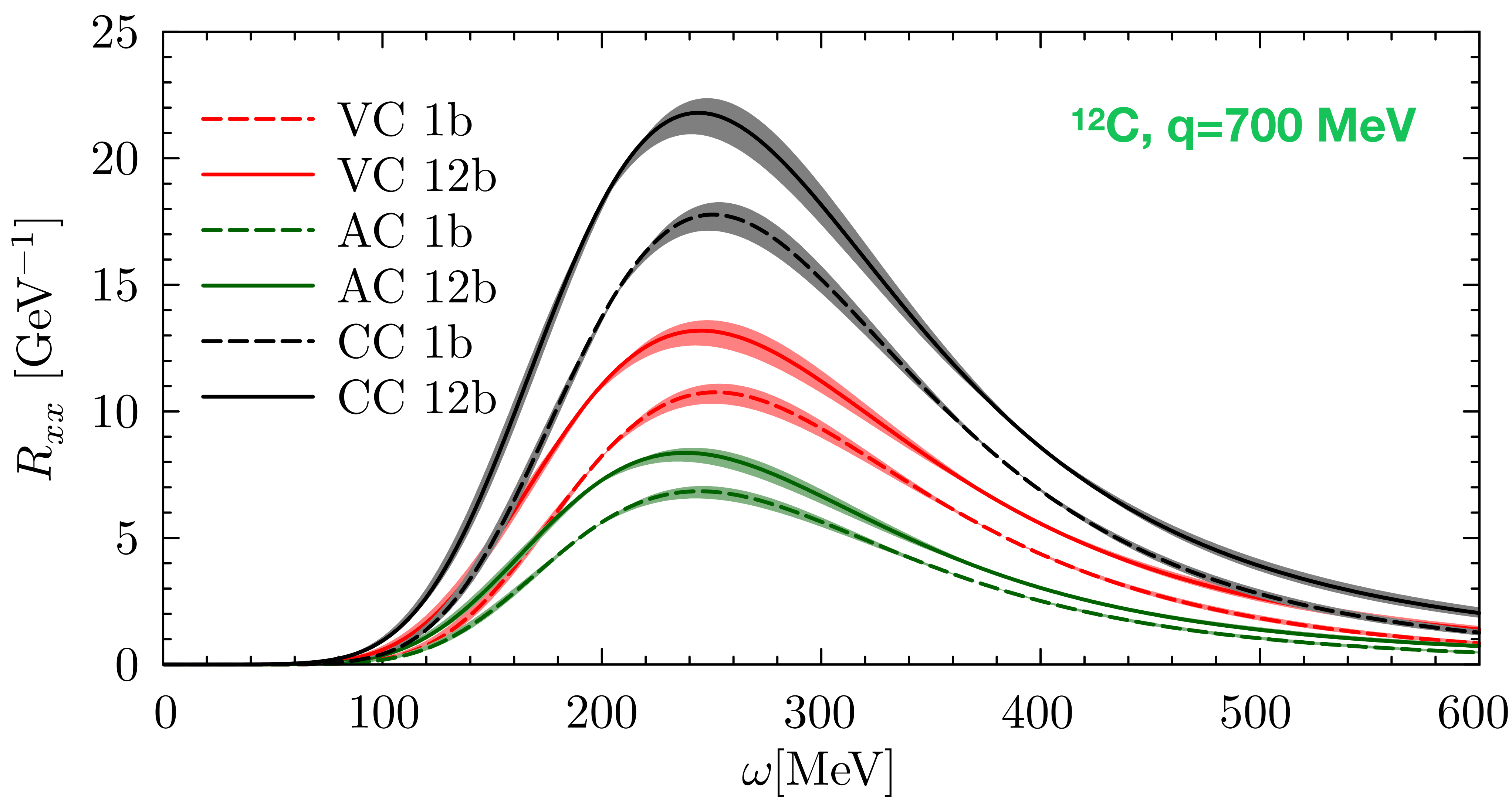
- Two-body currents have little effect in the vector term, but enhance the axial contribution at energy larger than quasi-elastic kinematics



^{12}C charged-current responses

- We recently computed the charged-current response function of ^{12}C

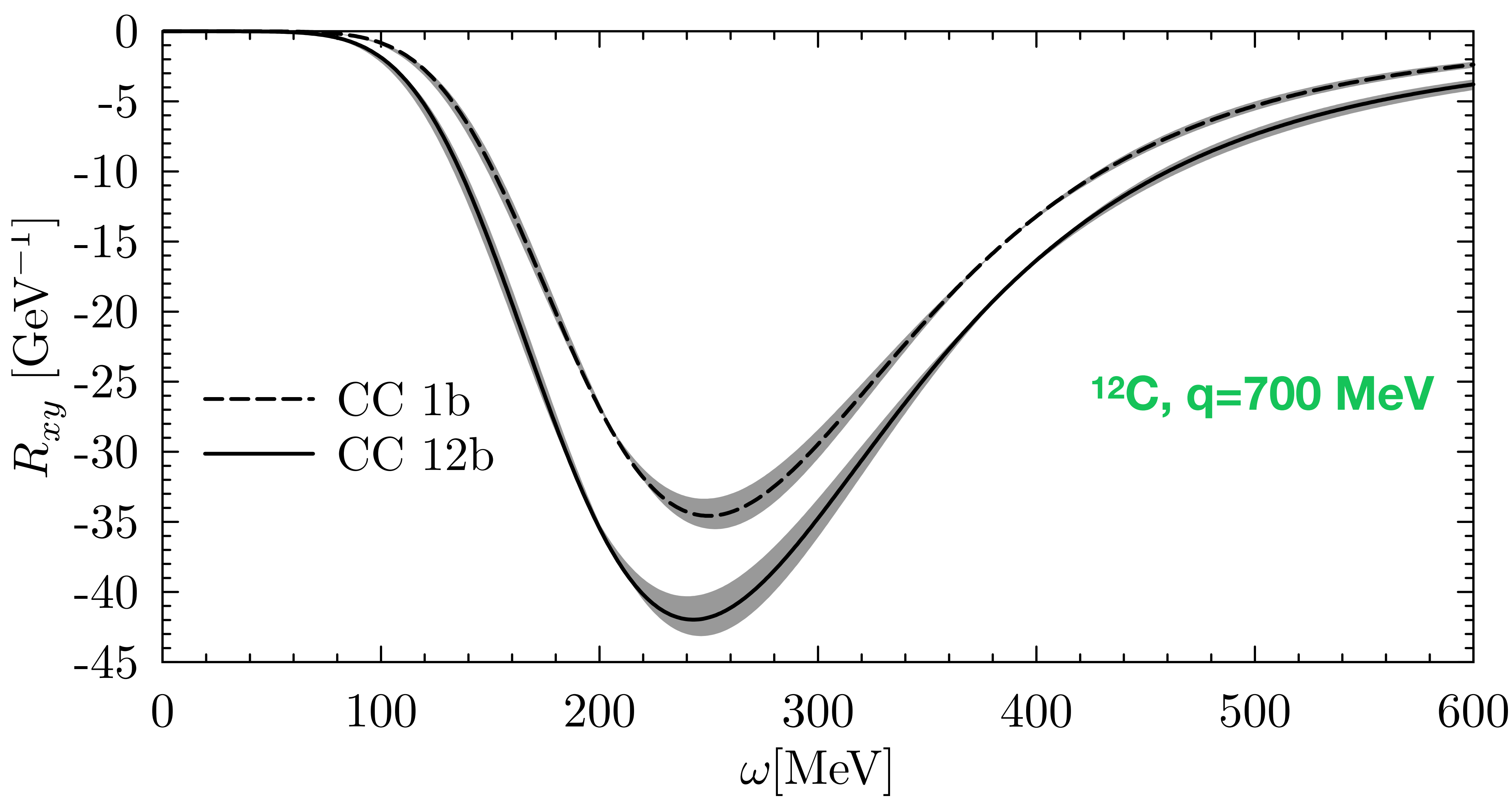
- Two-body currents have a sizable effect in the transverse response, both in the vector and in the axial contributions



^{12}C charged-current responses

- We recently computed the charged-current response function of ^{12}C

- Two-body currents have a sizable effect in the interference between the axial and vector current contributions, important to asses neutrino/antineutrino event rates



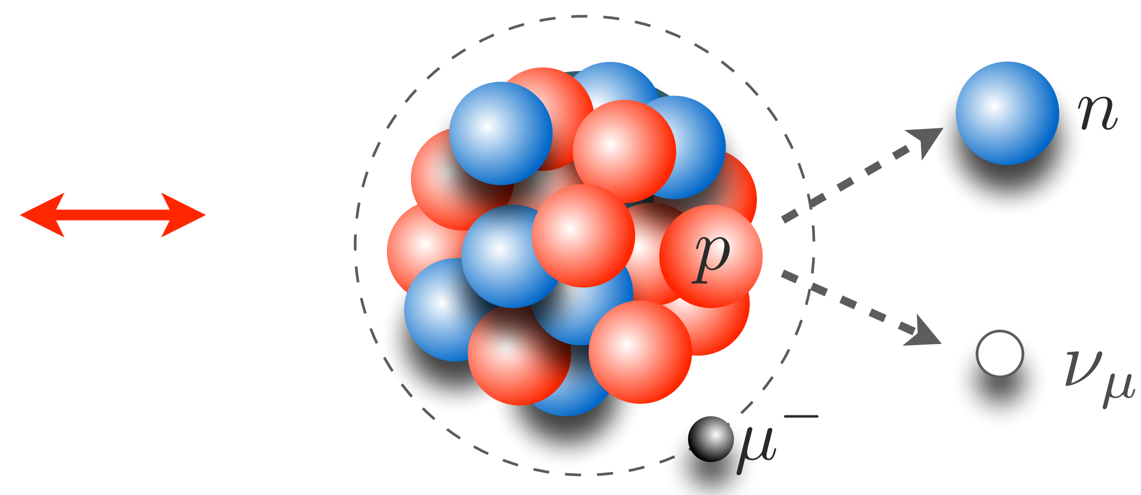
Muon capture in ^{4}He

Negative muons can be captured into high-lying atomic orbitals, from where they rapidly cascade into the 1s orbital. Two possible outcomes:

- **They decay** via the process $\mu^- \to e^- \nu_e \nu_\mu$ with a rate which is almost the same as in free space

- **They are captured by the nucleus** in a weak-interaction process resulting in the change of one of the protons into a neutron

The muon rest mass is converted in energy shared by the emitted neutrino and recoiling final nucleus



A comparison with experimental data is important to **validate our model of nuclear dynamics** at intermediate values of momentum transfer ~ muon mass.

- Role of **two-body terms** in the axial-current operator
- Test possible **parameterizations of the nucleon axial form factor**

Muon capture in ^{4}He

AL, N. Rocco, R. Schiavilla arXiv:1903.08078

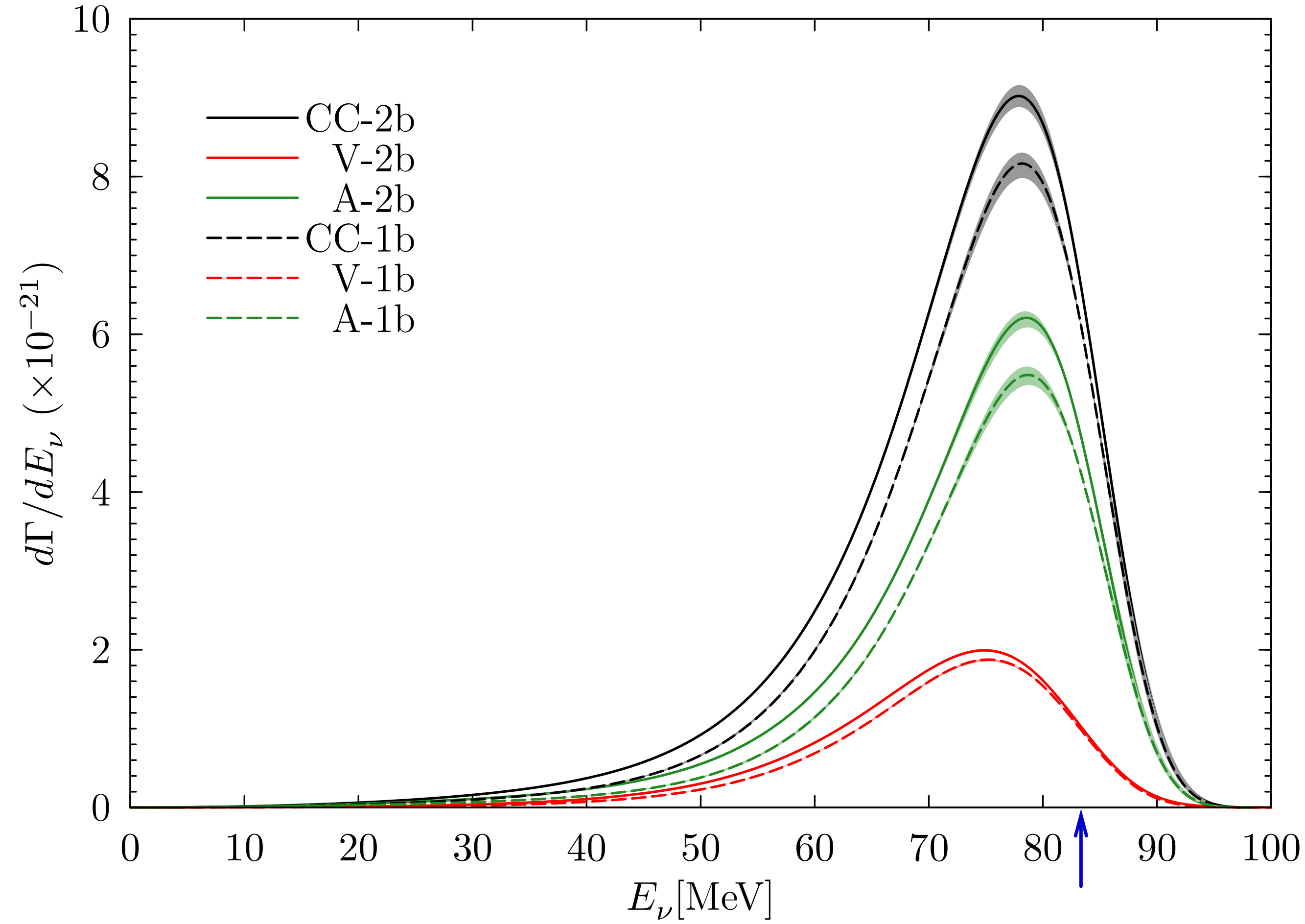
The differential capture rate can be computed interpolating the response functions at

$$\begin{cases} \omega = m_\mu + m_n - m_p - E_\nu \\ |\mathbf{q}| = E_\nu \end{cases}$$

The neutrino energy is in the range

$$0 \leq E_\nu \leq E_\nu^{\max} \approx 83.6\,\mathrm{MeV}$$

and the distribution is is skewed towards the high end of the neutrino energy.



The total rate is obtained integrating the differential one

	V-1b	V-2b	A-1b	A-2b	CC-1b	CC-2b	$\widetilde{\text{CC}}$-1b	$\widetilde{\text{CC}}$-2b
$\Gamma(\mathrm{s}^{-1})$	65 ± 1	73 ± 1	171 ± 6	200 ± 6	265 ± 9	306 ± 9	310 ± 12	355 ± 12

Theory results remarkably close to the ones by Foldy-Walecka (almost 50 years old)!

	Exp [47]	Exp [48]	Exp [49]	Th [50]	Th [51]
$\Gamma(\mathrm{s}^{-1})$	336 ± 75	375^{+30}_{-300}	364 ± 46	345 ± 110	278

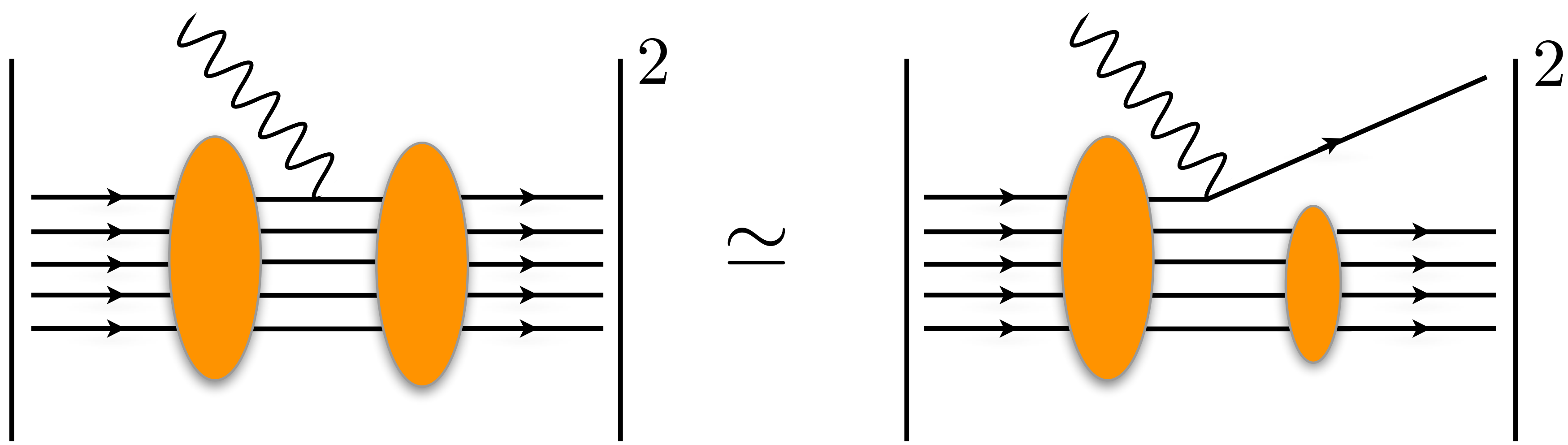
More exclusive processes at relatively-large momentum transfers

"Standard" factorization scheme

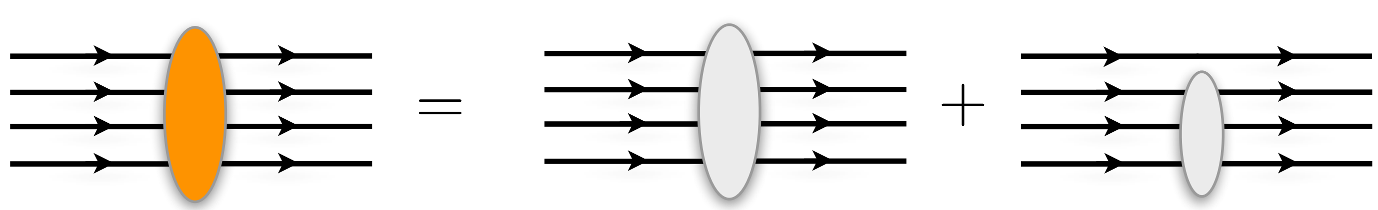
At **large momentum transfer**, scattering off a nuclear target reduces to the sum of scattering processes involving **individual bound nucleons**

$$J^\mu \to \sum_i j_i^\mu \qquad |\psi_f^A\rangle \to |p\rangle \otimes |\psi_f^{A-1}\rangle \qquad E_f = E_f^{A-1} + e(\mathbf{p})$$

The incoherent contribution to the response function is diagrammatically represented as



Excitations of the A-1 final state with **two nucleons in the continuum** are included



“Standard” factorization scheme

The response functions entail the structure and dynamics of the target

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

A **single-particle state completeness** is used to isolate the current matrix element

$$\langle\psi_f^A|J_\alpha|\psi_0^A\rangle \to \sum_k [\langle\psi_f^{A-1}| \otimes \langle k|]\,|\psi_0^A\rangle\langle p|\sum_i j_\alpha^I|k\rangle\,.$$

The **incoherent contribution** of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k},E) \sum_i \langle k|{j_\alpha^i}^\dagger|k+q\rangle\langle k+q|j_\beta^i|k\rangle\delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$

The **hole spectral function** yields the probability of removing a nucleon with momentum **k** from the target ground state leaving the residual system with excitation energy *E*.

$$P_h^{1h}(\mathbf{k},E) \to \sum_f |\langle\Psi_0^A|[|k\rangle \otimes |\Psi_f^{A-1}\rangle]|^2\delta(E - E_f^{A-1} + E_0^A)$$

Hole SF from correlated-basis function

The hole SF of finite nuclei is expressed as a sum of two contributions, displaying distinctly different energy and momentum dependences

$$P_h(\mathbf{k},E) = P_h^{1h}(\mathbf{k},E) + P_h^{\rm corr}(\mathbf{k},E)$$

The 1h terms corresponds to discrete excitations of the A-1 final states

$$P_h^{1h}(\mathbf{k},E) \to \sum_{\bar f} |\langle \Psi_0^A|[|k\rangle \otimes |\Psi_{\bar f}^{A-1}\rangle]|^2 \delta(E - E_{\bar f}^{A-1} + E_0^A)$$

Computing this term in principle requires evaluating single-nucleon overlaps. Within the CBF theory, it is obtained from a modified mean-field scheme

$$P_h^{1h}(\mathbf{k},E) = \sum_{\alpha\in\{\rm F\}} Z_\alpha |\phi_\alpha(\mathbf{k})|^2 F_\alpha(E - e_\alpha) \ ,$$

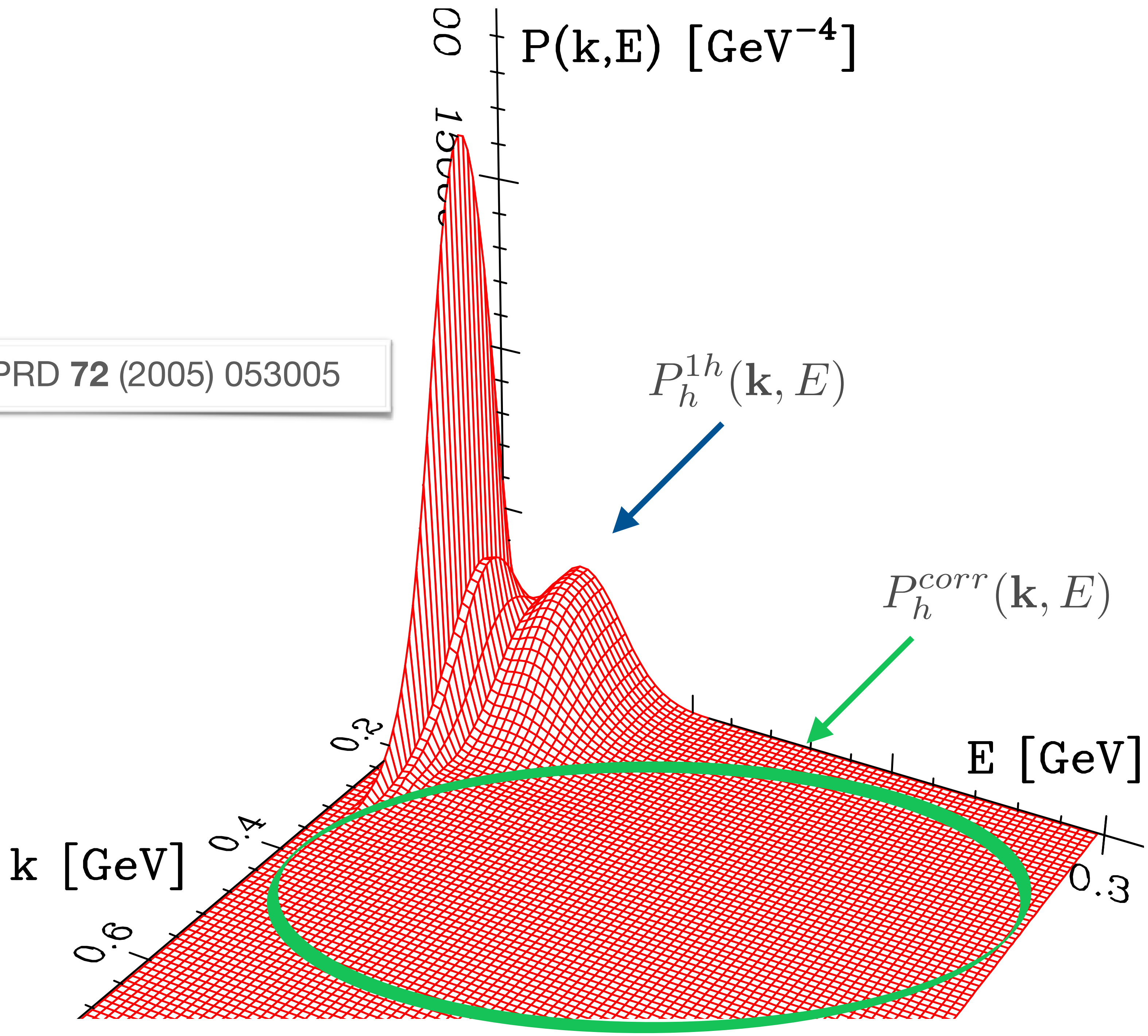
The high-momentum component, corresponding to the A-1 final state in the continuum, is obtained from CBF by calculations in infinite nuclear matter

$$P_h^{\rm corr}(\mathbf{k},E) = \int d^3R \ \rho_A(\mathbf{R}) P_{h,\,NM}^{\rm corr}(\mathbf{k},E;\rho_A(\mathbf{R}))\,,$$

Hole SF from correlated-basis function

The hole SF of finite nuclei is expressed as a sum of two contributions, displaying distinctly different energy and momentum dependences

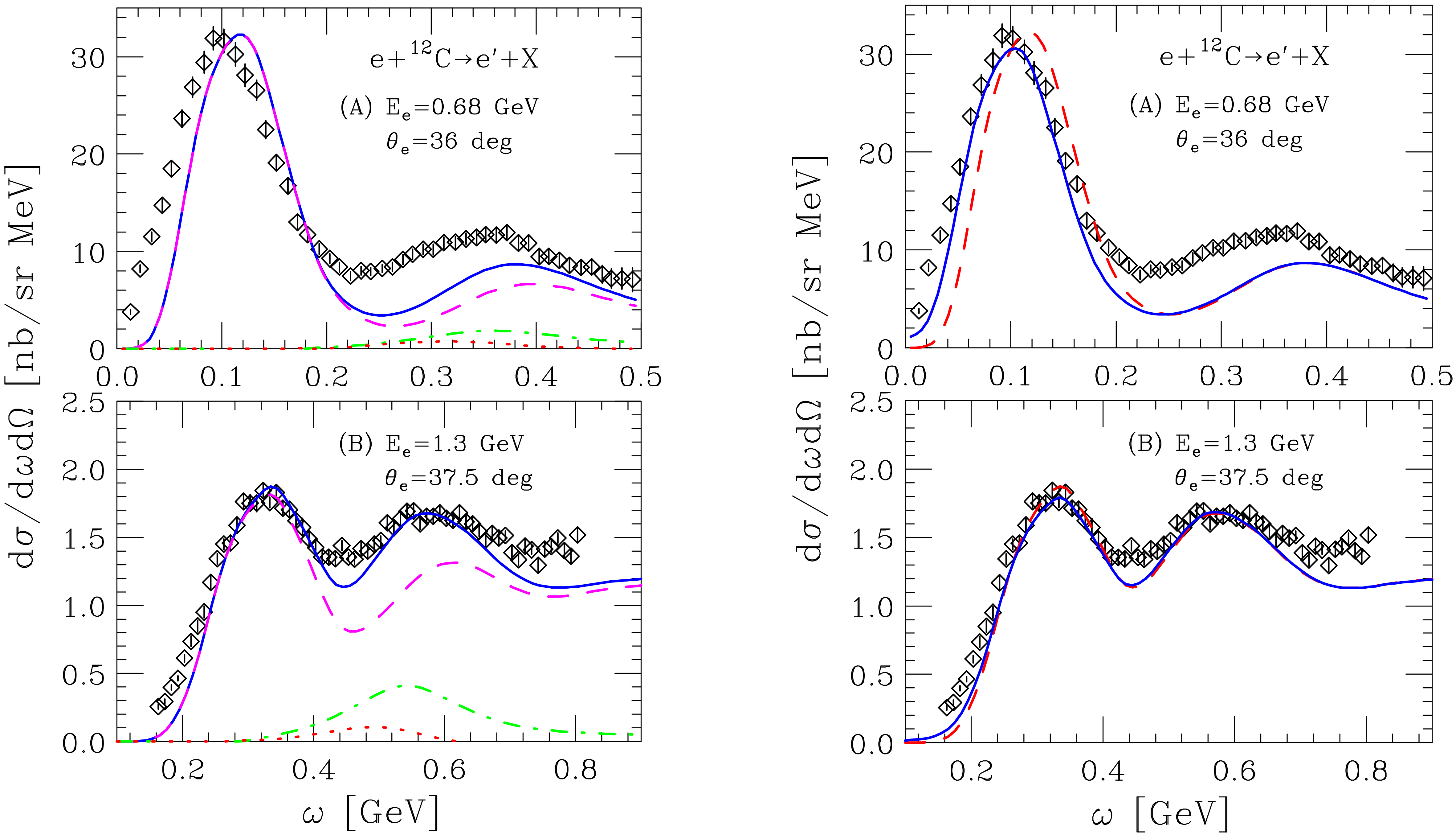
O. Benhar et al. PRD **72** (2005) 053005



“Extended” factorization schemes

Using relativistic MEC requires the extension of the factorization scheme to two-nucleon emissions

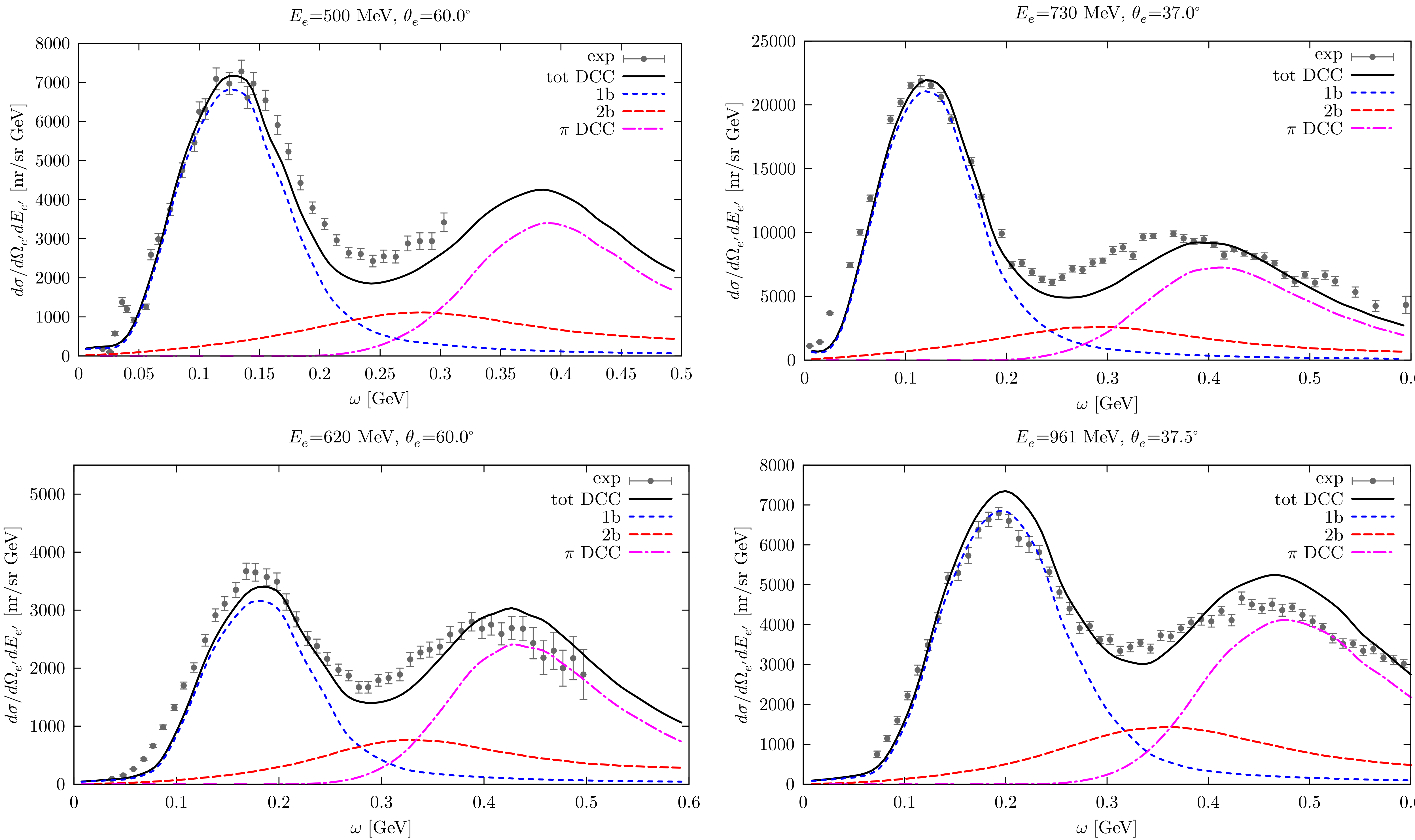
$$|\Psi_f^A\rangle \rightarrow |p_1 p_2\rangle \otimes |\Psi_f^{A-2}\rangle$$



N. Rocco, AL et al. PRL. **116** 192501 (2016)

Inclusion of pion-production mechanisms

The factorization scheme can be **further extended** to include “real” pions in the final state ⟷ $|\Psi_f^A\rangle \rightarrow |p_1 p_\pi\rangle \otimes |\Psi_f^{A-2}\rangle$



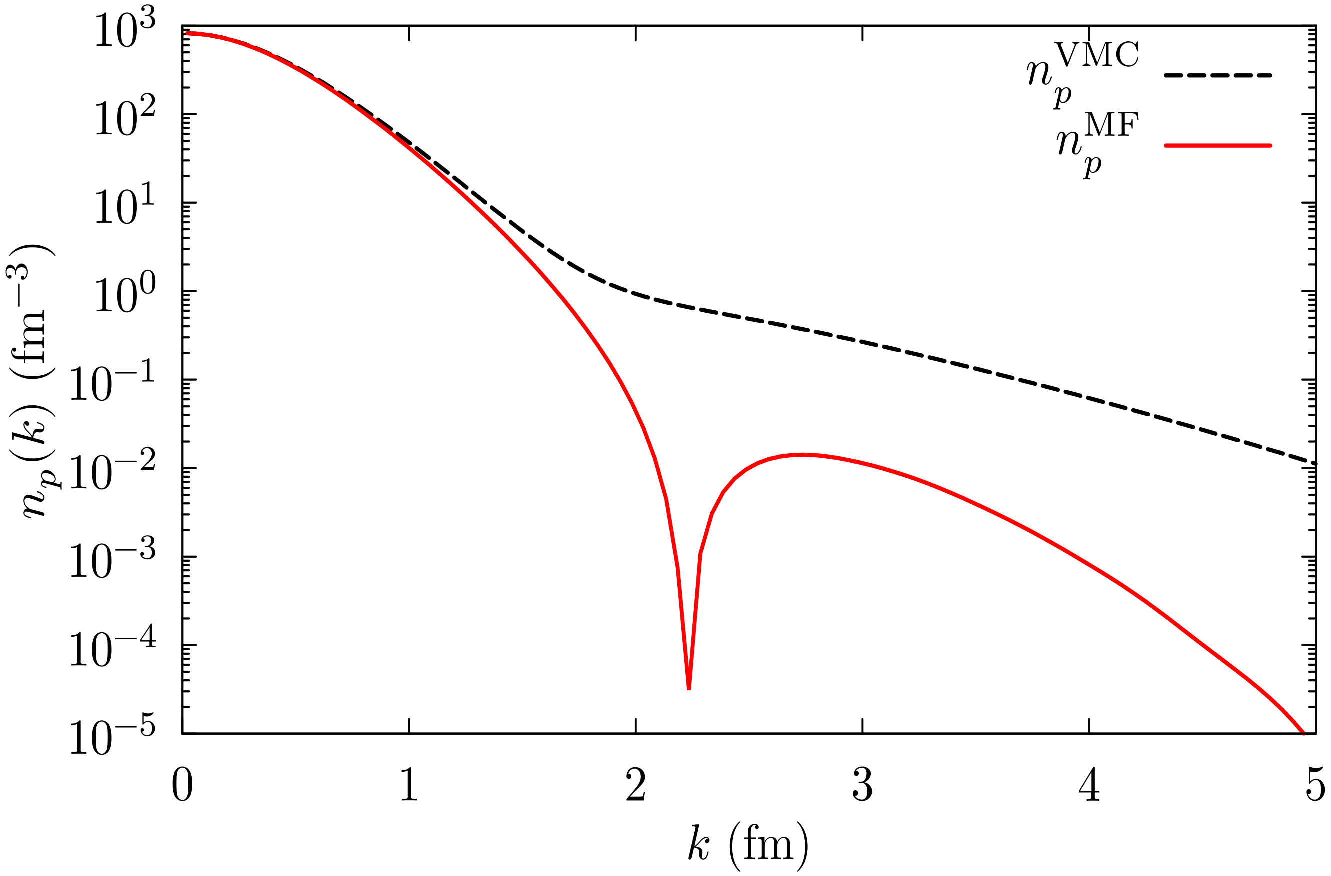
The VMC Spectral Function of ^{4}He

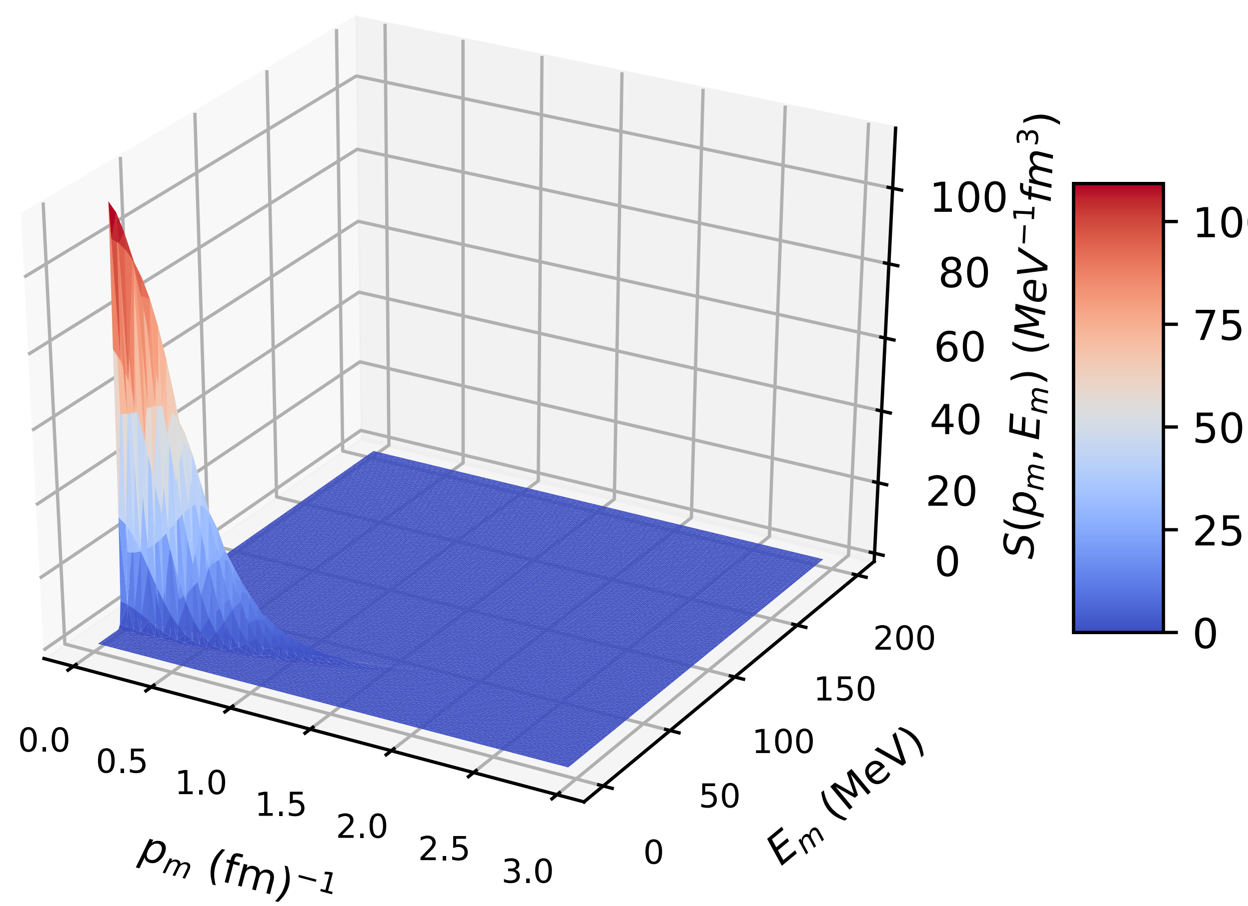
Since there are no excited states in ^{3}H, the 1h contribution is simply given by

$$P_h^{1h}(\mathbf{k}, E) = \sum_{\bar{f}} |\langle \Psi_0^A | [|k\rangle \otimes |\Psi_{\bar{f}}^{A-1}\rangle]|^2 \delta(E - E_{\bar{f}}^{A-1} + E_0^A)$$

$$= \langle \Psi_0^{^4\text{He}} | [|k\rangle \otimes |\Psi_0^{^3\text{H}}\rangle]|^2 \delta\left(E - E_0^{^3\text{H}} - \frac{k^2}{2M^{^3\text{H}}} + E_0^{^4\text{He}}\right)$$

The single-nucleon overlap can be (and have been) computed by Bob Wiringa within VMC (center of mass motion fully accounted for)





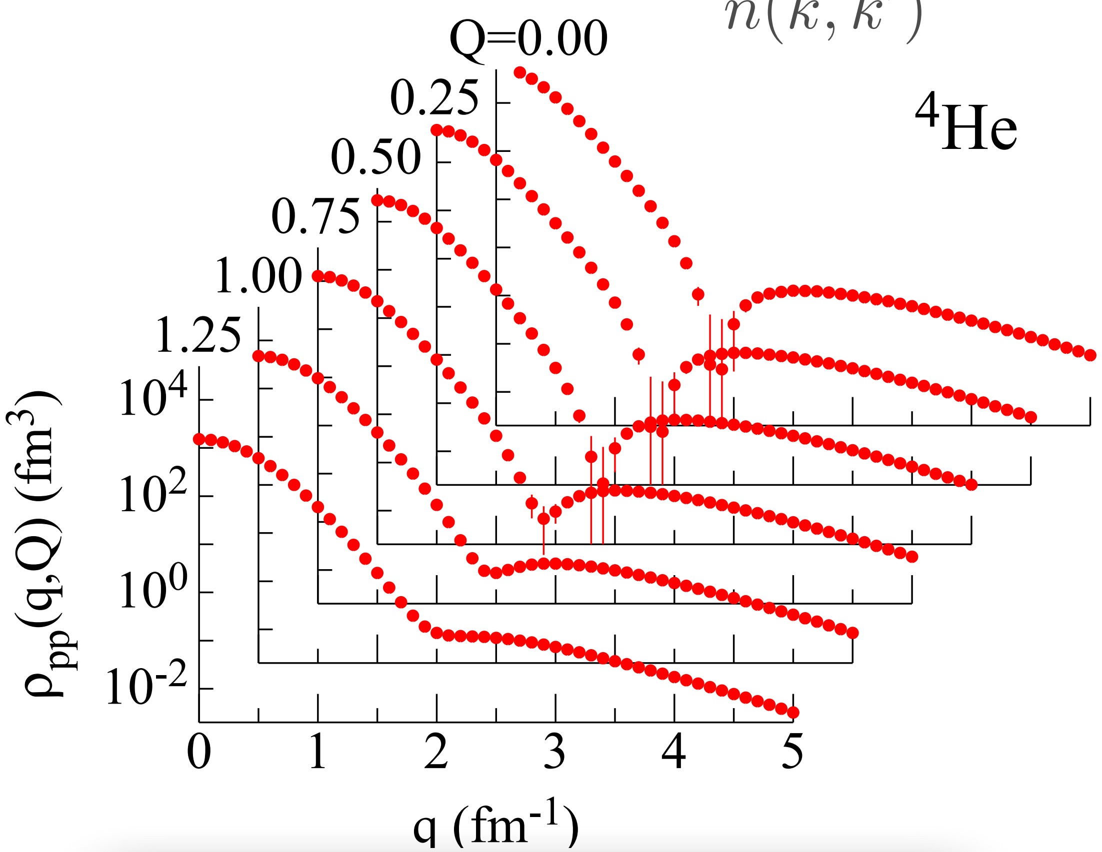
The VMC Spectral Function of ^{4}He

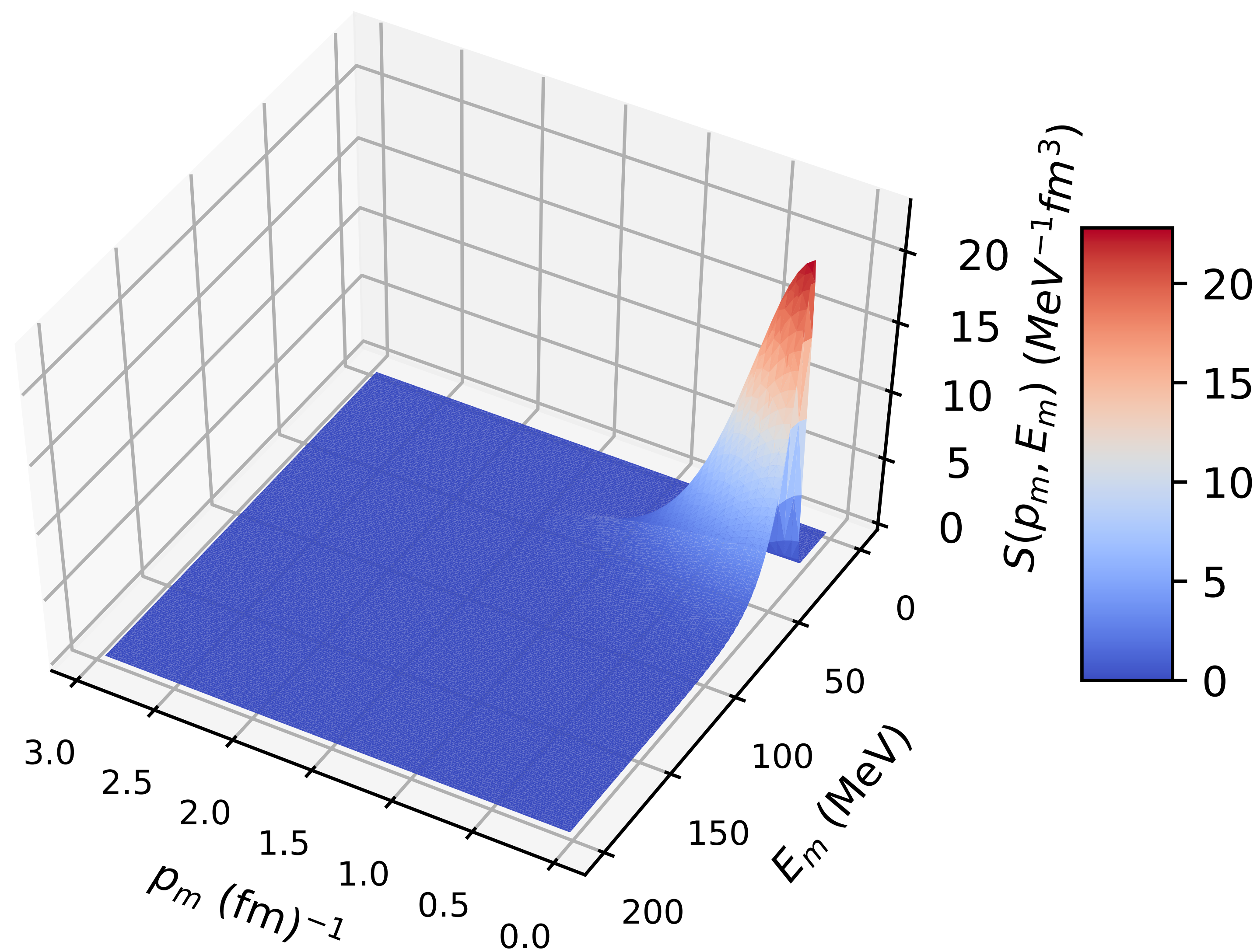
To determine the correlation component we utilize the two-nucleon momentum distributions computed within VMC

$$P_h^{corr}(\mathbf{k},E) \simeq \sum_f |\langle\Psi_0^A|[|kk'\rangle \otimes |\Psi_f^{A-2}\rangle]|^2 \delta\left(E - E_f^{A-2} + E_0^A - e(\mathbf{k}') - \frac{(\mathbf{k}+\mathbf{k}')^2}{2M_{^2\mathrm{H}}}\right)$$

$$\simeq \langle\Psi_0^A|\,|kk'\rangle\langle kk'|\,|\Psi_0^A\rangle \delta\left(E - \epsilon^{A-2} + E_0^A - e(\mathbf{k}') - \frac{(\mathbf{k}+\mathbf{k}')^2}{2M_{^2\mathrm{H}}}\right)$$

$n(k,k')$





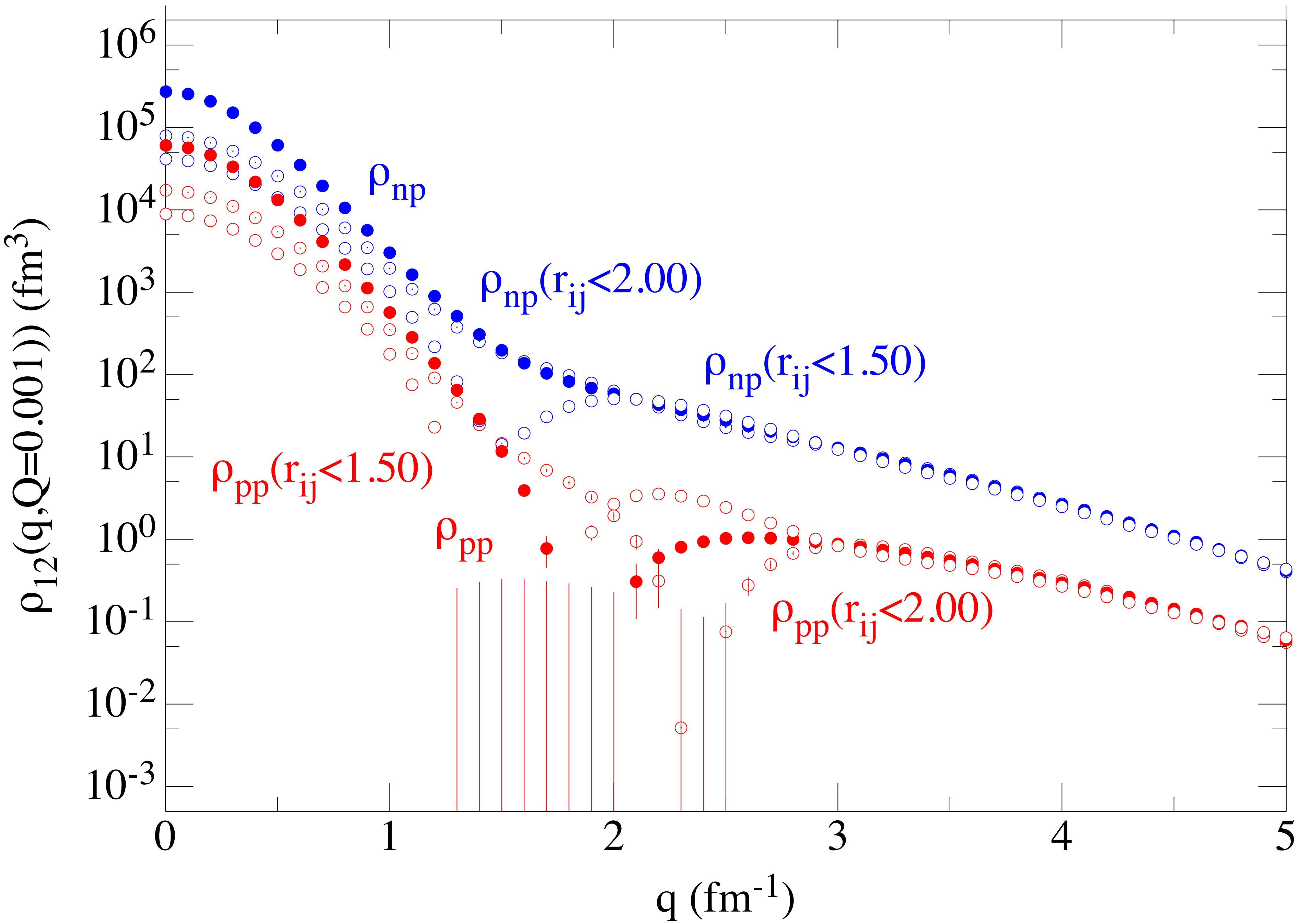
R.B. Wiringa, et al, PRC **89**, 024305 (2014)

The VMC Spectral Function of ^{4}He

Ideally, one should orthogonalize with the single-nucleon overlap

$$|k'\rangle \otimes |\Psi_f^{A-2}\rangle \rightarrow |k'\rangle \otimes |\Psi_f^{A-2}\rangle - |\Psi_{\bar{f}}^{A-1}\rangle\langle\Psi_{\bar{f}}^{A-1}|[|k'\rangle \otimes |\Psi_f^{A-2}\rangle]$$

Inspired by the contact formalism, we put a **cut on the relative distance** of the pair

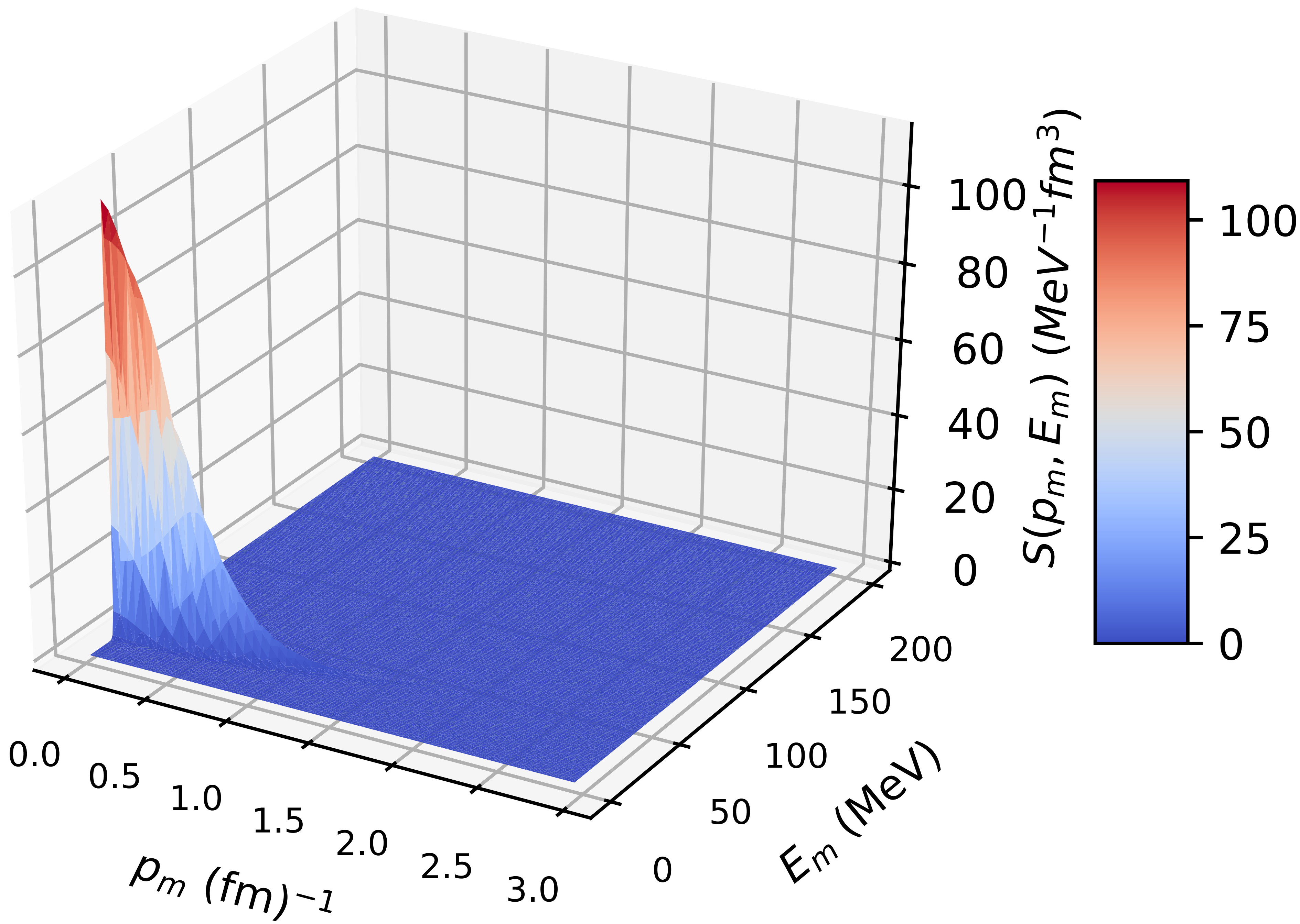


The VMC Spectral Function of ^{4}He

Ideally, one should orthogonalize with the single-nucleon overlap

$$|k'\rangle \otimes |\Psi_f^{A-2}\rangle \rightarrow |k'\rangle \otimes |\Psi_f^{A-2}\rangle - |\Psi_{\bar{f}}^{A-1}\rangle\langle\Psi_{\bar{f}}^{A-1}|[|k'\rangle \otimes |\Psi_f^{A-2}\rangle]$$

Inspired by the contact formalism, we put a **cut on the relative distance** of the pair



Implementation in generators

The **extended factorization scheme** can be readily implemented in neutrino event-generators

- Single-nucleon knock out:

$$P_{N_1}(p_1) = \int d^3k_1 d^3k_2 d^3k_1' d^3k_2' F(\omega; k_1, k_2, k_1', k_2' \to p_1)$$

- Two-nucleon knock-out

$$P_{N_1 N_2}(p_1, p_2) = \int d^3k_1 d^3k_2 d^3k_1' d^3k_2' \, F(\omega; k_1, k_2, k_1', k_2' \to p_1, p_2)$$

- Pion production processes

$$P_{N_1 \pi_1}(p_1, p_{\pi_1}) = \int d^3k_1 d^3k_2 d^3k_1' d^3k_2' \, F(\omega; k_1, k_2, k_1', k_2' \to p_1, p_{\pi_1})$$

Classical Monte Carlo cascade models do not modify the inclusive cross section

Test the reaction mechanisms on inclusive cross sections before implementing them

Conclusions & Plans

Conclusions

- GFMC calculations of ^{12}C electromagnetic responses in good agreement with experiments;
- Two-body currents enhance the electromagnetic, neutral- and charged-current responses;
- Total muon capture rate in ^{4}He in good agreement with available experimental data,
- Implemented pion-production amplitudes for electron and neutrino scattering; good agreement with data
- Started VMC calculations of the hole spectral function

Ongoing Plans

- Muon capture rate of ^{4}He employing chiral-EFT potentials and consistent electroweak currents;
- Complete calculations of the charged-current neutrino and anti-neutrino - ^{12}C cross sections;
- GFMC calculations of the spectral function of light nuclei using imaginary-time techniques

$$\int dE e^{-E\tau} P_h(\mathbf{k}, E) \sim \frac{\langle \Psi_0 | a^\dagger_{\mathbf{k}} e^{-(H-E_0)\tau} a_{\mathbf{k}} | \Psi_0 \rangle}{\langle \Psi_0 | e^{-(H-E_0)\tau} | \Psi_0 \rangle}$$

- Cluster expansion formalism to compute the interference between one and two-body currents

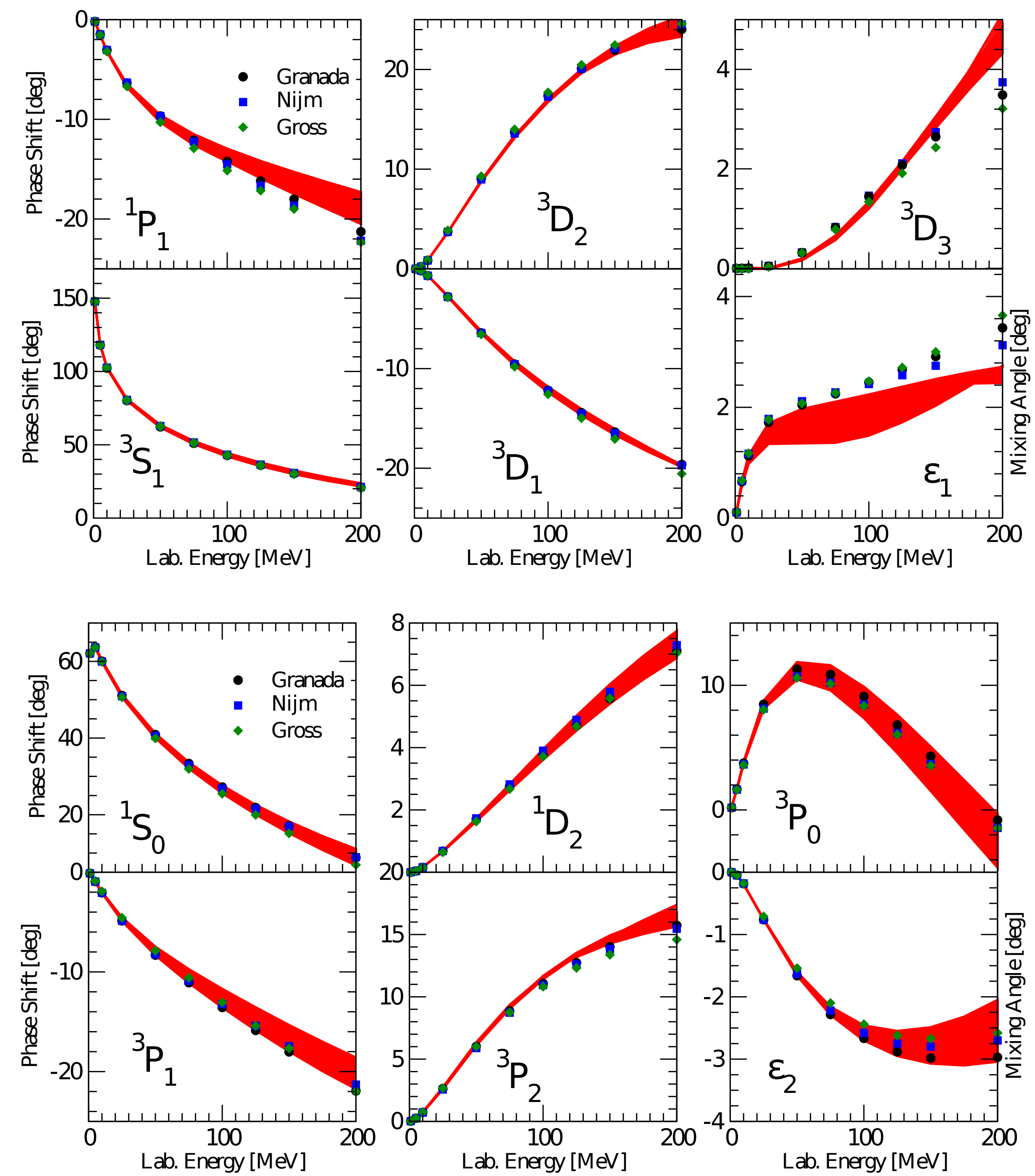
Thank you

Δ-full local chiral potential

We have complemented the historical "Argonne" approach by considering a local chiral Δ -full potential giving an excellent fit to the NN scattering data that can be readily used in QMC.

- Closer connection with QCD
- Consistent MEC available
- Theoretical uncertainty estimation

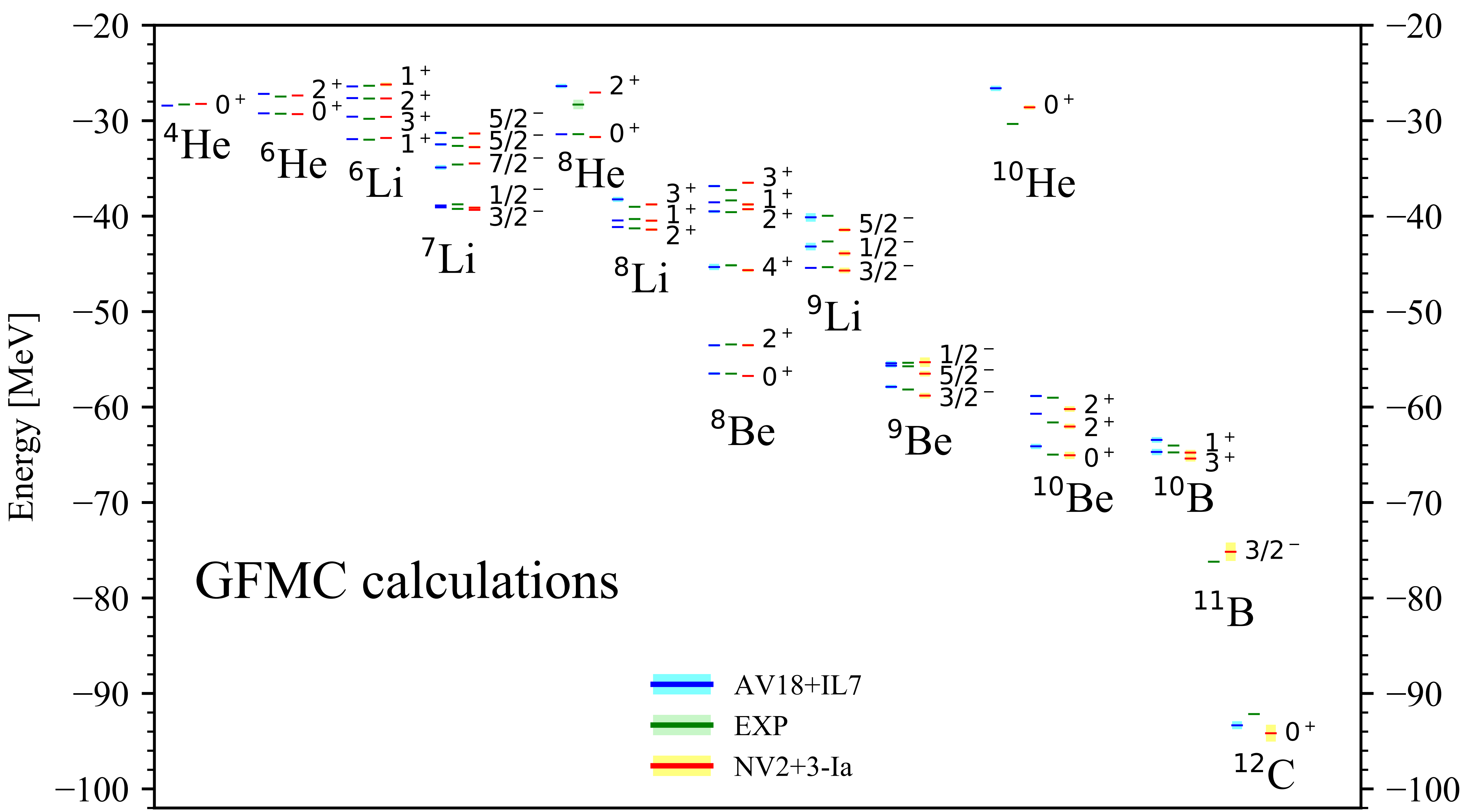
model	order	E_{Lab} (MeV)	N_{pp+np}	χ^2/datum
b	LO	0–125	2558	59.88
b	NLO	0–125	2648	2.18
b	N2LO	0–125	2641	2.32
b	N3LO	0–125	2665	1.07
a	N3LO	0–125	2668	1.05
c	N3LO	0–125	2666	1.11
$\widetilde{a}$	N3LO	0–200	3698	1.37
$\widetilde{b}$	N3LO	0–200	3695	1.37
$\widetilde{c}$	N3LO	0–200	3693	1.40



M. Piarulli et al. PRC **94** (2016) 054007

Δ-full local chiral potential

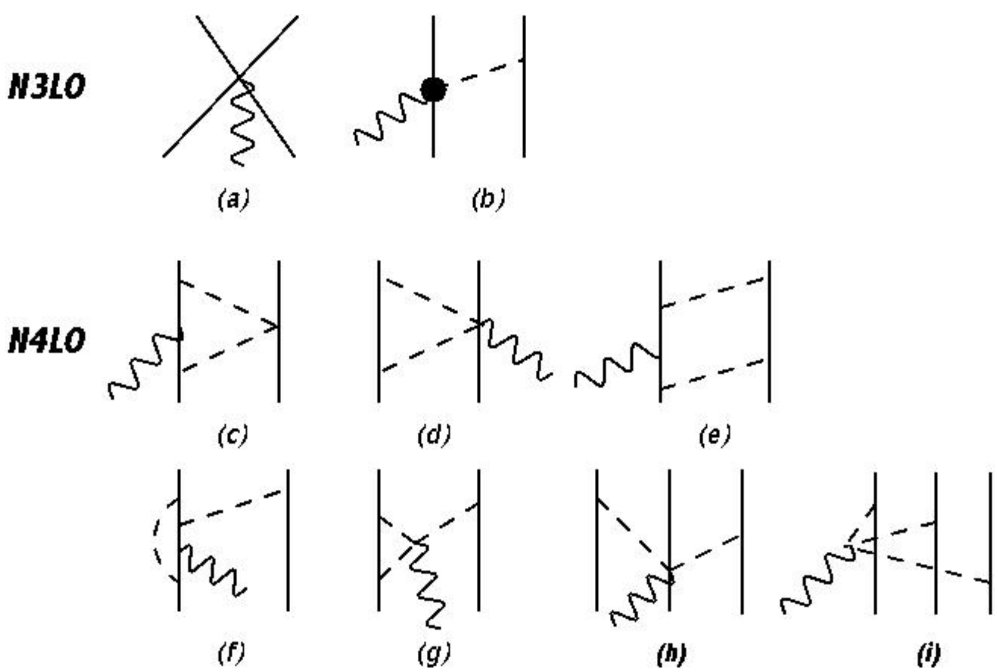
The experimental A≤12 ground- and excited state energies are very well reproduced by the local Δ-full NN+NNN chiral interaction

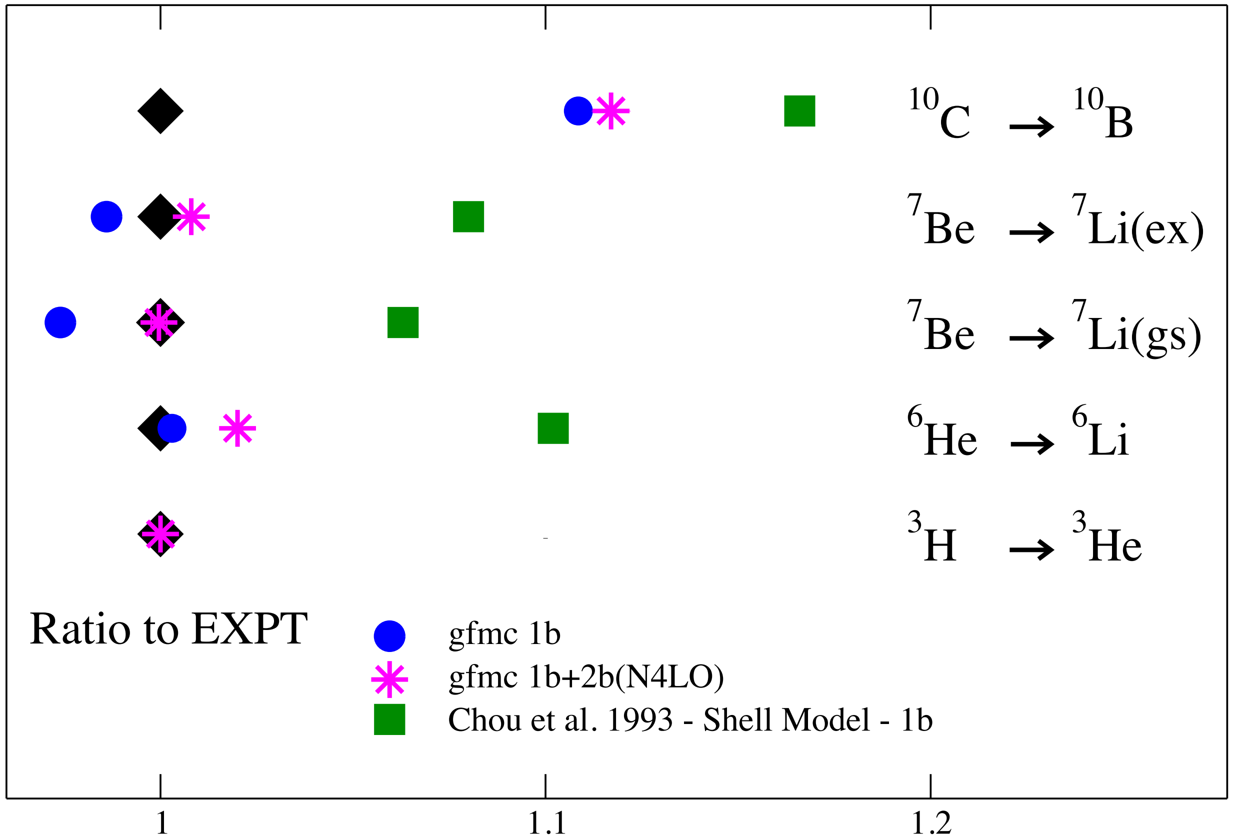


M. Piarulli, et al. PRL **120** (2018) 052503

Chiral-EFT currents

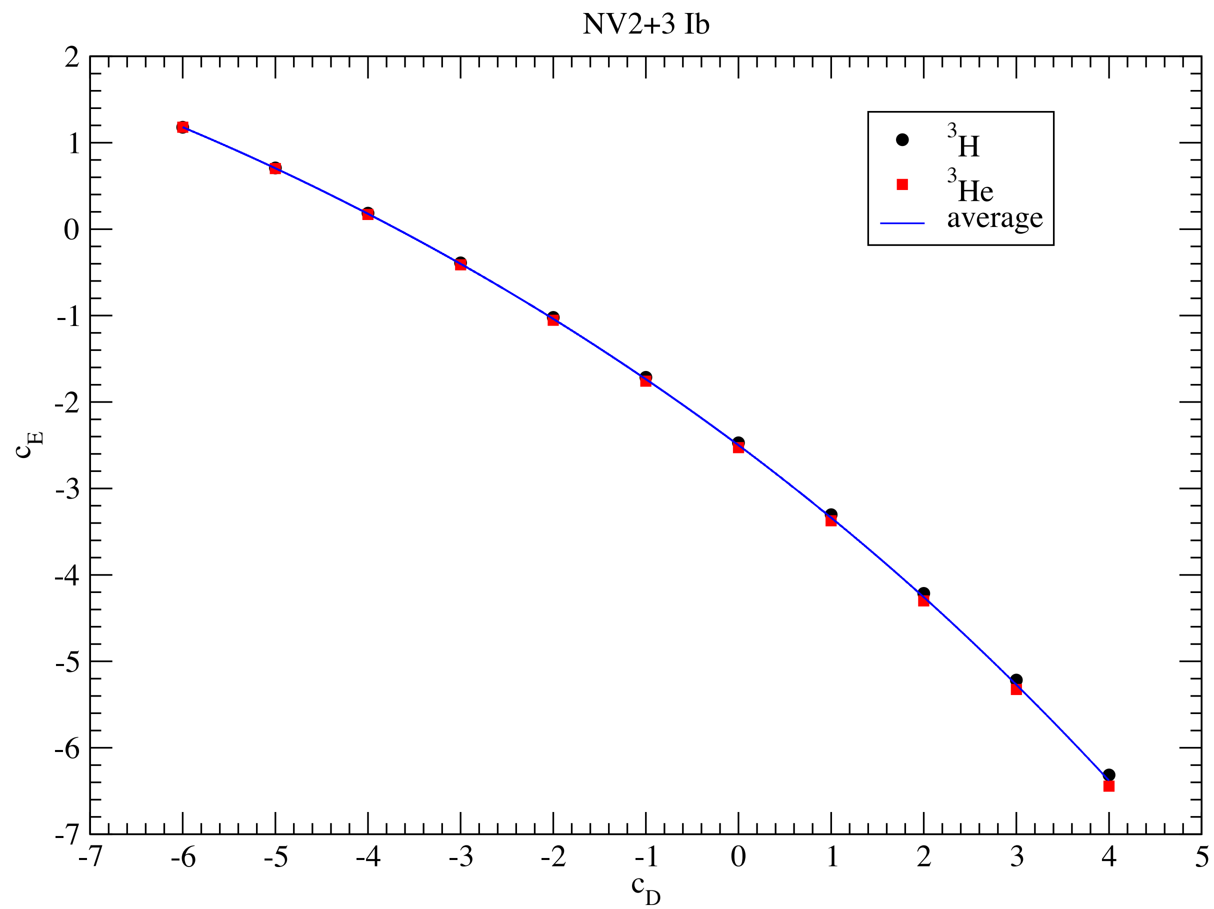
- Chiral currents consistent with the Δ-full local chiral potential have been developed
- Mixed-approach calculations indicate a slight enhancement of the decay rates from MEC





S. Pastore et al. PRC **97**, 022501 (2018)

Nuclear spectra and decays



A.Baroni et al., PRC **98**, 044003 (2018)

- Triton and ^{3}He average binding energies provide a correlation line between c_D and c_E

- Triton decay is most sensitive to c_D