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MADX-SC Development (for Booster Simulations)

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IOTA Collaboration Meeting & High Intensity Beams in Rings Workshop

10-11 June 2019

Issues & Goals

stage	E_inj (MeV)	Np/batch (e12)	ΔQx/ΔQy
PIP-I	400	4.5	0.25/0.31
PIP-I+	400	5.6	0.31/0.38
PIP-II	800	6.5	(0.36/0.44)*

*) would be with 400MeV injection

Losses at nominal (PIP-I) intensity were ~8%, can increase at high intensity operation

Simulations goals:

- understand experimental observations
- make projection for high intensity

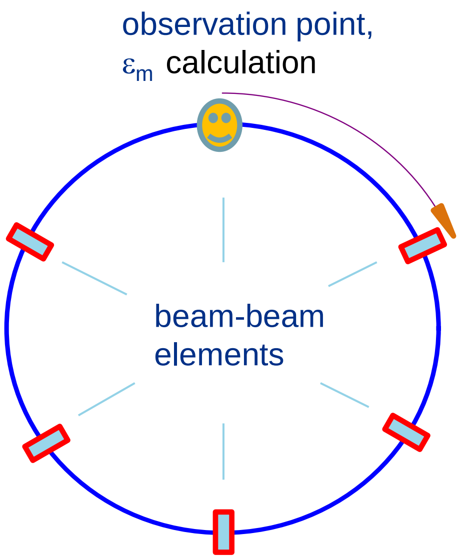
Tools used:

- Synergia (A. Macridin, E. Stern)
- MADX-SC (Y.A., A. Valishev with a lot of help from F. Schmidt)

MADX with Space Charge (MADX-SC)*

"Adaptive" SC simulations:

- Beam shape is simplified (Gaussian for now) to use analytics for SC kick
- Beam sizes are periodically updated (e.g. every turn) based on the ensemble evolution during tracking (c.o.m. position can be also updated).



$$\sigma_u^2 = \sum_m \beta_{um}\varepsilon_m, \quad u = x, y, z$$

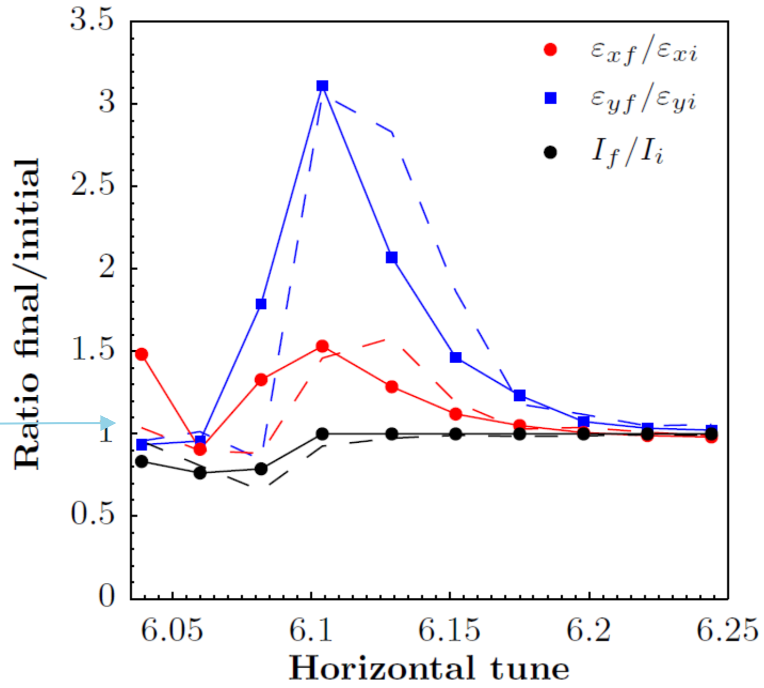
"Old" version:

- 2D SC kick calculated using Erskine-Basetti formula – no associated longitudinal kick (no symplecticity).
- Exponential fitting of 1-dimensional distributions in the transverse action variables – requires stable closed optics which may not exist at strong SC
- Periodicity of SC is imposed – particle-envelope resonance is suppressed

*) Important contribution was made by V. Kapin and A. Valishev

Old MADX-SC Benchmarking vs PS Data

Blowup at Q_{x0} = 6.035 was understood as the statistical noise effect

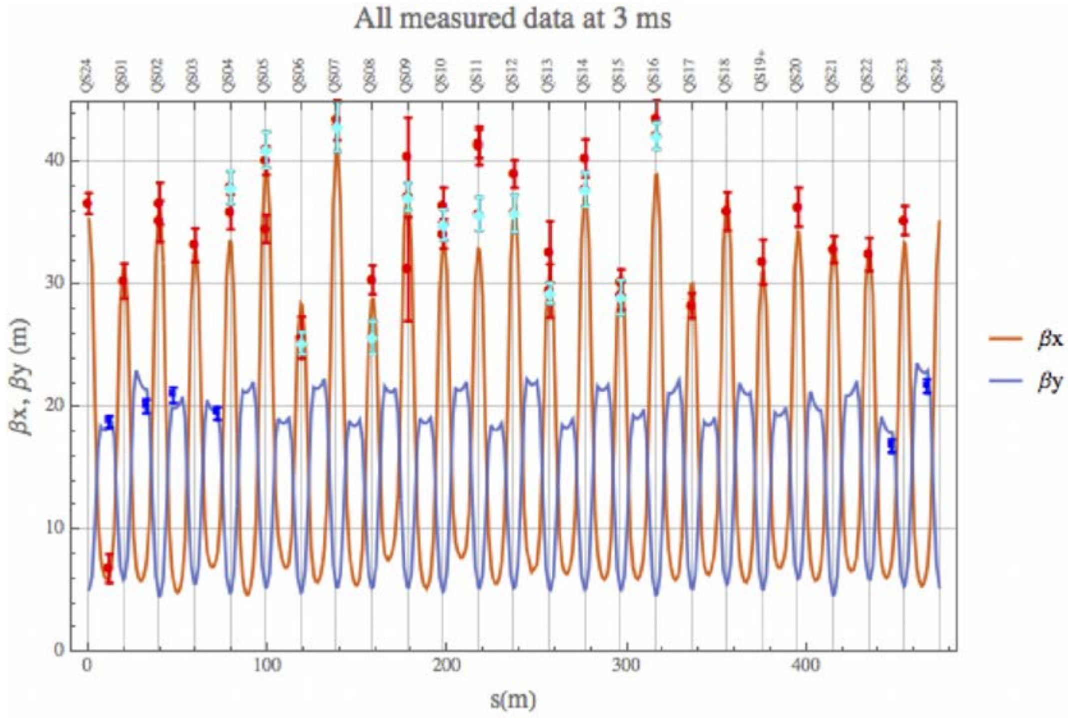


Q_{y0} = 6.476,
SC tuneshifts :
$\Delta Q_x \approx$-0.05, $\Delta Q_y \approx$-0.07.

PS beam emittance evolution over $5{\cdot}10^5$ turns at 2GeV vs. Q_{x0}. Dashed lines present experimental results, solid lines with dots present MADX simulations with adaptive SC.

NB:
good agreement for small SC does not guarantee validity for high SC

Booster "Flat" Optics Conundrum



Old HEP optics model (MADX) confirmed by K-modulation measurements shows strong perturbation by the extraction dogleg.

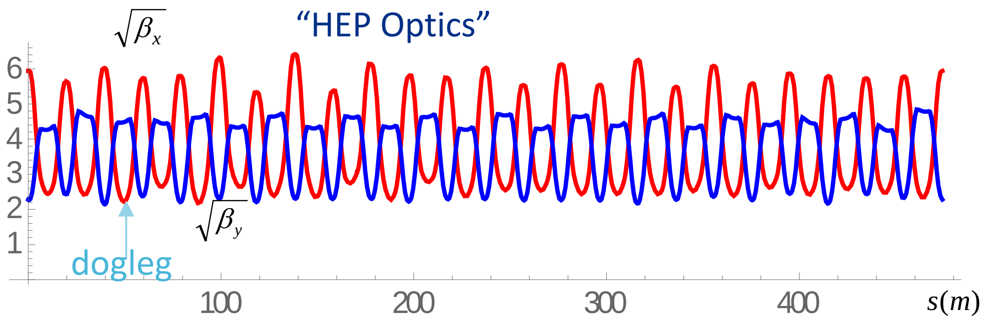
This perturbation can be corrected with tuning quads → "flat optics"

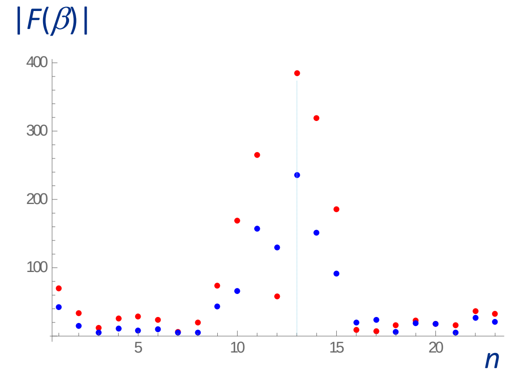
Optics Functions w/o SC

Fourier Sectra of β-functions

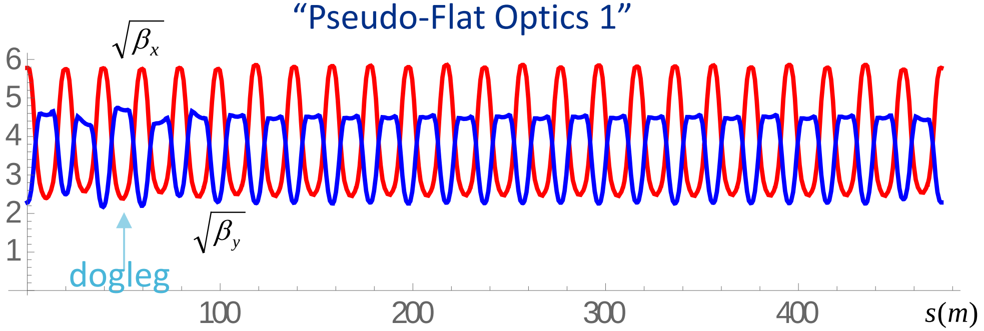
Red: hor
Blue: ver

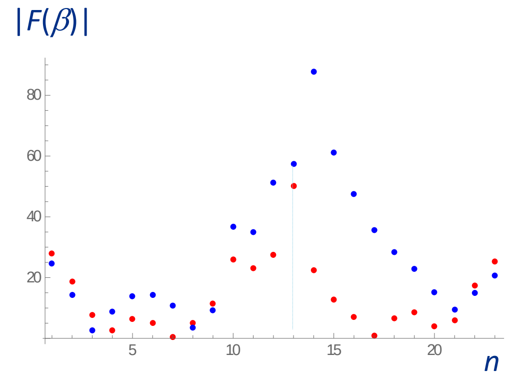
“HEP Optics”



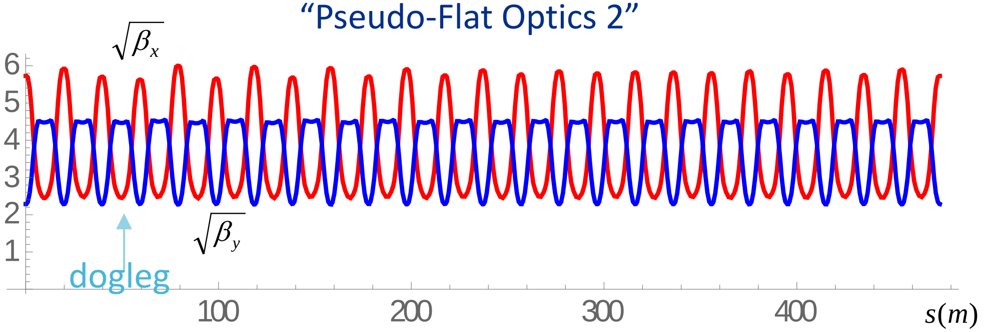


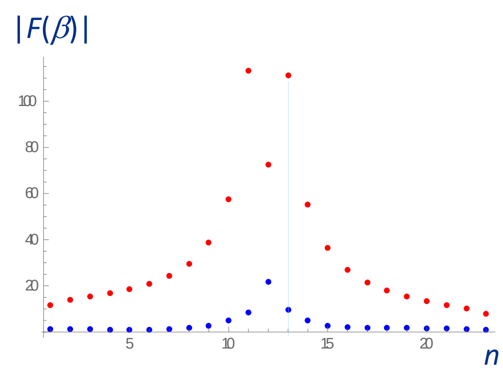
“Pseudo-Flat Optics 1”





“Pseudo-Flat Optics 2”





Tracking Simulations

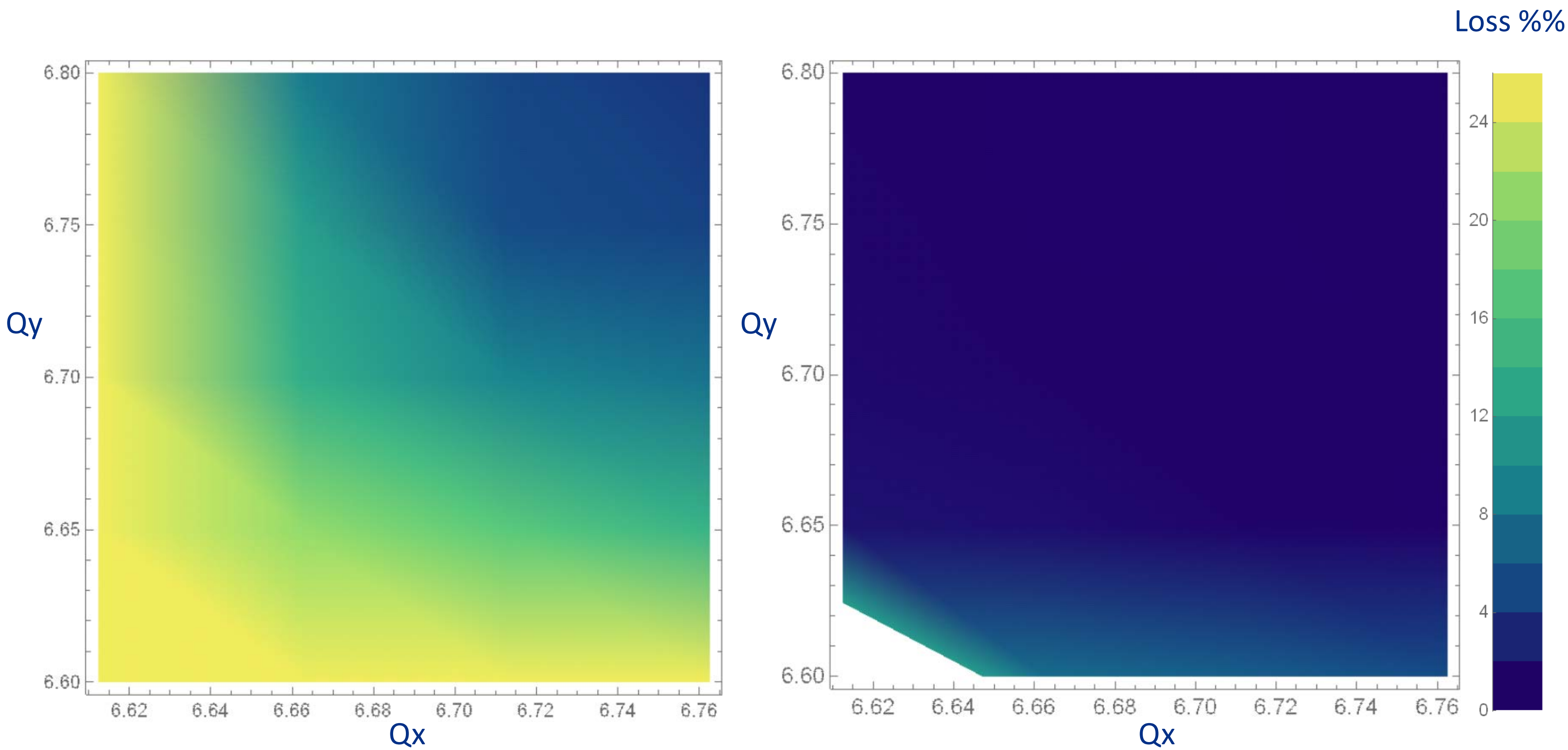
Beam parameters (used by A. Macridin in Synergia simulations):

Energy = 415 MeV
$\varepsilon_{\perp N}^{(r.m.s.)}$=2.34$\mu$m ($\varepsilon_{\perp N}^{(95\%)}$=14$\pi$ mm·mrad)
σ_z= 0.831532m, σ_p/p= 0.00185,

Space charge tuneshifts to 0.24, 0.32 for Np=5.6e10/bunch

Tracking 5k particles for 2000 turns at fixed energy $\rightarrow$ the effect of space charge (if any) is significantly exaggerated.

MADX-SC Simulations for HEP and "Flat" Lattice



Losses over 2000 turns as function of bare lattice tunes at nominal Np=5.6e10/bunch.
At Qx=6.7, Qy=6.8: HEP → 3.8%, "flat" → 0%

But operations showed no improvement with "flat" lattice!
Is anything wrong with MADX-SC?

New algorithm

- Gaussian fit of the Σ-matrix

$$\Sigma_{i,j} = \frac{1}{N}\sum_{k=1}^{N} \zeta_i^{(k)} \zeta_j^{(k)}, \quad \underline{\zeta}^{(k)} = \underline{z}^{(k)} - \langle \underline{z} \rangle$$

- Σ-matrix propagation from observation point (1) to SC elements (2) using linear(ized) transport matrix T
- does not require stable optics to exist,
- allows for nonstationary distribution - envelope resonances!

$$\Sigma^{(2)} = \mathrm{T} \cdot \Sigma^{(1)} \cdot \mathrm{T}^{t}$$

- Particle tracking with symplectic 3DoF SC kick (for Gaussian beam profile in all 3DoF for now)

Gaussian Fit

Y.A. “Computing Eigen-Emittances from Tracking Data”, arXiv:1409.5483, 2014; NAPAC-2016-THPOA17

The rigorous minimization process for $\int [G(\underline{z})-F(\underline{z})]^2 d^n z$ where provides equation for fitted Σ- matrix which can be solved by iterations

$$G(\underline{z})=\frac{1}{N}\sum_{k=1}^{N}\delta_{6D}(\underline{z}-\underline{z}^{(k)})$$

$$F(\underline{\zeta})=\frac{\eta}{(2\pi)^{n/2}\sqrt{\det\Sigma}}\exp[-\frac{1}{2}(\underline{\zeta},\Sigma^{-1}\underline{\zeta})]$$

$$\Sigma_{ij}=\frac{1}{N}\sum_{k=1}^{N}\zeta_i^{(k)}\zeta_j^{(k)}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]\Big/\left(\frac{1}{N}\sum_{k=1}^{N}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]-\frac{\eta}{2^{n/2+1}}\right)$$

where *n* is the dimensionality of the problem (any, e.g. 6) and η is the fraction of particles in the core. It can be fitted in the process as well:

$$\eta=\frac{2^{n/2}}{N}\sum_{k=1}^{N}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]$$

Problem:

Effective weight $W_k=\exp[-(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})/2]$ provides too aggressive suppression of contribution of moderate amplitude particles → reduction in the effective number of macro-particles → higher statistical fluctuations, in particular “fake coupling”

Solution:

Introduce softer weights retaining the general form of the equation for fitted Σ

Introducing Weights for General Distribution

We can define fitted Σ matrix using weight function *W*(*z*) as

$$\Sigma_{ij}^{(\mathrm{fit})} = \frac{\frac{1}{N}\sum_{k=1}^{N} z_i^{(k)} z_j^{(k)} W(z^{(k)})}{\frac{1}{N}\sum_{k=1}^{N} W(z^{(k)}) - p}$$

where the correction term *p* was introduced as it appears in the rigorous solution on the previous slide. To get the correct Σ matrix element for a sample realizing the distribution function *F*(*z*)

$$p = \int_{\Omega} W(z)F(z)d^n z - \int_{\Omega} z_i^2 W(z)F(z)d^n z \Big/ \int_{\Omega} z_i^2 F(z)d^n z,$$

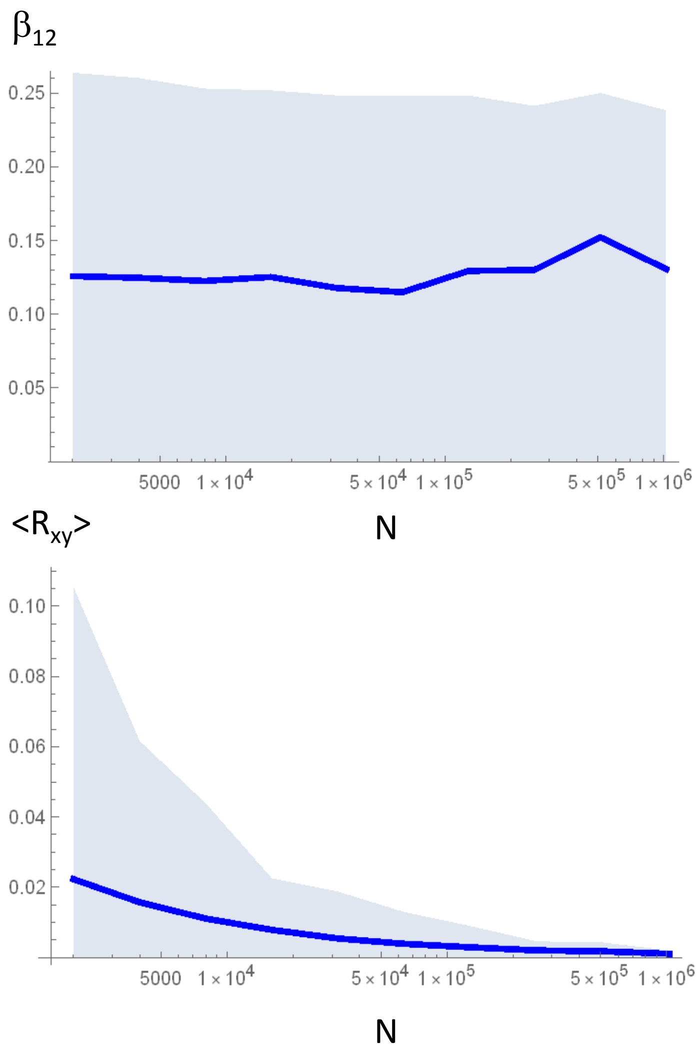
For a *n*-dimensional Gaussian distribution *F*(*z*) and weight function $W = \exp[-\alpha(\underline{\zeta}, \Sigma^{-1}\underline{\zeta})]$ we get

$$p = \frac{2\alpha}{(1+2\alpha)^{n/2+1}}$$

With α=1/2 p=1/2$^{n/2+1}$ and we retrieve the "rigorous" result. A smaller value α=1/5 looks like the optimum.

In principle, for every dimension we can use different *F*, *W* and *p*

Statistical Effects due to Small N Macroparticles



The most annoying is "fake coupling ".

Cross-plane beta-function β_{12} can be considered as a measure of coupling.

When reconstructed from a Σ-matrix obtained from particle distribution with equal emittances it does vanish for $N \to \infty$

Luckily, we are not using β_{12}

Correlation factor

$$R_{xy} = \langle xy \rangle / \sqrt{\langle x^2 \rangle \langle y^2 \rangle}$$

i.e. beam ellipse tilt, is vanishing as $1/N^{1/2}$, but is rather large for practical N.

It can be suppressed by symmetry in the initial distribution but will likely reappear

Sigma-Matrix Propagation

$$\Sigma_{i,j} = \frac{1}{N}\sum_{k=1}^{N} \zeta_i^{(k)} \zeta_j^{(k)}, \quad \underline{\zeta}^{(k)} = \underline{z}^{(k)} - \langle \underline{z} \rangle$$

Two options implemented:

- Periodic Σ mode (next slide)
- Free Σ mode: fitted Σ-matrix propagated from observation point (1) to SC elements (2) around the ring using linear(ized) transport matrix T

$$\Sigma^{(2)} = \mathrm{T} \cdot \Sigma^{(1)} \cdot \mathrm{T}^{\mathrm{t}}$$

Linearization of the SC force:
- averaging over transverse variables (Sacherer, 1971) gives factor 1/2 compared with small amplitudes in Gaussian beam,
- averaging over longitudinal coordinate gives another factor $1/\sqrt{2}$ in the case of Gaussian profile.

The total factor $1/2^{3/2}$ makes the SC tuneshift of envelope oscillations in a Gaussian bunch much smaller than the tuneshift for small-amplitude particles weakening the effect of (Gluckstern's) particle-envelope resonance.

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Periodic Sigma-Matrix

- Using (fitted) Σ-matrix find the eigen-mode emittances ε_m, m=1,2,3, which are imaginary parts of eigenvalues of matrix

$$\Omega = \Sigma \mathrm{S} \qquad \mathrm{S} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- The periodic Σ matrix is

$$\tilde{\Sigma}_{ik} = \sum_{m=1}^{3} \varepsilon_m (v'_{2m-1,i} v'_{2m-1,k} + v''_{2m-1,i} v''_{2m-1,k}) \qquad \underline{v}_i^{/} \equiv \mathrm{Re}\, \underline{v}_i, \quad \underline{v}_i^{//} \equiv \mathrm{Im}\, \underline{v}_i$$

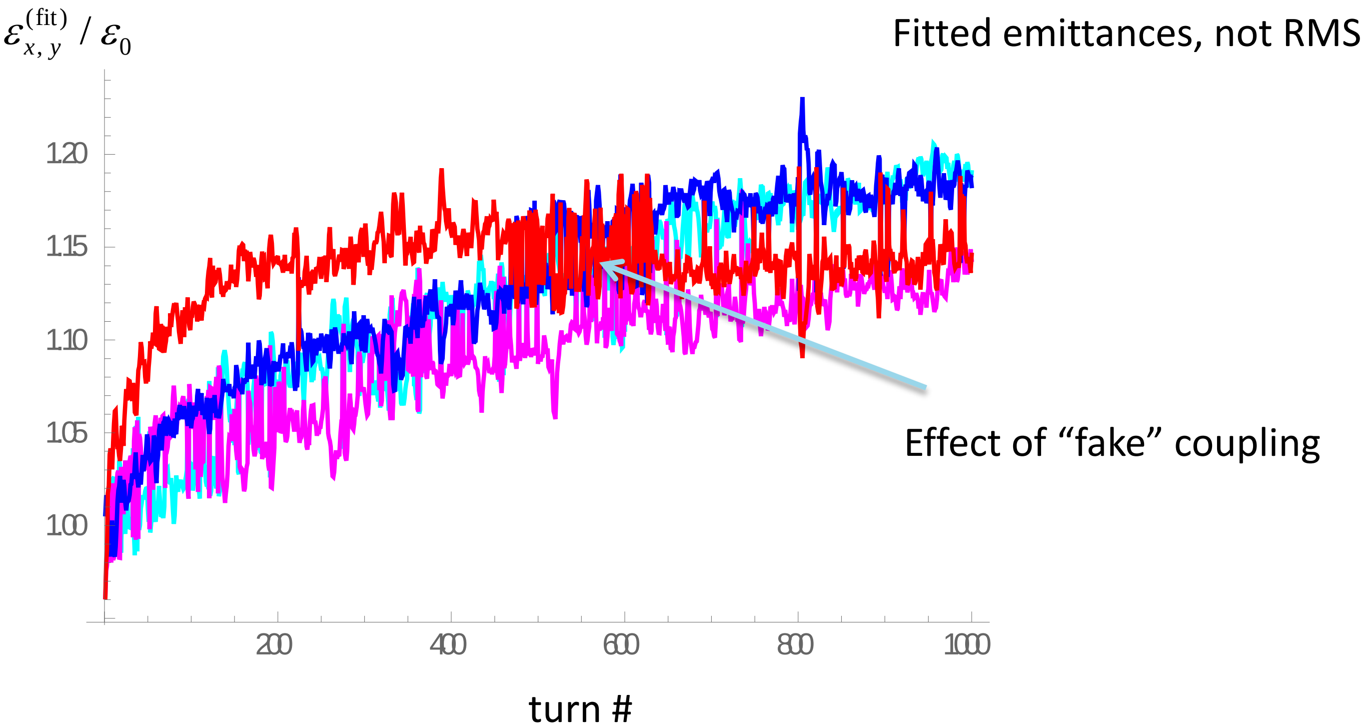
where $v_{m,k}$ means k-th component of m-th eigenvector of the 1-turn transfer matrix T

$$\mathrm{T}\, \underline{v}_k = \lambda_k \underline{v}_k, \quad \lambda_{2m} = \lambda^*_{2m-1}, \quad \underline{v}_{2m} = \underline{v}^*_{2m-1}$$

- The periodic Σ matrix provides a quasi-stationary solution, the envelope oscillations hence the particle-envelope resonances are suppressed.

Toy Lattice with High Space Charge

12-cell FODO with 1% error in 1 quad. Bare lattice Qx=3.72, Qy=3.845, ΔQ_{SC}=-0.9, ε_0=0.86e-6m (rms)

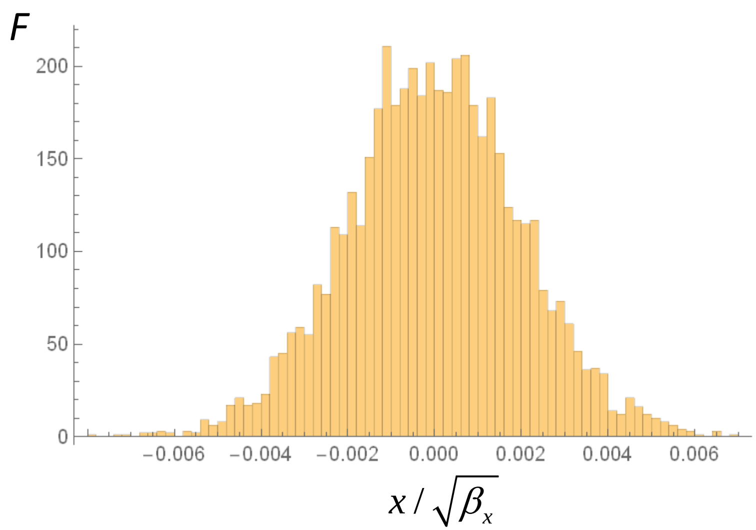


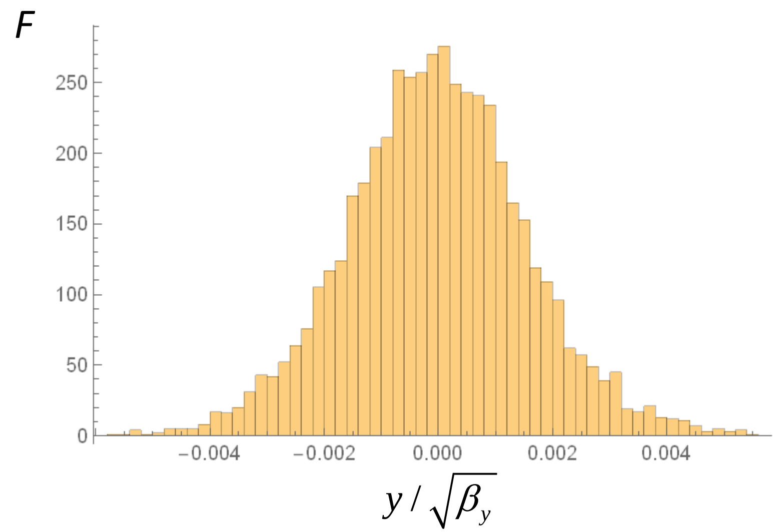
Red and blue: horz and vert emittance in a free Σ mode (beam sizes allowed to oscillate with account for *nonlinear* SC force).

Magenta and cyan: horz and vert emittance in a periodic Σ mode (periodicity of the beam sizes is imposed every turn).

Gluckstern's Resonance for Booster Parameters

Manifests itself in the "flat" lattice only for artificially large mismatch:
at 12.5% beam size modulation the losses are 0.2% in 2000 turns,
the RMS emittance growth is
{2.3e-6, 2.2e-6} $\rightarrow$ {2.9e-6, 2.5e-6} or {24%, 13%} increase





There is only insignificant halo generation. With well-corrected lattice even higher space charge can be tolerated.

Therefore losses in the "flat" lattice most likely had a different origin (LLRF) identified by C. Bhat, C.-Y. Tan, V. Lebedev

Outlook

We intend to continue development of MADX-SC:

- Longitudinal flat profile option (just rectangular for now)
- Introduction of the beam ellipse tilt: coupling via SC, “self-skewing” etc.

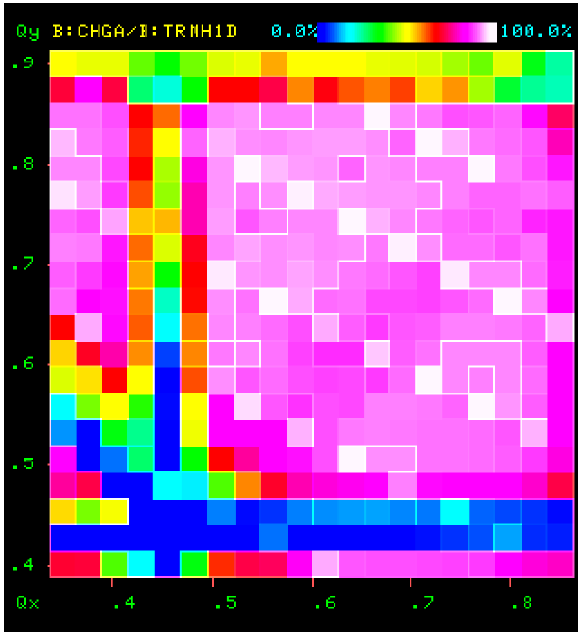
- Requires to find method of minimization of the “fake” coupling w/o suppression of the real one

- Self-consistent optics with coupling and strong SC (based on 4D perturbation theory)
- Accelerated convergence of the Σ matrix fitting (sometimes it is long)
- Introduction of wakes?

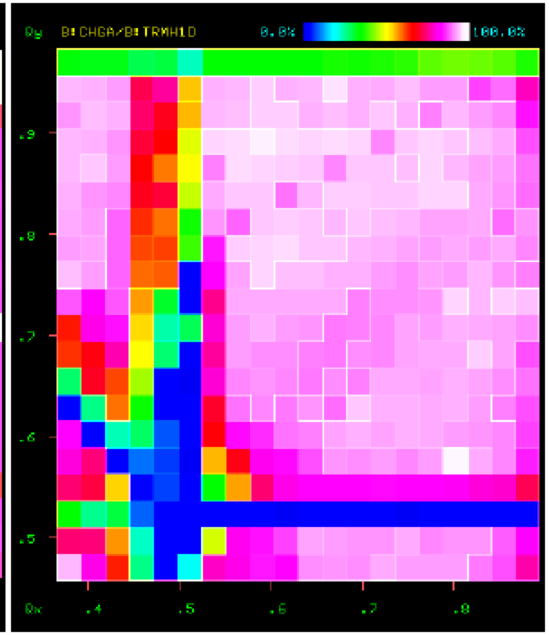
Additional Slides

Tune scan

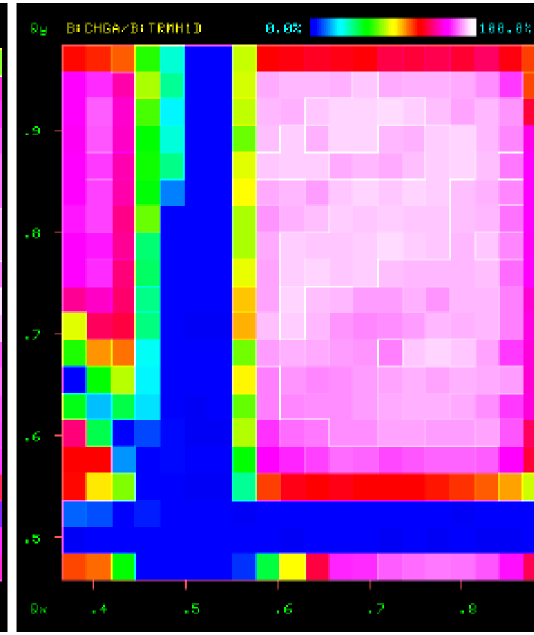
(low intensity)



Pseudo-flat lattice 1



Pseudo-flat lattice 2



HEP

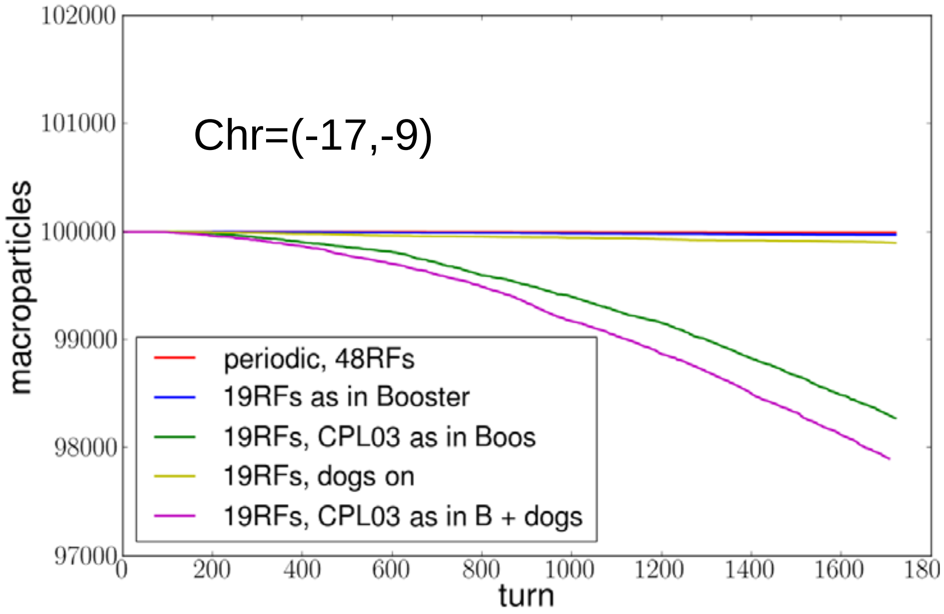
Pseudo-flat lattice 2 has smaller vertical 1/2 integer resonance and slightly larger horizontal 1/2 integer resonance.
Both pseudo-flat lattices are much improved over HEP lattice.

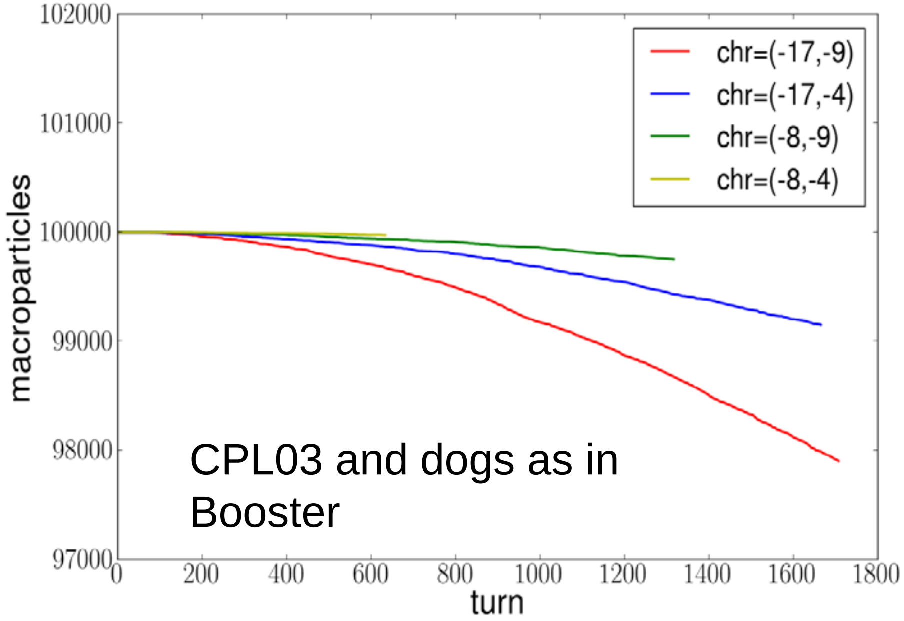
Pseudo-Flat Optics 2 looks like a victory, but there is no better working point than with HEP lattice at high intensity!

Synergia Simulations (A. Macridin)

Q_x=0.734 Q_y=0.82

n=7e10 pp bunch





- Position of the CPLO3 corrector package is the main culprit for beam loss

- horizontal chromaticity has a large influence on loss

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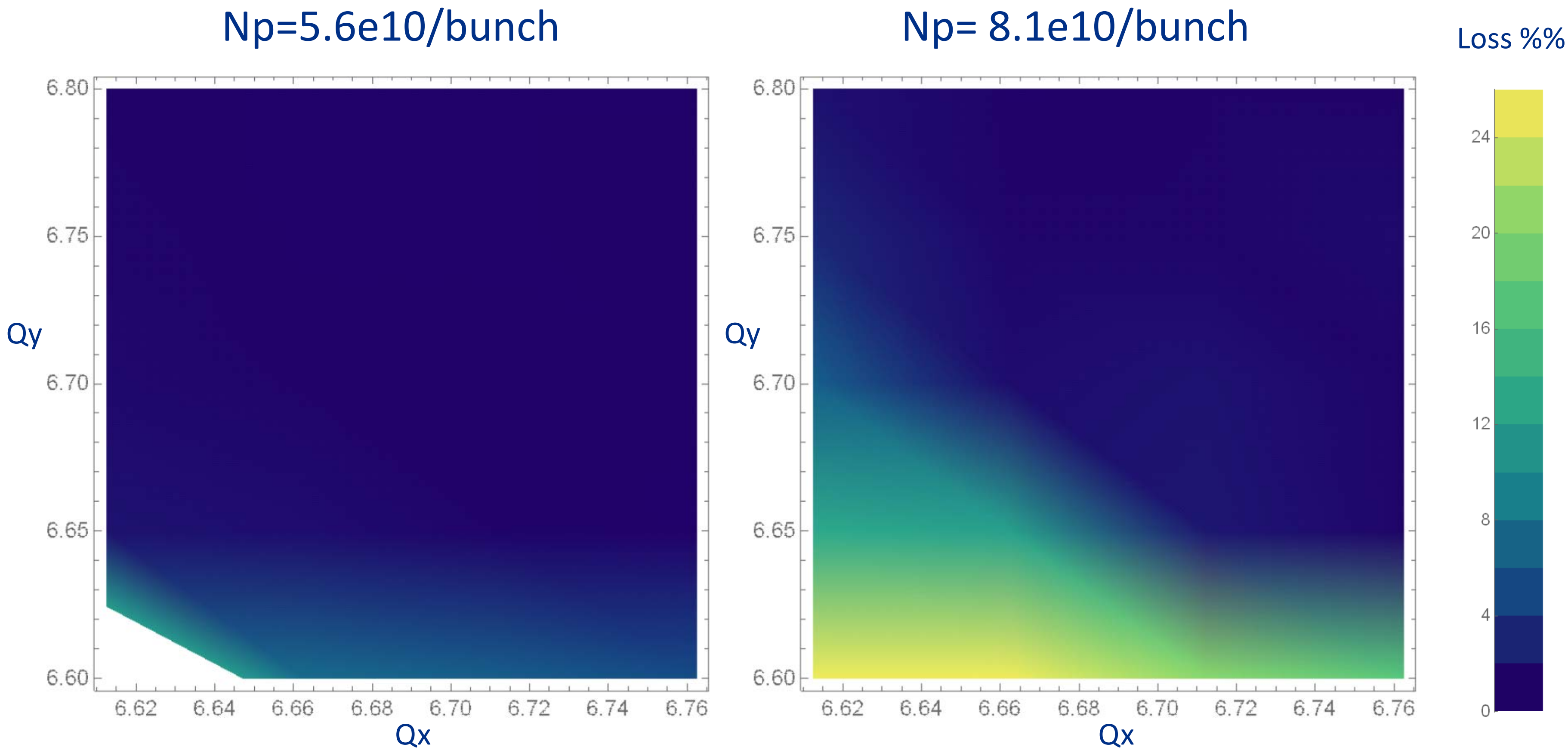
MADX-SC Simulations for Flat Lattice 2



Losses over 2000 turns as function of bare lattice tunes at nominal and PIP-II intensities. Qx+2Qy corrected.

At Qx=6.7, Qy=6.8 losses are negligible: 0% → 0.07%

Optimum Choice of the Weight Parameter α

Fitting should provide ~ correct average kick for various beam profiles.

Consider round flattop (KV) beam of radius "*a*":

$$\langle x^2 \rangle = \langle y^2 \rangle = a^2/4, \quad \rho = \lambda / \pi a^2, \quad \lambda = \text{linear density}$$

While for a Gaussian beam of the same r.m.s. *x* and *y* sizes

$$\rho(r) = \frac{\lambda}{2\pi\sigma_\perp^2} \exp(-\frac{r^2}{2\sigma_\perp^2}) \;\; \rightarrow \;\; \rho(0) = \frac{\lambda}{2\pi\sigma_\perp^2} = \frac{2\lambda}{\pi a^2}$$

TS for small amplitudes will be twice that in KV beam - σ should be larger than the r.m.s. value *a*/2 .

Introducing weights as $W = \exp[-\alpha(\underline{\zeta}, \Sigma^{-1}\underline{\zeta})]$ we obtain in the 4D case

α	σ_{fit}/a	
0	1/2	simple r.m.s.
0.198	0.582	correct average kick
0.25	0.597	used now
0.5	0.650	"rigorous" fit
0.908	$1/\sqrt{2}$	correct TS

Fitted σ is larger than the r.m.s. value *a*/2 - the property of the so-called platycurtic (negative excess curtosis) distributions.

α=1/5 looks like the optimum – it suppresses halo contribution (but not too drastically) and ensures ~ correct average kick for various beam profiles.

Symplectic 3DoF SC Kick (in Long Bunch)

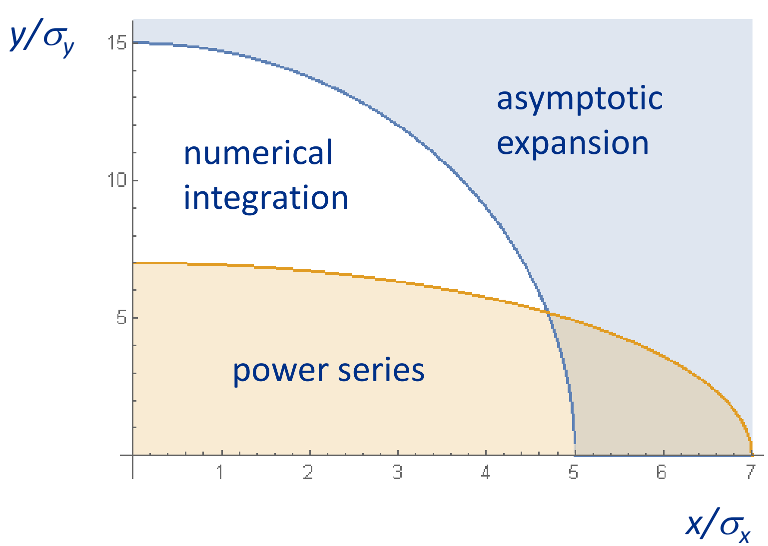
Symplecticity is automatically achieved if all kick components are derived from the same SC potential

$$\varphi(x,y,z,t) \cong \lambda(z - v_0 t)\cdot \Phi(x,y)$$

where $\lambda(z)$ is linear SC density. A convenient representation of Φ

$$\Phi(x,y) = \int_0^1 \left\{ \exp\left(-\frac{x^2 t}{2\sigma_x^2} - \frac{y^2 r^2 t}{2\sigma_y^2 [1+(r^2-1)t]} \right) - 1 \right\} \frac{dt}{t\sqrt{1+(r^2-1)t}}, \qquad r = \frac{\sigma_y}{\sigma_x}$$

It satisfies the boundary condition and can be complemented with a longitudinal wake which is independent of the transverse position (not to break the symplecticity).



Regions of good precision for power series and asymptotic expansion for aspect ratio $r = \sigma_y / \sigma_x = 1/3$.
For $(x, y) \in$ the white region the numerical integration has to be used.

With parameters set to ensure > 6 digits of precision the speed is ~ the same as with Erskine-Basetti 2D formulas.

Summary from 2018 Workshop on MW rings

- From the standpoint of transverse dynamics with space charge there should be no problem with PIP-II intensity at the present injection energy when using "flat" optics.
- However, we could not reduce losses with these apparently better optics

We tried:

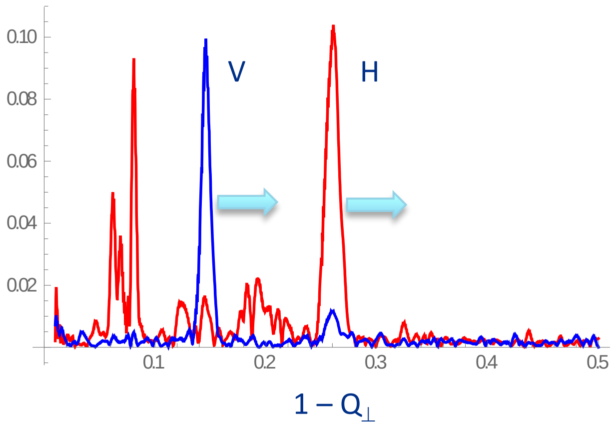
- injection orbit and optics matching
- aperture scans
- decoupling (though Qx+Qy has not been looked at since 2011)
- correction of the 3rd order using upright and skew sextupoles
- reduced chromaticity
- to see head-tail instability
- to detect dipole noise using TBT data

(quad noise seems unlikely)

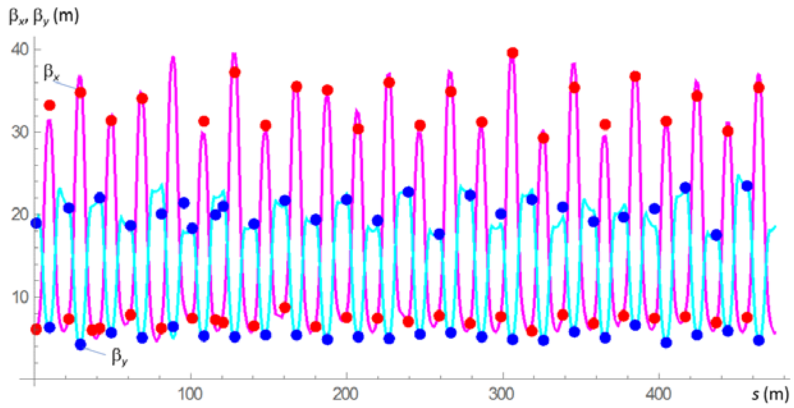
- all to no or very limited success.

Had we missed anything important?

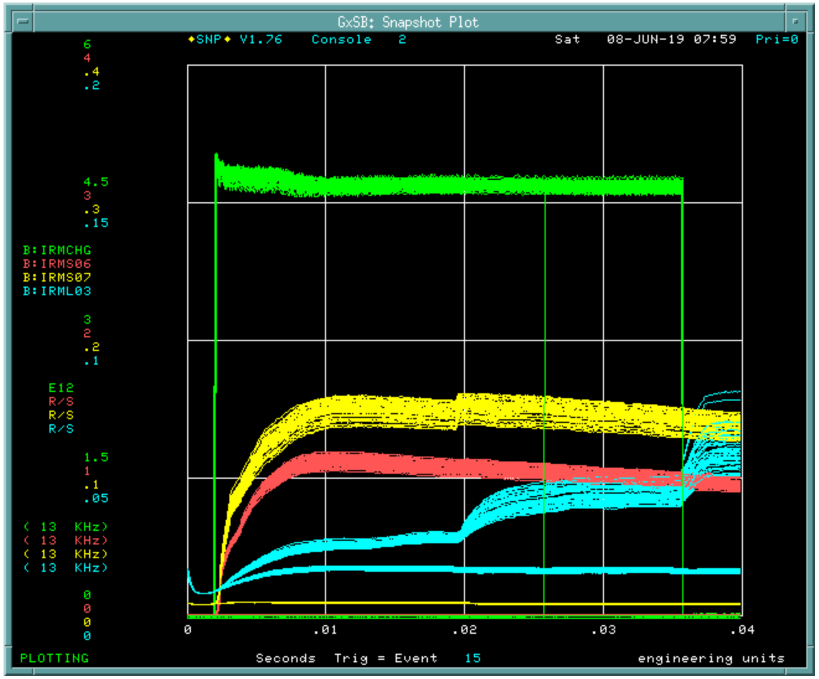
TBT spectra



Some Recent Booster Observations



Optics functions obtained with the MADX Booster model (solid lines: magenta – horizontal, cyan – vertical) and from the TBT measurements (dots).



Beam intensity (green) and losses at some locations.

Injection losses are reduced to < 3%.

Losses at ~6ms can be a sign of the horz multi-bunch instability – we had it before – which can be easily cured by chromaticity and/or the damper.