

IOTA Nonlinear Integrable Optics Experiment: NL Magnet

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06/10/2019

Research supported by DOE GARD and NIU Accelerator Cluster Funds
(NIU: Swapan Chattopadhyay, PI and Bela Erdelyi, Co-PI) & (NIU: Swapan
Chattopadhyay, PI)



Northern Illinois
University



Acknowledgments

Northern Illinois University
S. Chattopadhyay

Fermilab

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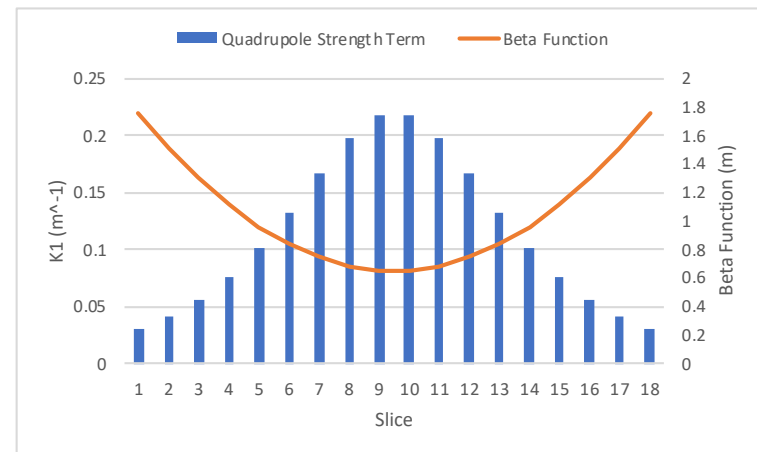
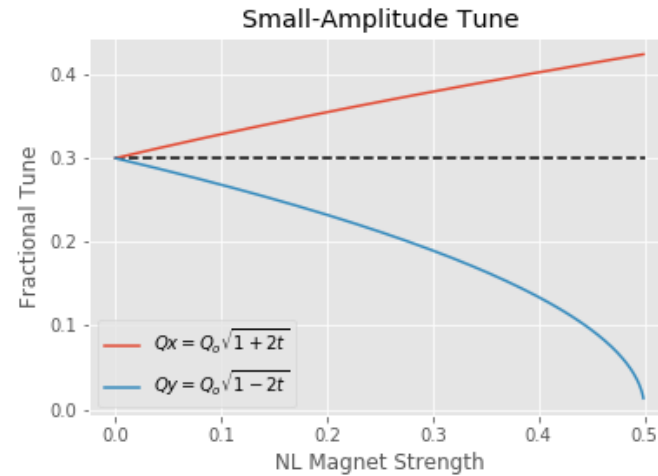
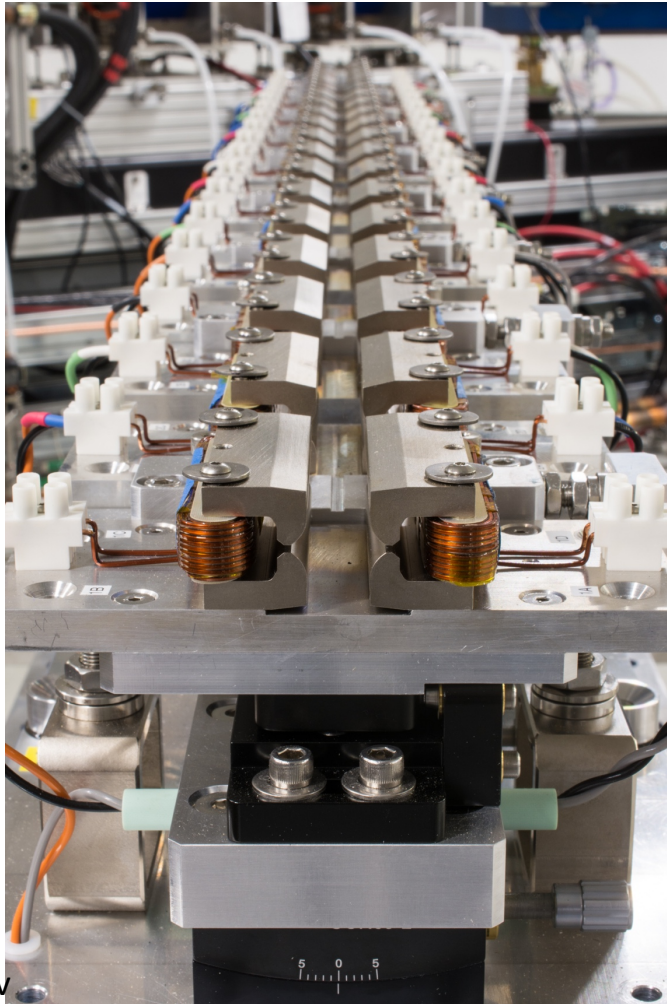


Outline

- Alignment of the Nonlinear Magnet
 - Calibration
- Limiting Factors of IOTA
- Measurements
 - Amplitude Dependent Tune Map



Nonlinear Magnet



fast.fnal.gov

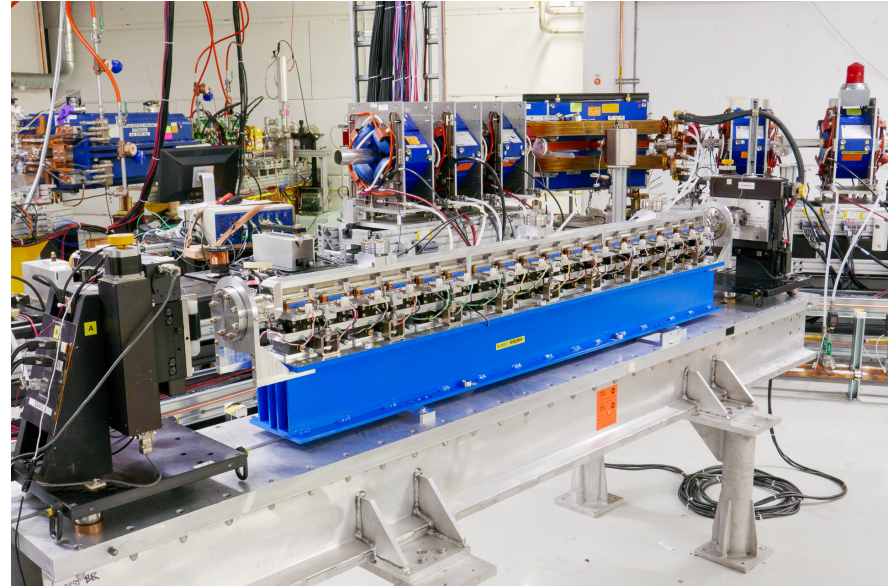
V. Danilov and S. Nagaitsev, Phys. Rev. ST-AB **13**, 084002 (2010)s

A. Romanov



Element Centering

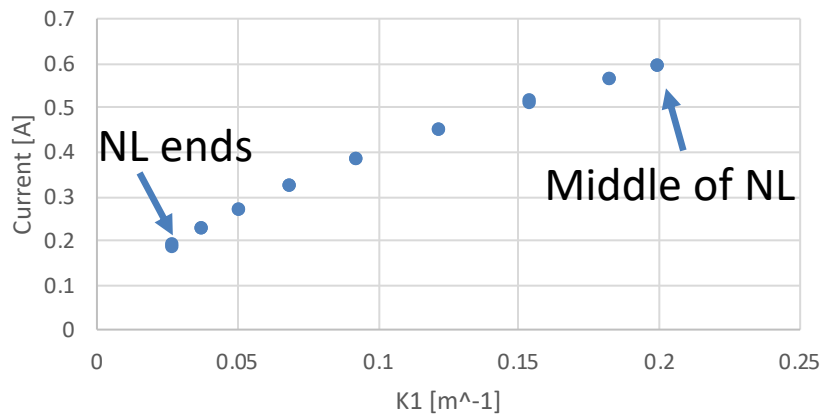
- Alignment experts from AP-STD, Joe DiMarco, measured the center of the 18 magnets individually
- Used stretched wire method
 - copper-beryllium wire
- Magnetic center is to within ± 50 μm



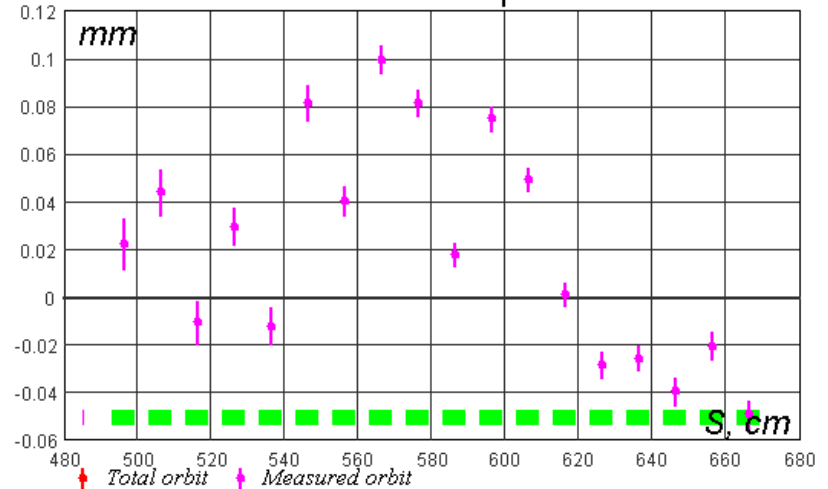
Verification

- Used beam based Orbit Response Measurement
 - Alignment is good up to 100 μ m
- Powered individual magnets for a tune response measurement
 - Fitted calibration strength coef.

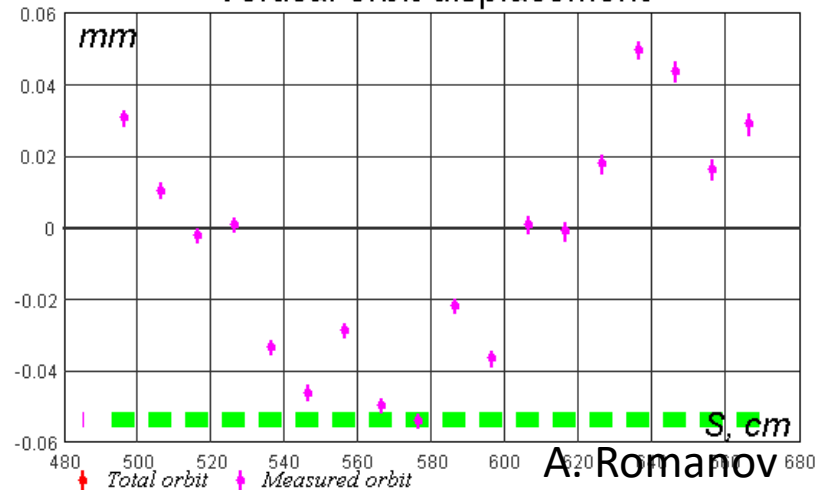
NL Calibration



Horizontal orbit displacement



Vertical orbit displacement



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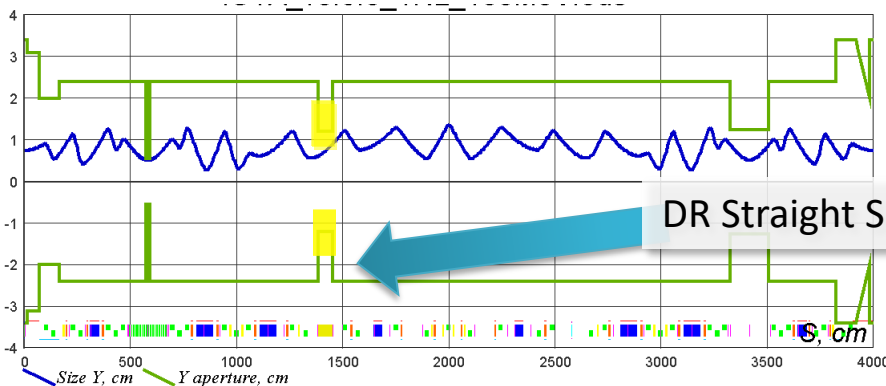
Limiting Factors

- Mechanical Restrictions

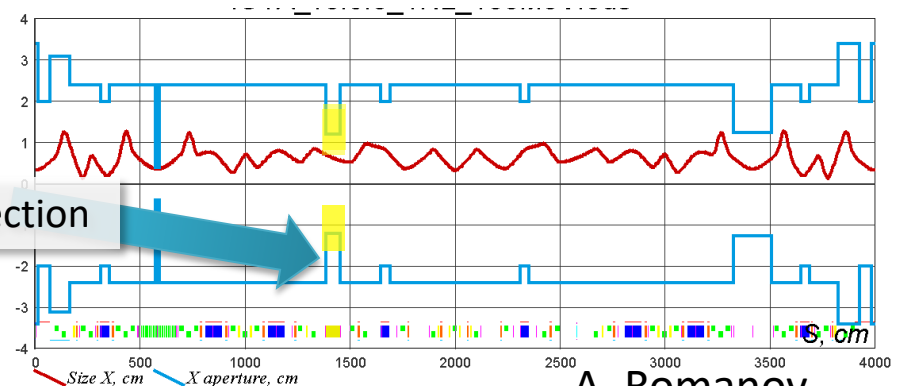
- DR straight beam pipe has a ~6mm misalignment
 - Planning to fix this during shutdown
- By design the smallest restriction should be at the middle of nonlinear magnet, 5.5mm vertically
- Beam measurements indicate an unexpected restriction of 6mm in DR straight section

- T-insert transfer map must be precise, up to 1%

- During run, lattice tuning up to 10%, will be improved next run



Vertical Mechanical Restriction



Horizontal Mechanical Restriction

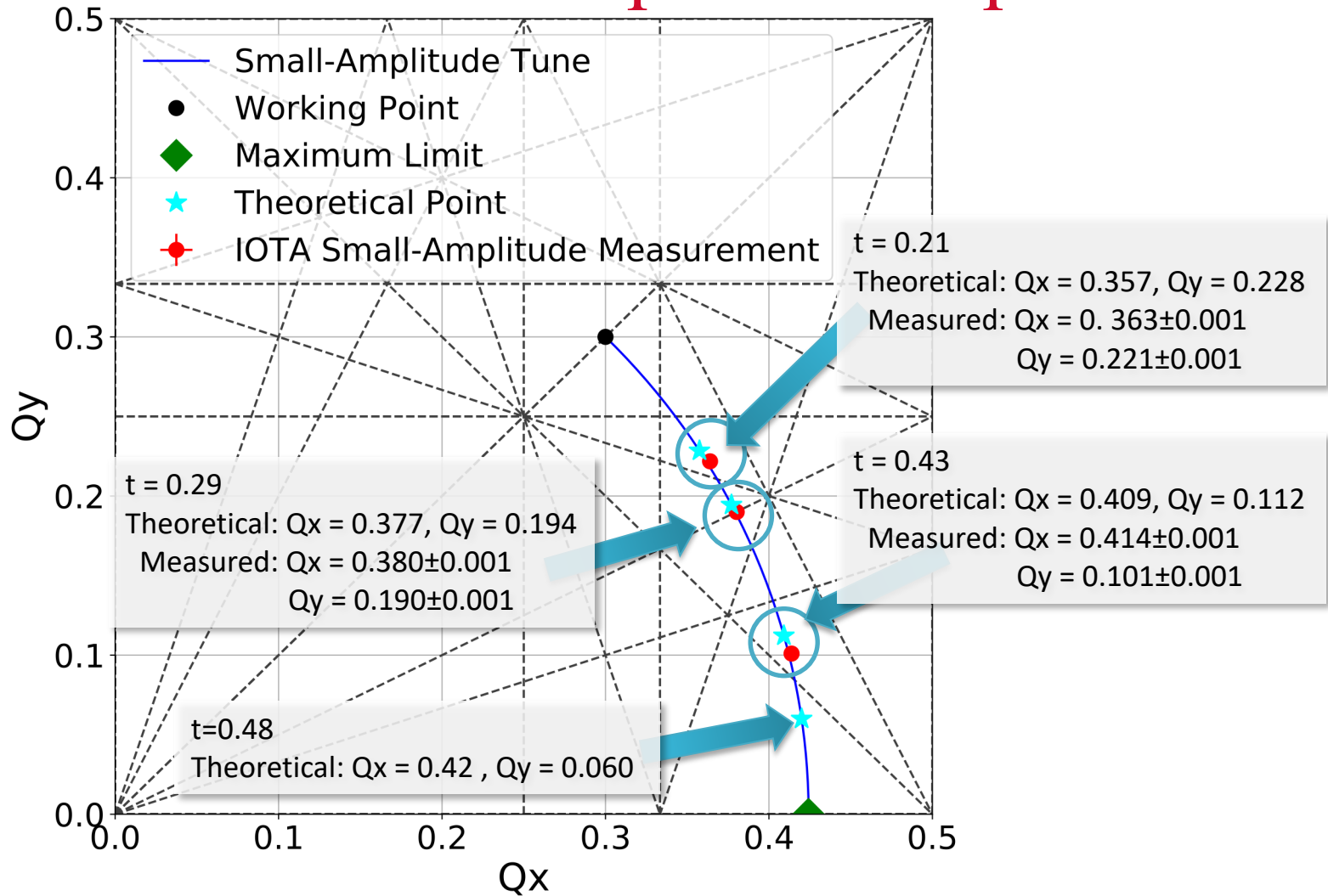
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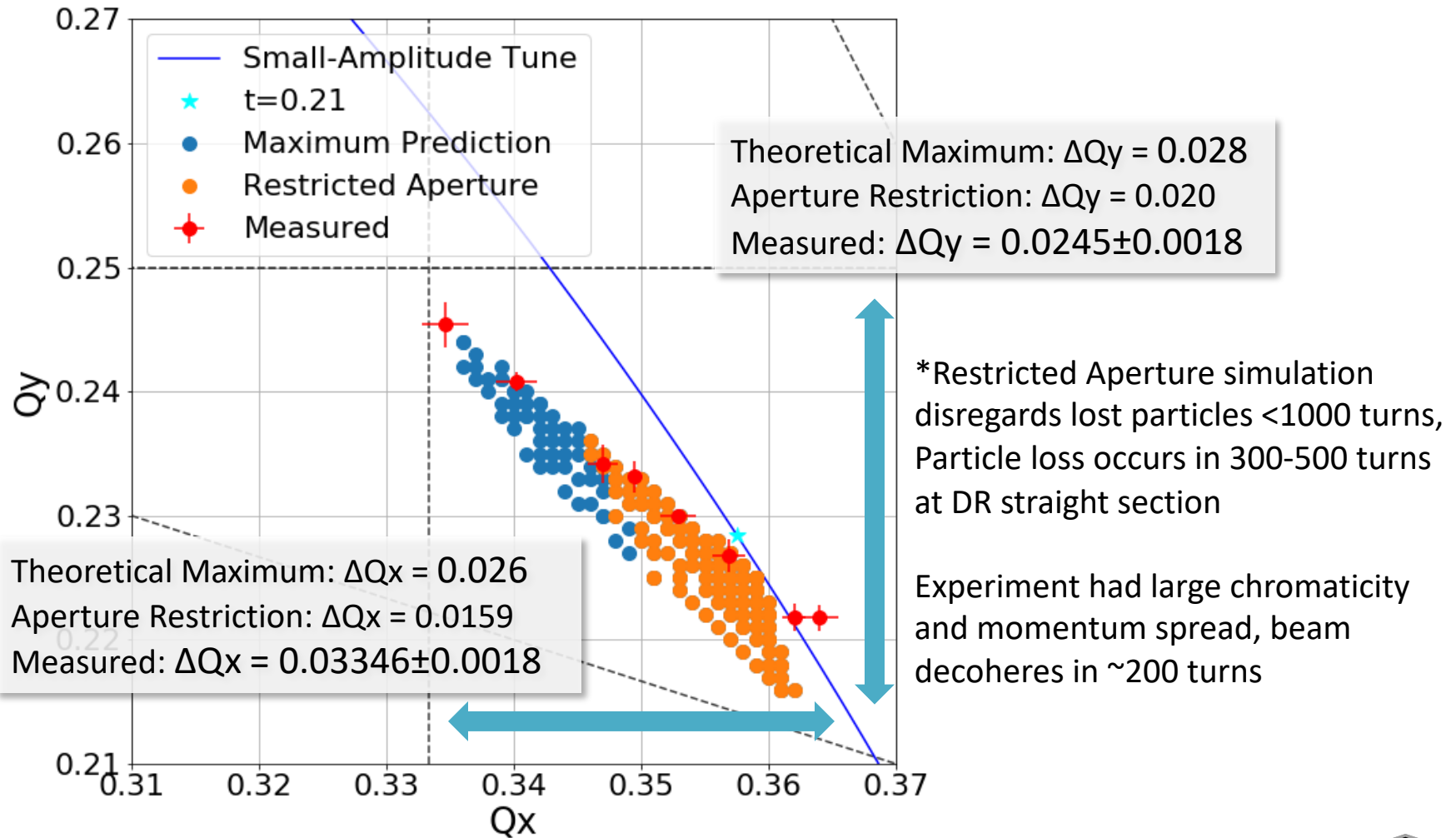
Tune Measurements

- Kicked a 100 MeV electron beam vertically from 0.3kV to 4.8kV
- Changed strength of nonlinear magnet to $t = 0.21, 0.29, 0.43$
- Calculated tunes via FFT algorithm from data collected via 21 Beam Position Monitors (BPM) around the ring

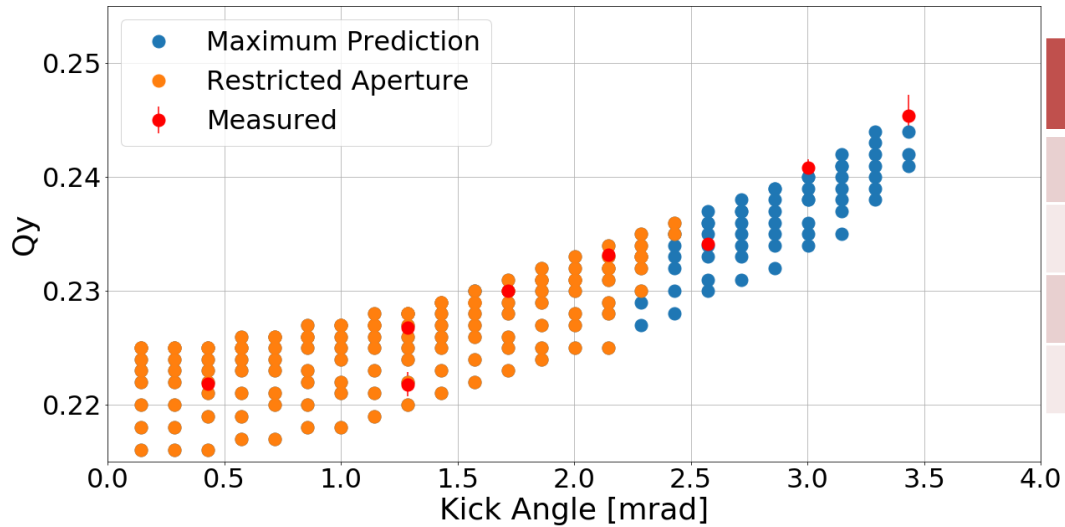
Small-Amplitude Map



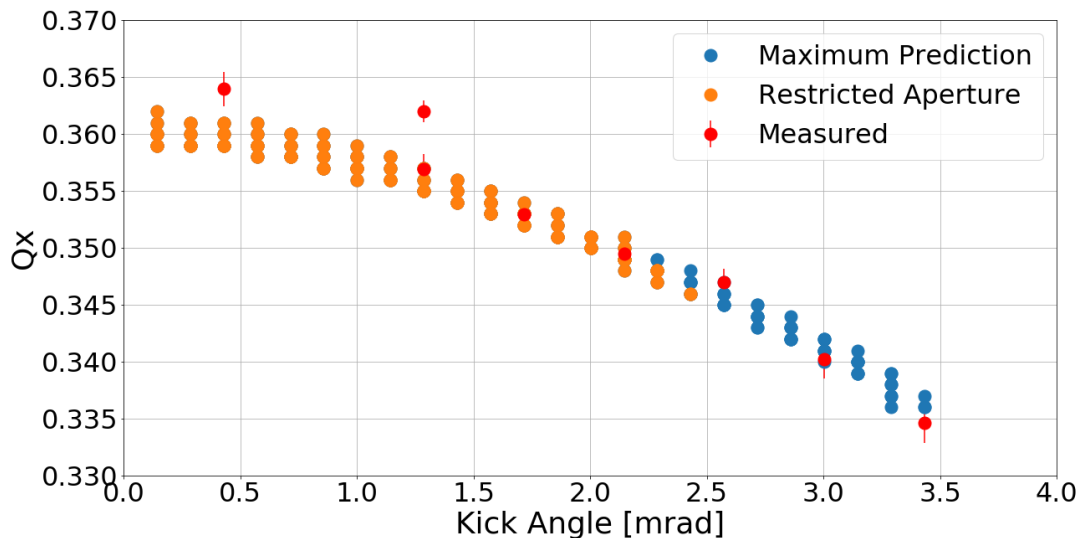
Amplitude Dependent Tune Map, $t=0.21$



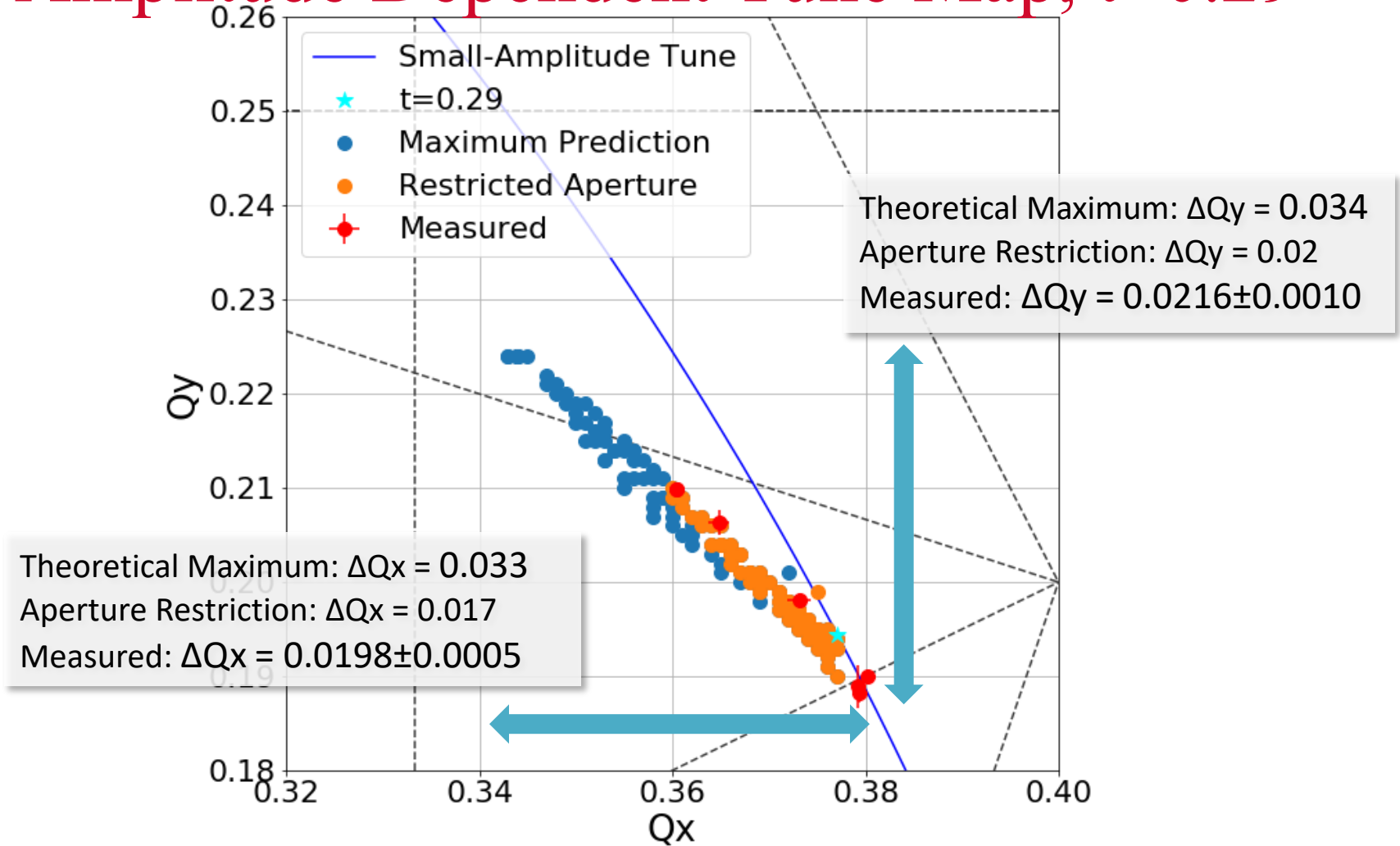
Amplitude Dependent Tune Shift, $t = 0.21$



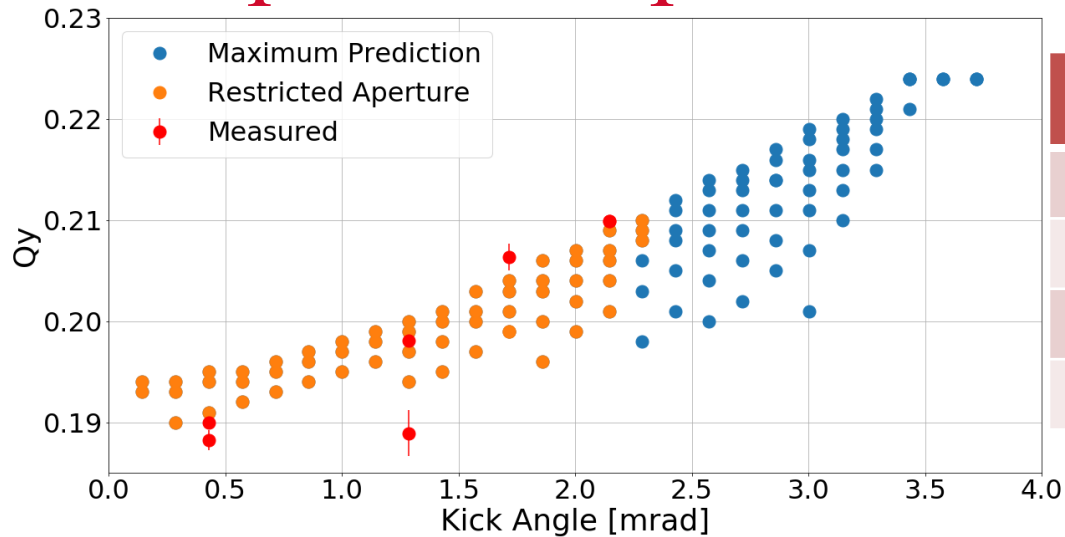
Parameter	Meas.	Aperture Restriction	Model
Max Kick [kV]	4.80	3.39	4.80
Max Kick Ang. [mrad]	3.43	2.43	3.43
Max Amp at NL [mm]	4.48	3.18	4.48
DR-Restriction [mm]	6	6	10



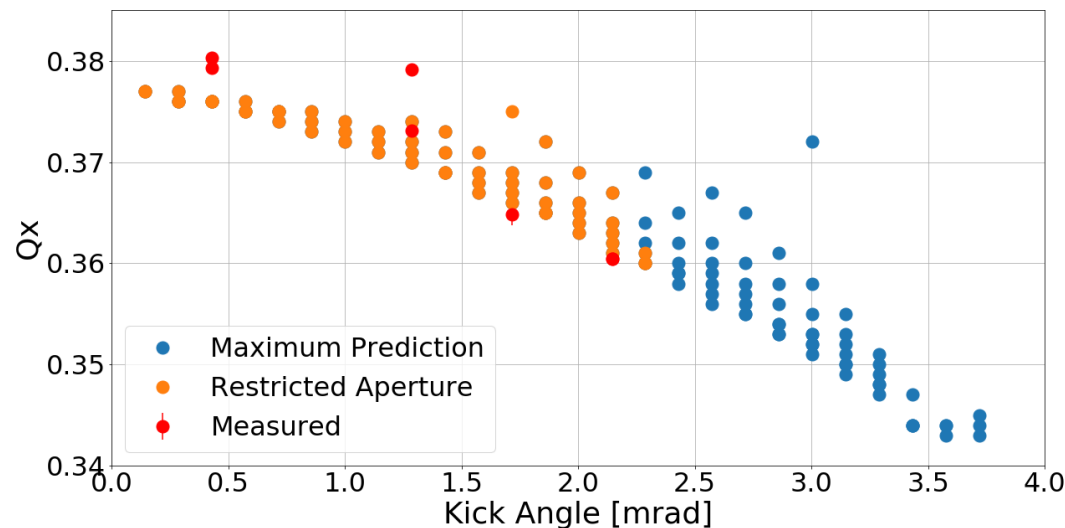
Amplitude Dependent Tune Map, $t=0.29$



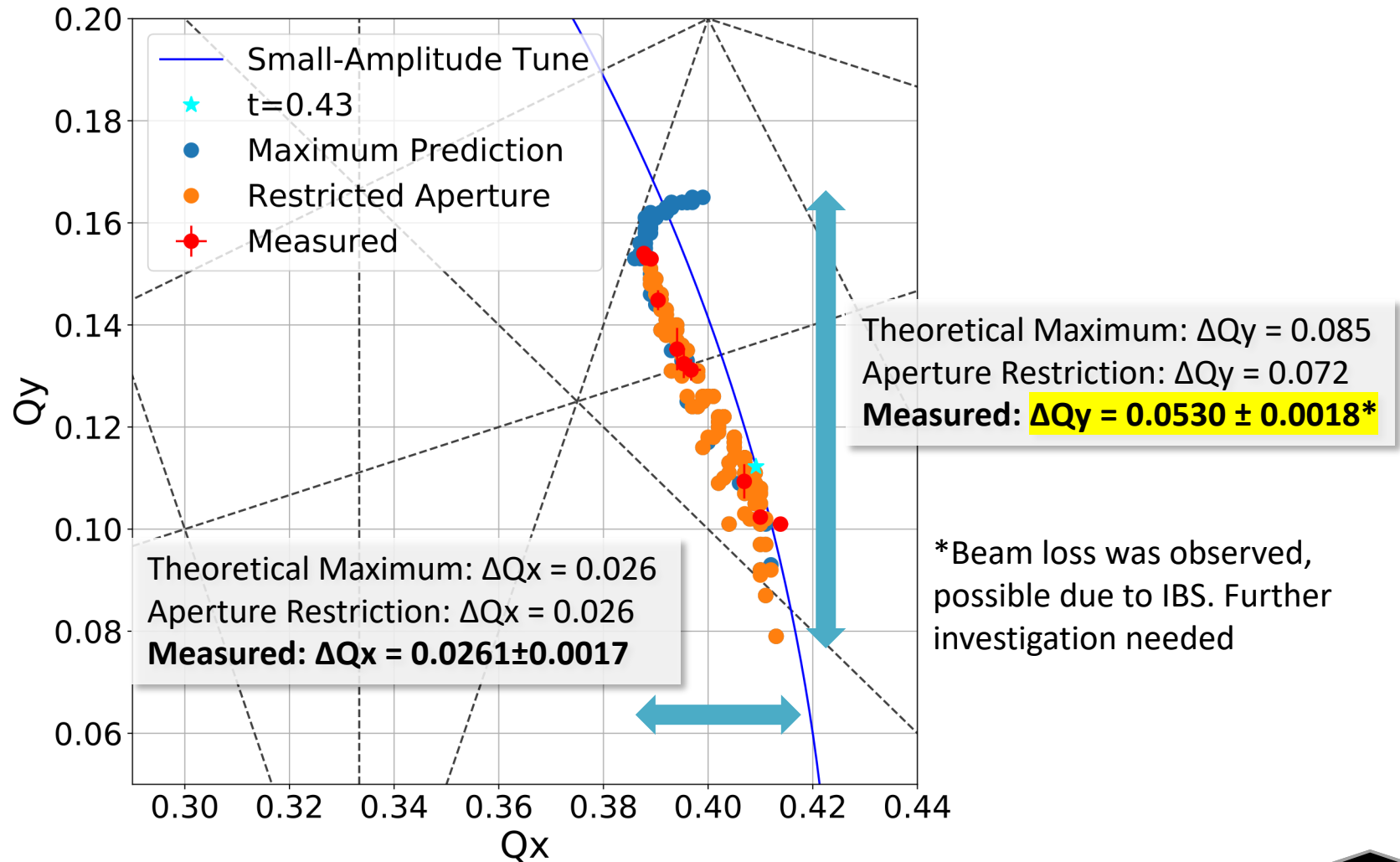
Amplitude Dependent Tune Shift, $t = 0.29$



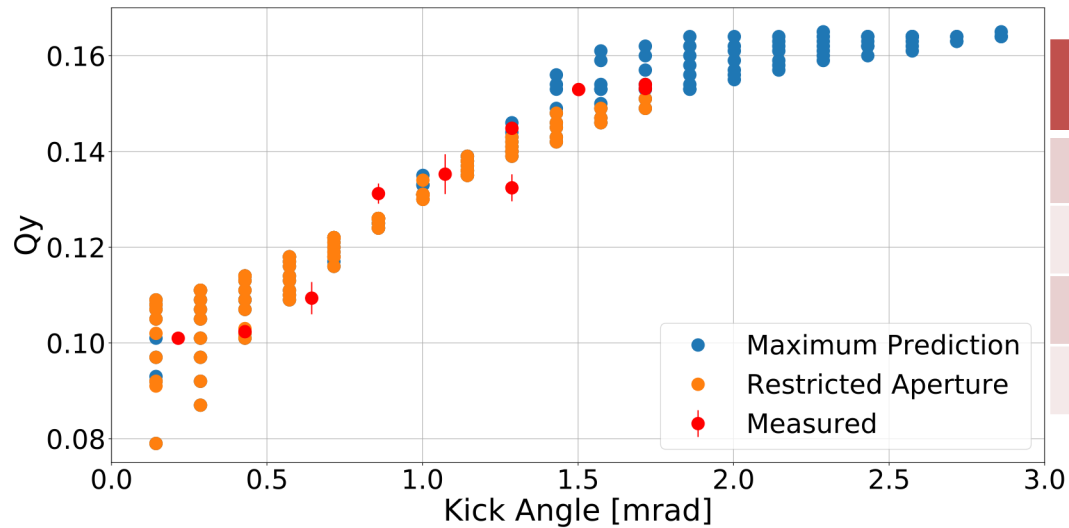
Parameter	Meas.	Aperture Restriction	Model
Max Kick [kV]	3.00	4.68	5.18
Max Kick Ang. [mrad]	2.14	2.28	3.71
Max Amp at NL [mm]	5.19	2.73	5.19
DR-Restriction [mm]	6	6	10



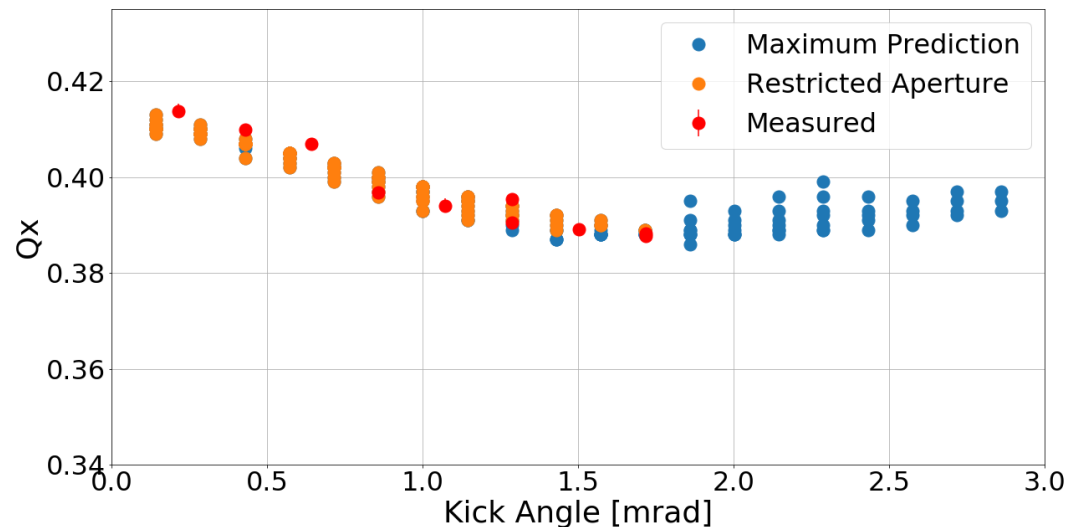
Amplitude Dependent Tune Map, $t=0.43$



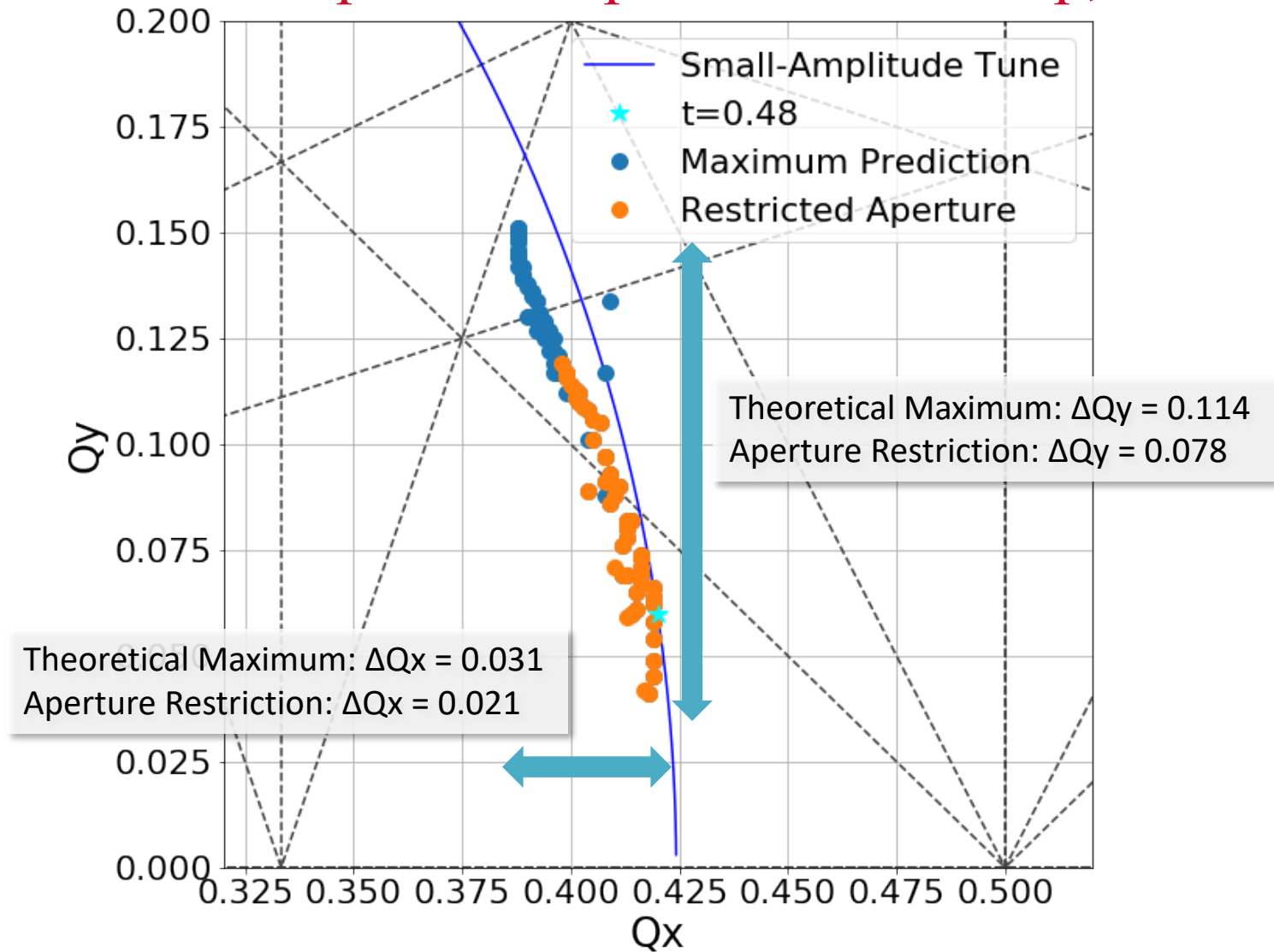
Amplitude Dependent Tune Shift, $t = 0.43$



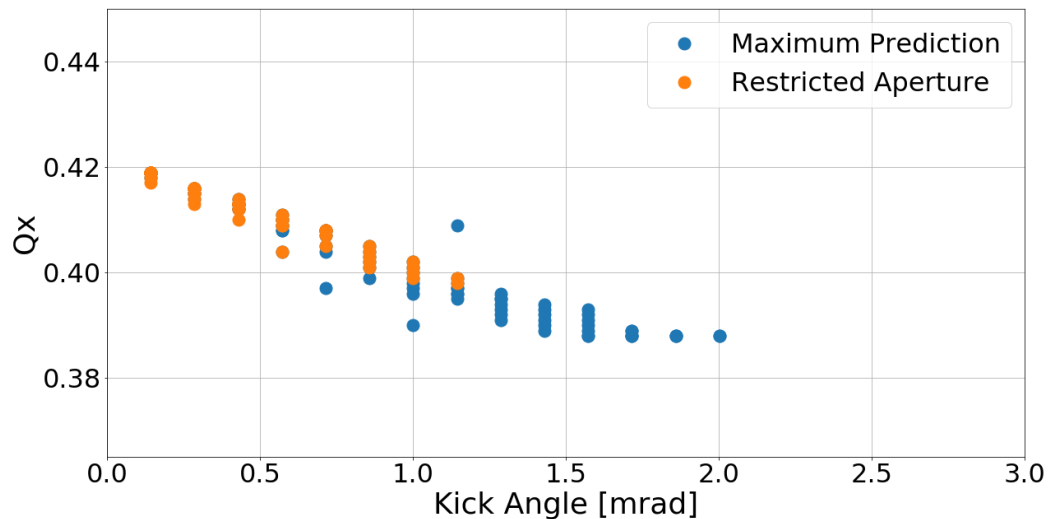
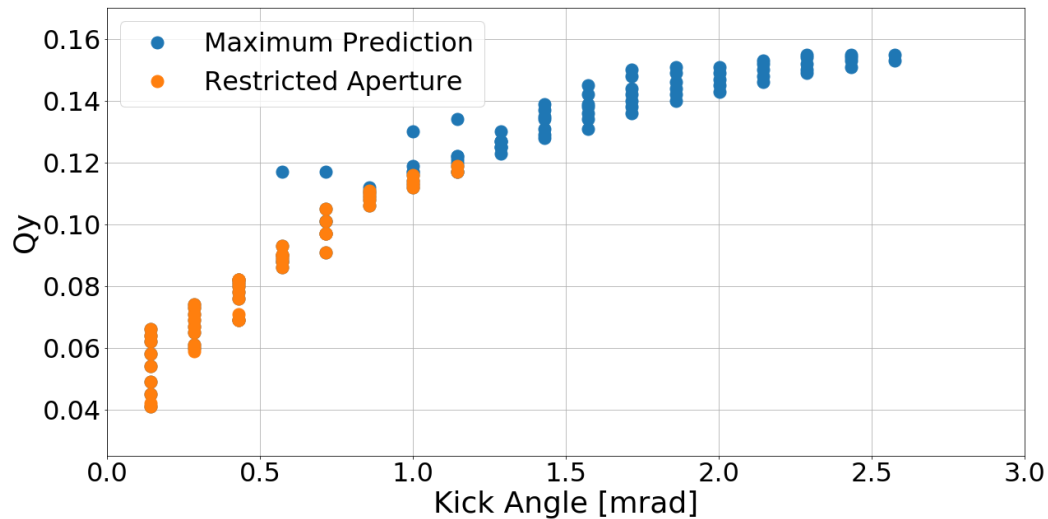
Parameter	Meas.	Aperture Restriction	Model
Max Kick [kV]	2.4	2.4	4.0
Max Kick Ang. [mrad]	1.71	1.71	2.86
Max Amp at NL [mm]	3.65	3.65	5.19
DR-Restriction [mm]	6	6	10



Simulation Amplitude Dependent Tune Map, $t=0.48$



Simulation of $t = 0.48$, Amplitude Dependent Tune Shift



Parameter	Aperture Restriction	Model
Max Kick [kV]	1.60	3.60
Max Kick Ang. [mrad]	1.14	2.57
Max Amp at NL [mm]	3.37	5.10
DR-pipe radius [mm]	6	10



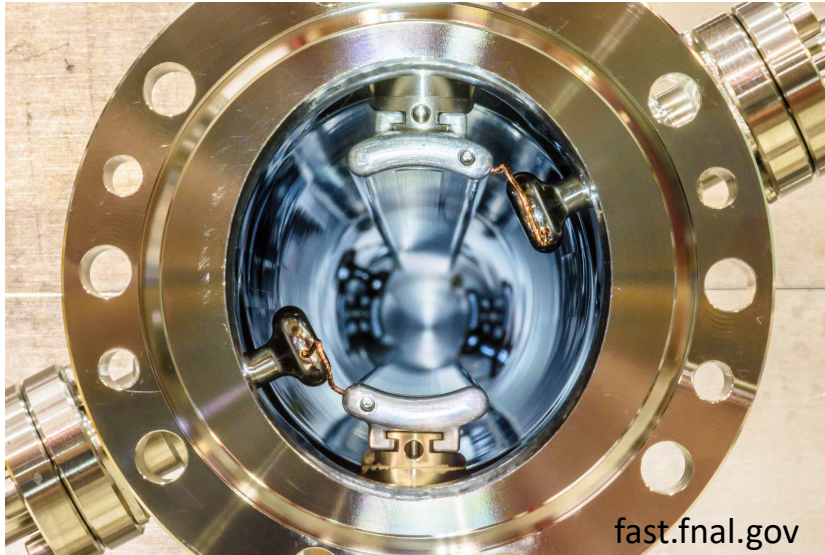
Summary and Outlook

- Measured tunes in good agreement with MAD-X Simulation
- Largest observed tune shift of $\Delta Q_x = 0.0261 \pm 0.0017$ and $\Delta Q_y = \mathbf{0.0530 \pm 0.0018}$
 - At larger strength value of $t=0.48$, simulation shows a tune shift of $\Delta Q_y \approx 0.11$
- Next Run will have ring improvements, allowing further exploration of the dynamic aperture
 - Realignment
 - Fix DR beam pipe
 - BPM Improvements
 - Additional sextupoles
 - Working horizontal kicker
- Further studies in understanding beam loss from previous run needs to be done

Back up Slides



Stripline Kicker



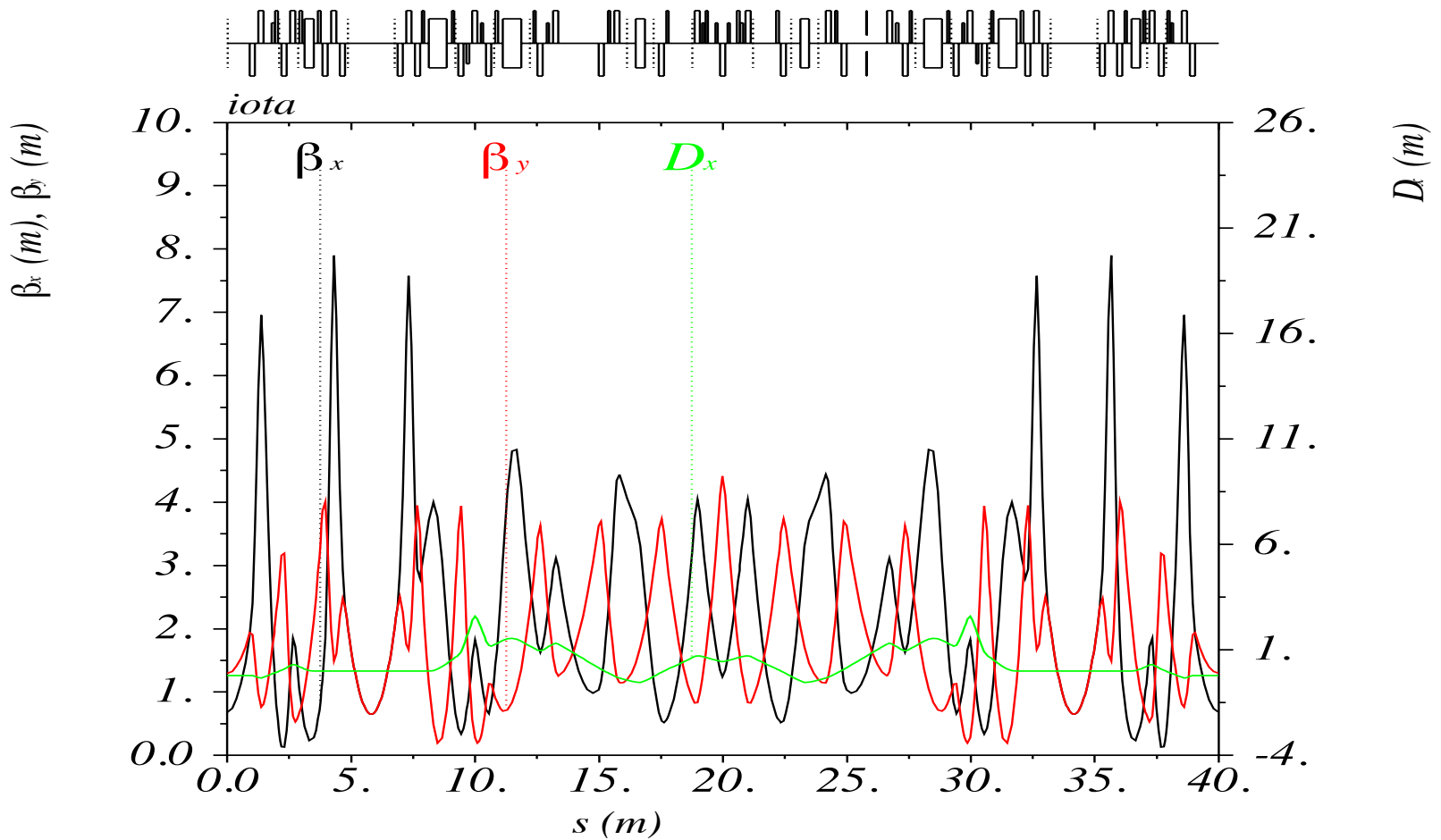
Parameters of vertical (horizontal) kicker

Plate inner radius	20 mm
Plate thickness	6 mm
Plate opening angle	65 deg
Length of plates	1050 (580) mm
Max plate Voltage	±25kV
Max. kick angle	16 (8) mrad

Kicker Strength:
$$\alpha = \frac{2VL}{r} \frac{4}{\pi} \sin \frac{\theta}{2}$$

S. Antipov, et.al, [arXiv:1607.00023](https://arxiv.org/abs/1607.00023)

IOTA Lattice



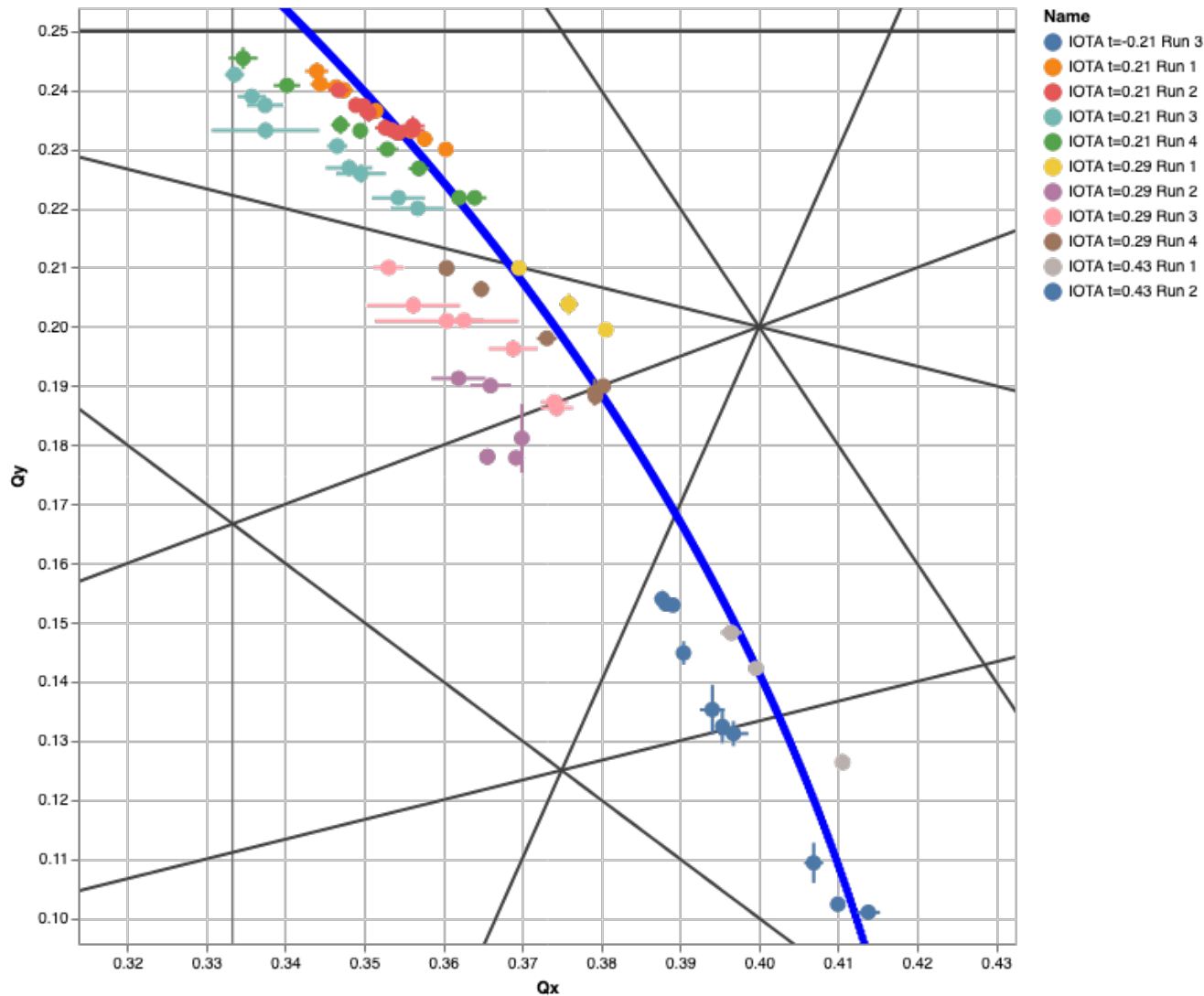
IOTA Parameters

PARAMETER	VALUE
Beam Energy	150 MeV
Circumference, C_0	39.97 m
Revolution period, T_0	133.3 μ s
RF voltage, frequency, revolution harmonic	1 kV, 30.0 MHz, 4
Number of particles, beam current	2×10^9 , 2.4 mA
Betatron tunes, $Q_{x,y}$	4-6
Synchrotron tunes, Q_s	5.3×10^{-4}
Momentum compaction, α_p	0.067
Radiation damping times $\tau_{x,y}, \tau_z$	0.9 s, 0.24 s
Maximum beta-function, β_x, β_y	8.5, 4 m
Equilibrium beam emittance, $\epsilon_{x,y}$	0.04 μ m
Beam energy spread, bunch length, σ_E, σ_z	1.35×10^{-4} , 10.8 cm
Vacuum	6×10^{-10} Torr

S. Antipov et al, J. Instrum. **12**, T03002 (2017)



All Measurements



Linear Focusing Accelerator

All present circular accelerators are built around linear focusing optics.

$$H = \frac{(p_x^2 + p_y^2)}{2} + K_x(s) \frac{x^2}{2} + K_y(s) \frac{y^2}{2}$$

The transverse motion, has two degrees of freedoms that the equation of motion is a linear oscillator, leads to two invariants, which are called action or integrable of motion.

$$\left. \begin{aligned} z_N(s) &= \frac{z(s)}{\sqrt{\beta_z(s)}} & p_{zN}(s) &= \sqrt{\beta_z(s)} p_z(s) - \frac{\beta'_z(s)}{2\beta_z(s)} z(s) \\ \psi_z(s) &= \int_0^s \frac{ds'}{\beta_z(s')} & J_z &= \frac{1}{2\pi} \oint p_{zN} dz_N \end{aligned} \right\} H = \nu_x J_x + \nu_y J_y$$



Time-Independent Hamiltonian

Start with a Hamiltonian with equal transverse focusing.

$$H = \frac{(p_x^2 + p_y^2)}{2} + K(s)\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + V(x, y, s)$$

Have H time-independent in normalized coordinates and introduce a 'new' time, which is the betatron phase

$$z_N(s) = \frac{z(s)}{\sqrt{\beta_z(s)}} \quad p_{zN}(s) = \sqrt{\beta_z(s)}p_z(s) - \frac{\beta'_z(s)}{2\beta_z(s)}z(s) \quad \frac{d\psi}{ds} = \frac{1}{\beta(s)}$$

$$H_N = \frac{(p_{xN}^2 + p_{yN}^2)}{2} + \frac{x_N^2 + y_N^2}{2} + \beta(\psi)V(x_N\sqrt{\beta(\psi)}, y_N\sqrt{\beta(\psi)}, s(\psi))$$

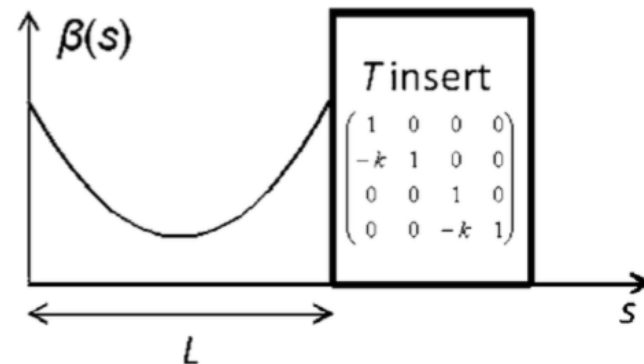
$$H_N = \frac{(p_{xN}^2 + p_{yN}^2)}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N, \psi)$$

H is now an integral of motion

Implementation Into An Accelerator

Start with a linear focusing accelerator with equal horizontal and vertical optics.

- The linear optics can be built with standard optics, but must have a $n\pi$ phase advance, “T-insert”
- Drift region L , matched beta function ($L=1.8\text{m}$ in IOTA)



Add a nonlinear potential $V(x,y,s)$ in the drift region

2nd Integral of Motion– Elliptical Potential

- Find a potential that satisfies the Laplace equation, and is separable in some variables and has a second integral of motion that is quadratic in momentum
- First studied by Gaston Darboux in 1901, yielded to the Bertrand-Darboux partial differential equation:

$$xy(U_{xx} - U_{yy}) + (y^2 - x^2 + c^2)U_{xy} + 3y_x - 3xU_y = 0$$

- A general solution satisfying the equation:

$$U(x, y) = \frac{f(\xi) + g(\eta)}{\xi^2 + \eta^2}$$

$$f_2(\xi) = \xi \sqrt{\xi^2 - 1} [d + t \operatorname{acosh}(\xi)]$$

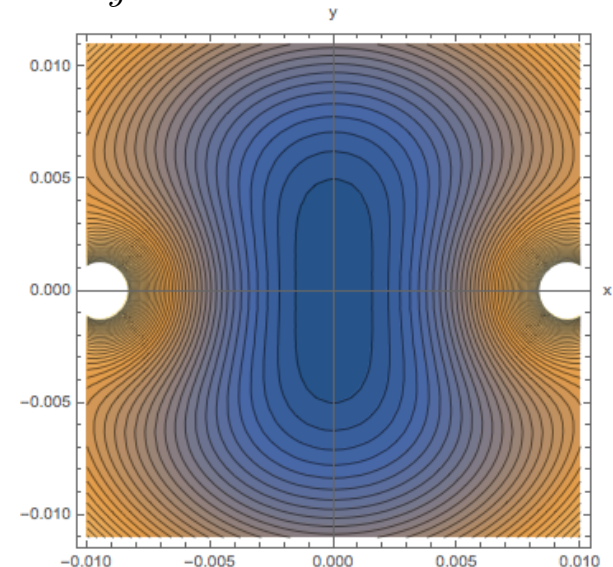
$$g_2(\eta) = \eta \sqrt{1 - \eta^2} [b + t \operatorname{acos}(\eta)]$$

- The second invariant is then:

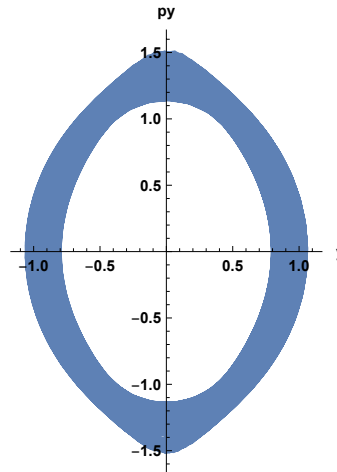
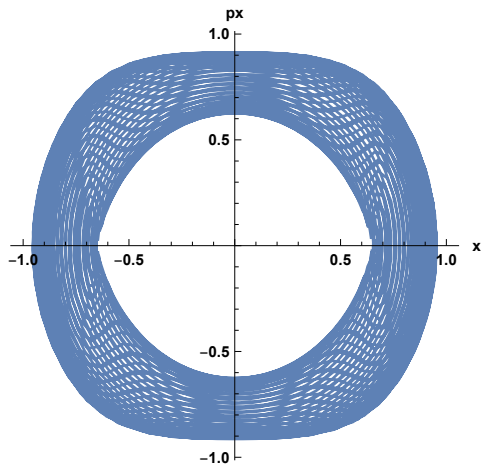
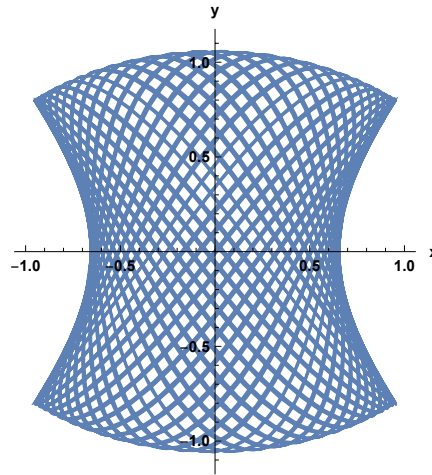
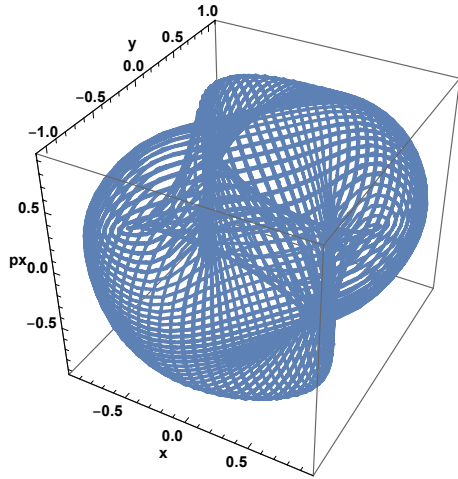
$$I(x, y, p_x, p_y) = (xp_y - yp_x)^2 + c^2 p_x^2 + 2c^2 \frac{f(\xi)\eta^2 + g(\eta)\xi^2}{\xi^2 - \eta^2}$$

Which has the form of $f = \frac{f_1}{2} + f_2$ and $g = \frac{g_1}{2} + g_2$, where $f_1(\xi) = c^2 \xi^2 (\xi^2 - 1)$ and $g_1(\eta) = c^2 \eta^2 (1 - \eta^2)$.

V. Danilov and S. Nagaitsev, Phys. Rev. ST-AB **13**, 084002 (2010)



Projections



Mathematica Toy Model
 $c = 1$ Dimensional Parameter
 $t = 0.47$ Strength of NL magnet
 $b = -\pi/2 * t$
 $d = 0$

$X_0 = 0.95$
 $Y_0 = -0.8$
 $P_{x0} = 0$
 $P_{y0} = 0.1$