Lattice QCD in 10 Minutes

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Outline

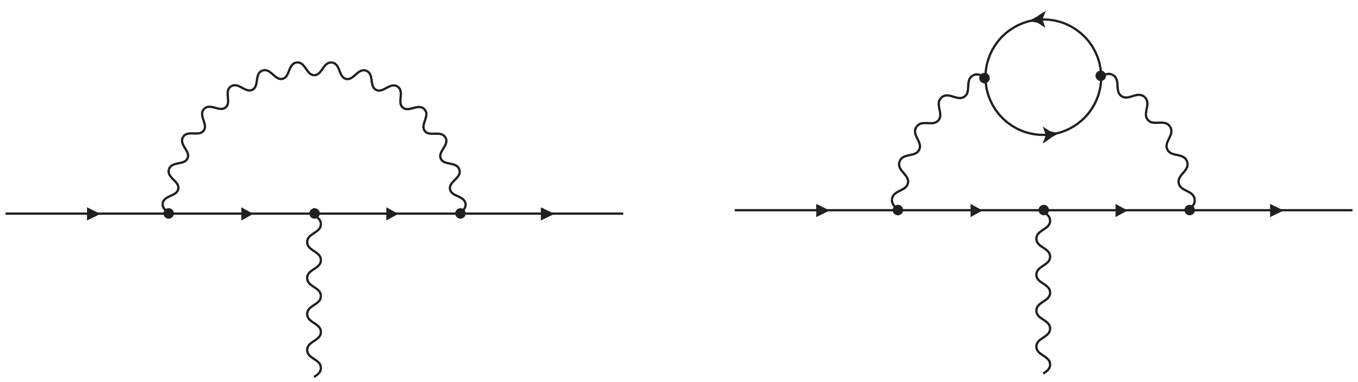
1. Lattice QCD: What, why, how?
2. High-precision: theory and experiment

Lattice QCD: What, Why, How?

- The Standard Model is a quantum field theory
- Physical predictions come from the path integral

$$\mathcal{Z} = \int \mathcal{D}[\text{fields}]e^{-iS[\text{fields}]}$$

- Weak coupling:



- A particle-centric picture: particles interact weakly with quantum fluctuations
- Predictions agree impressively with precision experiments
- Examples:
  - QED contribution to electron / muon (g-2)

Lattice QCD: What, Why, and How?

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- Physical predictions come from the path integral

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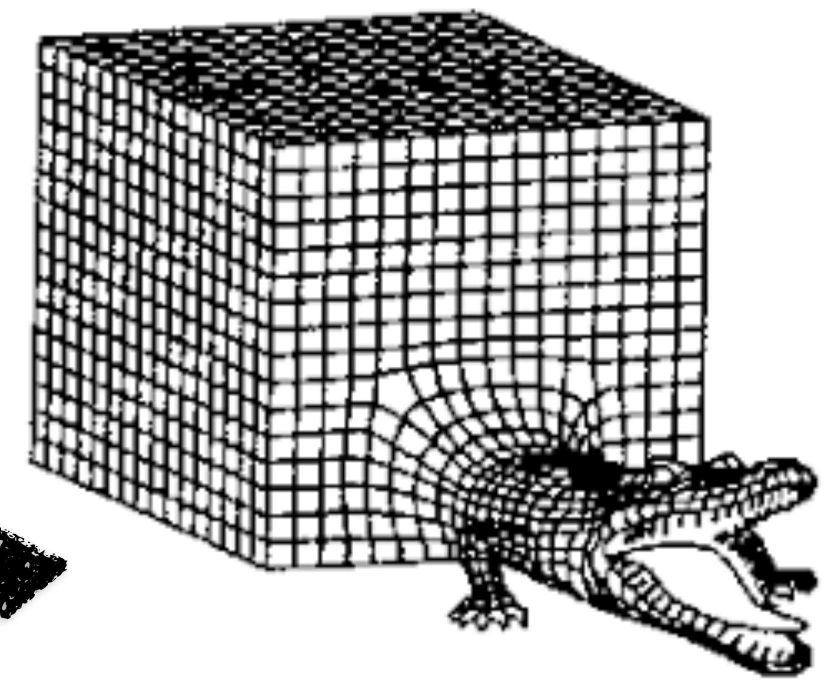
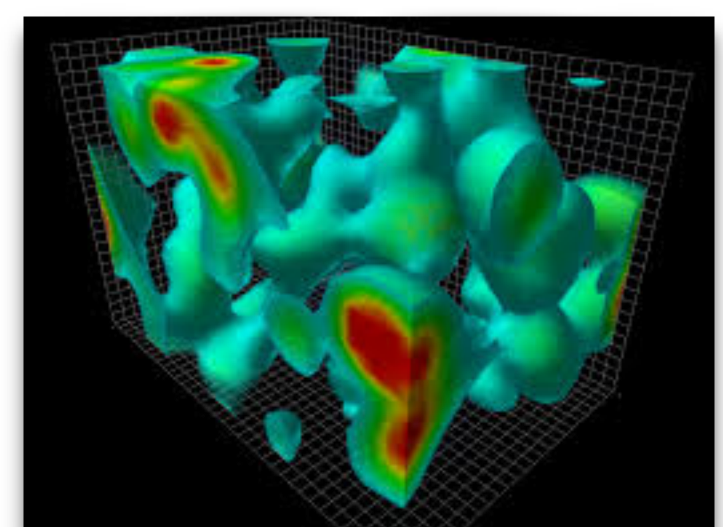


- Strong coupling:
- A field-centric picture: particles emerge from correlated quantum fluctuations
- Lattice-regulated field theory:
  - Approximate spacetime as a finite lattice
  - Rotate to “Euclidean time”
  - Recognize Z as manifestly finite (but high-dimensional) sum
  - Evaluate using Monte Carlo to evaluate correlation functions

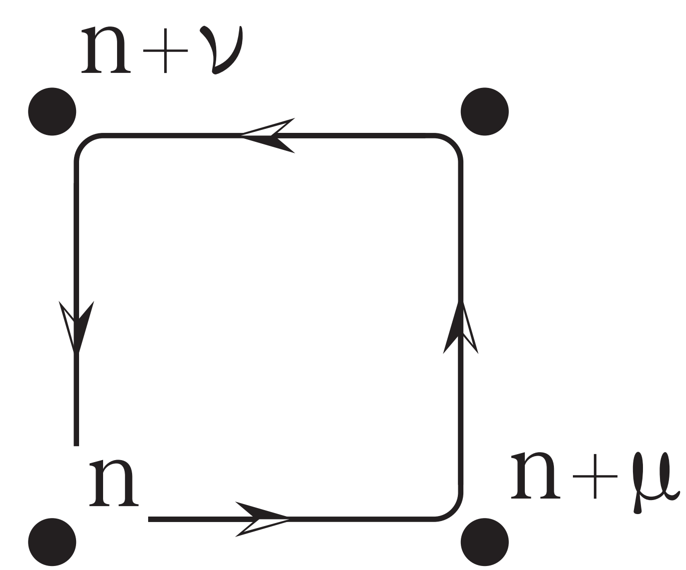


Lattice QCD: What, Why, and How?

Wilson's Lattice Gauge Theory, circa 1975

- At finite lattice spacing, gauge fields are group elements
- Each group element becomes a "color-transport" link
- Products / loops of links ≡ "Plaquettes"

$$U_\mu(n) = e^{iaA_\mu}$$



$$\begin{aligned} S_G &= \frac{2}{g^2} \sum_{n,\mu<\nu} \Re\,\mathrm{Tr}[1 - U_{\mu\nu}(n)] \\ &= \frac{a^4}{2g^2} \sum_{n,\mu<\nu} \mathrm{Tr}[F_{\mu\nu}(n)F^{\mu\nu}(n)] + \mathcal{O}(a^2) \\ &= \text{continuum QCD} + \text{irrelevant operators} \end{aligned}$$

Vanish as a→0

Modern Lattice QCD: Not your parents' lattice QCD

✓Improved actions (better continuum limit)

✓Dynamical quarks with physical masses

✓Continuum and infinite-volume extrapolations

✓Full systematic error budgets

✓A*b initio* calculations of the hadron spectrum and matrix elements with great accuracy and precision

Example Calculation: Particle masses and matrix elements

- Consider QCD in the isospin limit ($m_u = m_d$). Neglect heavier quarks.
- This theory has two free parameters: the gauge coupling and the quark mass
- Calculate two hadronic quantities and match to experiment.
- Then, ***all other hadronic quantities are theoretical predictions***

$$\Gamma(\pi \to \mu\bar{\nu}_\mu) = F_\pi^2 \underbrace{G_F^2}\underbrace{V_{ud}^2}\underbrace{\frac{m_\mu^2 m_\pi}{4\pi}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}$$

Decay const = (QCD physics)

$$\langle 0|\overline{u}\gamma^\mu\gamma^5 d|\pi(p)\rangle = iF_\pi p^\mu$$

Fermi constant = (EW physics)

CKM matrix element

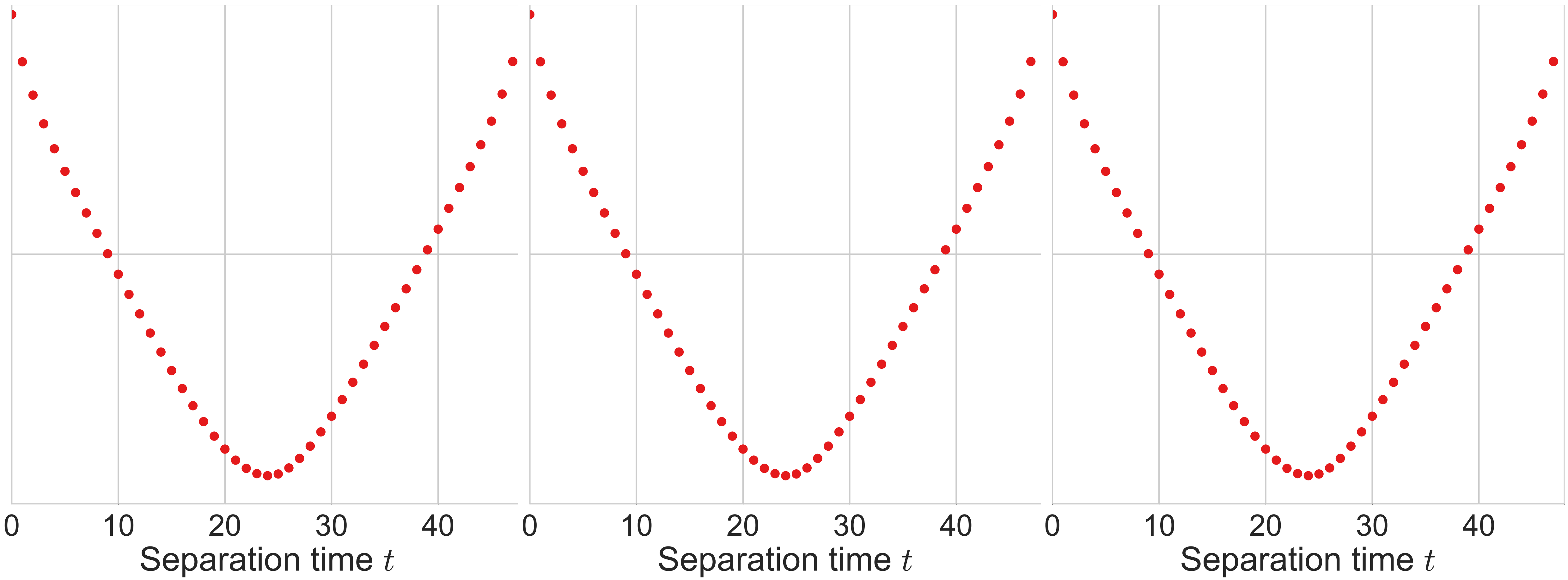
Kinematics & phase space

Example Calculation: Particle masses and matrix elements

$$\int d^3x \langle P(x;t) A^{\mu}(\mathbf{0},0)\rangle \sim \langle 0|P|\pi\rangle F_{\pi} e^{-m_{\pi} t}$$

Exponential decay on a periodic lattice $e^{-m_{\pi} t} \to e^{-m_{\pi}|t|}$

"Measure amplitude and mass from a fit to lattice data"

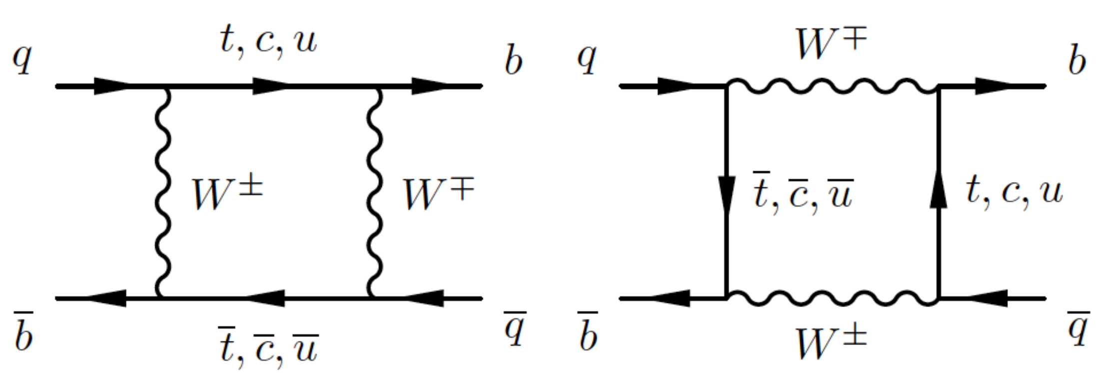


Lattice meets experiment at Fermilab

Flavor physics— a success story

Energy

EW-scale (~100s GeV): the full Standard Model



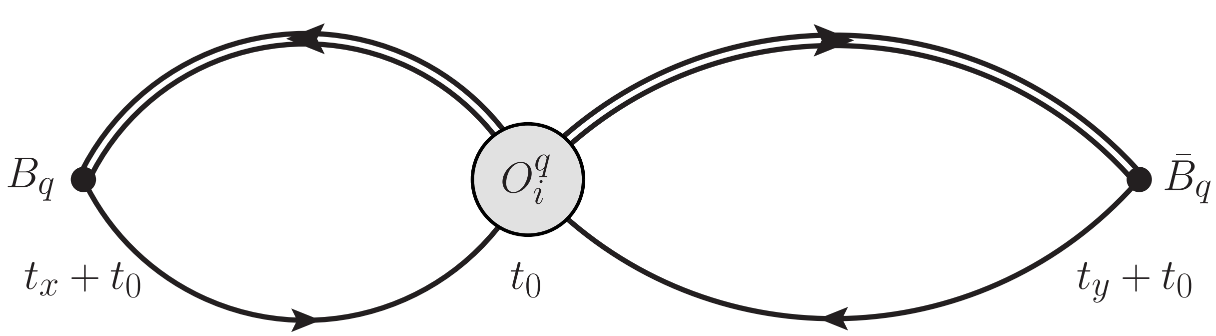
Experiment

Produce heavy mesons and measure:

» Mass differences, ΔM_q

» Decay-width differences, $\Delta\Gamma_q$

QCD-scale (~3 GeV): effective four-fermion interactions



Theory: Lattice QCD

Calculate three-point functions and extract:

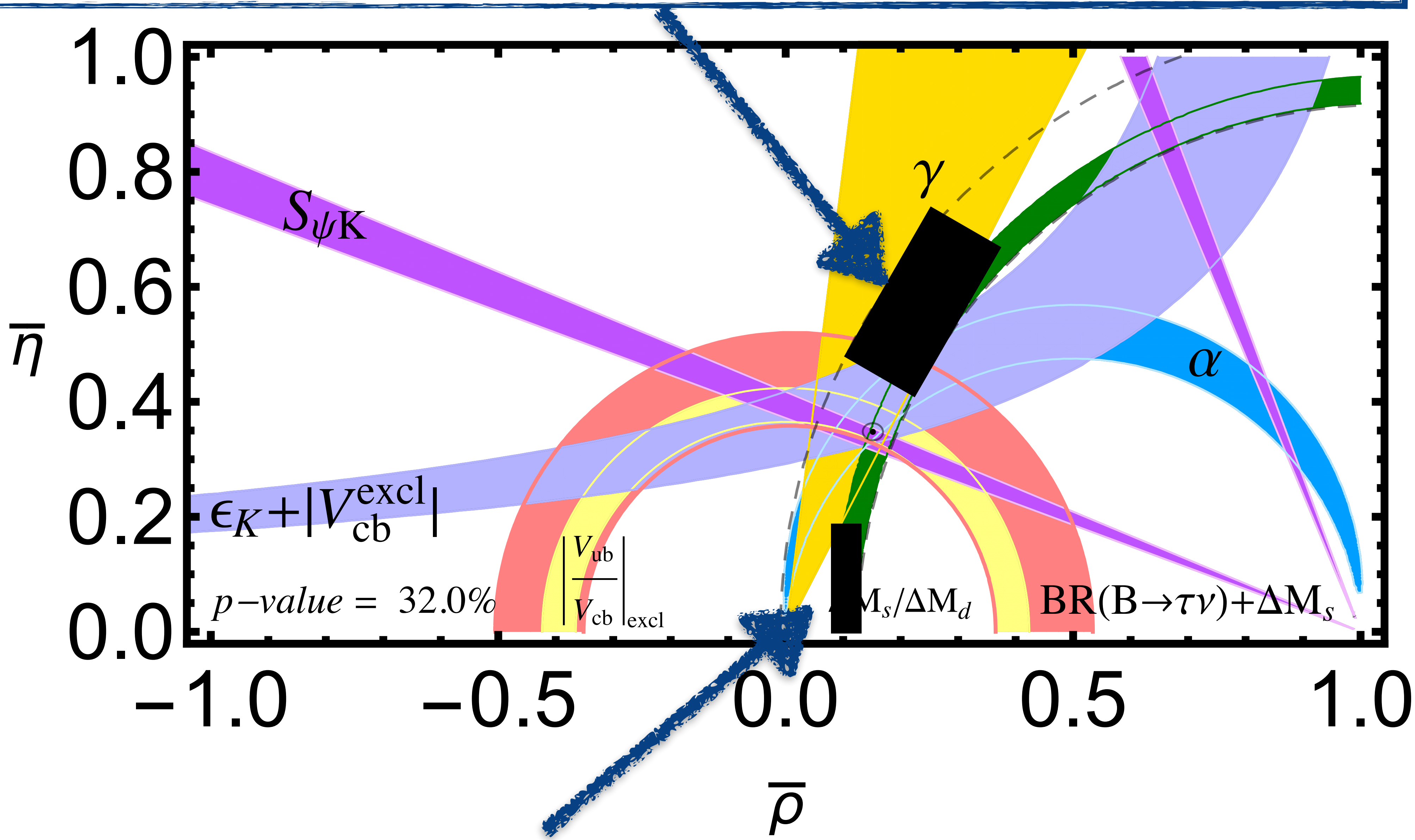
» Mesons masses

» Amplitudes

Lattice meets experiment at Fermilab

Mixing of heavy mesons — a success story

- » $\Delta\Gamma_d$: Delphi, BABAR, Belle, D0, LHCb
- » $\Delta\Gamma_d$: CDF, ATLAS, CMS, LHCb
- » ΔM_d: Belle, BABAR, LHCb
- » ΔM_s: CDF, LHCb



arXiv:1602.03560 from Fermilab Lattice + MILC Collaboration

Lattice meets experiment at Fermilab

The anomalous magnetic moment of the muon - partial results

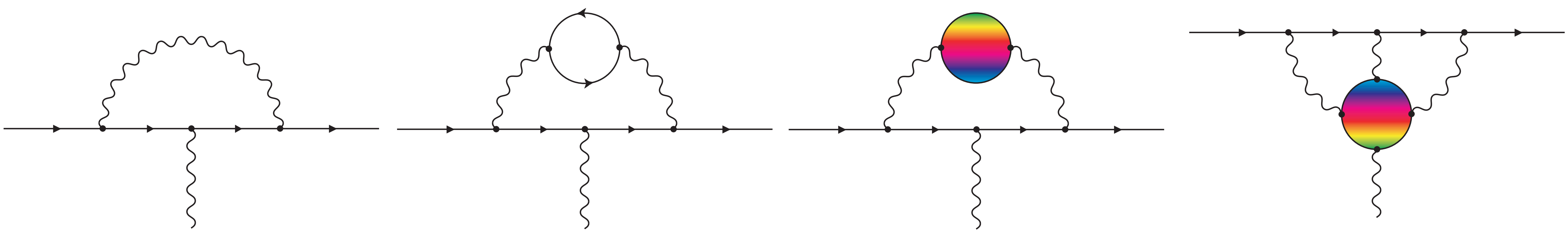
$$a_\mu^{EXP} = 116\,592\,089\,(63) \times 10^{-11}$$

$$a_\mu^{QED} = 116\,584\,718.\,95\,(8) \times 10^{-11}$$

- QED dominates to better than 1 in 10,000
- QCD corrections: 60 ppm
  - Hadronic vacuum polarization ~ 59 ppm
  - Hadronic light-by-light ~ 1 ppm
- Weak corrections: 1.3 ppm

Hadronic vacuum polarization

Hadronic “light-by-light”



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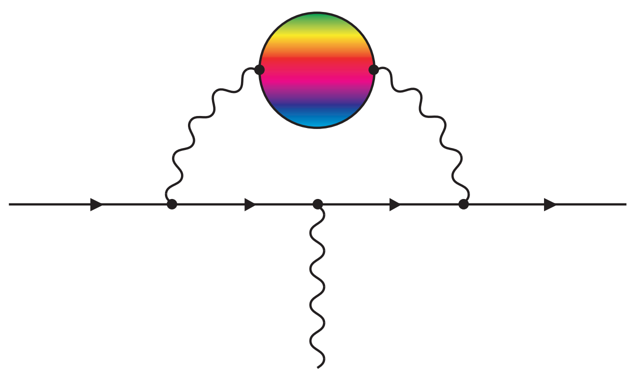
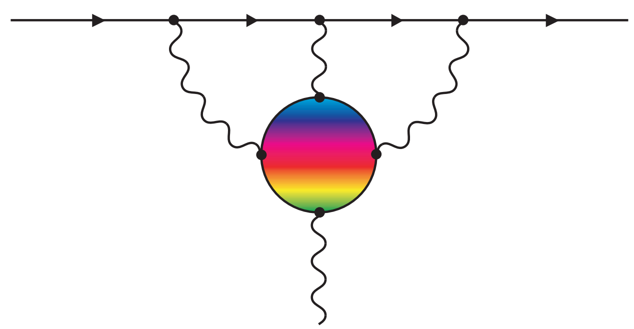
Hadronic vacuum polarization



Hadronic "light-by-light"

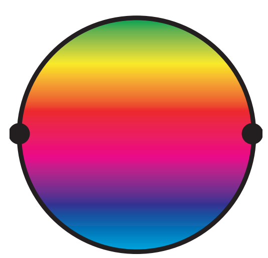


Can estimate HVP effect from experiment:

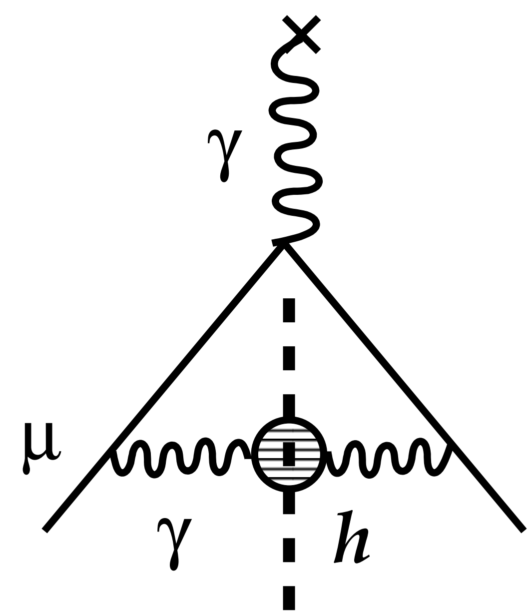
$$a_\mu^{\mathrm{HVP,LO}} \sim \text{``} \int \frac{\sigma_{tot}(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \text{''}$$

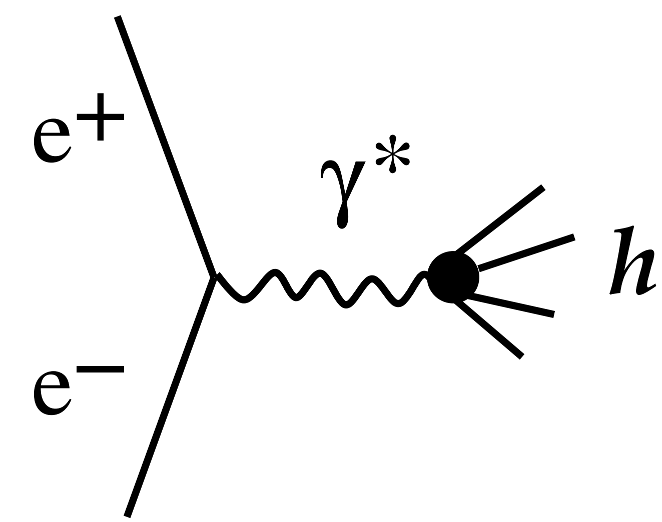
Theory: Calculate with lattice QCD

$$a_\mu^{\mathrm{HVP,LO}} \sim \int dQ^2 \times$$



Vector-vector two-point function



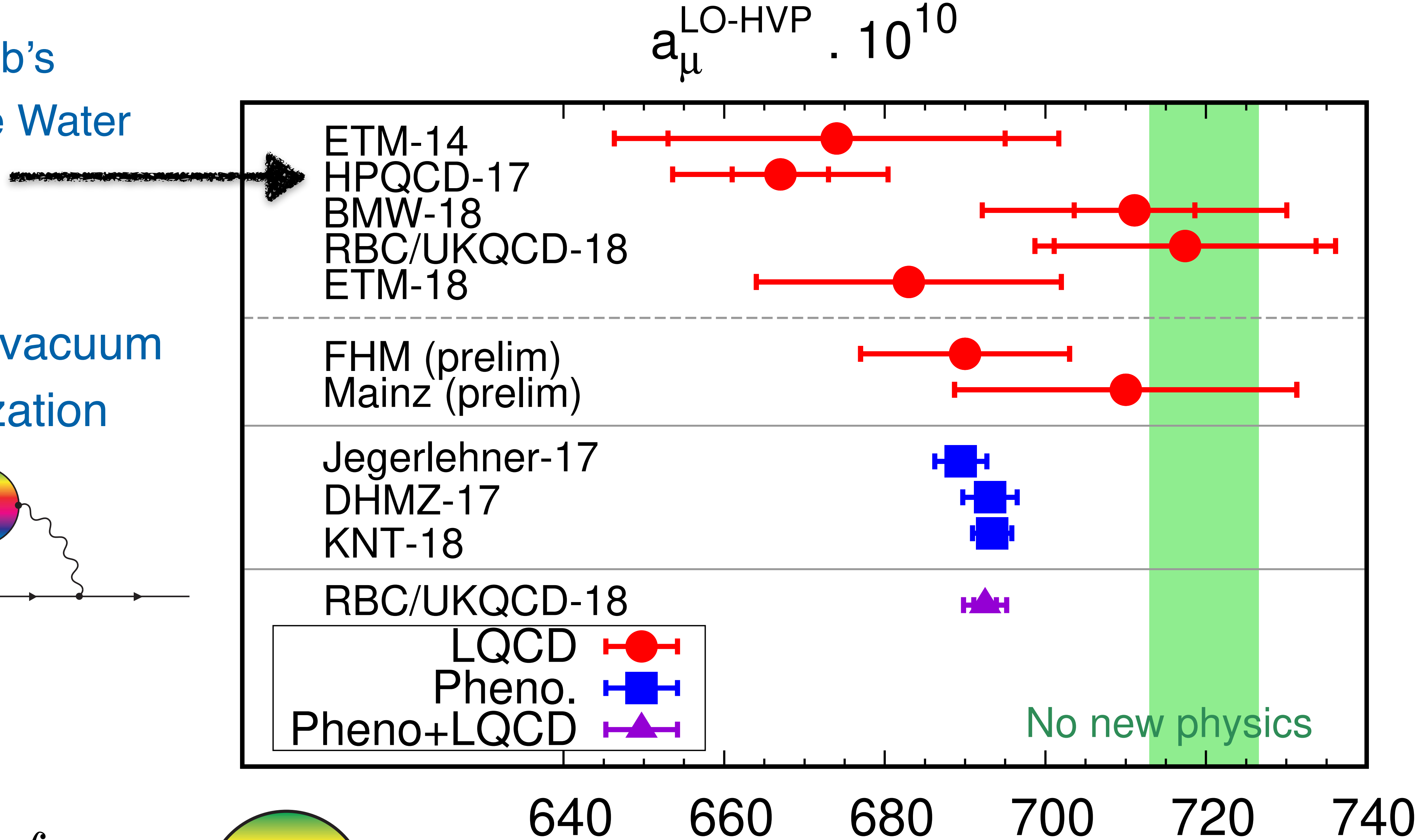


Lattice meets experiment at Fermilab

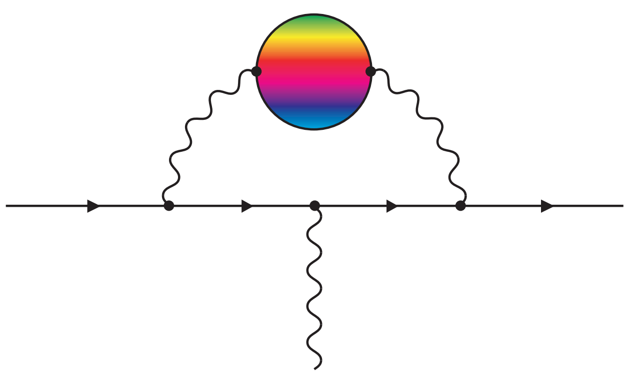
The anomalous magnetic moment of the muon - partial results

with Fermilab's

Ruth Van de Water



Hadronic vacuum polarization



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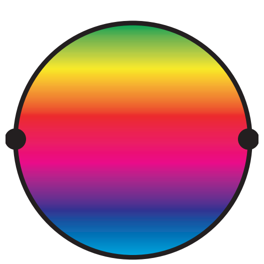
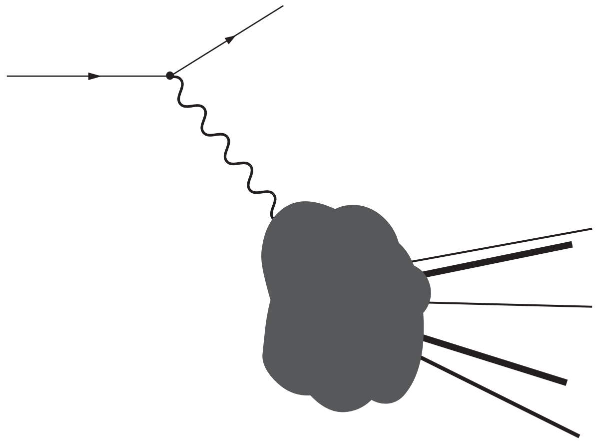


Figure: K. Miura in arXiv:1901.09052

Lattice meets experiment at Fermilab
Neutrino scattering — a frontier for lattice QCD



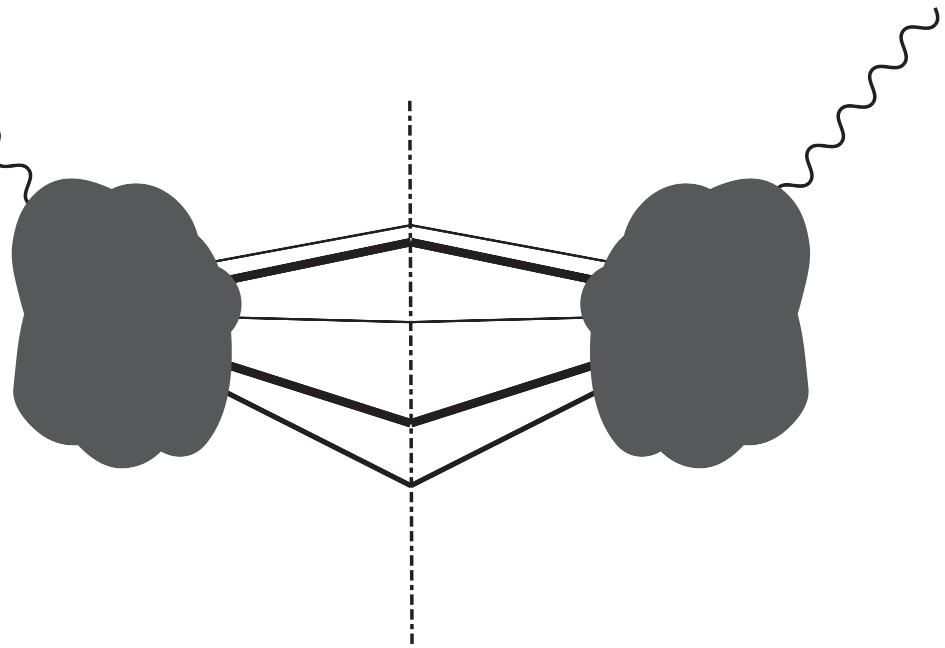
Suppose a neutrino strikes an argon nucleus…

… at high energies. Then the probe interacts with a single parton. The physics is deep inelastic scattering.

… at low energies. Then the probe interacts with hadronic constituents. The physics is “shallow inelastic scattering.”

$\Longrightarrow$ Non-perturbative physics accessible from lattice QCD

$$d\sigma(E_\nu) \propto$$



Lattice meets experiment at Fermilab

Neutrino scattering — a frontier for lattice QCD

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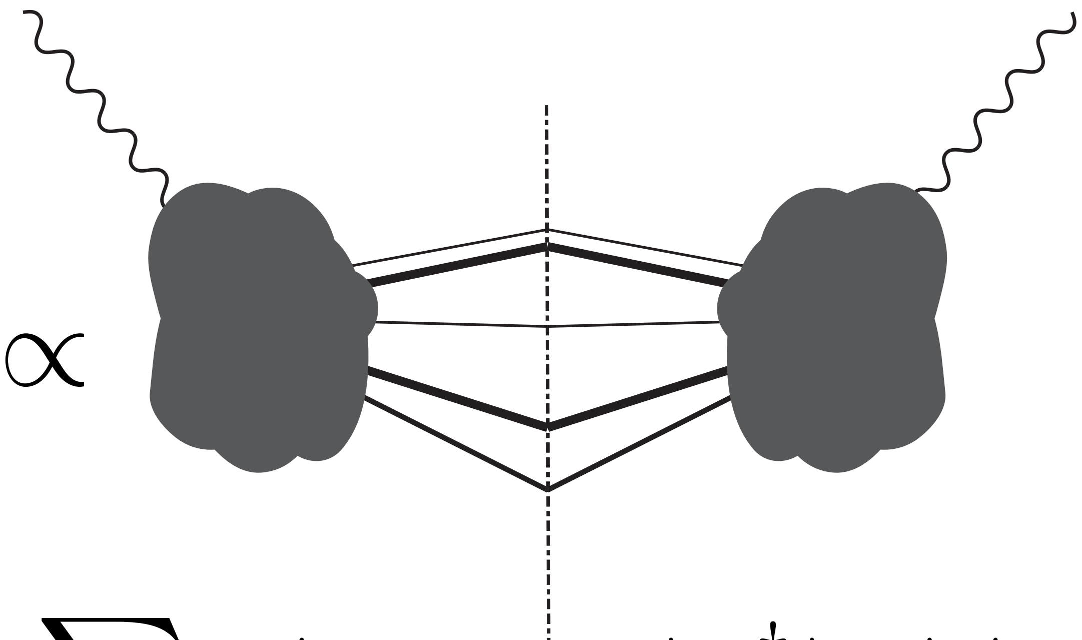

"Hadron tensor" $W_{\mu\nu}$
(Contains "structure functions"
aka "nuclear responses")

$$W_{\mu\nu} \propto \sum_{\substack{\text{hadronic} \\ \text{states X}}} \langle \text{nucleon} | J_\nu^\dagger | X \rangle \langle X | J_\mu | \text{nucleon} \rangle$$

$$= \langle N | J_\nu^\dagger J_\mu | N \rangle$$

- » Four-point correlation function
- » Calculate with lattice QCD

A tricky lattice calculation, but we're thinking about it!

More info: USQCD community whitepaper arXiv:1904.09931

References

- Lattice QCD textbooks:
  - Montvay & Müster, “Quantum Fields on a Lattice”
  - DeGrand & DeTar, “Lattice Methods for Quantum Chromodynamics”
  - Gattringer & Lang, “Quantum Chromodynamics on the Lattice”
  - Knechtli, Günter, & Peardon, “Lattice Quantum Chromodynamics: Practical Essentials”
- Flavour Lattice Averaging Group: http://flag.unibe.ch/2019/
  - 2019 FLAG reprot arxiv.1902.08191
    - “The PDG of flavor physics on the lattice”
    - More than 450 pages with many useful summaries
    - A good introduction / overview of the literature
  - B-mixing matrix elements referenced here: arXiv 1602.03560
- Muon (g-2) on the Lattice
  - Plenary from Lattice2018 conference: K. Miura arXiv:1901.09052
  - E.g., arxiv:1806.08190 for recent work from Fermilab
- Neutrino Physics on the Lattice
  - “Lattice QCD and Neutrino-Nucleus Scattering” whitepaper arXiv:1904.09931