Perturbative QCD in 10 Minutes

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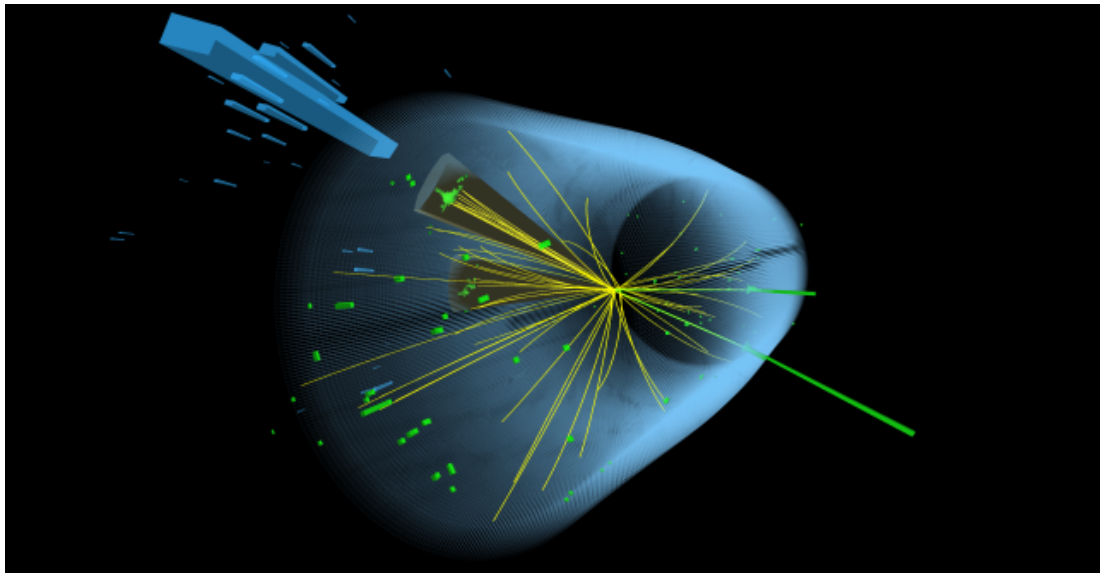
New Perspectives 2019

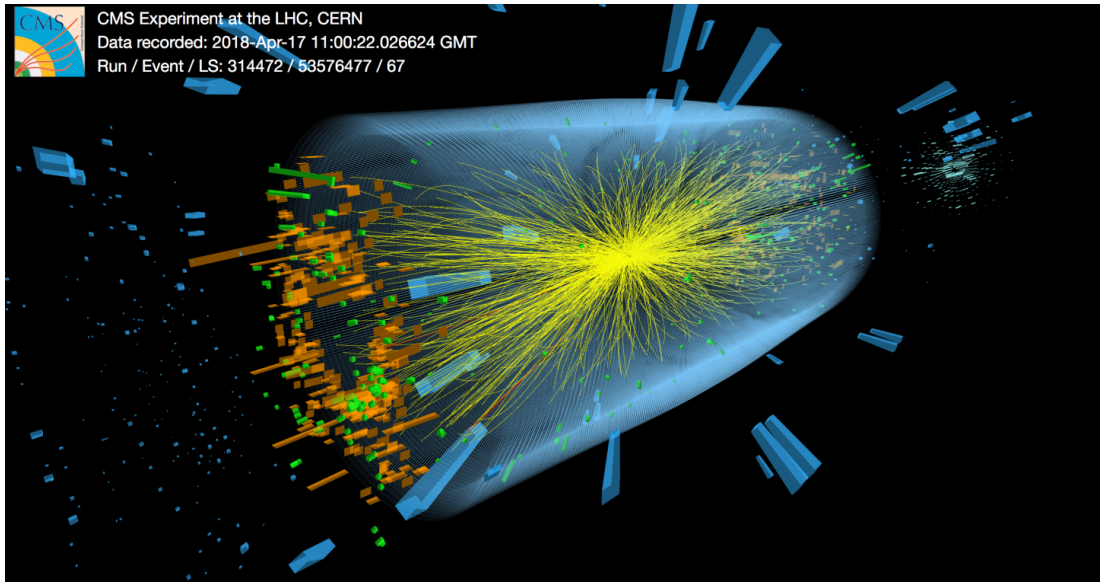
11 June 2019

Introduction

Goals of the Perturbative QCD Program:

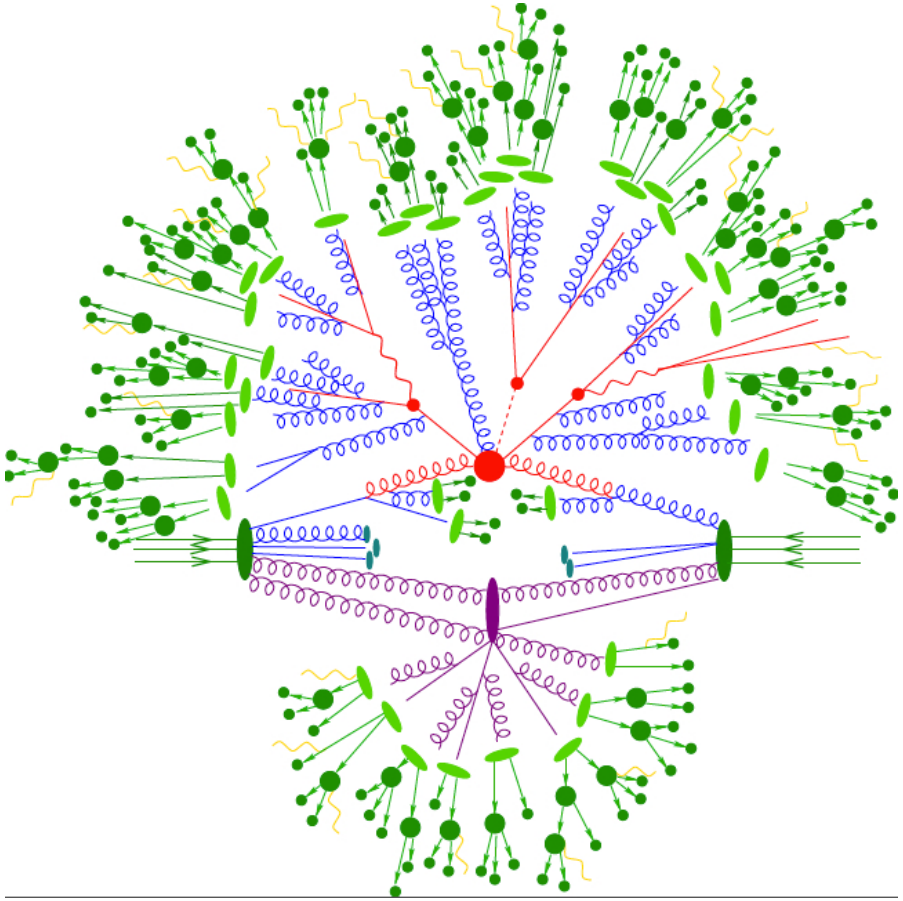
- Simulate events / Predict observables at the LHC according to the Standard Model
- Understand the background for beyond the Standard Model searches
- Obtain high precision predictions better than experimental uncertainty





Pieces of an Event

Factorization of an event:

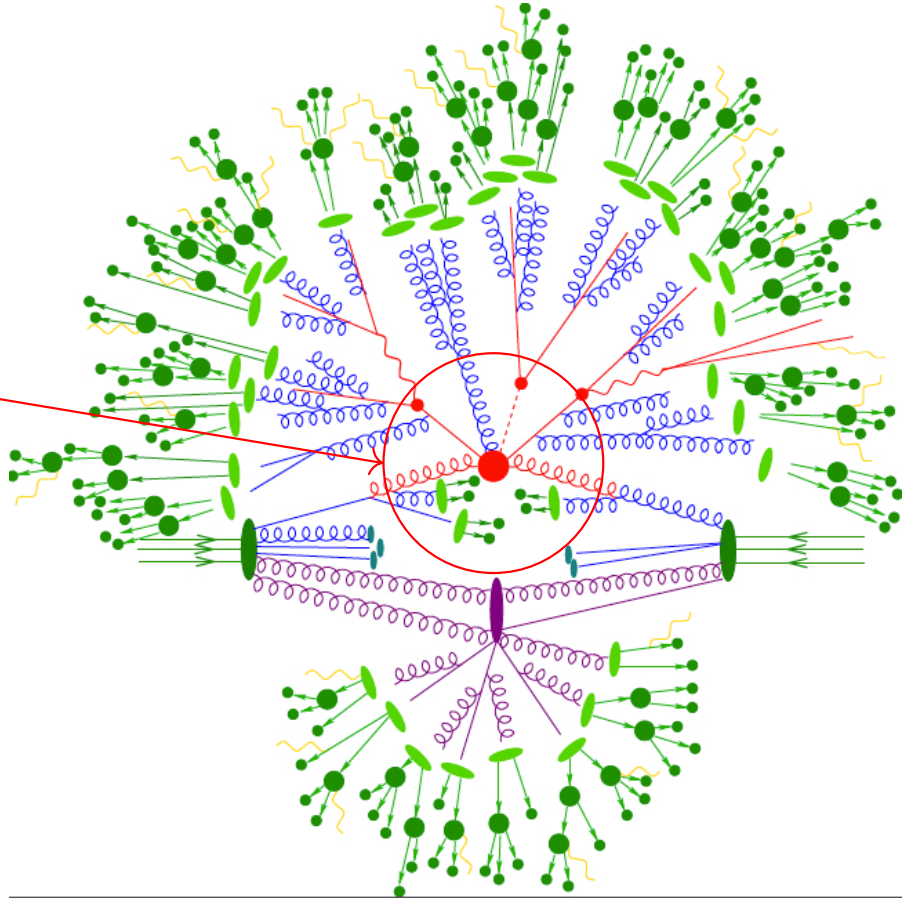


[Image from the Sherpa Authors]

Pieces of an Event

Factorization of an event:

- Hard Process

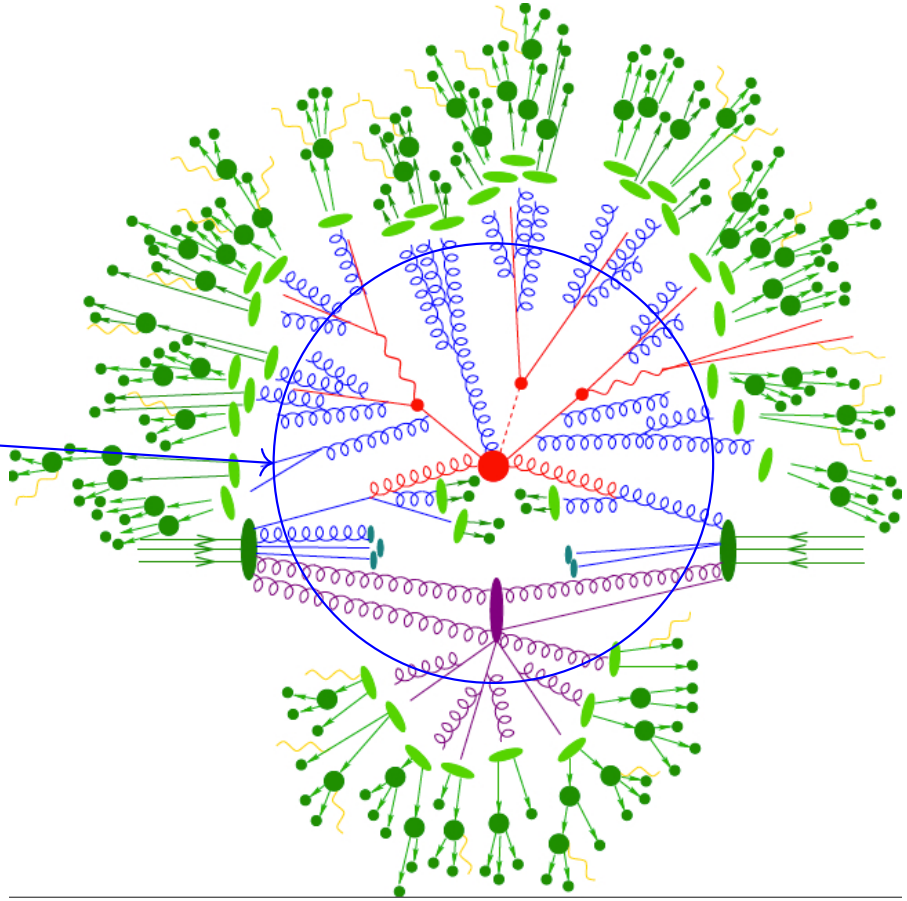


[Image from the Sherpa Authors]

Pieces of an Event

Factorization of an event:

- Hard Process
- Parton Shower (Resummation)

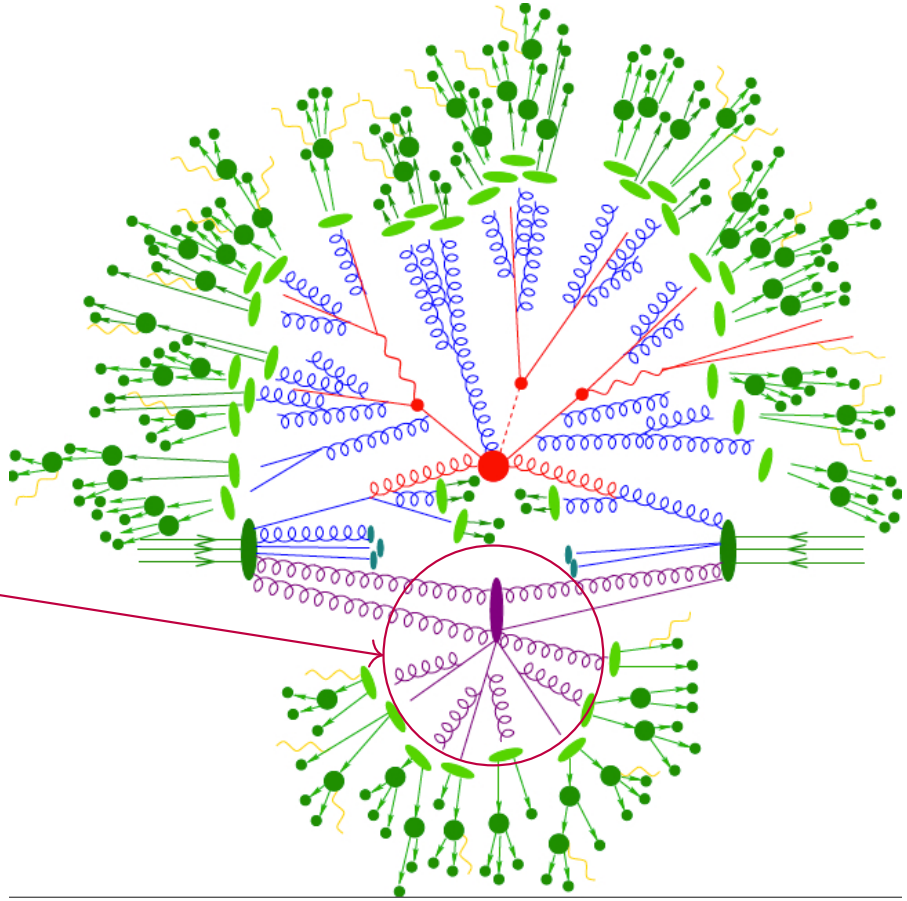


[Image from the Sherpa Authors]

Pieces of an Event

Factorization of an event:

- Hard Process
- Parton Shower (Resummation)
- Multiple Parton Interations

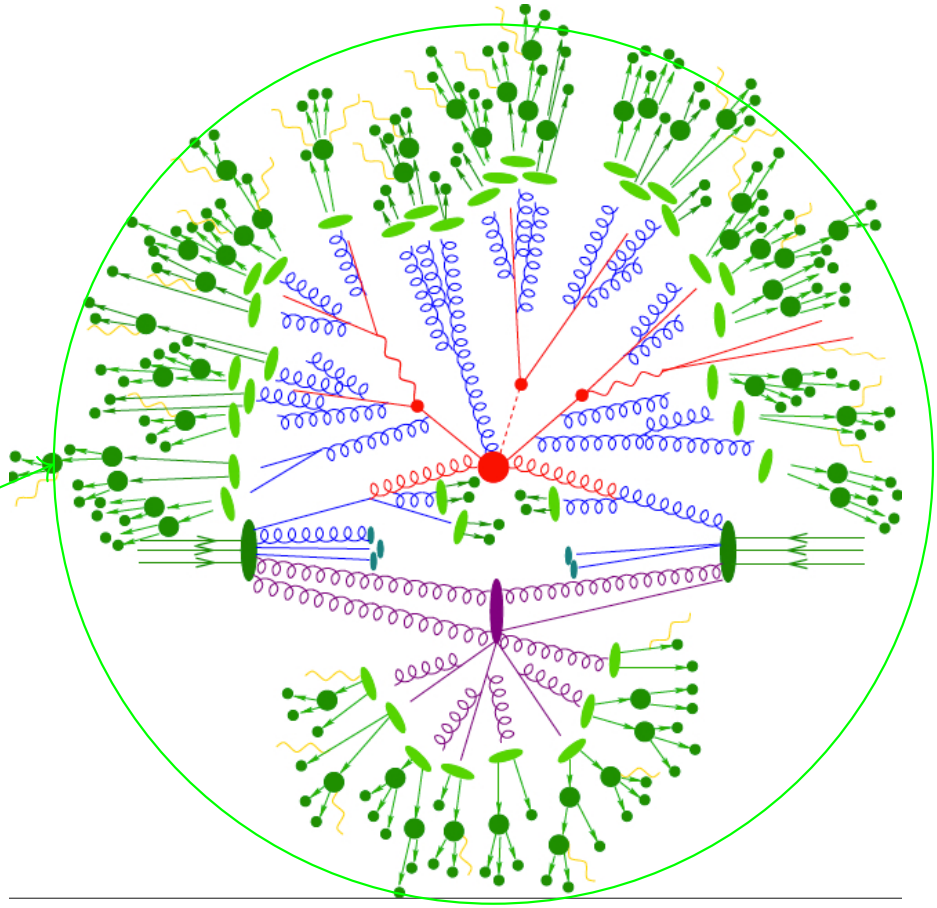


[Image from the Sherpa Authors]

Pieces of an Event

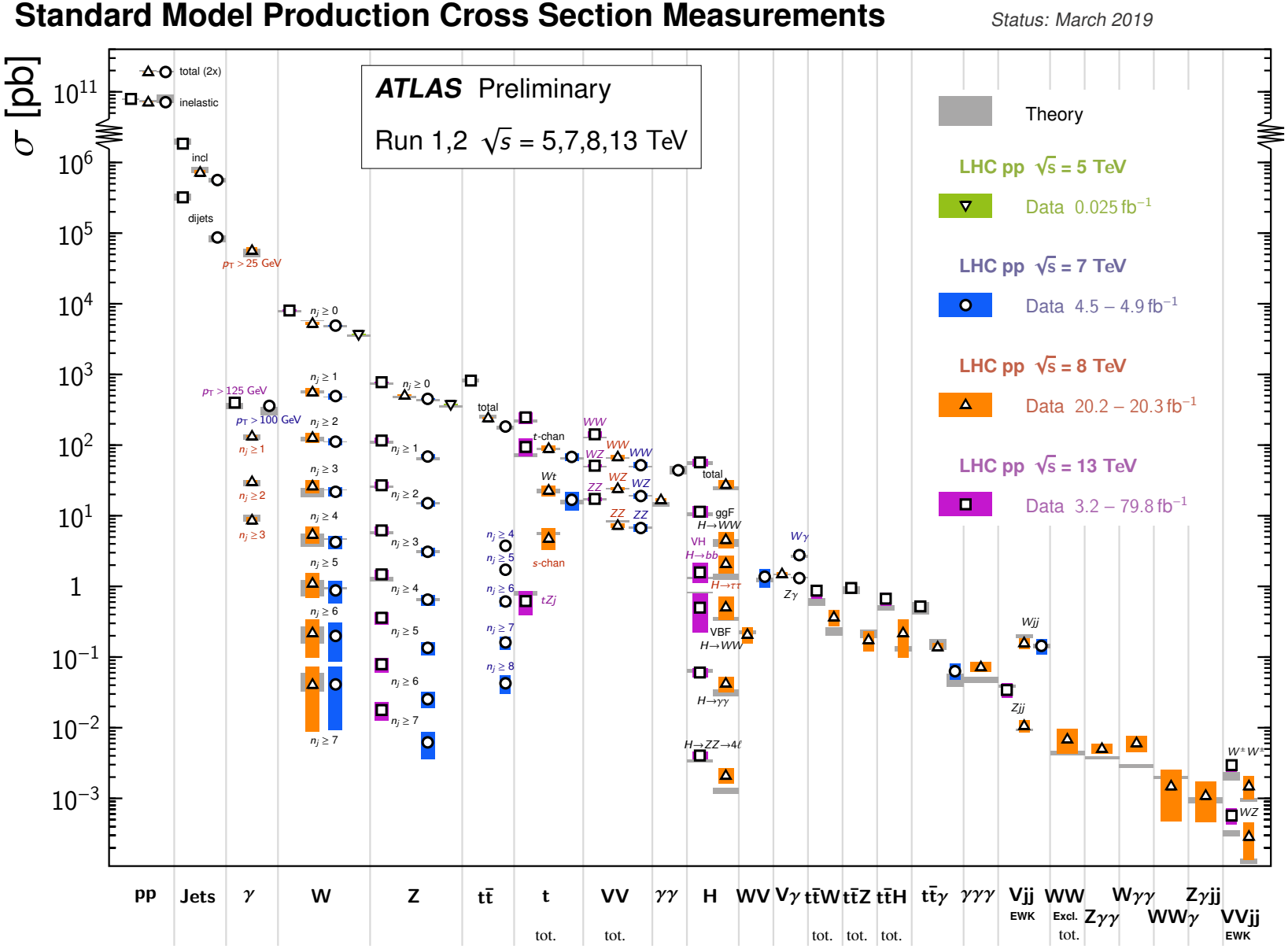
Factorization of an event:

- Hard Process
- Parton Shower (Resummation)
- Multiple Parton Interations
- Hadronization

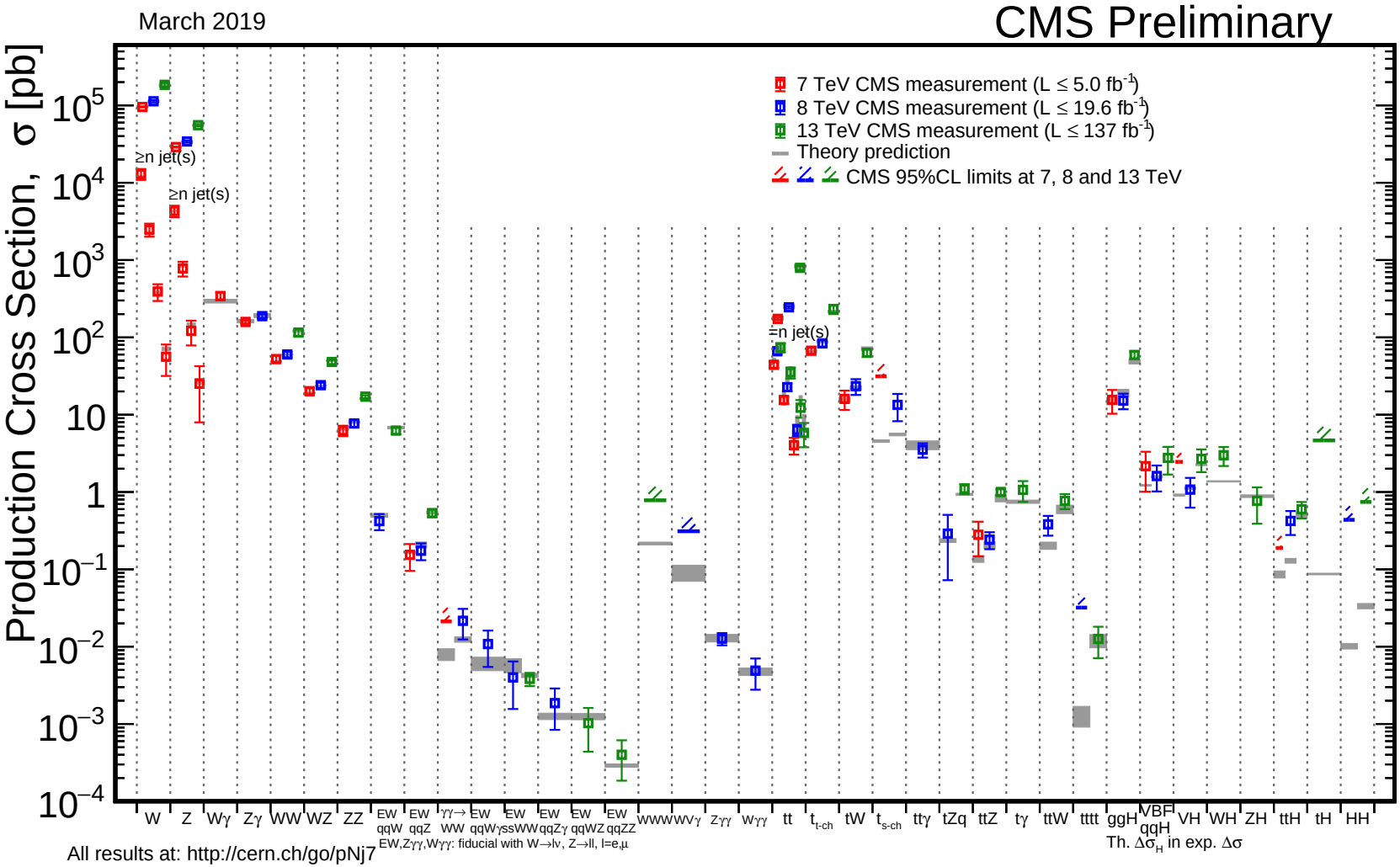


[Image from the Sherpa Authors]

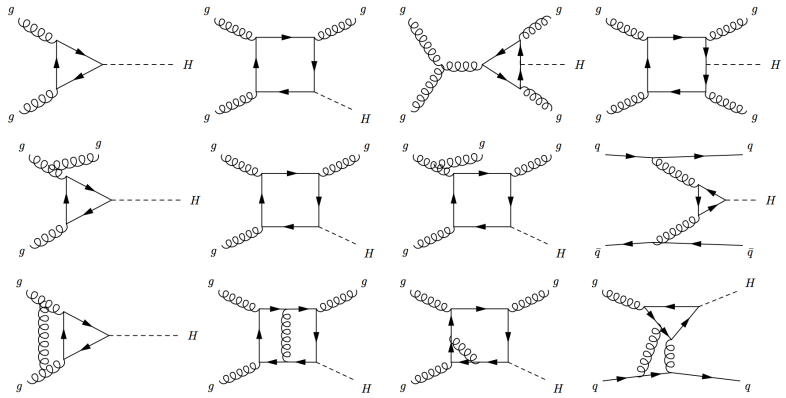
Hard Process



Hard Process



Perturbative Series



Sample of diagrams evolved in higher order corrections to Higgs Production

[Buschmann et. al. arXiv:1410.5806]

- Calculation done as expansion in coupling constant (α_s)
- Represented diagramatically by Feynman Diagrams
- Cancellation of IR poles between virtual and real diagrams
- Cancellation of UV poles in virtual digrams by renormalization
- High orders have large number of diagrams and complicated integrals. Ex. Higgs Total Cross-Section at N^3LO (4th term in series) has about 174,938 diagrams and about 300 master integrals.

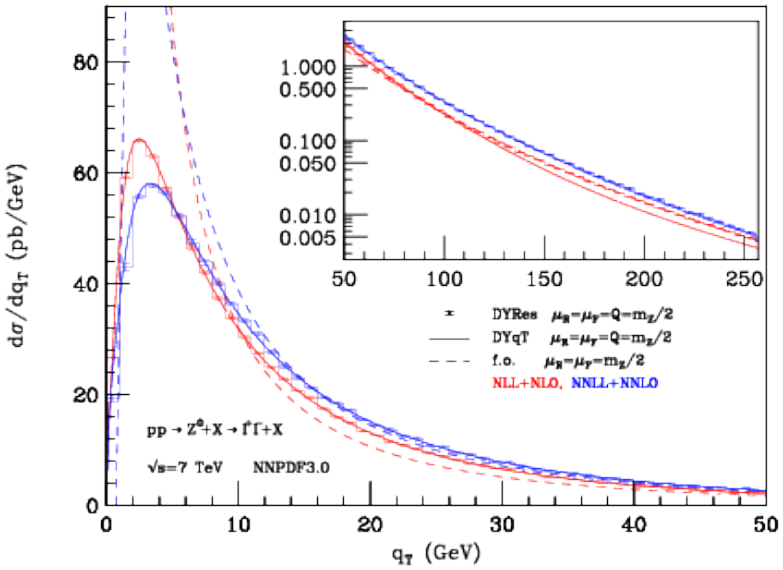
Breakdown of Fixed Order

Fixed order calculations cannot be used for every observable.
Example: Vector Boson Transverse Momentum at Hadron Colliders

- Fixed Order:

$$\frac{d\sigma}{dq_T} \propto \sum_i \alpha_s^i \log^{2i} \frac{q_T^2}{Q^2}$$

- Results in integrable singularity when virtual correction included
- Each term in expansion larger than previous term



Resummation

	LL	NLL	NNLL	NNNLL		N^kLL
LO	1					
NLO	$\alpha_s \log^2$	$\alpha_s \log$	α_s			
NNLO	$\alpha_s^2 \log^4$	$\alpha_s^2 \log^3$	$\alpha_s^2 \log^2$	$\alpha_s^2 \log$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	
N^kLO	$\alpha_s^k \log^{2k}$	$\alpha_s^k \log^{2k-1}$	$\alpha_s^k \log^{2k-2}$	$\alpha_s^k \log^{2k-3}$		α_s^k

The organization of fixed-order and resummed calculations. Going across a row is the fixed-order calculation included at a given order of α_s, while going down the columns is the resummed calculation up the the given power of the logs included.

Resummation

Transverse Momentum Resummation

$$\frac{d\sigma}{dQ^2 dq_T^2 dy} = \frac{\sigma_0}{(2\pi)^2} \int d^2b e^{i\vec{q}_T \cdot \vec{b}} \; e^{-\mathcal{S}(Q,b)} \; \sum_{ij} C_i\left(x_1\right) C_j\left(x_2\right)$$

Resummation

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- Impact Parameter (b):
  Is a measure of the distance of closest approach of the two protons. Fourier conjugate of transverse momentum.

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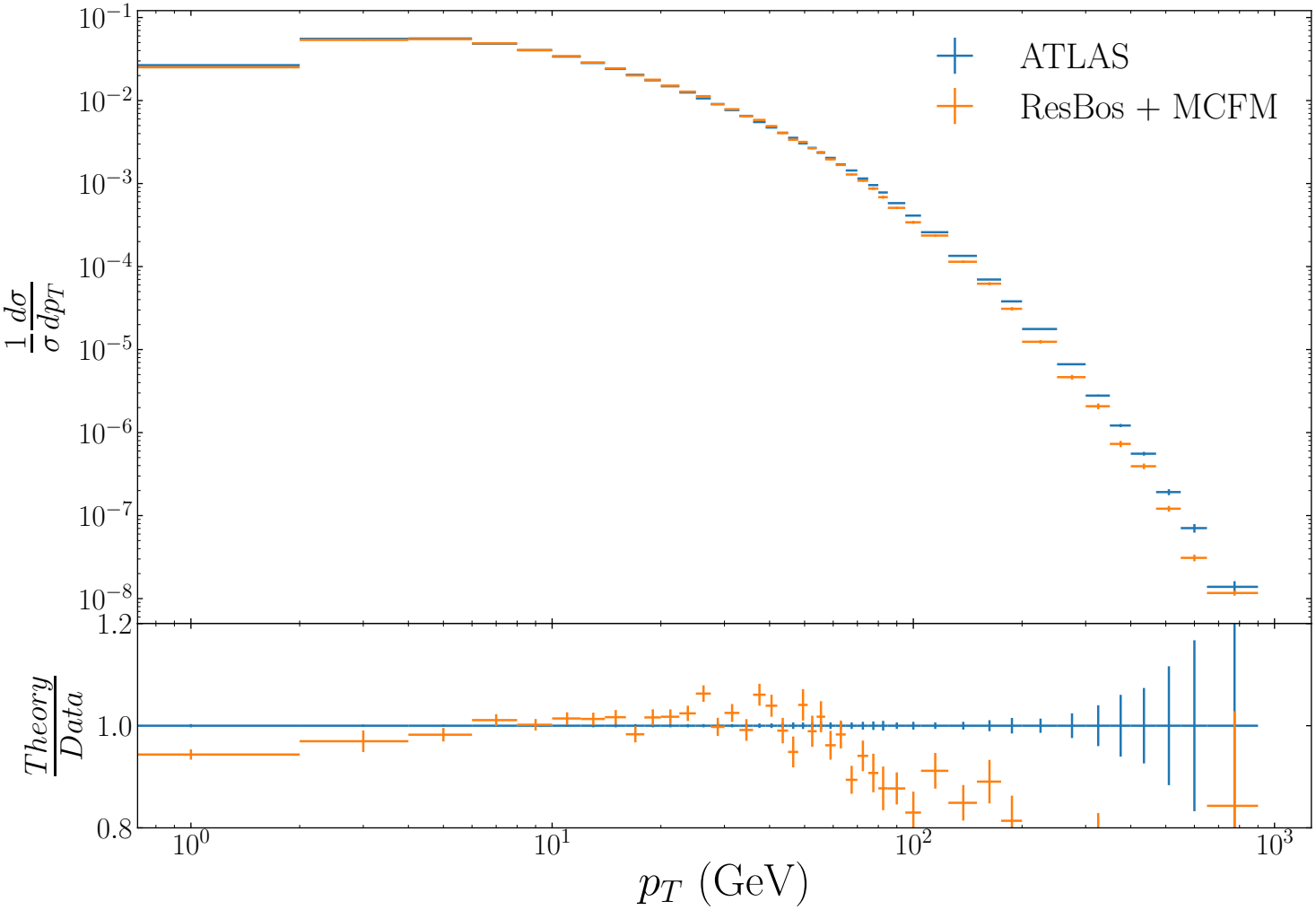
- Sudakov Factor:

$$S(Q, b, C_1, C_2) = \int \frac{d\mu^2}{\mu^2} \left(A \log \frac{Q^2}{\mu^2} + B \right)$$

Resummation



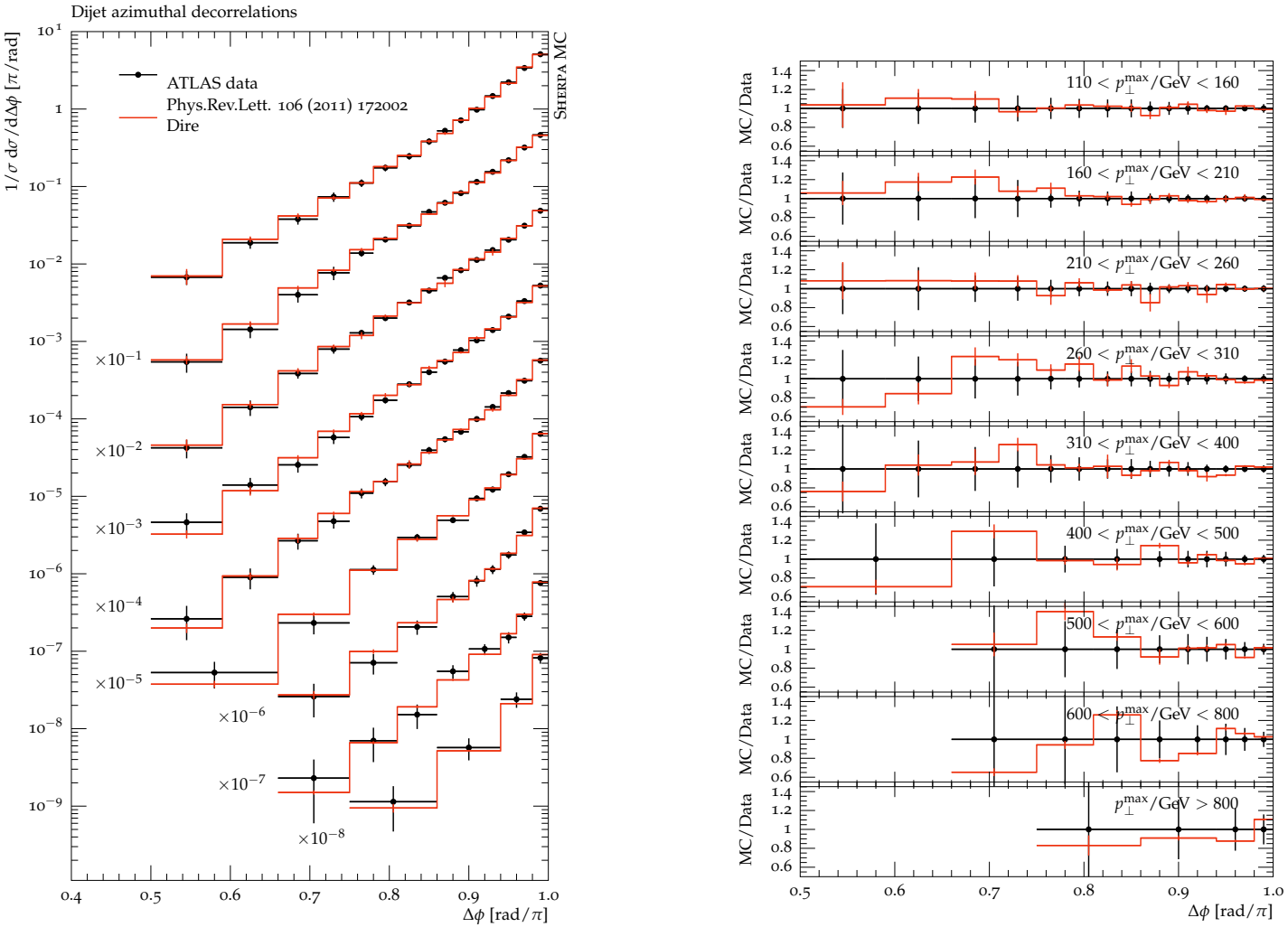
Parton Shower

Evolution Equation

$$\frac{df_a(x,t)}{d\ln t} = \sum_{b=q,g} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ab}(z) f_b\left(\frac{x}{z}, t\right)$$

- $f_a(x,t)$ is the obervable being evolved
- $P_{ab}(z)$ is the evolution kernel
- Solve using Markovian Monte-Carlo algorithms
- Treat P_{ab} as a probability
- Can think of as numerical resummation
- Easier to apply on arbitrary processes

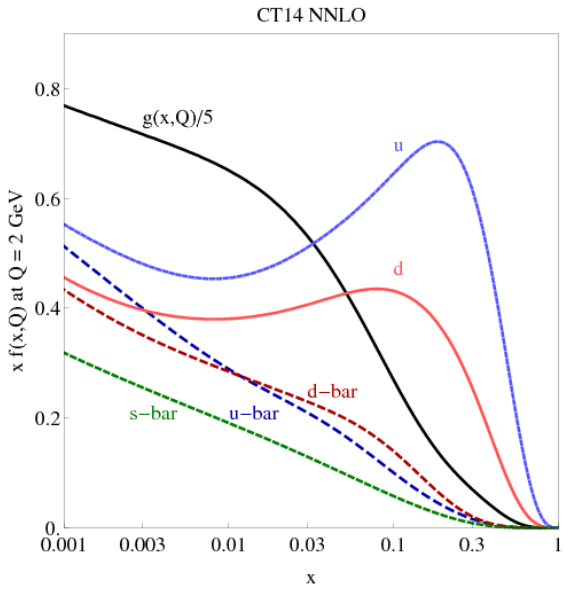
Example: Azimuthal angle between dijets



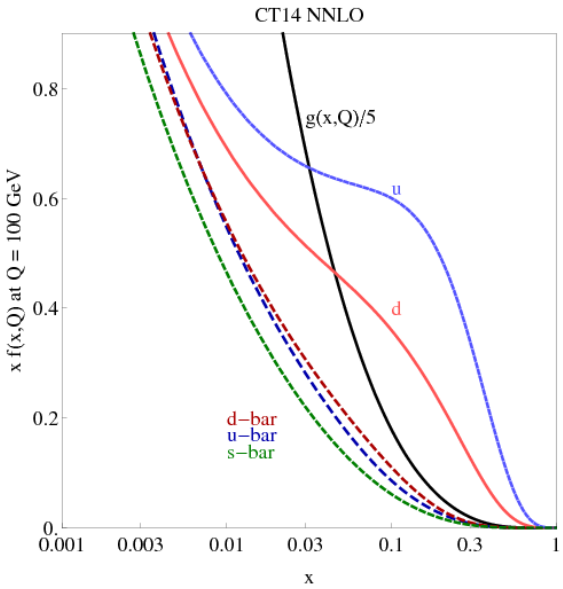
[Höche and Prestel, arXiv:1506.05057]

Parton Distribution Functions

- Probability of taking a parton with momentum fraction x out of a proton
- Has a non-perturbative piece fit to data, and a perturbative piece



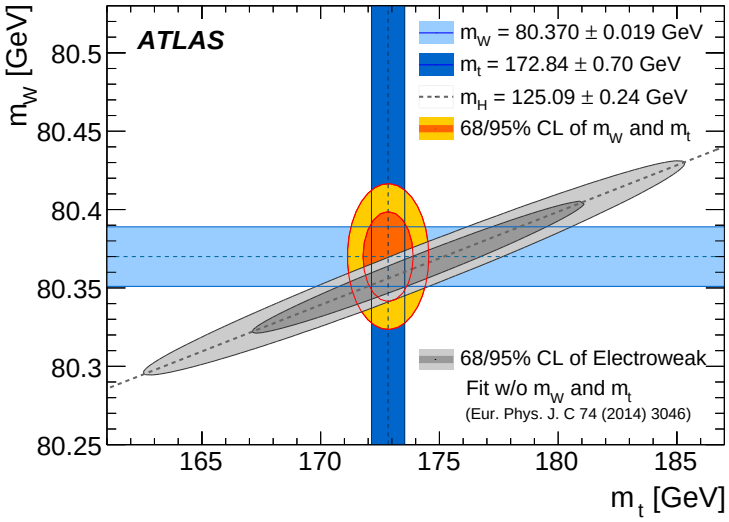
[Dulat et. al., arxiv:1506.07443]



[Dulat et. al., arxiv:1506.07443]

- Governed by DGLAP evolution equations
- Defined in the infinite momentum frame of the proton
- Depends on the hard scale of the interaction

W Mass



[ATLAS, arXiv:1701.07240]

Combined categories	Value [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.	χ^2/dof of Comb.
m_t, W^+, e-μ	80370.0	12.3	8.3	6.7	14.5	9.7	9.4	3.4	16.9	30.9	2/6
m_t, W^-, e-μ	80381.1	13.9	8.8	6.6	11.8	10.2	9.7	3.4	16.2	30.5	7/6
m_t, $W^\pm$, e-μ	80375.7	9.6	7.8	5.5	13.0	8.3	9.6	3.4	10.2	25.1	11/13
p_T^ℓ, W^+, e-μ	80352.0	9.6	6.5	8.4	2.5	5.2	8.3	5.7	14.5	23.5	5/6
p_T^ℓ, W^-, e-μ	80383.4	10.8	7.0	8.1	2.5	6.1	8.1	5.7	13.5	23.6	10/6
p_T^ℓ, $W^\pm$, e-μ	80369.4	7.2	6.3	6.7	2.5	4.6	8.3	5.7	9.0	18.7	19/13
p_T^ℓ, $W^\pm$, e	80347.2	9.9	0.0	14.8	2.6	5.7	8.2	5.3	8.9	23.1	4/5
m_t, $W^\pm$, e	80364.6	13.5	0.0	14.4	13.2	12.8	9.5	3.4	10.2	30.8	8/5
m_t-$^\ell$, W^+, e	80345.4	11.7	0.0	16.0	3.8	7.4	8.3	5.0	13.7	27.4	1/5
m_t-$^\ell$, W^-, e	80359.4	12.9	0.0	15.1	3.9	8.5	8.4	4.9	13.4	27.6	8/5
m_t-$^\ell$, $W^\pm$, e	80349.8	9.0	0.0	14.7	3.3	6.1	8.3	5.1	9.0	22.9	12/11
p_T^ℓ, $W^\pm$, μ	80382.3	10.1	10.7	0.0	2.5	3.9	8.4	6.0	10.7	21.4	7/7
m_t, $W^\pm$, μ	80381.5	13.0	11.6	0.0	13.0	6.0	9.6	3.4	11.2	27.2	3/7
m_t-p_T^ℓ, W^+, μ	80364.1	11.4	12.4	0.0	4.0	4.7	8.8	5.4	17.6	27.2	5/7
m_t-p_T^ℓ, W^-, μ	80398.6	12.0	13.0	0.0	4.1	5.7	8.4	5.3	16.8	27.4	3/7
m_t-p_T^ℓ, $W^\pm$, μ	80382.0	8.6	10.7	0.0	3.7	4.3	8.6	5.4	10.9	21.0	10/15
m_t-p_T^ℓ, W^+, e-μ	80352.7	8.9	6.6	8.2	3.1	5.5	8.4	5.4	14.6	23.4	7/13
m_t-p_T^ℓ, W^-, e-μ	80383.6	9.7	7.2	7.8	3.3	6.6	8.3	5.3	13.6	23.4	15/13
m_t-p_T^ℓ, $W^\pm$, e-μ	80369.5	6.8	6.6	6.4	2.9	4.5	8.3	5.5	9.2	18.5	29/27

[ATLAS, arXiv:1701.07240]

- Important precision test of SM
- Dominated by PDF and theory uncertainty
- Calculate $\frac{d\sigma/dp_T^W}{d\sigma/dp_T^Z}$ accurately to use data driven methods to extract M_W

Conclusions

- Many pieces involved in calculating Standard Model predictions
- Important for both precision observables and BSM searches
- Fixed order calculations work wonderfully for certain observables, but other techniques are needed for those dominated by soft gluon radiation
- Parton Distribution Functions one of largest Standard Model uncertainty

BACKUP

MCFM

Fixed order program developed at Fermilab: mcfm.fnal.gov

Authors:

- John Campbell
- Keith Ellis
- Walter Giele
- Tobias Neumann
- Ciaran Williams

ResBos

Resumation calculation developed by Joshua Isaacson: ResBos2

- Contains resummation to N^3LL with matching to $NNLO$ through MCFM
- Mainly focused on color singlet proesses, but being extended to processes with jets

Sherpa

Event Generator: Stefan Höche is now at Fermilab: Sherpa

- Contains matrix element generator
- Contains parton shower code
- Is a complete event generator

Multiple Parton Interactions and Hadronization