Reconstructing proton-proton collision positions at the Large Hadron Collider with a D-Wave quantum computer

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New Perspectives 2019, Fermi National Accelerator Laboratory, June 11th , 2019

Introduction

- Counter-rotating beams of bunches of protons cross, producing multiple collisions of protons
- Clustering resulting tracks along beam axis determines the p-p collision points
- Currently performed classically via deterministic annealing
- We use *quantum annealing*, p-p collision points = centroid of tracks belonging to a single cluster
- Centroid-based clustering is NP-hard, often heuristics are applied to find local minima solutions

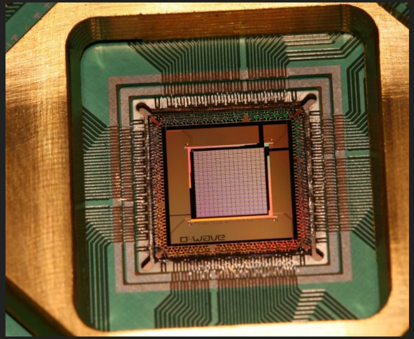
Quantum Computing

What is a Quantum Computer?

A quantum computer is a machine that utilizes the unique properties of quantum mechanics to perform calculations

- Two Paradigms
  - **Quantum Circuits**
  - **Quantum Annealers**

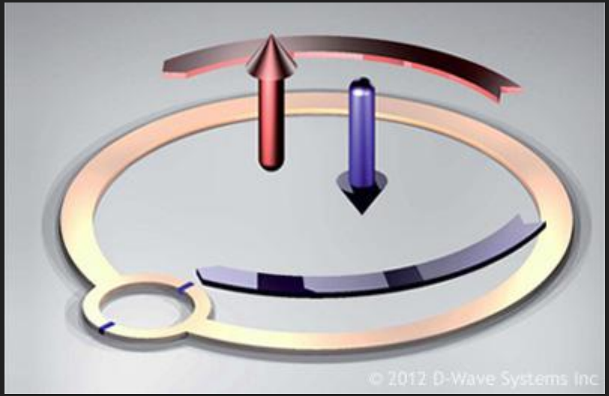
Quantum Annealers — D-Wave



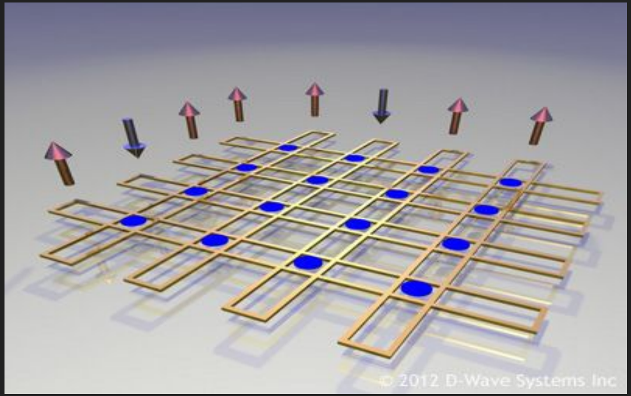
- Not a universal quantum computer
  - Cannot implement Shor's Algorithm, Grover's Algorithm, …
  - Can still do prime number factorization!
- Quantum Processing Unit (QPU) made of ~2048 rf-SQUIDs (radio frequency-superconducting quantum interference device) acting as qubits
  - Programmable external biases and couplings between qubits are made available
  - Not a fully connected graph of qubits
- System can be modeled as an **Ising model**

$$H_p = \sum_i h_i \sigma_z^i + \sum_i \sum_{j>i} J_{ij} \sigma_z^i \sigma_z^j$$

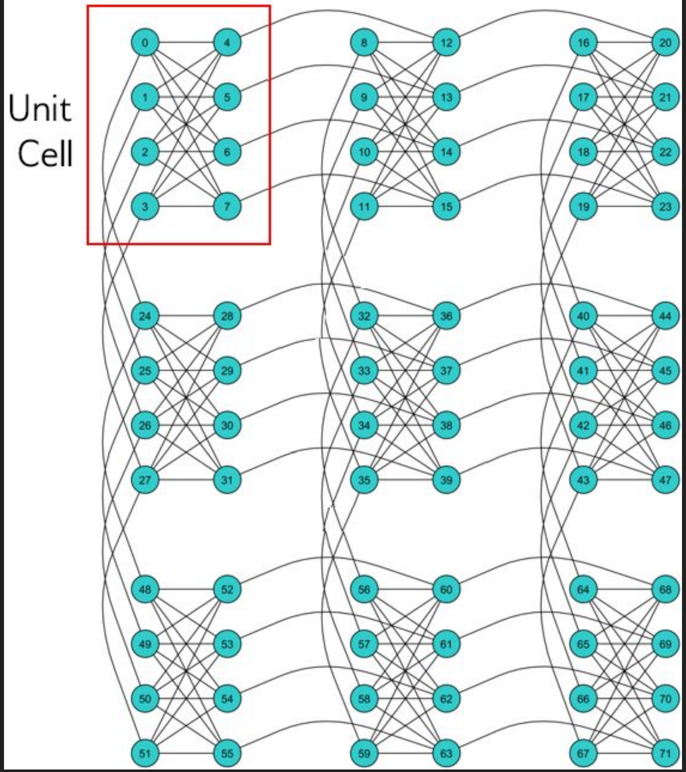
Qubits on Hardware



Top: A flux qubit made from an RF-SQUID. Bottom: The unit cell as it appears on the hardware. [Link to D-Wave]



Chimera Graph

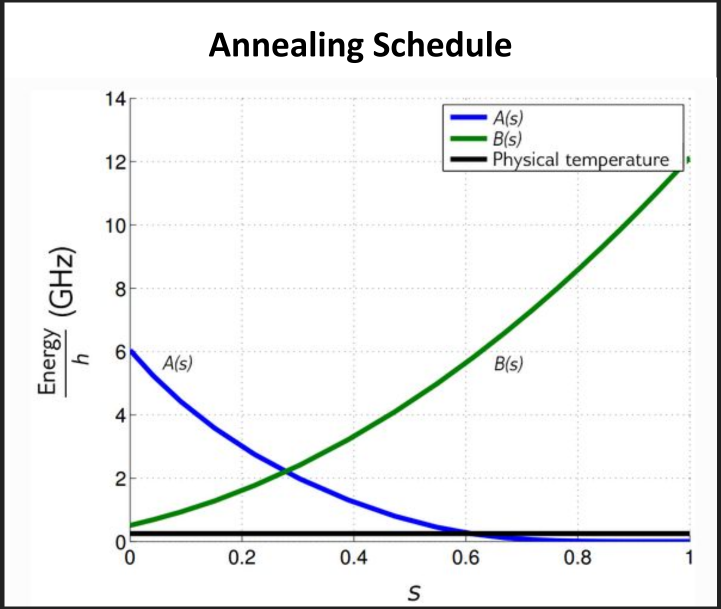


The chimera graph showcasing the limited connectivity of the qubits. [Link to D-Wave]

Quantum Annealers — Annealing Schedule

- Practical approximation to an adiabatic quantum computer
- *Adiabatic Theorem - A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum* [1]
- Final state is the ground state and the optimal solution to the problem Hamiltonian

$$\mathcal{H} = -\frac{A(s)}{2}\left(\sum_i \sigma_x^i\right) + \frac{B(s)}{2}\left(H_p\right)$$

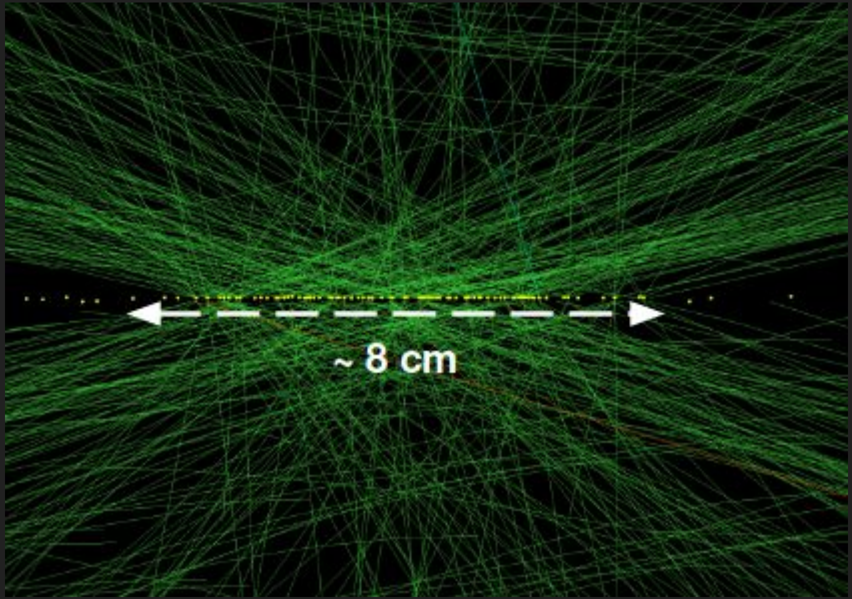


The annealing schedule and functions A(s) and B(s). [Link to D-Wave]

The Algorithm

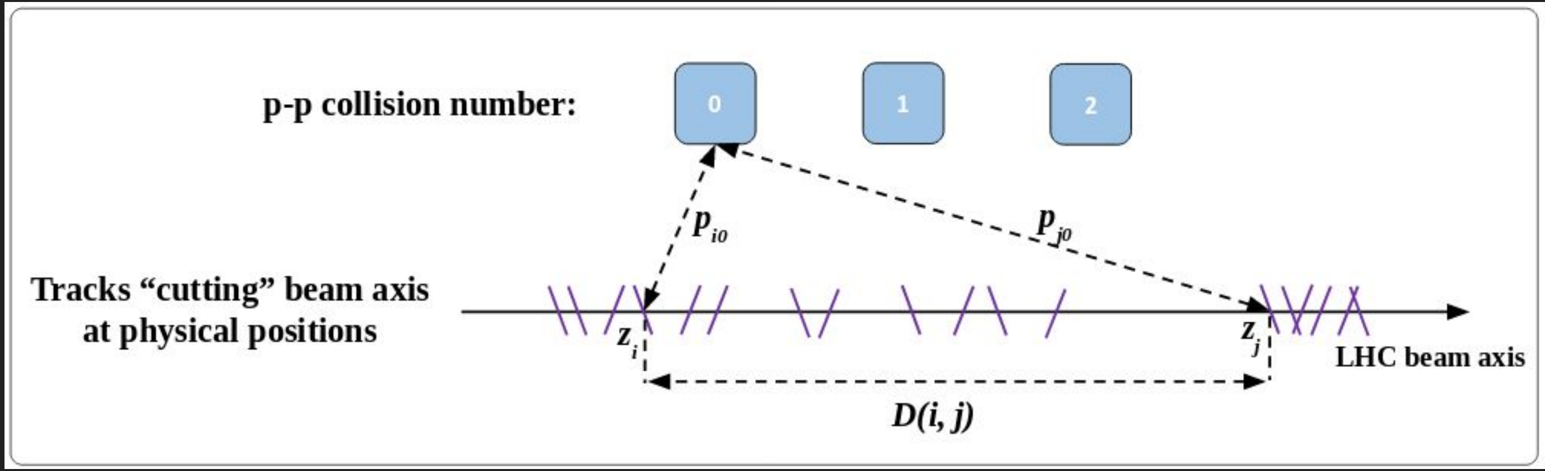
The Problem

- Cluster reconstructed particle tracks to determine location of proton-proton collision
- Currently a classical algorithm performs this analysis - deterministic annealing
- Quantum annealing seems like a natural step forwards
- Maps naturally onto the Ising Hamiltonian



An event in CMS with 78 p-p collisions. Green lines are charged particle tracks, yellow dots are p-p collisions.

The Formulation



A graphical representation of the algorithm

The Formulation

$$H_p = \sum_{k}^{n_V} \sum_{i}^{n_T} \sum_{j>i}^{n_T} p_{ik}\, p_{jk}\, g\left(D(i,j); m\right) + \lambda \sum_{i}^{n_T} \left(1 - \sum_{k}^{n_V} p_{ik}\right)^2$$

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p_{ik} is the probability that the i^{th} track belongs to the k^{th} cluster. $p_{ik} \in \{0, 1\}$

The Formulation

$$D(i,j) = \frac{|z_i - z_j|}{\sqrt{\delta z_i^2 + \delta z_j^2}}$$

z_i is the location of closest approach to the beam axis for the particle track

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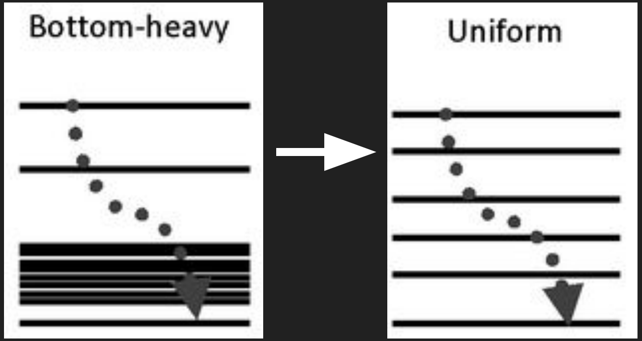
Distortion function used to distribute energy levels more uniformly

$$g(x;m) = 1 - e^{-mx}$$



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Constraint to ensure a track belongs to a single cluster

z_i is the location of closest approach to the beam axis for the particle track

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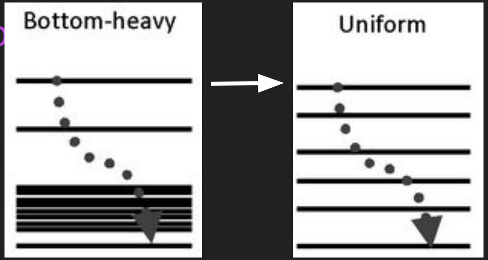

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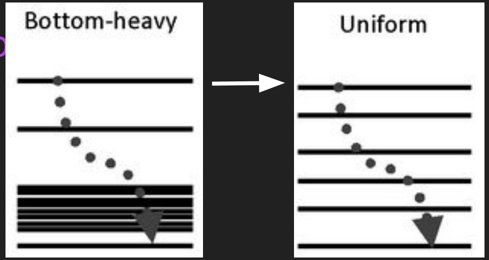
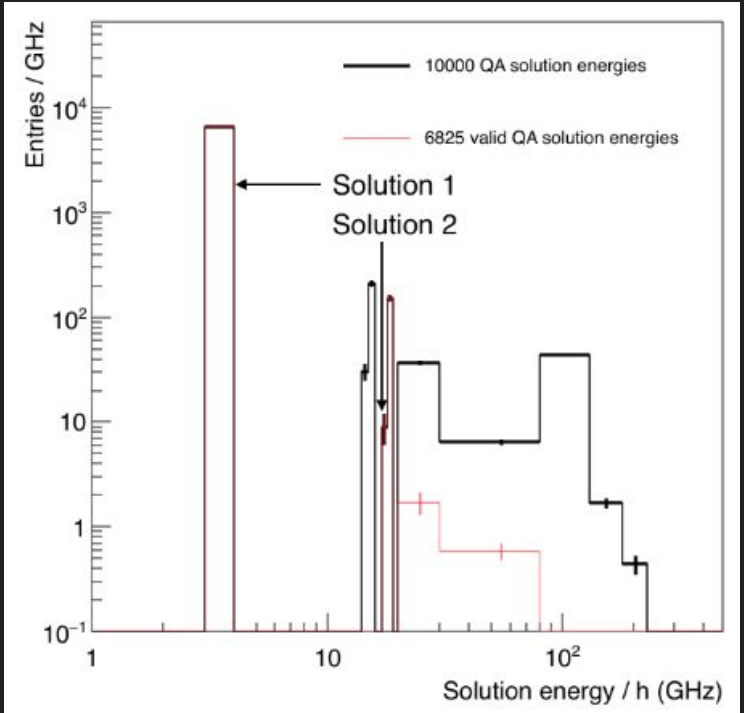
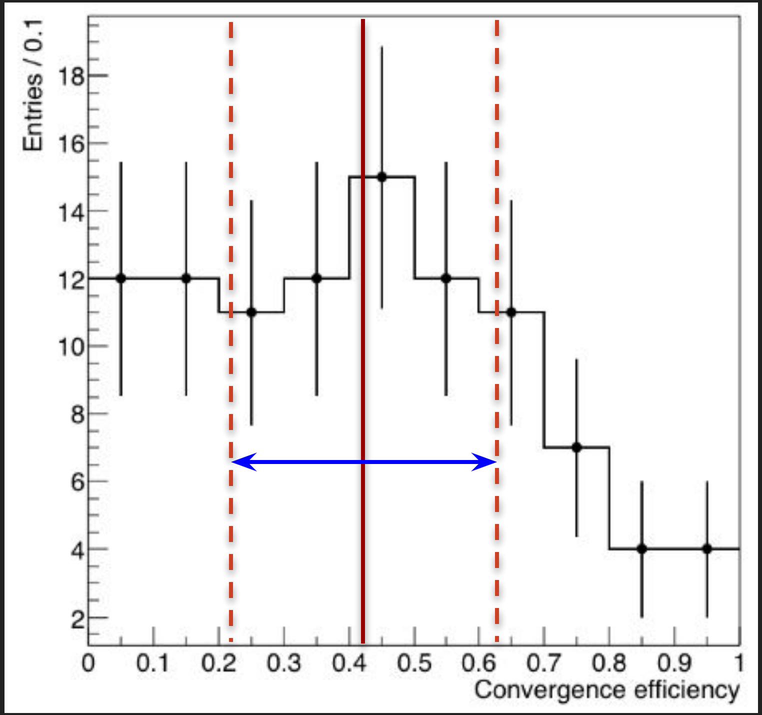


Penalty strength parameter

Results



The energy spectrum of solutions for one event with 3 p-p collisions and 15 tracks explored by the QPU with 10,000 samples



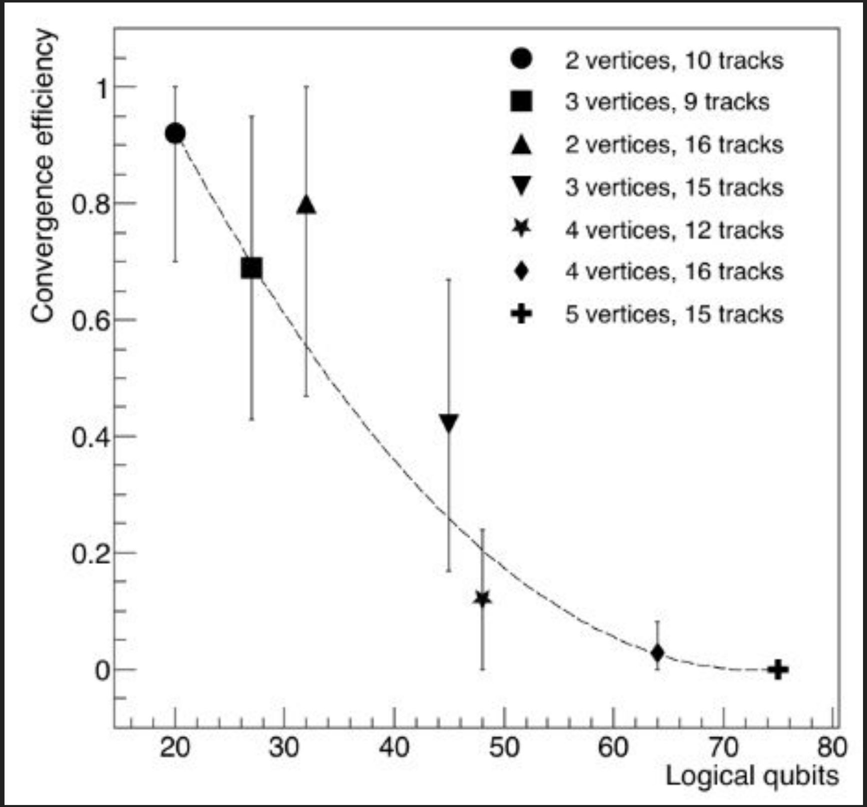
A histogram of QPU convergence efficiency for 3 p-p collisions and 15 tracks using 100 events.

Results

- Efficiency decreases with problem complexity
- Could have been used for Tevatron
- Number of samples, N, required for 95% confidence in at least one correct answer with mean efficiency ε:

$$N = \log_{1-\varepsilon} 0.05$$

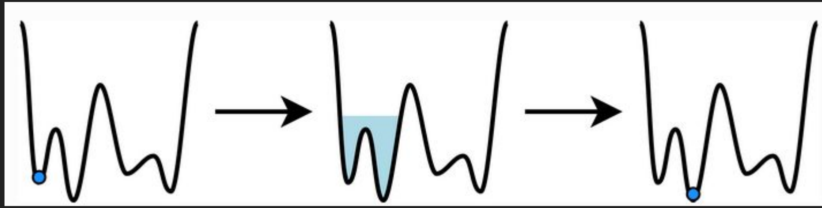
  - 2 samples (330 μs) required for 2 vertices, 10 tracks or 16 tracks
  - 10000 samples (1.6 s) required for 5 vertices, 15 tracks



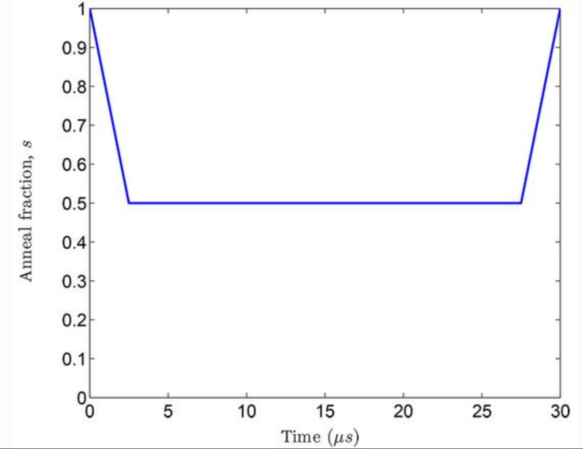
Plot of convergence efficiencies for various event topologies

Outlook — 3 Obstacles for Reaching LHC Event Complexities

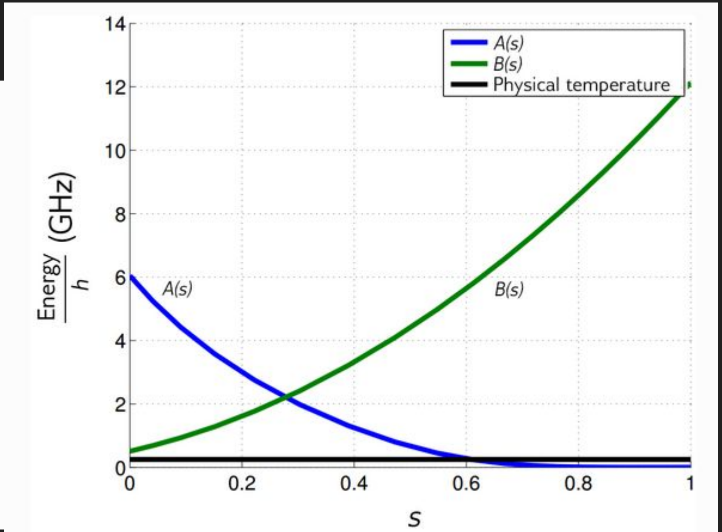
- Limitation on convergence efficiency
  - Modifications to distortion function
  - “Reverse Annealing” used to go from local minima solutions to global minima



Reverse Annealing process.. [Link to D-Wave]



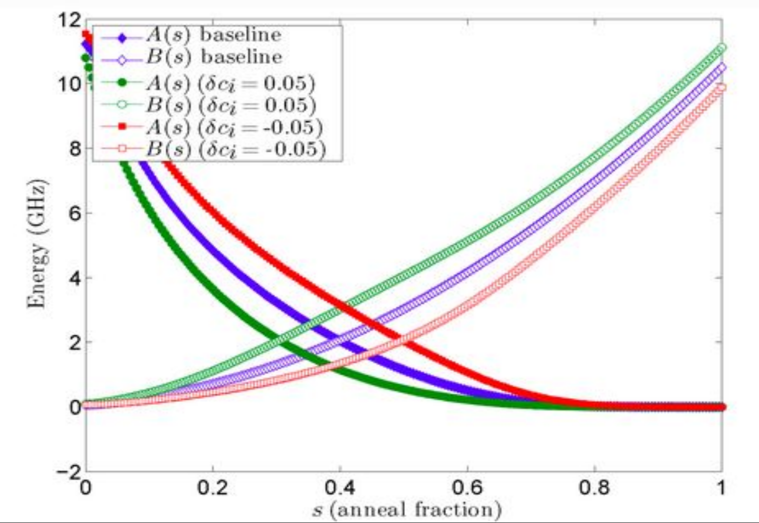
Reverse Annealing Schedule. [Link to D-Wave]



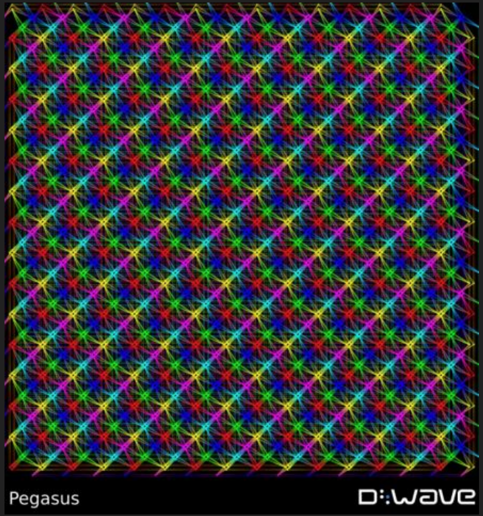
The annealing schedule and functions A(s) and B(s). [Link to D-Wave]

Outlook — 3 Obstacles for Reaching LHC Event Complexities

- Failure of graph embedding
  - Limited connectivity leads to long chains of physical qubits
  - Deterministic embedding may be possible
  - Offsetting longer chains of qubits to anneal later in the schedule



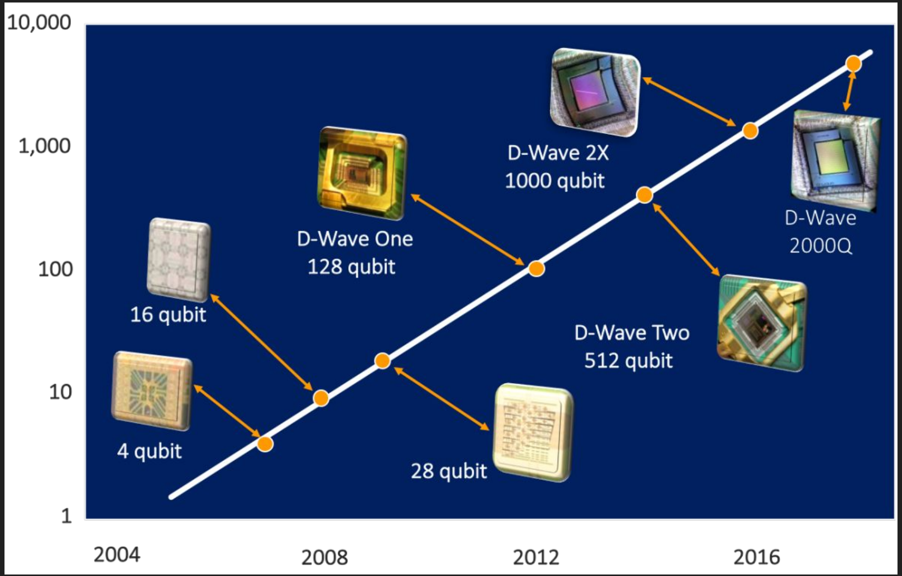
Effect of anneal offsets on A(s) and B(s). [Link to D-Wave]



Pegasus architecture showing qubit connectivity. [Link to D-Wave]

Outlook — 3 Obstacles for Reaching LHC Event Complexities

- Number of qubits available
  - 20 p-p collision positions and 1,000 tracks will require 20,000 logical qubits with current formulation
  - Hierarchical clustering with current formulation
  - Rose’s Law - Number of qubits doubles every two years



Summary

- Determining p-p collision points with track clustering is possible with QA
- Could have been used at Tevatron, does not currently scale to LHC
- 3 obstacles for reaching LHC

Questions?

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Backup Slides

Data

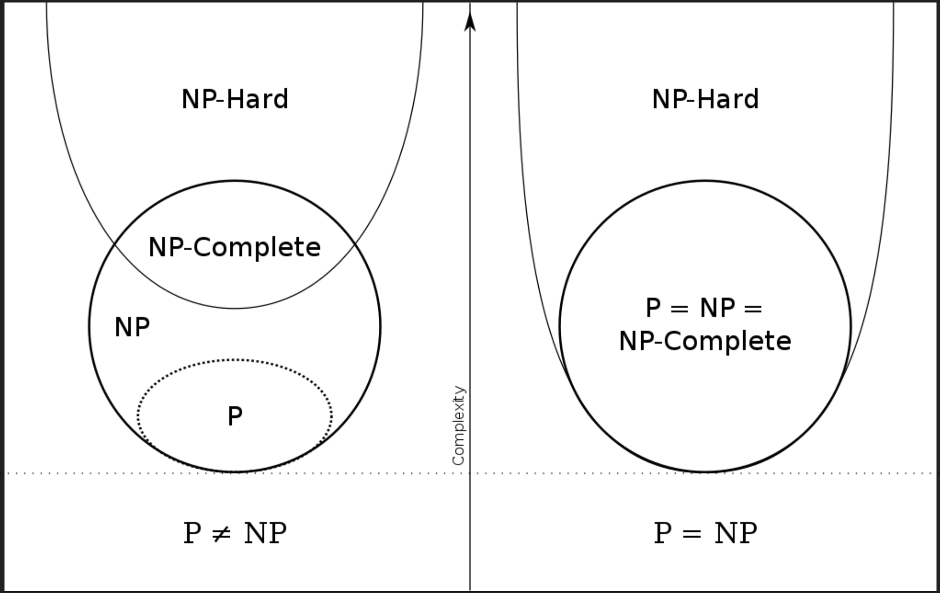
- Artificial events generated from known CMS event distributions
- Multiple event topologies are explored
- https://twiki.cern.ch/twiki/bin/view/CMSPublic/TrackingPOGPerformance2017MC#Expected_resolutions_on_track_pa

Is D-Wave Quantum?

- **Entanglement in a Quantum Annealing Processor, T. Lanting *et al.*** DOI: 10.1103/PhysRevX.4.021041
  - Showed quantum entanglement and coherence existed for 2 qubit and 8 qubit systems
- **Quantum annealing with manufactured spins, M. W. Johnson et al.** *Nature* **volume 473**, pages 194–198 (12 May 2011)
  - Showed quantum annealing performs better than thermal annealing
  - Has a temperature dependence that is quantum

P vs NP vs NP-hard vs NP-complete

- **P** - can be solved and verified in polynomial time
- **NP** - can be verified in polynomial time
- **NP-Hard** - is “harder” than any other NP problem. “Hard” to solve, “hard” to check (for now)
- **NP-Complete** - is “harder” than any other NP problems and is in NP



Euler diagram for P, NP, NP-Complete, NP-Hard [wikipedia]