

$$\Rightarrow \mathcal{M}_{\nu, \mathbf{k}}^{s_i s_f}(\mathbf{q}) = \frac{1}{N\Omega} \langle s_f | \hat{\mathcal{O}}_{\chi}^{\alpha}(\mathbf{q}) | s_i \rangle \langle \nu, \mathbf{k} | \sum_{lj} \hat{S}_{lj}^{\alpha} e^{i\mathbf{q} \cdot \mathbf{x}_{lj}} | 0 \rangle$$