

Entanglement and collective flavor oscillations in a dense neutrino gas

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October 10, 2019

The N3AS collaboration

- Network in Neutrinos, Nuclear Astrophysics, and Symmetries
- Multi-institutional network (3 centers + 8 sites) dedicated to recruiting and training postdocs, fostering collaborative efforts, and advancing research in the following areas:
 - Neutrino physics and astrophysics
 - Dense matter
 - Dark matter
- Funded by National Science Foundation (NSF) and Heising-Simons Foundation
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Today's talk(s)

- **Talk by Michael Cerveria:** eigenvalues and eigenstates of neutrino many-body Hamiltonian, for a two-flavor, single-angle system
- **This talk:**
 - Using eigenvalues and eigenstates to adiabatically evolve a neutrino many-body system
 - Introduce measures to quantify entanglement in the system
 - Comparison between flavor evolution in the many-body approach and in the mean-field description

References

-  Michael J. Cervia, Amol V. Patwardhan, A. B. Balantekin, S. N. Coppersmith, and Calvin W. Johnson
Phys. Rev. D 100, 083001 (2019), arXiv:1908.03511
-  Amol V. Patwardhan, Michael J. Cervia, and A. B. Balantekin
Phys. Rev. D 99, 123013 (2019), arXiv:1905.04386
-  Michael J. Cervia, Amol V. Patwardhan, and A. B. Balantekin
IJMPE 28 (2019) 1950032, arXiv:1905.00082

For extra credit



Ermal Rrapaj

arXiv:1905.13335



Savas Birol, Y. Pehlivan, A. B. Balantekin, and T. Kajino

Phys. Rev. D 98, 083002 (2018), arXiv:1805.11767



Y. Pehlivan, A. B. Balantekin, Toshitaka Kajino, and Takashi Yoshida

Phys. Rev. D 84, 065008 (2011), arXiv:1105.1182



Alexandre Faribault, Omar El Araby, Christoph Sträter, and Vladimir Gritsev

Phys. Rev. B 83, 235124 (2011), arXiv:1103.0472



Pieter W. Claeys

arxiv:1809.04447 (PhD thesis)

Outline

- 1 Many-body treatment of neutrino oscillations
- 2 Adiabatic evolution and entanglement measures
- 3 Comparison with mean-field calculations

Neutrino oscillations: flavor/mass isospin operators

- Denote Fermionic operators for neutrino flavor/mass states as $a_\alpha(\mathbf{p})$, $a_j(\mathbf{p})$, where $\alpha = e, x$, and $j = 1, 2$

$$a_e(\mathbf{p}) = \cos \theta a_1(\mathbf{p}) + \sin \theta a_2(\mathbf{p})$$

$$a_x(\mathbf{p}) = -\sin \theta a_1(\mathbf{p}) + \cos \theta a_2(\mathbf{p})$$

- Introduce the mass-basis isospin operators

$$J_{\mathbf{p}}^+ = a_1^\dagger(\mathbf{p})a_2(\mathbf{p}) , \quad J_{\mathbf{p}}^- = a_2^\dagger(\mathbf{p})a_1(\mathbf{p}) ,$$

$$J_{\mathbf{p}}^z = \frac{1}{2} \left(a_1^\dagger(\mathbf{p})a_1(\mathbf{p}) - a_2^\dagger(\mathbf{p})a_2(\mathbf{p}) \right) ,$$

which obey the usual $SU(2)$ commutation relations

$$[J_{\mathbf{p}}^+, J_{\mathbf{q}}^-] = 2\delta_{\mathbf{p}\mathbf{q}}J_{\mathbf{p}}^z , \quad [J_{\mathbf{p}}^z, J_{\mathbf{q}}^\pm] = \pm\delta_{\mathbf{p}\mathbf{q}}J_{\mathbf{p}}^\pm .$$

Neutrino Hamiltonian: single-angle approximation

- Many-body neutrino Hamiltonian with vacuum and ν - ν interactions (two-flavor, single-angle):

$$H_\nu = \sum_{p=1}^M \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J},$$

where p is an index for the ω s in the system, M in number

Neutrino Hamiltonian: eigenvalues and eigenstates

In terms of the parameters Λ_q (see Michael's talk), the eigenvalues are given by

$$E(\Lambda_1, \dots, \Lambda_N) = - \sum_p \frac{\omega_p}{2} + \mu \frac{N}{2} \left(\frac{N}{2} + 1 \right) - \mu \sum_p \omega_p \Lambda_p.$$

Eigenstates are given by $e_\kappa |\nu_1, \dots, \nu_1\rangle$, where the operator e_κ may be obtained by recursively applying the following identities, for $k = 1, \dots, \kappa$.

$$P_f(\Lambda_1, \dots, \Lambda_N) = \sum_{p_1=1}^M \cdots \sum_{p_f=1}^M J_{p_1}^- \cdots J_{p_f}^- \sum_{m=1}^f \Lambda_{p_m} \prod_{\substack{l=1 \\ l \neq m}}^f \frac{1}{\omega_{p_l} - \omega_{p_m}}.$$

$$e_k(\Lambda_1, \dots, \Lambda_N) = \frac{1}{k} \sum_{i=1}^k (-1)^{i-1} e_{k-i} P_i,$$

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Adiabatic evolution of a many-body neutrino system

- Eigenvalues and eigenvectors facilitate calculating the adiabatic evolution of the many-body neutrino system, starting from any given initial condition, as μ is varied
- Consider an initial many-body state, $|\Psi_0\rangle \equiv |\Psi(\mu_0)\rangle$
 - Example: in the (two-)flavor-basis, $|\nu_e\nu_x\nu_e\nu_e\rangle$
- May be decomposed into the basis of energy eigenstates:
$$|\Psi(\mu_0)\rangle = \sum_n c_n |e_n(\mu_0)\rangle$$
- If μ were to change sufficiently slowly then the system adiabatically evolves into

$$|\Psi(\mu)\rangle \simeq \sum_n c_n e^{-i \int_{\mu_0}^{\mu} \frac{E_n(\mu')}{d\mu'/dt} d\mu'} |e_n(\mu)\rangle$$

Adiabatic evolution of a many-body neutrino system

- Adiabatically evolve with Schrödinger's Eq.

$$|\Psi(\mu)\rangle \approx V \left[\exp \left(-i \int_{\mu_0}^{\mu} \Sigma(\mu') \frac{dr}{d\mu'} d\mu' \right) \right] V_0^T |\Psi_0\rangle,$$

- V and V_0 are (real) unitary transformations between energy eigenstates and mass-basis product states, parametrized by the 2^N solutions $\vec{\Lambda} \equiv (\Lambda_1, \dots, \Lambda_N)$, at times t and 0 , respectively
- $\Sigma \equiv V H V^T$ real, diagonal matrix of eigenvalues; any energy degeneracies split by differing $\vec{\Lambda}$ parameters
- V, Σ obtained efficiently using methods described in arXiv:1905.04386 (Michael's talk)
- Functional form of $\mu(r)$ adopted from single-angle bulb-model calculations—results not qualitatively dependent on this choice

Quantum entanglement in many-body neutrino systems

- In general, for $\mu > 0$, the eigenstates of the Hamiltonian are not factorizable into tensor products of individual neutrino states, and may therefore be described as entangled
- A system may initially start in a pure state—which happens to be a particular superposition of energy eigenstates. However, as the coefficients describing the superposition change with time (as do the eigenstates themselves), the system can become entangled. This is a feature unique to many-body systems, and cannot be observed in mean-field calculations
- Such entanglement may be quantified in terms of measures such as entropy of entanglement, length of individual neutrino polarization vectors, etc.

Summary of entanglement measures

Density Matrix, Polarization Vector, & Entanglement Entropy

Consider a pure, many-body neutrino state $\rho = |\Psi\rangle\langle\Psi|$.

Single-neutrino reduced density matrix: $\rho_q \equiv \text{Tr}_{1, \dots, \widehat{q}, \dots, N}[\rho]$, given by ($\widehat{}$ denotes exclusion)

$$\rho_q = \sum_{i_1, \dots, \widehat{i_q}, \dots, i_N=1}^2 \langle \nu_{i_1} \dots \widehat{\nu}_{i_q} \dots \nu_{i_N} | \rho | \nu_{i_1} \dots \widehat{\nu}_{i_q} \dots \nu_{i_N} \rangle,$$

- $S(\omega_q)$, Entropy of entanglement between neutrino q and rest:

$$S(\omega_q) = -\text{Tr}[\rho_q \log \rho_q]$$

- “Polarization vector” of neutrino q , $\vec{P}(\omega_q) = 2 \langle \vec{J}_q \rangle$, related to the reduced density matrix as:

$$\rho_q = \frac{1}{2}(\mathbb{I} + \vec{P}(\omega_q) \cdot \vec{\sigma})$$

Relations between entanglement measures

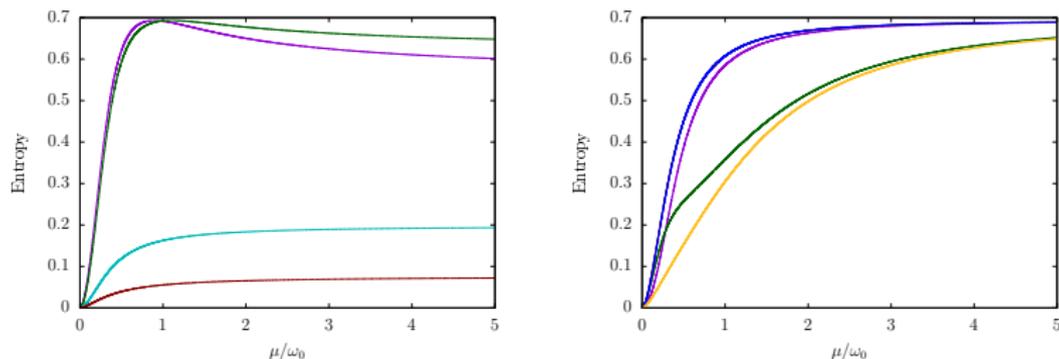
Entanglement entropy has a one-to-one, inverse relationship with the magnitude of the polarization vector

$$S(P_q) = -\frac{1-P_q}{2} \log\left(\frac{1-P_q}{2}\right) - \frac{1+P_q}{2} \log\left(\frac{1+P_q}{2}\right)$$

with $P_q = |\vec{P}(\omega_q)|$

- $P = 1 \iff S = 0$ (Unentangled)
- $P = 0 \iff S = \log(2)$ (*Maximally* Entangled)

Entanglement: a preview



[Cervia, AVP, Balantekin, Phys. Rev. D 100, 083001 (2019)]

Figure: Entropy of entanglement between the neutrino at frequency ω_4 and the rest of the ensemble, for all eigenstates of an $N = 4$ neutrino system, corresponding to $\kappa = 1$ (left) and 2 (right).

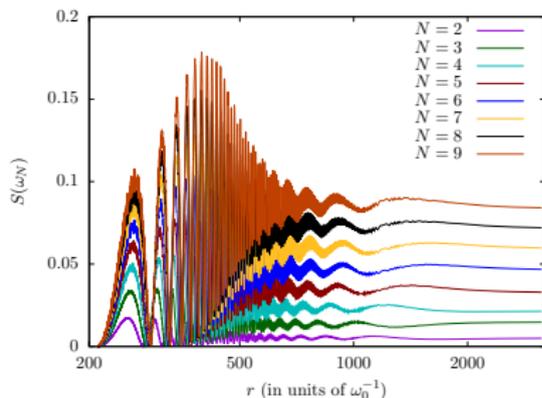
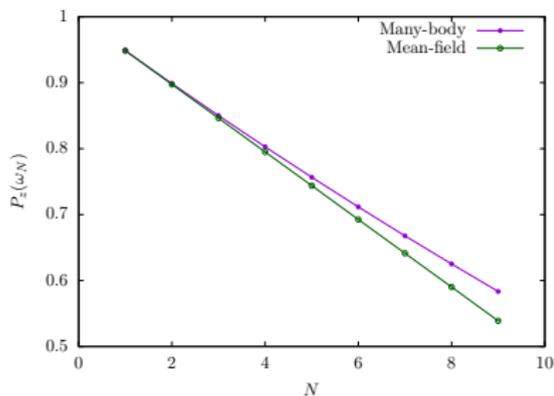
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Example: evolution of all-electron flavor initial state

Comparison of many-body and mean-field calculations

- System with frequencies $\omega_1, \dots, \omega_N$ where $\omega_p = p\omega_0$
- Evolve from $|\Psi_0\rangle = |\nu_e \dots \nu_e\rangle$ for systems of varying sizes ($N = 2, \dots, 9$)
- As $\mu \sim 0$ ($r \gg R_\nu$), H diagonal in mass-basis, therefore plot final spectra in the mass-basis: $P_z = n(\nu_1) - n(\nu_2)$



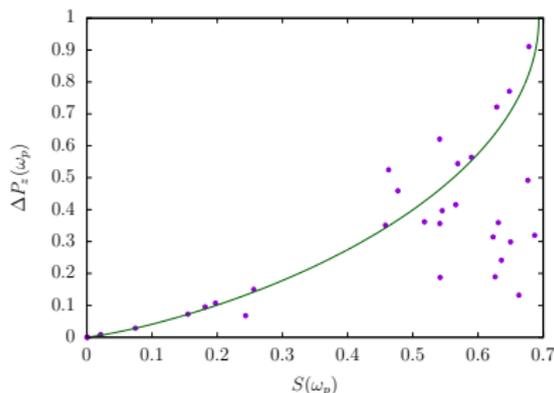
NB: In the mean-field case, $S = 0$ always

Correlation of P_z -discrepancies and entanglement entropy

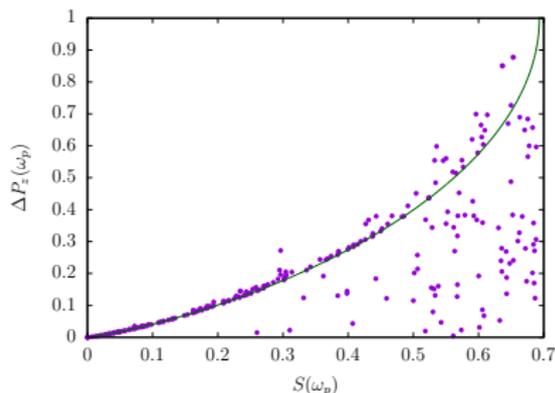
Calculate $\Delta P_z(\omega) \equiv |P_z^{\text{MF}}(\omega) - P_z^{\text{MB}}(\omega)|$ at $r \gg R_\nu$ (i.e., $\mu \approx 0$)

- For $N = 4$: all initial conditions with definite flavor ν_e, ν_x (e.g., $|\nu_e, \nu_x, \nu_x, \nu_x\rangle$)
- For $N = 8$: same ICs as $N = 4$, but with four additional ν_e appended to left or right of spectrum

($N = 4$)



($N = 8$)

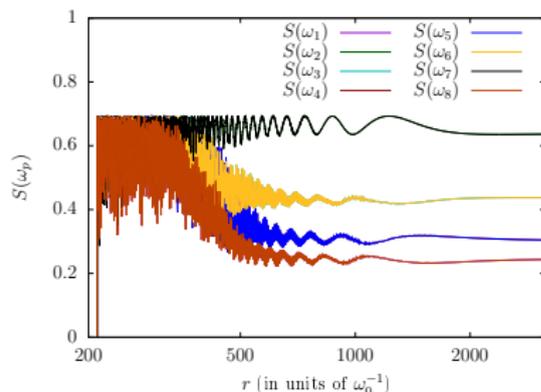
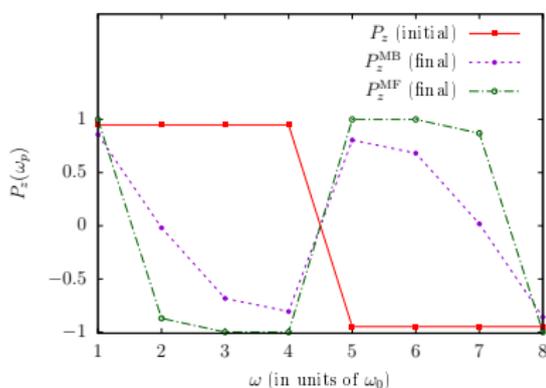


Trendline: $y(S) \equiv P^{\text{MF}}(S) - P^{\text{MB}}(S) = 1 - P(S)$

Example: initial condition with both neutrino flavors

Comparison of final P_z spectra between many-body and mean-field

- Evolve $|\Psi_0\rangle = |\nu_e \nu_e \nu_e \nu_e \nu_x \nu_x \nu_x \nu_x\rangle$ until $r \gg R_\nu$

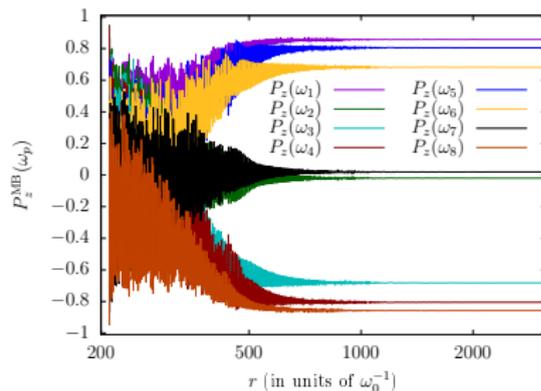
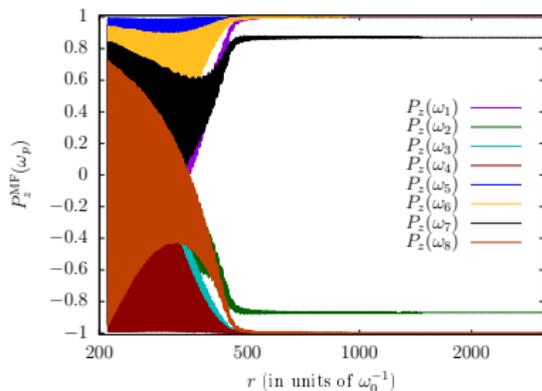


[Cervia et al., Phys. Rev. D 100, 083001 (2019)]

Spectral swap-like features persist in the many-body calculations, but are less sharp relative to mean-field calculations

Comparison of P_z evolution with r

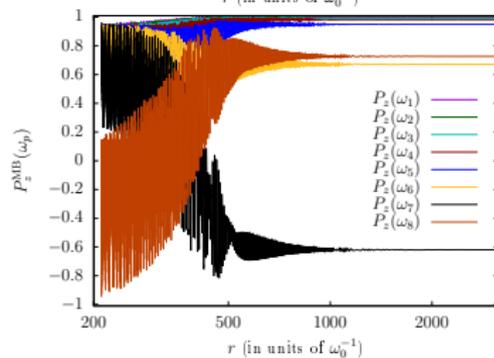
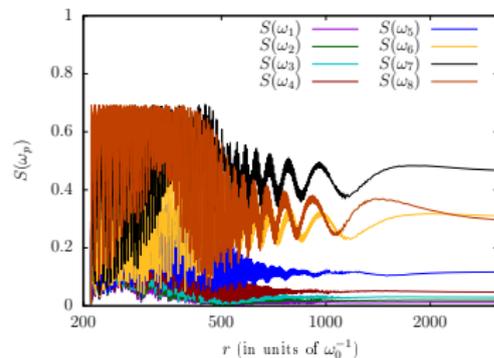
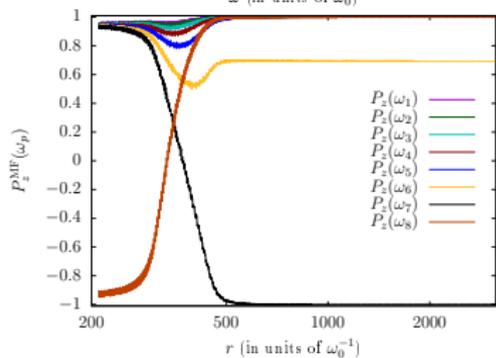
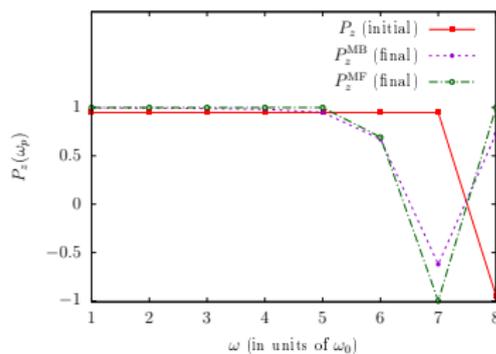
- Same initial conditions, $|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_e\nu_x\nu_x\nu_x\nu_x\rangle$



[Cervia et al., Phys. Rev. D 100, 083001 (2019)]

Comparison with mean field

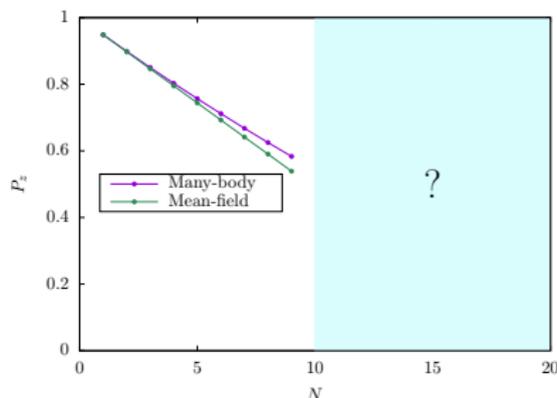
Different initial condition: $|\Psi_0\rangle = |\nu_e \nu_e \nu_e \nu_e \nu_e \nu_e \nu_e \nu_e \nu_x\rangle$



Conclusions

- Calculations of collective neutrino flavor evolution typically rely on a 'mean-field', i.e., effective one-particle description
- Important to test the efficacy and/or limitations of the mean-field by performing many-body calculations
- Evolution in the many-body case can be studied by calculating the eigenvalues and eigenvectors of the Hamiltonian by solving the Bethe Ansatz equations (or an equivalent set of equations)
- For certain simple systems, qualitative differences in flavor evolution observed between many-body and mean-field treatments, resulting from entangled states which are absent in the mean-field limit

Future Work

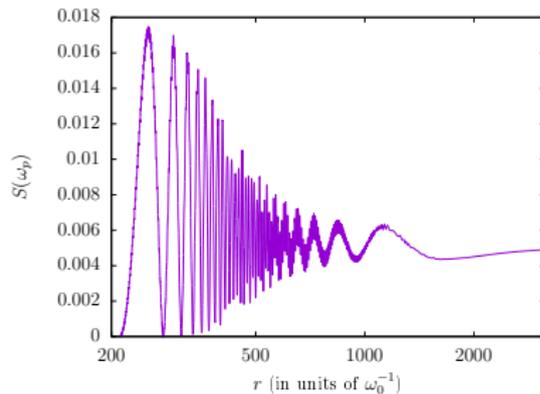
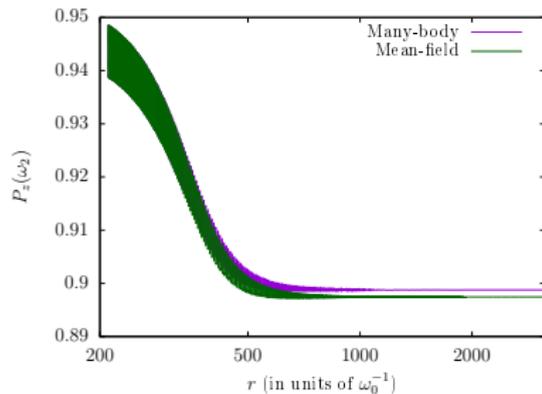


- Next steps in calculations
 - Larger N + inclusion of $\bar{\nu}$ ($\omega < 0$)
 - Matter (MSW) potential
 - Multiple neutrinos in frequency bins ($j_p > 1/2$)
 - Beyond single-angle approximation $\mu \rightarrow \mu_{\mathbf{p}\mathbf{q}}$
- Couple to baryons—how are nucleosynthetic yields affected?

Comparison of Intermediate P_z Spectra

While $r \gtrsim R_\nu$, $N = 2$ mono-flavor initially

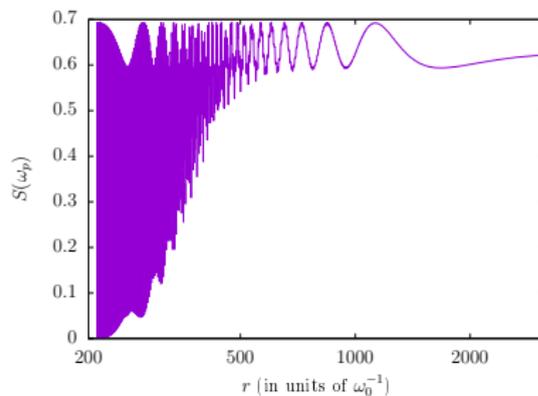
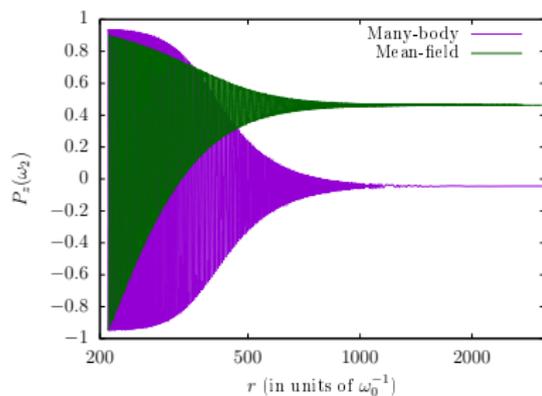
- $|\Psi_0\rangle = |\nu_e \nu_e\rangle$, and observe P_z before $r \gg R_\nu$



Comparison of Intermediate P_z Spectra

While $r \gtrsim R_\nu$, $N = 2$ different-flavor initially

- $|\Psi_0\rangle = |\nu_e \nu_x\rangle$, and observe P_z before $r \gg R_\nu$



Entanglement in Individual Eigenstates

- Eigenstates for $N = 5$, entanglement of N -th ν with the rest
- Highest/lowest-weight states are trivial

